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## Simulation-based multi-objective model for supply chains with disruptions in transportation

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## ABSTRACT

Unpredictable disruptions (e.g., accidents, traffic conditions, among others) in supply chains (SCs) motivate the development of decision tools that allow designing resilient routing strategies. The transportation problem, for which a model is proposed in this paper, consists of minimizing the stochastic transportation time and the deterministic freight rate. This paper extends a stochastic multi-objective minimum cost flow (SMMCF) model by proposing a novel simulation-based multi-objective optimization (SimMOpt) solution procedure. A real case study, consisting of the road transportation of perishable agricultural products from Mexico to the United States, is presented and solved using the proposed SMMCF-Continuous/SimMOpt solution framework. In this case study, time variability is caused by the inspection of products at the U.S.-Mexico border ports of entry. The results demonstrate that this framework is effective and overcomes the limitations of the multi-objective stochastic minimum cost flow problem (which becomes intractable for large-scale instances).

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### 1. Introduction

The United States is the most important customer for the Mexican ornamental flowers industry due to the geographical location of these two countries [1,2]. Road transportation is an affordable transportation method that is available to Mexican suppliers. According to the U.S. Bureau of Transportation Statistics [3], truck and rail transportation of products from Mexico to the United States increased by 3.7 percent, from \$34.3 billion in May 2012 to \$35.57 billion, in May 2013. Industry analysts expect this trend to continue growing in the near future.

Disruptive events are common in most transportation systems. Accidents, traffic conditions, and weather conditions (among others) are causes of disruptive events. Disruptive events are particularly important when transporting perishable products. The availability of inspection lanes and the process of drug trafficking

scan and detection add variability to the crossing time on the U.S.-Mexico border ports of entry. The variability on the waiting time due to security inspections can be translated into important economic impacts [4]. Thus, disregarding factors such as the variability on transportation time results in poorly designed supply chains, which in turn lead to important economic losses [5].

Mathematical programming models have been developed to solve the problem of determining the optimal flow of units along the available transportation routes. Many of these models consider single objective functions, which compute the transportation cost and ignore the variability of time attributes on arcs. A more comprehensive model, however, might require additional considerations such as the extremization of different objectives (e.g., the transportation time, the transportation freight rates, among others). Frameworks based on the multi-objective shortest path problem (MSPP) or the multi-objective minimum cost flow (MMCF) can extremize several objective metrics; in many of such frameworks, nonetheless, deterministic attributes have been assumed. In real systems, the duration of the transportation disruptions and the availability of servers (e.g., cargo inspection agents) are stochastic factors that are time-dependent.

To incorporate the inherent uncertainties of disruptive events into the complex supply chain design optimization process, this paper presents a model that extends previous research work (stochastic multi-objective minimum cost flow (SMMCF) Discrete model for solving the stochastic MMCF [6] problem with discrete

*Abbreviations:* U.S., United States; SC, Supply Chain; SMMCF, Stochastic Multi-Objective Minimum Cost Flow; SimMOpt, Simulation-Based Multi-Objective Optimization; MSPP, Multi-Objective Shortest Path Problem; MMCF, Multi-Objective Minimum Cost Flow; MCF, Minimum Cost Flow; CPM, Cost Per Mile; SPP, Shortest Path Problem; SA, Simulated Annealing; TM, Trade Mark; NFL, No Free Lunch Theorem; BMCF, Bi-Objective Minimum Cost Flow; NP, Non-Deterministic Polynomial; DHS, Department of Homeland Security; Sched, Schedule; CONACYT, Mexico National Science and Technology Council

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attributes) by considering stochastic continuous time disruptions and proposes a novel simulation-based multi-objective optimization (SimMOpt) solution procedure for solving the continuous version of the SMMCF problems (SMMCF-Continuous). The previous SMMCF-Discrete model considers a discrete distributed inspection time at the border ports of entry. The proposed novel SMMCF-Continuous model and SimMOpt solution procedure considers instead a more realistic, continuously distributed inspection time and identifies near-optimum solutions to large-scale instances of the problem of designing efficient routing plans for the international trade of perishable products.

The stochastic nature of the disruptive time is considered only for arcs that connect nodes that represent inspection operations at the ports of entry. The variability on the inspection time depends on the transportation service mode used to bring products across the border. Thus, cargo inspection time depends on the transportation mode employed. The general SMMCF model considers stochasticity related to the time attributes of some arcs connecting particular nodes, while keeping the rest of the attributes of the arcs deterministic. The main advantage of the SMMCF-Continuous model over the SMMCF-Discrete model is that the SMMCF-Continuous allows attributes, such as time, to be modeled by employing continuous probability distributions. Discretization of attributes that represent time does not accurately represent the real system. The main contribution of the proposed SimMOpt solution procedure is to overcome this limitation of the SMMCF-Discrete.

Important challenges arise when solving the SMMCF-Continuous problem. The number of possible values that coefficients representing stochastic elements can take increases exponentially. The large number of decision variables and multiple objective functions subject to constraints can become a problem in terms of convergence and computational time when using search algorithms. Some algorithms for solving this problem are subject to local-optima entrapment. Many become costly in terms of computational effort when searching solutions that depend on a large number of variables and subject to a number of hard constraints. Section 5 describes how the SimMOpt solution procedure overcomes these challenges.

There are many real applications that can be modeled as a stochastic minimum cost flow problem. For example, in real situations, the freight rate, which is assumed deterministic by the general SMMCF, follows a probability distribution. Fig. 1 shows the main components of the freight rate [7]. Fig. 2 shows the value of the U.S. based freight cost per mile (CPM) index between December 2013 and December 2014 [7]. These figures show how the

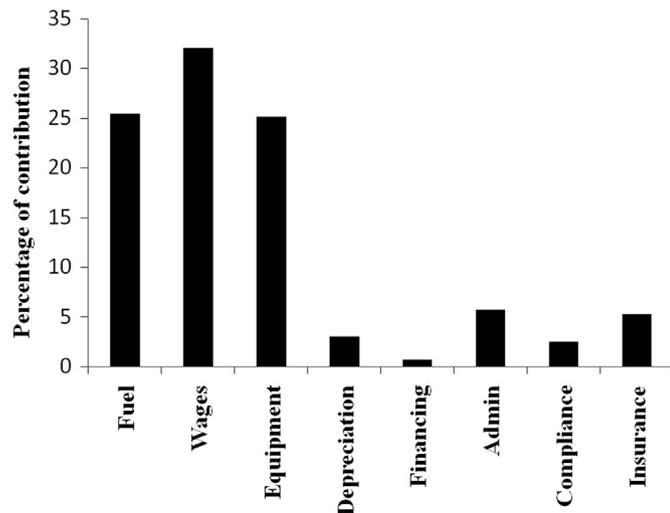


Fig. 1. Freight Rate Components (based on [7]).

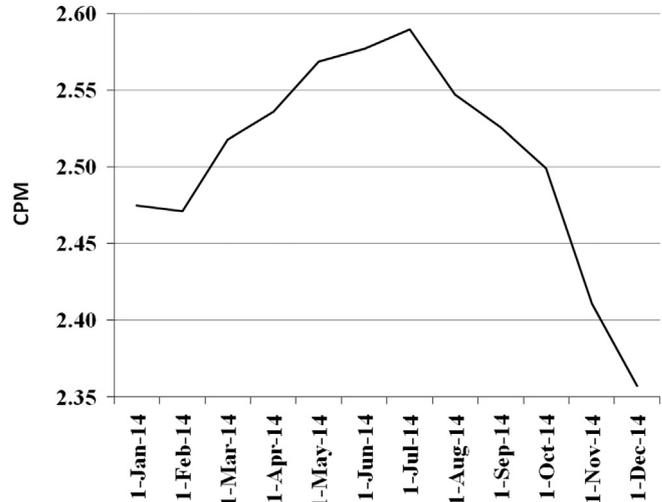


Fig. 2. U.S. CPM Index (based on [7]).

freight rates commonly change through time and how some factors contribute to the variability. The continuously distributed variation of the freight rate can be realistically modeled with the SMMCF-Continuous.

SMMCF-Discrete becomes ineffective when the statistical distribution of any of the stochastic attributes is modeled by employing a continuous probability distribution. The discretization of such distribution(s) into few classes can result in important inaccuracies while modeling the effects of time variability, for example. The SimMOpt solution procedure presented in this paper aims to overcome this limitation.

This paper is structured as follows: Section 2 presents a literature review of stochastic multi-objective minimum cost flow models and simulation models developed to treat multi-objective and/or stochastic versions of the transportation flow problem. Section 3 presents a description of the general SMMCF model and the SimMOpt solution procedure. Section 4 presents the mathematical formulation of the SMMCF-Discrete. Section 5 presents the implementation of the proposed SimMOpt solution procedure using MATLAB<sup>®</sup>. Section 6 describes a real case study. Sections 7 and 8 present the results from using the SMMCF-Discrete model and the SimMOpt solution procedure to solve the case study. Sections 9 and 10 present the concluding remarks and future research.

## 2. Literature review

Different models and procedures have been developed for finding near-optimum solutions to the problem of transporting products. These models typically assume a finite vehicle capacity, a maximum flow capacity related to arcs, and a predefined number of visits to customers, among others. This section describes two of the most relevant methods used to solve this type of problem.

The first type of methods, based on linear optimization, serves as foundation for the development of the SMMCF-Discrete model. A review of this approach is presented in Section 2.1. Models that consider stochastic attributes related to each arc are of particular interest in this brief literature review.

The second type, based on simulation-optimization models, is the main foundation of the SimMOpt solution procedure. A review of this procedure is presented in Section 2.2.

**Table 1**  
MMCF/SPP (shortest path problem) models.

Author	Year	Problem	Stochastic	Method
Sedeño-Noda, Gonzalez Martin [8]	2000	MCF		Two-Phases
Prsbylski et al. [9]	2006	MCF		Two-Phases
Opasanon, Miller-Hooks [10]	2006	SPP	✓	Label Correcting
Fonseca et al. [11]	2009	MCF		Inner Point
Eusebio, Figueira [12]	2009	MCF		Two-Phases
Raith, Ehrgott [13]	2009	MCF		Two-Phases

### 2.1. Review of the minimum cost flow and shortest path problems

The objective of a version of the minimum cost flow problem is to find the best way to send a definite number of items in a network while minimizing the net cost of the transportation. The objective of a version of the shortest path problem is to find a path between nodes such that the sum of the costs of using the arcs included in the route is minimized. The minimum cost flow problem is a generalization of the shortest path problem. For a detailed description of the multi-objective minimum cost flow and shortest path problems and their solution procedures, refer to the literature works listed in Table 1. It can be observed in Table 1 that only one model considers stochastic attributes that represent the cost related to allowing a certain amount of items flow through particular arcs. Table 1 shows that only few works have made major contributions to the development of transportation models that consider variability on the cost attributes.

Regarding methods for the optimization of several objective functions, Talbi [14] and Rangaiah [15] show a schematic diagram of the multi-objective optimization models.

### 2.2. Review of developments on simulation-based optimization applied to supply chain problems

For many applications, deterministic optimization techniques attempt to minimize or maximize the outputs of a predefined objective function. The complexity of a problem increases when feasible solutions, that is  $X=(x_1, x_2, \dots, x_n)$ , are restricted to a set of linear/nonlinear constraints (i.e.,  $X \subseteq \mathbb{R}^n$ ). Thus, this kind of problem is commonly "simulated" when it becomes analytically complex.

The input and output variables of most real systems are often considered stochastic due to the large range of uncontrollable factors that intervene in the behavior of the system. The stochastic behavior of these variables is often modeled so they follow probability distributions. When variables follow continuous distributions, or the space of the value to these variables is extremely large, classical optimization techniques may become inadequate during the optimization of the performance metric of the system. The concept of simulation-based optimization becomes especially useful when the stochastic behavior of the variables makes classical optimization techniques infeasible or computationally expensive. Fu [16] defines simulation optimization as "the optimization of performance measures based on output from stochastic (primarily discrete-event) simulations."

Fu [16] describes the simulation optimization complexity as the computational expense derived from the large number required replications (samples) to reduce the variation of the performance metric value. Several optimization algorithms have been developed to minimize or maximize these performance measures while reducing the number of alternatives to be evaluated. The complexity of a problem increases as the number of performance measures increases. To cope with complexity, the combination of performance measures into a weighted single objective problem is

commonly used. A basic version of the SimMOpt solution procedure follows this approach [17].

Optimization techniques allow discrete-event simulation models to evaluate a sequence of combinations of values for the decision variables of the system. These sequences are generated and evaluated to provide a near-optimum solution [18].

Simulation-based optimization has turned into the most novel topic in simulation [18]. Swisher et al. [19] provide a categorization of the literature on discrete-event simulation optimization. Jacobson et al. [20] cover thoroughly the literature preceding 1988. Carson et al. [21] show a schematic diagram of the simulation-based optimization methods.

#### 2.2.1. Simulation-based optimization and the simulated annealing (SA) algorithm

The presented optimization procedure is based on Simulated Annealing (SA). SA is a local search method, analogous to the annealing process where material is gradually cooled so that a minimal energy state is achieved [22],[23]. Kirkpatrick et al. [24] describe the SA algorithm as a variation of a local search algorithm suitable for deterministic objective functions.

A prominent characteristic of the SA algorithm is that its convergence properties can be proven for different settings of the parameters of the algorithm. To ensure that convergence to a global optimum occurs, the temperature parameter must be decreased quite slowly [22]. Hence, it can be considered slow in converging when compared to some other metaheuristic approaches [16]. SA is similar to other methods when its performance is averaged among all possible problems (No Free Lunch Theorem-NFL) [25].

Geman et al. [26] demonstrated the convergence of the SA algorithm to a global optimum is possible as the temperature was decreased slowly. Nonetheless, Akley [27] suggested a decreasing annealing schedule to decrease the temperature more rapidly.

The parameters value settings used to define the annealing schedule, the initial starting solution, the temperature decay rate, and the number of iterations significantly affect the effectiveness of the SA algorithm [22].

The SA parameters must be tuned as function of the objective function complexity. Any set of parameters may become sensitive to the quality of the initial solution. Thus, it is recommended to restart the optimization algorithm several times to eliminate any dependency of the optimized solution on the initial solution [22].

Setting the parameters of the SA algorithm requires experience since the objective function complexity is often unknown and in most cases arbitrary values of the SA parameters are not effective. The complexity of the objective function can be obtained by graphing the function for different values to the decision variables. In most cases, this is very expensive or impossible to do. Such is the case of the case study considered in this paper due to the large number of decision variables.

The sensitivity of the SA algorithm to changes in its parameters makes it hard to reuse best-found parameters for their use in different problems, unless similarities in complexity can be assumed.

#### 2.2.2. Simulation-based optimization with hypotheses tests

The SMMCF-Discrete model considers randomness related to inspection time by including into the objective function the possible values of the stochastic attributes, for each scenario determined with the  $\epsilon$ -constraint method [6]. The values of these stochastic attributes are assumed to follow discrete probability distributions. However, the SMMCF-Discrete model is not appropriate when the values of the attributes follow continuous probability distributions. The SimMOpt solution procedure considers continuously distributed attributes by using sets of replications

(i.e., sample of values of the objective functions).

Wang et al. [28] proposed hypotheses testing to determine if a neighbor (new candidate) solution is statistically better than some other solution. This approach, incorporated into the SimMOpt solution procedure, allows reducing continuous variability through several replications, for each set of values for the decision variables. The number of replications used in the SimMOpt solution procedure was determined in such a way that the central limit theorem allowed performing hypotheses testing for two large samples.

### 3. Description of the general SMMCF model and SimMOpt approach

The general SMMCF represents the transportation system of products from supplying nodes to demanding nodes. The model assumes deterministic freight rates and deterministic travelling times affected by disruptive stochastic inspection times related to some arcs connecting nodes along segments of the forming routes.

The basic MCF problem can be modeled as a network containing a set of origin nodes with certain capacity which is intended to satisfy the demand of a set of destination nodes. The allocation of flow (the number of items moved through a given arc) should minimize the cost related to the transportation, so that these items being moved reach the destination nodes. The transportation cost related to each arc is represented as an attribute of the arc. The general SMMCF model considers *two components of the transportation cost: freight rate (monetary value) and transportation time (non-monetary value)*.

The variability in time caused by disruptive events should not be ignored, and special attention is required when dealing with perishable products. This variation has a major effect on the shelf-life of the perishable products. In the techniques described in this paper, the *freight rate component* is represented by a *deterministic attribute* of arcs and the *time component* is represented by a *deterministic attribute and a stochastic attribute*. The deterministic time attribute represents the time to travel between inter-connected nodes. The stochastic time attribute represents the variability on the time required to inspect cargoes at the ports of entry in the U.S.-Mexico border. This randomness ultimately derives from the transportation modes used to transport products across the border and the time of the resulting required inspections.

There are several road transportation modes that are available when moving products across the U.S.-Mexico border. The selection of the transportation mode affects the variation on the inspection time at the border crossing ports. Variability on inspection time builds up time disruptions along the supply chain. Summarizing, the stochastic time attributes in the proposed model represent the variation on inspection time due the selection of certain transportation modes.

Some approaches assume that the total cost can be considered as a single performance metric. In many cases, the total cost involves a series of components that are not always proportionally related. Not all of these components can easily be expressed in terms of equivalent units. When components cannot be expressed in terms of similar units, when priorities must be considered, or when proportionality cannot be assumed between the components of cost, multi-objective models provide an alternative to quantify performance metrics. In the case study presented in this paper, arcs with large values on the freight rate attribute cannot be assumed to have large values on corresponding time attribute (refer to Section 6). Proportionality cannot be assumed because increments in the value of freight rate attributes are not directly or inversely related to increments in the value of time attributes.

This model supports the decision making process from the perspective of the shipper. The minimization of the freight cost depends on the routing strategy to particular warehouses and ultimately to the customer. The routes represent a collection of connected nodes and arcs. The selection of a route in the case study implies the decision of sending units to particular inter-connected warehouses in the Mexican side and the selection of a transportation mode to cross the U.S.-Mexico border. Freight rates for the segments of routes (i.e., arcs) were obtained from transportation providers. Estimated deterministic times were also obtained from transportation service providers. The observations that constitute the stochastic components of time were gathered as explained in Section 6.

The MCF assumptions are considered by the general SMMCF model. Thus, the case study ignores the effect of the arriving time to the inspection facility at the port of entry. However, the SimMOpt solution procedure has the ability to iteratively adapt the parameters of the distributions used to model disruptive time variability. This ability can be further used to model the effect of arriving time to the inspection lanes, which cause peak hours at the inspection operations.

### 4. The SMMCF-discrete model

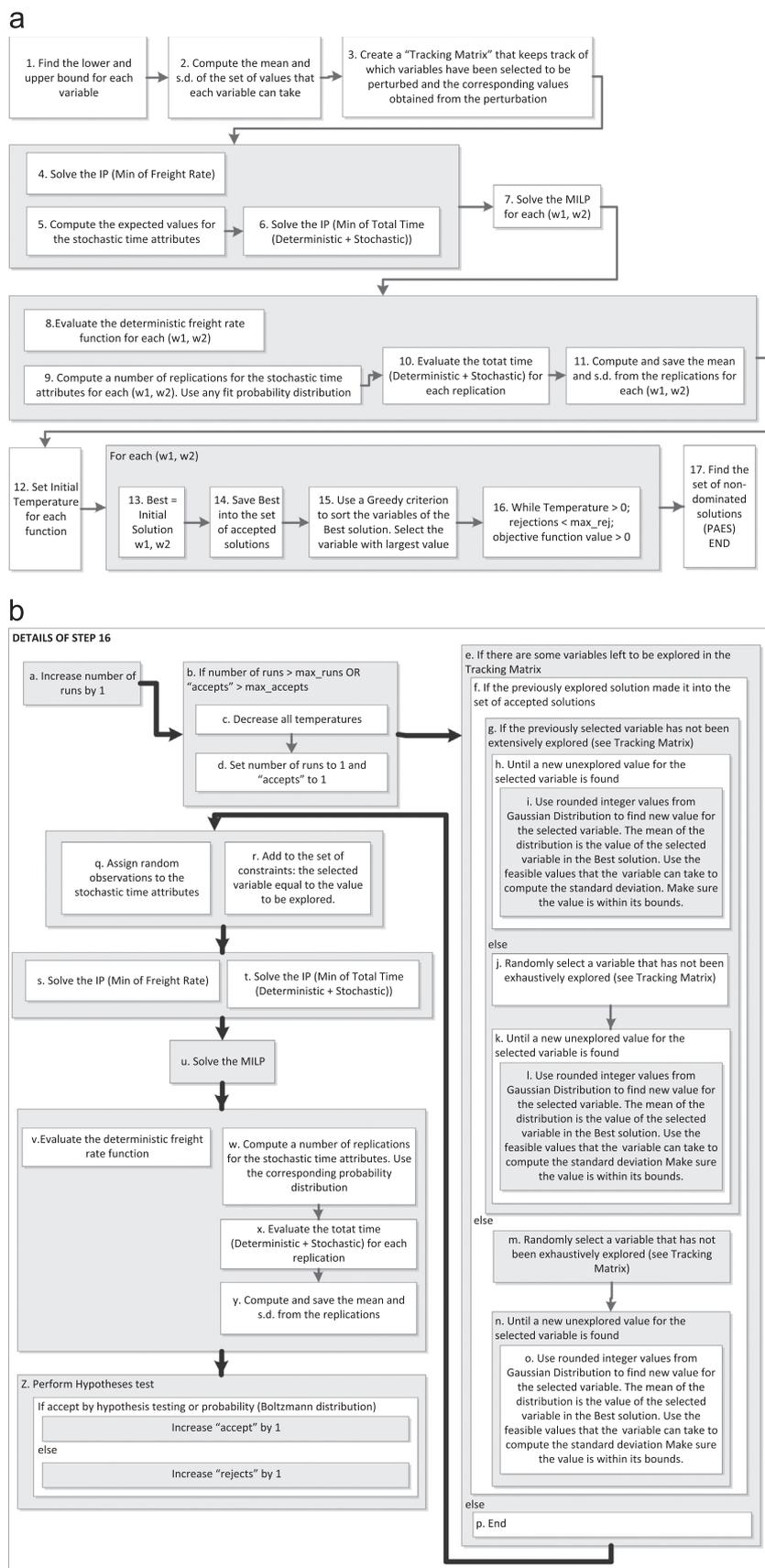
The variability on the attributes of arcs on the network presented in the case study represents the stochastic inspection time caused by selecting a mode from the various border crossing services available to suppliers. The SMMCF-Discrete model assumes that this variation on the crossing time can take *three* possible values that follow corresponding empirical discrete probability distributions.

The formulation of the two-phased model for the SMMCF-Discrete problem presented by Bustos et al. [6] is summarized in Appendix A. The notation was slightly changed to present a condensed formulation. For a detailed description of the SMMCF-Discrete, refer to Bustos et al. [6].

The first group of terms in Eq. (2) (see Appendix A) represents the known (input) time and freight rate values of the objective function. This group includes the terms for which all values are deterministic. According to the  $\epsilon$ -constraint method, the components in the first term that are related to a particular goal remain in the objective function, the rest of the components are considered in additional constraints. In the  $\epsilon$ -constraint method, a common scale before forming a weighted sum in the objective function is not necessary. The flow conservation constraints between nodes connected by arcs with stochastic and deterministic attributes are considered in Eq. (3) (see Appendix A). The second group of terms in Eq. (2) (see Appendix A) represents the part for which values of time are not certain, but possible values and corresponding probabilities are known.

The SMMCF-Discrete model relies on the two-phased method to solve the problem and uses the  $\epsilon$ -constraint method to develop the possible scenarios according to the values of time on the stochastic attributes of certain arcs. The  $\epsilon$ -constraint method defines a grid in the objective space and solves the single objective problem constrained by each cell on the grid. The rest of the objectives are considered as constraints. All the optimal Pareto solutions can be found only if the grid has a high resolution. This means that at most one Pareto optimal solution is found in each cell. The number of  $L$  scenarios can be determined by  $L = 3^\psi$ , where  $\psi$  is the number of stochastic variables (i.e., stochastic arcs).  $L$  increases exponentially to a point where the problem becomes intractable [29]. Bender's decomposition was used to deal to a certain degree with the exponential growth in the number of scenarios.

Ruhe [30] shows that an exponential number of non-



**Fig. 3.** a. Structure of the SimMOpt solution procedure. b. Structure of the SimMOpt solution procedure (Details of Step 16).

dominated solutions for the bi-objective case of the MMCF model exists. The continuous version of the MMCF model becomes intractable. The bi-objective minimum cost flow (BMCF) is a generalization of the bi-objective shortest path problem, which is NP-Hard. Ehrgott [31] describes the computational complexity of combinatorial problems and demonstrates that the MSPP is NP-Complete. Being the MCF a generalization of the SPP, it can be concluded that the MMCF is also NP-Complete.

## 5. The SimMOpt solution procedure

The SimMOpt solution procedure proposed in this paper finds near-optimum solutions for realistic situations where random variables are used to model attributes on arcs that represent disruptive time following continuous probability distributions.

The SMMCF-Discrete computes the stochastic portion of time by multiplying each possible value of the stochastic time attribute by the corresponding probability. For the SMMCF-Continuous, the space of the time attribute is infinite and the probability for each particular value approaches zero. The SimMOpt solution procedure computes a number of replications instead.

The SimMOpt solution procedure relies on multi-objective SA to assess the objectives functions. Section 5.1 describes the SimMOpt solution procedure in detail.

### 5.1. SimMOpt and SA multi-objective optimization

Fig. 3 describes the SimMOpt solution procedure. Flow on arcs is represented as decision variables that the approach optimizes to find the set of non-dominated solutions.

The SimMOpt solution procedure considers and adapts some the features of the model described by Wang et al. [28].

#### 5.1.1. Initial solutions

Wang et al. [28] suggest generating a large number of random initial solutions. The case study in this paper has a large number of decision variables with stochastic coefficients (i.e., attributes) in one of the objective functions. It is extremely time consuming to find a large set of initial solutions by using randomization of the values of the decision variables subject to a set of constraints as described by Wang et al. [28]. For this reason, goal programming weights method is used to find a set of initial solutions.

The values for the stochastic time attributes considered for the initial solutions are the expected values of the corresponding distributions. In this way, getting initial solutions that consider unlikely values of the stochastic time is avoided. Multi-criteria optimization emulates deterministic conditions by using values inferred from distributions of the stochastic coefficients.

The scaling factors used in the goal programming weights method are the optimal values of each of the objective functions when single criteria optimization is performed. These factors are then multiplied by the corresponding weights and deviation variables in the multi-criteria optimization. The deviation variables are defined as the variables that allow a given objective function to deviate from their individual optimal solutions when multiple competing objectives are being optimized. These deviation variables model the trade-off commonly required when optimizing competing solutions.

Once the optimization technique has found the optimal values for the decision variables given a configuration of weights for the deviation variables corresponding to the competing objective functions, these values are input into the objective functions and several replications are computed for random values from the corresponding continuous probability distributions for the stochastic coefficients. The variability on the coefficient values of the

stochastic objective function is reduced by computing a set of replications.

One sample of each configuration and the corresponding statistics are computed for each configuration of weights. Thus, these configurations determine the resolution of the set of Pareto non-dominated solutions.

#### 5.1.2. Neighbor solutions

The SimMOpt solution procedure uses a neighbor function to determine the value of a selected variable and finds the optimal values of the rest of decision variables by using multi-criteria optimization to minimize multiple functions given some weights for each scaled deviation variable.

The variable to be perturbed is selected according to the performance of the objective functions. A greedy criterion is followed to iteratively select a variable to be perturbed. The rest of the variables are optimized considering an increased set of constraints that now includes the perturbed variable equal to the corresponding value.

Then discretization of the Gaussian probability density function is used to perturb the selected variable. The perturbation is constrained so that the selected value remains within feasibility bounds and redundant exploration is avoided.

Random values from fit continuous probability distributions are considered for the stochastic time attributes when optimizing multiple functions. Scaling and replication are performed just like with the initial solutions.

The corresponding mean value and standard deviation of the sample of replications are computed. Then, the sample of replications for the best solution and the sample for the neighbor solution can be compared by using hypotheses testing when the evaluated function contains stochastic coefficients.

#### 5.1.3. SA temperature

The temperatures for each objective function are separately determined. This obviates the need to scale the objective functions with respect to each other. However, scaling of deviation variables was required when optimizing the rest of the variables given a value from the perturbation of a selected variable. The initial temperature for the stochastic time objective function is computed as described by Wang et al. [28].

#### 5.1.4. Probability of acceptance

The overall probability used in the SA is the product of individual probabilities for each function [32].

$$P_{all} = \min \left( 1, \prod_{n=1}^{NO.F.s} \exp \left( \frac{(f_n(x's_{i-1}) - f_n(x's_i)) / T_n}{T_n} \right) \right) \quad (1)$$

#### 5.1.5. Exploitation condition

This condition is considered by using three procedures. The first two procedures consist in selecting a variable in a particular solution that has been accepted. Initially, most contributing variables are explored according to a greedy criterion, and if promising solutions are found, the selected variable will remain the same unless exploration of any new values becomes redundant or infeasible. The third procedure consists in perturbing a variable in a particular solution that has been accepted. The SimMOpt solution procedure uses the Gaussian distribution to perform local searches. A discretization of the Gaussian distribution allows exploring with a high probability those integer values close to the value of the selected variable in the current best solution.

#### 5.1.6. Exploration condition

This condition is considered by using three procedures

intended to avoid local-optima entrapment. The first procedure consists of a random acceptance of solutions leads to exploration of different regions of the solution surface. In the second procedure, as the values of variables become explored, the iterated perturbation of a selected variable forces search values that have a low probability of being selected given a mean (i.e., value of the selected variable in the current best solution) and deviation from the Gaussian distribution. This perturbation has the effect of contrastingly changing the values of the rest of the variables. This leads to exploring distant regions of the solution surface. The third procedure is when a variable has been found to be fully explored. The random selection of a new variable leads to the exploration of different regions in the solution surface. The extended SimMOpt approach tracks at any given time the values of the variables that have been already explored. These conditions and convergence are highly dependent of the appropriate tuning of the schedule parameters. In this paper, the case study is simulated under different configurations of the annealing cooling schedule. The approach to the solution of the case study and the assessment of the performance of the SimMOpt solution procedure is explored for several schedules and reported for the most effective configuration. The effectiveness of the solutions is reported in terms of the trade-off between computational effort and convergence to the global-optima of the discrete version obtained by using the SMMCF-Discrete [6]. The configurations of the schedules are described in Section 8.

## 6. Case study

The case study presented in this paper is a real-life case. Data was obtained from the Department of Homeland Security (DHS); the data corresponds to July 5th, 2010 from 09:00 to 22:00 h. The case study presents the distribution problem faced by two suppliers of ornament flowers in Tenango, State of Mexico and Tecamachalco, Puebla both in Mexico. The destination nodes represent the most important customers for these flower producers. These

customers are located in Chicago, Illinois. Nodes that represent warehouses on the Mexican side of the border are connected by arcs with deterministic freight rate attributes and deterministic time attributes. The freight rates that are used in the model were obtained by the Mexican Institute of Transportation from cargo transportation providers. The nodes connected by arcs with stochastic time attributes represent the multi-modal transportation of products across the border line. The case study consists of a network with 29 nodes and 158 arcs. The items are shipped from two origin nodes and must reach the two destination nodes in Chicago. From the 158 arcs, 12 represent the border-crossing disruptive inspection operations. Nodes 1 and 2 represent suppliers with capacities of 7 and 10 units, respectively. Nodes 28 and 29 represent customers with demands of 8 and 9 units, respectively.

Sections 7 and 8 show the results of experimental runs for the case study. Fig. 4 shows the network. From this figure three ports of entry can be identified. The arcs with stochastic attributes represent the inspection operations required by the different transportation modes [6].

## 7. The SMMCF-discrete results

The disruptive inspections are modeled by stochastic attributes on the arcs connecting particular node  $i$  and particular node  $j$ . The real system was simplified so that the stochastic attributes were considered as values of discrete probability distributions followed by the times required for the inspection of cargo. Bustos et al. [6] shows the Pareto frontier for the non-dominated solutions for the discrete stochastic problem solved with the SMMCF-Discrete. It is statistically demonstrated that the Pareto frontiers from the discrete stochastic version of the case study and from a deterministic version based on expected values are differently distributed.

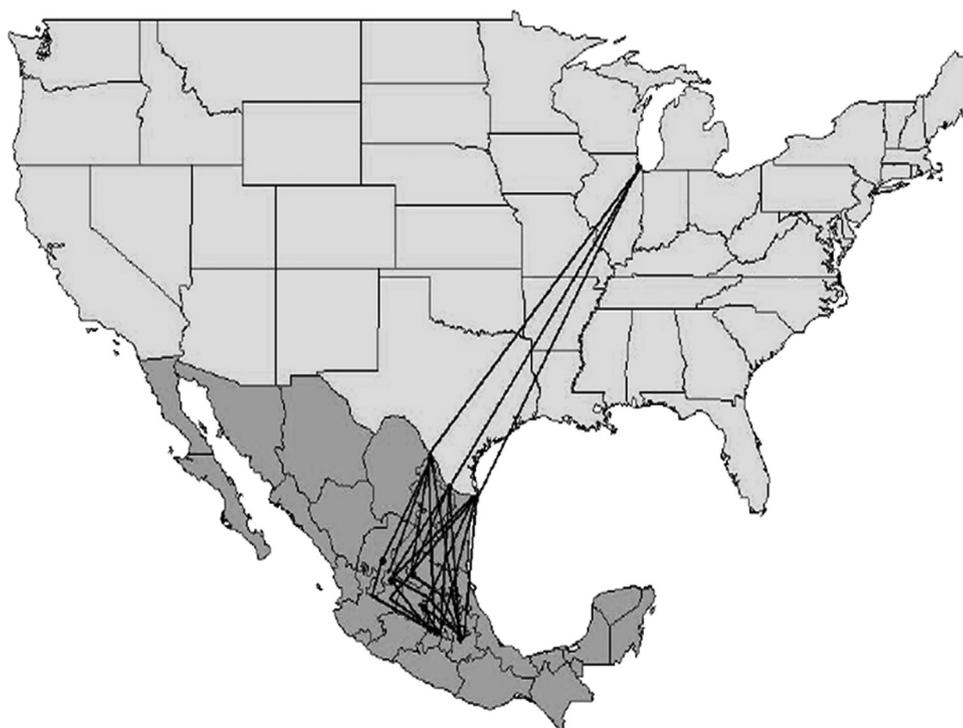
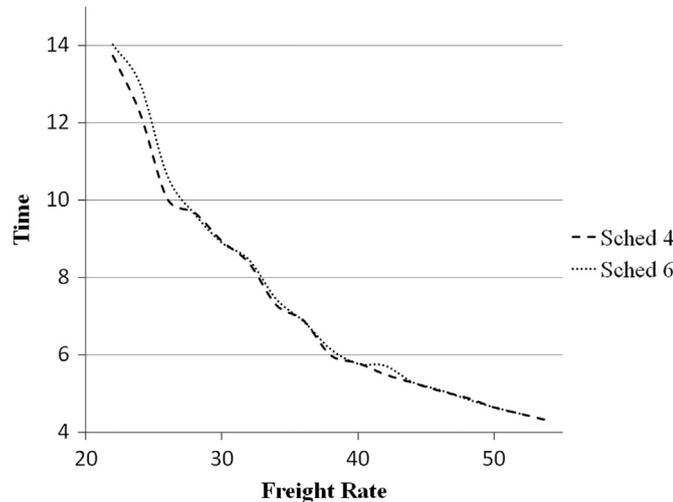


Fig. 4. Basic map of transportation of ornament flowers (based on Bustos et al. [6]).

**Table 2**  
Experiments for instance 'A'.

	Sched 1	Sched 2	Sched 3	Sched 4	Sched 5	Sched 6
<b>Cooling factor</b>	0.7	0.7	0.2	0.2	0.02	0.02
<b>Maximum rejections</b>	8	4	8	4	8	4
<b>Maximum runs</b>	20	10	20	10	20	10
<b>Maximum accepts</b>	8	4	8	4	8	4

The Pareto frontiers corresponding to 'Sched 4' to 'Sched 6' are shown in Fig. 5.



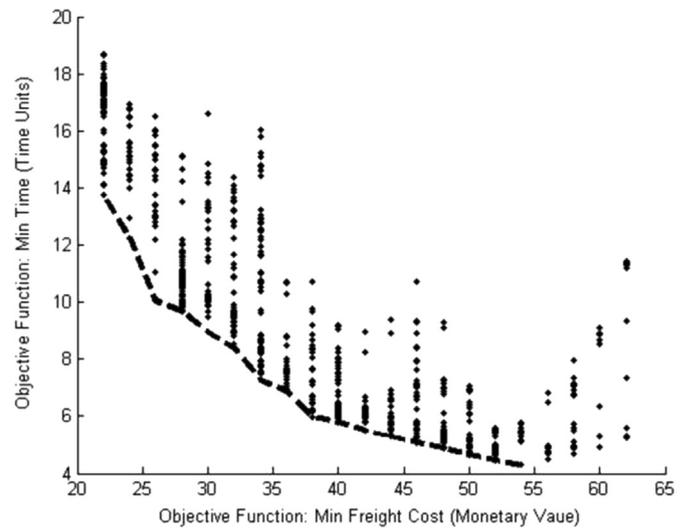
**Fig. 5.** Pareto frontiers for Sched. 4 and 6. (Note: Lead time: Time Units. Freight Cost: Monetary).

## 8. The results from the SimMOpt solution procedure

In order to test the model, an instance 'A' was initially considered. This instance consisted of 7 nodes and 6 stochastic arcs. The capacity of suppliers (nodes 1 and 2) is 3 and 7 units, respectively. The demand from customers (nodes 6 and 7) is 6 and 4 units, respectively. The SA schedule parameters were set based on the results of different experiments. The set of parameters is shown in Table 2. The number of replications for each solution being explored was set to 30.

In Fig. 5, it can be observed that the schedule parameters have an effect on the characteristics of the solution. When schedule 4 was used with the SimMOpt solution procedure, 17 non-dominated solutions were found. The computation time for schedule 4 was 20.2 min. When schedule 6 was used with the extended SimMOpt solution procedure, 16 non-dominated solutions were found. The exploration was insufficient to generate all the non-dominated solutions found with schedule 4. The computational time for schedule 6 was 54.98 min. The initial solutions for both schedules were the same as they were computed by using the expected values from the continuous probability distributions followed by the stochastic time attributes. Overall, it can be concluded that schedule 4 performed better than schedule 6 in terms of the number of non-dominated solutions found. Convergence suffered from a rapid cooling when considering schedule 6 [26]. Another conclusion is that schedule 1 performed as good as schedule 4. However, schedule 1 required 48.68 min. Thus, many unsuccessful iterations in terms of finding non-dominated solutions were computed. All the computed solutions by using schedule 4 are shown in Fig. 6. When 30 replications were computed none of the initial solutions remained as members of the set of efficient solutions for any of the schedules.

The solutions computed for the 'A' instance are lined up



**Fig. 6.** Pareto frontier for Sched 4. (Note: Delivery lead time: Time Units and Cost: Monetary).

because the deterministic coefficients of the freight rate objective function are integers. The coefficients in the time objective function are not integers as they follow continuous probability distributions. The decision variables are also integers as they represent the flow of items along arcs. Thus, the freight rate objective function is not continuously distributed in instance 'A'.

For the case study, after performing goodness of fit tests on the observed values of inspection time (see Section 6), triangular distributions with different parameters were considered for the modeling of the stochastic time attributes on arcs representing the border-crossing inspection operations. The coefficients in the freight rate objective function are not integers in the case study.

The "intlinprog" Matlab<sup>®</sup> function was used to solve the integer programming (IP) problems mentioned in Fig. 3a and b. The "intlinprog" is used due to its computational efficiency to solve problems with large number of variables and constraints. Table 3 shows the SA schedules considered for the case study. The number of replications for each solution being explored was set to 30. This table includes the computational time and the percentage of exploration for the last replication performed for several schedules. The percentage of exploration is measured as the total number of values assigned to a selected variable (ref. Fig. 3b step "h") divided by the space of all feasible values that a decision variable can take.

Figs. 7 and 8 show the Pareto frontiers for schedules 1 and 8, respectively.

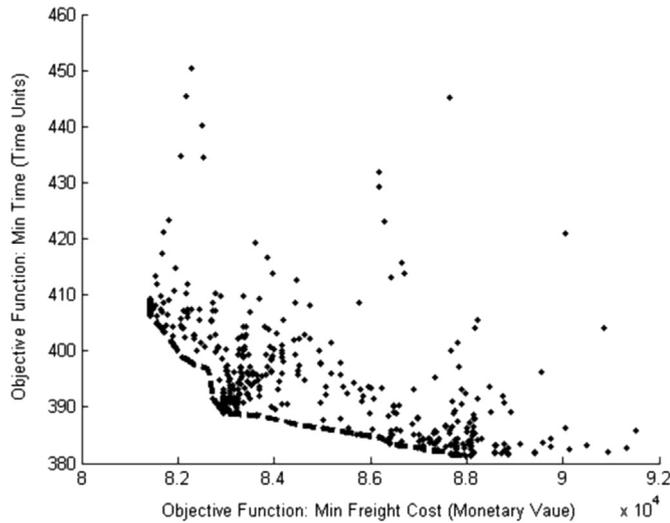
With schedule 8, the SimMOpt solution procedure found 34 non-dominated solutions in 16.27 hours. With schedule 1, it found 19 non-dominated solutions in 1.35 hours. A trade-off analysis is necessary when assessing the quality of the solutions. The set found by SMMCF-Discrete contains 21 efficient solutions. Figs. 9 and 10 show the Pareto frontiers for the SMMCF-Discrete model and the SimMOpt solution procedure with schedules 1 and 8, respectively. The "SMMCF Pareto" series in Figs. 9 and 10 show the frontiers described by the non-dominated solutions found with the SMMCF-Discrete model. The dotted lines in these figures show the non-dominated solutions found with the SimMOpt solution procedure (schedules 1 and 8, respectively) when continuous probability distributions are considered.

A test on the convergence of the solution obtained with schedule 1 to the global-optima found by using SMMCF-Discrete [6] was performed. The p-values were obtained for the F-tests on the values of the evaluated freight rate function and the statistics from the evaluated replications of the time objective function for the non-dominated solutions obtained with the SimMOpt solution

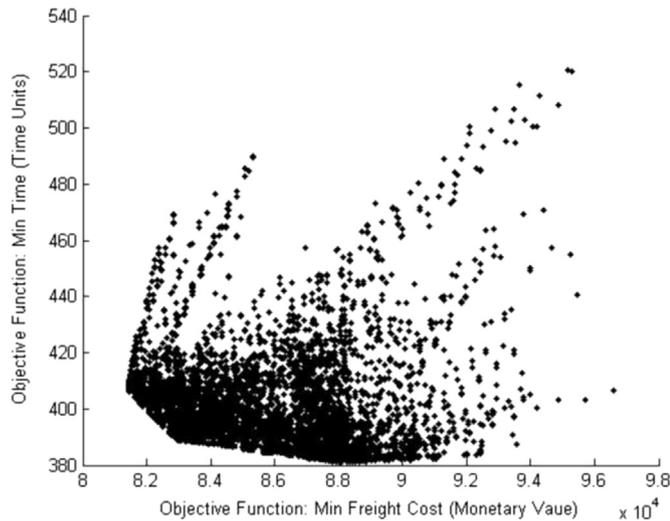
**Table 3**  
Experiments for instance 'Case study'.

	Sched 1	Sched 2	Sched 3	Sched 4	Sched 5	Sched 6	Sched 7	Sched 8
<b>Cooling factor</b>	0.02	0.02	0.2	0.2	0.4	0.4	0.6	0.8
<b>Maximum Rejections</b>	4	8	4	8	4	8	4	8
<b>Maximum Runs</b>	10	20	10	20	8	10	10	10
<b>Maximum Accepts</b>	4	8	4	8	4	8	4	8
<b>Comp. Time (hrs)</b>	1.36	2.89	2.54	5.35	1.77 <sup>a</sup>	5.77	3.16	16.28
<b>% Exploration</b>	3.64%	7.89%	4.04%	18.11%	9.81%	28.23%	4.65%	75.04%

<sup>a</sup> Data corresponding to 10 Max Runs.

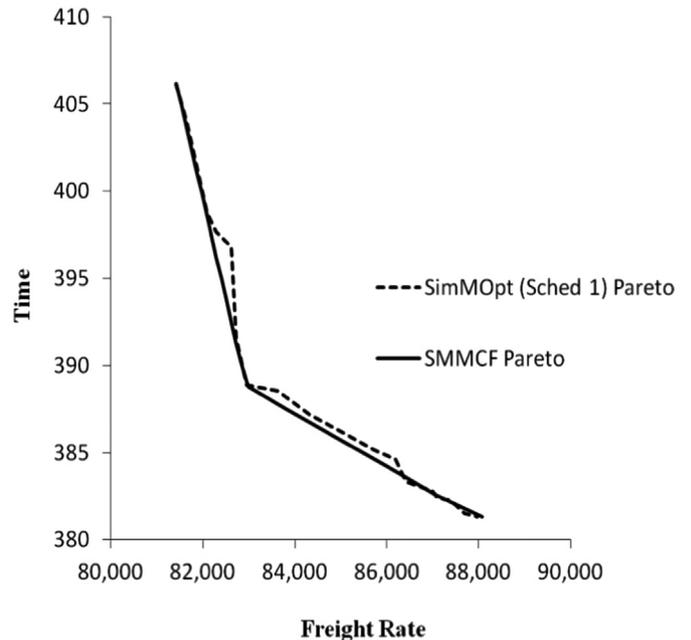


**Fig. 7.** Pareto frontier for Schedule 1. (Note: Delivery lead time: Hours and Cost: Monetary).

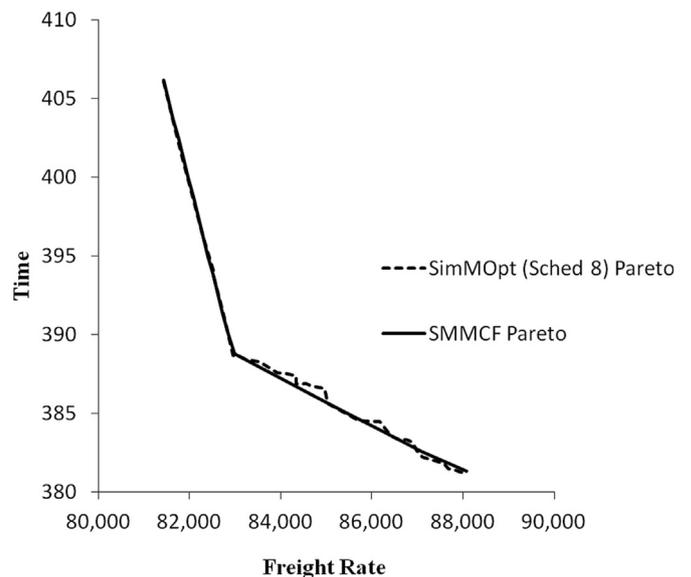


**Fig. 8.** Pareto frontier for Schedule 8. (Note: Delivery lead time: Hours and Cost: Monetary).

procedure and the solutions obtained with the SMMCF-Discrete. The respective values are 0.42 and 0.78. At a significance level 0.05, it can be concluded that the variances are equal. Similarly, the p-values for the t-test assuming equal variances are 0.064 and 0.15. At a significance level of 0.05, a significant difference between the two sets is not indicated. A Kolmogorov-Smirnov test was performed on the results for both objective functions. For the freight cost values, the test considered 15 intervals. The corresponding critical value is 0.133. For the time objective function, 10 intervals were considered. The corresponding critical value is



**Fig. 9.** Pareto frontier for the SMMCF-Discrete & SimMOpt solution procedure (Schedule 1). (Note: Lead time: Hours. Cost: Monetary).



**Fig. 10.** Pareto frontier for the SMMCF-Discrete & SimMOpt solution procedure (Schedule 8). (Note: Lead time: Hours. Cost: Monetary).

0.168. At a 0.05 significance level, it can be assumed an equal distribution for both samples.

For the case study, it can be demonstrated that the set of efficient solutions from the SimMOpt solution procedure that

considers the SMMCF-Continuous model are statistically equal to set of solutions for the SMMCF-Discrete model. The SMMCF-Discrete can be considered as the benchmark since this is a simplified version of the model. The optimality of the set of solutions found with the SMMCF-Discrete model is discussed by Bustos et al. [6].

## 9. Conclusions

The results when considering a deterministic MMCF problem (using expected values of the probability distributions) and when considering the SMMCF-Discrete model can be proved to be statistically different. This implies that solving problems where variability is reduced to expected values of distributions yield results that do not guarantee optimum or near-optimum solutions to MMCF problems with stochastic attributes on arcs.

Discretization of continuously distributed variables implies assuming oversimplifications that do not emulate the real system. This is aggravated when the discrete distributions consider a limited number of classes or when the more realistic continuous distribution show special behavior, such as the multimodal distributions.

The SMMCF-Discrete model is suitable when the variability of attributes of arcs can be modeled by using discrete probability distribution with few classes. The SMMCF-Discrete model was used to solve the case study when only three possible values were considered for the coefficients that represent the stochastic time attributes. The number of stochastic arcs in this case study is 12. Therefore, the number of scenarios considered by the SMMCF-Discrete model was  $3^{12}=531,441$ . When the number of stochastic arcs or the number of classes in the discrete probability distributions used to model the attributes increases, the number of scenarios increases greatly. Thus, the SMMCF-Discrete model becomes inefficient.

Using probability distributions and replicates of performance functions is a convenient alternative to model stochastic attributes. This paper shows the results from testing the efficiency of the SimMOpt solution procedure which has the ability to consider continuously distributed components in the SMMCF-Continuous problem. The results from the SimMOpt solution procedure are compared to the results from the SMMCF-Discrete model. The efficiency is assessed in terms of computational effort, percentage of exploration and convergence to the global-optimum.

The results from the SimMOpt solution procedure are statistically equal to optimal solutions found with the SMMCF-Discrete.

The schedule 1 outperformed the rest of the schedules (with only 3.64% of exploration and a CPU time of 1.36 hours) and was used to obtain these results. The computational time of schedule 1 is 8.29% of that required by a more exhaustive exploration from using schedule 8. Schedule 1 considers relatively low parameters when compared against some other tested schedules. Thus, optimality of the results might not be a consequence of exhaustive exploration. However, the SimMOpt solution procedure requires experimentation to find an adequate schedule for each particular implementation.

In conclusion, this paper presents a novel methodology that uses a hybrid multi-objective simulated annealing (MOSA) to solve MCF problems with continuously distributed stochastic attributes on arcs.

## 10. Future work

One feature of the SimMOpt solution procedure is that the parameters of the probability distributions can be easily adapted to change according to the time of the day. This would allow modeling additional intrinsic factors and its effects, such as the effects of peak hours on disruptive events. The effect on the quality of the solutions and computational times when considering this feature will be assessed in the future. A variant to this feature would be to change the parameters of the probability distribution according to the current flow of items to model saturation of arcs at the border ports of entry.

The convergence to global-optimum by using the proposed SimMOpt solution procedure was previously discussed. However, a thorough analysis of the results requires the evaluation of the computational effort required to obtain converging results from using population-based metaheuristics such as ant colony optimization, particle swarm optimization, and artificial bee colony.

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## Appendix A

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$b$	Index for the origin node, $b \in V$
$d$	Index for the destination node, $d \in V$
$C_{bd}^p$	Cost of using the arc from $b$ to $d$ according to criterion $p$ , $p = 1, 2, \dots, r$ , $(b, d) \in A$
$Y_{bd}$	Flow from node $b$ to node $d$ , $b, d \in V$
$h_b$	Capacity/Demand of node $b$ , $b \in V$
$E[C_{bd}^p]$	Expected value of cost of using the arc that goes from $b$ to $d$ according to criterion, $p = 1, 2, \dots, r$ , $(b, d) \in A$
$P_q$	Probability of scenario $q$ , $q = 1, 2, \dots, Q$
$C_{qbd}^p$	Cost of using arc that goes from $b$ to $d$ according to criterion $p$ in a $q$ scenario, $p=1, 2, \dots, r, (b, d) \in A$ , $q = 1, 2, \dots, Q$
$P[C_{qbd}^p]$	Probability of cost value resulting from using arc from $b$ to $d$ according to criterion $p$ in a $q$ scenario, $p=1, 2, \dots, r, (b, d) \in A$ , $q = 1, 2, \dots, Q$
$Y_{qbd}$	Flow from $b$ to $d$ in a $q$ scenario, $(b, d) \in A$ , $q = 1, 2, \dots, Q$ , Second-stage decision variables given $q$ scenario.

---

$$\min Z = \left( \sum_{(b,d) \in A} E[C_{bd}^p] Y_{bd} \right) + \left( \sum_q \sum_{(b,d) \in A} P_q [C_{qbd}^p] Y_{qbd} \right) \quad (2)$$

subject to:

$$\sum_{d=1}^l Y_{bd} - \sum_{d=1, \dots, l} Y_{db} + \sum_{d=1+1, \dots, \theta} Y_{qbd} - \sum_{d=1+1, \dots, \theta} Y_{qdb} = h_b \quad (3)$$

for  $b = 1, \dots, g$

$b = 1, \dots, l$  indexes of deterministic arcs

$b = 1, \dots, g$   $g \leq \beta$  for the second phase

$b = l+1, \dots, \beta$  indexes of stochastic arcs

$d = 1, \dots, l$  indexes of deterministic arcs

$d = 1, \dots, k$   $k \leq \theta$  for the second phase

$d = l+1, \dots, \theta$  indexes of stochastic arcs

$q = 1, \dots, Q$  scenarios

$Y_{bd} \geq 0$

$Y_{qbd} \geq 0$

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