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Original Article

A mathematical model for root canal preparation using Endodontic file

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ABSTRACT

Aim: To develop the mathematical model for root canal preparation using Endodontic file.
Method: The process of obturation consist of the filling of the root canal cavity using heated and softened gutta-percha to get the fluid tight seal in between the canal wall and filled gutta-percha. Any obturation process to perform before it requires the preparation of the root canal which involves the removing dead tissue, substrates and debris from the decayed root canal. The various geometric parameters viz. Taper, relative angle for orientation of cross-section between two planes, pitch and polar symmetry constant of the Endodontic file etc. Geometric parameters are derived. The various performance parameters like Endodontic file life assessment, cutting force components (tangential and normal), substrate removal rate, torque exerted by Endodontic file etc are also derived successfully. The Endodontic file life is estimated based on two approaches viz. speed and volumetric wear approach.
Conclusion: The mathematical model described is helpful for the Endodontic experts, researchers, design engineers etc. However the applicability of the described mathematical model limited to assumption of study. The gap between root canal to be prepared and Endodontic file is zero while preparation. The Endodontic file weight, speed of rotation and substrate removal rate is assumed to be constant. The mathematical model for Endodontic file discussed above proved to be efficient tool for studying the root canal preparation. The various geometric parameters and the various performance parameters are derived.

1. Introduction

The root canal treatment is one of clinical intervention which helps to recover the decayed and painful tooth. The root canal treatment is done to save the infected or partially damaged teeth. In dentistry, root canal treatment is known as obturation. The process of obturation consist of the filling of the root canal cavity using heated and softened gutta-percha to get the fluid tight seal in between the canal wall and filled gutta-percha.[11] Any obturation process to perform before it requires the preparation of the root canal which involves the removing dead tissue, substrates and debris from the decayed root canal to develop the logical cavity so that any practitioner can fill it effectively (Typical/logically prepared root canal cavity[10] view is shown in Fig. 3) The nitinol made files are used to perform the preparation process (Geometry of Endodontic file used for root canal preparation is shown in Fig. 1). The nitinol is bio-compatible alloy widely used as bio-material for manufacturing nitinol Endodontic file in field of Endodontics. Nowadays single nitinol instruments can be used for different types of working motions for preparation of root canal.[1–4] (see Table 1)

During use of Endodontic file the root canal cavity offers the resistance to file rotations (Endodontic file inserted into root canal for preparation is shown in Fig. 2) which can be minimized through use of the lubricant. Due to dependability of failure on various parameters the failure prediction becomes difficult.[5,6] One must strongly agree for shape of the root canal cavity is highly influencing factor for locking action and breaking of file in canal which can be defined in term of torsion. When file rotates in the canal with curvature the bending takes place and hence failure. The combined torsion and bending model can be used to predict the failure stress for file. The fatigue failure phenomenon is observed in case of the Endodontic file, because of the decrease in the cyclic fatigue resistance, torsional fatigue resistance of file causes respective cyclic fatigue and torsional fatigue failure.[7] There is need to develop mathematical model which can accurately estimates the geometrical parameters and performance parameters. The geometrical and performance parameters are helpful to trace the behavior of Endodontic file in the root canal during preparation process.

There are various approaches in literature had been used viz. Empirical models which are based on curve fitting and arbitrary values, Regression models and mathematical modeling using MATLAB software. Even though there is formulation for calculating the life of the

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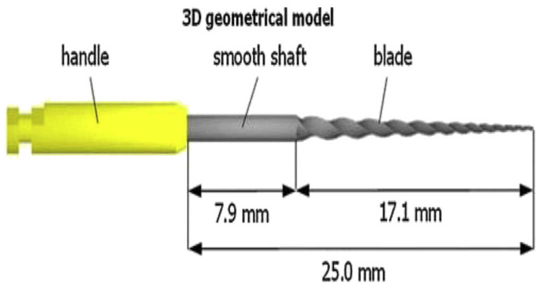

Fig. 1. Geometry of Endodontic file used for root canal preparation.[1]

Endodontic file, based on this we cannot decide the dimensions of the file which avoid failures. The dental practitioners may sometime use the file to perform more than one root canal treatment and failure prediction becomes difficult. The different brands of Endodontic file have their own commercial design and nomenclature too which restricts us to compare various brands to opt for best treatment performance.

2. Materials and methods

The mathematical model for the root canal preparation process using Endodontic file is divided into two main parts viz. A mathematical model describing geometric parameters and A mathematical model describing the performance parameters.

2.1. A mathematical model describing geometric parameters for Endodontic file

The mathematical models of the geometric parameters are important in view of forming the root canal cavity. The parameters such as taper of Endodontic file, relative angle defining orientations of cross-section of Endodontic file, radius of file etc. are most influencing geometric parameters to form the logical cavity. The relation of the radius of file and taper of file is important in inter-converting the available relation. The simplified geometry of Endodontic file for mathematical model is shown in Fig. 4.

2.2. A model for taper of Endodontic file

The Endodontic file used for the cleaning and shaping the canal having tapered cross section. The radius at any point along the length of file is given by

$$r(z) = r_0 + 0.5 \int_0^z t(z)dz \tag{1}$$

2.3. A model for relative angle $\Psi(z)$

The relative angle $\Psi(z)$ for orientation of cross-section between two planes which are z distance apart can be expressed as,

$$\Psi(z) = \int_0^z \frac{\tan[\alpha(z)]}{r(z)} \tag{2}$$

In term of the Endodontic file pitch and constant of polar symmetry we get the following relation,

$$\Psi(z+p) - \Psi(z) = \frac{360°}{C} \tag{3}$$

2.4. Radius at particular distance z from tip of the Endodontic file

The radius at particular distance z from tip of the Endodontic file is given using following relation by considering the taper of file,

$$r(z) = r_0 + 0.5\, t_z \tag{4}$$

2.5. A model to evaluate relative angle Ψ

The relative angle Ψ can be calculated using relation,

$$\Psi(z) = \frac{2\, tan\alpha}{t} \ln\left(1 + \frac{t}{2r_0} z\right) \tag{5}$$

2.6. A mathematical model describing performance parameters for Endodontic file

The performance parameters described in this section dealing with Endodontic file failure phenomenon. The Endodontic file gets failed because of fatigue caused due to locking action of the canal to Endodontic file motion.[8] Therefore, various fatigue failure related parameters are investigated. The three dimensional Cartesian frame of reference system considered for evaluation of performance parameters is shown in Fig. 5.

2.7. The cutting force analysis model

The cutting force has two components one is tangent to circular cross section of file and another one is normal component.

The tangential component f_t of the cutting force of Endodontic file cutter is given as

$$f_t = f_c\, cos\beta - f_{fc}\, sin\beta - f_{fw}\, cos\beta_c - f_w\, sin\beta_c \tag{6}$$

The normal component of Cutting Force of Endodontic file cutter is given by relation

$$f_n = f_c\, sin\beta - f_{fc}\, cos\beta - f_{fw}\, cos\beta_c + f_w\, sin\beta_c \tag{7}$$

The cutting force is given by relation

$$f_c = r_c * \frac{\pi}{4} [r_0 + 0.5\, t_z]^2 \tag{8}$$

There is friction between the Endodontic file and root canal wall whose contact area is a_c is called as cutting contact area and force due to friction is called as friction force applied on Endodontic file at cutting area, is give by

$$f_{fc} = \mu\, f_c = \mu\, r_c \frac{\pi}{4} [r_0 + 0.5\, t_z]^2 \tag{9}$$

The normal force component also acts on the wear area and denoted by f_w

$$f_w = R\, a_w * f_w = \mu R a_w \tag{10}$$

The tangential and normal components can be given in term of the taper t_z and outermost radius of file r_0

$$f_t = r_c * \frac{\pi}{4} [r_0 + 0.5\, t_z]^2\, (sin\beta - \mu sin\beta) + \mu\, R\, a_w \tag{11}$$

$$f_n = r_c * \frac{\pi}{4} [r_0 + 0.5\, t_z]^2\, (sin\beta + \mu sin\beta) + R\, a_w \tag{12}$$

Assumed that during root canal cleaning and shaping process the parameter: weight of file, speed of rotation, derbies flow rate are constant. As canal preparation continues the cutting depth deceases the linear wear of file increases. There are two approach to find the life of Endodontic file first is based on speed of file and second is volumetric wear of Endodontic file.

Table 1
Nomenclature of endodontic file.

Ψ = The relative angle of orientation for cross section of Endodontic file	r_0 = Outermost radius of Endodontic file
z = Distance from tip of Endodontic file	f_n = Normal component of cutting force of Endodontic file
C = Polar symmetry constant for cross-section of Endodontic file	f_t = Tangential cutting force component of Endodontic file
t = Taper of the Endodontic file	f_{fc} = Friction force applied on Endodontic file at cutting contact area
β = Cutting angle	f_{fw} = Friction force applied on Endodontic file at wear flat area
μ = Coefficient of friction between Endodontic file and root cancel wall	f_w = Normal force component acting on wear flat area
a_w = Wear area	f_c = Cutting force of Endodontic file
R = Resistance offered by canal wall for pressing action of Endodontic file	N = Endodontic file lifen = Speed of Endodontic file
a_{wf} = Endodontic file's wear flat area	n = Speed of Endodontic file
w = Compaction force applied on Endodontic file	W_v = Volumetric wear of file
C_1 = Propotionality constant of compaction force applied on Endoddontic file and normal component	w_d = Dimensionless file wear
C_2 = Propotionality constant of volumetric wear and work friction	f_h = Horizontal force exerted by cutter of Endodontic file
C_3 = Proportionality constant of penetration rate per rotation and preparation depth of canal	τ = Time in hour
F = Number of cutters of Endodontic contributing removal of substrate	SRR = Substrate removal rate
r_c = Radial location of cutter under consideration of Endodontic file	δ = Preparation depth of canal
p = Resistance offered by root canal wall to Endodontic file	

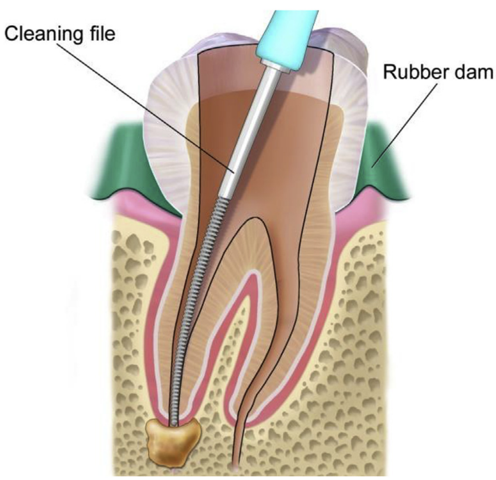

Fig. 2. Endodontic file inserted into root canal for preparation.[2]

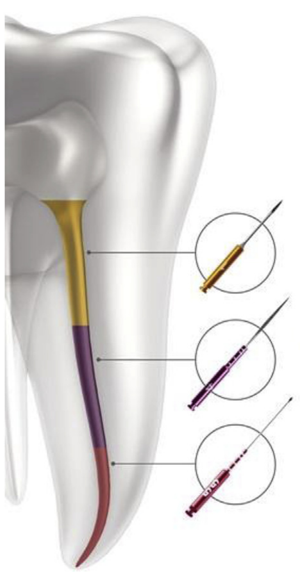

Fig. 3. Typical/logically prepared root canal cavity with files used for preparation.[2]

2.8. Life assessment using speed approach

During life assessment of the Endodontic file it was assumed that the weight on the Endodontic file, rotary speed and debris flow rate coming out from root canal is constant. The life of Endodontic file is given by

$$N = 2\mu r_c n\tau \tag{13}$$

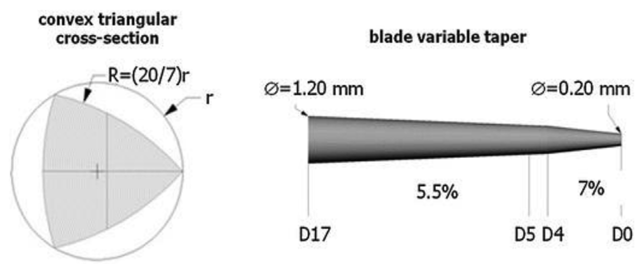

Fig. 4. Simplified geometry of Endodontic file for mathematical model.

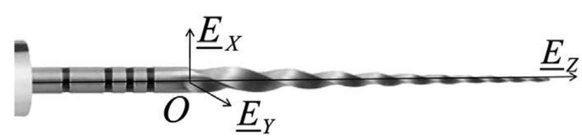

Fig. 5. Three dimensional Cartesian frame of reference system considered for evaluation of performance parameters.

2.9. Life assessment using volumetric wear approach

The volumetric wear is given by

$$W_v = 2\pi C_2 r_c \mu f_n n\ \tau \tag{14}$$

By introducing the compacting force w applied on Endodontic file and proportionality constant C_1 between compaction force applied on the Endodontic file and normal component of cutting force we get,

$$W_v = (2\pi C_2 r_c \mu f_n n\ \tau) * \frac{w}{C_1} \tag{15}$$

$$W_v = \int_0^x a_{wf}(x)dx \tag{16}$$

$$dx = d_c cos\beta\ dw \tag{17}$$

$$W_v = \frac{d_c^2 cot\alpha}{2}\int_0^w \frac{\sqrt[2]{1-4w(1-w)}}{2} - \sqrt[2]{w(1-5w+8w^2-4w^3} \tag{18}$$

$$W_v = \frac{d_c^2 cot\alpha}{2} w_d \tag{19}$$

Where w_d is dimensionless volumetric wear function and is give by

$$w_d = \frac{Actual\ W_v}{Voulume\ ofcube\ having\ side\ equal\ to\ readius\ of\ file} \tag{20}$$

Therefore, the Endodontic file life in term of the dimensionless volumetric wear function w_d and using volumetric wear approach is given by

$$\tau = \frac{d_c^2 cot\alpha}{2} * \frac{w_d}{2\pi C_2 r_c \mu f_n n} \tag{21}$$

2.10. A model for substrate removal rate (SRR)

By rearranging equation (12) the cutting area is given as

$$a_c = \frac{f_n - R\, a_{wf}}{r_c(sin\beta + \mu cos\beta)} \tag{22}$$

We know that, cutting area is function of the depth of preparation and therefore,

$$a_c = f(\delta) \tag{23}$$

Solving for δ and substituting for a_c we get,

$$\delta = \frac{4w_d(f_n - Ra_{wf})}{d_c r_c(sin\beta + \mu cos\beta)} \tag{24}$$

By introducing the proportionality constant C_3 between penetration rate per rotation and canal preparation depth we get,

$$\text{Substrate removal rate (SRR)} = C_3 \delta n F \tag{25}$$

2.11. A model to calculate torque exerted by Endodontic file

Consider the respective equations (26), (27), (28) of cutting area, tangential and normal component of the cutting force we have,

$$a_c = \frac{f_n - Ra_{wf}}{r_c(sin\beta + \mu cos\beta)} \tag{26}$$

$$f_t = r_c * \frac{\pi}{4}[r_0 + 0.5\, t_z]^2\ (sin\beta - \mu sin\beta) + \mu\, R\, a_w \tag{27}$$

$$f_n = r_c * \frac{\pi}{4}[r_0 + 0.5\, t_z]^2\ (sin\beta + \mu sin\beta) + R\, a_w \tag{28}$$

The horizontal component of cutting force and tangential components can be related as,

$$f_h = \frac{f_t}{cos\Psi} \tag{29}$$

After introducing equations (26), (27) and (28) into equation (29) we get,

$$f_h = f_n \frac{1 - \mu tg\beta}{\mu + tg\beta} - \frac{2 - \mu - tg\beta}{\mu + tg\beta} p\, a_w \tag{30}$$

The torque equation is give by,

$$T = \frac{1 - \mu tg\beta}{\mu + tg\beta} n \int_0^{r_f} f_n\, \rho\, dr - \frac{2 - \mu - tg\beta}{\mu + tg\beta} p \int_0^{r_f} a_w\, \rho\, r\, dr \tag{31}$$

where,

$$a_w(r) = a_w r_c \frac{f(r)}{f(r_c)}$$

$$f_n(r) = \frac{w}{C_1(r)\,\delta(r)}$$

2.12. The locking action exerted by canal wall

The canal wall having the locking action can be represented in term of resistance offered p and is given by relation,

$$p = \left(e^{\frac{180^\circ * t}{C\ tan\alpha}} - 1\right) z + \frac{2r_0}{t}\left(e^{\frac{180^\circ * t}{C\ tan\alpha}} - 1\right) \tag{32}$$

3. Discussion

The mathematical model for Endodontic file discussed above proved to be efficient tool for studying the root canal preparation. The various geometric parameters viz. Taper, relative angle $\Psi(z)$ for orientation of cross-section between two planes, pitch and polar symmetry constant of the Endodontic file etc. Geometric parameters are discussed. The various performance parameters like Endodontic file life assessment, cutting force components (tangential and normal), substrate removal rate, torque exerted by Endodontic file etc are derived successfully.

The model for radius of Endodontic file is given in Equations (1) and (4) which helps estimate the radius at any distance z from the tip of Endodontic file. The radius of the Endodontic file is important geometrical parameter since the preparation and shaping of the root canal cavity is dependent on proper use of diameter. The Endodontic file specifications available in the manufacturer is in term of diameter of file which varies along the length of file since file is tapered. The relative angle $\Psi(z)$ for orientation of cross-section between two planes which are z distance apart is estimated using Equations (2), (3) and (5). Tangential cutting force component, normal cutting force component and the cutting force are given by Equations (6), (7) and (8) respectively. The significance of the cutting force is in removing the root canal substrate. The initial separation of the substrate layer is done with help of cutting force applied by Endodontic file.

The life assessment of Endodontic file using speed approach and volumetric wear approach is given by Equations (13) and (14) respectively. Sometimes during the use of Endodontic file in preparation of the root canal, the file get failed after its use repeatedly. To know exact failure point there is need to assess the life of Endodontic file in term of number of rotations that file perform before failure. If dental practitioner know the life of file then the breakage of file will be avoided in the canal during its use. The said failure of the file is earlier if locking action by the root canal on file is more. The locking pressure exerted by the root canal opposing to rotation of Endodontic file is give by Equation (32). However, the removal of the substrate rate is important factor to evaluate the digging capacity of the Endodontic file. The substrate removal rate is given by Equations (22), (23) and (25). The torque of the Endodontic file is give by Equation (31). The higher value of torque represents higher cutting force. Moreover if the torque value is more, one can ensure the rotation of Endodontic file in root canal even for considerable locking action.

Thus, the performance parameters are most contributing parameters for efficient removal of the substrate, dead tissue and debris from root canal to be prepared and obturated. On the other hand, the geometrical parameters are most contributing for shaping canal and forming the logical cavity volume. After preparation of the logical cavity the practitioner can fill it with help of biocompatible material eg. gutta-percha.[9]

4. Conclusion

The mathematical model described is helpful for the Endodontic experts, researchers, design engineers etc. However the applicability of the described mathematical model limited to assumption of study. The gap between root canal to be prepared and Endodontic file is zero while preparation. The Endodontic file weight, speed of rotation and substrate removal rate is assumed to be constant. The mathematical model for Endodontic file discussed above proved to be efficient tool for studying the root canal preparation. The various geometric parameters and the various performance parameters are derived.

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Declaration of competing interest

None.

References

1. Capar ID, Ertas H, Ok E, Arslan H, Ertas ET. Comparative study of different novel nickel-titanium rotary systems for root canal preparation in severely curved root canals. *J Endod.* 2014;40:52–56.
2. Saber SE, Nagy MM, Schafer E. Comparative evaluation of the shaping ability of WaveOne, Reciproc and OneShape single-file systems in severely curved root canals of extracted teeth. *Int Endod J.* 2015;48:09–14.
3. Burklein S, Hinschitza K, Dammaschke T, Schafer E. Shaping ability and cleaning effectiveness of two single-file systems in severely curved root canals of extracted teeth: reciproc and waveone versus Mtwo and Protaper. *Int Endod J.* 2012;45:449–461.
4. Yared G. Canal preparation using only one Ni-Ti rotary instrument: preliminary observations. *Int Endod J.* 2008;41:39–44.
5. Galle EM, Woods HB. Variable weight and rotary speed for lowest drilling cost. *AAODC Annual Meeting.* American Petroleum Institute; 1960 New Orleans.
6. Galle EM, Woods HB. Best constant weight and rotary speed for rotary rock bits. *Drilling and Production Practice.* 1963;48:44–61.
7. Vieira EP, Nakagawa RKL, Buono VTL, Bahia MGA. Torsional behaviour of rotary Ni-Ti ProTaper Universal instruments after multiple clinical use. *Int Endod J.* 2009;42(10):47–53.
8. Lokhande PR, et al. A review of contemporary fatigue analysis and biomaterials studies in endodontics. *Mater Sci Forum.* 2019;969https://doi.org/10.4028/www.scientific.net/MSF.969.193.
9. Lokhande PR. A comparative mircoleakage assessment in root canals obturated by three obturation techniques using fluid filtration system. *Biomed Pharmacol J.* 2019http://dx.doi.org/10.13005/bpj/1709.
10. Lokhande PR, et al. A review of contemporary research on root canal obturation and related quality assessment techniques. *Lecture Notes Mech Eng.* 2018https://doi.org/10.1007/978-981-13-2697-4_55.
11. Lokhande PR, et al. A review of contemporary researches on biomedical image analysis. *Commun Comput Inf Sci.* 2019https://doi.org/10.1007/978-981-13-9184-2_7.