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A CUSUM test for panel mean change detection

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ABSTRACT

A test for panel structural mean change is developed from the CUSUM of the panel processes. Limiting null distribution and consistency of the test are established. The test is shown to have stable finite sample sizes than the existing test of Horvath and Huskova (2012) based on the squared CUSUM. If the mean changes are not cancelled in that their average is away from zero, the proposed test has better power than the existing test. On the other hand, if the mean changes are nearly cancelled, the existing test has better power. The proposed tests are illustrated by a real data set analysis.

1. Introduction

Structural change problems in panel data models are important issues for economic or financial data analysis because a big macroeconomic policy change or a financial crisis has simultaneous influence on many economic or financial variables. Researches for structural change problems in panel models have been activated due to a vast amount of data in the modern economic world or financial markets. Some studies were made by Bai (1997), Bai et al. (1998) and Han and Park (1989) for change point estimation and testing of multivariate time series models and by Emerson and Kao (2001, 2002) for testing of structural change of a time trend regression in panel data.

Recently, structural change detection problems in mean and variance of panel data have been investigated by some authors. Bai (2010) studied estimation for common change point in mean and variance in panel data. Horvath and Huskova (2012), following Bai (2010)'s quasi-maximum likelihood argument, developed a test for panel mean change, based on the *squared* cumulative sum (squared CUSUM) of the panel processes. Li, Tian, Xiao, and Chen (2015) and Shi (2015) proposed tests for panel variance changes, based on the CUSUM and the squared CUSUM, respectively, of the squared panel processes.

We note that the test of Horvath and Huskova (2012), being based on the squared CUSUM, has good power against average squared change away from 0. However, in practice, one may be more interested in detecting average change than average squared change. For that purpose, we will construct a simple panel mean change detection test based on the CUSUM of the panel processes. As well as the limiting null distribution, consistency of the proposed test will be established against average change away from 0. The proposed test, being based on the CUSUM, has good power against average changes away from 0.

Aimed at different targets of average squared changes and average changes, none of the existing test of Horvath and Huskova (2012) and the proposed test does not dominate the other one in power performance. The existing test has power

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advantage over the proposed test against changes which cancel to near-zero sum. The proposed test has power advantage over the existing test against non-cancelling changes.

We claim that non-cancelling changes are more frequent than cancelling change in practice. The mean changes in the panel system are usually caused by a big shock which shifts all means to a common direction: a good big shock shifts all panel units to a good direction and a bad big shock acts reversely. For example, the Korean IMF economic crisis in the years 1997–1998 caused bad effects on almost all Korean stock prices, foreign exchange rates, house values, etc. Similar bad effects of the world wide financial crisis in the years 2007–2008 can be observed on stock prices and house values.

A Monte Carlo experiment compares the two tests. It shows better power performance of the proposed test than the existing test against non-cancelling changes and reversed power performance against cancelling changes. Moreover, it reveals that the proposed test has significantly better size performance in case of serially correlated panels.

The panel volatility change tests of Li et al. (2015) and Shi (2015) are compared in the context of mean changes of the squared process showing the same relative performance as the panel mean change tests: in case of non-cancelling volatility changes, the test of Li et al. (2015) based on the CUSUM of squared process has better power than the test of Shi (2015) based on the squared CUSUM of squared process; in case of cancelling volatility changes, the test of Shi (2015) has better power than the test of Li et al. (2015).

The remaining of the paper is organized as follows. Section 2 discusses the mean change detection with main theoretical results. Section 3 deals with Monte Carlo comparisons. Section 4 compares the tests for volatility change detections. Section 5 illustrates the proposed tests with a real data set. Section 6 concludes.

2. Mean change detection

We consider a panel data model consisting of n panels and T observations on each panel unit as given by

$$X_{it} = \mu_i + \sigma_i(\delta_i \mathbb{I}\{t > t_0\} + u_{it}), \quad 1 \le i \le n, \ 1 \le t \le T \tag{1}$$

where $t_0 \in \{1, \dots, T\}$ is unknown common change point, $\mathbb{I}\{t > t_0\}$ is the indicator function of $\{t > t_0\}$, and, for each i, $\{u_{it}\}$ is a stationary process with $Eu_{it} = 0$ and $Var(u_{it}) = 1$. As in Horvath and Huskova (2012), Li et al. (2015), and Shi (2015), we assume that $\{u_{it}, -\infty < t < \infty\}$ are cross-sectionally independent but can be serially correlated. We have $E(X_{it}) = \mu_i$ for $1 \le t \le t_0$; $E(X_{it}) = \mu_i + \sigma_i\delta_i$ for $t_0 < t \le T$; and $Var(X_{it}) = \sigma_i^2$. The mean change parameter δ_i is standardized mean change in $E(X_{it})$. We wish to test the null hypothesis

$$H_0 : \delta_i = 0 \quad \text{for all } 1 \le i \le n$$

that the mean $E(X_{it})$ will not change during the observation period.

Horvath and Huskova (2012) developed a test by applying the quasi-maximum likelihood argument of Bai (2010) which is based on the **squared cumulative sum** process

$$\bar{H}_{nT}(z) = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \left\{ \frac{1}{\upsilon_i^2} Z_{iT}^2(z) - \frac{[Tz](T-[Tz])}{T^2} \right\}, \quad 0 \le z \le 1, \tag{2}$$

where

$$Z_{iT}(z) = \frac{1}{\sqrt{T}} \sum_{t=1}^{[Tz]} (X_{it} - \bar{X}_i), \quad \text{with } \bar{X}_i = \frac{1}{T} \sum_{t=1}^{T} X_{it},$$

is the cumulative sum process and $\upsilon_i^2 = \lim_{T\to\infty} Var\left(\frac{1}{\sqrt{T}} \sum_{t=1}^{T} X_{it}\right)$ is the long-run variance of X_{it} for $i = 1, \dots, n$. Note that $Z_{iT}(z)$ is standardized by the long-run standard deviation υ_i in order to adjust serial correlation in X_{it}.

As proved by Horvath and Huskova (2012, Theorem 3) for consistency of the sup test based on the squared CUSUM process, say Sup H, against changes such that $n^{-1/2}T \sum_{i=1}^{n} \delta_i^2 \to \infty$, the test has good power for detecting average squared changes $\bar{\delta}^Q = n^{-1} \sum_{i=1}^{n} \delta_i^2$. However, even though the test Sup H has good power against average squared changes, the Sup H test remains to be improved for detecting other changes in some class of important alternatives of nonnegative (or nonpositive) panel mean changes such as those caused by the world wide financial crisis or the Korean IMF economic crisis mentioned in Section 1.

Such nonnegative or nonpositive panel mean changes may be more well detected by a test designed to detect average change $\bar{\delta} = n^{-1} \sum_{i=1}^{n} \delta_i$ than by the Sup H test designed to detect average squared change $\bar{\delta}^Q$. Average change is well detected by a test designed to detect a common change $\delta_1 = \cdots = \delta_n = \delta$. From (1), we have $(X_{it} - \mu_i)/\sigma_i = \delta_i \mathbb{I}\{t > t_0\} + u_{it}$. Note that, if μ_i are all known, then the test problem for the common change δ becomes that of the summed univariate time series model $\sum_{i=1}^{n} (X_{it} - \mu_i)/\sigma_i = \sum_{i=1}^{n} \delta \mathbb{I}\{t > t_0\} + \sum_{i=1}^{n} u_{it}$ for which the Sup test and the CUSUM test are all based on the cumulative sum process $B_{nT}(z) = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \left\{ \frac{1}{\upsilon_i} \frac{1}{\sqrt{T}} \sum_{t=1}^{[Tz]} (X_{it} - \mu_i) \right\}$. For the real situation of unknown μ_i, the unknown μ_i are replaced by $\bar{X}_i$ and a natural test for the common change is constructed from the **cumulative sum** process

$$\bar{B}_{nT}(z) = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} \left\{ \frac{1}{\upsilon_i} Z_{iT}(z) \right\}. \tag{3}$$

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Being designed for detecting a common change, the sup test based on the CUSUM process, say $\sup B$, has consistency against average changes $|\bar{\delta}|$ such that $\sqrt{nT}|\bar{\delta}| \to \infty$ as will be shown in Theorem 2.2.

The long-run variance parameter v_i^2 is consistently estimated by the usual kernel estimator

$$\hat{v}_i^2 = \hat{\gamma}_i(0) + 2\sum_{j=1}^{\ell}\left(1 - \frac{j}{\ell+1}\right)\hat{\gamma}_i(j), \quad \hat{\gamma}_i(j) = T^{-1}\sum_{t=1}^{T-j}(X_{it} - \bar{X}_i)(X_{i,t+j} - \bar{X}_i)$$

where ℓ is a bandwidth parameter. This long-run variance estimator $\hat{v}_i^2$ may be preferred to other kernel estimators because $\hat{v}_i^2$ is always positive, which guarantees existence of $\hat{v}_i$. The long-run variance estimators are plugged-in in (2) and (3) to give $\hat{H}_{nT}(z)$ and $\hat{B}_{nT}(z)$, respectively, and the sup tests

$$\hat{H}_{nT}(z) = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left\{\frac{1}{\hat{v}_i^2}Z_{iT}^2(z) - \frac{[Tz](T-[Tz])}{T^2}\right\}, \quad \mathrm{Sup}\, H = \sup_{0\le z\le 1}|\hat{H}_{nT}(z)|, \tag{4}$$

$$\hat{B}_{nT}(z) = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left\{\frac{1}{\hat{v}_i}Z_{iT}(z)\right\}, \quad \mathrm{Sup}\, B = \sup_{0\le z\le 1}|\hat{B}_{nT}(z)|. \tag{5}$$

Limiting null distribution of $\hat{B}_{nT}(z)$ and consistency of the Sup B test are established in the following theorems for which we assume linear innovations

$$u_{it} = \sum_{l=0}^{\infty} c_{il}\epsilon_{i,t-l}, \quad 1 \le i \le n,\ 1 \le t \le T$$

with the following assumptions (A1)–(A5) below:

(A1) $E(\epsilon_{i0}) = 0$, $E(\epsilon_{i0}^2) = 1$, $E|\epsilon_{i0}|^\kappa < \infty$ and $\limsup_{n\to\infty} n^{-1}\sum_{i=1}^{n} E|\epsilon_{i0}|^\kappa < \infty$ for some $\kappa > 4$,
(A2) The sequences $\{\epsilon_{it}, -\infty < t < \infty\}$ are independent of each other,
(A3) For every i the random variables $\{\epsilon_{it}, -\infty < t < \infty\}$ are i.i.d.,
(A4) $|c_{il}| \le c_0(l+1)^{-\alpha}$ for all $1 \le i \le n$, $0 \le l < \infty$, with some c_0 and $\alpha > 2$,
(A5) Let $a_i = \sum_{l=0}^{\infty} c_{il}$ for each $1 \le i \le n$ and there exists $b > 0$ such that $a_i \ge b$ for all i.

Theorem 2.1. *Consider model (1) with H_0. Under conditions (A1)–(A5), as $\min\{n, T\} \to \infty$,*

$$\sup_{0\le z\le 1}|\hat{B}_{nT}(z)| \xrightarrow{\mathrm{d}} \sup_{0\le z\le 1}|B^0(z)|$$

where $B^0(z)$ is a standard Brownian bridge.

Theorem 2.2. *Consider model (1) and assume that conditions (A1)–(A5) hold. If*

$$\sqrt{nT}|\bar{\delta}| \to \infty \quad \text{as } \min\{n, T\} \to \infty$$

and for $t_0 = t_0(T)$,

$$0 < \liminf_{T\to\infty}\frac{t_0}{T} \le \limsup_{T\to\infty}\frac{t_0}{T} < 1,$$

then as $\min\{n, T\} \to \infty$,

$$\sup_{0\le z\le 1}|\hat{B}_{nT}(z)| \xrightarrow{\mathrm{p}} \infty.$$

3. Monte Carlo comparison

We compare the two tests Sup H and Sup B of (4) and (5) in a Monte Carlo experiment whose setup is similar to that of Horvath and Huskova (2012). We choose the data generating process

$$x_{it} = \rho x_{i,t-1} + u_{it}, \qquad X_{it} = \delta_i\mathbb{I}\{t > t_0\} + x_{it}, \quad i = 1,\ldots,n,\ t = 1,\ldots,T$$

with standard normal error u_{it}. As in Horvath and Huskova (2012), the mean changes of X_{it}, if any, occur at time $t_0 = bT$ for half of the panel units $i = 1,\ldots,n$ for which we consider $b = 0.1$ and $b = 0.5$. Mean change parameters δ_i are chosen from uniform distributions:

$$\mathrm{NC}: \delta_i = 0; \quad \mathrm{MC1}: \delta_i \sim U(-0.2, 0.2); \quad \mathrm{MC2}: \delta_i \sim U(0, 0.2); \quad \mathrm{MC3}: \delta_i \sim U(0.1, 0.3).$$

Table 1
Sizes (%) of level 5% tests for mean changes based on 10 000 replications.

n	T	$\rho = 0.1$		$\rho = 0.3$			
		Sup B	Sup H	Sup B	Sup H	Sup B	Sup H
50	50	4.8	6.2	5.0	10.8	5.3	20.4
50	100	5.0	5.7	5.2	8.9	5.5	16.5
50	200	5.1	5.4	5.5	8.5	6.0	15.5
100	50	4.9	6.8	4.9	13.5	5.2	31.3
100	100	4.7	6.6	4.8	12.2	5.2	25.6
100	200	5.4	6.7	5.8	11.7	6.4	24.4
200	50	4.8	7.9	4.9	20.8	5.3	52.2
200	100	5.3	7.3	5.3	16.5	5.7	40.5
200	200	5.2	7.1	5.5	15.9	5.9	37.1

Table 2
Powers (%) of level 5% tests for mean changes based on 10 000 replications.

n	T	Breaks at $0.1T$						Breaks at $0.5T$					
		MC1		MC2		MC3		MC1		MC2		MC3	
		Sup B	Sup H	Sup B	Sup H	Sup B	Sup H	Sup B	Sup H	Sup B	Sup H	Sup B	Sup H
50	50	4.7	4.9	5.8	4.9	10.1	5.2	4.9	9.5	21.0	14.0	59.2	24.4
50	100	4.9	4.8	7.2	4.9	16.6	5.2	5.7	15.6	36.0	14.0	88.4	59.2
50	200	5.4	5.0	11.0	4.9	34.9	6.2	7.7	36.9	59.0	32.0	99.4	96.9
100	50	4.8	4.9	6.9	4.8	16.6	5.0	5.3	10.9	38.0	14.0	86.5	36.7
100	100	4.6	5.0	10.0	4.9	34.0	5.3	5.5	23.0	56.0	16.0	99.4	84.2
100	200	5.5	5.7	17.5	5.8	69.6	8.0	7.3	58.4	85.0	50.0	100.0	99.9
200	50	4.8	5.1	10.2	5.1	34.9	5.3	5.3	15.2	66.0	23.0	99.2	60.4
200	100	5.2	5.1	17.0	5.1	68.7	5.9	6.0	36.3	91.0	36.0	100.0	98.2
200	200	5.2	5.8	33.3	5.8	97.6	9.0	7.2	82.5	100.0	87.0	100.0	100.0

Note that NC has no mean change. Other cases have mean changes: MC1 has cancelling mean changes in that $\bar{\delta} = n^{-1}\sum_{i=1}^{n}\delta_i \cong 0$; MC2 and MC3 have non-cancelling positive mean changes with $\bar{\delta} \cong 0.1$ and $\bar{\delta} \cong 0.2$, the latter being more homogeneous than the former.

As in Horvath and Huskova (2012), we consider $n = 50, 100, 200$; $T = 50, 100, 200$. We set $\rho = 0, 0.1, 0.3, 0.5$ for the case of NC for size study. For the other MC1, MC2, MC3 cases for power study, we set $\rho = 0$. The data X_{it} are simulated using $x_{i0} = 0$ and standard normal error u_{it} generated by RNNOA, a FORTRAN subroutine in IMSL.

Tables 1, 2 report sizes and powers of the two level 5% tests Sup B and Sup H which are computed using 10 000 independent replications $\{X_{it}, i = 1, \ldots, n, t = 1, \ldots, T\}$. For the long-run variance estimator, we use the bandwidth parameter $\ell = [10(T/100)^{1/4}]$. We use the 1/4-order bandwidth because, in the semiparametric unit root test literature, the 1/4-order bandwidth is generally recommended for consistent longrun variance estimators, see Schwert (1989).

For each test, as the critical value, we use the right 5% empirical quantile of 100 000 values of the test simulated under NC with $\rho = 0$.

Our major findings from these tables are:

1. Sup B has significantly better size than Sup H in case of serial correlation of NC with $\rho = 0.1, 0.3, 0.5$,
2. For MC1 of cancelling $U(-0.2, 0.2)$ mean changes, Sup H has substantially better power than Sup B,
3. For MC2, MC3 of non-cancelling $U(0, 0.2)$, $U(0.1, 0.3)$ mean changes, Sup B has substantially better power than Sup H.

More detailed comparison may be also interesting. For size performance, the proposed test Sup B has stable size value close to the nominal level 5% for all combinations of (n, T, ρ) considered here. On the other hand, the existing test Sup H has size distortion which gets more severe as ρ increases: for $n = 200$, Sup H has empirical size greater than 15%, 37%, respectively, if $\rho = 0.3, 0.5$, respectively. For given $\rho > 0$, size of Sup H gets more deviated from the nominal level for the larger n or for the smaller T.

For MC2 and MC3 of non-cancelling changes $\bar{\delta} > 0$, the proposed test Sup B has substantially better power for all n, T considered here than Sup H. When breaks occur at early time ($b = 0.1$), Sup H has almost no power while Sup B has some power.

On the other hand, for MC1 of cancelling changes $\bar{\delta} \cong 0$, the proposed test Sup B has almost no power even for larger n or larger T while the existing test Sup H has good power increasing as n or T increase if break occurs at the middle of data span.

From this Monte Carlo study, we may conclude that the proposed test Sup B has substantially better size performance than the existing test Sup H while having better power against non-cancelling homogeneous changes and having almost no power against cancelling changes.

Table 3
Sizes (%) of level 5% tests for volatility changes based on 10 000 replications.

n	T	$\rho = 0.1$		$\rho = 0.3$		$\rho = 0.5$	
		Sup BQ	Sup HQ	Sup BQ	Sup HQ	Sup BQ	Sup HQ
50	50	5.0	4.6	5.1	5.2	5.6	7.4
50	100	5.0	4.7	5.0	5.3	5.3	6.9
50	200	5.0	4.9	4.9	5.2	5.1	7.3
100	50	5.3	5.2	5.5	6.3	6.8	9.9
100	100	4.5	4.9	4.9	5.9	5.7	9.6
100	200	5.2	4.9	5.1	6.1	5.8	8.8
200	50	4.8	4.8	5.1	6.9	7.9	13.8
200	100	4.7	5.3	5.2	7.2	6.5	12.6
200	200	5.1	5.3	5.3	6.9	6.2	11.9

4. Volatility change detection

It would be interesting to compare the test, Sup BQ say, of Li et al. (2015) and the test, Sup HQ say, of Shi (2015) for volatility change detection in the context of mean change detection described in Sections 2 and 3. The test Sup BQ is constructed from the CUSUM of $(X_{it} - \bar{X}_i)^2$ in the same way as Sup B is constructed from the CUSUM of $(X_{it} - \bar{X}_i)$. Similarly, the test SupHQ is constructed from the squared CUSUM of $(X_{it} - \bar{X}_i)^2$ in the same way as Sup H is constructed from the squared CUSUM of $(X_{it} - \bar{X}_i)$.

We consider a panel data model having variance changes $\nu_1, \ldots, \nu_n$ at common time t_0 as given by

$$X_{it} = \mu_i + \sigma_i \left(1 + \nu_i \mathbb{I}\{t > t_0\}\right) u_{it}, \quad 1 \le i \le n,\ 1 \le t \le T$$

where u_{it} is a linear process with the same condition as that for model (1). We have

$$Var(X_{it}) = \sigma_i^2 \quad \text{for } 1 \le t \le t_0; \qquad Var(X_{it}) = \sigma_i^2 (1 + \nu_i)^2 \quad \text{for } t_0 < t \le T.$$

We wish to test the null hypothesis $H_0 : \nu_i = 0$ for all $1 \le i \le n$ of no change volatility. We have

$$(X_{it} - \mu_i)^2 = \sigma_i^2 [(1 + \delta_i \mathbb{I}\{t > t_0\}) + a_{it}], \quad 1 \le i \le n,\ 1 \le t \le T,$$

where $\delta_i = 2\nu_i + \nu_i^2$ and $a_{it} = (1 + \delta_i \mathbb{I}\{t > t_0\})\,(u_{it}^2 - 1)$ is a zero-mean process. Noting that the squared process has mean changes $\delta_i = 2\nu_i + \nu_i^2$ at time t_0, we have

$$E(X_{it} - \mu_i)^2 = \sigma_i^2, \quad 1 \le t \le t_0; \qquad E(X_{it} - \mu_i)^2 = \sigma_i^2 (1 + \delta_i), \quad t_0 < t \le T.$$

Since detection of obvious change is not of statistical interests, we may assume that $|\nu_i| \ll 1$ for the usual statistical comparison of the two tests. Then the mean change in the squared process satisfies $\delta_i \cong 2\nu_i$ and we expect the same relative power performance of Sup BQ and Sup HQ as that of Sup B and Sup H discussed in Sections 2 and 3. If volatility changes cancel each other so that $\bar{\nu} = n^{-1} \sum_{i=1}^{n} \nu_i \cong 0$, then so is the average mean change $\bar{\delta}$ in the squared process and expect better power for Sup HQ than for Sup BQ. If volatility changes do not cancel each other significantly, we expect the reverse: better power for Sup BQ than for Sup HQ. This point will be investigated in a Monte Carlo simulation below.

The two tests Sup BQ and Sup HQ are compared in a Monte Carlo experiment whose setup is similar to those of Li et al. (2015) and Shi (2015). We choose the data generating process

$$x_{it} = \rho x_{i,t-1} + u_{it}, \qquad X_{it} = x_{it} + \nu_i \mathbb{I}\{t > t_0\} u_{it}, \quad i = 1, \ldots, n,\ t = 1, \ldots, T$$

with standard normal error u_{it}. Volatility changes, if any, occur for half of n panel units at a time $t_0 = bT$ for which we consider $b = 0.1$ and $b = 0.5$. The volatility change parameters ν_i are chosen from:

$$\text{NC} : \nu_i = 0; \qquad \text{VC1} : \nu_i \sim U(-0.2, 0.2); \qquad \text{VC2} : \nu_i \sim U(0, 0.2).$$

Note that NC has no volatility change, VC1 has cancelling volatility changes ν_i on $[-0.2, 0.2]$ in that $\bar{\nu} \cong 0$, and VC2 has non-cancelling positive volatility changes ν_i on $[0, 0.2]$. Other experimental setup is the same as that in Section 3. Sizes and powers of the volatility change tests SupBQ and Sup HQ are reported in Tables 3, 4, respectively.

We observe that both Sup BQ and Sup HQ have good size performance even though Sup HQ has some size distortion for $\rho = 0.5$. Relative power performance of the volatility change tests Sup BQ and Sup HQ is similar to that of the mean change tests Sup B and Sup H: for the cancelling volatility change of VC1, Sup HQ has good power while Sup BQ has no power; for the non-cancelling volatility change of VC2, Sup BQ has better power than Sup HQ.

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Table 4
Powers (%) of level 5% tests for volatility changes based on 10 000 replications.

n	T	Breaks at $0.1T$				Breaks at $0.5T$			
		VC1		VC2		VC1		VC2	
		Sup BQ	Sup HQ	Sup BQ	Sup HQ	Sup BQ	Sup HQ	Sup BQ	Sup HQ
50	50	4.9	4.6	6.7	4.6	5.8	13.0	29.9	12.1
50	100	5.3	4.9	8.5	5.0	7.0	31.7	53.8	26.7
50	200	5.4	6.0	13.6	5.9	9.9	73.2	83.3	64.5
100	50	5.6	5.2	8.9	5.2	6.4	19.9	50.6	17.2
100	100	4.9	5.2	12.8	5.1	6.9	49.2	81.5	41.8
100	200	6.0	6.6	24.6	5.9	10.2	93.3	98.3	87.8
200	50	4.8	4.8	13.0	4.8	6.1	29.4	80.8	25.7
200	100	5.1	5.9	25.4	5.6	7.0	76.3	98.5	67.0
200	200	6.2	7.5	52.9	6.9	10.6	99.7	100.0	99.0

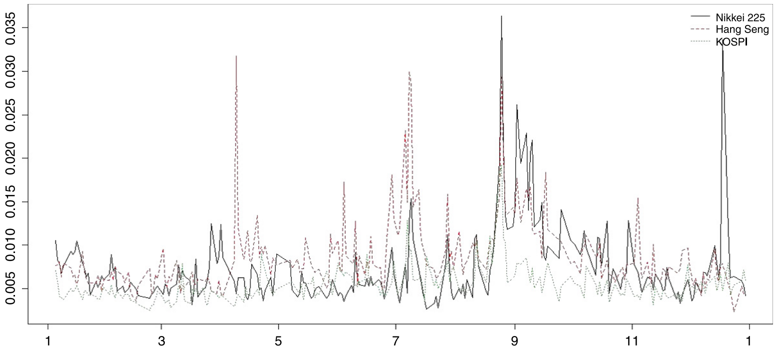

Fig. 1. Daily 5 min realized volatilities of the KOSPI, the NIKKEI, and the Hang Seng index.

5. An example

The proposed tests are illustrated by a real data set consisting of the daily 5 min realized volatilities of 3 eastern Asian country stock price indices for the year 2015: the KOSPI, the NIKKEI, and the Hangseng index. The volatility data set is downloaded from the Oxford-Man realized library (http://realized.oxford-man.ox.ac.uk/). Some example analyses for univariate realized volatilities are available in Hwang and Shin (2013, 2015, in press) and Song and Shin (2015) for structural breaks, in Yun and Shin (2015) for an overnight issue, and in Cho and Shin (2016) for an integrated model.

Fig. 1 shows time series plots of the 3 series, which reveals simultaneous increases both in levels and variations in September–October. Values of the test statistics are: Sup $B = 5.517$, Sup $H = 5.887$, Sub$BQ = 2.413$, Sup $HQ = 1.410$ Level 5%, 1% critical values of (Sup B, Sup$H$, Sup BQ, Sup HQ) are (2.29, 2.29, 1.68, 1.68), (2.74, 2.74, 2.24, 2.24), respectively. For the mean, we find significant breaks at level 1% by both Sup$B$ and Sup H. For the volatility, SupBQ indicates breaks at 1% level while Sup HQ fails to detect significance at 5% level. It seems that the non-cancelling high values in variations around September–October may have been more well detected by Sup BQ than Sup HQ.

6. Conclusion

We have considered a panel mean change test based on the CUSUM of the panel processes. The test has better size performance than the existing test of Horvath and Huskova (2012). Compared with the existing test, the proposed test has better power against non-cancelling changes but has worse power against cancelling changes. Therefore, when we have prior information that the means are shifted to a common direction as in the world wide financial crisis, we may prefer the proposed test to the existing one. When we have the prior information of cancelling changes, the existing test should be preferred. When we do not have information on the direction of mean shifts, we need to consider both of the proposed test and the existing one because none of the two dominates the other one in power performance. It would be better if we have a theoretical explanation on the Monte-Carlo difference between the proposed test Sup$B$ and the existing test Sup H. Investigation of the theoretical point may be a good topic of future research.

7. Proofs

Proof of Theorem 2.1. Under H_0, note that for $0 \le z \le 1$,

$$\frac{1}{\hat{v}_i}\frac{1}{\sqrt{T}}\sum_{t=1}^{[Tz]}(X_{it}-\bar{X}_i) = \frac{\sigma_i}{\hat{v}_i}\frac{1}{\sqrt{T}}\sum_{t=1}^{[Tz]}\left(u_{it}-\frac{1}{T}\sum_{t=1}^{T}u_{it}\right)$$

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and

$$v_i^2 = \lim_{T\to\infty} Var\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T} X_{it}\right) = \sigma_i^2 \lim_{T\to\infty} Var\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T} u_{it}\right) =: \sigma_i^2\varphi_i^2$$

where $\varphi_i^2 = \lim_{T\to\infty} Var\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T} u_{it}\right)$ is the long-run variance of u_{it}, $i = 1, \dots, n$. According to Phillips and Solo (1992) we have

$$\sum_{t=1}^{[Tz]}\left(u_{it} - \frac{1}{T}\sum_{t=1}^{T} u_{it}\right) = a_i\left(\sum_{t=1}^{[Tz]}\epsilon_{it} - \frac{[Tz]}{T}\sum_{t=1}^{T}\epsilon_{it}\right) + \eta_{iT}(z)$$

where a_i is defined in (A5), noting $a_i^2 = \varphi_i^2$, and where

$$\eta_{iT}(z) = \left(1 - \frac{[Tz]}{T}\right)\tilde{u}_{i0} - \tilde{u}_{i,[Tz]} + \frac{[Tz]}{T}\tilde{u}_{iT}$$

with $\tilde{u}_{it} = \sum_{l=1}^{\infty}\tilde{c}_{il}\epsilon_{i,t-l}$ and $\tilde{c}_{il} = \sum_{k=l+1}^{\infty} c_{ik}$. Under condition (A4), $\limsup_{i\to\infty}\sum_{l=0}^{\infty}|\tilde{c}_{il}| < \infty$ and thus $\eta_{iT}(z) \xrightarrow{p} 0$ as $T \to \infty$ for each $i = 1, \dots, n$. Thus, uniformly in z,

$$\frac{1}{\varphi_i}\frac{1}{\sqrt{T}}\left(\sum_{t=1}^{[Tz]} u_{it} - \frac{[Tz]}{T}\sum_{t=1}^{T} u_{it}\right) \xrightarrow{d} B_i^0(z) \quad \text{as } T \to \infty$$

where $B_i^0(z)$, $i = 1, \dots, n$, are independent standard Brownian bridges. Thus as $T \to \infty$,

$$\left|\frac{\sigma_i}{\hat{v}_i}\frac{1}{\sqrt{T}}\sum_{t=1}^{[Tz]}\left(u_{it} - \frac{1}{T}\sum_{t=1}^{T} u_{it}\right) - \frac{\sigma_i\varphi_i}{\hat{v}_i}B_i^0(z)\right| \xrightarrow{p} 0.$$

Since $\sigma_i\varphi_i/\hat{v}_i \xrightarrow{p} 1$, we have $\left|\frac{1}{\hat{v}_i}\frac{1}{\sqrt{T}}\sum_{t=1}^{[Tz]}(X_{it} - \bar{X}_i) - B_i^0(z)\right| \xrightarrow{p} 0$. Therefore, uniformly in z, $\left|\hat{B}_{nT}(z) - \frac{1}{\sqrt{n}}\sum_{i=1}^{n} B_i^0(z)\right| \xrightarrow{p} 0$ as $\min\{n, T\} \to \infty$. Since $\frac{1}{\sqrt{n}}\sum_{i=1}^{n} B_i^0(z) \xrightarrow{d} B^0(z)$ uniformly in z as $n \to \infty$, along with the tightness argument similar to the proof of Lemma 2 of Horvath and Huskova (2012), we have the desired result. □

Proof of Theorem 2.2. For $t_0 = t_0(T)$ we denote

$$\underline{\tau} := \liminf_{T\to\infty}\frac{t_0}{T}, \qquad \bar{\tau} := \limsup_{T\to\infty}\frac{t_0}{T}.$$

By the assumption, $0 < \underline{\tau} \le \bar{\tau} < 1$. We observe $Z_{iT}(z)$ for $z \in [0, t_0/T]$ and $z \in (t_0/T, 1]$, respectively. It can be shown straightforwardly that

$$Z_{iT}(z) = \frac{\sigma_i}{\sqrt{T}}\left(\sum_{t=1}^{[Tz]} u_{it} - \frac{[Tz]}{T}\sum_{t=1}^{T} u_{it}\right) + \frac{\sigma_i}{\sqrt{T}}\lambda_{iT}(z)$$

where

$$\lambda_{iT}(z) = \begin{cases} -\delta_i\dfrac{[Tz](T - t_0)}{T} & \text{if } 0 \le [Tz] \le t_0 \\ -\delta_i\dfrac{t_0(T - [Tz])}{T} & \text{if } t_0 < [Tz] \le T. \end{cases}$$

Note that $Z_{iT}(z)$ has the same limiting distribution as that of $\sigma_i\varphi_i B_i^0(z) + \frac{\sigma_i}{\sqrt{T}}\lambda_{iT}(z)$ by the proof of Theorem 2.1. Hence

$$\hat{B}_{nT}(z) = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\frac{\sigma_i\varphi_i}{\hat{v}_i}B_i^0(z) + \frac{1}{\sqrt{nT}}\sum_{i=1}^{n}\frac{\sigma_i}{\hat{v}_i}\lambda_{iT}(z) + o_p(1)$$

and thus, using the fact that $\sigma_i\varphi_i/\hat{v}_i \xrightarrow{p} 1$,

$$\hat{B}_{nT}(z) = B^0(z) + \frac{1}{\sqrt{nT}}\sum_{i=1}^{n}\varphi_i\lambda_{iT}(z) + o_p(1). \tag{6}$$

Note that

$$\left|\frac{1}{\sqrt{nT}}\sum_{i=1}^{n}\varphi_i\lambda_{iT}(z)\right| \ge \frac{b}{\sqrt{nT}}\left|\sum_{i=1}^{n}\lambda_{iT}(z)\right| + o_p(1) \tag{7}$$

where b is the common lower bound of φ_i $(=a_i)$ in (A5) for all i. In case that $0 \le [Tz] \le t_0$, we have

$$\frac{1}{\sqrt{nT}}\left|\sum_{i=1}^{n}\lambda_{iT}(z)\right| = \frac{1}{\sqrt{nT}}\left|\sum_{i=1}^{n}(-\delta_i)\frac{[Tz](T-t_0)}{T}\right| = [Tz]\frac{\sqrt{n}}{\sqrt{T}}\left|\bar{\delta}\right|\left(\frac{T-t_0}{T}\right) \ge [Tz]\frac{\sqrt{n}}{\sqrt{T}}\left|\bar{\delta}\right|(1-\bar{\tau})$$

since $(T-t_0)/T \ge \liminf_{T\to\infty}(T-t_0)/T = 1-\bar{\tau}$. Also in case that $t_0 < [Tz] \le T$, we have

$$\frac{1}{\sqrt{nT}}\left|\sum_{i=1}^{n}\lambda_{iT}(z)\right| = \frac{1}{\sqrt{nT}}\left|\sum_{i=1}^{n}(-\delta_i)\frac{t_0(T-[Tz])}{T}\right| = (T-[Tz])\frac{\sqrt{n}}{\sqrt{T}}\left|\bar{\delta}\right|\frac{t_0}{T} \ge (T-[Tz])\frac{\sqrt{n}}{\sqrt{T}}\left|\bar{\delta}\right|\underline{\tau}$$

since $t_0/T \ge \liminf_{T\to\infty} t_0/T = \underline{\tau}$. Thus

$$\sup_{0\le z\le 1}\frac{1}{\sqrt{nT}}\left|\sum_{i=1}^{n}\lambda_{iT}(z)\right| \ge \max\left\{\sup_{0\le z\le t_0/T}[Tz]\frac{\sqrt{n}}{\sqrt{T}}\left|\bar{\delta}\right|(1-\bar{\tau}),\ \sup_{t_0/T<z\le 1}(T-[Tz])\frac{\sqrt{n}}{\sqrt{T}}\left|\bar{\delta}\right|\underline{\tau}\right\}. \tag{8}$$

Note that since $1-\bar{\tau} > 0$ and $\underline{\tau} > 0$, we have

$$\lim_{\min\{n,T\}\to\infty}\max\left\{\sup_{0\le z\le t_0/T}[Tz]\frac{\sqrt{n}}{\sqrt{T}}\left|\bar{\delta}\right|(1-\bar{\tau}),\ \sup_{t_0/T<z\le 1}(T-[Tz])\frac{\sqrt{n}}{\sqrt{T}}\left|\bar{\delta}\right|\underline{\tau}\right\} = C\lim_{\min\{n,T\}\to\infty}\sqrt{nT}|\bar{\delta}|$$

for some positive C. By the assumption, the right-hand side is ∞. Thus by (6)–(8) along with the above limit, we conclude that $\sup_{0\le z\le 1}\left|\hat{B}_{nT}(z)\right| \xrightarrow{p} \infty$. □

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