



# Design and use of curriculum resources for teachers and teacher educators: Example of the Chinese abacus at primary school



Ghislaine Gueudet<sup>a,b,\*</sup>, Caroline Poisard<sup>a,c</sup>

<sup>a</sup> Université de Brest, Université de Rennes, CREAD, F-29200 Brest, France

<sup>b</sup> ESPE Bretagne site de Rennes, 153 rue Saint-Malo, 35043 Rennes Cedex, France

<sup>c</sup> ESPE Bretagne site de Quimper, 8 rue de Rosmadec, 29196 Quimper Cedex, France

## ARTICLE INFO

### Keywords:

Abacus  
Documentation work  
Operational invariants  
Resources  
Teachers' professional development

## ABSTRACT

In this paper we discuss a particular perspective on the design and use of curriculum resources brought through applying the documentational approach to didactics. We present this theoretical approach and the associated “reflective investigation” method. We consider that teachers and teacher educators develop documents, associating resources and a scheme of use of these resources.

We provide an illustrative example of curriculum resources around the use of the Chinese abacus. We present our results, concerning the documents developed by a primary school teacher and teacher educator using these resources. We analyze simultaneously the teacher's capacity to use curriculum resources and curriculum resources influence on instruction.

## 1. Introduction

The field of curriculum ergonomics introduced in this special issue proposes to conceptualize the interactions between curriculum design and use (Choppin, Roth McDuffie, Drake, & Davis, 2018). This article is a contribution to this conceptualization. We use the theoretical frame of the documentational approach to didactics (DAD, Gueudet & Trouche, 2009) and the associated method for studying the interactions between users (teachers, but also teacher educators) and various kinds of resources, including curriculum materials. We focus on a particular case, concerning the Chinese abacus at primary school, with a triple aim: (1) investigate the interactions between a user and her resources and their consequences in terms of professional growth; (2) illustrate the use of DAD for this kind of investigation; and (3) enlighten possible contributions of DAD to curriculum ergonomics.

In the following section we introduce DAD, initiate the discussion on its possible contributions to curriculum ergonomics and present the research questions studied here. In Section 3, we describe our methods and in Section 4, the resources designed for primary school teachers around the use of the abacus (the abacus kit, available online), and an online « training path » designed for teacher educators. This training path provides the structure and content of a blended teacher education course about the use of the abacus. In Section 5, we present and analyze the appropriation of the resources designed by a teacher, and then her appropriation of the training path as a teacher educator. In Section 6 we present our conclusions.

## 2. Theoretical approach and research questions

In this section we firstly briefly introduce DAD (Gueudet & Trouche, 2009), and discuss its possible contribution to curriculum

\* Corresponding author at: ESPE Bretagne site de Rennes, 153 rue Saint-Malo, 35043 Rennes Cedex, France.

E-mail addresses: [Ghislaine.Gueudet@espe-bretagne.fr](mailto:Ghislaine.Gueudet@espe-bretagne.fr) (G. Gueudet), [caroline.poisard@espe-bretagne.fr](mailto:caroline.poisard@espe-bretagne.fr) (C. Poisard).

ergonomics. Then we present our research questions.

### 2.1. The documentational approach, central concepts

The documentational approach to didactics focuses on the interactions between teachers and different kinds of *resources*, and on the consequences of these interactions in terms of professional development and professional growth (as defined by Perrin-Glorian, Deblois, & Robert, 2010). ‘Resources’ in this approach takes on a broad meaning as proposed by Adler (2000): everything likely to resource the teacher’s practice, hence to contribute to the teacher’s professional growth. This includes curriculum materials, but also cultural resources like language, discussions with students or colleagues. This perspective invites attention to be given to the multiple forms of resources that come into play in teachers’ work.

A teacher might sometimes look for new resources for a precise teaching objective, discover new resources during a discussion with colleagues or when visiting a website (outside of any precise teaching objective). He/she can use these resources immediately or keep them for later potential use. When preparing a lesson, the teacher will also look through resources that he/she has already used before; he/she will combine different resources, modify them, use them in class and can also share them with colleagues. All this activity constitutes the documentation work of the teacher; it holds a central place in the teacher’s professional activity, in and out of the classroom.

Referring to a perspective introduced by Rabardel (1995), the documentational approach considers that, alongside this work, the teacher develops a mixed entity, called a *document*. A document comprises resources (modified, combined by the teacher), and a *scheme of use* of these resources (Vergnaud, 1998). A scheme of use is a stable organization of the activity for a given aim. A scheme of use comprises several components: the aim of the activity, the rules of actions, operational invariants and possibilities of inferences (to adapt to future similar situations). The operational invariants can be theorems-in-action (propositions considered as true) or concepts-in-action (concepts considered as relevant).

For example in a previous study (Poisard, Bueno-Ravel, & Gueudet, 2011) we observed a teacher working with a grade 8 class, who developed a stable organization for the aim: “Introducing the abacus”. The scheme developed by this teacher comprised this aim and included rules of action, like “ask the students to write instructions for use of the abacus and share them with the class”. The stable organization also included operational invariants like “asking the students to write for a presentation to the class to support learning and understanding” (theorem-in-action) and “presentation for the other students” (concept-in-action). This teacher developed a document by associating a set of resources - material abacus, virtual abacus, posters (where students write instructions)- and this scheme.

An operational invariant can have many different origins: it can be related to mathematical knowledge, or pedagogical choices; it can be shared with many other teachers; or it can be more individual. In all these cases, it guides the teacher’s activity. In what follows we will not describe complete schemes, but will focus on operational invariants. According to Vergnaud (1998), operational invariants are the most important component of the scheme, the knowledge developed from the activity and guiding it. Moreover we will not distinguish between theorems-in-action and concepts-in-action, since both are always associated.

The process of development of a document by a teacher is called a *documentational genesis* and comprises two intertwined processes. On the one hand, the features of the resources influence the schemes developed by the teacher: this is the *instrumentation* process. On the other hand, the teacher already has many existing schemes, influencing his/her use of the resource: this is the *instrumentalization* process. The documentational geneses constitute a central aspect of teachers’ professional development (Gueudet & Trouche, 2009). Moreover, Psycharis and Kalogeria (in press) have evidenced that the same holds for teacher educators.

In this article we focus on *curriculum resources* as defined by Pepin and Gueudet (2018): “curriculum resources [are] all the material resources that are developed and used by teachers and students in their interaction with mathematics in/for teaching and learning, inside and outside the classroom” (p.132). We distinguish between material resources, meaning tangible (the material abacus) and virtual resources, meaning here digital (the virtual abacus), following the vocabulary introduced in Poisard (2017).

### 2.2. Contributions of DAD to curriculum ergonomics

Choppin et al. (2018) identify several themes that define curriculum ergonomics. In this paper, we try to deepen the possible contribution of DAD to two of these themes: (1) teachers’ relationship with and capacity to use materials; (2) the ways curriculum materials influence instruction. We also propose to extend the scope of these themes to include teacher education and teacher educators.

DAD studies how teachers choose resources, modify them during their preparation work, both in class and after using them in class. This documentation work is central in teachers’ work: with a DAD perspective, teachers (and teacher educators) are considered as designers of their teaching. At the same time, DAD considers that teachers interact with resources, and that the features of these resources influence the teachers’ practices.

With a DAD perspective, the two themes mentioned above are strongly linked. The notions of document and documentational genesis can be considered as a conceptualization addressing both themes, where the operational invariants play a central role. The instrumentalization processes describe how teachers appropriate and transform resources, according to their already existing operational invariants. Teachers’ capacity to use materials depends on these existing operational invariants. On the other hand, the instrumentation processes describe how the repeated use of resources leads to the development of new operational invariants, shaped by the features of these resources. Teachers are designers; nevertheless their design and more generally their instructional practices are influenced by the features of the resources they use (Remillard, 2012).

**Table 1**  
The columns of a “documents table”.

Aim of the teacher’s activity	Main resources used	Operational invariants	Usual or new in the teacher’s practice
-------------------------------	---------------------	------------------------	--

### 2.3. Research questions

We consider in this paper the case of the teaching and learning of decimal place-value and decimal numbers at primary school in France, and of curriculum resources designed for teachers and teacher educators around the use of the abacus. Referring to DAD, our research questions are the following:

- Which documents are developed by a primary school teacher using curriculum resources designed to support the use of the abacus (material and virtual) to teach numbers and operations?
- Which documents are developed by a teacher educator using training resources to set up a teacher education course concerning the use of the abacus (material and virtual) to teach numbers and operations?

Answering these research questions allows us to pursue the three objectives mentioned in the introduction. Firstly, we investigate the interactions between a user and her resources, through the concept of document and in particular of operational invariant. This naturally illustrates the use of DAD, our second objective. Finally, analyzing documents also means analyzing documentational geneses, with their two components: instrumentalization and instrumentation processes. Instrumentalization processes contribute to the understanding of teacher’s capacity to use curriculum resources, while instrumentation processes enlighten the influence of resources on instruction. Thus the analysis in terms of documents deepens the understanding of how DAD contributes to central themes in curriculum ergonomics.

### 3. Methodology

The documentational approach is associated with a specific method, the *reflective investigation*.

Reflective investigation follows two principles. First, the teacher (or teacher educator) is shadowed in and out of the class over a significant period of time. Indeed, long-term observation, during which all the curriculum resources used or produced are collected, is necessary in order to observe regularities or evolutions in the teacher’s activity. Second, the teacher is actively engaged in the data collection, in a reflective stance. Indeed, only the teacher can answer questions such as: “Which resources did you use to design this lesson?”

These principles can lead to the collection of several kinds of data: curriculum resources used or designed; interviews with the teacher or teacher educator; and videos of lessons and/or of teacher education courses.

To analyze this data, we always compare the teacher’s declarations with the resources produced and the videos that inform us on his/her actual choices. Drawing on this comparison, the researcher fills in a first version of a *documents table* (Table 1). This version is proposed to the teacher, who may add to it or amend it. In the third column entitled “Operational invariants”, we (or the teacher his/herself) note some convictions of the teacher expressed during the interviews, and likely to explain how the resources are used. In the fourth column, the teacher notes if the operational invariants mentioned are usual or new.

We applied this method in the case of Rose. Rose was very experienced, having been a teacher for thirty years and a teacher educator for twenty. We followed her during the academic year 2015–2016; she was then a teacher in grade 4 part time, and part time, a teacher trainer. She used the resources of the abacus kit (see § 4.2) in her class. We had an initial discussion with her to present the Chinese abacus and the resources before the lesson, and interviewed her after her lesson (Interview 1, 1st June 2016). After collecting all the resources she used and produced (including students’ productions), we shadowed her for a further two academic years, during which she became a facilitator<sup>1</sup> and used the *abacus training path* (see § 4.3). We had again an initial discussion with her to present the training path, an interview after the first time she organized the training (Interview 2, 23rd June 2017), and a second interview the second year she organized the training (Interview 3, 26th February 2018). We also collected all the resources she used and produced.

Using this data we filled in two documents tables: one for Rose as a teacher, and one for Rose as a facilitator. We submitted these two tables to Rose during a last interview (Interview 4, 26th June 2018). She amended the tables and added some details and explanations, in particular declaring whether the operational invariants were usual or new. Drawing on these tables, we identified essential elements of documents developed by Rose: the aim of her activity, the resources she used for this aim, and the associated operational invariants. We also noted if the operational invariants intervening were already present and guided Rose’s appropriation of the resources (instrumentalization) or if they were developed during the interaction with these resources (instrumentation).

<sup>1</sup> By facilitator, we mean a teacher who evolved as a teacher educator on a full time position proposed by the employer (not by a university).

#### 4. Presentation of the curriculum resources designed for teachers and teacher educators

In this section we briefly outline the principles of the Chinese abacus before presenting the curriculum resources designed for teachers and teacher educators.

##### 4.1. The Chinese abacus, material and virtual

The Chinese abacus (*suan-pan*) is a traditional calculating instrument. The virtual abacus (Fig. 1) is software (developed by the Sésamath association) that reproduces the features of the Chinese abacus, and adds new features: in particular, numbers can be shown in digits (see below). For our study, the Chinese abacus is introduced in class for teaching and learning number sense, place-value system and calculation.

The Chinese abacus is separated in two parts by a central bar called *the reading bar*. Only the beads on this bar are considered as *activated*; hence in Fig. 1 on the left, the material abacus displays zero. There are two kinds of beads: 5-unit counters (two of them, in the upper part for the virtual abacus) and 1-unit counters (five of them, in the lower part of the abacus). The Chinese abacus comprises 13 vertical rods. Each rod corresponds to a rank of the place-value system: units, tens, hundreds, etc. There are several possibilities for displaying the same number.

On the virtual abacus, the beads can be moved using the mouse. Three upper buttons allow users to display the number written in digits (“see number” button), represent the number using the least beads (“placement” button), or return to zero (“display zero” button). Complementing the virtual abacus, we also use a software called “j3p abacus”. The j3p abacus can be used by a teacher for programming questions concerning two tasks: to set a number, or to read a number. The j3p software provides feedback with the right answer.

##### 4.2. Resources designed for primary school teachers

From 2008–2015 we worked within a group comprising teachers, teacher educators, and researchers, with the aim of designing resources around the Chinese abacus for primary school teachers. The group first designed and tested lessons using the abacus. It produced then a structured set of resources called *the abacus kit*, which is freely available online<sup>2</sup> and continues to evolve. We describe here the abacus kit in 2015, as used by Rose (§ 5), which comprises four sections.

The first section presents how the material and virtual abacii work. We discuss the software, abacus frame sheet, instruction to build an abacus, and a tutorial. The second section is entitled *teacher resources*, which include a teacher booklet, tutorial, interviews with two teachers, slideshow of presentation of the abacus, two quizzes, lesson plan templates, and didactical analyzes grid. In fact, several of the teacher resources come from the training path we designed (§ 4.3). The third section is entitled *class resources* and comprises lesson plans, students’ worksheet, a numbers dictionary, and an abacus book. Finally, section four addresses how to go further and includes research and professional articles and presentation of the training path.

Further explanations regarding the specific choices we made for these resources are as follows.

The first section of the abacus kit comprised information on the abacus itself, both the material and virtual versions. The kit also comprised information on how to build an abacus. Indeed, some of the previous work of the second author of this paper was about the interest to build and study calculating instruments for the learning of mathematics (Poisard, 2018).

The third section comprised examples of lessons with the abacus, with comments concerning the didactical choices governing these lessons. We give some examples below.

- (1) Each lesson used both the material and the virtual abacus. Indeed, it is important to allow students to manipulate the material abacus. At the same time, the virtual abacus offers opportunities for more student autonomy thanks to the *display number* button: the students can check by themselves which number is inscribed on the abacus.
- (2) The abacus was used as one amongst many possible representations of a number: other representations included fingers, digits, oral number systems, etc. (Poisard, 2017). One of the resources offered, the *numbers dictionary*, associated different representations of the same number – including the abacus (Fig. 2).
- (3) Each lesson proposed activities on paper. Indeed, it was observed in classes that some students developed trial and errors strategies when working with the virtual abacus. They moved the beads and observed the number written in digits until they reached the correct number, but did not control the value of the beads. Work on paper is necessary to avoid these strategies.

The abacus kit also comprised (in the second and third sections) resources intended to support the design by the teachers of their own lessons, consisting of a lesson plan template and a grid for analyzing a lesson. Moreover, some worksheets representing abacii without beads (to set a number) and with beads (to read a number) were available in the abacus kit for printing and/or modifying.

Soon after the design of a first version of the abacus kit, we decided to design an associated teacher education course. We describe it in the following section.

<sup>2</sup> [http://seminaire-education.espe-bretagne.fr/?page\\_id=611](http://seminaire-education.espe-bretagne.fr/?page_id=611)

#### 4.3. Designing a blended “training path” around the Chinese abacus

M@gistère is a national online platform in France managed by the ministry of education and by local authorities in each region. It offers different kinds of online resources for teacher education, ranging from resources for a distance self-training to training paths, providing the structure and the content of blended teacher education courses (Gueudet & Trouche, 2011).

For this platform, we designed (between 2014 and 2015) a training path entitled “The Chinese abacus at school” (Fig. 3). This training path proposed a particular organization for a blended teacher education program, grounded in the principles of the documentary approach.

The central idea governing the training offered was the design and implementation by the trainees of a lesson using the Chinese abacus. The training program, lasting nine hours (the maximum length authorized for M@gistère programs for primary school teachers), comprised five successive steps, face to face or at a distance, described below in Table 2. Moreover, during the whole training, an online forum was available to trainees and the path had a specific platform on which to share lesson plans and discuss them.

In terms of resources offered, the *Chinese abacus at school* path and the abacus kit are closely linked. In 2014, we integrated in the training path all the resources available in the abacus kit at that time. Moreover we designed resources specifically for the training path, in particular, a video showing an interview with two teachers discussing their uses of the abacus with their students. In addition, there were videos of classroom situations and templates to describe the plan of a lesson with the abacus. The resources designed for the training path were added to the abacus kit in 2015.

The training path is intended to be used by teacher educators. This means that they can modify the path in terms of architecture (number, type of sessions, etc.) and the available resources as well. Hence, the path can be considered as a set of resources for the teacher educators.

### 5. Use and transformation of the abacus resources: a case study

We present here the results of our study, concerning the documents developed by Rose when she used the abacus kit to teach in her grade 4 class (§ 5.1) and then when she used the “*Chinese abacus at school*” training path to design and implement a teacher education course (§ 5.2).

#### 5.1. Documents developed by Rose as a teacher

##### 5.1.1. Presentation of the lesson designed by Rose

As a teacher, Rose’s initial motivation for learning more about the Chinese abacus was to help a student of her class who had significant difficulties in mathematics: Rose estimated the student’s achievements as those of a grade 1 student. She contacted us for this reason. We (Rose and the second author of this paper) had a one and a half hour discussion during which we presented the abacus kit to Rose. The main mathematical notions discussed were the place-value system (natural and decimal numbers), addition, and subtraction. Rose borrowed ten of the material abacii from the teacher training institute and started very quickly to introduce the abacus in her class. In the school, she had access to a specific room where thirteen computers and fifteen laptops were available for the whole school. Her class was equipped with a computer, a video-projector, and a viewer. She designed six sessions with the aim for students (written in her preparation): “to better understand the system of exchanges in place-value system with big numbers (S1 to S3) and decimal numbers (S4 to S6)” (Table 3).

##### 5.1.2. Documents developed by Rose using the abacus kit resources

According to our analysis, Rose developed different documents throughout her use of the abacus kit resources. Due to length limitations, we will only focus on five of these documents, summarized in the “documents table” below (Table 4). The operational invariants paraphrase her declarations during the interview after the lesson (Interview 1, 1st June 2016).

Below, we comment on the documents mentioned in the table. All the quotes come from the last interview, where the tables are discussed with the teacher (Interview 4, 26th June 2018).

- (1) Document with the aim of: “making students manipulate”. According to Rose’s declarations, we consider that she already had an operational invariant like “*it is important for primary school students to manipulate*”. She considered that manipulation was likely to be especially helpful for her student who was encountering difficulties. This operational invariant guided her choices for the teaching of several mathematical themes, but not for her teaching of numbers, before she used the resources of the abacus kit. These resources were coherent with this operational invariant (see the choices presented in § 4.2), and contributed to her adopting them. For her, the two kinds of manipulations - with the material abacus and with the virtual abacus - were important for her student. “Virtual manipulation” was new in her practice.
- (2) Document with the aim of: “making students practice”. This document combines work on the computer and work on slates. For Rose, the “j3p abacus”, with the possibility it provides of giving feedback to students, is important to enable students to train in mathematics, for her “students never train enough [...] we often run after time to stabilize competencies”. Moreover, Rose used the abacus poster (that was supposed to be used in a large format with magnets instead of beads). She used it as a slate for students: each student had a laminated abacus sheet to set numbers on at any time during mathematics (in A4 format). This is a specific appropriation for a particular use in class, changing the format and the use. Rose is used to organizing quick

mathematical activities. In an instrumentalization process, she transforms the resources of the abacus kit to adapt them to her usual practice.

- (3) Document with the aim of: “making students inquire and argue”. For her, “Inquiry-based teaching is something I practice for several years. [...] Sometimes you must let the students search, and sometimes you have to give them the techniques. Here, it was appropriate to let them search and make propositions.” Moreover, she considers that it is especially interesting to share the observations made during the inquiry in whole class discussions. Whole class discussions about the way to set and read a number are central for Rose. Hence in her lesson, students make hypotheses and try to give answers. To support a whole class discussion, the virtual abacus is shown with a video-projector or a material abacus is shown with a viewer. The virtual abacus has the specificity to be used to verify the answer. Indeed, the virtual abacus displays numbers written in digits. The different ways to set numbers are discussed with students, following an inquiry perspective (Artigue & Blomhøj, 2013).
- (4) Document with the aim of: “adapting the teaching to different students”. Rose often organizes the work of her students in workshops. For Rose, the workshops allow her “to sit in a group and watch them doing [...] when we only look at the final production, we miss things.” During the workshop sessions, the class is divided in groups, and the students work on all workshops during a specific session. During the collective introductions, some worksheets called *quick calculations*, designed by Rose a few years ago and adapted for the abacus, are used to work on reading numbers. The calculations are carried out during one week. First, the answer is hidden from students. Later, they can see whether their answer is right or wrong, allowing them to work on the questions during the entire week. The workshops and the “quick calculation” worksheets are important for Rose: they allow her to differentiate her teaching. During one week, every student learns at her/his own pace. Moreover, using a lesson example of the abacus kit designed for a grade 1 class, she asked her low-achieving student to create a counting book with different representations of numbers. At the same time, high-achieving students had to study the question: “if all the beads are activated, what is the number set on the abacus?” (Fig. 4).

We infer here an operational invariant guiding Rose’s action: “it is important that students can work at their own pace”. This operational invariant was probably present for many years in Rose’s schemes. We infer that she also developed a new operational invariant, concerning her use of the abacus kit: “to differentiate in class, I use activities already designed in the kit at different levels”.

- (5) Document with the aim of: “observing and analyzing students’ work”. For Rose, students’ work is a very important resource; she thinks teachers should spend more time looking at students’ work and analyzing students’ mistakes to help them improve their learning. The operational invariant here can be formulated as: “students’ mistakes help me to design and modify my teaching resources”. Rose modified some worksheets she previously gave to students (before using the abacus). She used a *place-value chart* (*tableau de numération*). She noticed that the place-value chart can be difficult to fill in for some students and tried to modify it to help them. The integration of the abacus in her resources led Rose to raise the issue of the place of the decimal point (a comma in French) in a place-value chart. She was used to write the comma in the units column. Working with the abacus, she noticed that mathematically, the comma separates the whole part and the decimal part of a number. For this reason, she moved the decimal point from the unit column to the line that separates the units and the decimal part of numbers on her place-value chart (Fig. 5). She observed that it solved some of the students’ difficulties.

We can see here the double process of the documentational geneses, linked with the two themes of curriculum ergonomics we retained. Rose, as an experienced teacher, had a well-developed capacity to use curriculum resources, which is manifest in a number of ways: she used the abacus kit resources; she added paper-and-pencil worksheets; she used the video projector to stimulate discussion between students and she used the virtual abacus to verify answers and for students’ autonomy. We claim that all these choices concerning the abacus were guided by Rose’s operational invariants in an instrumentalization process. On the other hand, the features of the resources influenced the development of Rose’s operational invariants, and thus of her instructional practices. This influence is illustrated by the examples given above, about a new practice of virtual manipulation, and about the place of the comma in decimal numbers. Because the abacus resource was new in her practice, she developed new schemes; furthermore, some previous operational invariants were reinforced or transferred to new mathematical content. As a result, Rose developed several documents from the abacus kit and she developed her practice for the teaching of mathematics.

## 5.2. Documents developed by Rose as a teacher educator

### 5.2.1. Description of the teacher education program designed by Rose

The year following the design of the grade 4 lesson plan, Rose became a facilitator and decided to use the Chinese abacus training path to organize a teacher education course. We analyze more specifically the second year Rose organized it. In France, primary teachers have to attend 18 h per year of in-service training, including 9 h of mathematics. Rose organized a 9 h training (Table 5) for 9 primary school teachers (teaching from grade 1 to grade 5).

As said above, we consider the abacus training path as a set of resources for teacher educators. From these resources, Rose developed documents that we identified from our data.

### 5.2.2. Documents developed by Rose using the “Chinese abacus” training path

The Documents Table below (Table 6) summarizes three of the documents developed by Rose in her role as a teacher educator. The operational invariants in the table paraphrase her declarations in the interview after the training organized in 2017–2018 (Interview 3, 26th February 2018).

Below, we comment on the documents mentioned in the table. All the quotes come from the last interview, where we presented to

Rose a first version of the table (Interview 4, 26th June 2018).

- (1) Document with the aim of: “making trainees practice and manipulate”. For Rose, it is very important that teachers – like students – can manipulate the material abacus: “Yes, manipulation is essential [...] Moreover, to put trainees in activity allow them to live, to feel difficulties”.  
We consider that “*it is important that trainees manipulate*” is an operational invariant for Rose. Her choice for this training partly came from this operational invariant, already present and guiding her practice as a sport trainer: we interpret this as an instrumentalization process. On the other hand, the resources in the *Chinese abacus at school* training path are coherent with this operational invariant. For this reason, the first session organized by Rose was very close to what we proposed in the training path. It was new for her in a training concerning mathematics, we consider this as an instrumentation process.
- (2) Document with the aim of: “showing and discussing videos of classroom situations and students’ work”. She added some slides linked to what she had tested in her class the previous year, to the slideshow: some grade 4 videos and some students’ works (about the number set when all beads are activated). As a trainer, Rose stated that it was important to draw on her experience as teacher; we interpret this as an operational invariant: “*My teacher’s experience is a value for trainees.*” This operational invariant guided this adaptation of the slideshow. Rose also used some of her students’ productions to discuss them with the trainees during the face to face sessions. We said above that students’ work was an important resource for Rose as a teacher; they were also important resources for her as trainer. She stated that discussing students’ productions with the trainees was an important aspect of professional development. For Rose: “to show yourself in an activity is essential, to show it works in class, the closer to reality as possible [...] It is not perfect, it is just an example of what can be done”. Concerning videos of classroom implementations, more generally “*To interest teachers, we have to show classroom implementations and discuss these with them*” was clearly an operational invariant guiding Rose’s choices. This is linked to Rose’s view on teachers’ professional development: “trainees should participate, not only listening but also doing. I support commitment by restoring what they have done in their class.” In the path she designed, Rose also added her own classroom videos and the videos made in her trainees’ classrooms to the training path we had designed: this is an instrumentalization process, the training path opens up the possibility for such modifications.
- (3) Document with the aim of: “supporting trainees to test in their classes”. An important choice by Rose in her training, not planned in the training path, was to visit the trainees in their classes during their abacus lessons; she even recorded these lessons on video. We consider that this change is the outcome of an instrumentalization process. The temporality of the training path throughout the school year is very linked to the possibility of testing in class. For the first year in Rose’s path, step 1 was in January and the second face to face was step 5 in June. For the second year, Rose thought it important to move step 1 earlier, so it took place in November and the second face to face encounter was step 4 in April. During this step 4, she used videos recorded in the trainees’ classes. For Rose, “*face to face steps are central to share implementations and give the possibility to other teachers to implement in their class as well*”. With this organization, teachers have the possibility to work with Rose and with their colleagues after the second face to face step. Rose thinks she “fostered trainees’ commitment” by the fact that she contacted trainees to ask them if they would like her to come into their class to support their introduction of the Chinese abacus.

Considering the two themes we retained from curriculum ergonomics, first we observe that Rose as an experienced teacher educator has a well-developed capacity to use the training path resources. This was linked with her already existing operational invariants, for example concerning the importance to discuss classroom videos. She used the videos from the path, but also added her own videos. According to DAD, this is the outcome of an instrumentalization process. At the same time we notice that Rose has no experience as trainer for distant training. She does not yet have any operational invariant corresponding to the specific aspects of distance training. For this reason, Rose has chosen to not use the online forum and the online platform (available on the path). She has not yet developed a capacity to use resources specific to distant training.

Moreover we observe that the use of the resources proposed in the path influence the activity of Rose as a teacher educator. For the first time, she introduced the material and virtual manipulation in a training concerning mathematics. The features of the resources, here the material and virtual abacii, clearly influenced her practice as a teacher educator.

## 6. Conclusions and contribution of DAD to curriculum ergonomics

We had two research questions: one about the documents developed by Rose as a teacher, along her documentation work with the resources of the abacus kit; and the other about the documents developed by Rose as a teacher educator, along her documentation work with the resources of the *Chinese abacus at school* training path.

These two questions were associated with the three aims of this paper: investigate the interactions between a teacher or teacher educator and resources; illustrate the use of DAD; and explore the possible contributions of DAD to curriculum ergonomics. In this conclusion we go back to the answers to these questions, and how they contribute to each of these three aims.

We identified five documents developed by Rose as a teacher, for five different aims of her activity: make students manipulate, make them practice, make them inquire and argue; adapt the teaching to different students, and observe and analyze students’ work. These aims were already present in Rose’s work, but she developed new documents (described in §5.1.2). These documents include the abacus and resources from the abacus kit, for her activity corresponding to these aims. Similarly, we identified three documents developed by Rose as a teacher educator, for the following aims: make trainees practice and manipulate; show and discuss videos of classroom situations; and support trainees to try them out in their classes. These documents (described in § 5.2.2) included resources from the *Chinese abacus in school* path. We do not claim that we made a complete analysis of Rose’s documentational geneses; rather,

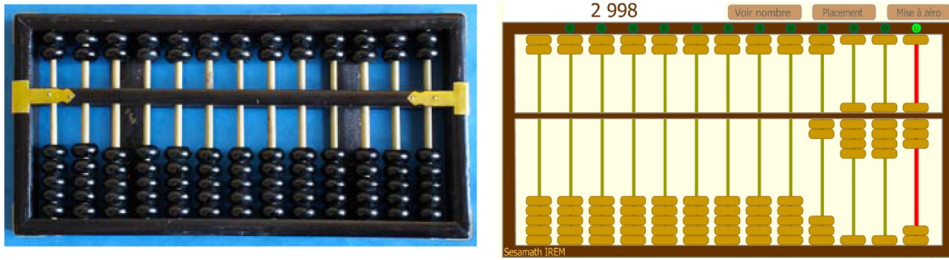


Fig. 1. The Chinese abacus: material on the left and virtual on the right.

0 zéro ZÉRO				
1 un UN				
2 deux DEUX				
3 trois TROIS				

Fig. 2. “Numbers dictionary”, extract, here for numbers from 0 to 3.

### CHRONOLOGIE DU PARCOURS

- Accueil du parcours
- Etape 1 : Présentation du boulier chinois à l'école
- Etape 2 : Appropriation des ressources
- Etape 3 : Conception de séquences
- Etape 4 : Premières séances boulier en classe
- Etape 5 : Bilan des séances en classe
- Conclusion

### TOUT AU LONG DU PARCOURS

- Mutualisation trame et analyse
- Forum du parcours

## Le boulier chinois à l'école

### Accueil du parcours

**PRÉSENTATION DE VOTRE FORMATION**

Nous présentons le parcours "boulier chinois à l'école" en trois parties : le contexte du parcours, les objectifs de la formation et le descriptif des étapes.

Diaporama sonorisé de présentation du parcours (8'45)

#### 1. Contexte du parcours

- Groupe de recherche MARENE
- Travail collaboratif de production et diffusion de ressources en mathématiques pour l'école
- Ressources matérielles et virtuelles

Texte du diaporama sonorisé de présentation

### BARRE DE PROGRESSION

Maintenant

Placez la souris sur le bdc pour plus d'info.

Suivi des participants

Actualiser la barre de suivi  
Mise à jour : 09h40

### NEWS

**DERNIÈRES NOUVELLES**

Ajouter un nouveau sujet..  
(Aucune brève n'a encore été publiée)

### MES FORMATEURS

- Brigitte Sansavy
- Carolina Poisard
- Damien Gouyou-Beauchamps
- Dominique Tournès

Fig. 3. “The Chinese abacus at school”, a training path on the online platform M@gistère.

we focused on her most important documents.

Aside from the aim of the activity, and the resources used, we identified operational invariants – the most important part of schemes - involved in these documents. These analyses helped us to understand the interactions between resources and operational



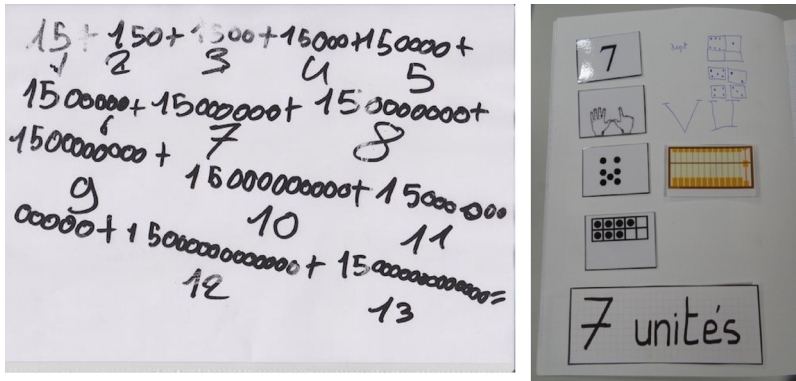


Fig. 4. Students' works: all beads activated (left) – counting book (right).

**Table 2**  
The five steps of the training path “the Chinese abacus at school”.

Total 9h	Step 1	Step 2	Step 3	Step 4	Step 5
Type	Face to face, 1h30	Distance, 2 h	Face to face, 1h30	Distance, 2 h30	Face to face, 1h30
Aim of step	To understand the way of use of the Chinese abacus.	To appropriate the resources of the kit	To collaboratively design sessions	To implement in class of sessions	To analyze and summarize sessions implemented
Resources	Resources of the kit and videos	Quiz 1: way of use of the Chinese abacus	Examples of lessons Lesson plan template	Quiz 2: Study of the Chinese abacus in class	Grid of didactical analysis

**Table 3**  
The lesson designed by Rose with the abacus.

N°	Session's title/aim	Student's resources	Organization
1	To discover the Chinese abacus and research the ways of using it	Material and virtual abacus, viewer, video-projector, number system sheet, slate	Collective, groups of two or three students
2	To read and set natural numbers on the Chinese abacus.	Same as S1 and worksheet accompanying the kit, laminated abacus sheet used like a slate	Three workshops: groups of five or six students
3	To read and set natural numbers on the Chinese abacus: different ways to set numbers, exchanges, to order numbers.	Same as S1 and: j3p abacus, worksheets of the kit	Classroom (laptops) and computers room, three workshops: groups of three students
4	To search how to set decimal numbers on the Chinese abacus: understand zeros in numbers, to verify with the virtual abacus, differentiation (j3p)	Same as S3	Collective introduction (quick calculation). Two workshops: groups of three students
5	To use the Chinese abacus to work on ordering decimal numbers.	Same as S1 and: worksheets of the kit	Two workshops: groups of three students
6	Synthesis: quiz 1 of the training path submitted to students.	Material and virtual abacus, viewer, video-projector, worksheets of the kit	Whole class with an oral discussion

invariants in the case of Rose. They also illustrated the use of DAD to analyze teachers' and teacher educators' interactions with resources and their consequences.

Moreover, they contribute to our exploration of the contribution of DAD to two important themes of curriculum ergonomics: (1) teachers' capacity to use curriculum resources; and (2) curriculum resources influence on instruction. Rose is an experienced teacher and teacher educator. As such, she has many already existing operational invariants, for the aims mentioned above. These operational invariants played an important role in the way she modified and implemented these resources. For example she created slates for her students; we claim that this as a consequence of an operational invariant about the interest of quick mathematical activities to make student practice. For the trainees, she added videos from her own class and from their classes to the available resources, consistent with her operational invariant “my experience as a teacher is of value to trainees”. This follows our claim that teachers and teacher educators' capacity to use curriculum resources depends on their operational invariants.

At the same time, teachers' work with resources leads to the development of new operational invariants, which can be influenced by the features of the resources. For example, in her design of a place-value card for her grade 4 students, Rose considered the place of the comma, and developed what we consider as a new operational invariant: “the comma must not be placed in a column, but on the top of the separation between two columns”. The development of this operational invariant conceptualizes how her awareness was raised by her documentation work with the abacus, and why this produced an evolution of her instructional practices.

**Table 4**  
The documents table, for Rose using the abacus kit resources as a teacher.

Aim of Rose's activity as a teacher	Main resources used by Rose as a teacher	Operational invariants identified	Usual or new in Rose's practice
(1) To make students manipulate	Material and virtual abacus Worksheets Video-projection, viewer	At primary school, the use of material is important to support learning.	Usual for other domains in mathematics (measuring, fractions in particular) but new with regards to place value system. Also new: the possibility of virtual manipulation
(2) To make students practice	Virtual abacus j3p (online interactive exercises with feedback) Abacus slate for every student (for oral quick mathematical activities)	Interactive online exercises and quick oral activities permit the organization of autonomous work for students and opportunities for practice.	
(3) To make students inquire and argue	Virtual abacus (to set numbers in digits to verify answers) Video-projection (to support whole class discussions)	Organizing inquiry in class is interesting, for the teaching of mathematics. The inquiry must be concluded by a whole-class discussion.	Usual and reinforced here
(4) To adapt the teaching to different students (high achieving or low achieving students)	Worksheets (written quick mathematical activities over one week) Workshops organization	To differentiate in class, I use activities already designed in the kit at different levels. It is important that students can work at their own pace.	Usual about the organization in workshops. New about the use of activities designed for different levels.
(5) To observe and analyze students' works	Workshops organization Students' work and possible mistakes Modification of Roses' students worksheets (place-value chart)	It is important for the teacher to have frequent overviews of students' understanding. Students' mistakes help me to design and modify my teaching resources.	Usual, but development of new operational invariants about place value system and decimal numbers written in digits

MILLIERS			UNITÉS			PARTIE DÉCIMALE		
C	D	U	Centaines	Dizaines	Unités	Dixièmes	Centièmes	Millièmes
(... × 100 000)	(... × 10)	(... × 1 000)	(... × 100)	(... × 10)	(... × 1)	(... × 0,1) (... × $\frac{1}{10}$ )	(... × 0,01) (... × $\frac{1}{100}$ )	(... × 0,001) (... × $\frac{1}{1000}$ )

Fig. 5. Extract of the place-value chart produced by Rose after using the abacus.

**Table 5**  
The abacus training designed by Rose (second year).

Total 9h	Step 1	Step 2	Step 3	Step 4	Step 5
Type	Face to face, 1h30	Distance, 2 h	Distance, 2 h	Face to face, 1h30	Distance, 2 h
Aim of steps	To understand the way of using the Chinese abacus.	To appropriate the resources.	To design and implement sessions in class.	To share implementation in class of sessions. Achievements and difficulties.	To implement sessions in class.
Resources/ Sharing	Resources of the kit and videos. Rose's videos recorded in her class the previous year.	Individually and in school project Emails and visits in class of Rose as a facilitator.	Individually and in school project Emails and visits in class of Rose as a facilitator.	Discussions. Rose's videos recorded in trainees classes. Students' works.	Emails and visits in class of Rose as a facilitator.

In this paper we only discussed the contributions of DAD to two of the themes of curriculum ergonomics. We claim that DAD can also contribute to other themes. Concerning issues of alignment between design intentions and curriculum use, DAD could lead to propositions of design modes producing resources more likely to lead to such an alignment. As a matter of fact, how teachers use resources depends on their operational invariants. If the design processes take these operational invariants into account, the teachers are more likely to adopt these resources and to align with the designers' intentions. Moreover, concerning the theme of digital resources and their consequences in terms of dissolving boundaries between design and use, DAD also proposes a conceptualization of this phenomenon. The documents developed by teachers entail resources they design. Some of the resources designed by Rose have

**Table 6**  
Documents developed by Rose as a teacher educator.

Aim of Rose's activity as a trainer	Main resources used by Rose as a trainer	Operational invariants identified	Usual or new in Rose's practice
(1) To make trainees practice and manipulate	Material and virtual abacus Video-projection, viewer	It is important that trainees manipulate the material abacus and use the virtual abacus to verify answers.	Usual from her sportive trainer experience, new for a training on mathematics
(2) To show and discuss videos of classroom situations and students' work	Videos and works from the training path but also from her own practice as a teacher	My teacher's experience is a value for trainees. To interest teachers, we have to show classroom implementations and discuss these with them.	Usual from her facilitator experience
(3) To support trainees to test in their classes	Material abacii loaned to trainers Rose's observation and help during sessions (no use of the forum) Videos and works used for the second face to face	Face to face steps are central in sharing implementations and giving other teachers the possibility of implementing in their class as well.	Usual from her vision of "professional development" New operational invariants concerned how to support trainees in their use of the abacus

been integrated in the abacus kit. The digital resources designed by teachers can easily become resources for other teachers, who can use them but also contribute to the design of sets of resources.

### Acknowledgments

We would like to thank all the teachers and students who made this study possible, and all supports we had from different institutions to work in our research groups: MARENE (UBO, ESPE de Bretagne, CREAD, IFÉ); REVIMM (UBO, ESPE de Bretagne, CREAD) and MITEC (UBO, IREM de Brest). This work was initiated in the frame of the project Mallette Mathématiques pour l'école, funded by DGESCO, France.

### References

- Adler, J. (2000). Conceptualising resources as a theme for teacher education. *Journal of Mathematics Teacher Education*, 3, 205–224.
- Artigue, M., & Blomhøj, M. (2013). Conceptualizing inquiry-based education in mathematics. *ZDM Mathematics Education*, 45, 797–810.
- Choppin, J., Roth McDuffie, A., Drake, C., & Davis, J. (2018). Curriculum ergonomics: Conceptualizing the interactions between curriculum design and use. in press *International Journal of Educational Research*.
- Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for teachers? *Educational Studies in Mathematics*, 71(3), 199–218.
- Gueudet, G., & Trouche, L. (2011). Mathematics teacher education advanced methods: An example in dynamic geometry. *ZDM, The International Journal on Mathematics Education*, 43(3), 399–411.
- Pepin, B., & Gueudet, G. (2018). Curriculum resources and textbooks in mathematics education. In S. Lerman (Ed.). *Encyclopedia of mathematics education* (pp. 132–135). Cham: Springer International Publishing.
- Perrin-Glorian, M.-J., Deblois, L., & Robert, A. (2010). Individual practising mathematics teachers. Studies on their professional growth. In K. Krainer, & T. Wood (Eds.). *The International handbook of mathematics teacher education. Vol. 3 participants in mathematics teacher education* (pp. 35–59). Rotterdam: Sense Publishers.
- Poisard, C. (2018). *La didactique des mathématiques pour décrire et analyser des activités d'animation scientifique*. Paris, France: Colloque Espace Mathématiques Francophone.
- Poisard, C. (2017). *Introducing an old calculating instrument in a new technologies environment: A praxeological analysis of students' tasks using different registers*. *ReSMITCE Review of Science, Mathematics and ICT Education*.
- Poisard, C., Bueno-Ravel, L., & Gueudet, G. (2011). Comprendre l'intégration de ressources technologiques en mathématiques par des professeurs des écoles. *Recherches en didactique des mathématiques*, 31(2), 151–189.
- Psycharis, G., & Kalogeria, E. (2018). Studying the process of becoming a teacher educator in technology-enhanced mathematics. in press *Journal of Mathematics Teacher Education* online.
- Rabardel, P. (1995). *Les hommes et les technologies: approche cognitive des instruments contemporains*. Paris: Armand Colin.
- Remillard, J. (2012). Modes of engagement: Understanding teachers' transactions with mathematics curriculum resources. In G. Gueudet, B. Pepin, & L. Trouche (Eds.). *Mathematics curriculum material and teacher development: From text to 'lived' resources* (pp. 105–122). New York: Springer.
- Vergnaud, G. (1998). Towards a cognitive theory of practice. In A. Sierpiska, & J. Kilpatrick (Eds.). *Mathematics education as a research domain: A search for identity* (pp. 227–241). Dordrecht: Kluwer Academic Publisher.