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Strategic technology licensing in a supply chain☆

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ABSTRACT

This paper deals with R&D investment and technology licensing in a supply chain formed of an original equipment manufacturer (OEM) and a contract manufacturer (CM). The R&D is conducted by the CM and the OEM agrees to pay a share of the cost. At the R&D stage, we assume that there are some uncertainties both in terms of performance of the developed technology and market uncertainties. These uncertainties are resolved in the sales stage, as technology matures and information about consumers' preferences become available. Further, the OEM can license the technology to a third party and share the revenues with the CM. We characterize equilibrium pricing and licensing strategies in two scenarios, namely, the licensing decision is made before or after the uncertainties are resolved. A comparison of the two equilibria indicates that the OEM is indifferent between making the licensing decision in the first or the second stage in most cases. But when the market potential, competition intensity, royalty rate and revenue sharing rate are moderate, there exists a small region in the parameter space where the OEM prefers to make the licensing decision in Stage 2. Interestingly, we obtain that for a large region of the parameter space, the two partners have the same preferences in terms of licensing. It is also found that different probability distribution of stochastic technology efficiency results in different licensing strategies.

1. Introduction

Cooperation in research and development (R&D) is popular among technology-intensive firms pursuing time and cost reduction, better product design, and higher quality objectives (Albino, Carbonara, & Giannoccaro, 2007). Coordinated investment in R&D is often preferred to competitive investment because: (i) it achieves higher economics of scale and scope; (ii) it reduces risk and wasteful duplication of R&D efforts; and (iii) it leads to higher total investments, and therefore higher knowledge, as appropriability and free riding are no more an issue (Ge & Hu, 2008; Harabi, 2002).

Cooperation in R&D between firms can be horizontal or vertical. In the former, companies competing in the same product market coordinate their R&D efforts by, e.g., jointly investing in a research laboratory; see the seminal papers by d'Aspremont and Jacquemin (1988) and Kamien, Muller, and Zang (1992). Vertical cooperation refers to firms belonging to a supply chain, e.g., an upstream company and a downstream firm that collaborate in R&D to realize a collectively better outcome. For instance, Dell helped in 2002 its supplier Lexmark to enhance its printer technology with an innovative Dell-developed cartridge replenishment software, which eventually benefited both firms (Bhaskaran & Krishnan, 2009). Toyota Motor Co. Ltd has been cooperating with its suppliers to improve product performance since 1970.[1] Kisiel (2007) mentions that auto manufacturers have also involved suppliers during the production process, which allowed early detection of problems and the use of better components. Vonortas (1997) found that vertical cooperation dominated other types of cooperation in the US during the period 1985–1995, a result also obtained by Arranz and de Arroyabe (2008).

In this paper, a downstream firm (an original equipment manufacturer, OEM) pays part of the R&D investment cost incurred by an upstream firm (a contract manufacturer, CM) to develop a new technology or a new product. This cost-sharing mechanism is in line with what has been observed empirically. For instance,

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General Motors Corporation provides an annual budget of 200–400 million dollars for its Six Sigma Program, with a significant portion of which being dedicated to improve its suppliers' component quality (Snee & Hoerl, 2003).

Additionally to R&D cooperation, we assume that the OEM can license the new technology and share the revenues with the CM. Technology licensing means that an organization sells the rights to use its technology in the form of patents, processes and technical know-how to another firm for payment of royalties and/or other compensation (McDonald & Leahey, 1985). Technology licensing has for long been viewed by most high-tech enterprises as a quick and effective means for improving technology and innovation development (Benassi & Di Minin, 2009; Fosfuri, 2006; Lichtenthaler, 2011; Zhao, Chen, & Hong, 2014). Arora, Fosfuri, and Gambardella (2004) reports that over 15,000 licensing transactions in technology occurred worldwide already in the period 1985–1997 with a total value of over $320 billion. Technology licensing yields considerable additional revenues to firms, see, e.g., Arora, Fosfuri, and Rø nde (2013); Kim and Vonortas (2006); Lichtenthaler (2011); Zhao et al. (2014). For instance, IBM, Texas Instruments and Dow Chemical are known to collect hundreds of millions of dollars in annual licensing revenues (Arora et al., 2013; Lichtenthaler, 2011). It also yields non-monetary benefits such as enabling the licensor to establish industry standards or enter new markets (Gambardella, Giuri, & Luzzi, 2007; Lichtenthaler, 2011). However, there may be a negative side to licensing as licensees can develop products that end up competing with the licensor's products (Avagyan, Esteban-Bravo, & Vidal-Sanz, 2014; Bagchi & Mukherjee, 2014; Erkal & Minehart, 2014; Fosfuri, 2006; Kim, 2009). To illustrate, the company RCA that once licensed its color TV technology to a number of Japanese companies for originally exclusive exploitation in Japan ended up facing competition in the U.S. market from these firms that quickly assimilated RCA's technology (Hill, Hwang, & Kim, 1990). Consequently, the decision of licensing involves a trade-off between the revenues from licensing fees and the potential losses in sales revenues due to the competition from the licensee. Moreover, there is a dense literature that dealt with the design of licensing contracts, that is, the determination of fixed fees, royalties, and also about the coexistence of royalties and fixed fees; see, e.g., Bagchi and Mukherjee (2014); Rostoker (1984); Savva and Taneri (2014); Zhao et al. (2014). This work investigates the OEM's licensing strategy based on the royalty contract.

Rewards from technology investment are far from being fully predictable (Bhaskaran & Krishnan, 2009; Ma, Grubler, & Nakamori, 2009). In the R&D stage, the firm cannot be sure to fully succeed in effectively designing and efficiently manufacturing new products. On the top of this technology (or performance) uncertainty, the firm faces market uncertainty as, at least initially, it does not have reliable data about consumer's preference and demand (Bacon, Beckman, & Mowery, 1994; Bhattacharya, Krishnan, & Mahajan, 1998). These uncertainties, called as technology efficiency uncertainty for short, are resolved in the sales stage as the firm has access to more accurate information and the technology and market mature in this stage. This two-stage structure has also been adopted in, e.g., Xiao and Xu (2012) and Ge, Hu, and Xia (2014), however assuming away the above mentioned uncertainties and retaining different licensing and pricing contracts.

Overall, as for the high-tech industries, technology licensing brings about additional licensing revenue on one hand, but it also aggravates competition from the licensees. Thus, technology licensing is a strategic decision for the firm. In conjunction with the fact that technology and market uncertainties exist in the initial stage but are resolved in the second stage, the timing of the licensing decision-making is another issue worth studying. The timing of R&D collaboration decision has been of interest to practitioners and scholars, but most contributions, apart from Allain, Henry, and Kyle (2015), are concerned by investment timing (Harrison & Sunar, 2015; Perdikaki, Kostamis, & Swaminathan, 2016) and licensing timing (Crama, De Reyck, & Taneri, 2016). We enlarge the focus of this literature by attempting to respond to the following research questions:

1. What is the optimal timing for the OEM to make the licensing decision, and what is the optimal licensing strategy?
2. How does technology efficiency uncertainty impact the licensing strategy as well as investment and pricing decisions?
3. Under what conditions, licensing is a win-win situation for both channel members?

To address these questions, we consider a two-echelon supply chain playing a two-stage game, where an OEM and a CM jointly conduct technology investment to expand their market. The OEM makes the licensing decision and controls the share it pays of the CM's investment cost in R&D and its margin. The CM decides the investment level in R&D and its wholesale margin. Two scenarios are considered. In the first scenario, the licensing decision is made in the R&D stage, and therefore we must account for both technology and market uncertainties. In the second scenario, we suppose that the OEM can delay its licensing decision to the sales stage when the uncertainties are already resolved.

By determining and contrasting the strategies and outcomes in the two scenarios, we obtain the following insights: (1) In most cases, the OEM is indifferent between making the licensing decision in the first or the second stage. But when the market potential, competition intensity, royalty rate and revenue sharing rate are moderate, there exists a small region in the parameter space where the OEM prefers to make the licensing decision in Stage 2. Further, there also exist some parameter regions where making the licensing decision in Stage 1 is preferred. (2) If the uncertainty, or the technology competition intensity, or market potential is high, then the OEM does not license the technology. However, it does if its share in the licensing revenues, or royalty rate is high. (3) Technology efficiency uncertainty improves technology investment, expected retail margin and profits for both players, but has a non-monotonic impact in terms of investment sharing rate. (4) If the licensing option is made in the second stage, then no licensing will occur if the technology efficiency is high, and the reverse if it is low. (5) Different probability distributions of the stochastic technology efficiency may lead to different technology licensing strategies. Finally, we obtain that in a large region in the parameter space, the optimal licensing strategy is profit improving for both players.

The contribution of this paper is twofold. First, we propose a model to deal with licensing issues, including licensing strategy and timing of decision-making in the presence of uncertainties, which have been ignored in the licensing literature. We believe that our approach fills a gap in the research on technology licensing. Second, the theoretical results provide guidance for firms to make licensing decision, and the sensitivity analysis gives them hints on how to adjust their strategies in different market environments.

The rest of the paper is organized as follows. Section 2 describes the model. In Section 3, we derive the equilibria in the two scenarios and we compare them in Section 4. This work is extended in Section 5 and concluded in Section 6.

2. Model

Consider a two-stage game in a supply chain formed of an original equipment manufacturer and a contract manufacturer. In the first stage, the two players jointly invest in R&D to improve the OEM's product quality, which is sold in the market in the second stage. From now on, we shall use indifferently Stage 1 or R&D stage and Stage 2 or sales stage.

The outcome of R&D investment is uncertain both in terms of resulting technical performance and market acceptance (see, e.g., Bhattacharya et al. (1998); Oosterhuis, Van Der Vaart, and Molleman (2011), Perdikaki et al. (2016); Wang and Nguyen (2017)). As mentioned before, we assume that technology and market uncertainties are resolved in the sales stage, as the product's performance can be accurately tested by experts and consumers' defense groups, and the firm disposes of much better information about demand. Let the random variable Θ, following a two-point distribution, characterize technology and market uncertainties, with $P\{\Theta=\theta_2\}=\alpha, P\{\Theta=\theta_1\}=1-\alpha$ and $0\le\theta_1<\theta_2\le1$, $0\le\alpha\le1$. Hereafter, we call it as random technology efficiency for short. The corresponding mean and variance are $\mu=\alpha\theta_2+(1-\alpha)\theta_1$ and $\sigma^2=\alpha(1-\alpha)(\theta_2-\theta_1)^2$, respectively.

Denote by x the R&D activities and suppose that the cost is convex increasing and well approximated by the following simple quadratic function:

$$C(x)=x^2,$$

which is commonly used in the literature to characterize diminishing returns from investment (e.g., Caulkins, Feichtinger, and Grass (2017); d'Aspremont and Jacquemin (1988)). We assume that this total cost is shared by the two partners, with the OEM picking up a share ϕ and the CM the remaining $1-\phi$.

Denote by m the OEM's margin and by w the CM's wholesale margin. The retail price to consumer is then given by $p=m+w$. We suppose that the demand is decreasing in the retail price p, and increasing in the technology quality,[2] which is measured by Θx. If the OEM licenses the technology to another supplier operating in the same market, then it gets some revenues from the licensee, and loses some demand to it. Let Λ be the indicator function characterizing the technology licensing decision, that is,

$$\Lambda=\begin{cases}1, & \text{licensing,}\\ 0, & \text{no licensing.}\end{cases} \tag{1}$$

The supply chain's revenue from licensing is royalty based and given by $\pi\Lambda\Theta x$, where π is the royalty rate.[3] The total licensing revenue is shared between the supply chain's members, with the OEM getting the exogenously given percentage τ and the CM the rest, i.e., $1-\tau$. This revenue sharing mechanism is widely used in supply chains, see, e.g., Cachon and Lariviere (2005); Cai, Hu, and Tadikamalla (2017), and in particular in the Dell-Lexmark example mentioned in the introduction.

On the negative side of licensing, some consumers will buy from the licensee instead of purchasing the product from the OEM. To measure this loss, denote by M the market potential in the absence of any technology investment. By conducting R&D, the supply chain expects to increase this market potential by $\vartheta\Theta x$, where ϑ is a nonnegative scaling parameter. Denote by δ the competition intensity or supplier substitution rate ($0<\delta<1$) such that the licensee's sales could be measured by $\delta\vartheta\Lambda\Theta x$. Consequently, the market size is given by $M+\vartheta\Theta x-\delta\vartheta\Lambda\Theta x$. To keep the model parsimonious, we normalize from now on ϑ to one (Shum, Tong, & Xiao, 2016). We assume the demand function to be linear, which is common in the economics and management science literature and given by

$$D=M+\Theta x(1-\delta\Lambda)-(m+w). \tag{2}$$

[2] We use indifferently the terms technology quality, product performance and (simply) product's quality.

[3] Rostoker (1984) reports that 39% of licensing cases are based on royalty contract alone, 13% are fixed-fee alone, and 46% combines royalty and fixed-fee together.

Table 1
Notations.

Decision variables			
ϕ:	cost sharing rate	x:	technology investment
m:	retail margin	w:	wholesale price
Probability distribution parameter			
Θ:	random technology efficiency	θ_1,θ_2:	realizations of Θ
α:	$P\{\Theta=\theta_2\}$	μ,σ^2:	mean and variance of Θ, respectively
Demand parameters			
M:	market potential	δ:	competition intensity
Licensing parameters			
τ:	revenue sharing rate	π:	royalty rate
Nomenclature			
L:	licensing	NL:	no licensing
Z:	depending on realizations		

Note that in the above equation, the marginal impact of retail price on demand has been normalized to one. Also, it is well known that technology investment improves the product quality and thus increases consumers' willingness to pay for the product (Wang & Shin, 2015). One example is Caroma dual-flush with its half-flush and full-flush technology that reduces water usage by up to 67 percent compared to traditional toilets. It appeals to consumers because they will actually save money on energy and water bills over the long term, and this increases demand (Yenipazarli, 2017). Further, without loss of generality, we normalize the unit production cost to zero. All the notations are listed in Table 1.

Assuming profit maximization behavior, the objective functions of the OEM and CM are then given by

$$\Pi_o=mD+\tau\pi\Lambda\Theta x-\phi x^2, \tag{3}$$

$$\Pi_c=wD+(1-\tau)\pi\Lambda\Theta x-(1-\phi)x^2. \tag{4}$$

We formulate the problem as a two-stage game, with each stage being played à la Stackelberg with the OEM acting as leader and the CM as follower.

Remark 1. In the supply chain and marketing channels literature, the typical assumption is that the manufacturer determines first its wholesale price and next the retailer announces its retail price (see, e.g., the survey in Ingene, Taboubi, & Zaccour (2012)). Still, the sequence can be reversed for some reasons, e.g., a powerful retailer. Here, we have in mind the example of cooperation between Apple (the OEM) and Foxconn (the CM). Apple acts as leader and decides the quality and retail price of products, and Foxconn, as follower, is responsible of assembling mobile phones according to Apple's request, and charges the wholesale price of component.

A formal description of the two scenarios follows.

Licensing decision in stage 1: In this scenario, the optimization problems in the two stages are defined as follows:

Stage 1: The OEM announces licensing decision (Λ) and investment cost sharing rate ϕ. The CM then determines technology investment x. Each player maximizes its individual expected profits, that is,

$$\max_{\Lambda,\phi}\ E[\Pi_o]=E[mD+\tau\pi\Lambda\Theta x-\phi x^2],$$
$$\max_{x}\ E[\Pi_c]=E[wD+(1-\tau)\pi\Lambda\Theta x-(1-\phi)x^2]. \tag{5}$$

Stage 2: The technology efficiency realizes as θ_2 with probability α and θ_1 with probability $1-\alpha$. Knowing this, the OEM determines first its retail margin m, and next the CM sets its wholesale price w. The optimization problems are given by

$$\max_{m} \ \Pi_o = md + \tau\pi\Lambda\theta x - \phi x^2,$$
$$\max_{w} \ \Pi_c = wd + (1-\tau)\pi\Lambda\theta x - (1-\phi)x^2, \tag{6}$$

where θ and d denote the realizations of the random technology efficiency and the random demand, respectively.

Licensing decision in stage 2: In this scenario, the optimization problems in the two stages are defined as follows:

Stage 1: The OEM decides the cost sharing rate ϕ, and the CM sets the technology investment x afterwards. The optimization problems are

$$\max_{\phi} \ E[\Pi_o] = E[mD + \tau\pi\Lambda\Theta x - \phi x^2],$$
$$\max_{x} \ E[\Pi_c] = E[wD + (1-\tau)\pi\Lambda\Theta x - (1-\phi)x^2]. \tag{7}$$

Stage 2: The technology efficiency realizes as θ_2 with probability α and θ_1 with probability $1-\alpha$. Then, the OEM decides whether or not to license the technology, i.e., chooses Λ, and the retail margin m. Next, the CM determines the wholesale price w. The optimization problems are given by

$$\max_{\Lambda, m} \ \Pi_o = md + \tau\pi\Lambda\theta x - \phi x^2,$$
$$\max_{w} \ \Pi_c = wd + (1-\tau)\pi\Lambda\theta x - (1-\phi)x^2. \tag{8}$$

To save on notation, we introduce the auxiliary variable $\xi := \mu^2 + \sigma^2 = \alpha\theta_2^2 + (1-\alpha)\theta_1^2$.

Note that under our assumption $0 \le \theta_1 < \theta_2 \le 1$, we clearly have $\xi \le 1$.

3. Equilibria

In this section, we characterize the equilibria in both scenarios. For each of them, we verify under what conditions licensing is optimal to the OEM and eventually if this suits the CM.

3.1. Licensing decision in stage 1

The following proposition characterizes the subgame-perfect equilibrium strategies and outcomes for a given Λ:

Proposition 1. *Assuming an interior solution and if the technology licensing option is made in Stage 1, then the equilibrium strategies for a given Λ are given by*

$$\phi = \frac{8\Lambda\pi(\xi(5-7\tau)(1-\delta\Lambda)^2 - 16(1-3\tau)) + M(1-\delta\Lambda)(\xi(1-\delta\Lambda)^2+48)}{16(5M(1-\delta\Lambda)+8\Lambda\pi(1+\tau))}, \tag{9}$$

$$x = \frac{\mu(5M(1-\delta\Lambda)+8\Lambda\pi(1+\tau))}{2(16-3\xi(1-\delta\Lambda)^2)}, \tag{10}$$

$$w = \frac{8\Lambda\mu\theta\pi(1+\tau)(1-\delta\Lambda)+M(5\mu\theta-6\xi)(1-\delta\Lambda)^2+32M}{8(16-3\xi(1-\delta\Lambda)^2)}, \tag{11}$$

$$m = 2w, \tag{12}$$

and the expected profits by

$$E[\Pi_o] = \frac{16\Lambda\pi\mu^2(1+\tau)(5M(1-\delta\Lambda)+4\Lambda\pi(1+\tau))+M^2(25\mu^2-24\xi)(1-\delta\Lambda)^2+128M^2}{64(16-3\xi(1-\delta\Lambda)^2)}, \tag{13}$$

Table 2
Sensitivity analysis in scenario 1 for $\Lambda = 1$.

	ϕ	x	m	w	$E[\Pi_o]$	$E[\Pi_c]$
M	?	+	+	+	+	+
δ	?	–	–	–	–	–
π	?	+	+	+	+	+
τ	+	+	+	+	+	–
μ	?	+	+	+	+	+
σ^2	?	+	+	+	+	+

$$E[\Pi_c] = \frac{16\Lambda\pi\mu^2(M(3-2\tau)(1-\delta\Lambda)+4\Lambda\pi(1-\tau^2))+M^2(5\mu^2-6\xi)(1-\delta\Lambda)^2+32M^2}{32(16-3\xi(1-\delta\Lambda)^2)}. \tag{14}$$

Proof. See Appendix A. □

The results in the above proposition call for the following comments. First, the proposition is stated under the assumption of an interior solution, that is, $0<\phi<1$, and $x, m, w > 0$. It is straightforward to verify that x, m and w are strictly positive. For $\Lambda = 0$, we have

$$\phi|_{\Lambda=0} = \frac{\xi + 48}{80},$$

which clearly shows that $0 < \phi|_{\Lambda=0} < 1$. For $\Lambda = 1$, we verify that $\phi < 1$ for all parameter values, and obtain the following corollary.

Corollary 1. *If $\tau > \bar{\tau}$, the OEM supports CM's technology investment, i.e., $\phi > 0$; otherwise, it is willing to participate in the investment only if $\pi < \underline{\pi}$, where $\bar{\tau} = \frac{16-5\xi(1-\delta)^2}{48-7\xi(1-\delta)^2}$, $\underline{\pi} = -\frac{M}{8A}(1-\delta)(\xi(1-\delta)^2 + 48)$ and $A = \xi(5-7\tau)(1-\delta)^2 - 16(1-3\tau)$.*

Proof. $\phi|_{\Lambda=1}$ can be rewritten as $\phi|_{\Lambda=1} = \frac{8\pi A + M(1-\delta)(\xi(1-\delta)^2+48)}{16(5M(1-\delta)+8\pi(1+\tau))}$. Clearly, $\phi|_{\Lambda=1} > 0$ if $A > 0$. Further, A is increasing in τ, i.e., $\frac{\partial A}{\partial \tau} > 0$, and $A|_{\tau=0} = 5\xi(1-\delta)^2 - 16, A|_{\tau=1} = 2(16 - \xi(1-\delta)^2) > 0$. If $A|_{\tau=0} > 0$, $A > 0$ for $\tau \in (0, 1)$, then $\phi > 0$. If $A|_{\tau=0} < 0$, there exists a threshold $\bar{\tau}$, below which $A < 0$, then $\phi > 0$ only if $\pi < \underline{\pi}$, above which, $A > 0$, then $\phi > 0$. □

It can be inferred that $\bar{\tau} < 1/3$. This means that when $\tau > 1/3$, the OEM is willing to conduct R&D investment with the CM. However, when $\tau < 1/3$, the OEM may support CM's investment only if π is not too large, i.e., $\pi < \underline{\pi}$; otherwise, it may not cooperate with the CM. It is natural that the OEM is actively engaged in investment cooperation if the revenue sharing rate is attractive enough, i.e., $\tau > 1/3$. This result coincides with the findings of Jørgensen, Taboubi, and Zaccour (2003) and Buratto, Grosset, and Viscolani (2007) who states that the leader will participate in a cooperative marketing program only if its revenue sharing rate is larger than 1/3. Further, to stimulate cooperator's innovation, some firms share a larger portion (50–70%) of the total revenue to their partners (Bhaskaran & Krishnan, 2009). But when the revenue sharing is relatively low, i.e., $\tau < 1/3$, and the royalty rate is relatively large, the CM obtains more licensing revenue and becomes interested in increasing its investment, which in turn inevitably incurs high investment cost for the OEM. Thus, the OEM will not participate in the R&D cooperation when facing a low revenue sharing and a large royalty rate.

Second, we observe that the leader's margin is twice the follower's margin, that is, $m = 2w$. This result is classical in the marketing channels literature, see, e.g., Ingene et al. (2012); Martín-Herrán and Taboubi (2015). Finally, a sensitivity analysis of strategies and expected payoffs for $\Lambda = 1$ leads to the results in Table 2. The computational details are in the Appendix A.

In Table 2, "+" and "–" stand for the parameter exerting a positive and negative impact on equilibria, respectively, and "?"

means that the impact is non-monotonic, that is, the parameter may have positive or negative impact on the equilibria depending on different values. As it can be seen from Table 2, all equilibrium strategies, except the cost-sharing rate ϕ, increase with M, π, τ, μ and σ^2, but decrease with δ. Specifically, ϕ is increasing in τ, but its variations are non-monotonic with respect to M, δ, π, μ and σ^2. Intuitively, large market potential M attracts high investment, enabling the channel members to charge a high margin, which leads to high profits. In the face of a high competition intensity δ, the CM has to decrease investment to reduce demand loss, resulting in low margins and eventually less profits. The larger the royalty π or the share of licensing revenues τ that the OEM keeps, the larger the incentive to invest in R&D as high licensing revenue, $\tau\pi\theta x$, are generated and such high profit can offset the increased investment cost. A high value of μ means that the demand is greatly boosted by the investment, which induces more investment and enables the channel members to set a high retail margin and wholesale price. One interesting result is that high variance σ^2 also triggers high investment, retail margin and wholesale price, which eventually creates high payoffs for the channel members. Large variance means that the chances of either a large or small technology efficiency are higher. Note that the channel members are always able to use investment and pricing as levers to retain high profit margin and demand in the case of a lower technology efficiency. Thus, the equilibrium investment, retail margin, wholesale price and expected profits are enhanced by a high variance. This is similar to the results of Shum et al. (2016) who also finds that high uncertainty in cost reduction always benefits the firm when studying the impact of uncertain cost reduction and strategic customer behavior on a firm's pricing strategy and profit.

We next turn to the impacts of M, δ, π, μ, σ^2 on ϕ. The impact of M and π on the equilibrium cost sharing rate is described in the following corollary.

Corollary 2. *If $\tau > 2/3$, the OEM will provide more investment support to the CM when facing a low market potential or a high royalty rate. Otherwise, it will reduce investment cost sharing.*

From Table 2, we see that the relationship between M and π on the contribution of the OEM to the investment cost ϕ is ambiguous. The above corollary is clarifying this relationship by introducing in the picture the share of the OEM in the revenues, which we know that it has a positive impact on the investment (see Table 2). When this share is large, i.e., $\tau > 2/3$, the OEM can afford to boost its contribution to the R&D cost. However, if its share in the revenue is not large enough, i.e., $\tau < 2/3$, the OEM is willing to incur more investment cost to stimulate the CM's investment if the market potential is high so that the revenues from licensing are still attractive. Similarly, when τ is large enough ($\tau > 2/3$), a high royalty rate is an incentive for the OEM to boost CM's investment effort because it leads to a larger market and more revenues from licensing. The reverse is taking place when τ is not sufficiently large.

The derivative of ϕ with respect to δ (see Appendix A.2) indicates that ϕ is decreasing with δ if $\tau < 2/3$. The reason behind this is that with a relatively low revenue sharing rate, i.e., $\tau < 2/3$, when δ increases, the equilibrium values of x, m, w decrease. This leads to a low demand and profit, along with a low licensing fee sharing. Therefore, it is intuitive that the OEM offers a low cost support. However, if $\tau > 2/3$, ϕ is increasing in δ when π is relatively large. In this situation, although low investment and margins are generated by a high δ, the OEM would be still willing to provide a high cost sharing to stimulate the CM's investment such that it can benefit from the high licensing fee sharing in the presence of a high royalty rate and revenue sharing rate.

Additionally, as seen from the derivatives of ϕ with respect to μ and σ^2 (see Appendix A.2) that if $\tau < 5/7$, ϕ increases with μ and σ^2; otherwise, ϕ may decrease with μ and σ^2 if π is relatively large. Facing a relatively low revenue sharing rate ($\tau < 5/7$), when mean and variance increase, the OEM still prefers to share more cost to boost CM's investment because it will generate more demand and eventually profit for the OEM. On the contrary, with a high enough revenue rate ($\tau > 5/7$) and a high royalty rate, ϕ is greatly lifted by τ (see Table 2). High investment is generated by high mean and variance, leading to a fast increase in the corresponding cost, which pushes the OEM to reduce the cost sharing rate to avoid this high cost. On the other hand, a high royalty rate helps the OEM maintaining a high licensing revenue.

Table 2 also shows that the expected profit of OEM is positively impacted by π, τ, μ and σ^2, while CM's expected profit experiences a positive effect of π, μ and σ^2 and a negative one from τ. Both channel members benefit from a high royalty, and it is natural that the OEM is better off with a high revenue sharing, but the CM is worse off. Mean and variance lift up the profits of OEM and CM, indicating that large uncertainty in technology efficiency benefits both channel members.

Comparing the expected profits with and without technology licensing yields the OEM's technology licensing strategy.

Proposition 2. *When made in the first stage, the equilibrium technology licensing strategy is defined by*

$$\Lambda = \begin{cases} 1, & \pi \geq \bar{\pi}_o, \\ 0, & \pi < \bar{\pi}_o, \end{cases} \tag{15}$$

where

$$\bar{\pi}_o = \frac{5M(\sqrt{9\xi^2(1-\delta)^2 - 48\xi(\delta^2 - 2\delta + 2) + 256} - (16-3\xi)(1-\delta))}{8(16-3\xi)(1+\tau)}. \tag{16}$$

Proof. Use (13) to compute

$$E[\Pi_o]|_{\Lambda=1} - E[\Pi_o]|_{\Lambda=0}$$
$$= \frac{16\pi\mu^2(1+\tau)(5M(1-\delta) + 4\pi(1+\tau)) + M^2(25\mu^2 - 24\xi)(1-\delta)^2 + 128M^2}{64(16-3\xi(1-\delta)^2)}$$
$$- \frac{M^2(25\mu^2 - 24\xi) + 128M^2}{64(16-3\xi)}.$$

It is straightforward to verify that

$$E[\Pi_o]|_{\Lambda=1} - E[\Pi_o]|_{\Lambda=0} \geq 0 \Leftrightarrow \pi \geq \bar{\pi}_o.$$

□

The above proposition shows, not unexpectedly, that the decision of licensing depends on all parameter values. In short, the main message is that licensing requires a sufficiently high value of π to offset the profit losses from the decreased demand for the OEM when it opts for licensing. Further, the threshold $\bar{\pi}_o$ increases in mean μ, variance σ^2, competition intensity δ, and market potential M, but decreases in revenue sharing rate τ. The increase in μ and σ^2 means that higher technology efficiency and volatility are deterrent for licensing. One interpretation is that high mean and variance greatly pull up investment, and then demand, but it may lead to a large demand loss if the OEM licenses the technology. The corresponding profit is lower than that without licensing because in the latter case a higher profit is created by a high investment. Thus, the OEM prefers to give up technology licensing in this situation. The comparative static analysis of $\bar{\pi}_o$ with respect to τ, δ and M shows that the licensing region enlarges with τ and shrinks with δ and M. Clearly, high external revenue drives the OEM to choose licensing, and extensive technology competition reduces this incentive. As for the case without licensing, the OEM can reap considerable profits from the increased investment when the market potential is large. This is more attractive relative to the revenue obtained from technology licensing. Thus, the OEM prefers no licensing.

The optimal technology licensing strategy and the corresponding equilibria are obtained by combining the results in Propositions 1 and 2.

3.2. Licensing decision in stage 2

As in the previous scenario, we start by solving the second-stage problem in (8). Recall that in this scenario, the technology efficiency is θ_2 with probability α and θ_1 with probability $1-\alpha$. Next, the OEM decides whether to license the technology, i.e., chooses Λ, and the retail margin m. After that the CM determines the wholesale price w. The following lemma characterizes the second-stage equilibrium.

Lemma 1. *If the technology licensing decision is made in Stage 2, then the equilibrium strategies are as follows:*

$$\Lambda^* = \begin{cases} 1, & x < \bar{x}, \\ 0, & x \geq \bar{x}, \end{cases} \tag{17}$$

$$\mathbf{m}^*(\theta) = \begin{cases} \frac{1}{2}(\theta x(1-\delta)+M), & x < \bar{x}, \\ \frac{1}{2}(\theta x+M), & x \geq \bar{x}, \end{cases} \tag{18}$$

$$\mathbf{w}^*(\theta) = \begin{cases} \frac{1}{4}(\theta x(1-\delta)+M), & x < \bar{x}, \\ \frac{1}{4}(\theta x+M), & x \geq \bar{x}, \end{cases} \tag{19}$$

where

$$\bar{x} = \frac{2(4\tau\pi - M\delta)}{\delta\theta(2-\delta)}.$$

Proof. See Appendix A. □

Lemma 1 shows that the technology licensing option is contingent to the investment decision x made in the first stage. The lemma shows that if x is larger than a threshold $\bar{x}$, which we assume to be positive, i.e., $4\tau\pi > M\delta$, then the OEM would not license the technology. If $4\tau\pi < M\delta$, scenario 2 coincides with scenario 1 without licensing. One way of summarizing the result regarding the licensing decision is by stating that the OEM will license a technology that does not require a high investment. This result can be explained as follows: although high technology investment will bring high licensing revenue, it inevitably causes a high demand loss and a high investment cost. Since the latter dominates the former, the OEM is better-off not licensing. However, if the investment is low, technology licensing creates additional revenue on one hand, and induces low demand loss and low investment cost on the other hand, which can be offset by the increased licensing revenue. As such, the OEM prefers technology licensing when the CM's investment is low. Note that the threshold is increasing in the revenue sharing rate τ and in the marginal licensing revenue π, but is decreasing in the competition intensity δ and in the stochastic technology efficiency. Additionally, the licensing region when $\Theta = \theta_2$ is smaller than when $\Theta = \theta_1$, that is, the higher the technology efficiency, the smaller is the licensing region. This is because the market demand is greatly expanded by a high technology efficiency. Technology licensing in this situation will lead to a large demand loss, which prevents the OEM from licensing the technology. Further, as in the previous scenario, the equilibrium retail margin is twice the wholesale price. However, as expected, these strategies depend here on the realization of the stochastic technology efficiency and not on its statistics.

The first stage is played sequentially, with the OEM (the leader) announcing first the investment sharing rate ϕ and next the CM (the follower) decides on the investment x. As usual, we start by first determining the follower's reaction function. Given the OEM's investment sharing rate ϕ, and taking the second-stage responses into account, the CM's problem is to determine the technology investment x to maximize its expected profit, that is,

$$\max_{x} \; E[\Pi_{c_1}] = E[w^*(\Theta)D + (1-\tau)\pi\Lambda^*\Theta x - (1-\phi)x^2]. \tag{20}$$

Accounting for the results in the second stage, the above expected payoff can then be rewritten as follows:

$$E[\Pi_{c_1}] = \begin{cases} \frac{1}{16}\xi x^2(1-\delta)^2 + \frac{M}{8}\mu x(1-\delta) \\ +\pi\mu x(1-\tau) - (1-\phi)x^2 + \frac{M^2}{16}, & x < \bar{x}|_{\theta=\theta_2}, \\ \frac{\alpha}{16}(M+\theta_2 x)^2 - (1-\phi)x^2 \\ +(1-\alpha)\left(\frac{1}{16}(M+\theta_1 x(1-\delta))^2 + \pi\theta_1 x(1-\tau)\right), & \bar{x}|_{\theta=\theta_2} \leq x \leq \bar{x}|_{\theta=\theta_1}, \\ \frac{1}{16}\xi x^2 + \frac{M}{8}\mu x - (1-\phi)x^2 + \frac{M^2}{16}, & x > \bar{x}|_{\theta=\theta_1}. \end{cases} \tag{21}$$

The CM's first-stage investment response is given below by solving the optimization problem (20).

Lemma 2. *The CM's first-stage best investment response is*

$$x^* = \begin{cases} \frac{\mu(8\pi(1-\tau)+M(1-\delta))}{16(1-\phi)-\xi(1-\delta)^2}, & 0 < \phi < \phi_1, \\ \frac{M\theta_1(1-\alpha)(1-\delta)+8\pi\theta_1(1-\tau)(1-\alpha)+M\alpha\theta_2}{16(1-\phi)-\theta_1^2(1-\alpha)(1-\delta)^2-\alpha\theta_2^2}, & \phi_1 \leq \phi \leq \phi_2, \\ \frac{M\mu}{16(1-\phi)-\xi}, & \phi_2 < \phi < 1, \end{cases} \tag{22}$$

where

$$\phi_1 = 1 - \frac{\mu\delta(2-\delta)(8\pi(1-\tau)+M(1-\delta))}{32(4\tau\pi-\delta)} - \frac{\xi(1-\delta)^2}{16},$$

$$\phi_2 = 1 - \frac{M\delta\mu\theta_1(2-\delta)+2\xi(4\tau\pi-M\delta)}{32(4\tau\pi-M\delta)}. \tag{23}$$

Proof. See Appendix A. □

Lemma 2 shows that the investment in R&D has three different values depending on cost sharing rate ϕ. Moreover, if there is a low or high cost-sharing rate, i.e., $0<\phi<\phi_1$ or $\phi_2<\phi<1$, the optimal investment depends on the mean and variance. Specifically, it increases with μ and σ^2. When the cost-sharing rate is moderate, that is, $\phi_1 \leq \phi \leq \phi_2$, then the investment depends on the realizations θ_1, θ_2, and is increasing in both of them.

Incorporating the CM's best responses given in Lemma 2 in the OEM's objective function, we then need to solve the following optimization problem:

$$\max_{\phi} \; E[\Pi_{o_1}] = E[m^*(\Theta)D + \tau\pi\Lambda^*\Theta x^* - \phi x^{*2}], \tag{24}$$

where

$$E[\Pi_{o_1}] = \begin{cases} \frac{\mu^2(8\pi(1-\tau)+M(1-\delta))}{8(16(1-\phi)-\xi(1-\delta)^2)^2}(8\pi\xi(1-\delta)^2(1-2\tau) \\ +M(1-\delta)(8(4-5\phi)-\xi(1-\delta)^2) \\ -64\pi(\phi(1+\tau)-2\tau)) + \frac{M^2}{8}, & 0<\phi<\phi_1, \\ \frac{1}{8(16(1-\phi)-(1-\alpha)(1-\delta)^2\theta_1^2-\alpha\theta_2^2)^2} \\ \times((1-\delta)^2(1-\alpha)^2\theta_1^4(8\pi(1-\alpha)(8\pi(1-2\tau)(1-\tau) \\ -M\tau(1-\delta)) + M^2\alpha(1-\delta)^2) \\ -2\alpha M\theta_2(1-\delta)^2(1-\alpha)^2(4\pi\tau+M(1-\delta))\theta_1^3 \\ +(1-\alpha)(M^2(1-\delta)^2(\alpha\theta_2^2+8\phi(5\alpha-1)-32\alpha) \\ -8M\pi(1-\alpha)(1-\delta)(\alpha\tau\theta_2^2+16\phi(3-2\tau) \\ -16(2-\tau)) - 64\pi^2(1-\tau)(1-\alpha)(\alpha\theta_2^2(2\tau-1) \\ +8\phi(1+\tau)-16\tau)) \\ \theta_1^2 + 2\alpha M\theta_1\theta_2(1-\alpha)(64\pi(2-\phi(3-2\tau)) \\ -\alpha\theta_2^2(4\pi\tau+M(1-\delta)) \\ -8M(5\phi-4)(1-\delta)) \\ -M^2(32(1-\phi)(\alpha\theta_2^2-8(1-\phi)) \\ -\alpha^2\theta_2^2(\theta_2^2(1-\alpha) \\ +8(4-5\phi)))) & \phi_1 \leq \phi \leq \phi_2, \\ \frac{M^2(32(1-\phi)(8(1-\phi)-\xi)+\mu^2(8(4-5\phi)-\xi)+\xi^2)}{8(16(1-\phi)-\xi)^2}, & \phi_2<\phi<1, \end{cases} \tag{25}$$

Solving the OEM's optimization problem in (24) yields the following first-stage optimal solution.

Proposition 3. *The OEM's first-stage optimal investment cost sharing rate is given by*

$$\phi^* = \begin{cases} \frac{\xi+48}{80} & \pi < \pi_2, \\ \frac{1}{16\theta_1(1-\alpha)(5M(1-\delta)+8\pi(1+\tau))+5\alpha\theta_2 M} \\ ((1-\delta)^2(1-\alpha)^2(8\pi(5-7\tau)+M(1-\delta))\theta_1^3 \\ +\alpha\theta_2 M(1-\delta)^2(1-\alpha)\theta_1^2 \\ +(1-\alpha)(8\alpha\pi\theta_2^2(5-7\tau) \\ +128\pi(3\tau-1)+M(1-\delta)(\alpha\theta_2^2+48))\theta_1 & \pi_2 \le \pi \le \pi_1, \\ +(\alpha\theta_2^2+48)\alpha\theta_2 M), \\ \frac{8\pi(\xi(5-7\tau)(1-\delta)^2-16(1-3\tau))+(1-\delta)M(\xi(1-\delta)^2+48)}{16(5M(1-\delta)+8\pi(1+\tau))}, & \pi_1 < \pi. \end{cases} \tag{26}$$

where

$$\pi_1 = \frac{M\delta(5\mu\theta_2(1-\delta)(2-\delta)-12\xi(1-\delta)^2+64)}{8(32\tau-6\xi\tau(1-\delta)^2-\mu\theta_2\delta(2-\delta)(1+\tau))}, \tag{27}$$

$$\pi_2 = \frac{M\delta(5\mu\theta_1(2-\delta)+64-12\xi)}{16\tau(16-3\xi)}. \tag{28}$$

Proof. See Appendix A. □

Proposition 3 shows that if the royalty rate from licensing is sufficiently low, i.e., $\pi < \pi_2$, then the equilibrium cost-sharing rate is the same as the one obtained in Scenario 1 without licensing, and it is always strictly larger than 0.6. If π is high enough, that is, $\pi > \pi_1$, then the equilibrium cost-sharing rate corresponds to the one in Scenario 1 with licensing. When the external margin is moderate, that is, $\pi_2 \le \pi \le \pi_1$, then the equilibrium cost-sharing rate depends on the realizations θ_1 and θ_2. The equilibrium strategies are obtained by combining the results in Lemmas 1, 2 and Proposition 3, and the corresponding equilibrium payoffs are denoted as $E[\Pi_o^*], E[\Pi_c^*]$ for OEM an CM, respectively.

4. Comparison of the two scenarios

In this section, we compare the equilibrium payoffs obtained in the two scenarios. Since the decision of licensing is taken by the OEM and it is the leader of the game, we first check when licensing is profitable to the OEM. Second, we see if this decision suits the CM or not, keeping in mind that, as a follower, it cannot change it. As one could easily expect, the results depend on the parameter values and could be presented in different ways. However, we believe that the most comprehensive approach is to focus on the royalty parameter π. We have already defined three threshold values, namely, $\bar{\pi}_o, \pi_1, \pi_2$ in (16), (27) and (28), respectively. Further, we introduce the following thresholds for the OEM

$\tilde{\pi}_o$: value such that $E[\Pi_o]|_{\Lambda=1} = E[\Pi_o^*]$,

$\hat{\pi}_o$: value such that $E[\Pi_o]|_{\Lambda=0} = E[\Pi_o^*]$,

and the following values for the CM:

$$\bar{\pi}_c = \frac{M}{8(16-3\xi)(1-\tau^2)}((16-3\xi)(1-\delta)(2\tau-3) + \sqrt{A_1+A_2-A_3}),$$

$\tilde{\pi}_c$: value such that $E[\Pi_c]|_{\Lambda=1} = E[\Pi_c^*]$,

$\hat{\pi}_c$: value such that $E[\Pi_c]|_{\Lambda=0} = E[\Pi_c^*]$,

where

$$A_1 = 3(1-\delta)^2(3\xi^2(2\tau-3)^2+128\tau(3\xi-8)),$$

$$A_2 = 16\delta(2-\delta)(3\tau^2(13\xi-48)+39\xi-64),$$

$$A_3 = 32(3\xi-8)(9+4\tau^2).$$

The following proposition characterizes the conditions under which the OEM licenses or not the technology and the decision stage.

Proposition 4. *The optimal technology licensing strategy depends on external royalty rate as follows:*

- ***R1**: If $\bar{\pi}_o < \pi < \pi_2$, or $\pi > \max\{\pi_1, \bar{\pi}_o\}$, then the OEM licenses its technology, and there is no difference if this decision is made in Stage 1 or 2.*
- ***R2**: If $\pi < \min\{\pi_2, \bar{\pi}_o\}$, or $\pi_1 < \pi < \bar{\pi}_o$, then the OEM does not license its technology, and there is no difference if this decision is made in Stage 1 or 2.*
- ***R3**: If $\max\{\pi_2, \tilde{\pi}_o\} < \pi < \pi_1$, then the OEM licenses its technology in Stage 1.*
- ***R4**: If $\max\{\pi_2, \hat{\pi}_o\} < \pi < \min\{\pi_1, \tilde{\pi}_o\}$, then the licensing decision is made in Stage 2, and it is licensing if $\Theta = \theta_1$, and no licensing if $\Theta = \theta_2$.*
- ***R5**: If $\pi_2 < \pi < \min\{\pi_1, \hat{\pi}_o\}$, then the OEM does not license its technology and makes this decision in Stage 1.*

Proof. Straightforward and based on comparing profit values. □

The above proposition is based on comparing the expected profits of the OEM. We see that we have five different regions in the parameter space. In the first two regions (R1 and R2), the timing of the licensing decision does not matter. Regions R3 and R5 characterize the cases where the decision is made in Stage 1. Finally, R4 gives the values of π where the licensing decision is made in Stage 2. Here, a low-realization of technology efficiency (θ_1) leads the OEM to license the technology, whereas high-realization value (θ_2) is a disincentive for licensing. The reason is that with a high technology efficiency θ_2, the market is greatly expanded, and the OEM prefers no licensing to avoid the large demand loss from technology licensing. When facing the low-realization θ_1, the OEM prefers to license the technology to gain more external revenue.

As the ordering of the different values showing up in the proposition depends on the other parameter values, it is hard to clearly interpret the results. In the numerical subsection, we will provide a figure that will allow to visualize at a glance the result.

The next proposition characterizes the preferences of the two channel members in terms of licensing decision (licensing or not) and its timing (Stage 1 or Stage 2). This proposition, which is based on straightforward payoffs comparisons, is stated for completeness. The results are by no way easy to interpret, and a visual representation is provided below. Still, one notes that in Cases 1 to 5 both players' interests are fully aligned, whereas in the remaining six cases, the preferences differ.

Proposition 5. *The channel members' preference on licensing strategy depends on external royalty rate as follows:*

- Case 1. *If $\max\{\pi_2, \tilde{\pi}_o\} < \pi < \pi_1$, then both players prefer licensing in Stage 1.*
- Case 2. *If $\max\{\pi_2, \hat{\pi}_o, \hat{\pi}_c\} < \pi < \min\{\pi_1, \tilde{\pi}_c\}$, then both players prefer licensing decision to be taken in Stage 2; licensing if $\Theta = \theta_1$, and no licensing if $\Theta = \theta_2$.*
- Case 3. *If $\pi_2 < \pi < \min\{\pi_1, \hat{\pi}_o, \hat{\pi}_c\}$, then both players prefer a no licensing decision in Stage 1.*
- Case 4. *If $\bar{\pi}_o < \pi < \pi_2$, or $\pi > \max\{\pi_1, \bar{\pi}_o, \bar{\pi}_c\}$, then both players prefer licensing, and there is no difference if this decision is made in Stage 1 or 2.*
- Case 5. *If $\pi < \min\{\pi_2, \bar{\pi}_c\}$, or $\pi_1 < \pi < \min\{\bar{\pi}_o, \bar{\pi}_c\}$, then both players prefer no licensing, and there is no difference if this decision is made in Stage 1 or 2.*
- Case 6. *If $\max\{\pi_2, \hat{\pi}_o, \tilde{\pi}_c\} < \pi < \min\{\pi_1, \tilde{\pi}_o\}$, then the OEM makes the licensing decision in Stage 2, and chooses licensing if $\Theta = \theta_1$, and no licensing if $\Theta = \theta_2$, while the CM prefers licensing in Stage 1.*
- Case 7. *If $\hat{\pi}_o < \pi < \min\{\pi_1, \hat{\pi}_c\}$, then the OEM makes the licensing decision in Stage 2, and it is licensing if $\Theta = \theta_1$, and no li-*

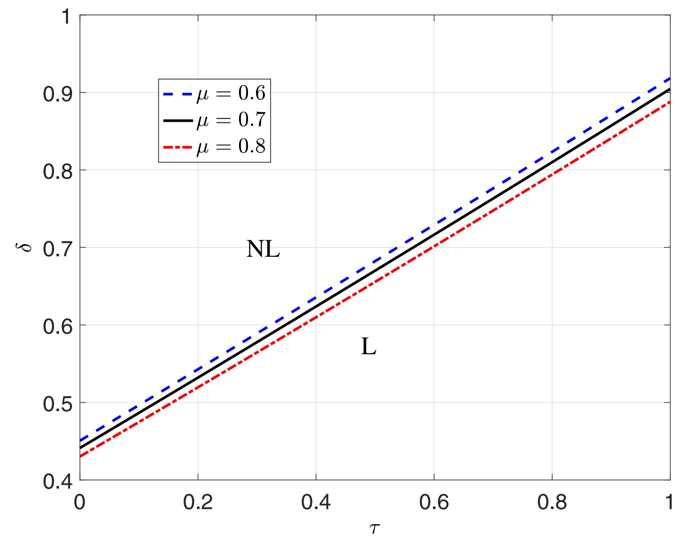

Fig. 1. Licensing decision when mean μ varies.

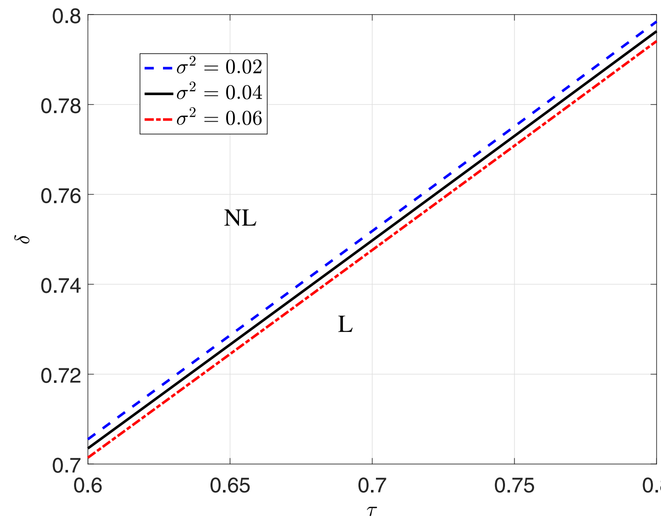

Fig. 2. Licensing decision when variance σ^2 varies.

censing if $\Theta = \theta_2$, while the CM prefers no licensing in Stage 1.

Case 8. *If $\max\{\pi_2, \widetilde{\pi}_c\} < \pi < \widehat{\pi}_o$, then the OEM prefers no licensing in Stage 1, while the CM prefers licensing in Stage 1.*

Case 9. *If $\max\{\pi_2, \widehat{\pi}_c\} < \pi < \min\{\pi_1, \widehat{\pi}_o, \widetilde{\pi}_c\}$, then the OEM licenses the technology in Stage 1, while the CM prefers licensing decision in Stage 2, with licensing if $\Theta = \theta_1$, and no licensing if $\Theta = \theta_2$.*

Case 10. *If $\max\{\pi_1, \bar{\pi}_o\} < \pi < \bar{\pi}_c$, then the OEM prefers licensing, and there is no difference if this decision is made in Stage 1 or 2, while the CM prefers no licensing, and there is no difference if this decision is made in Stage 1 or 2.*

Case 11. *If $\max\{\pi_1, \bar{\pi}_c\} < \pi < \bar{\pi}_o$ or $\bar{\pi}_c < \pi < \min\{\pi_2, \bar{\pi}_o\}$, then the OEM prefers no licensing, and there is no difference if this decision is made in Stage 1 or 2, while the CM prefers licensing, and there is no difference if this decision is made in Stage 1 or 2.*

4.1. Numerical illustrations

Although all our results are analytical, we wish to provide in this section few numerical examples to give a visual illustration of (i) how some parameter values affect the licensing decision; and (ii) the shape of the different regions identified in Propositions 4 and 5. We retain the following constellation of parameter values as a benchmark:

$$\theta_1 = 0.5, \theta_2 = 1, M = 1, \delta = 0.5, \tau = 0.4, \alpha = 0.6,$$
$$\pi = 0.3, \mu = 0.8, \sigma^2 = 0.06.$$

Fig. 1 shows the impact of varying δ and τ on licensing option for different values for μ. (Note that varying μ while keeping unchanged the value of $\sigma^2 = 0.06$, requires that we adjust consequently the values of θ_1 and θ_2.) In Fig. 1, the plane is divided by a solid curve into two regions, above which no licensing is the optimal choice and below which licensing is the best option. This reflects that it is beneficial to choose licensing when τ is high and δ is low. These results are intuitive as high revenue rate motivates the OEM to license the technology, while higher technology competition deters the OEM from doing so. In particular, the licensing region shrinks, and no-licensing region expands with the increase of mean μ. High mean value of technology efficiency boosts the demand and increases profit, which dominates the external revenue from licensing.

Fig. 2 exhibits the impact of varying δ and τ on licensing decision for different values of variance σ^2. Again, we need to adjust θ_1 and θ_2 when varying σ^2 while keeping $\mu = 0.8$. We see that a larger variance σ^2 expands the no-licensing region and shrinks the licensing region, meaning that larger volatility reduces the OEM's motivation to license the technology.

To further verify the theoretical results obtained in Section 3, we look at the impact of varying M, δ, π and τ on the equilibria in Table 3, while keeping the Table 4 same parameter values in the benchmark except for $\pi = 0.2$ and $\tau = 0.69$.

As seen from Table 3, the OEM prefers to make the licensing decision in the second stage. The corresponding equilibrium x, m are larger, but ϕ is lower, as compared with those of scenario 1 with licensing. But relative to scenario 1 without licensing, x, m of scenario 2 are lower, and ϕ is larger. Although the equilibrium strategies of scenario 2 are moderate, the OEM's payoff is the largest. The interpretation is that, as for scenario 2, high demand is generated by the high investment, along with the high margin and low cost sharing, jointly contributing to a high payoff for the OEM, in comparison to scenario 1 with licensing. Even though the cost sharing rate is relatively large with respect to scenario 1 without licensing, the investment is relatively low, thus, the investment cost is not too high. Besides, additional licensing fees are reaped, eventually benefiting the OEM.

Table 3 shows that only when the parameters M, δ, π and τ are moderate, it is better to make the licensing decision in Stage 2, and the equilibrium licensing strategy depends on the realizations of the stochastic technology efficiency. Otherwise, in most of the parameter space, there is no difference when the licensing decision is made in Stage 1 or 2. In fact, there exists a small region (R4) where making licensing decision in the second stage is preferred. Particularly, when M, δ increase, the OEM tends to choose no licensing. However, when π and τ raise, the OEM tends to license the technology. As seen from Table 3, when the market potential M increases, as compared with the case under licensing, the cost sharing rate ϕ under no licensing is always lower, the retail margin m is larger, and the investment x increases faster. This leads to high demand and low investment cost, eventually profiting the OEM. It is easy to follow that facing a high competition intensity δ, the OEM is unwilling to license the technology to avoid the high demand loss. A related work conducted by Allain et al. (2015) indicates that competition in pharmaceutical industry has significant impact on the timing of licensing, but this effect differs by the type of competitor. Specifically, the technology licensing delays with an

Table 3
Sensitive analysis on expected equilibria of two scenarios.

		Scenario 1 ($\Lambda=1$)					Scenario 1 ($\Lambda=0$)					Scenario 2					Decision	Licensing
		ϕ	x	m	$E[\Pi_o]$	$E[\Pi_c]$	ϕ	x	m	$E[\Pi_o]$	$E[\Pi_c]$	ϕ	x	m	$E[\Pi_o]$	$E[\Pi_c]$	stage	decision
M	0.8	0.6206	0.1216	0.4243	0.0943	0.0454	0.6088	0.1151	0.4460	0.0915	0.0446	0.6206	0.1216	0.4243	0.0943	0.0454	1/2	L
	0.9	0.6197	0.1281	0.4756	0.1171	0.0567	0.6088	0.1295	0.5018	0.1158	0.0565	0.6197	0.1281	0.4756	0.1171	0.0567	1/2	L
	1.0	0.6188	0.1345	0.5269	0.1425	0.0692	0.6088	0.1439	0.5576	0.1430	0.0697	0.6117	0.1430	0.5501	0.1431	0.0696	2	Z
	1.1	0.6181	0.1410	0.5782	0.1705	0.0830	0.6088	0.1583	0.6132	0.1730	0.0843	0.6088	0.1583	0.6133	0.1730	0.0843	1/2	NL
δ	0.40	0.6181	0.1497	0.5359	0.1463	0.0707	0.6088	0.1439	0.5576	0.1430	0.0697	0.6181	0.1497	0.5359	0.1463	0.0707	1/2	L
	0.45	0.6184	0.1420	0.5312	0.1444	0.0699	0.6088	0.1439	0.5576	0.1430	0.0697	0.6184	0.1420	0.5312	0.1444	0.0699	1/2	L
	0.50	0.6188	0.1345	0.5269	0.1425	0.0692	0.6088	0.1439	0.5576	0.1430	0.0697	0.6117	0.1430	0.5501	0.1431	0.0696	2	Z
	0.55	0.6194	0.1272	0.5229	0.1408	0.0685	0.6088	0.1439	0.5576	0.1430	0.0697	0.6088	0.1439	0.5576	0.1430	0.0697	1/2	NL
π	0.19	0.6184	0.1310	0.5262	0.1416	0.0689	0.6088	0.1439	0.5576	0.1430	0.0697	0.6088	0.1439	0.5576	0.1430	0.0697	1/2	NL
	0.20	0.6188	0.1345	0.5269	0.1425	0.0692	0.6088	0.1439	0.5576	0.1430	0.0697	0.6117	0.1430	0.5501	0.1431	0.0696	2	Z
	0.21	0.6192	0.1380	0.5276	0.1434	0.0695	0.6088	0.1439	0.5576	0.1430	0.0697	0.6192	0.1380	0.5276	0.1434	0.0695	1/2	L
	0.22	0.6196	0.1415	0.5283	0.1444	0.0699	0.6088	0.1439	0.5576	0.1430	0.0697	0.6196	0.1415	0.5283	0.1444	0.0699	1/2	L
τ	0.65	0.5901	0.1326	0.5266	0.1421	0.0695	0.6088	0.1439	0.5576	0.1430	0.0697	0.6088	0.1439	0.5576	0.1430	0.0697	1/2	NL
	0.69	0.6188	0.1345	0.5269	0.1425	0.0692	0.6088	0.1439	0.5576	0.1430	0.0697	0.6117	0.1430	0.5501	0.1431	0.0696	2	Z
	0.73	0.6468	0.1362	0.5273	0.1429	0.0688	0.6088	0.1439	0.5576	0.1430	0.0697	0.6468	0.1362	0.5273	0.1429	0.0688	1/2	L
	0.77	0.6742	0.1378	0.5276	0.1434	0.0685	0.6088	0.1439	0.5576	0.1430	0.0697	0.6742	0.1378	0.5276	0.1434	0.0685	1/2	L

Table 4
Impact of probability distribution on licensing strategy.

τ	π	θ_1	θ_2	α	μ	σ^2	Decision stage	Licensing decision
Example 1								
0.62	0.216	0.5	1	0.6	0.8	0.06	2	Z
		0.31	0.92	0.8	0.8	0.06	1	L
Example 2								
0.67	0.198	0.5	1	0.6	0.8	0.06	1	NL
		0.31	0.92	0.8	0.8	0.06	2	Z

increase in the number of entrants or a decrease in the number of incumbents in the market. Different from their work, our results indicate that when the competition intensity is at a moderate level, the OEM will make the licensing decision in the second stage, and the licensing strategy depends on realizations of technology efficiency. Otherwise, the OEM is indifferent between making the licensing decision in the first or the second stage. In particular, the OEM prefers licensing with a relatively low competition intensity, whereas prefers no licensing when facing a relatively high competition intensity. The reason for the difference with the results in Allain et al. (2015) is that they concern the impacts of market structure (incumbent or entrant) on the timing of technology licensing, but ours focus on the competition on the timing of licensing decision-making. In addition, it is also intuitive that the OEM is willing to license the technology when the royalty rate and revenue sharing rate are relatively high. This well supports the prevalent application of technology licensing in several sectors such as transportation, information technology and equipment where royalty rates are relatively high Sen and Stamatopoulos (2016). The sensitivity analysis not only provides some managerial insights, but also offers significant guidances to firms on how to adjust decisions when facing different market environment.

Finally, it should be mentioned that, in most cases, the CM also benefits from the OEM's licensing strategy.

Apart from the above four parameters, there are three probability distribution parameters of the stochastic technology efficiency, i.e., θ_1, θ_2 and α. Although they only affect the equilibrium solution in Scenario 1 in terms of mean and variance, they have a significant influence on the results in Scenario 2, and in particular on the licensing decision. To give an intuition about the effects of probability distribution parameters on optimal licensing strategy, we report two examples in Table 2 where in both cases the distribution's mean μ and variance σ^2 are kept at 0.8 and 0.06, respectively.

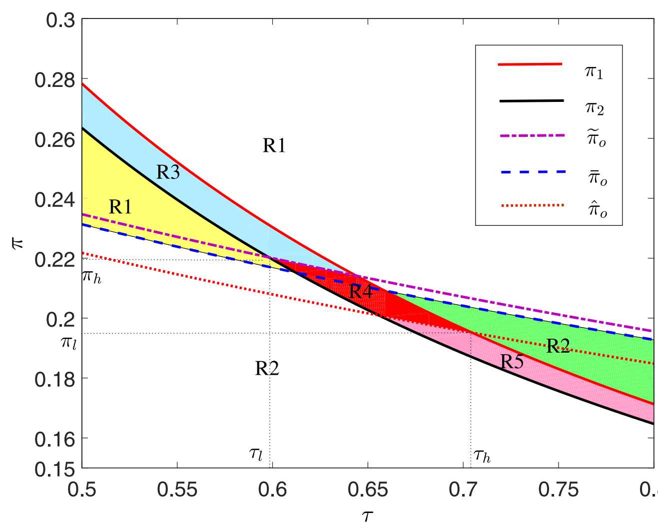

Fig. 3. Different regions characterized in Proposition 4.

Examples 1 and 2 show that when we modify the probability distribution, keeping all other parameters at their benchmark values, the timing of licensing decision changes. In Example 1, the shift is from Stage 2 to Stage 1 and is the other way around in Example 2. Further, in Example 1 the decision changes from depending on realization to licensing, whereas in Example 2, the change is from no licensing to depending on the realization of the stochastic technology efficiency. In a nutshell, the clear-cut conclusion is that the probability distribution of stochastic technology efficiency significantly affects, not only quantitatively but also, qualitatively the OEM's licensing strategy.

Based on the benchmark's parameter values, Figs. 3 and 4 illustrate Propositions 4 and 5, respectively.

Recall that Proposition 4 states that the OEM's licensing strategy depends on the relationship between π and $\pi_1, \pi_2, \bar{\pi}_o, \tilde{\pi}_o, \hat{\pi}_o$, and depicts five regions according to their relationships. As seen from Fig. 3, there exist two thresholds for π, namely, $\pi_h = 0.22$ and $\pi_l = 0.195$, and two thresholds for τ, i.e., $\tau_h = 0.705$ and $\tau_l = 0.598$. The main takeaways from this figure are: (i) Loosely speaking, if $\pi \le \bar{\pi}_o$, then the optimal decision is no licensing and this seems to be fairly intuitive. Indeed, if the royalty is too low, then there is no point for the OEM to license its technology and expose itself by the same token to competition. (ii) If $\pi \ge \bar{\pi}_o$,

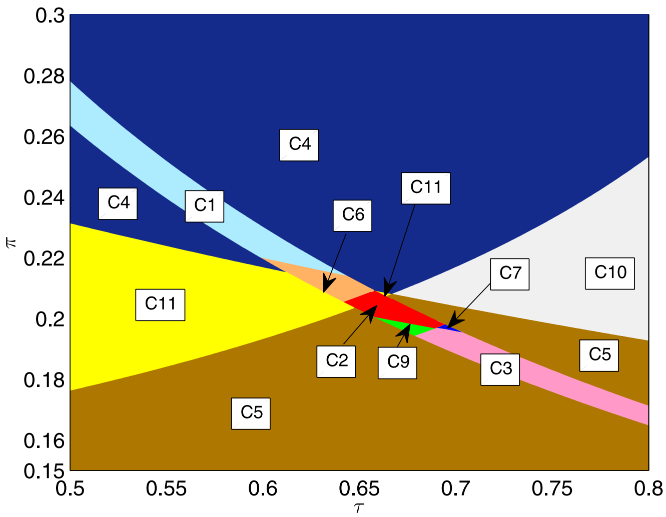

Fig. 4. Different regions characterized in Proposition 5.

then we have the mirror case where licensing is profitable. (iii) There is an in-between region (R4) where the decision depends on the realization of the stochastic technology efficiency. As we can see, this region is relatively small. In other words, the OEM should not make licensing decision in the second stage in most parameter space. (iv) The value of $\bar{\pi}_o$ depends on the royalty π and the revenue sharing parameter τ. The higher the value of the royalty, the less share of revenue it takes to the OEM to license the technology. Finally, (v) we note that in most of the space, it does not matter if the licensing decision is made in Stage 1 or in Stage 2. Overall, according to the results in (iii) and (iv), it is a good choice to make licensing decision in the first stage.

Fig. 4 illustrates the results stated in Proposition 5, with Case i referred to by Ci. Recall that in Cases 1–5, the two partners in the supply chain have their objectives aligned in terms of licensing decision. We see that these cases occupy a large part of the space, which is good news in terms of avoiding any possible conflicts. This is also supported by the numerical results in Table 3 where the CM benefits from the OEM's licensing strategy in most parameter space. Note that for this parameter constellation, Case 8 does not materialize.

5. Extensions

In this section, we relax some of the assumptions we made and assess the robustness of our results.

5.1. A general distribution function

We first assume that the random variable Θ lies in the interval $[0, 2\mu]$, with $f(\Theta)$ being the probability density function and $F(\Theta)$ the cumulative density function.

The equilibria of scenario 1 only depend on the statistics of the random variable, i.e., the mean μ and variance σ^2. As such, a general distribution does not affect the equilibria of scenario 1. The corresponding equilibria under a general distribution are the same with those under a two-point distribution.

We next turn to the equilibria of scenario 2. We first solve the second-stage problem in (8) and obtain

$$\Lambda^* = \begin{cases} 1, & \theta < \bar{\theta}, \\ 0, & \theta \geq \bar{\theta}, \end{cases} \tag{29}$$

$$m^*(\theta) = \begin{cases} \frac{1}{2}(\theta x(1-\delta) + M), & \theta < \bar{\theta}, \\ \frac{1}{2}(\theta x + M), & \theta \geq \bar{\theta}, \end{cases} \tag{30}$$

$$w^*(\theta) = \begin{cases} \frac{1}{4}(\theta x(1-\delta) + M), & \theta < \bar{\theta}, \\ \frac{1}{4}(\theta x + M), & \theta \geq \bar{\theta}, \end{cases} \tag{31}$$

where

$$\bar{\theta} = \frac{2(4\tau\pi - M\delta)}{\delta x(2-\delta)}.$$

These equilibria are the same as in Lemma 1. Specifically, when the technology efficiency is relatively large, i.e., $\theta > \bar{\theta}$, technology licensing is preferred; otherwise, no licensing is better, which is in line with the results obtained with a two-point distribution. Similarly, if $4\pi\tau > M\delta$, the equilibria of scenario 2 are the same as those of scenario 1 without licensing.

Next, we deal with the first-stage problem, and start by determining the follower's (CM's) reaction function. For mathematical tractability, we take the example of a uniform distribution. The mean and variance are

$$E[\Theta] = \mu, \quad Var[\Theta] = \sigma^2 = \frac{\mu^2}{3}.$$

With maximizing the expected profit, the CM is searching for the best technology investment x, i.e.,

$$\max_x \; E[\Pi_{c_1}] = E[w^*(\Theta)D + (1-\tau)\pi\Lambda^*\Theta x - (1-\phi)x^2], \tag{32}$$

where $E[\Pi_{c_1}]$ can be rewritten as

$$E[\Pi_{c_1}] = \begin{cases} \frac{x^2}{12}(\mu^2 - 12(1-\phi)) + \frac{M\mu}{8}x + \frac{M^2}{16} & x \geq \frac{4\pi\tau - M\delta}{\mu\delta(2-\delta)}, \\ -\frac{(M\delta + 8\pi(4\tau-3))(4\pi\tau - M\delta)^2}{24\mu\delta^2(2-\delta)^2 x}, & \\ (\frac{\mu^2}{12}(1-\delta)^2 - (1-\phi))x^2 & x < \frac{4\pi\tau - M\delta}{\mu\delta(2-\delta)}. \\ +(\frac{M}{8}(1-\delta) + \pi(1-\tau))\mu x + \frac{M^2}{16}, & \end{cases}$$

Solving the optimization problem (32) yields the following CM's first-stage investment strategy:

$$x^*(\phi) = \begin{cases} x_1^*(\phi), & x_1^*(\phi) \geq \frac{4\pi\tau - M\delta}{\mu\delta(2-\delta)}, \\ \frac{\mu(8\pi(1-\tau) + M(1-\delta))}{16(1-\phi) - \xi(1-\delta)^2}, & \text{otherwise}, \end{cases} \tag{33}$$

where $x_1^*(\phi)$ satisfies that

$$\frac{x_1^*(\phi)}{6}(\mu^2 - 12(1-\phi)) + \frac{M\mu}{8} + \frac{(M\delta + 8\pi(4\tau-3))(4\pi\tau - M\delta)^2}{24\mu\delta^2(2-\delta)^2 x_1^{*2}(\phi)} = 0.$$

Incorporating the CM's best response $x^*(\phi)$ into the OEM's objective function, we solve the following optimization problem:

$$\max_\phi \; E[\Pi_{o_1}] = E[m^*(\Theta)D + \tau\pi\Lambda^*\Theta x^*(\phi) - \phi x^{*2}(\phi)], \tag{34}$$

where

$$E[\Pi_{o_1}] = \begin{cases} \frac{1}{6}(\mu^2 - 6\phi)x^{*2}(\phi) + \frac{1}{4}M\mu x^*(\phi) & \\ +\frac{M^2}{8} + \frac{(4\pi\tau - M\delta)^3}{12\mu\delta^2(2-\delta)^2 x^*(\phi)}, & x_1^*(\phi) \geq \frac{4\pi\tau - M\delta}{\mu\delta(2-\delta)}, \\ \frac{\mu^2(8\pi(1-\tau) + M(1-\delta))}{8(16(1-\phi) - \xi(1-\delta)^2)^2} & \\ (8\pi\xi(1-\delta)^2(1-2\tau) & \\ +M(1-\delta)(8(4-5\phi) & \\ -\xi(1-\delta)^2) & \\ -64\pi(\phi(1+\tau) - 2\tau)) + \frac{M^2}{8}, & \text{otherwise}. \end{cases}$$

The optimal first-stage cost sharing rate can be obtained by solving the OEM's optimization problem in (34), i.e.,

$$\phi^* = \begin{cases} \phi_1^* & x_1^*(\phi_1^*) \geq \frac{4\pi\tau - M\delta}{\mu\delta(2-\delta)}, \\ \frac{8\pi(\xi(5-7\tau)(1-\delta)^2 - 16(1-3\tau)) + (1-\delta)M(\xi(1-\delta)^2 + 48)}{16(5M(1-\delta) + 8\pi(1+\tau))}, & \text{otherwise}, \end{cases} \tag{35}$$

Table 5
Sensitive analysis on expected equilibria of two scenarios.

		Scenario 1 ($\Lambda = 1$)		Scenario 1 ($\Lambda = 0$)		Scenario 2		Decision stage	Licensing decision
		$E[\Pi_o]$	$E[\Pi_c]$	$E[\Pi_o]$	$E[\Pi_c]$	$E[\Pi_o]$	$E[\Pi_c]$		
M	0.9	0.1172	0.0567	0.1163	0.0566	0.1172	0.0567	1/2	L
	1.0	0.1426	0.0692	0.1436	0.0699	0.1439	0.0696	2	Z
	1.1	0.1706	0.0831	0.1737	0.0846	0.1738	0.0847	2	Z
	1.2	0.2012	0.0981	0.2068	0.1007	0.2068	0.1007	1/2	NL
δ	0.46	0.1441	0.0698	0.1436	0.0699	0.1441	0.0698	1/2	L
	0.50	0.1426	0.0692	0.1436	0.0699	0.1439	0.0696	2	Z
	0.54	0.1412	0.0687	0.1436	0.0699	0.1437	0.0698	2	Z
	0.58	0.1389	0.0682	0.1436	0.0699	0.1436	0.0699	1/2	NL
π	0.18	0.1408	0.0686	0.1436	0.0699	0.1436	0.0699	1/2	NL
	0.20	0.1426	0.0692	0.1436	0.0699	0.1439	0.0696	2	Z
	0.22	0.1445	0.0700	0.1436	0.0699	0.1445	0.0700	1/2	L
	0.24	0.1465	0.0707	0.1436	0.0699	0.1465	0.0707	1/2	L
τ	0.62	0.1419	0.0698	0.1436	0.0699	0.1436	0.0699	1/2	NL
	0.69	0.1426	0.0692	0.1436	0.0699	0.1439	0.0696	2	Z
	0.76	0.1434	0.0686	0.1436	0.0699	0.1434	0.0686	1	NL
	0.83	0.1442	0.0680	0.1436	0.0699	0.1442	0.0680	1/2	L
μ	0.6	0.1347	0.0662	0.1346	0.0664	0.1347	0.0662	1/2	L
	0.7	0.1384	0.0676	0.1386	0.0680	0.1390	0.0675	2	Z
	0.8	0.1426	0.0692	0.1436	0.0699	0.1439	0.0696	2	Z
	0.9	0.1476	0.0711	0.1498	0.0724	0.1500	0.0722	2	Z

Table 6
One numerical example for a generalized cost function.

	ϕ	x	$E[m]$	$E[w]$	$E[\Pi_o]$	$E[\Pi_c]$	Decision stage	Licensing decision
Scenario 1 ($\Lambda = 1$)	0.6300	0.1404	0.5211	0.2746	0.1362	0.0660		
Scenario 1 ($\Lambda = 0$)	0.6228	0.1614	0.5565	0.2944	0.1369	0.0665	2	Z
Scenario 2	0.6259	0.1601	0.5480	0.2780	0.1370	0.0664		

where ϕ_1^* satisfies that

$$\left(\frac{1}{3}(\mu^2 - 6\phi)x_1^*(\phi_1^*) + \frac{1}{4}M\mu\frac{(4\pi\tau - M\delta)^3}{12\mu\delta^2(2-\delta)^2 x_1^{*2}(\phi_1^*)}\right)\frac{\partial x_1^*(\phi)}{\partial\phi}|_{\phi=\phi_1^*} = 0.$$

Substituting ϕ^* into (33), the equilibrium investment x^* is obtained, and Λ^*, m^*, w^* as well as the equilibrium payoffs $E[\Pi_c^*], E[\Pi_o^*]$ are calculated. Indeed, although it is hard to give the analytical expressions for $x^*(\phi)$, ϕ^*, we can resort to numerical simulations to solve them.

One numerical example is conducted based on the following parameters:

$$M = 1, \delta = 0.5, \tau = 0.69, \pi = 0.2, \mu = 0.8.$$

The sensitivity analysis with respect to M, δ, π, τ and μ is carried out in Table 5.

Similar to the results in Table 3, we can conclude from Table 5 that only when the parameters M, δ, π and τ are moderate, the OEM prefers to make the licensing decision in Stage 2, and the equilibrium licensing strategy depends on the realizations of the random technology efficiency. In most parameter space, it does not matter if the licensing decision is made in Stage 1 or 2. Specifically, the OEM prefers no licensing when facing a large M or δ, but prefers licensing in the presence of a large π or τ. It also means that there exists a small parameter region where making the licensing decision in the second stage is better. Additionally, when mean μ increases, the OEM tends to make the decision in the second stage. Note that Θ follows a uniform distribution in [0, 2μ], thus a high mean μ means a high variance σ^2. Since this high uncertainty is resolved in the second stage, it is in the best interest of the OEM to make the licensing decision in Stage 2, and consequently get higher profit.

To sum up, this extension indicates that the uniform distribution does not qualitatively alter the theoretical results obtained with a two-point distribution.

5.2. A more general cost function

In this subsection, we adopt a more general investment cost function, that is,

$$C(x) = ax + bx^2,$$

with a, $b > 0$ being cost coefficients. This function has an additional linear term with respect to the one we had before, which means that the marginal cost at zero is not zero anymore but a positive quantity given by a. Keeping everything else as in Section 2, we follow the same procedure to determine the corresponding equilibria, which we do not present for space saving,[4] but look at the implications of having this more general cost functions on them. A numerical example in Table 6 where the parameters are given by

$$\mathbf{M} = 1, \theta_1 = 0.5, \theta_2 = 1, \alpha = 0.6,$$
$$\delta = 0.5, \pi = 0.2, \tau = 0.69, c_0 = 0.1, a = 0.1, b = 0.5.$$

Table 6 shows that for this parameter constellation, the OEM prefers to make the licensing decision in the second stage, and the equilibrium licensing strategy depends on the realizations of the random technology efficiency. Varying the parameters, we also find that in most parameter space, the OEM is indifferent between making the licensing decision in Stage 1 or 2. One fact should be pointed out is that when the parameters, such as M, δ, π, τ, c_0, a and b, are moderate, there exists a small parameter region where the OEM prefers to make the licensing decision in Stage 2. This is in line with the results under a specified quadratic cost function.

[4] The results are available from the authors upon request.

Furthermore, taking both of the generalized distribution and cost functions into account, we also find our theoretical results are robust.

6. Conclusion

In this paper, we considered a simple model of R&D cooperation in a supply chain. We characterized pricing, investment and cost-sharing equilibrium strategies in two scenarios, namely, a scenario where licensing decision can be taken before R&D and market uncertainties are resolved, and a scenario where this decision can be postponed to the sales stage where these uncertainties are resolved. Our focus is on the strategic licensing decision of the OEM. The main results can be summarized as follows: (1) When the market potential, competition intensity, royalty rate and revenue sharing rate are moderate, there exists a small parameter region where making the licensing decision in Stage 2 is preferred. But for most parameter space, it does not matter if the licensing decision is made in Stage 1 or 2. Besides, there also exist some cases where the OEM prefers to make the licensing decision in Stage 1. (2) Large revenue sharing rate or royalty rate spurs the OEM to license the technology, but a large uncertainty, or competition, or market potential prevent it from doing it. (3) Technology efficiency uncertainty promotes technology investment, expected retail margin and profits for CM and OEM, but exerts a non-monotonic effect on investment sharing rate. (4) If the licensing option is made in the second stage, the OEM chooses no licensing if the technology efficiency is high, otherwise, it chooses licensing. (5) Different probability distribution of stochastic technology efficiency may result in different licensing strategies. (6) In most cases, the channel members have the same preferences in terms of licensing strategy.

In this work, the CM cooperates with the OEM based on the revenue sharing and cost sharing contracts. The OEM is able to flexibly adjust the cost sharing rate according to the CM's revenue sharing schedule. However, in some cases, the follower prefers a retained revenue rather than a cost sharing mechanism. How to design an optimal retained revenue such that the CM is willing to be involved in this cooperation is worth considering for the OEM. In particular, the revenue sharing mechanism is modeled here as a parameter, and considering it as a strategic variable could provide some interesting insights. Finally, giving a strategic role to the licensee instead of modeling its presence only through an impact on OEM's demand is clearly of interest.

Appendix

A.1. Proof of Proposition 1

By backward induction, we first solve the CM's second-stage problem:

$$\max_w \ \Pi_c = w(M-(m+w)+\theta x \\ -\delta\Lambda\theta x)+(1-\tau)\pi\Lambda\theta x-(1-\phi)x^2. \tag{36}$$

Using the first-order condition, we get

$$w(\theta)=\frac{1}{2}\theta x(1-\delta\Lambda)+\frac{1}{2}(M-m). \tag{37}$$

Taking it into account, we then solve the OEM's second-stage problem:

$$\max_m \ \Pi_o = m(M-(m+w)+\theta x-\delta\Lambda\theta x)+\tau\pi\Lambda\theta x-\phi x^2. \tag{38}$$

Similarly, the optimal retailer margin m is calculated as

$$m(\theta)=\frac{1}{2}\theta x(1-\delta)+\frac{M}{2}. \tag{39}$$

With the second-stage response functions (37) and (39), the CM's first-stage problem is

$$\max_x \ E[\Pi_c] = E[w(\Theta)(M-(m(\Theta)+w(\Theta))+\Theta x-\delta\Lambda\Theta x) \\ +(1-\tau)\pi\Lambda\Theta x-(1-\phi)x^2]. \tag{40}$$

The corresponding solution is easily obtained as

$$x=\frac{\mu(8\pi\Lambda(1-\tau)+M(1-\delta\Lambda))}{16(1-\phi)-\xi(1-\delta\Lambda)^2}. \tag{41}$$

Next, with the consideration of (37), (39) and (41), and given Λ, solving the following OEM's first-stage problem:

$$\max_\phi \ E[\Pi_o]=E[m(\Theta)D+\tau\pi\Lambda\Theta x-\phi x^2], \tag{42}$$

yields

$$\phi=\frac{8\Lambda\pi(\xi(5-7\tau)(1-\delta\Lambda)^2-16(1-3\tau))+M(1-\delta\Lambda)(\xi(1-\delta\Lambda)^2+48)}{16(5M(1-\delta\Lambda)+8\Lambda\pi(1+\tau))}. \tag{43}$$

Substituting (43) into (37), (39) and (41) yields the equilibrium solutions as given in (9)–(11).

A.2. Details of derivatives in Table 2

The derivatives of equilibrium strategies and payoffs given in Table 2 are as follows:

$$\frac{\partial\phi}{\partial M}=\frac{2\pi(1-\delta)(16-3\xi(1-\delta)^2)(2-3\tau)}{(8\pi(1+\tau)+5M(1-\delta))^2},$$

$$\frac{\partial\phi}{\partial\delta} \\ =\frac{-5M^2\xi(1-\delta)^3+16\pi M((8\tau-7)\xi(1-\delta)^2+16(3\tau-2))+64\xi\pi^2(1+\tau)(7\tau-5)(1-\delta)}{8(8\pi(1+\tau)+5M(1-\delta))^2},$$

$$\frac{\partial\phi}{\partial\pi}=\frac{2M(1-\delta)(3\tau-2)(16-3\xi(1-\delta)^2)}{(8\pi(1+\tau)+5M(1-\delta))^2},$$

$$\frac{\partial\phi}{\partial\tau}=\frac{2\pi(8\pi+3M(1-\delta))(16-3\xi(1-\delta)^2)}{(5M(1-\delta)+8\pi(1+\tau))^2}>0,$$

$$\frac{\partial\phi}{\partial\mu}=\frac{\mu(1-\delta)^2(8\pi(5-7\tau)+M(1-\delta))}{8(5M(1-\delta)+8\pi(1+\tau))},$$

$$\frac{\partial\phi}{\partial\sigma^2}=\frac{(1-\delta)^2(8\pi(5-7\tau)+M(1-\delta))}{16(5M(1-\delta)+8\pi(1+\tau))},$$

$$\frac{\partial x}{\partial M}=\frac{5\mu(1-\delta)}{2(16-3\xi(1-\delta)^2)}>0,$$

$$\frac{\partial x}{\partial\delta}=-\frac{\mu(15M\xi(1-\delta)^2+48\pi\xi(1+\tau)(1-\delta)+80M)}{2(16-3\xi(1-\delta)^2)^2}<0,$$

$$\frac{\partial x}{\partial\pi}=\frac{4\mu(1+\tau)}{16-3\xi(1-\delta)^2}>0,$$

$$\frac{\partial x}{\partial\tau}=\frac{4\mu\pi}{16-3\xi(1-\delta)^2}>0,$$

$$\frac{\partial x}{\partial\mu}=\frac{(8\pi(1+\tau)+5M(1-\delta))(16+3(\mu^2-\sigma^2)(1-\delta)^2)}{2(16-3\xi(1-\delta)^2)^2}>0,$$

$$\frac{\partial x}{\partial\sigma^2}=\frac{3\mu(1-\delta)^2(8\pi(1+\tau)+5M(1-\delta))}{2(16-3\xi(1-\delta)^2)^2}>0,$$

$$\frac{\partial m}{\partial M}=\frac{(5\mu\theta-6\xi)(1-\delta)^2+32}{4(16-3\xi(1-\delta)^2)}>0,$$

$$\frac{\partial m}{\partial\delta}=-\frac{2\mu\theta(3\pi\xi(1+\tau)(1-\delta)^2+20M(1-\delta)+16\pi(1+\tau))}{(16-3\xi(1-\delta)^2)^2}<0,$$

$$\frac{\partial m}{\partial\pi}=\frac{2\mu\theta(1+\tau)(1-\delta)}{16-3\xi(1-\delta)^2}>0,$$

$$\frac{\partial m}{\partial\tau}=\frac{2\mu\theta\pi(1-\delta)}{16-3\xi(1-\delta)^2}>0,$$

$$\frac{\partial m}{\partial\mu}=\frac{\theta(1-\delta)(5M(1-\delta)+8\pi(1+\tau))(16+3(\mu^2-\sigma^2)(1-\delta)^2)}{4(16-3\xi(1-\delta)^2)^2}>0,$$

$$\frac{\partial m}{\partial\sigma^2}=\frac{3\mu\theta(1-\delta)^3(5M(1-\delta)+8\pi(1+\tau))}{4(16-3\xi(1-\delta)^2)^2}>0,$$

$$\frac{\partial E[\Pi_o]}{\partial M}=\frac{((25\mu^2-24\xi)(1-\delta)^2+128)M+40(1-\delta)\pi\mu^2(1+\tau)}{32(16-3\xi(1-\delta)^2)}>0,$$

$$\frac{\partial E[\Pi_o]}{\partial\delta}=-\frac{\mu^2(8\pi(1+\tau)+5M(1-\delta))(10M+3\pi\xi(1+\tau)(1-\delta))}{4(16-3\xi(1-\delta)^2)^2}<0,$$

$$\frac{\partial E[\Pi_o]}{\partial \pi} = \frac{\mu^2(1+\tau)(5M(1-\delta)+8\pi(1+\tau))}{4(16-3\xi(1-\delta)^2)} > 0,$$

$$\frac{\partial E[\Pi_o]}{\partial \tau} = \frac{\mu^2\pi(5M(1-\delta)+8\pi(1+\tau))}{4(16-3\xi(1-\delta)^2)} > 0,$$

$$\frac{\partial E[\Pi_o]}{\partial \mu} = \frac{\mu(5M(1-\delta)+8\pi(1+\tau))^2(16-3\sigma^2(1-\delta)^2)}{32(16-3\xi(1-\delta)^2)^2} > 0,$$

$$\frac{\partial E[\Pi_o]}{\partial \sigma^2} = \frac{3\mu^2(1-\delta)^2(5M(1-\delta)+8\pi(1+\tau))^2}{64(16-3\xi(1-\delta)^2)^2} > 0,$$

$$\frac{\partial E[\Pi_c]}{\partial M} = \frac{M((5\mu^2-6\xi)(1-\delta)^2+32)+8\pi\mu^2(1-\delta)(3-2\tau)}{16(16-3(1-\delta)^2)} > 0,$$

$$\frac{\partial E[\Pi_c]}{\partial \delta} = -\frac{\mu^2(M\pi(16+3\xi(1-\delta)^2)(3-2\tau)+2(1-\delta)(12\xi\pi^2(1-\tau^2)+5M^2))}{2(16-3(1-\delta)^2)^2} < 0,$$

$$\frac{\partial E[\Pi_c]}{\partial \pi} = \frac{\mu^2(8\pi(1-\tau^2)+M(3-2\tau)(1-\delta))}{2(16-3(1-\delta)^2)} > 0,$$

$$\frac{\partial E[\Pi_c]}{\partial \tau} = -\frac{\mu^2\pi(4\pi\tau+M(1-\delta))}{16-3(1-\delta)^2} < 0,$$

$$\frac{\partial E[\Pi_c]}{\partial \mu} = \frac{\mu(5M(1-\delta)+8\pi(1+\tau))(8\pi(1-\tau)+M(1-\delta))(16-3\sigma^2(1-\delta)^2)}{16(16-3\xi(1-\delta)^2)^2} > 0,$$

$$\frac{\partial E[\Pi_c]}{\partial \sigma^2} = \frac{3\mu^2(1-\delta)^2(5M(1-\delta)+8\pi(1+\tau))(8\pi(1-\tau)+M(1-\delta))}{32(16-3\xi(1-\delta)^2)^2} > 0.$$

A.3. Proof of Proposition 2

The derivatives of $\bar{\pi}_o$ with regard to μ, σ^2, δ, τ are

$$\frac{\partial \bar{\pi}_o}{\partial \mu} = \frac{30M\delta\mu(2-\delta)}{\sqrt{(16-3\xi)^3(1+\tau)^2(16-3\xi(1-\delta)^2)}} > 0,$$

$$\frac{\partial \bar{\pi}_o}{\partial \sigma^2} = \frac{15M\delta(2-\delta)}{\sqrt{(16-3\xi)^3(1+\tau)^2(16-3\xi(1-\delta)^2)}} > 0,$$

$$\frac{\partial \bar{\pi}_o}{\partial \delta} = \frac{5M(3\xi(1-\delta)+\sqrt{(16-3\xi)(16-3\xi(1-\delta)^2)})}{8(1+\tau)\sqrt{(16-3\xi)(16-3\xi(1-\delta)^2)}} > 0,$$

$$\frac{\partial \bar{\pi}_o}{\partial \tau} = -\frac{5M\sqrt{9\xi^2(1-\delta)^2-48\xi(\delta^2-2\delta+2)+256}-(16-3\xi)(1-\delta)}{8(16-3\xi)(1+\tau)^2} < 0.$$

A.4. Proof of Lemma 1

Given x and ϕ, the second-stage responses with and without technology licensing in stage 2 are

$$m = \begin{cases} \frac{1}{2}\theta x(1-\delta) + \frac{M}{2}, & \Lambda = 1, \\ \frac{1}{2}\theta x + \frac{M}{2}, & \Lambda = 0, \end{cases} \quad (44)$$

$$w = \begin{cases} \frac{1}{4}\theta x(1-\delta) + \frac{M}{4}, & \Lambda = 1, \\ \frac{1}{4}\theta x + \frac{M}{4}, & \Lambda = 0. \end{cases} \quad (45)$$

Substituting (44) and (45) into the OEM's profit yields

$$\Pi_{o_2} = \begin{cases} (\frac{1}{8}\theta^2(1-\delta)^2-\phi)x^2 + (\frac{M}{4}\theta(1-\delta)+\tau\pi\theta)x + \frac{M^2}{8}, & \Lambda = 1, \\ (\frac{1}{8}\theta^2-\phi)x^2 + \frac{M}{4}\theta x + \frac{M^2}{8}, & \Lambda = 0. \end{cases}$$

Comparing the profits with and without licensing, the OEM prefers to license the technology ($\Lambda^* = 1$) when $x < \bar{x}$, otherwise, no licensing ($\Lambda^* = 0$) is a better option for the OEM. The corresponding retail margin and wholesale price are obtained in (30) and (31).

A.5. Proofs of Lemma 2 and Proposition 3

It is easy to obtain the corresponding result using first-order conditions. It is omitted here for space saving.

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