Shear modulus and material damping of municipal solid waste based on large-scale cyclic triaxial testing

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Shear modulus and material damping of municipal solid waste based on large-scale cyclic triaxial testing

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Abstract: Representative dynamic properties of municipal solid waste (MSW) are required to perform reliable seismic analyses of MSW landfills. A comprehensive large-scale cyclic triaxial laboratory testing program was performed on MSW retrieved from a landfill in the San Francisco Bay area to evaluate the small-strain shear modulus, and strain-dependent normalized shear modulus reduction and material damping ratio relationships of MSW. The effects of waste composition, confining stress, unit weight, time under confinement, and loading frequency on these dynamic properties were evaluated. The small-strain shear modulus depends primarily on waste composition, confining stress, unit weight, and time under confinement. The normalized shear modulus reduction and material damping curves for MSW depend on waste composition and confining stress. Based on the results of this study and a review of literature, strain-dependent shear modulus reduction and material damping relationships are recommended for use in landfill design.

Key words: cyclic triaxial testing, dynamic properties, earthquakes, solid waste landfill, solid waste properties.

Résumé : Les propriétés dynamiques représentatives des résidus solides municipaux (« MSW ») sont requises pour réaliser des analyses sismiques des enfouissements sanitaires de « MSW ». On a réalisé un vaste programme d'essais triaxiaux cycliques à grande échelle en laboratoire sur des « MSW » prélevés d'un site dans la région de la Baie de San Francisco pour évaluer le module de cisaillement à petite déformation, la réduction du module de cisaillement normalizé dépendant du temps, et les relations du rapport d'amortissement du matériau de « MSW ». On a évalué les effets de la composition des résidus, de la contrainte de confinement, du poids volumique, du temps sous confinement, et de la fréquence des chargements sur ces propriétés dynamiques. Le module de cisaillement à faible déformation dépend principalement de la composition des résidus, de la contrainte de confinement, du poids volumique, et du temps sous confinement. La réduction du module de cisaillement normalizé et les courbes d'amortissement des « MSW » dépendent de la composition des résidus et de la contrainte de confinement. En partant des résultats de cette étude et de la revue de la littérature, on recommande d'utiliser la réduction du module de cisaillement dépendant du temps et les relations d'amortissement du matériau pour la conception de sites d'enfouissement.

Mots-clés : essais triaxiaux cycliques, propriétés dynamiques, séismes, enfouissement de matières solides, propriétés de résidus solides.

[Traduit par la Rédaction]

Introduction

An engineer's ability to evaluate the likely seismic performance of a municipal solid waste (MSW) landfill is limited currently by the lack of reliable data on the strain-dependent dynamic properties of MSW. Required dynamic properties include the: (*i*) small-strain shear modulus ($G_{\max}$) or shear wave velocity (V_s); (*ii*) strain-dependent normalized shear modulus reduction ($G/G_{\max}$) relationship, and (*iii*) strain-dependent material damping ratio (λ) relationship. In this study, MSW was retrieved from the Tri-Cities landfill, located in Fremont, California, and was tested in large-scale cyclic triaxial devices to investigate the importance of a number of factors, including waste composition, confining stress, unit weight, time under confinement, and loading frequency on these dynamic properties of MSW. Following a review of previous studies, the results from this investigation are presented. Relationships for MSW are recommended for use in seismic analyses of MSW landfills.

Previous investigations of the dynamic properties of municipal solid waste

The small-strain shear modulus ($G_{\max}$) is related to the shear wave velocity (V_s) and the mass density (ρ) of a material through the relationship:

$$[1] \qquad G_{\max} = \rho V_s^2$$

The V_s of waste materials in a landfill can be measured in situ by various seismic methods. The downhole method, the crosshole method, the suspension logging method, the spec-

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tral analysis of surface waves (SASW), and controlled source surface wave profiling methods have been used successfully to estimate V_s in situ. The SASW method has been popular for landfills, because it is reliable, nonintrusive, and cost effective. For preliminary purposes and when landfill-specific data are not available, Kavazanjian et al. (1996) recommended typical V_s profiles for southern California MSW landfills. Subsequently, after evaluating additional field data presented by other researchers, Kavazanjian (1999) found that these V_s profiles could be used for landfills in temperate climates as well as in arid climates.

Strain-dependent normalized shear modulus reduction (G/G_{max}) and material damping relationships for MSW have been recommended by a number of researchers (such as Idriss et al. 1995; Matasovic and Kavazanjian 1998; Augello et al. 1998*b*; Morochnik et al. 1998; and Elgamal et al. 2004). These studies were primarily based on back-analyses of the seismic response of the OII landfill in southern California. For example, Idriss et al. (1995) performed one-dimensional and two-dimensional equivalent-linear finite element analyses of a single cross section with best estimates for the properties of the landfill, and the native soil, to back-calculate the seismic response of the OII landfill during four earthquakes. Shear modulus reduction and material damping curves were recommended for use in practice. Elgamal et al. (2004) used system identification techniques to estimate shear modulus reduction and material damping curves using the ground motions recorded at the OII landfill during six earthquakes. The results of their analyses suggest an average constant damping of about 5%, with no significant reduction in shear modulus for strains between 0.001% and 0.2%.

Matasovic and Kavazanjian (1998) performed two-dimensional equivalent-linear time domain response analyses using the computer program QUAD4M (Hudson et al. 1994) and back-calculated the east–west component of five earthquakes that were recorded by the two accelerometers. Best-estimate geometry, V_s profile, unit weight, and Poisson's ratio (ν) were used in the analyses. The best-fit OII solid waste shear modulus reduction and material damping curves were determined on the basis of qualitative examination of the observed and predicted acceleration response spectra at the top of the landfill. The resulting maximum shear strains from these earthquakes were estimated to be 0.1%. At larger strains (0.1% to 7%), the results from large (457 mm diameter) cyclic simple shear tests performed on reconstituted specimens from the OII landfill were used.

Augello et al. (1998*b*) back-calculated the response of the OII landfill using the recorded ground motions at the top of the landfill from five earthquake events. Two-dimensional finite element analyses were performed in the two orthogonal horizontal directions at the site and numerous V_s profiles, Poisson's ratio values, shear modulus reduction curves, and material damping curves were used. Comparisons between the observed and calculated motions were made using an objective statistical analysis technique. The results of the analyses indicated that the selection of shear modulus reduction and material damping curves had the largest overall effect on the calculated response. Curves that were similar to those recommended by Vucetic and Dobry (1991) for clays with plasticity index (PI) = 30 best captured the observed response. The maximum shear strain level calculated in the waste fill during these earthquake events was only about 0.2%.

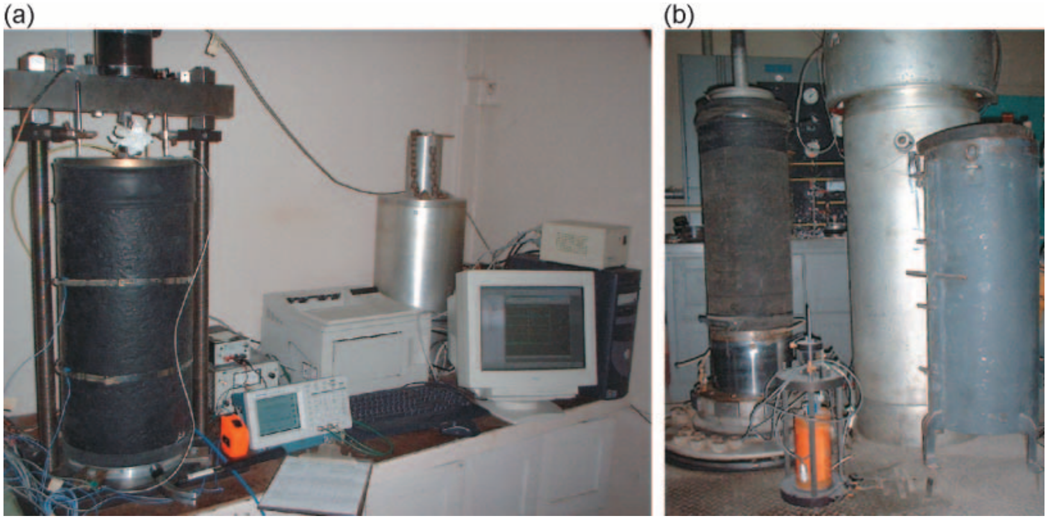
Although much insight has been garnered through these studies, the OII landfill is somewhat atypical, because it includes unusually large amounts of soil material as well as commercial and industrial waste (Matasovic and Kavazanjian 1998). In addition, the shear modulus reduction and material damping curves recommended by these research groups differ significantly. Limited laboratory investigations have been performed on MSW primarily due to the difficulties in performing such tests. Difficulties include the health and safety issues associated with sampling and testing waste material, sample disturbance, and the large size of specimens required to include the larger waste particles. In addition to the Matasovic and Kavazanjian (1998) cyclic simple shear tests on waste from the OII landfill, the only other large-scale cyclic testing of MSW currently available in the literature are a limited number of cyclic triaxial tests performed by Towhata et al. (2004) at shear strains of approximately 0.3%. The need for a systematic study of the factors that may affect the dynamic properties of MSW provided the motivation for this laboratory testing program.

The selection of an appropriate MSW unit weight (γ) profile and the dynamic strength of MSW can also significantly impact the results of dynamic analyses of landfills. A recent study by Zekkos et al. (2006) presented a comprehensive framework for developing a landfill-specific MSW unit weight profile. Generalized MSW unit weight profiles that depend primarily on waste composition and compaction effort during placement of waste were also provided for use in preliminary analysis. Less is understood regarding the dynamic strength of MSW. The dynamic strength of waste fill has been back-calculated in a number of studies to be at least equal to the static strength recommended by Kavazanjian et al. (1995) (i.e., c = 24 kPa and $\phi = 0^\circ$ for $\sigma_n <$ 30 kPa, and c = 0 kPa and $\phi = 33^\circ$ for $\sigma_n >$ 30 kPa). For example, Augello et al. (1998*a*) recommend that a reasonable range of dynamic friction angles is from 33° to 38°. Tests performed by Zekkos et al. (2007) also indicate that the dynamic strength of MSW is about 20% higher than its static strength due to loading rate effects. Also, their results indicate that the Kavazanjian et al. (1995) strength recommendation provides a conservative estimate of the static strength of MSW.

Waste characterization and laboratory testing program

Two large-diameter (760 mm) borings were augered to depths of 10 and 32 m using a bucket auger at the Tri-Cities landfill in Fremont, located in the San Francisco Bay area. Shallow and deep samples, and young and old samples of waste were retrieved and stored separately in 39 large, sealed 55 gallon bulk waste material drums. Excessive grinding of the waste particles was not observed, so the collected waste materials are assumed to be unprocessed. Two to four drums of waste were collected at each sampling interval of 3 m. The in situ unit weight of waste was measured using the procedures described in Zekkos et al. (2006), and ranged from 10 kN/m^3 near the surface to 16 kN/m^3 at depth. The waste material was transported to the Richmond

Fig. 1. Cyclic triaxial (CTX) test devices used: (*a*) bench-top device and (*b*) floor-based device, with the pressure chamber and the specimen preparation mold. Municipal solid waste (MSW) specimen shown with black membrane. A 100 mm diameter specimen triaxial cell is shown in the foreground for comparison.



Field Station of the University of California at Berkeley, where it was characterized. Waste characterization included separating the waste material into material larger and smaller than 20 mm. This segregation is considered useful because: (*i*) material smaller than 20 mm is composed of soil-like material that is derived primarily from daily cover, other soil materials, and some fine waste inclusions, whereas material larger than 20 mm generally consists of waste material; (*ii*) material smaller than 20 mm can be characterized using conventional soil mechanics index tests, such as sieve analyses and Atterberg limits; and (*iii*) material that is finer than 20 mm can also be tested using geotechnical testing equipment of conventional size.

The waste samples that were collected as part of this study form three general classes. Class A is relatively "deep old waste" and included sample groups A1–A4. Class B is "deep old waste with fibrous smaller than 20 mm material" and included sample group B1. Class C is "shallow fresh waste" and included sample groups C1–C6. Classes A and B waste were placed in 1987, and Class C waste was placed after 1999. The percentage by weight of the finer than 20 mm material and the amount of plastic, paper, wood, gravel, and other constituents of the coarser than 20 mm material were measured for a total of six waste sample groups. At the Tri-Cities landfill, about 50%–75% of the total waste sample by weight was smaller than 20 mm material, and the remaining coarser material consisted primarily of paper, plastic, wood, and gravel. Other constituents such as metals, glass, stiff plastics, and textiles comprised a significantly lower percentage of the material by weight and by volume. Details of the field investigation and waste characterization may be found in Zekkos (2005).

Two large-scale cyclic triaxial (CTX) test devices (d = 300 mm, h = 600–630 mm upon specimen preparation) were used to evaluate the dynamic properties of the collected MSW. The CTX device has proved to be a viable means for measuring dynamic material properties (e.g., Kramer 1996). The bench-top device (Fig. 1*a*) provides high-resolution data with more precise control, but is limited to testing specimens under vacuum confinement (i.e., isotropic confining stresses up to about 90 kPa). Linear variable differential transformers (LVDTs) were placed directly on the top cap of the specimens in diametrically opposite locations to reliably measure axial strains as small as 0.0003%. The floor-based device (Fig. 1*b*) is the larger capacity CTX testing device used by Seed et al. (1984). It can test specimens at higher confining stresses, but in its present configuration, tests must be performed under an anisotropic confining stress such that the axial stress always remains higher than the horizontal stress during sinusoidal loading. The majority of the tests were performed at confining stresses less than 100 kPa using the bench-top device to examine the effects of specimen composition, unit weight, loading frequency, and time under confinement. Additional tests were performed using the floor-based device to evaluate the effects of higher confining stresses on the dynamic properties. Replicate tests performed using the floor-based device at the same mean confining stresses used in the bench-top device indicated good consistency between the results of the two CTX testing devices.

Specimen preparation procedures for the reconstitution of MSW specimens were developed as part of this investigation and are described in detail by Zekkos (2005). Waste constituents were carefully mixed and then compacted in a 305 mm diameter steel mold in 8–9 layers using a 100 N weight that was dropped repeatedly from a constant height to achieve a target unit weight or compaction effort. Specimens were prepared with 100%, 62%–76%, and 8%–25% by weight smaller than 20 mm material. The larger than 20 mm fraction is largely fibrous and has a lower particle unit weight than the smaller than 20 mm fraction. Bulky particles (e.g., wood and gravel) included in the larger than 20 mm material were screened to a maximum particle size of about 40 mm, whereas the paper and plastic constituents that are softer, more flexible, and fold easily were screened to a maximum allowable particle size of about 80 mm. It is generally accepted that soil particles that are no larger than one-sixth of the diameter of the test specimen can be in-

Table 1. Characteristics of municipal solid waste cyclic triaxial test specimens.

Specimen	γ_t (kN/m^3)[a]	Moisture content (%)	Composition (% by weight) < 20 mm	Paper	Soft plastics	Wood	Gravel
A3–1L	13.5	12.3	100	0	0	0	0
A3–2L	13.0	9.2	100	0	0	0	0
A3–3L	9.3	7.5–8.7	100	0	0	0	0
A3–4L	13.0	10.9–11.4	100	0	0	0	0
A3–5L	13.0	11.4	100	0	0	0	0
A3–6L	9.7	12.4	75.9	12.9	3.7	7.4	0
A3–7L	10.4	11.6	62.1	14.0	2.7	11.2	10
A3–8L	8.2	9.9	62.1	14.0	2.7	11.2	10
A3–9L	10.2	9.7	62.1	14.0	2.7	11.2	10
A3–11L	10.4	12.4	62.1	14.0	2.7	11.2	10
A3–12L	5.0	25.3	13.7	56.3	5	13.1	11.9
A3–13L	4.3	15.5	13.7	56.3	5	13.1	11.9
A3–14L	10.6	12.7	62.1	14.0	2.7	11.2	10
A3–15L	5.1	19.6	11.3	55.7	5.2	14.5	13.3
C6–1L	12.7	14.0	100	0	0	0	0
C6–2L	12.6	13.3	100	0	0	0	0
C6–3L	8.1	12.2	62.1	17.9	4.7	4.7	10.6
C6–4L	10.2	12.3	62.1	17.9	4.7	4.7	10.6
C6–7L	10.6	10.7	62.1	17.9	4.7	4.7	10.6
C6–8L	11.6	13.5	62.1	17.9	4.7	4.7	10.6
C6–10L	11.1	15.2	62.1	17.9	4.7	4.7	10.6
C6–11L	11.2	15.2	62.1	17.9	4.7	4.7	10.6
C3–1L	9.0	23	100	0	0	0	0
C3–2L	8.9	20.7	71.6	14.3	3.7	5.5	4.9
C3–3L	5.0	22.6	20.5	41.5	19	10	10

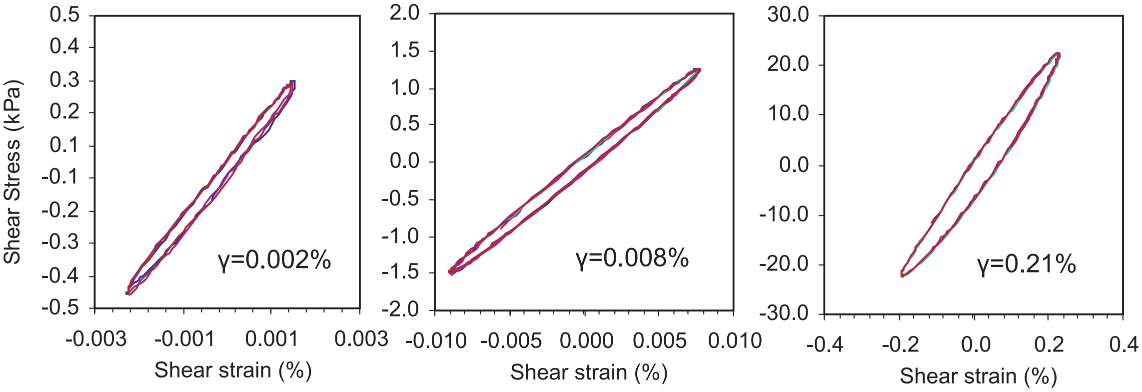
[a] Unit weight upon specimen preparation.

cluded without biasing the results. Hence, bulky waste materials were restricted to a maximum size of 40 mm. Pliable, elongated materials, such as plastic and paper, were allowed to have a maximum dimension of 80 mm, because these particles had high aspect ratios and were flexible. There should be a limit on the effect of waste constituent particle size on the overall waste response. Once particles become significantly larger than those in the waste matrix, especially if fibrous, their actual size should not be critical. It is encouraging that the laboratory-derived shear modulus and damping curves for MSW resulting from this work are consistent with those developed through back-analysis of MSW landfill responses during earthquakes, because these latter curves do not include particle size restrictions. It is likely that by using a 300 mm large-scale triaxial testing device, the removal of bulky and pliable particles larger than 40 mm and 80 mm, respectively, did not greatly affect these test results.

More than 90 CTX test series have been performed at different times and confining stresses on 25 large-scale triaxial specimens of MSW from the Tri-Cities landfill using material from 3 different sample groups. Primary characteristics of the MSW specimens are provided in Table 1. Group A3 is material sampled from borehole BH-2 at relatively large depth and was 15-years old at the time of drilling (placed in 1987). Group C6 includes material sampled from borehole BH-1 at a different location of the landfill at relatively shallow depths and was 2-years old at the time of drilling (placed in 2000). Group C3 was selected for testing because it was judged to be composed of material that differed the most from the other two groups that were tested. It originated from borehole BH-2 at a depth of 3.5–4.5 m, was less than 1-year old at the time of drilling (placed in 2002), and was visually identified as having significantly more paper constituents in the smaller than 20 mm fraction than sample groups A3 and C6. The organic content of the finer than 20 mm material, as measured by the loss of mass due to heating from a temperature of 105° to 440°, was 15%–30%, 10%–16%, and 20%–36% for the A3, C6, and C3 sample groups, respectively.

Stress-controlled CTX tests under a constant horizontal confining stress and a sinusoidal axial loading were performed. The axial load and axial deformation of the specimen were measured. In addition, the radial deformation was recorded for some of the specimens using elastomer strain gauges (Riemer and Safaqah 2007). From the ratio of the axial and radial deformation, Poisson's ratio (ν) was measured to be about 0.3 for specimens that included 100% smaller than 20 mm and 0.2 for specimens that also included material larger than 20 mm (Zekkos 2005). Poisson's ratio was found to reduce as waste composition in the larger than 20 mm material increased and total unit weight decreased. From the axial stress and strain, Young's modulus (E) is directly measured, and using the estimates of ν, the shear stress, shear strain, and shear modulus G can be calculated. Although the value of ν does affect the absolute value of G (on the order of ±10% using the values estimated in this study), it has a negligible effect on the calculated strain-dependent normalized shear modulus reduction and material damping ratio curves.

Fig. 2. Three shear stress versus shear strain loops from cyclic triaxial tests on specimen C3–2L which included 71.6% of <20 mm material.



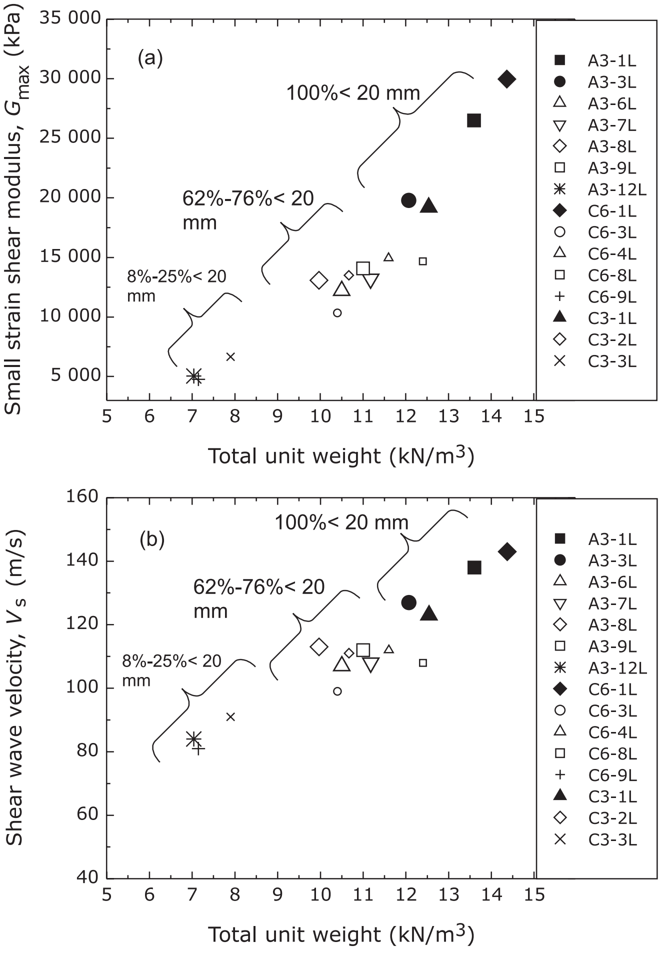
Representative cyclic loops from a series of large-scale CTX tests performed on specimen C3–2L, which included 71.6% by weight smaller than 20 mm material, are shown in Fig. 2. Reliable measurements of G and material damping are obtained at shear strains of 0.002%, 0.008%, and 0.21% in this case. Specimens with lower composition in larger than 20 mm material were stiffer, so that G could be evaluated at smaller strains. In almost all cases, the shape of the obtained $G/G_{\max}$ curve indicated that the $G_{\max}$ was reliably measured. In the subsequent sections, the effects of waste composition, confining stress, unit weight, time under confinement, loading frequency on the $G_{\max}$, the strain-dependent normalized shear modulus reduction and material damping curves are evaluated. All tests were performed at a field moisture content that was measured to be about 12%–20%. These values are consistent with moisture content values at or below field capacity of waste. The effect of moisture content on the dynamic properties was not evaluated as part of this study.

Small-strain shear modulus $G_{\max}$

Figure 3a presents the laboratory-measured $G_{\max}$ values as a function of specimen composition and total unit weight for tests performed at a sinusoidal loading frequency of 1 Hz on specimens that remained for 24 h at a constant mean confining stress of approximately 75 kPa. Data shown with full symbols represent tests performed on specimens with 100% smaller than 20 mm material, whereas data shown with hollow symbols represent tests performed on specimens that include larger than 20 mm material. Specimens with increasing amounts of the larger than 20 mm material have lower values of unit weight and $G_{\max}$. The decrease in the value of $G_{\max}$ as the proportion of material larger than 20 mm increases is a result of the reduction in the specimen unit weight and the reduction of the V_s of the specimen, as shown in Fig. 3b. Interestingly, the test results for all three sample groups with varying age and composition are consistent and follow the same relationship of unit weight versus $G_{\max}$ at the same confining stress of about 75 kPa.

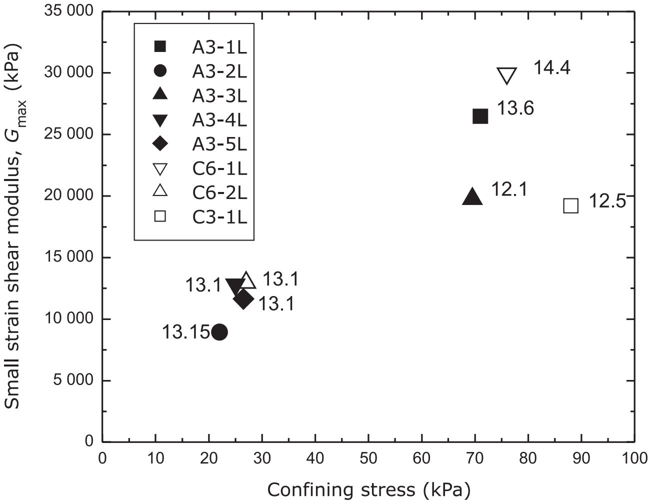
Figure 4 presents testing results on specimens from all three sample groups that include 100% smaller than 20 mm material. Only tests that were performed at a sinusoidal loading frequency of 1 Hz on specimens that remained under constant isotropic confining stress for 24 h are presented in Fig. 4. At a mean confining stress of about 25 kPa, the $G_{\max}$ is about 12 MPa; whereas at a mean confining stress of about 75 kPa, the $G_{\max}$ is about 30 MPa for the denser waste. As shown in Fig. 4, lower values of $G_{\max}$ are estimated for specimens that have lower unit weight.

Fig. 3. Influence of total unit weight on the variation of: (a) small-strain shear modulus, and (b) shear wave velocity measured at a confining stress of 75 kPa.



The $G_{\max}$ increases significantly with time under confinement. The $G_{\max}$ was measured from the cyclic triaxial tests for time under confinement up to 48 d. Complementary V_s measurements were also performed using accelerometers

Fig. 4. Influence of confining stress on the small-strain shear modulus of specimens with 100% smaller than 20 mm material. Values next to the data indicate the specimen total unit weight (in kN/m^3).



mounted on the specimen. A hammer was used to generate horizontally polarized shear waves at the base of the specimen (Fig. 1*a*) that propagated along the height of the specimen. The two methods agreed on the effect of time under confinement on the G_{max}. Figure 5 presents G_{max} at different times under confinement normalized by the G_{max} value at 24 h under confinement measured from the two methods. No systematic differences in the trend of the data were found for the three sample groups and the varying waste compositions. The relationship presented in Fig. 5 can be expressed by the following equation:

$$[2] \qquad \frac{G_{\max}(t)}{G_{\max}(t = 24\ \mathrm{h})} = 0.32\log(t) + 0.63$$

where t is the time under confinement, in hours, $G_{max}(t)$ is the small-strain shear modulus at time t, and $G_{max}(t = 24\ h)$ is the small-strain shear modulus at $t = 24$ h evaluated using the same method (i.e., either cyclic triaxial testing or shear wave velocity measurements).

Cyclic tests at loading frequencies of 0.01, 0.1, 1, and 10 Hz have also been performed on specimens of varying composition, and the results are presented in Fig. 6. For the three sample groups, the magnitude of the shear modulus (G) increases roughly linearly with the logarithm of the loading frequency for frequencies ranging between 0.01 and 10 Hz regardless of waste composition according to the following equation:

$$[3] \qquad \frac{G(f)}{G(f = 1\ \mathrm{Hz})} = 0.092\log(f) + 1.0$$

where $G(f)$ is the value of the shear modulus at relatively small strains (~0.001%) for a loading frequency f, and $G(f = 1\ Hz)$ is the value of the shear modulus at the same strain level for a frequency of 1 Hz.

Fig. 5. Influence of the time under confinement on the small-strain shear modulus of municipal solid waste (MSW).

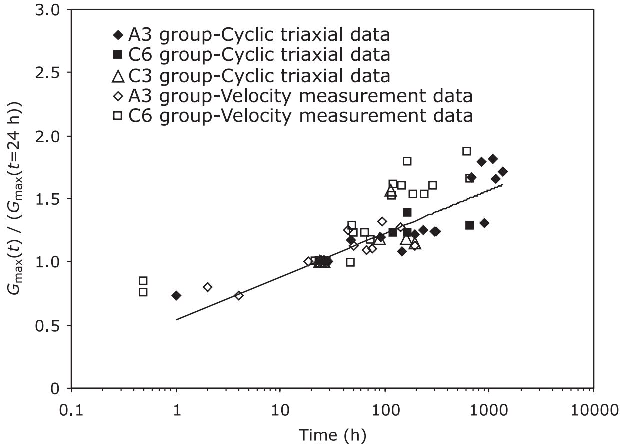
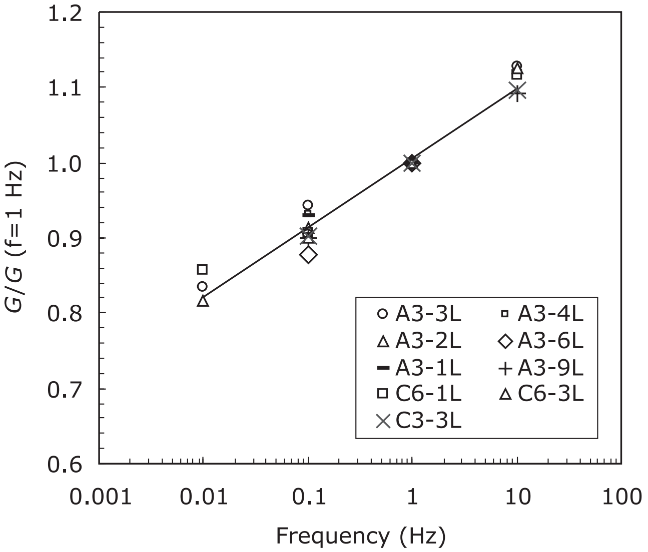
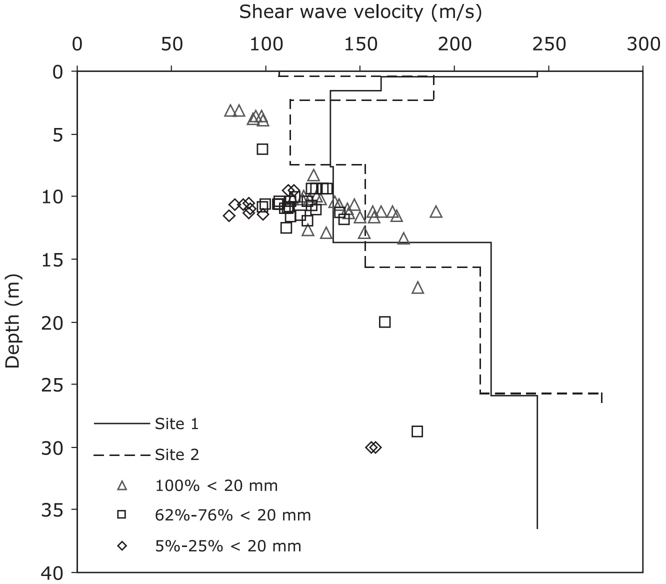


Fig. 6. Effect of loading frequency on the shear modulus of municipal solid waste (MSW).



From the estimated G_{max} values during the cyclic triaxial tests and the unit weight of the specimen, the shear wave velocity of the specimen was calculated. The shear wave velocities estimated in the laboratory were compared to the in situ measurements of the shear wave velocity performed by Professor K. Stokoe and his colleagues at the landfill using the SASW method (Lin et al. 2004). The results are shown in Fig. 7 for all the cyclic triaxial tests performed (i.e., for times under confinement between 24 h and 48 d). The field shear wave velocity profiles were evaluated assuming a total unit weight of about 15 kN/m^3 and a Poisson's ratio of 0.2 and are located next to the boreholes where the samples were collected. The laboratory-measured V_s for specimens that include 100% smaller than 20 mm material at mean confining stresses corresponding to equivalent depths within the landfill are generally similar or higher than the V_s measured in the field. For depths up to 15 m, the laboratory-measured V_s of specimens with 62%–76% smaller than 20 mm material, which best matched the composition of the waste in the field, were found to be only slightly lower than the in situ V_s of the waste at the corresponding depths. Laboratory specimens with 8%–25% smaller than 20 mm material had significantly lower V_s than those measured in the field. At greater depths (i.e., higher confining stresses), the laboratory data shown are for specimens that have been under high confining stress for only 1 h, and,

Fig. 7. Comparison of laboratory-measured and field-measured shear wave velocities of municipal solid waste (MSW) at the Tri-Cities landfill (spectral analysis of surface waves (SASW) field V_s data from Lin et al. 2004).



thus, these values are systematically lower than the field measurements that were performed on waste that has been placed in the landfill for years. The differences are surmised to be largely due to the time under confinement effects, because these effects were demonstrated to be important (i.e., eq. [2] and the deeper, older waste did not show signs of significant degradation).

Shear modulus reduction and material damping curves

Cyclic triaxial tests (CTX) were performed on specimens from each of the three sample groups with 100%, 62%–76%, and 8%–25% by weight smaller than 20 mm material. The results indicate that for approximately the same proportions of smaller than 20 mm material, the shear modulus reduction curves and the material damping curves are similar for all three sample groups tested, even though the waste was sampled from different locations and depths of the landfill and had different ages. For example, Fig. 8 presents the shear modulus reduction and material damping curves for all three waste groups and for specimens that included 62%–76% by weight smaller than 20 mm material.

Figure 9 presents $G/G_{\max}$ data for CTX tests performed at confining stresses less than 100 kPa for all three sample groups as a function of the percent by weight of material that is smaller than 20 mm. As the relative amount of material larger than 20 mm increases, the volumetric threshold strain increases, the material response becomes more linear at intermediate strains, and consequently, the normalized shear modulus reduction curves shift significantly. As explained in subsequent sections, the observed shift to the right of the normalized shear modulus reduction curve to the right is attributed primarily to the fibrous nature of the larger than 20 mm fraction and the size of the fibers.

The material damping ratio (λ) curve is also affected by the specimen composition as shown in Fig. 10. Waste composition has an important effect on the λ at large strains (>0.01%). As the relative amount of material larger than 20 mm increases, λ at intermediate to large strain levels decreases (i.e., the response becomes more linear). The observed reduction in λ for specimens with increasing waste content is consistent with the more linear response in the $G/G_{\max}$ curves. At smaller strains (<0.005%), the laboratory results suggest that λ is not significantly reduced with strain, but remains roughly constant with values of about 3%–4%. This trend is more clearly observed in Fig. 11 where the λ curves developed from each individual cyclic test series are shown. The "plateau" of constant λ at small strains appears to extend to higher strains as the proportion of larger than 20 mm fibrous material increases. For relatively small shear strains (<0.001%), even though there is scatter in the data, the specimens with 100% smaller than 20 mm material yielded lower λ than specimens with higher composition of larger than 20 mm material.

Fig. 8. Normalized shear modulus reduction and material damping cyclic triaxial tests (CTX) test data for all waste groups and specimens with 62%–76% < 20 mm.

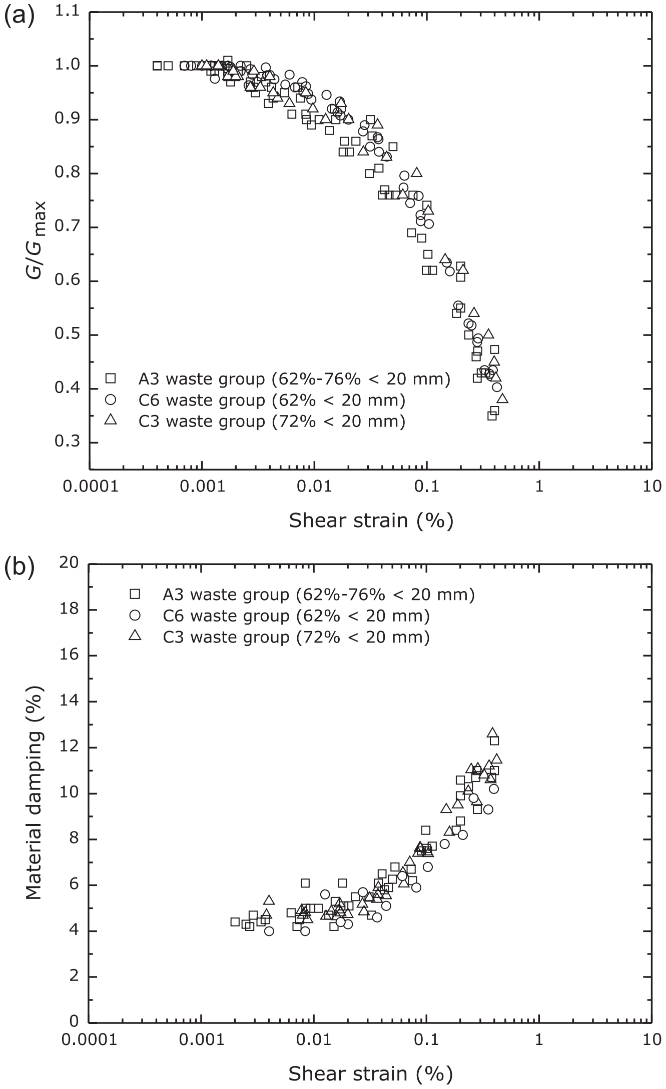


Fig. 9. Recommended strain-dependent normalized shear modulus reduction curves for municipal solid waste (MSW) based on results from all cyclic triaxial tests (CTX) tests at confining stress less than 125 kPa.

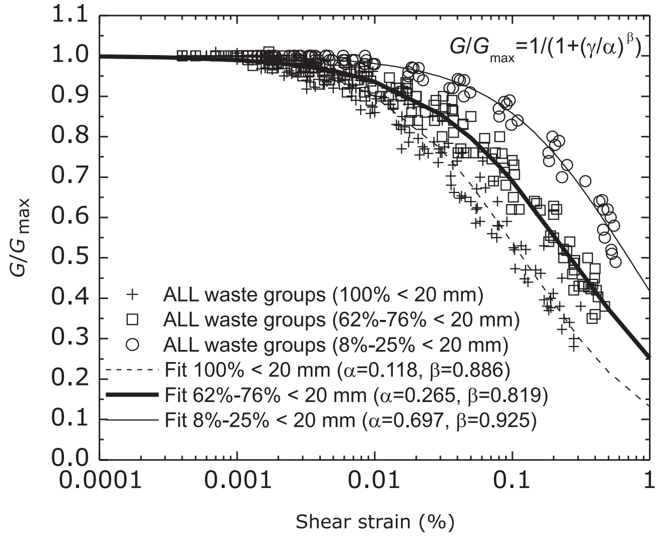


Fig. 10. Recommended strain-dependent material damping ratio curves for municipal solid waste (MSW) based on results from all cyclic triaxial tests (CTX) tests at confining stress less than 125 kPa.

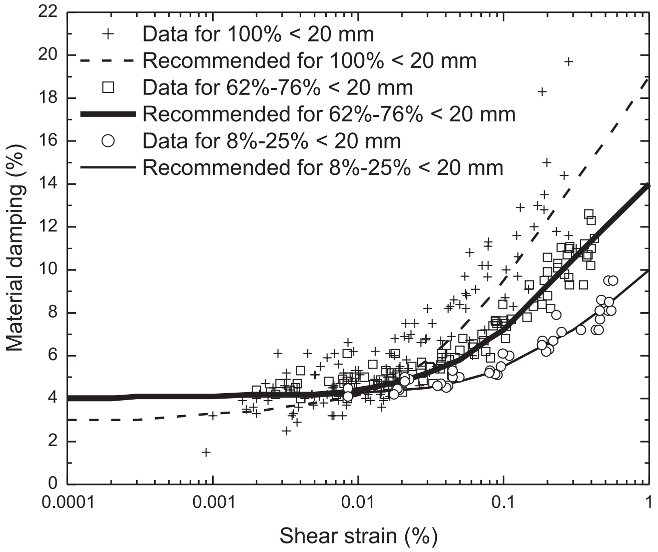
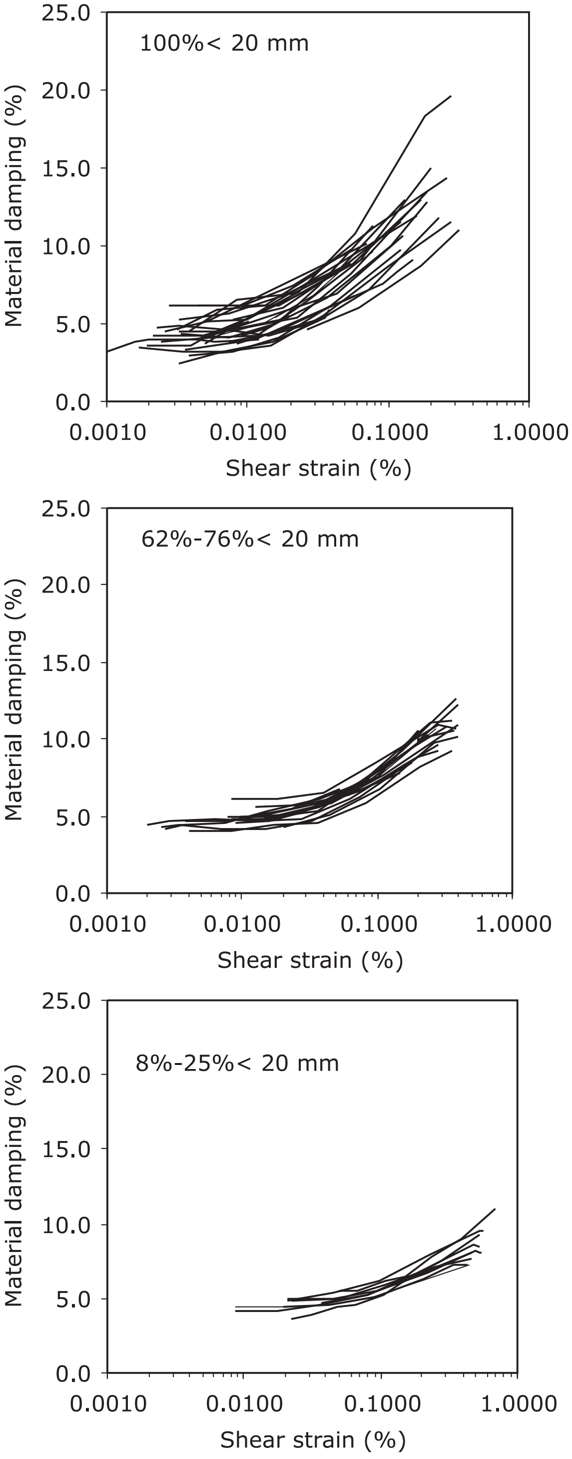


Fig. 11. Material damping curves from individual test series on specimens with 100%, 62%–76%, and 8%–25% smaller than 20 mm material



Additional tests were performed using the floor-based triaxial device to investigate the effect of confining stress. Cyclic triaxial tests using the floor-based device were performed under anisotropic stress conditions of varying stress ratios. Specimens were isotropically consolidated for at least 24 h at low confining stress (<100 kPa) and were subsequently anisotropically consolidated at higher confining stress for 1 h prior to the application of the cyclic loading. For the same mean confining stress, up to 0.1% strain, the G/G_{max} and λ values were similar for the floor-based and the bench-top triaxial devices. At shear strains larger than 0.1%, the anisotropically consolidated specimens yielded higher λ values than the isotropically consolidated tests. This difference is attributed primarily to the stress–strain response of MSW that affects the shape of the cyclic loops. More than 10% strain is required to reach peak shear stress in monotonic triaxial compression tests, whereas only 1% strain is required to reach peak stress in monotonic triaxial extension tests performed on waste (Zekkos 2005). Thus, at larger strains, the shear stress – shear strain cyclic loops measured for the isotropically consolidated MSW test specimens become slightly skewed during the extension phase of the testing as the peak stress is approached. This yields systematically lower damping for the isotropically consolidated CTX test specimens as compared to the anisotropically consolidated CTX test specimens. This limitation of CTX testing is being evaluated through comparison of these test results with those from cyclic simple shear tests being performed by another research team.

A number of specimens were tested at higher confining stresses using the floor-based device. Figure 12 presents a summary of all tests performed at higher confining stresses on specimens with 62%–76% smaller than 20 mm material. Based on the observations of the effect of confining stress on the $G/G_{\max}$ and λ curves, the data collected for 25–90 kPa are considered applicable for confining stresses up to about 125 kPa. The mean of the data for tests performed at confining stresses less than 125 kPa is shown with a thick continuous line along with the data points from tests performed at higher mean stresses. Hollow symbols are used for tests performed at a mean stresses between 125 and 350 kPa, whereas solid symbols are shown for the test data collected from tests performed at mean confining stresses higher than 350 kPa. As confining stress increases, the $G/G_{\max}$ curve shifts to the right, i.e., the volumetric threshold strain increases. Similar trends were observed for specimen A3–15L, which included 11.3% smaller than 20 mm material. Material damping was also observed to reduce with confining stress, but less systematically. Material damping was typically found to reduce about 1% and as much as 2% at high confining stresses when compared to the data at confining stresses of 25–90 kPa. However, in some cases, λ was not found to reduce with confining stress. Detailed results of the trends observed in the material damping data for individual specimens are presented in Zekkos (2005). In developing the damping curves shown in Fig. 12, the generally observed trend of decreasing λ with increasing confining stress was followed, which is consistent with the confining stress effect observed in shear modulus and with that observed for material damping in soil.

Based on the results of this investigation, relationships for $G/G_{\max}$ and λ as a function of shear strain for MSW are recommended for use in engineering practice. The shear modulus data were fitted to a hyperbolic model, which is expressed by the following equation:

$$[4] \qquad \frac{G}{G_{\max}} = \frac{1}{1 + (\gamma/\alpha)^{\beta}}$$

where $G_{\max}$ is the small-strain shear modulus, G is the secant shear modulus at shear strain γ, and α and β are two hyperbolic parameters, the value of which depends primarily on the specimen composition and confining stress. Table 2 presents the values of the two hyperbolic parameters α and β based on the amount of particles smaller than 20 mm in size for the data of Fig. 9, which are considered representative of confining stresses up to 125 kPa, or depths up to approximately 20 m. The correlation coefficient R^2 values of the proposed curves indicate a robust fit. Table 3 also includes the fitted hyperbolic parameters for the $G/G_{\max}$ curves for waste that includes 62%–76% smaller than 20 mm material at higher confining stresses (as shown in Fig. 12).

Fig. 12. Recommended strain-dependent curves for municipal solid waste (MSW) with 62%–76% smaller than 20 mm material at higher confining stresses: (*a*) normalized shear modulus reduction, and (*b*) material damping.

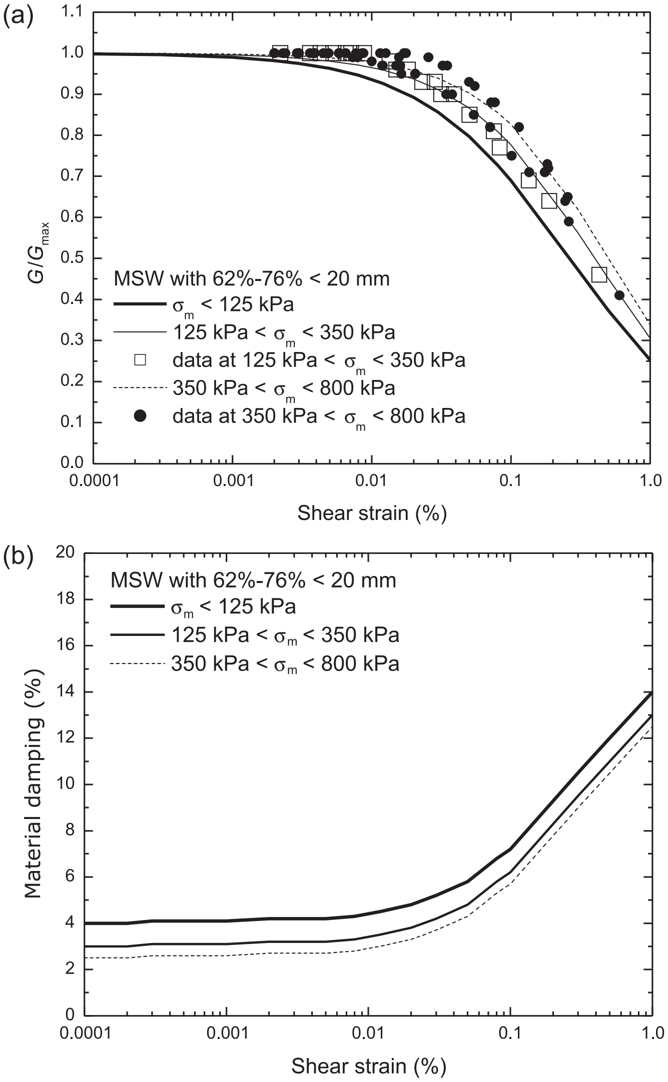


Table 2. Hyperbolic parameters (α and β*)* for capturing the shear modulus reduction relationships of municipal solid waste (MSW) for $\sigma_m <$ 125 kPa (and shear strains up to 1%).

	Percentage of material less than 20 mm		
	100	62–76	8–25
α	0.118	0.265	0.697
β	0.886	0.819	0.925
R^2	0.972	0.978	0.981

The recommended values of material damping ratio as a function of shear strain and waste composition for confining stresses up to 125 kPa (as shown in Fig. 10) are presented in Table 4. The recommended material damping curves at

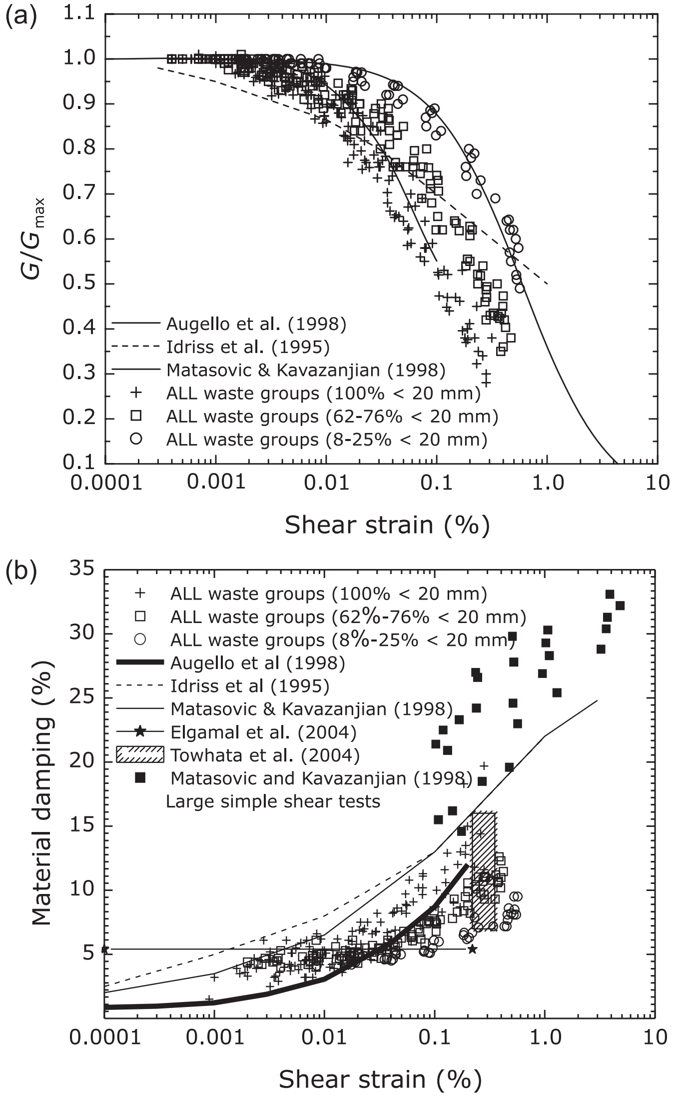
Table 3. Hyperbolic parameters (α and β) for capturing the shear modulus reduction relationships of municipal solid waste (MSW) with 62%–76% <20 mm at different confining stresses (and shear strains up to 1%).

	$\sigma_m < 125$ (kPa)	$350 < \sigma_m < 125$ (kPa)	$800 < \sigma_m < 350$ (kPa)
α	0.265	0.40	0.50
β	0.819	0.90	0.97

Table 4. Recommended material damping ratio values for municipal solid waste for $\sigma_m < 125$ kPa.

Shear strain, γ (%)	Percentage of material less than 20 mm		
	100	62–76	8–25
0.0001	3	4	4
0.0002	3	4	4
0.0003	3	4.1	4.1
0.001	3.3	4.1	4.1
0.002	3.4	4.2	4.1
0.003	3.6	4.2	4.1
0.005	3.8	4.2	4.1
0.008	4	4.3	4.2
0.012	4.3	4.5	4.3
0.02	5	4.8	4.4
0.03	5.8	5.2	4.5
0.05	7.2	5.8	4.8
0.08	8.7	6.8	5.2
0.1	9.5	7.2	5.5
0.3	14	10.5	7.2
0.5	16	12	8.3
1.0	19	14	10

Fig. 13. Cyclic triaxial test results and comparison with the literature: (*a*) normalized shear modulus reduction curve, and (*b*) material damping curve as a function of shear strain.



higher confining stresses are shown in Fig. 12. The recommended material damping relationships for waste material with 62%–76% smaller than 20 mm at higher confining stresses (i.e., 125 kPa < σ_m < 350 kPa and σ_m > 350 kPa) are developed by reducing the values of material damping at each strain level presented for lower confining stress by 1% and 1.5%, respectively.

Discussion – comparison with the literature

The shear modulus reduction and material damping data for MSW from this study at confining stresses less than 125 kPa, which were shown in Figs. 9 and 10, are compared to relationships previously recommended by others in Fig. 13. As shown in Fig. 13*a*, the G/G_{max} curve recommended by Augello et al. (1998*b*) falls between the G/G_{max} data for specimens with 100% and 62%–75% smaller than 20 mm material; whereas the G/G_{max} curve recommended by Matasovic and Kavazanjian (1998) falls close to the G/G_{max} data for specimens with 8%–25% smaller than 20 mm material. Both curves provide reasonable bounds to the laboratory data produced by this study. However, the G/G_{max} relationship presented by Idriss et al. (1995) is not consistent with the trends in these data.

As shown in Fig. 13*b*, the material damping curve recommended by Augello et al. (1998*b*) captures the laboratory data well at intermediate strains, but lies below the data at small strains. Conversely, both the Matasovic and Kavazanjian (1998) and Idriss et al. (1995) curves capture the data better at small strains, but fall above the laboratory data at larger strains. The relatively constant values of λ at smaller strains observed in the laboratory data are consistent with the analytical results by Elgamal et al. (2004), wherein a 5% constant λ for strains up to 0.01% for the waste of the OII landfill was estimated. In addition, small-strain material damping values estimated in two landfills in Atlanta, Georgia by Rix et al. (1998) using the SASW method and a technique that allowed the simultaneous inversion of surface wave velocity and attenuation measurements ranged between 1.5%–7.5% for the Bolton Landfill and 5.5%–10% for the Sanifill Landfill.

The results of this laboratory testing program of MSW are also consistent with the limited laboratory results available in the literature. The range of triaxial test results performed by Towhata et al. (2004) on MSW from Germany is shown in Fig. 13*b*. Tests were only performed at shear strains of approximately 0.3%. The higher values of λ within the range shown represent tests on specimens that included only the finer soil-like material; whereas lower λ values were measured when plastic fibers were added. The results of Towhata et al. (2004) are in good agreement with the results of the present study, with the higher λ values being in agreement with the results of this study for specimens with 100% smaller than 20 mm material, and the lower λ values being in agreement with the results for specimens with 62%–76% and 8%–25% smaller than 20 mm material. Conversely, the large-scale cyclic simple shear test results on waste from the OII landfill by Matasovic and Kavazanjian (1998) that included mostly more than 80% by weight finer material have systematically higher λ values than those measured in this study for specimens with 100% smaller than 20 mm material from the Tri-Cities landfill. The trends in the data are consistent, but offset in magnitude with the Matasovic and Kavazanjian (1998) laboratory data falling above their own recommended material damping curve.

It is judged that the proposed set of shear modulus reduction and material damping relationships can be used to select landfill-specific relationships based on waste composition and confining stress for use in dynamic analyses of MSW landfills. For example, as previously mentioned, waste characterization of the samples from the Tri-Cities landfill indicated that the placed waste material consists of approximately 50%–75% by weight of smaller than 20 mm material. Thus, for the performance of dynamic analyses of this landfill, the best-estimate curves would be those recommended for 62%–76% smaller than 20 mm material at depths consistent with the confining stress ranges shown in Fig. 12.

As previously noted, moisture content was not evaluated as part of this study, and all the tests were performed at a field moisture content that was approximately 10%–25% depending on the waste composition and depth, i.e., below the waste field capacity. This range of moisture contents is representative of landfills where the introduction of liquids are restricted (“dry tomb” concept). The effects of higher moisture content on the dynamic properties of MSW is largely unknown at present. In addition, the Tri-Cities landfill waste was generally found to be relatively undegraded waste, even the oldest waste, which was darker in color. Thus, the effects of waste decomposition on the properties of MSW could not be investigated in this study. It may be postulated, however, that as degradation occurs, the fiber composition of the waste material will reduce, resulting in a material response closer to that observed for the waste that included material that was mostly smaller than 20 mm. The occurrence and rate of degradation are dependent on a number of factors including climate, landfill operation procedures, presence of moisture, and composition of waste. So, considering all these factors may prove difficult. Further research is required to evaluate the potential effects of moisture content and degradation on MSW properties.

Scale effects – effects of processing of waste

Cyclic triaxial tests were also performed on small diameter (68 mm) waste specimens. Tests were initially performed on specimens with 100% smaller than 20 mm material. The estimated $G_{\max}$ and $G/G_{\max}$ and λ curves were found to be essentially the same as those developed from the large triaxial specimens that included 100% smaller than 20 mm material. For example, shear modulus reduction data from both the small and large CTX devices are shown to be consistent in Fig. 14*a*.

Cyclic triaxial tests were performed near the end of the testing program on processed waste. Processing involved the careful reduction of the size of the larger than 20 mm waste fibrous material (i.e., paper, plastic, and wood) to particles with a maximum size of about 20 mm. In addition, gravel material was replaced with material smaller than 20 mm. A series of CTX tests were performed on specimens with 62% and 14% smaller than 20 mm material with the large CTX tests performed on unprocessed waste and the small CTX tests performed on processed waste with identical initial composition. The results indicate that processed waste had important differences in the dynamic properties when compared to the waste tested in the large diameter specimens. For example, as shown in Fig. 14*b*, the curves for the processed waste do not shift to the right as much as the unprocessed waste tested in the large CTX device, although the $G/G_{\max}$ curves of the processed waste exhibit the same trend of the volumetric threshold strain increasing as the amount of the fibrous materials increases. This difference is more pronounced for specimens with greater amounts of material larger than 20 mm, which tends to be fibrous in nature. Additionally, the material damping curves were also found to be different, as shown in Fig. 14*c*. The small CTX tests on processed waste exhibit the same trend of reduction in material damping at large strains as the composition in fibrous materials increases. However, processed waste appears to yield lower λ than the original waste tested in the large-scale devices with the same composition in fibrous material. Lastly, as shown in Fig. 14*d*, significant differences in the measured $G_{\max}$ for the processed and unprocessed waste were observed. The $G_{\max}$ for the processed waste was measured to be nearly twice that for the unprocessed waste tested in the large CTX test device.

These results suggest that testing of small specimens with processed waste cannot completely capture the dynamic properties of the original waste material with larger sized particles. However, while not fully representative of the original waste, tests on processed waste can still provide insight on the dynamic response of waste material, as some key aspects, such as the effect of the fibrous nature of larger waste particles, can be partially captured. This comparison suggests that the trends of increased volumetric threshold strain (i.e., a more linear normalized shear modulus reduction curve and less material damping) at large strains with increasing composition in fibrous material observed in the large CTX testing is largely a result of two factors: (*i*) the fibrous nature of the larger than 20 mm material, and (*ii*) the relative size of the fibers with respect to the matrix material. Future research may provide a better understanding of these issues.

Fig. 14. Comparison of test results for small and large cyclical triaxial (CTX) test specimens of municipal solid waste (MSW): (*a*) $G/G_{\max}$ versus γ for 100% smaller than 20 mm material, (*b*) $G/G_{\max}$ versus γ for processed and unprocessed waste, (*c*) material damping versus γ for processed and unprocessed waste, and (*d*) small-strain shear modulus for processed and unprocessed waste.

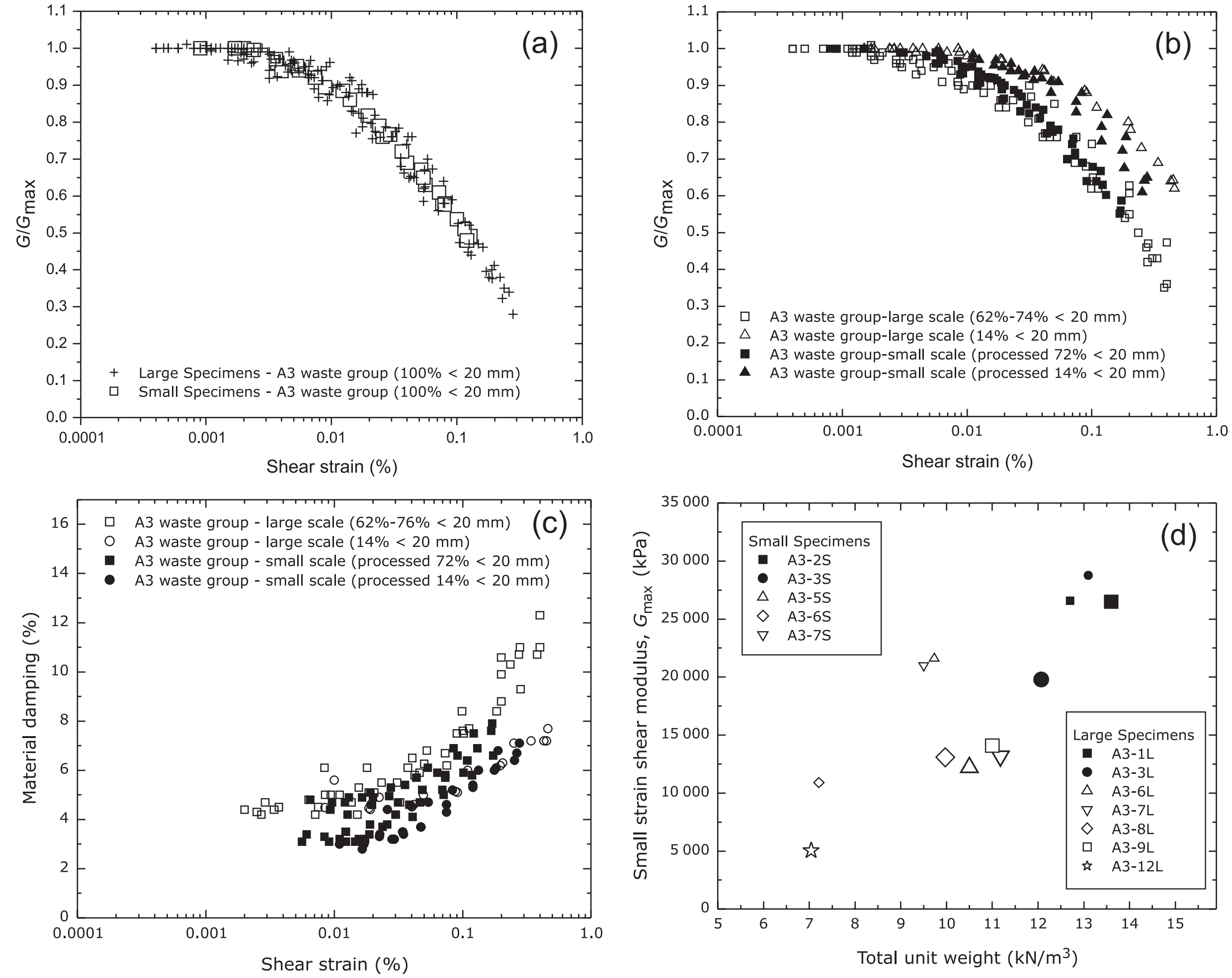


Table 5. Effect of different parameters on the dynamic properties of municipal solid waste (MSW).

	Small-strain shear modulus, $G_{\max}$	Normalized shear modulus reduction curve, $G/G_{\max}$ versus γ	Material damping curve, λ versus γ
Composition	Most important	Most important	Most important
Confining stress	Important	Important	Likely important
Unit weight	Important	Not important	Not important
Confinement time	Important	Not important	Not important
Loading frequency	Less important	Not important	Not important

Conclusions

More than 90 cyclic triaxial tests have been performed on 300 mm diameter specimens from three different sample groups of solid waste from the Tri-Cities landfill, and the effects of different parameters on the dynamic properties of MSW were investigated. The relative importance of these parameters on the small-strain shear modulus, shear modulus reduction, and material damping of MSW are summarized in Table 5.

Of the parameters investigated, waste composition was found to have the most important effect on the dynamic properties of MSW. As the relative amount of fibrous material that is larger than 20 mm increases in the MSW, the absolute value of $G_{\max}$ reduces, the normalized shear modulus reduction curve shifts to the right (i.e., volumetric threshold strain increases), and the material damping at larger strains reduces significantly. The value of $G_{\max}$ is also significantly affected by confining stress, by the unit weight, and by the time under confinement, and to a lesser degree by loading frequency. The strain-dependent normalized shear modulus reduction and material damping relationships are affected primarily by waste composition (i.e., amount of fibrous material larger than 20 mm) and confining stress. They are not significantly affected by unit weight, time under confinement, and loading frequency. The normalized shear modulus reduction curves shift to the right as the amount of material larger than 20 mm increases or as confining stress increases. Correspondingly, material damping reduces at larger strains as the amount of material larger than 20 mm increases or as confining stress increases.

The laboratory test results from this investigation were found to be in good agreement with the limited laboratory and in situ test results available in the literature and to some degree consistent with some of the recommended relationships estimated through back-calculation of the seismic response of the OII landfill. Smaller-scale CTX tests on processed waste captured only part of the observed influence of waste composition on strain-dependent shear modulus reduction and material damping responses observed on the large-scale CTX tests on unprocessed waste. The small-strain shear modulus of processed waste was measured to be significantly greater than that measured on unprocessed waste. The amount, characteristics, and relative size of the fibrous materials in the waste were found to be important, and tests on processed waste cannot fully capture all of these influences.

Strain-dependent normalized shear modulus reduction and material damping ratio relationships for MSW, which are a function of waste composition and confining stress, are recommended for the performance of dynamic analyses of MSW landfills. An important advantage of the proposed set of relationships is that they provide a basis for the selection of the most appropriate dynamic curves for use in engineering analyses of a specific landfill. Future large-scale laboratory testing on waste material from additional landfills, particularly landfills with old, degraded waste in different geographic regions and climatic conditions, will help clarify the general applicability of the recommended curves.

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