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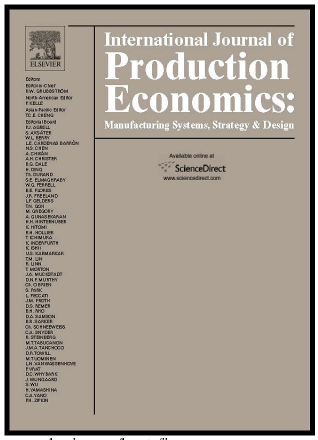

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A Revenue-Sharing Option Contract toward Coordination of Supply Chains

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Abstract

In this paper, a novel mixed revenue-sharing option contract is introduced to coordinate a retailer-manufacturer supply chain. A European call option mechanism and a revenue-sharing mechanism are combined to cover drawbacks of the classic contracts. The option can increase the profit of the chain and the revenue-sharing can reduce double marginalization effects. In addition, an instantaneous purchase and a shortage penalty mechanism are introduced. The proposed mixed contract is modeled through a game theoretic approach to examine several possible situations in order to obtain the order quantity of the retailer and the production quantity of the manufacturer in the Nash equilibrium. Also, both the retailer and the manufacturer are considered as the leader of the chain to recommend an appropriate contract conditions for various types of industries and markets. Finally, the best conditions for achieving the supply chain coordination are provided in different situations. Results also demonstrate that the mixed contract dominates a wholesale and a basic option contract. The proposed coordination mechanism is applied in a real fashion apparel supply chain in Iran and a comprehensive sensitivity analysis is implemented on some parameters of the contract to provide some managerial insights for the supply chain members.

Keywords: Option contract; revenue-sharing contract; mixed revenue-sharing option contract; game theory; supply chain coordination; double marginalization.

1. Introduction

Supply management plays a remarkable role in a supply chain (SC) because it influences profit of members of the SC. Today's fluctuating economies require considering uncertainties and risks associated with several parameters. Several types of risks have been identified threatening SCs from several resources while an appropriate risk hedging strategy mitigates the effects of them through following two stages: recognition of source of uncertainty in the network and individualization of the most correct way for reducing such level of uncertainty (Cucchiella & Gastaldi 2006). Risks in SCs have been classified from different points of view. Supply network design, supplier relationship, supplier selection process, supplier order allocation and supply contract are the most critical areas which risk hedging policies could be applied (Tang 2006). The current study focuses on the supply contract as a critical area in a SC since it can improve the profit of the whole SC.

The supply contracts indicate the parameters (such as quantity, price, time and quality) for a supplier to satisfy the demand of a buyer (Gan et al. 2009). In other words, a supply contract is a coordination mechanism that provides incentives to all the parties so that the decentralized chain behaves as nearly or exactly the same as the integrated one (Tsay 1999). In traditional supply contracts, partners across the SC make decisions, independently, or make their decisions so that maximize their own profit. Both of these strategies can lead to local optimum solutions which may provide lower total profit for the SC (Bresnahan & Reiss 1985; Lee et al. 1997; Li et al. 2013; Corbett et al. 2004). Other issue which is created by local optimum decisions is double marginalization. It occurs if both parties charge a markup which will result a higher final retail price and a lower total demand in comparison with vertically integrated contracts. In contrast, SC coordination improves the profit of the SC. To achieve a perfect coordination, participants must work as a unified system. SC coordination could be achieved via several approaches including SC contracts, information technology, information sharing and joint decision making (Kanda & Deshmukh 2008). In this paper, the SC is coordinated via a supply contract.

Several types of coordination contracts have been introduced in the literature. The basic contracts include buyback, revenue-sharing, and quantity flexibility on which other types of contracts are based (Chopra & Meindl 2007). In a revenue-sharing contract, the supplier sells products to the buyer at a low wholesale price and gets a fraction of revenue of the buyer instead. The concept of the revenue-sharing mechanism is incorporated in the proposed contract in this paper.

In addition to the common contracts listed above, option contract as another coordination mechanism has been recently introduced (Huang 2009; Zhao, Yang, et al. 2013; Gomez_Padilla & Mishina 2009; Burnetas & Ritchken 2005; Nomikos et al. 2013). The option contract is based on a financial derivative called an option. Large amount of uncertainties of prices in the financial markets has made the financial risk hedging tools the most powerful ones. Therefore, the option mechanism, as the most well-known financial derivative, is used as a supply contract in operations management. Two types of the option contracts are put and call options. A call option gives the holder of the option the right to buy an asset by a certain date at a certain price (Hull 2012). The important parameters of the option contract are an expiration date (maturity) of the contract, an option price as a fee that the buyer pays for each purchased contract, and exercise price as the final price of the underlying commodities which the buyer will pay at the time of exercising. In a European option, the buyer could exercise the option only at the maturity of the contract. Unlike the financial options, which use the price feasibility as the condition of an option exercising, the real options use both the existence of demand and the price feasibility as the sufficient conditions of an option exercising. To mitigate the double marginalization effects, which could not be avoided by the option mechanism, a revenue-sharing mechanism is incorporated into the option mechanism. Moreover, two novel mechanisms are added to the basic definition of the contracts. The first one allows the retailer to purchase the extra products of the manufacturer in a higher price during the selling season. The second one fines the manufacturer if he cannot meet his commitments to the retailer.

In this paper, a mixed mechanism is introduced which aims at coordination of a retailer-manufacturer SC. In the proposed framework, the demand and the price of the underlying commodity are assumed to be stochastic. In the proposed contract, the manufacturer charges a low exercise price and gets a fraction of the revenue of the retailer at the end instead. The coordination mechanism is modeled as a Stackelberg game in which the order quantity of the retailer and the production quantity of the manufacturer are strategies of the members. The Nash equilibrium of this game provides optimal strategies of the parties. Moreover, as the optimal strategies of the parties strongly depend on the leader of the chain, both the retailer-led and the manufacturer-led situations are modeled. The mixed coordination mechanism is applied in real-world case study. A fashion apparel SC is considered in which a company produces clothing (i.e., the manufacturer) and another company sells the products in the final market (i.e., the retailer). A hypothetical centralized SC is also investigated to show the perfect coordination situation. Moreover, a wholesale contract is applied to the SC to obtain a nadir solution for the coordination problem. The final results are compared with the centralized SC and the wholesale contract to evaluate the performance of the novel mixed contract. A number of sensitivity analysis are also implemented to validates the performance of the model and provide some managerial insight for decision makers.

Remaining of the paper is organized as follows. In Section 2, a review of the literature in several types of contracts is presented. In section 3, the game theoretic model is presented in cases of the retailer-led, the manufacturer-led, and the centralized SC. The mixed contract is analyzed in a real-world case study in fashion apparel sector in Section 5. A comprehensive sensitivity analysis is also implemented in this section and some concluding remarks and directions for future extensions are presented in the last section.

2. Literature Review

The critical role of the supply contracts in economic relations causes a new research stream in the SC management. Although several types of contracts are introduced in the body of literature in economic studies, few previous studies investigate the supply contracts within the SC settings. Review papers in supply contracts have categorized them from several points of view. Tsay et al. (1998) addresses a qualitative overview of several types of the contracts under the deterministic or stochastic demand. Also, Lariviere (1998) provides a quantitative analysis of various types of contracts when the demand is random. SC coordination conditions and the role of supply contracts to achieve coordination are addressed in Cachon (2003). Although all the basic contracts create some advantages for the parties, there are some shortcomings which may stimulate the parties to use wholesale contracts without any coordination mechanism. Therefore, hybrid contracts are proposed to remove the shortcomings of the pure basic contracts. In this regard, the literature review is divided to three following parts: option contracts, revenue-sharing contracts and hybrid contracts.

2.1. The option mechanism

Option mechanism as the most well-known financial derivative, has been recently utilized as a risk-hedging contract in SCs. A growing number of studies have investigated it under different assumptions while in the most of them the SC is assumed to involve two echelons. Wang & Tsao (2006) introduce a single-period bidirectional option contract within which the retailer can increase or decrease the order quantity after the demand realization. They analyze the problem from the buyer's perspective and show that their proposed contract can increase the profit of the buyer. A retailer-led option contract is modeled by Wang & Liu (2007) in which the retailer aims to indicate the upstream production quantity via the proposed contract. Their results show that the proposed contract improves profit of both parties of the SC. Jiao et al. (2007) model several types of uncertainties in a production environment of a flexible manufacturing system through an option contract. They model the uncertain demand as a stochastic process which is known as Geometric Brownian Motion. Another two-echelon SC is proposed by Gomez_Padilla & Mishina (2009) in which an option contract is used to coordinate the SC in two following cases: single supplier-single retailer and multiple suppliers-single retailer. Their results demonstrate improvement in profit of the chain as well as that of both parties. A cooperative game theory approach to the option mechanism is addressed in Zhao et al., (2010) within which the retailer and the manufacturer adopt the order and the production quantity, simultaneously. They utilize a wholesale mechanism as a benchmark for the proposed contract to select the best parameters in order to coordinate the chain by a negotiation mechanism. They find out that a higher negotiation power and less risk-aversion resulted more profit for the corresponding party. Wang & Chen (2013) propose a Stackelberg game framework to model an option contract in cases of centralized and decentralized SCs. They demonstrate that the model has a unique solution and the SC could not be coordinated through the option contract. In another study, Zhao, Ma, et al. (2013) introduce a bidirectional option contract as a risk hedging mechanism versus demand random fluctuations. They attain a closed-form solution for the initial order and order exercising quantities. Another recent study on option contract is performed by Chen et al. (2014), which considered the risk preferences of parties within the SC. They investigate the effects of risk preferences on the optimal order and production decisions. Supplier disruption is another type of uncertainty threatening the SC. Xu & Nozick (2009) address a stochastic programming model for supplier selection coordinated with an option contract. They express that the option contract is appropriate for a fluctuating environment while it may cause extra costs, otherwise. Another application of the option contracts is introduced by Liang et al. (2012) who model a relief material SC by means of a special type of option contract. In their proposed framework, the option exercising condition is disaster occurrence whether it is profitable or not. They perform the option pricing through a binomial tree method. In a similar study, Wang et al. (2015) demonstrate that an option contract dominates both the pre-purchasing with buyback and instant purchasing with return policy in a humanitarian SC.

2.2. The revenue-sharing mechanism

Another basic contract considered in this paper as a complementary mechanism is revenue-sharing. A comprehensive study is accomplished by Cachon & Lariviere (2005), within which the benefits and shortcomings of a revenue-sharing contract are enumerated. They investigated the performance of a combination of a revenue-sharing mechanism with other basic contracts. Yao et al. (2008) model a two-echelon SC with one supplier and two competing retailers which are coordinated through a revenue-sharing contract. They utilize Bayesian Nash game to attain the equilibrium according to the demand variability and price-sensitivity factor. The results confirm the performance improvement by using the revenue-sharing contract in a competitive environment. Another study, which investigates the revenue-sharing mechanism to coordinate SCs, is performed by Linh & Hong (2009), where the revenue-sharing ratio and wholesale price are determined through a two-period newsboy problem. They find out that the wholesale prices are set to be lower than the retail prices and the optimal revenue-sharing ratio is increasing in the wholesale prices. A game theory approach is used by Palsule-Desai (2013) to model a novel revenue-dependent revenue-sharing contract to coordinate a SC wherein the actual revenue-sharing ratio depended on the quantum of revenue generated. They prove that the revenue-dependent contract is preferred to a revenue-independent one because of surplus gain could be achieved from revenue-dependency. A recent study on a two-way revenue-sharing contract in a dual channel SC is performed by Xu et al. (2014) where all the members are assumed to be risk-averse. They demonstrate how risk preferences could change the parameters of the proposed coordination contract. The reader can find other studies in the revenue-sharing contract in Giannoccaro & Pontrandolfo (2004), Gerchak & Wang (2004), Wang et al. (2004), and Chakraborty et al. (2015).

2.3. The combined contracts

Due to the drawbacks of the basic contracts, hybrid contracts have attracted attention of academicians and practitioners. Also, the coordination contracts could be combined with other contracts to increase the flexibility of the business relations. Here, we focus on the hybrid contracts which include option and revenue-sharing contracts. It is worth mentioning that a number of these studies have investigated a contract selection between different types of contracts. A wholesale-option contract is proposed by Xu (2010) while both of the supplier's yield and market demand are assumed to be random. Also a similar study is performed by Buzacott et al. (2011) with supply and demand-side uncertainty while a mean-variance approach is considered instead of an expected value one. Xia et al. (2011) compare an option contract with a wholesale one. They find the disruption risk reduction as the most important strength of the option versus the wholesale contract. They demonstrate that the supplier with a higher disruption risk level provides more profit while profit of a reliable supplier depends on the contract type. A similar wholesale-option contract is addressed by Chen & Shen (2012) in which the order quantity of the retailer and the production quantity of the manufacturer are decided so that satisfy a service requirement. They mention that the expected profit of the retailer is non-increasing in the service requirement whereas the supplier's expected profit is non-decreasing in it. Another mixed wholesale-option contract is proposed by Jörnsten et al. (2013) with similar assumptions and under random discrete demand condition. They demonstrate that the mixed contract is preferred to a pure option contract when the manufacturer had a bound on how much variance he/she is willing to accept and is risk-averse. The reader can find more studies considered the option and the wholesale contracts in Cheng et al. (2003) and Burnetas & Ritchken (2005).

Unlike the previous studies, which considered option and wholesale contracts as components of the combined contract, following studies utilize other contracts for the combination. A vulnerable option contract is introduced by Babich (2006) in which both parties could postpone their decisions where the price is assumed uncertain and the demand is deterministic. They consider two competing suppliers while both of them may be disrupted. They, finally, investigate effects of suppliers' default risk and competition on the key decisions of both parties. Alongside the option, forward is another financial derivative which might be combined with an option contract in SC management. Li et al. (2009) introduce a combination of a forward and an option mechanism in order to coordinate the SC under the demand and the price uncertainties. They assume an asymmetric information environment because of the closeness of the retailer to the market. A comparison between option contract and an advance price discount contract is performed in Liu et al. (2014). They investigate risk hedging and channel coordination through both of them while the manufacturer is loss-averse. Another mixed contract is proposed by Sarathi et al. (2014) who tries to coordinate the SC via a combined revenue-sharing-quantity discount contract. They also assume that the demand is price sensitive and stock dependent. The reader is referred to Gerchak & Wang, (2004), Xiong, Chen, & Xie, (2011), Y. Xu & Bisi (2012), Zhang (2013) and Liu et al. (2013) to find other mixed contracts.

According to the presented comprehensive literature review, following drawbacks are extracted. In studies considered option contract, double marginalization issue has been neglected while the option contract cannot mitigate this effect. In addition, a game theory approach is not commonly used in previous studies. This point shows that a main side of the contract is neglected while it is not at all a realistic assumption. Although different situations are conceivable in the real SCs including retailer-led, manufacturer-led, previous studies just considered one of them commonly, the retailer-led. Also, most of previous studies considered the price of the product as a deterministic parameter which is an unrealistic assumption. To cope with mentioned drawbacks, this paper proposes a mixed revenue-sharing option contract.

Remarkable novelties of this study could be summarized as follows. Firstly, a novel mixed contract is proposed for the first time consists of a revenue-sharing mechanism and a European call option mechanism. The option mechanism is used to coordinate the SC through increasing profit of the whole SC and the revenue-sharing is mainly used to mitigate the double marginalization effect. Therefore, the proposed mixed contract can dominate other option contracts. Secondly, a Stackelberg game framework is used to model the real-world situation in which the contract could be applied. Unlike the most of the previous studies, the production strategy of the manufacturer is adopted through the game. Thirdly, both possible situations including a retailer-led and a manufacturer-led SC are taken into consideration. The best strategy according to the preferences of the leader is indicated in each situation, which makes the application of the model wider. Fourthly, a novel instantaneous purchase and shortage penalty mechanisms are proposed to improve the performance of the contract. Instantaneous purchase occurs when the production quantity is greater than the order quantity and the instantaneous purchase is feasible for the retailer. Shortage penalty mechanism is used when the production quantity is less than the order quantity and the demand is greater than the production quantity. In this situation, the manufacturer should pay a penalty to the retailer. The shortage penalty forces the manufacturer to produce enough products to satisfy the market demand as much as possible. Finally, a real-world case study of a fashion apparel SC in Iran is considered to show the applicability of the mechanism.

3. Decentralized supply chain

Considering conditions of the real world contracts, there are sequential positions for decision makers to indicate the production and the order quantities within the contract. Also, most of commercial organizations try to maximize their benefit regarding other participants in the contract. In other words, they indicate the production and order quantities taking other participants' decisions into account. Therefore, a dynamic game theory approach has been used in order to model the proposed revenue-sharing option contract, namely a Stackelberg game.

A two-echelon single product SC has been considered including a retailer and a manufacturer coordinated via a revenue-sharing option contract. The proposed SC tries to satisfy the random demand, ξ, with a cumulative distribution function $F(\xi)$. Demand information is assumed to be symmetric. Notations that are used in the proposed mathematical model are introduced as follows:

Parameters:	
ρ	Unit market price of the underlying product which is a random variable with $(.)$ and $g(.)$ as the cumulative distribution function and the density function, respectively. $\bar{\rho}$ is expected value of the market price.
w	Unit price of the product in the instantaneous purchase mechanism
b	Unit shortage cost
β	Penalty of the manufacturer due to the lack of adherence to commitments
r	Fraction of revenue-sharing
ζ	Market demand of the underlying product which is a random variable with $F(.)$ and $f(.)$ as the cumulative distribution function and the density function, respectively
e	Unit exercise price of the option contract
o	Unit option price
c	Unit production cost
υ	Unit salvage value of the product at the end of the contract
θ	Wholesale price in the wholesale mechanism
Variables:	
Q_M	Production quantity of the manufacturer
Q	Order quantity of the retailer
Q_S	Production quantity of the centralized SC

There are some constraints which should be guaranteed to avoid trivial and unreasonable cases in the proposed contract. 1) $o + \upsilon < c$ which prevents the manufacturer to arbitrage with the option, 2) $(1 - r)\bar{\rho} + b - e > e$ which avoids unreasonable situation in which the retailer prefers returning products and encountering with shortage rather than satisfying the market demand, 3) $r\bar{\rho} + \beta - c > c - \upsilon$ which ensures that the participating in the contract is profitable for the manufacturer, 4) $o + e < \bar{\theta}$ which ensures that the retailer prefers to use option contract rather than the wholesale mechanism, 5) $o + \upsilon < e$ avoids unreasonable cases in which the manufacturer prefers to sell the products in salvage value rather than satisfy the options purchased by the retailer, 6) $o + e < w$ that prevents the retailer to use mainly the instantaneous purchase mechanism instead of option mechanism, 7) The probability density function of market demand is assumed to be downward sloping ($f'(.) \leq 0$), 8) The probability of instantaneous purchase is assumed to be less than 0.5 which means $\alpha > 0.5$, and 9) All the cost coefficients are assumed greater than zero.

The considered two-echelon SC might be a retailer-led or a manufacturer-led one. If the manufacturer has a unique product or technology, he will enjoy a monopolistic market. In this case, the manufacturer-led SC is used. On the other hand, as the markets become more customer-oriented, the power of the retailers becomes more. Therefore, nowadays, most of SCs are retailer-led and that is the reason why this case is modeled in this paper. The retailer-led SC and the manufacturer-led SC are taken into consideration in Sections 3.1 and 3.2, respectively.

3.1. Retailer-led supply chain

In this section, the market is assumed to be customer-oriented. Therefore, the retailer is considered to be the leader of the chain because of being closer to the market. The retailer adopts his order strategy, and then the manufacturer adopts his production strategy according to the retailer's. Sequence of events in the proposed mechanism is as follows: firstly, the retailer, as the leader of the chain, decides the order quantity of options, Q, and pays the option price, o, to the manufacturer. Secondly, the manufacturer decides the production quantity, Q_M, according to the contract parameters and the order quantity of the retailer. Thirdly, after demand realization, the retailer exercises the purchased options at exercise price, e, according to the realized demand ξ and initially purchased options. Fourthly, in case of $Q_M \geq Q$, the retailer can purchase the extra products of the manufacturer, instantaneously, according to the realized demand quantity in price w. To do so, the retailer compares the possible losses from instantaneous purchase with lost sale. If instantaneous purchase is beneficial, the retailer will purchase the extra products of the manufacturer. Otherwise, the retailer will reject the extra orders of customers. In case of $Q_M \leq Q$, the retailer will exercise the options up to Q_M and if $\xi \geq Q_M$, the manufacturer should pay a penalty (β) for shortage. Fifthly, the manufacturer can sell the extra products to the market in salvage value υ. In terms of revenue-sharing mechanism, the manufacturer sells the product at a lower exercise price in comparison with the classic option contract and the retailer pays a fraction of his revenue, r, to the manufacturer. It is worth mentioning that in financial markets, there are standard couples of the option price and the exercise price. Therefore, they are considered input parameters in this problem. It is also worth mentioning that two cases of $Q_M \geq Q$ and $Q_M \leq Q$ are the initial assumption and the retailer or the manufacturer does not decide which case is used in decision making. According to the illustrated mechanism and dynamic game theory concepts, the utility function of the retailer is expressed in Eq. (1).

$$\begin{aligned}
\Pi_R &= ((1-r)\rho - e)\min\left\{\min\left\{Q_M, Q\right\}, \zeta\right\} - oQ + \beta\min\left\{\max\left\{\zeta - Q_M, 0\right\}, \max\left\{Q - Q_M, 0\right\}\right\} + \\
&Pr(\rho - w \geq -b)\left[((1-r)\rho - w)\min\left\{\max\left\{\zeta - Q, 0\right\}, \max\left\{Q_M^*(Q) - Q, 0\right\}\right\}\right. \\
&\left. -b\max\left\{\zeta - Q_M^*(Q), 0\right\}\right] + Pr(\rho - w \leq -b)\left[-b\max\left\{\zeta - \min\left\{Q_M, Q\right\}, 0\right\}\right]
\end{aligned} \tag{1}$$

The first term is the profit of selling the products to the market. Second term is initial price of the purchased options. The third one shows penalty of the manufacturer in case of shortage and $Q_M \leq Q$, which should be paid to the retailer. The fourth term is a summation of the profit of the retailer from instantaneous purchase and hid shortage when the instantaneous purchase is beneficial. The fifth term show shortage cost of the retailer in case of infeasibility of the instantaneous purchase. In addition, profit of the manufacturer is modeled as follows:

$$\begin{aligned}\Pi_M = &(r\rho+e)\min\{\zeta,\min\{Q_M,Q\}\}+oQ-cQ_M-\beta\min\{\max\{\zeta-Q_M,0\},\max\{Q-Q_M,0\}\}\\ &+Pr(\rho-w\geq -b)\left[(r\rho+w)\min\{\max\{\zeta-Q,0\},\max\{Q_M-Q,0\}\}+\upsilon\max\{Q_M-\zeta,0\}\right]\\ &+Pr(\rho-w\leq -b)\left[\upsilon\max\{Q_M-\min\{\zeta,\min\{Q_M,Q\}\},0\}\right]\end{aligned} \quad (2)$$

where the first term is a summation of the revenue of the manufacturer from option exercising and the fraction of retailer's revenue which is shared with the manufacturer. The second term is the initial price of the purchased options. The third term is the production costs of products. The fourth term is the penalty of shortage in case of $Q_M \leq Q$. The fifth term is a summation of revenue from instantaneous purchase and salvage of extra products when the instantaneous purchase is beneficial and the last term illustrates the salvage value of extra products when the instantaneous purchase is not beneficial.

As seen in Eq. (1) and Eq. (2), there are complicated terms such as $min\{Q, Q_M\}$ which raise computational complexity of the model, significantly. Therefore, two cases are considered; 1) the manufacturer produces more than the order quantity of the retailer and 2) vice versa. Computational complexity of the problem could be decreased by this assumption and it divides the problem to two sub problems.

In the retailer-led supply chain, firstly, the retailer decides the order quantity. Then, the manufacturer decides the production quantity so that satisfies the constraint $Q_M \geq Q$ or $Q_M \leq Q$ in each case. Also, in the manufacturer-led supply chain, the manufacturer decides the production quantity, firstly. Then the retailer decides the order quantity so that satisfies the constraints $Q_M \geq Q$ or $Q_M \leq Q$ in each case.

3.1.1. Case 1: $Q_M \geq Q$

In the first case, the production quantity of the manufacturer is assumed greater than the order quantity of the retailer. According to the proposed coordination mechanism, the retailer can exercise whatever is necessary according to the realized demand up to initial purchased options. Sequence of events in the retailer-led supply chain in case of $Q_M \geq Q$ is as follows: firstly, the retailer decides the order quantity of options. Secondly, the manufacturer decides the production quantity subject to the constraint $Q_M \geq Q$. Thirdly, after demand realization, the retailer exercises the purchased options. Fourthly, the retailer can purchase the extra products of the manufacturer, instantaneously, according to the realized demand quantity. To do so, the retailer compares the possible losses from instantaneous purchase with lost sale. If instantaneous purchase is beneficial, the retailer will purchase the extra products of the manufacturer. Otherwise, the retailer will reject the extra orders of customers. Fifthly, the manufacturer sells the extra products to the market in a salvage value. In terms of revenue-sharing mechanism, the manufacturer sells the product at a lower exercise price in comparison with the classic option contract and the retailer pays a fraction of his revenue, r, to the manufacturer. According to the proposed coordination mechanism illustrated before, profit of the retailer as the leader is calculated in Eq. (3):

$$\begin{aligned}\Pi_R &= -e\min\left\{\zeta,Q\right\} - oQ + Pr(\rho - w \geq -b)\Big[(1-r)\rho\min\left\{\zeta,Q_M^*(Q)\right\} \\ &-w\min\left\{\max\left\{\zeta - Q,0\right\},\max\left\{Q_M^*(Q)-Q,0\right\}\right\} - b\max\left\{\zeta - Q_M^*(Q),0\right\}\Big] \\ &+Pr(\rho - w \leq -b)\Big[(1-r)\rho\min\left\{\zeta,Q\right\} - b\max\left\{\zeta - Q,0\right\}\Big]\end{aligned} \quad (3)$$

where the first and the second terms are exercising and initial ordering cost of options. Revenue of the retailer from selling the products to the market is expressed in the third and the sixth terms of the equation in two different events. In the case of profitability of instantaneous purchase with probabilityPr$(\rho - w \geq -b)$, the retailer purchases extra products of the manufacturer according to the realized demand level at price w which its cost is mentioned in the fourth term. In the case of infeasibility of instantaneous purchase with probability Pr$(\rho - w \leq -b)$, the retailer does not use instantaneous purchase. The fifth and the seventh terms of the equation are shortage cost of the retailer in the two illustrated events, respectively.

Additionally, profit of the manufacturer as the follower is expressed in Eq. (4):

$$\begin{aligned}\Pi_M(Q_M) &= e\min\left\{\zeta,Q\right\} + oQ - cQ_M + Pr(\rho - w \geq -b)\Big[r\rho(\min\left\{\zeta,Q_M\right\}) \\ &+w\min\left\{\max\left\{\zeta - Q,0\right\},\max\left\{Q_M - Q,0\right\}\right\} + \upsilon\max\left\{Q_M - \zeta,0\right\}\Big] \\ &+Pr(\rho - w \leq -b)\Big[r(\rho\min\left\{\zeta,Q\right\}) + \upsilon\max\left\{Q_M - \min\left\{\zeta,Q\right\},0\right\}\Big]\end{aligned} \quad (4)$$

where the first and the second terms are the revenue from exercising the purchased options and initial price of them, respectively. The third term is the production cost. The fourth and the seventh terms of the equation are fractions of retailer's revenue which are shared with the manufacturer according to the used revenue-sharing mechanism in two illustrated events for the retailer. The fifth term is the revenue from instantaneous purchase in the case of profitability, and finally, the sixth and the eighth terms are salvage value of extra products for the manufacturer in the two above mentioned events, respectively. Nash equilibrium is obtained from proposition 1.

Proposition 1. In a revenue-sharing option contract with given (r, o, e), $Q_M \geq Q$, and $\frac{2\alpha}{1-\alpha} < \frac{r\bar{\rho}+w}{\upsilon}$ while the retailer is leader of the chain, and let $\Pr(\rho - w \leq -b) = \alpha$:

I. Optimal order quantity of the retailer, Q^*, is found from the solution of the following equation:

$$(-e+(1-\alpha)w+\alpha((1-r)\bar{\rho}+b))(1-F(Q))+(1-\alpha)((1-r)\bar{\rho}-w+b)\int_{Q_M^*(Q)}^{\infty} Q_M^{*}{}'(Q)dF(\zeta)-o=0$$

(5)

where, $Q_M^*(Q)$ denotes optimal production strategy of the manufacturer which is a function of Q and $Q_M^{*\prime}(Q)=\frac{\partial Q_M^*}{\partial Q}$. Q_M^* is resulted from Eq. (7).

II. Let Q_M' be obtained from solving following equation:

$$(1-\alpha)(-r\bar{\rho}-w)[1-F(Q_M)]+\upsilon F(Q_M)+\alpha\upsilon[Q_M-Q]f(Q_M)-(1-\alpha)\upsilon Q_M f(Q_M)=c \quad (6)$$

Optimal production quantity of the manufacturer is obtained from the following equation:

$$Q_M^* = \begin{cases} Q_M', & Q^* < Q_M' \\ Q^*, & Q^* \geq Q_M' \end{cases} \quad (7)$$

Proof. See Appendix A. ,

Eqs. (5) and (6) in proposition 1 are the first-order optimality condition of the members' profit function. These equations cannot be solved analytically. Therefore, achieving the Nash equilibrium of the game depends on solving these equations according to the real parameters and probability functions. To show applicability of this proposition, an exact solution of the problem addressed in the proposition 1 is presented here in which the market demand follows a uniform distribution $D\sim U(a,d)$. The exact solution of Eqs. (5) and (6) according to this assumption is as follows:

$$Q_M^* = \frac{-(1-\alpha)(r\bar{\rho}+w)d+\upsilon(a+\alpha Q)+c(d-a)}{-(1-\alpha)(r\bar{\rho}+w)+2\alpha\upsilon} \quad (8)$$

$$Q^* = \frac{d.Z-\frac{X}{Y}.[2b\alpha^2\upsilon^2-a\alpha\upsilon^2-c\alpha\upsilon(d-a)]+o(d-a)}{Z-\frac{X}{Y}\upsilon^2\alpha^2} \quad (9)$$

where X, Y, and Z are defined as follows. This terms are used to simplify the Eq. (9).

$$X=(1-\alpha)((1-r)\bar{\rho}-w+b)) \quad (10)$$

$$Y=[-(1-\alpha)(r\bar{\rho}+w)+2\alpha\upsilon]^2 \quad (11)$$

$$Z = e - (1-\alpha)w + \alpha((1-r)\bar{\rho} + b) \tag{12}$$

According to this complicated results from the uniform distribution, which is the simplest distribution, more complex distributions need numerical analysis to solve the Eqs. (5) and (6).

3.1.2. Case 2: $Q_M \leq Q$

This case illustrates default risk which may be caused by the upstream firm (e.g. the manufacturers or suppliers). Like the first case, revenue and cost terms of the profit of participants are expressed. But as the production quantity of the manufacturer is assumed to be less than order quantity of the retailer, instantaneous purchase is impossible in this case. Sequence of events in the retailer-led supply chain in case of $Q_M \leq Q$ is as follows: firstly, the retailer decides the order quantity of options. Secondly, the manufacturer decides the production quantity subject to the constraint $Q_M \leq Q$. Thirdly, after demand realization, the retailer exercises the purchased options. Fourthly, the retailer will exercise the options up to Q_M and if $\xi \geq Q_M$, the manufacturer should pay a penalty (β) for shortage. Fifthly, the manufacturer sells the extra products to the market in salvage value. In terms of revenue-sharing mechanism, the manufacturer sells the product at a lower exercise price in comparison with the classic option contract and the retailer pays a fraction of his revenue, r, to the manufacturer. Therefore, profit equations are simpler than those of the first case. In addition, if the retailer exercises the purchased option more than Q_M, the manufacturer will be penalized for the lack of adherence to the commitments. Eq. (13) shows the retailer's profit:

$$\begin{aligned}\Pi'_R &= (1-r)\rho \min\left\{Q_M^*(Q), \zeta\right\} - e\min\left\{Q_M^*(Q), \zeta\right\} - oQ - b\max\left\{\zeta - Q_M, 0\right\} \\ &+ \beta \min\left\{\max\left\{\zeta - Q_M, 0\right\}, \max\left\{Q - Q_M\right\}\right\}\end{aligned} \tag{13}$$

where the first term is the revenue from the sold products. The second and the third terms are the exercise and the option price, respectively, and the last term is the shortage cost of the retailer due to the unsatisfied demand. Eq. (14) illustrates the manufacturer's profit:

$$\begin{aligned}\Pi'_M &= r\rho \min\left\{Q_M, \zeta\right\} + e\min\left\{Q_M, \zeta\right\} + oQ - cQ_M + \upsilon \max\left\{Q_M - \zeta, 0\right\} \\ &- \beta \min\left\{\max\left\{\zeta - Q_M, 0\right\}, \max\left\{Q - Q_M\right\}\right\}\end{aligned} \tag{14}$$

where the first term is the fraction of the retailer's profit, which is paid to the manufacturer. The second and the third terms are revenue from option exercising and initial ordering, respectively. The fourth term is the production costs and the fifth one is the salvage value which is gained from selling the extra products. The last term is the shortage penalty for the manufacturer due to the lack of adherence to the commitments. Nash equilibrium of the dynamic game in this case is provided in proposition 2.

Proposition 2. In a revenue-sharing option contract with given (r, o, e), $Q_M \leq Q$, while the retailer is leader of the chain:

I. Optimal order quantity of the retailer, Q^*, is found from the following equation:

$$Q^* = F^{-1}\left(\frac{\beta - o}{\beta}\right) \quad (15)$$

II. Let Q'_M be the solution of following equation:

$$Q'_M = F^{-1}\left(\frac{r\bar{\rho} + e - c + \beta}{r\bar{\rho} + e - \upsilon + \beta}\right) \quad (16)$$

Optimal production quantity of the manufacturer is calculated as follows:

$$Q^*_M = \begin{cases} Q'_M, & Q^* > Q'_M \\ Q^*, & Q^* \leq Q'_M \end{cases} \quad (17)$$

Proof. See Appendix A. ,

3.2. Manufacturer-led supply chain

Unlike the previous section, the manufacturer could be the leader of the chain in many cases. In monopolistic markets where a company has a unique technology or product, manufacturer plays role of the leader. In addition, unlike the most of customer-oriented markets around the world, there are several cases without this strategy. Thus, current section investigates a manufacturer-led case of the proposed mixed contract via a Stackelberg game in which the manufacturer decides the production strategy, firstly, and the retailer decides the order strategy according to the production strategy of the manufacturer and the market situation.

A general formulation of the profit of both parties has been presented in Eqs. (1) and (2). As mentioned before, because of computational complexity of these functions, the problem is divided into two cases including $Q_M \geq Q$ and $Q_M \leq Q$. It is worth mentioning that the formulations of the two extracted cases are the same as those of the retailer-led SC.

3.2.1. Case 1: $Q_M \geq Q$

Profit functions for the manufacturer and the retailer are presented in Eqs. (3) and (4) when the production quantity of the manufacturer is greater than the order quantity of the retailer. Sequence of events in the manufacturer-led supply chain in case of $Q_M \geq Q$ is similar to the sequence of the problem in the retailer-led supply chain except for the first two steps. In other words, the manufacturer firstly decides the production quantity. Then the retailer decides the order quantity subject to the constraint $Q_M \geq Q$. According to the dynamic game theory concepts, first, the retailer problem is solved regardless of the strategy of the manufacturer, and then the manufacturer problem is solved according to the optimal strategy of the retailer. Nash equilibrium of the corresponding game is resulted proposition 3.

Proposition 3. In a revenue-sharing option contract with given (r, o, e), $Q_M \geq Q$, and $\frac{2\alpha}{1-\alpha} < \frac{r\bar{\rho}+w}{\upsilon}$, where the manufacturer is leader of the chain, then:

I. The optimal production strategy of the manufacturer Q_M^* is obtained by solving the following equation:

$$(1-\alpha)(r\bar{\rho}+w)[1-F(Q_M)]+\upsilon F(Q_M)+\alpha\upsilon[Q_M-Q]f(Q_M)-(1-\alpha)\upsilon Q_M f(Q_M)=c \quad (18)$$

II. Let Q' be the solution of the following equation:

$$Q' = F^{-1}\left(1-\frac{o}{-e+(1-\alpha)w+\alpha\left((1-r)\bar{\rho}+b\right)}\right) \quad (19)$$

The optimal order quantity of the retailer in Nash equilibrium is obtained from the following equation:

$$Q^* = \begin{cases} Q', & Q' < Q_M^* \\ Q_M^*, & Q' \geq Q_M^* \end{cases} \quad (20)$$

Proof. See Appendix B. ,

Because Eq. (18) does not have a closed-form solution, to show the applicability of this model, a special case of this proposition is presented here. Like the case presented after proposition 1, the market demand is assumed to follow a uniform distribution $D \sim U(a, d)$. According to this assumption, the exact solution of Eqs. (18) and (19) is as follows:

$$Q_M^* = \frac{-(1-\alpha)(r\bar{\rho}+w)d+\upsilon(a+\alpha Q^*)+c(d-a)}{-(1-\alpha)(r\bar{\rho}+w)+2\alpha\upsilon} \quad (21)$$

$$Q' = \left[1-\frac{o}{-e+(1-\alpha)w+\alpha\left((1-r)\bar{\rho}+b\right)}\right](d-a)+a \quad (22)$$

Since Eq. (5), (6), and (18) do not represent an explicit solution for the strategies of the members, some more explanations are presented here. Because the profit functions for the propositions 1 and 3 are the same, so the only factor that can make their solution different is the sequence of decision making. Eq. (5) could be restate as follows:

$$Q' = F^{-1}\left(1 - \frac{o}{-e+(1-\alpha)w+\alpha\left((1-r)\bar{\rho}+b\right)} + \frac{(1-\alpha)((1-r)\bar{\rho}-w+b)\int_{Q_M^*(Q)}^{\infty} Q_M^{*\prime}(Q)dF(\zeta)}{-e+(1-\alpha)w+\alpha\left((1-r)\bar{\rho}+b\right)}\right) \quad (23)$$

A comparison between Eq. (23) and Eq. (19) shows the difference between the strategy of the retailer in two cases of the manufacturer-led and the retailer-led supply chain. The term $\frac{-(1-\alpha)((1-r)\bar{\rho}-w+b)\int_{Q_M^*}^{\infty} Q_M^{*\prime} dF(\xi)}{-e+(1-\alpha)w+\alpha((1-r)\bar{\rho}+b)}$ in Eq. (23) is the difference between these two equations. This term is demonstrated to be positive in the proof of proposition 1. Therefore, because $F^{-1}(.)$ is increasing the resulting order strategy will be greater in Eq. (23). This shows that the retailer orders more in the retailer-led supply chain. Table 2 is also shown the numerical evidence for this difference.

Also, to explain the Eqs. (6) and (18) (which are similar), the role of each term is explained here. The first term shows the revenue coefficient of the manufacturer when the instantaneous purchase is used to provide the more products than the order quantity. The second, third and fourth terms are related to the salvage value of the extra products. In fact, these terms as a whole show the salvage value coefficient of the manufacturer. It is worth mentioning that the third term is positive which shows that in case of not using instantaneous purchase (coefficient (α) shows not using instantaneous purchase) the revenue from salvage value increases. On the other hand, the fourth term shows that the salvage decreases in the case of using instantaneous purchase (coefficient $(1-\alpha)$ shows using instantaneous purchase). The other side of the equation is the cost coefficient of each product. Therefore, Eqs. (6) and (18) express that the maximum profit of the manufacturer will result if two sides of these equations are equal. It is interesting that the regular sale of the products to the retailer does not have any effect to the decision making.

3.2.2.Case 2: $Q_M \leq Q$

In this case, another situation is investigated in which the manufacturer produces less than the order quantity of the retailer. It seems to rarely happen because of the penalty that the manufacturer should pay for the lack of adherence to the commitment. On the other hand, according to the value of the parameters of the game, it may be feasible for the manufacturer to produce less than the order of the retailer. Sequence of events in the manufacturer-led supply chain in case of $Q_M \leq Q$ is similar to the sequence of the problem in the retailer-led supply chain except for the first two steps. In other words, the manufacturer firstly decides the production quantity. Then the retailer decides the order quantity subject to the constrain $Q_M \leq Q$.

Eqs. (13) and (14) show the profit functions of the retailer and the manufacturer in this situation, respectively. Similarly, in the manufacturer-led SC, firstly, optimal order quantity of the retailer is indicated, and then optimal production strategy of the manufacturer will be calculated according to the retailer's strategy. Proposition 4 summarizes results of the game in this situation.

Proposition 4. In a revenue-sharing option contract with given (r, o, e) and $Q_M \leq Q$, while the manufacturer is leader of the chain:

I. Optimal production strategy of the manufacturer (Q_M^*) as the leader is obtained from following equation:

$$Q_M^* = F^{-1}\left(\frac{r\bar{\rho}+e-c+\beta}{r\bar{\rho}+e-\upsilon+\beta}\right) \quad (24)$$

II. Let Q' be the solution of following equation:

$$Q' = F^{-1}\left(\frac{\beta-o}{\beta}\right) \quad (25)$$

Optimal order quantity of the retailer in Nash equilibrium is obtained from the following equation:

$$Q^* = \begin{cases} Q', & Q' > Q_M^* \\ Q_M^*, & Q' \leq Q_M^* \end{cases} \quad (26)$$

Proof. See Appendix B.,

The interesting point is independency of members' strategies in this situation. It means that when production quantity of the manufacturer is less than order quantity of the retailer, optimal strategies are independent.

3.3. Centralized supply chain

The best coordination situation occurs when two parties of the SC behave like a centralized firm. In this regard, the corresponding payoff function is presented in Eq. (27).

$$\Pi_{SC}(Q_{SC}) = \rho \min\{\zeta, Q_{SC}\} - cQ_{SC} - b\max\{\zeta - Q_{SC}, 0\} \quad (27)$$

The first term is the revenue from selling products to the market. The second term is the production costs and the last term is the shortage cost. Proposition 5 shows the optimal order and production quantity of the centralized SC.

Proposition 5. In the proposed revenue-sharing option contracts with given (r, o, e), under the assumption of a hypothetical centralized firm, the optimal strategy of the hypothetical centralized firm is calculated in the following equation:

$$Q_{SC}^* = F^{-1}\left(\frac{\bar{\rho}+b-c}{\bar{\rho}+b}\right) \quad (28)$$

Proof. See Appendix C.

3.4. Evaluation criteria

Two evaluation criteria are presented here to show the beneficial performance of the revenue-sharing option contract. A wholesale contract and a basic option contract are the evaluation criteria whose descriptions are presented in this section.

To evaluate the performance of the proposed revenue-sharing option contract, a wholesale mechanism is presented as a benchmark. In a wholesale mechanism, both parties decide their strategies, independently. In addition, the retailer buys products from the manufacturer in a wholesale price θ while is not given and is assumed to be a random variable with $\Phi(\theta)$ and $\varphi(\theta)$ as the cumulative distribution and the probability density functions, respectively. Payoff functions of the retailer and the manufacturer are presented in Eqs. (29) and (30).

$$\Pi_{WR} = \rho \min\left\{Q_{WR}, \zeta\right\} - \theta Q_{WR} + \upsilon \max\left\{Q_{WR} - \zeta, 0\right\} \quad (29)$$

$$\Pi_{WM} = \theta \min\left\{Q_{WM}, \zeta\right\} - c Q_{WM} + \upsilon \max\left\{Q_{WM} - \zeta, 0\right\} \quad (30)$$

In Eq. (29), the first term is the revenue from selling products to the market. The second term illustrates the wholesale price of the products paid to the manufacturer. The third and the fourth terms describe shortage cost and the salvage value of the extra products, respectively. In Eq. (30), the first term is the revenue from selling products to the retailer and the second term is the production costs. Also, the third and the fourth terms are the same as those in Eq. (29). Since the wholesale mechanism is thoroughly investigated in the literature, the optimal order and production quantities are $Q_{WR}^* = F^{-1}\left(\frac{\bar{\rho}-\theta}{\bar{\rho}-\upsilon}\right)$ and $Q_{WM}^* = F^{-1}\left(\frac{\theta-c}{\theta-\upsilon}\right)$, respectively (Zhao et al. 2010). See **Appendix C** for proof.

In addition to the wholesale contract, a simple option mechanism is also presented to evaluate the improvement resulted from the structural amendments in the proposed contract, it should be compared with the basic option mechanism. In the basic option contract, firstly, the retailer orders some products. In other words, the retailer purchases some call options and pays an option price. Then, the manufacturer produces exactly the order quantity of the retailer. In the selling season, the retailer exercises the purchased options according to the real market demand and pays an exercise price. Therefore, the production policy of the manufacturer is make-to-order, so the manufacturer never experience shortage during the horizon. Also, the retailer never has extra products because amount of option exercising is exactly equal to the real market demand. Since the order and the production quantity are equal in the basic option contract, only the order quantity of the retailer is decision variable. Considering all of these assumptions, the members' profit functions are expressed as follows:

$$\Pi_R = p.\min\{Q,x\} - o.Q - \omega.\min\{Q,x\} - \rho.\max\{x-Q,0\} \quad (31)$$

$$\Pi_M = \omega.\min\{Q,x\} + o.Q - c.Q + \upsilon.\max\{Q-x,0\} \quad (32)$$

In Eq. (31), the first term shows the income from selling the products in the final market. The second and the third terms are option and exercise price, which is paid to the manufacturer, and the last term is the shortage cost. In Eq. (32), the first and the second term are the exercise and option price. The third term is production cost and the last term is salvage value of the unexercised options. The optimal order quantity of the retailer in this problem is $Q^* = \Psi^{-1}(\frac{p-e+\rho-o}{p-e+\rho})$, which has been calculated in previous studies (see Zhao et al., (2010) for proof).

4. Case study

To demonstrate applicability of the proposed mixed contract, a fashion apparel SC in Iran is taken into consideration. In this industry, a manufacturer produces clothing. Type of clothing differs according to the season. Therefore, the selling season for each type is limited to a couple of months. In addition, a retailer is considered which sells the clothing to the final customers. For example, the retailer orders a number of warm clothing before the beginning of winter. The manufacturer adopts his production quantity and then produce before beginning of the winter. During the winter, the retailer exercises a number of them according to the realized market demand. Moreover, if the market demand exceeds the number of purchased options, the retailer can use the instantaneous purchasing. On the other hand, the manufacturer will be fined according to the shortage penalty mechanism if he produces less than the market demand. Table 1 shows the parameters of the mixed contract according to the real values in an apparel fashion industry in Iran for a particular type of clothing.

Table 1. Value of parameters of the contract (prices and costs are in ten thousand Rials)

Parameter	**Value**	**Parameter**	**Value**
Option price	2	Salvage value	5
Exercise price	10	Penalty	5
Exercise price (basic option contract)	14	Price of instantaneous purchase	20
Market demand	Uniform(10,20)	Production cost	8
Market price	Uniform(16,26)	Production cost (Wholesale)	10
Market price (basic option and wholesale)	Uniform(17,27)	Revenue-sharing fraction	0.15
Wholesale price	18		

The optimal order and the production quantity in the Nash equilibrium and corresponding profits of the retailer and the manufacturer are calculated. Table 2 shows Nash equilibrium of the Stackelberg game model for two cases of the retailer-led and the manufacturer-led SC. Without using the proposed mixed contract, a wholesale mechanism can be an alternative. In addition, a basic option contract is presented to show that the modification applied on the option contract can improve the performance of the option mechanism. Optimal quantities for the hypothetical centralized SC, the wholesale mechanism, and the basic option contract are presented in Table 3.

It is worth mentioning that the exercise price in the basic option contract is higher than that of the mixed contract because the basic option contract does not use revenue-sharing mechanism. Consequently, the market price of the basic option contract is higher than that of the mixed contract. This shows that the mixed contract decreases the double marginalization effect and increases the market demand in a long-term horizon. Also, production cost is higher in the wholesale mechanism in comparison with the option mechanism. This is a direct result of fast production in the wholesale mechanism because of lack of coordination.

Table 2. Results of Stackelberg game

	Stackelberg (Retailer-led)						Stackelberg (Manufacturer-led)					
	$Q_M \geq Q$			$Q_M \leq Q$			$Q_M \geq Q$			$Q_M \leq Q$		
	R	M	SC	R	M	SC	R	M	SC	R	M	SC
Profit	86.5	102.9	189.4	58.67	99.73	158.4	53.3	126.2	179.5	67.6	101.7	169.3
Optimal Quantity	18.6	18.6	-	16	16	-	10.63	10.63	-	17.71	17.71	-

Table 3. Results of wholesale mechanism and centralized SC

	Option Only			Wholesale			Centralized SC
	Partners		Total	Partners		Total	Total
	R	M	SC	R	M	SC	SC
Profit	81.48	85.61	167.09	56.69	77.15	133.84	301.09
Optimal Quantity	18.57	18.57	-	12.5	17	-	16.52

A comparison between results of the wholesale contract and several cases for the proposed mixed contract implies following findings:

- In the case of the retailer-led SC, the retailer should adopt a lower order quantity in comparison with the production quantity of the manufacturer to gain more profit ($Q_M \geq Q$). This strategy results in a remarkable improvement in the profit of the chain in comparison with the wholesale contract. Compared to the case of $Q_M \leq Q$, the profit of the manufacturer is improved remarkably, while the profit of the retailer is improved slightly. As the retailer take only his profit into account however much the profit of the manufacturer changes, he adopts the case of $Q_M \geq Q$,

- In the case of the manufacturer-led SC, the manufacturer should adopt a production quantity which is greater than the order quantity of the manufacturer. The results in the table prove this result in that the profit of the chain is increased in the case of $Q_M \geq Q$. Although, the manufacturer will choose the case of $Q_M \geq Q$ if he aims at the SC coordination, he may choose the case of $Q_M \leq Q$ because it improves his profit. Moreover, the case of $Q_M \geq Q$ improves the profit of all members and that of the chain in comparison with the wholesale contract,
- The optimal order and the optimal production quantities are equal in the equilibrium of the proposed mixed contract in most cases (see Table 2). This strongly depends on the value of several parameters of the contract. Some of these parameters are investigated in the next sections,
- A hypothetical centralized SC in table 3 shows the best coordination situation. All the coordination mechanisms try to achieve the profit of the centralized SC as the positive ideal solution. Therefore, a comparison between the profit of the centralized SC shows the real performance of the coordination mechanism,
- All the cases in the mixed contract increases the profit of the whole SC in comparison with the wholesale contract. This shows that the mixed contract can coordinate the SC,
- The basic option contract dominates the wholesale contract. This is a reliable reason for using option mechanism in the mixed contract to coordinate the SC,
- The best situations occur in the retailer-led and the manufacturer-led supply chains when $Q_M \geq Q$. Both of these cases dominates the basic option contract. This shows that use of revenue-sharing, instant purchase, and penalty can improve the performance of the option mechanism.

The option price (o), the exercise price (e), the revenue-sharing fraction (r), the instantaneous purchase price (w) and the penalty of shortage (β) are the most effective parameters of the mixed contract. Several experiments on these parameters are implemented to show the optimal behavior of the SC members under the different values of them. According to the results of the five situations illustrated in this paper, all of them could improve the profit of the SC in comparison with the wholesale mechanism.

4.1. Option and exercise price

An option price is an initial premium which the retailer pays to the manufacturer. Therefore, it can change the profit of both members and the whole SC. In addition, an exercise price is a final price of products at the exercise time of the contract. Also, summation of the option and the exercise price forms the final price of the product for the retailer. So, couple of option and exercise price could generate several option contracts with the same profit of the whole SC. This section aims to find the couples of (o, e) which make a same profit. The experiment is designed on the best situations in each of which $Q_M \geq Q$. Figures (1) and (2) shows the results of the experiment.

On the resulted surfaces, areas with the same color provide the same profit. Decision makers could select the best couple of (o, e) according to their preferences. In addition, a negotiation procedure could be effective to indicate the option and exercise price according to the negotiation power and the risk preferences. According to the surfaces presented in this section, option and exercise price are negatively correlated. Therefore, an increase in the option price and a decrease in the exercise price could provide a same profit for the SC.

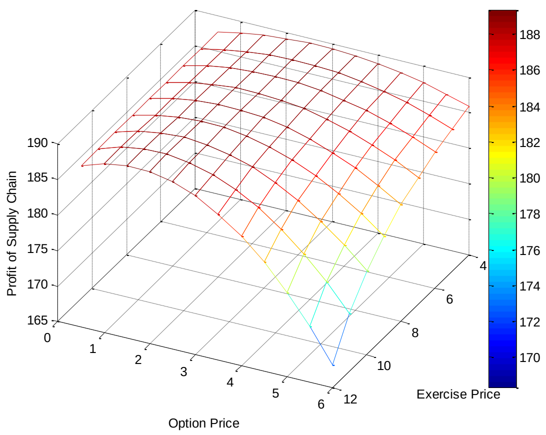

Figure 1. Profit of SC under different couples of (o, e)**for a retailer-led chain and** $Q_M \geq Q$.

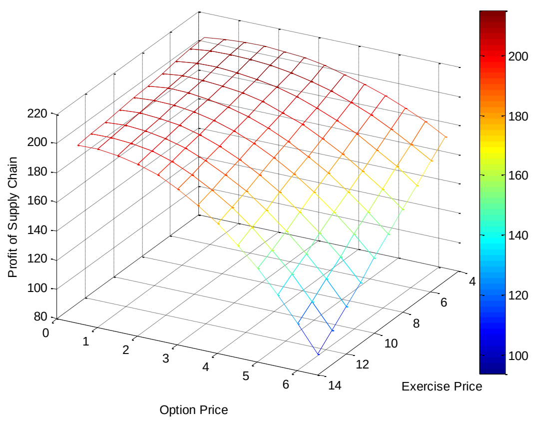

Figure 2. Profit of SC under different couples of (o, e)**for a Manufacturer-led chain and** $Q_M \geq Q$

4.2. Exercise Price and revenue-sharing fraction

The basic concept of a revenue-sharing contract is a reduction in the wholesale price by the manufacturer and sharing a fraction of the retailer's revenue with the manufacturer. In an option contract, wholesale price is more than sum of the exercise price and the option price. The exercise price is characterized as the wholesale price in this paper. The main aim of this section is obtaining couples of the exercise price and the revenue-sharing fraction to provide the same profit for the SC. A surface of the profit of the SC for different values of (e, r) is plotted in figures (3) and (4) for two cases of $\mathbf{Q_M \geq Q}$.

Similarly, parts of the surfaces with the same color show couples of (e, r) which provide a same profit for the SC. According to the figures, the revenue-sharing fraction is negatively correlated with the exercise price. In other words, an increase in the revenue-sharing fraction along with a decrease in the exercise price could make a same profit for the SC. Therefore, decision makers in both parties could choose couples of (e, r) according to their risk preferences and negotiation power.

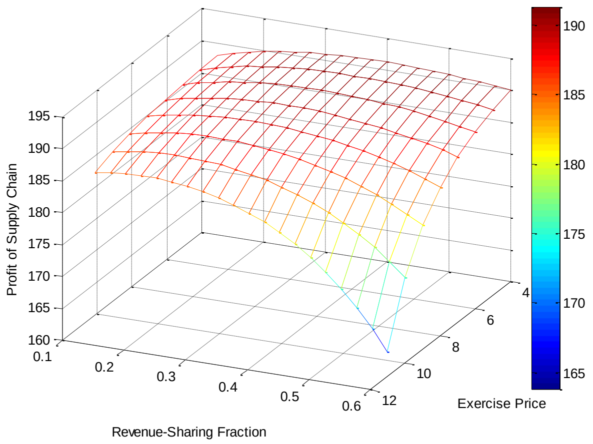

Figure 3. Profit of SC under different couples of (e, r) **for a retailer-led chain and** $\mathrm{Q_M} \geq \mathrm{Q}$

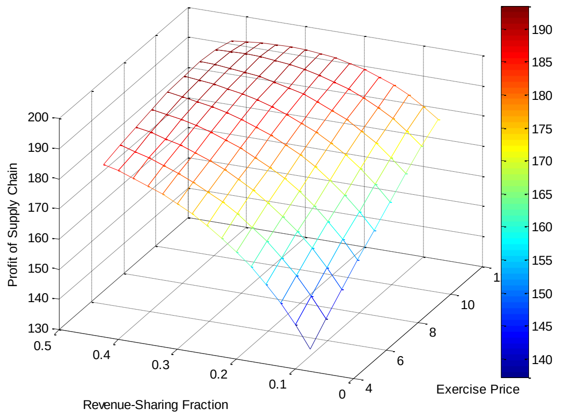

Figure 4. Profit of SC under different couples of (e, r) for a Manufacturer-led chain and $Q_M \geq Q$

Since the revenue-sharing mechanism decreases the wholesale price of the product for the retailer, it increases the profit of the retailer, directly. However, the retailer should pay a fraction of his revenue to the manufacturer to compensate the loss of the lower wholesale price. Moreover, the retailer could decrease the final price of the product for the customer. Therefore, according to the market mechanism which shows a negative correlation between the market price and the demand, the market demand rises by means of the revenue-sharing mechanism in a long-term horizon. In this paper, a short-term planning in a single period contract is performed while demand and price are assumed to be independent in short-term. This is a convincing reason for the both parties to use a revenue-sharing mechanism.

Feasibility of using the revenue-sharing mechanism is obvious from the Figures 3 and 4. To show this fact, the surface should be projected on the plane with revenue-sharing fraction and profit of supply chain as its axes. In Figure 3, the supply chain profit have a peak, which does not occur in zero. It means that there is a value of revenue-sharing fraction that increases the supply chain profit in all the values of the wholesale price. Therefore, use of revenue-sharing is profitable. In Figure 4, A similar projection results in an incresing supply chain profit curve. In other words, a higher revenue-sharing fraction increases the supply chain profit. Therefore, the revenue-sharing mechanism is profitable to use.

4.3. Instantaneous purchase price (w)

One of the main novelties of the proposed mixed contract is the instantaneous purchase during the selling season. An instantaneous purchase occurs when the market demand is greater than the order quantity of the retailer. But the necessary condition for the instantaneous purchase is $Q_M \geq Q$ and price feasibility. Due to the effectiveness of the retailer-led SC under the condition $Q_M \geq Q$, a sensitivity analysis is implemented on instantaneous purchase price in this case. Figure (9) shows the optimal strategy of the retailer and the manufacturer in the Nash equilibrium in different values of (w). Figures (5) and (6) show the results of the experiment.

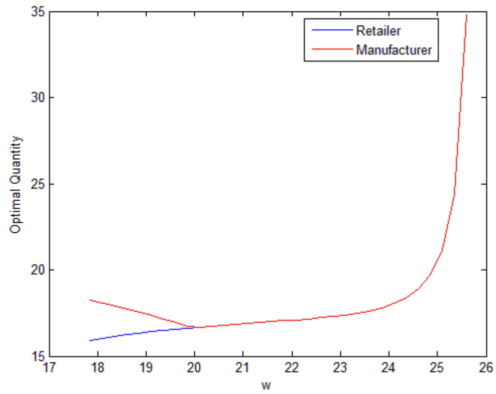

Figure 5. Effects of instantaneous purchase price on the optimal strategy of SC members

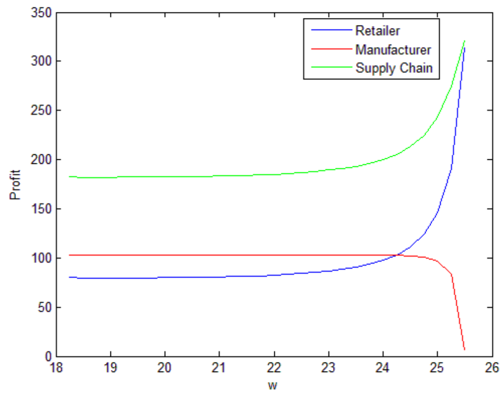

Figure 6. Effects of instantaneous purchase price on the profit of SC members and whole SC

According to the diagrams, the optimal order quantity of the retailer will increase by an increase in w . Also, the production quantity of the retailer will decrease subject to constraint $Q_M \geq Q$. Unlike the previous illustrated situations, the optimal order and the production quantity in the Nash equilibrium are not equal in small values of w. It demonstrates a significant effect of w on the optimal strategies of the members. In fact, instantaneous purchase is a motivation for the manufacturer to produce more. On the other hand, if the instantaneous purchase price be small, the retailer will prefer to order less and use instantaneous purchase. Otherwise, the retailer will order more and according to the constraint $Q_M \geq Q$, both quantities will become equal. Also, profit of the whole SC will increase by an increase in w. In addition, the manufacturer prefers the retailer to order less and use the instantaneous purchase while the retailer orders less in a lower w. Therefore, the manufacturer produces less in a higher w, but constraint $Q_M \geq Q$ enforces the manufacturer to produce more. Accordingly, the manufacturer loses and the retailer profits off more in high values of w.

4.4. Shortage Penalty (β)

Unlike the instantaneous purchase mechanism under condition $Q_M \geq Q$, a shortage penalty mechanism is considered under condition $Q_M \leq Q$ with rate β. This mechanism will be used when the manufacturer produces less than the order quantity of the retailer. However, the manufacturer will be fined only if the market demand is higher than the production quantity. As illustrated before, the manufacturer-led SC could improve the profit of the chain in case of $Q_M \leq Q$. A sensitivity analysis is implemented on the penalty rate of this mechanism, β. It is obvious that the shortage of the retailer is different. Figures (7) and (8) illustrate this analysis for the optimal strategies and the profits.

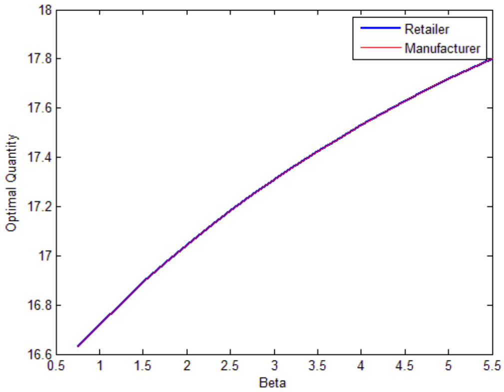

Figure 7. Effects of Shortage rate on the optimal strategy of SC members

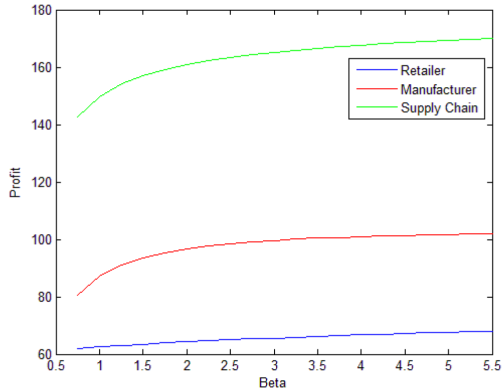

Figure 8. Effects of shortage rate price on the profit of SC members and whole SC

As is clear in Figure (7), the optimal order and the production quantities are equal. It shows that the manufacturer as the follower tries to produce more than the order quantity of the retailer and constraint $Q_M \leq Q$ causes equality between two optimal strategies. In addition, the more the shortage rate, the greater the order quantity and the production quantity are resulted. The more order quantity causes greater shortage for the manufacturer which increases the profit of the retailer. Therefore, the increasing behavior is rational. Also, profit of the retailer, manufacturer, and whole SC are increasing in penalty rate, respectively as illustrated in Figure (8). Therefore, profit of the whole SC could be increased by an increase in β. Thus, this mechanism can coordinate the chain more efficiently by a greater value of β.

5. Conclusion

SC coordination could be achieved with several mechanisms. One of the most well-known mechanisms is a coordination contract. Previous studies used revenue-sharing, quantity flexibility, buyback, and the option contract to coordinate SCs. All of them have several advantages and disadvantages. A mixed contract compensates drawbacks of the mentioned pure contracts. An option contract can coordinate the SC while it cannot mitigate the double marginalization effect. Therefore, in this paper, a mixed revenue-sharing option contract was introduced to reduce the double marginalization effects, which is created by the option mechanism, through the revenue-sharing mechanism in a retailer-manufacturer SC. In other words, the manufacturer sold products to the retailer at a lower exercise price and instead got a share of the revenue of the retailer. Apart from the conventional option mechanism, an instantaneous purchase and a shortage penalty mechanism were proposed to realize the contract more. Instantaneous purchase occurs when the manufacturer produces more than order quantity of the retailer. Under this assumption, a sufficient condition for using the instantaneous purchase is the price feasibility. A shortage penalty occurs when the production quantity of the manufacturer is less than the order quantity of the retailer and the manufacturer pays a penalty to the retailer for the lack of adherence to the commitments. The proposed mixed contract was modeled in both the retailer-led and the manufacturer-led SC settings. Stackelberg game was used to obtain the optimal order and the optimal production quantities in the Nash equilibrium in both of these cases.

From the perspective of application, a case study of a fashion apparel SC in Iran was presented to show the real-world application of the proposed mixed contract. Finally, the best conditions for the SC coordination were identified in comparison with the centralized SC and the wholesale mechanism as a benchmark. Results showed that, in the case of the retailer-led SC, the retailer should order less than the production quantity of the manufacturer. On the other hand, in case of the manufacturer-led SC, the manufacturer should produces more than the order quantity of the manufacturer to raise the profit of the whole chain. In the latter case, the manufacture may choose to produce less than the order quantity to gain a higher profit. In addition, a comparison between the mixed contract and some solution criteria showed that the mixed contract dominates a wholesale and a basic option contract.

This demonstrated that the modifications applied on the option mechanism improved performance of the contract. A comprehensive sensitivity analysis on the option price, the exercise price, the revenue-sharing fraction, the instantaneous purchase price and the shortage penalty rate was implemented for the best selected contracts. Couples of (e, r) and (o, e) were provided which made a same SC profit to choose the best couple according to the risk preferences and the negotiation power of the decision makers.

Several research gaps are existed to be developed in the future studies. Firstly, a bargaining model could be developed to obtain the best parameters of the contract including the option price, the exercise price and the revenue-sharing fraction. Secondly, the social optimum points for the strategic games could be developed to maximize the total profit of the SC. Thirdly, unlike the previous studies and this paper, a multi-period selling season could be considered which needs using an American option mechanism. Finally, because the addressed problem contains a high level of complexity, developing some numerical methods including a heuristic, a metaheuristic or a hybrid one is a remarkable gap in this area.

Appendix A:

Firstly, we should provide the expected value of the retailer's and the manufacturer's profit in two cases of $Q_M \geq Q$ and $Q_M \leq Q$. With some algebra, we obtain the expected values as:

$$
\begin{aligned}
&E(\Pi_M \mid Q_M \geq Q) = oQ - cQ_M + e\left[\int_{-\infty}^{Q} \zeta dF(\zeta) + \int_{Q}^{\infty} Q dF(\zeta)\right] \\
&(1-G(w-b))\left[r\bar{\rho}\left[\int_{-\infty}^{Q_M} \zeta dF(\zeta) + \int_{Q_M}^{\infty} Q_M dF(\zeta)\right] + w\left[\int_{Q}^{Q_M} (\zeta - Q) dF(\zeta) + \int_{Q_M}^{\infty} (Q_M - Q) dF(\zeta)\right]\right. \\
&\left.+\upsilon\int_{-\infty}^{Q_M} (Q_M - \zeta) dF(\zeta)\right] + G(w-b)\left[r\bar{\rho}\left[\int_{-\infty}^{Q} \zeta dF(\zeta) + \int_{Q}^{\infty} Q dF(\zeta)\right]\right. \\
&\left.+\upsilon\left[\int_{-\infty}^{Q} (Q_M - \zeta) dF(\zeta) + \int_{Q}^{Q_M} (Q_M - Q) dF(\zeta)\right]\right]
\end{aligned}
\tag{A.1}
$$

$$
\begin{aligned}
&E(\Pi_R \mid Q_M \geq Q) = -e\left[\int_{-\infty}^{Q} \zeta dF(\zeta) + \int_{Q}^{\infty} Q dF(\zeta)\right] - oQ \\
&+(1-G(w-b))\left[(1-r)\bar{\rho}\left[\int_{-\infty}^{Q_M^*(Q)} \zeta dF(\zeta) + \int_{Q_M^*(Q)}^{\infty} Q_M^*(Q) dF(\zeta)\right] - w\left[\int_{Q}^{Q_M} (\zeta - Q) dF(\zeta) + \int_{Q_M}^{\infty} (Q_M - Q) dF(\zeta)\right]\right. \\
&\left.-b\int_{Q_M^*(Q)}^{\infty} (\zeta - Q_M^*(Q)) dF(\zeta)\right] + G(w-b)\left[(1-r)\bar{\rho}\left[\int_{-\infty}^{Q} \zeta dF(\zeta) + \int_{Q}^{\infty} Q dF(\zeta)\right]\right. \\
&\left.-e\left[\int_{-\infty}^{Q} \zeta dF(\zeta) + \int_{Q}^{\infty} Q dF(\zeta)\right] - b\int_{Q}^{\infty} (\zeta - Q) dF(\zeta)\right]
\end{aligned}
$$

(A.2)

$$E(\Pi_M \mid Q_M \le Q) = (r\bar{\rho}+e)\left[\int_{-\infty}^{Q_M} \zeta dF(\zeta) + \int_{Q_M}^{\infty} Q_M dF(\zeta)\right] + oQ - cQ_M + \upsilon\left[\int_{-\infty}^{Q_M} (Q_M-\zeta) dF(\zeta)\right]$$
$$-\vartheta\left[\int_{Q_M}^{Q} (\zeta - Q_M) dF(\zeta) + \int_{Q}^{\infty} (Q-Q_M) dF(\zeta)\right]$$

(A.3)

$$E(\Pi_R \mid Q_M \le Q) = ((1-r)\bar{\rho}-e)\left[\int_{-\infty}^{Q_M^*(Q)} \zeta dF(\zeta) + \int_{Q_M^*(Q)}^{\infty} Q_M^*(Q) dF(\zeta)\right] - oQ - b\int_{Q_M}^{\infty} (\zeta - Q_M) dF(\zeta)$$
$$+\vartheta\left[\int_{Q_M}^{Q} (\zeta - Q_M) dF(\zeta) + \int_{Q}^{\infty} (Q-Q_M) dF(\zeta)\right] \quad \text{(A.4)}$$

Proof of proposition 1. We consider $Q_M \ge Q$ and $Q_M \le Q$ as constraints in the problem. To obtain the Nash equilibrium of the Stackelberg game in the retailer-led SC, firstly, the optimal production quantity of the manufacturer is obtained. Secondly, the optimal production quantity is substituted in the retailer's profit function, and thirdly, optimal order quantity of the retailer is calculated from the first order optimality condition. In this regard, firstly the constraints are ignored and then feasibility of the obtained solutions will be checked. To do so, the first order optimality condition provides the optimal production strategy of the manufacturer in the Nash equilibrium regardless of the order quantity. So, we have the first order derivative w. r. t. Q_M:

$$\frac{\partial \Pi_M}{\partial Q_M} = ((1-\alpha)(-r\bar{\rho}-w)+\upsilon)F(Q_M) + (2\alpha-1)\upsilon Q_M f(Q_M) - \alpha\upsilon Q f(Q_M)$$
$$+(1-\alpha)(r\bar{\rho}+w) - c = 0 \quad \text{(A.5)}$$

In addition, the sufficient condition which is the second order optimality condition is as follows:

$$\frac{\partial^2 \Pi_M}{\partial Q_M^2} = ((1-\alpha)(-r\bar{\rho}-w)+2\alpha\upsilon)f(Q_M) + ((2\alpha-1)\upsilon Q_M - \alpha\upsilon Q)f'(Q_M) \quad \text{(A.6)}$$

Since an explicit solution for the optimal production quantity of the manufacturer is not achievable, we have to present some reasonable inequalities to prove negativity of Eq. (A.6). α shows probability of not using the instantaneous purchase mechanism. It is reasonable to say that the instantaneous purchase mechanism occurs at less than half of the time ($\alpha > 0.5$). In addition, the density function of the market demand is assumed to be downward-sloping ($f'(.) < 0$). With some algebra, the second term of Eq. (A.6) is strictly negative. Also, with some algebra, the condition $\frac{2\alpha}{1-\alpha} < \frac{r\bar{\rho}+w}{\upsilon}$ should be satisfied to achieve negativity of the whole expression. Since α is greater than 0.5, therefore, the recent condition transform to $r\bar{\rho} + w > 2\upsilon$. Under the presented conditions, Eq. (A.6) is strictly negative and solution of Eq. (A.5) is optimal value of the manufacturer's profit function.

In terms of the retailer's optimal order quantity, we have:

$$\frac{\partial \Pi_R}{\partial Q_R} = \left(-e + (1-\alpha)w + \alpha((1-r)\bar{\rho} + b)\right)\left(1 - F(Q)\right) + (1-\alpha)((1-r)\bar{\rho} - w + b)\int_{Q_M^*(Q)}^{\infty} Q_M^{*\prime}(Q)dF(\zeta) - o = 0 \quad \text{(A.7)}$$

where $Q_M^{*\prime}(Q) = \frac{\partial Q_M^*}{\partial Q}$. It is worth mentioning that existence of $Q_M^{*\prime}$ in the profit function of the retailer shows interdependence of the optimal strategy of both parties. In order to approve that the obtained solution from Eq. (A.7) is the maximum profit, the second order optimality condition is provided here w. r. t. Q:

$$\frac{\partial^2 \Pi_R}{\partial Q^2} = -((1-\alpha)(w-e) + \alpha((1-r)\bar{\rho} + b - e))f(Q) + (1-\alpha)((1-r)\bar{\rho} - w + b)\left[\int_{Q_M^*}^{\infty} Q_M^{*\prime\prime} f'(\zeta)d\zeta - Q_M^{*\prime} Q_M^{*\prime} f(Q_M^*)\right] \quad \text{(A.8)}$$

Since $w > e + o$ and $(1-r)\bar{\rho} + b - e > e$, $-\left((1-\alpha)(w-e) + \alpha\left((1-r)\bar{\rho} + b - e\right)\right)f(Q)$ is negative. Also, as $f'(\xi)$ is assumed negative, $\int_{Q_M^*}^{+\infty} Q_M^{*\prime\prime} f'(\xi)d\xi - Q_M^{*\prime} Q_M^{*\prime} f(Q_M^*)$ is negative and if we prove that its coefficient is positive, Eq. (A.8) will negative under the mentioned assumptions. In this regard, we should prove that $(1-r)\bar{\rho} - w > -b$. $(1-\alpha)$ is probability of using instantaneous purchase mechanism by the retailer in the case of shortage. As the second term of Eq. (A.8) is multiplied by $(1-\alpha)$, instantaneous purchase price has the price feasibility condition for the retailer which means that $(1-r)\bar{\rho} - w > -b$ (right hand side is profit from using instantaneous purchase and the left hand side is profit of facing shortage) and $(1-r)\bar{\rho} > b - w$, consequently. In brief, we proved that both terms of the Eq. (A.8) are negative under the assumptions of the model. Therefore, Eq. (A.8) is strictly negative and solution of Eq. (A.7) is the optimal order quantity of the retailer.

As is seen from Eq. (A.5) and Eq. (A.7), explicit general solutions for Q_M^* and Q^* could not be obtained. However, for a special distribution function for the demand, explicit solutions are found in section 5. Since the retailer, as the leader of the chain, decides his order quantity before the manufacturer, the manufacturer should produce more than the order quantity of the retailer according to $Q_M \geq Q$. In this regard, let Q_M' be the solution of Eq. (A.5) and Q^* be the solution Eq. (A.7). Therefore, the optimal production quantity of the manufacturer is obtained from Eq. (7).

Proof of proposition 2. In the second case in which $Q_M \leq Q$ and the retailer plays the role of the leader, firstly, the optimal production strategy of the manufacturer is obtained regardless of the order quantity of the retailer. In this regard, the first order derivative of the manufacturer's profit (Eq. (A.3)) w. r. t. Q_M is presented in the following equation:

$$\frac{\partial \Pi_M}{\partial Q_M} = -(r\bar{\rho}+e-\upsilon+\beta)F(Q_M)+r\bar{\rho}+e-c+\beta=0 \quad \text{(A.9)}$$

In addition, the second order optimality condition is as follows to prove that the manufacturer's profit function is concave:

$$\frac{\partial^2 \Pi_M}{\partial Q_M^2} = -(r\bar{\rho}+e-\upsilon+\beta)f(Q_M) \quad \text{(A.10)}$$

It is assumed that $e > \upsilon + o$. Therefore, $e > \upsilon$. So, $r\bar{\rho} + e + \beta > \upsilon$ and $-(r\bar{\rho} + e - \upsilon + \beta)f(Q_M) < 0$ and the manufacturer's profit function is strictly concave.

In terms of the retailer's optimal order quantity, the first and the second order derivatives of the retailer's profit function (Eq. (A.4)) w. r. t. Q are as follows:

$$\frac{\partial \Pi_R}{\partial Q} = \beta - o - \beta F(Q) = 0 \quad \text{(A.11)}$$

$$\frac{\partial^2 \Pi_R}{\partial Q^2} = -\beta f(Q) \quad \text{(A.12)}$$

Because $\beta > 0$, the retailer's profit function is strictly concave, obviously. Therefore, the provided quantity of Q in Eq. (A.11) is the optimal order quantity ($Q^* = F^{-1}(\frac{\beta-o}{\beta})$). In addition, result of the first order optimality condition in Eq. (A.9) is the optimal production quantity which is $Q'_M = F^{-1}(\frac{r\bar{\rho}+e-c+\beta}{r\bar{\rho}+e-\upsilon+\beta})$. However, the feasibility of Q'_M should be checked according to the constraint $Q_M \leq Q$. Because the retailer is the leader of the chain, obtained Q^* is the optimal order quantity, but the optimal production quantity of the manufacturer could be found from Eq. (17).

Appendix B:

Proof of proposition 3. This proposition is related to the manufacturer-led SC. In this regard, to provide the optimal quantities in the Nash equilibrium of the Stackelberg game, firstly, the optimal order quantity of the retailer is obtained. Secondly, it is substituted in the manufacturer's profit function. Thirdly, the optimal production quantity of the manufacturer is obtained. It is worth mentioning that the problem will be solved regardless of the constraint $Q_M \geq Q$ and then the feasibility of the obtained solutions will be checked.

In order to provide the optimal order quantity of the retailer, the first and the second order derivatives of the retailer's profit function (Eq. (A.2)) w. r. t. Q are as followings:

$$\frac{\partial \Pi_R}{\partial Q} = ((1-\alpha)(w-e) + \alpha((1-r)\bar{\rho} + b - e))(1 - F(Q)) = 0 \quad \text{(B.1)}$$

$$\frac{\partial^2 \Pi_R}{\partial Q^2} = -((1-\alpha)(w-e) + \alpha((1-r)\bar{\rho} + b - e))f(Q) \quad \text{(B.2)}$$

Since $w > e + o$ and $0 \leq \alpha \leq 1$, $(1-\alpha)(w-e) > 0$. In addition, as it is assumed that $(1-r)\bar{\rho} + b - e > e$ and $0 \leq \alpha \leq 1$. So, $\alpha((1-r)\bar{\rho} + b - e > 0$. Therefore, the second order derivative of the retailer's profit function is negative and it is strictly concave. Consequently, the solution of Eq. (B.1) ($Q' = F^{-1}(1 - \frac{o}{-e+(1-\alpha)w+\alpha((1-r)\bar{\rho}+b)})$) is the optimal order quantity of the retailer after checking the feasibility.

To obtain the manufacturer strategy, the first and the second order optimality conditions are checked w. r. t. Q_M as followings:

$$\frac{\partial \Pi_M}{\partial Q_M} = ((1-\alpha)(-r\bar{\rho} - w) + \upsilon)F(Q_M) + (2\alpha - 1)\upsilon Q_M f(Q_M) - \alpha\upsilon Q f(Q_M) + (1-\alpha)(r\bar{\rho} + w) - c = 0 \quad \text{(B.3)}$$

$$\frac{\partial^2 \Pi_M}{\partial Q_M^2} = ((1-\alpha)(-r\bar{\rho} - w) + 2\alpha\upsilon)f(Q_M) + ((2\alpha - 1)\upsilon Q_M - \alpha\upsilon Q)f'(Q_M) \quad \text{(B.4)}$$

Proof of the negativity of Eq. (B.4) is similar to the proof of negativity of Eq. (A.6) in proposition 1. Therefore, Eq. (B.4) is strictly concave and solution of Eq. (B.3) is the optimal production quantity of the manufacturer in the Nash equilibrium. In order to check feasibility of the provided optimal quantities, constraint $Q_M \geq Q$ should be guaranteed. To do so, the optimal production quantity of the retailer is provided through solving Eq. (B.3) while there is not any explicit form for Q_M^*, obviously. As the manufacturer is the leader, the optimal order quantity of the retailer should be compared with the optimal production quantity of the manufacturer. Therefore, Q^* is provided from Eq. (20).

Proof of proposition 4. As seen in Eq. (15) and Eq. (16), the optimal strategies of the manufacturer and the retailer in the Nash equilibrium are independent. Therefore, proof of the optimality conditions for proposition 4 is as the same as proposition 2. On the other hand, as the manufacturer is the leader of the chain in this situation, the manufacturer decides his strategy before the retailer. In this regard, optimality condition $Q_M \leq Q$ should be checked to provide the optimal order quantity of the retailer. To do so, Q^* is obtained from Eq. (26).

Appendix C:

Proof of optimal order and production strategy of wholesale mechanism. In this mechanism, the retailer's and the manufacturer's strategies are independent. Therefore, the optimal strategy for each of them should be provided and optimality conditions should be checked.

Firstly, the expected values of Eq. (29) and Eq. (30) are obtained in the following equations:

$$E(\Pi_{WR})=\bar{\rho}\left[\int_{-\infty}^{Q_{WR}}\zeta dF(\zeta)+\int_{Q_{WR}}^{\infty}Q_{WR}dF(\zeta)\right]-\bar{\theta}Q_{WR}+\upsilon\int_{-\infty}^{Q_{WR}}(Q_{WR}-\zeta)dF(\zeta) \qquad \text{(C.1)}$$

$$E(\Pi_{WM})=\bar{\theta}\left[\int_{-\infty}^{Q_{WM}}\zeta dF(\zeta)+\int_{Q_{WM}}^{\infty}Q_{WM}dF(\zeta)\right]-cQ_{WM}+\upsilon\int_{-\infty}^{Q_{WM}}(Q_{WM}-\zeta)dF(\zeta) \qquad \text{(C.2)}$$

where Q_{WR} and Q_{WM} are the retailer's order and the manufacturer's production quantities, respectively. To obtain the optimal value of the order quantity of the retailer, the first and the second order derivatives w. r. t. Q_{WR} are provided in the following equations:

$$\frac{\partial\Pi_{WR}}{\partial Q_{WR}}=\bar{\rho}-\bar{\theta}-(\bar{\rho}-\upsilon)F(Q_{WR})=0 \qquad \text{(C.3)}$$

$$\frac{\partial^2\Pi_{WR}}{\partial Q_{WR}^2}=-(\bar{\rho}-\upsilon)f(Q_{WR}) \qquad \text{(C.4)}$$

It is obvious that $\bar{\rho}>\upsilon$. Therefore, $-(\bar{\rho}-\upsilon)<0$. Accordingly, the second order derivative of the retailer's profit function is strictly concave. Therefore, the solution of the Eq. (C.3) is the optimal retailer's order quantity which has been obtained as $Q_{WR}^{*}=F^{-1}(\frac{\bar{\rho}-\bar{\theta}}{\bar{\rho}+\upsilon})$.

To obtain the optimal production quantity of the manufacturer, the first and the second order derivatives w. r. t. Q_{WM} are expressed in the following equations:

$$\frac{\partial\Pi_{WM}}{\partial Q_{WM}}=\theta-c-(\theta-\upsilon)F(Q_{WM})=0 \qquad \text{(C.5)}$$

$$\frac{\partial^2\Pi_{WM}}{\partial Q_{WM}^2}=-(\bar{\theta}-\upsilon)f(Q_{WM}) \qquad \text{(C.6)}$$

Since $\bar{\theta}>\upsilon$, therefore, $-(\bar{\theta}-\upsilon)<0$. Therefore, the second order derivative of the manufacturer's profit function is negative and Eq. (C.2) is strictly concave. So, solution of Eq. (C.5) as $Q_{WM}^{*}=F^{-1}(\frac{\bar{\theta}-c}{\bar{\theta}-\upsilon})$ is the proven optimal production quantity of the manufacturer.

Proof of proposition 5. Firstly, the expected value of profit function of the hypothetical centralized SC has been presented in the following equation:

$$E(\Pi_{SC}(Q_{SC})) = \bar{\rho}\left[\int_0^{Q_{SC}} \zeta dF(\zeta) + \int_{Q_{SC}}^{\infty} Q_{SC} dF(\zeta)\right] - cQ_{SC} - b\int_{Q_{SC}}^{\infty} (\zeta - Q_{SC}) dF(\zeta) \quad \text{(C.7)}$$

In terms of the hypothetical centralized SC, to obtain the optimal quantity, the first and the second order derivatives of Eq. (C.7) are presented in the following equations:

$$\frac{\partial \Pi_{SC}}{\partial Q_{SC}} = \bar{\rho} + b - c - (\bar{\rho} + b)F(Q_{SC}) = 0 \quad \text{(C.8)}$$

$$\frac{\partial^2 \Pi_{SC}}{\partial Q_{SC}^2} = -(\bar{\rho} + b)F(Q_{SC}) \quad \text{(C.9)}$$

Since $\bar{\rho} > 0$ and $b > 0$, $-(\bar{\rho} + b)f(Q_{SC}) < 0$. Accordingly, the second order derivative of the profit function of the hypothetical centralized SC is negative and Eq. (C.7) is strictly concave while it is sufficient optimality condition. Finally, the solution of Eq. (C.8) ($Q_{SC}^* = F^{-1}(\frac{\bar{\rho}+b-c}{\bar{\rho}+b})$) is the proven optimal quantity. It is worth mentioning that the profit of the whole SC, which is used within the text, is summation of the retailer's and the manufacturer's profit functions. As both of them have been proven that are concave in all cases and summation of two concave functions are concave, therefore, the SC profit function in all cases is a concave function.

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