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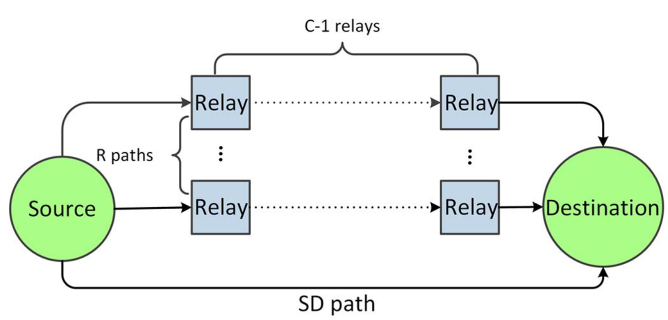

The structure of multi-hop parallel FSO cooperative communication system.

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Performance Analysis of Multihop Parallel Free-Space Optical Systems Over Exponentiated Weibull Fading Channels

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Abstract: The performances of multihop parallel free-space optical (FSO) cooperative communication systems with decode-and-forward protocol under exponentiated Weibull (EW) fading channels have been investigated systematically. With the max–min criterion as the best path selection scheme, the probability density function and the cumulative distribution function of the max–min EW random variable are derived. The analytical expressions for the average bit error rate (ABER) and outage probability with identically and independently distributed (i.i.d.) links are then obtained, respectively. Based on it, the ABER for a non-identically and independently distributed (non-i.i.d.) FSO system is also deduced with the help of the Gauss–Laguerre quadrature rule. The ABER performance of the considered system are further analyzed, in detail, under different turbulence conditions, receiver aperture sizes, and structure parameters (*R* and *C*). The comparison between i.i.d. and non-i.i.d. FSO systems over EW fading channels shows that the performances of both systems could be improved with large aperture diameters adopted for the structure parameters *R* and *C* selected. Monte Carlo simulation is also provided to confirm the correctness of the analytical ABER expressions. This work presents a generalized system model, and it can be used to analyze and design FSO communication systems.

Index Terms: Exponentiated Weibull distribution, free-space optical communication, multihop parallel relay, performance analysis.

1. Introduction

In recent years, free-space optical (FSO) communication has attracted significant attentions due to its very large bandwidth, license-free, excellent security and being a promising solution for the "last mile" problem [1]–[4]. It is widely accepted as a powerful complementary and/or alternative technology to traditional wireless communication for a long distance of applications.

However, the existence of atmospheric-induced turbulence will degrade the performance of FSO links particularly with transmission distance longer than 1 kilometer as a major limiting factor [5]–[7]. Among variety of fading mitigation techniques, relay-assisted FSO communication is a promising solution to overcome turbulence-induced fading and extend coverage, which has received considerable attentions. It was first proposed by Acampora and Krishnamurthy in [8]. However, their work focused mainly on networking and did not address the physical layer aspects. In [9]–[13], one relay-based FSO communication systems has been studied and analyzed over gamma-gamma (G-G) turbulence fading channels. In [4], [6] and [14]–[21], the standalone uses of serial relaying (i.e., multi-hop transmission) and parallel relaying (i.e., cooperative diversity) have been investigated extensively. In fact, the multi-hop transmission is always used to extend the coverage of FSO communication systems when the FSO link is longer than several kilometers in which increasing the transmit power to the required level is limited by the safety and technology requirements. Besides, it could also enhance the system performance against the turbulence-induced fading effects because the fading variance relies on the distance in FSO systems. The parallel relaying scheme normally uses multi-laser transmit apertures, which could be equipped in the source node, and its performance gains are less than those of multi-hop transmission since the parallel relay only contains two hops. Recently, a more practical FSO network combining serial and parallel relaying, called as multi-hop parallel FSO network, has been proposed by Mohammadreza A. Kashani and Murat Uysal in [22] with decode-and-forward (DF) protocol. The so-called multi-hop parallel relaying scheme demonstrates substantial performance improvements with respect to both standalone serial and parallel relaying schemes.

To predict the channel behavior under different atmospheric turbulence conditions, several mathematical models for randomly fading irradiance have been proposed up to now, which include the log-normal (LN), K and G-G distributions [23]. Traditionally, LN distribution is commonly used for the weak turbulence regime [9], [24] and K distribution is always adopted to model strong turbulence condition [24], [25]. Compared with LN and K distribution, G-G distribution offers a good agreement with the experiment data in weak-to-strong turbulence conditions [26]. However, for larger receiver apertures under moderate-to-strong turbulence conditions, G-G distribution does not work very well [27]. A novel PDF model called exponentiated Weibull (EW) has been proposed in [28] and [29], which showed that the EW distribution can provide excellent agreement with the PDF of irradiance for both simulation and experiment data in weak-to-strong turbulence regimes under all aperture averaging conditions. In [30], the FSO experiments conducted at Barcelona, Spain, between the rooftops of two buildings along a medium density residential terrain, showed that EW model is valid in all the tested conditions, and even outperforms the GG and LN distributions. Hence, the EW fading channel model can be regarded as an excellent candidate to accurately predict the probability of fade and the BER performance of laser beam propagating through atmospheric turbulence under the presence of aperture averaging. But only a few works [30]–[33] have been reported on the average bit error rate (ABER) and outage performance of FSO communication systems over EW distribution so far. In addition, all of them only studied the point-to-point (PP) scenario and did not consider the multi-hop parallel relaying scheme, which is more realistic in FSO systems for application.

Motivated by the above analysis, a generalized multi-hop parallel DF based FSO cooperative communication system with the max-min criterion as the best path selection scheme over EW fading channels has been proposed and studied in this paper. The probability density function (PDF) and the cumulative distribution function (CDF) of max-min EW random variable (RV) with regard to SNR have been derived for the first time. Then, the analytical expressions for ABER and outage probability with BPSK subcarrier intensity modulation with identically and independently distributed (i.i.d.) links are obtained, respectively. Furthermore, the ABER for non-identically and independently distributed (non-i.i.d.) FSO system is also derived in terms of the Gauss-Laguerre quadrature rule. The ABER performance of the considered systems have been analyzed deeply with different turbulence strengths, receiver aperture sizes and structure parameters. Besides, the ABER performance under EW fading channels has been compared with

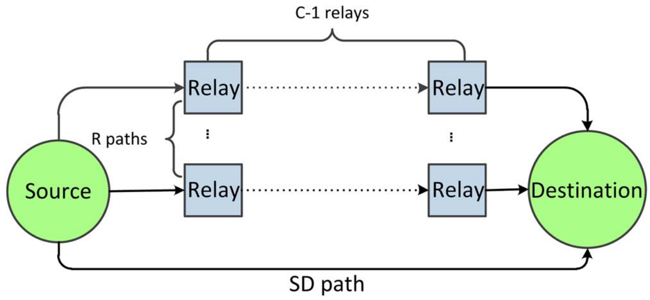

Fig. 1. Structure of multi-hop parallel FSO cooperative communication system.

that over G-G and LN fading channels. Monte Carlo (MC) simulation is also provided to confirm the correctness of the analytical ABER expressions.

2. System and Channel Models

Fig. 1 shows a BPSK modulated FSO cooperative communication system based on the symbol-wise DF with R parallel paths between the source and destination nodes. C hops are assumed to be in the cooperative path, that is, there are $C-1$ relays in each path. R and C are the structure parameters. In the chosen cooperative path, the relays utilize symbol-wise DF method to demodulate the data transmitted from the previous node. And only one relay could be allowed to pass the data to the next node at one time without using error correction coding and error detection coding. Besides, a direct link of source-to-destination (SD) is also considered in this system and the same receiver apertures are assumed for both the relay nodes and destination node.

2.1. Channel Model

In this paper, BPSK subcarrier intensity modulation is adopted in each PP link. At the transmitter, $\mathrm{s}(t)$ is the subcarrier signal and pre-modulated by the source data. Then, the subcarrier signal $\mathrm{s}(t)$ is used to modulate the intensity of a continuous wave laser beam. The transmitted power of the modulated laser beam $P^t_{m,n}$ of the n-th hop in the m-th cooperative path can be obtained from [34] as

$$P^t_{m,n} = P_{m,n}[1+\xi \mathrm{s}(t)] \tag{1}$$

where $m = 1\ldots R$ and $n = 1\ldots C$, $P_{m,n}$ is the average transmitter power in each link and ξ is the modulation index satisfying the condition $-1<\xi\mathrm{s}(t)<1$. At the receiver of each PP link, the received optical power is converted into the electrical signal and the instantaneous photocurrent can be expressed as

$$i_{m,n}(t) = P_{m,n}R_p I_{m,n}(t)[1+\xi\mathrm{s}(t)] + \mathrm{z}(t) \tag{2}$$

where R_p is the photodetector responsivity and $I_{m,n}(t)$ is the instantaneous channel gain in each link. $\mathrm{z}(t)$ is the zero-mean additive white Gaussian noise (AWGN) with variance σ_n^2. Generally, for high-intensity background radiation, the receiver noise can be well modeled as a signal-independent AWGN [31]. The instantaneous electrical SNR at the input of the electrical demodulator of an optical receiver can be written as

$$\gamma_{m,n} = \frac{\left(P_{m,n}R_p\xi\right)^2}{\sigma_n^2} I^2_{m,n} = \bar{\gamma}_{m,n} I^2_{m,n} \tag{3}$$

where $I_{m,n}$ is the RV by sampling $I_{m,n}(t)$ at time instant $t = t_0$, which obeys the PDF $f_{I_{m,n}}(I)$. $\bar{\gamma}_{m,n} = (P_{m,n}R_p\xi)^2/\sigma_n^2$ is the average electrical SNR of each link.

In this work, EW distribution is used to model the turbulence-induced fading and the corresponding PDF and CDF are given in [28]. Thus, the PDF $f_{I_{m,n}}(I)$ and the CDF $F_{I_{m,n}}(I)$ of each PP link can be written as

$$f_{I_{m,n}}(I) = \frac{\alpha_{m,n}\beta_{m,n}}{\eta_{m,n}}\left(\frac{I}{\eta_{m,n}}\right)^{\beta_{m,n}-1}\exp\left[-\left(\frac{I}{\eta_{m,n}}\right)^{\beta_{m,n}}\right]\left\{1-\exp\left[-\left(\frac{I}{\eta_{m,n}}\right)^{\beta_{m,n}}\right]\right\}^{\alpha_{m,n}-1} \tag{4}$$

and

$$F_{I_{m,n}}(I) = \left\{1-\exp\left[-\left(\frac{I}{\eta_{m,n}}\right)^{\beta_{m,n}}\right]\right\}^{\alpha_{m,n}}. \tag{5}$$

According to [28, Eqs. (10)–(12)] and [29, Eqs. (20)–(22)], it is easy to calculate the shape parameters $\alpha_{m,n}$, $\beta_{m,n}$ and the scale parameter $\eta_{m,n}$ in each link. However, the expressions do not work very well for point aperture case. Hence, the Levenberg-Marquardt least-square fitting algorithm was used to get the best fit estimation of the EW parameters.

Substituting (3) into (5), the CDF of each link with regard to SNR can be written as

$$F_{\gamma_{m,n}}(\gamma) = \left\{1-\exp\left[-\left(\frac{\gamma}{\bar{\gamma}_{m,n}\eta_{m,n}^2}\right)^{\frac{\beta_{m,n}}{2}}\right]\right\}^{\alpha_{m,n}}. \tag{6}$$

The PDF of each link can be obtained by differentiating (6) with regard to γ as follows:

$$f_{\gamma_{m,n}}(\gamma) = \frac{\alpha_{m,n}\beta_{m,n}\gamma^{\frac{\beta_{m,n}-2}{2}}}{2\left(\bar{\gamma}_{m,n}\eta_{m,n}^2\right)^{\frac{\beta_{m,n}}{2}}}\left\{1-\exp\left[-\left(\frac{\gamma}{\bar{\gamma}_{m,n}\eta_{m,n}^2}\right)^{\frac{\beta_{m,n}}{2}}\right]\right\}^{\alpha_{m,n}-1}\times\exp\left[-\left(\frac{\gamma}{\bar{\gamma}_{m,n}\eta_{m,n}^2}\right)^{\frac{\beta_{m,n}}{2}}\right]. \tag{7}$$

2.2. System Model

Considering the system structure studied, the SNR of each link can be regarded as a $R \times C$ matrix as follows:

$$\begin{bmatrix} \gamma_{1,1} & \gamma_{1,2} & \cdots & \gamma_{1,C} \\ \gamma_{2,1} & \gamma_{2,2} & \cdots & \gamma_{2,C} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{R,1} & \gamma_{R,2} & \cdots & \gamma_{R,C} \end{bmatrix}. \tag{8}$$

As known, DF protocol may suffer from the severely erroneous relaying of the data from relays. To solve this problem, the best path selection scheme based on the max-min criterion is adopted to enhance the system performance, which has obtained excellent performance in wireless radio-frequency (RF) communication systems.

Let's define a random variable $\gamma_{\max,\min}$ based on the max–min criterion which can be written as [35]

$$\gamma_{\max,\min} = \max\left(\min(\gamma_{m,n})\big|_{n=1}^{C}\right)\Big|_{m=1}^{R}. \tag{9}$$

It is known from (9) that the minimum $\gamma_{m,n}$ of each row is achieved first and then the maximum value of these lowest-values $\gamma_{\max,\min}$ can be selected. Assuming $\bar{\gamma}_{m,n} = \bar{\gamma}_{s,d} = \bar{\gamma}$, here $\bar{\gamma}_{s,d}$ is the average SNR for SD link. Besides, let $\alpha_{m,n} = \alpha_{s,d} = \alpha$, $\beta_{m,n} = \beta_{s,d} = \beta$, $\eta_{m,n} = \eta_{s,d} = \eta$ for i.i.d. links in the current system, here $\alpha_{s,d}$, $\beta_{s,d}$, $\eta_{s,d}$ are the parameters of the SD direct link. According to [36, Eqs. (6.54)–(6.58)], the CDF of $\gamma_{\text{max,min}}$ can be written as

$$F_{\gamma_{\text{max,min}}}(\gamma) = \left\{1 - \left[1 - F_{\gamma_{m,n}}(\gamma)\right]^{C}\right\}^{R} = \left[1 - \left(1 - \left\{1 - \exp\left[-\left(\frac{\gamma}{\bar{\gamma}\eta^2}\right)^{\frac{\beta}{2}}\right]\right\}^{\alpha}\right)^{C}\right]^{R} \tag{10}$$

and the PDF can be obtained by differentiating (10) with regard to γ as follows:

$$\begin{aligned} f_{\gamma_{\text{max,min}}}(\gamma) &= RC\left\{1 - \left[1 - F_{\gamma_{m,n}}(\gamma)\right]^{C}\right\}^{R-1} \times \left[1 - F_{\gamma_{m,n}}(\gamma)\right]^{C-1} \times f_{\gamma_{m,n}}(\gamma) \\ &= RC\left[1 - \left(1 - \left\{1 - \exp\left[-\left(\frac{\gamma}{\bar{\gamma}\eta^2}\right)^{\frac{\beta}{2}}\right]\right\}^{\alpha}\right)^{C}\right]^{R-1} \times \left(1 - \left\{1 - \exp\left[-\left(\frac{\gamma}{\bar{\gamma}\eta^2}\right)^{\frac{\beta}{2}}\right]\right\}^{\alpha}\right)^{C-1} \\ &\quad \times \frac{\alpha\beta\gamma^{\frac{\beta-2}{2}}}{2(\bar{\gamma}\eta^2)^{\frac{\beta}{2}}}\left\{1 - \exp\left[-\left(\frac{\gamma}{\bar{\gamma}\eta^2}\right)^{\frac{\beta}{2}}\right]\right\}^{\alpha-1} \times \exp\left[-\left(\frac{\gamma}{\bar{\gamma}\eta^2}\right)^{\frac{\beta}{2}}\right]. \end{aligned} \tag{11}$$

Let $\gamma_{s,d}$ denotes the instantaneous SNR of SD link. From (6), the CDF of random variable $\gamma_{s,d}$ can be written as

$$F_{\gamma_{s,d}}(\gamma) = \left\{1 - \exp\left[-\left(\frac{\gamma}{\bar{\gamma}_{s,d}\eta_{s,d}^2}\right)^{\frac{\beta_{s,d}}{2}}\right]\right\}^{\alpha_{s,d}}. \tag{12}$$

For the system investigated, the best path should include the hop with the highest value of the minimum of instantaneous SNRs in each path. For the SD direct link of this system, the following scheme can be used:

$$\begin{aligned} &\text{if } \gamma_{\text{max,min}} > \gamma_{s,d}, \quad \text{then use the cooperative path,} \\ &\text{if } \gamma_{\text{max,min}} < \gamma_{s,d}, \quad \text{then use the SD link.} \end{aligned} \tag{13}$$

According to (13), a new random variable γ_{max} can be defined as follows:

$$\gamma_{\text{max}} = \max(\gamma_{\text{max,min}}, \gamma_{s,d}). \tag{14}$$

Thus, the CDF of this random variable γ_{max} can be written as [36, Eq. (6.55)]

$$\begin{aligned} F_{\gamma_{\text{max}}}(\gamma) &= F_{\gamma_{\text{max},min}}(\gamma) \times F_{\gamma_{s,d}}(\gamma) = \left\{1 - \left[1 - F_{\gamma_{m,n}}(\gamma)\right]^{C}\right\}^{R} \times F_{\gamma_{s,d}}(\gamma) \\ &= \left[1 - \left(1 - \left\{1 - \exp\left[-\left(\frac{\gamma}{\bar{\gamma}\eta^2}\right)^{\frac{\beta}{2}}\right]\right\}^{\alpha}\right)^{C}\right]^{R} \times \left\{1 - \exp\left[-\left(\frac{\gamma}{\bar{\gamma}\eta^2}\right)^{\frac{\beta}{2}}\right]\right\}^{\alpha} \end{aligned} \tag{15}$$

and the corresponding PDF can be obtained from (15) as follows:

$$\begin{aligned} f_{\gamma_{\text{max}}}(\gamma) &= f_{\gamma_{m,n}}(\gamma) F_{\gamma_{\text{max,min}}}(\gamma) + F_{\gamma_{\text{max}}}(\gamma) f_{\gamma_{\text{max,min}}}(\gamma) \\ &= f_{\gamma_{m,n}}(\gamma)\left\{1 - \left[1 - F_{\gamma_{m,n}}(\gamma)\right]^{C}\right\}^{R-1} \\ &\quad \times \left\{1 - \left[1 - F_{\gamma_{m,n}}(\gamma)\right]^{C} + RC \times F_{\gamma_{m,n}}(\gamma) \times \left[1 - F_{\gamma_{m,n}}(\gamma)\right]^{C-1}\right\} \end{aligned} \tag{16}$$

where $F_{\gamma_{m,n}}(\gamma)$ and $f_{\gamma_{m,n}}(\gamma)$ are given in (6) and (7), respectively. Considering the difficulty of determining the accurate end-to-end SNR of the above-mentioned DF system, γ_{max} is adopted as the approximate end-to-end SNR [35].

For this generalized FSO system, the structure can be adjusted by changing the parameters R and C. With $R=1$ and $C=2$, it is a single relay communication system and with $R>1$ and $C=2$, it is a parallel relaying system. However, with $R=1$ and $C>2$, it is a multi-hop relaying system. Using (15) and (16), it is easy to describe the above three marginal cases.

In fact, the identically and independently distributed (i.i.d.) FSO system are considered in the above analysis. However, for the ABER performance FSO system with non-identically and independently distributed (non-i.i.d.) links, the corresponding analysis can be found in Appendix.

3. Performance Analysis

In this section, the analytical expressions of ABER and outage probability for the present FSO communication system with BPSK based subcarrier intensity modulation have been derived.

3.1. ABER Performance Analysis

In [37], the BER expression of BPSK has been given as

$$P_e(\gamma) = \mathrm{Q}(\sqrt{2\gamma}) \tag{17}$$

where $\mathrm{Q}(\cdot)$ is the Q-function which can be written as

$$\mathrm{Q}(x) = \left(\frac{1}{\sqrt{2\pi}}\right)\int_x^\infty e^{-t^2/2}\mathrm{d}t. \tag{18}$$

The analytical ABER expression of the above-mentioned system can be obtained from (16) and (17) as

$$\begin{aligned} P_e &= \int_0^\infty P_e(\gamma)\times f_{\gamma_{\max}}(\gamma)d\gamma \\ &= \int_0^\infty \mathrm{Q}(\sqrt{2\gamma})\times f_{\gamma_{m,n}}(\gamma)\left\{1-\left[1-F_{\gamma_{m,n}}(\gamma)\right]^C\right\}^{R-1} \\ &\quad \times\left\{1-\left[1-F_{\gamma_{m,n}}(\gamma)\right]^C + RC\times F_{\gamma_{m,n}}(\gamma)\times\left[1-F_{\gamma_{m,n}}(\gamma)\right]^{C-1}\right\}d\gamma. \end{aligned} \tag{19}$$

The generalized Gauss-Laguerre quadrature function [38], $\int_0^\infty x^\beta e^{-x} f(x)dx = \sum_{i=1}^n H_i f(x_i)$, can be used to efficiently and accurately approximate (19). Thus the (19) can be expressed by a truncated series

$$\begin{aligned} P_e &= \int_0^\infty P_e(\gamma)\times f_{\gamma_{\max}}(\gamma)d\gamma = -\int_0^\infty F_{\gamma_{\max}}(\gamma)dP_e(\gamma) = \frac{1}{2\sqrt{\pi}}\int_0^\infty \gamma^{-1/2}e^{-\gamma}F_{\gamma_{\max}}(\gamma)d\gamma \\ &\approx \frac{1}{2\sqrt{\pi}}\sum_{i=1}^n H_i\left[1-\left(1-\left\{1-\exp\left[-\left(\frac{x_i}{\bar{\gamma}\eta^2}\right)^{\frac{\beta}{2}}\right]\right\}^\alpha\right)^C\right]^R \times\left\{1-\exp\left[-\left(\frac{x_i}{\bar{\gamma}\eta^2}\right)^{\frac{\beta}{2}}\right]\right\}^\alpha \end{aligned} \tag{20}$$

where x_i is the i-th root of the generalized Laguerre polynomial $L_n^{(-1/2)}(x)$ and the weight H_i can be calculated by [39]

$$H_i = \frac{\Gamma\left(n+\frac{1}{2}\right)x_i}{n!(n+1)^2\left[L_{n+1}^{(-1/2)}(x_i)\right]^2}. \tag{21}$$

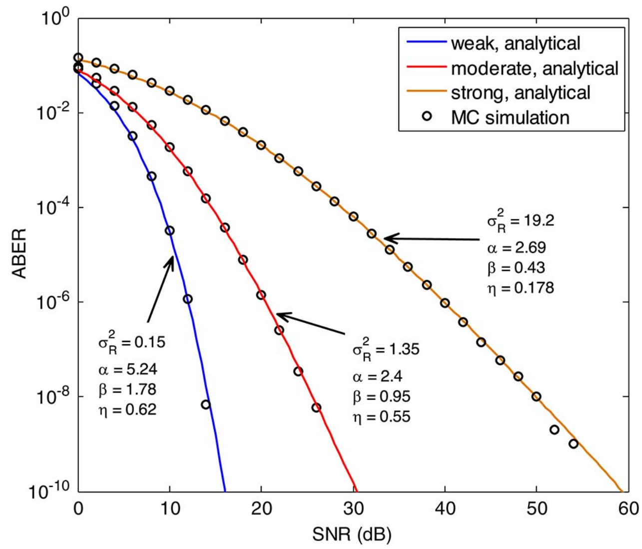

Fig. 2. ABER performance of the multi-hop parallel FSO system with $R = 3$, $C = 3$ under weak ($\sigma_R^2 = 0.15$), moderate ($\sigma_R^2 = 1.35$), and strong ($\sigma_R^2 = 19.2$) turbulence conditions.

3.2. Outage Probability Analysis

Substituting (3) into (15), the CDF of this system with regard to RV I can be written as

$$F_{I_{\max}}(I) = \left[1 - \left(1 - \left\{1 - \exp\left[-\left(\frac{I}{\eta}\right)^{\beta}\right]\right\}^{\alpha}\right)^{C}\right]^{R} \times \left\{1 - \exp\left[-\left(\frac{I}{\eta}\right)^{\beta}\right]\right\}^{\alpha}. \tag{22}$$

The outage probability is defined as the probability that the instantaneous SNR is lower than a specified threshold [33]. According to the max-min criterion, if the outage occurs, RV $\gamma_{\max}$ must fall below the threshold. Therefore, the outage probability of the system can be expressed as

$$P_{\text{out}} = P(\gamma_{\max} \le \gamma_{th}) = P\left(I \le \sqrt{\frac{\gamma_{th}}{\gamma_{\max}}}\right). \tag{23}$$

Substituting (22) into (23), the analytical outage probability can be achieved as

$$P_{\text{out}} = F_{I_{\max}}\left(\sqrt{\frac{\gamma_{th}}{\gamma_{\max}}}\right) = \left[1 - \left(1 - \left\{1 - \exp\left[-\left(\frac{1}{\eta^2\gamma_n}\right)^{\frac{\beta}{2}}\right]\right\}^{\alpha}\right)^{C}\right]^{R}\left\{1 - \exp\left[-\left(\frac{1}{\eta^2\gamma_n}\right)^{\frac{\beta}{2}}\right]\right\}^{\alpha} \tag{24}$$

where $\gamma_n = \gamma_{\max}/\gamma_{th}$ is the normalized electrical SNR defined in [23].

4. Analytical and Simulation Results

In this section, the analytical results of ABER and outage probability are obtained from (20) and (24), respectively. In computing the generalized Gauss-Lagurre approximations, we choose n to be 30. The inverse transform method [40] is used in the MC simulation to generate the random fading channels following the EW distribution. The parameters (α, β, η) used in the analytical calculation and MC simulation are all selected from the best PDF fitting in [28] and [29].

Fig. 2 shows the ABER performance of the multi-hop parallel FSO system over EW fading channels at different SNRs, which contains three cooperative paths (three hops in each path) and a SD direct link. Under weak, moderate and strong turbulence conditions, the corresponding Rytov variance σ_R^2 equals to 0.15, 1.35, and 19.2, respectively. It can be seen that the analytical ABER results have excellent agreements with the MC simulations. This confirms the

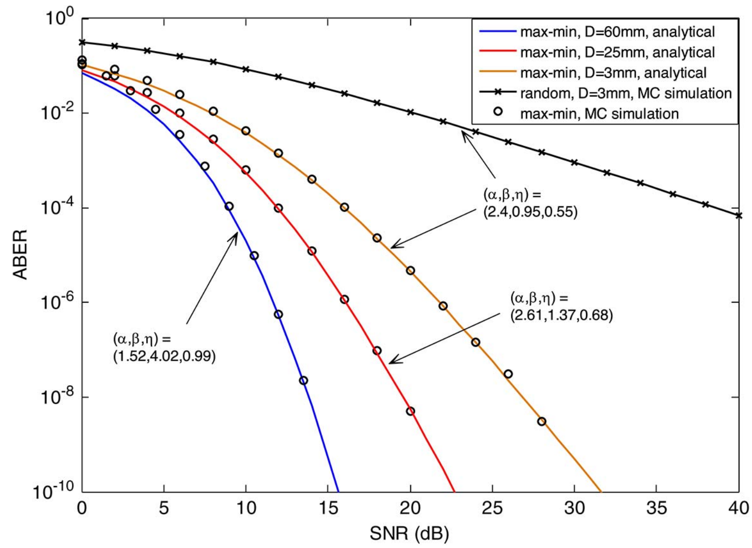

Fig. 3. ABER performance of multi-hop parallel FSO system with $R = 3$, $C = 3$ under moderate turbulence condition ($\sigma_R^2 = 1.35$) of three different apertures.

correctness of our ABER model. It is also seen from the figure that the ABER of this system increases with the increase of the value of Rytov variance (from weak to strong turbulence). This indicates that the performance of the present system is degraded with the increase of the strength of atmosphere turbulence, which has been also confirmed in the FSO PP link over EW distribution [33].

The ABER versus SNR performances of random path selection scheme and best path selection scheme under moderate turbulence condition ($\sigma_R^2 = 1.35$) with 3 mm aperture size have been shown in Fig. 3. The link distance L is equal to 1225 m and the coherence radius ρ_0 is 9.27 mm. The results of random path selection are obtained by MC simulation. It can be found that the ABER performance of the best path selection scheme is remarkably superior to that of the random path selection scheme over EW fading channels. For instance, when the SNR value is equal to 30 dB, the ABER of the random path selection scheme is 10^{-3} while the ABER of the best path scheme is about 10^{-9} in the moderate turbulence condition. This is because that the random path selection scheme does not provide any diversity gain compared with the best path selection scheme. The ABER performances at three receiving apertures with the max-min criterion have been also given for comparison in this figure, which match very well with the MC simulations. It can be seen that the ABER performance of this FSO system has been significantly improved by increasing the aperture size over EW distribution. It can be also seen from this figure that the diversity gain increases with the increase of aperture size. This is because that aperture averaging can be considered as a simple form of spatial diversity when the receiver lens aperture is larger than the fading correlation length [41].

In Fig. 4, the ABER performance of the multi-hop parallel FSO cooperative communication system under moderate turbulence condition ($\sigma_R^2 = 1.35$) with the same number of relays in each path but different number of cooperative paths ($R = 2$, $R = 3$, $R = 5$, and $R = 8$) has been plotted. The parameters (α, β, η) are equal to (2.4,0.95,0.55). Each link distance L is equal to 1225 m and the coherence radius ρ_0 equals to 9.27 mm. The aperture size of each receiver is 3 mm ($D = 3$ mm). It can be found that the ABER values decrease with the increase of the cooperative paths (R). For example, when the SNR value is equal to 20 dB, the ABERs for R of 2, 3, 5, 8 are approximately equal to 0.3×10^{-4}, 0.4×10^{-5}, 0.2×10^{-6}, and 0.3×10^{-8}, respectively. This is because adding more cooperative path in the system can improve the diversity gain.

Fig. 5 shows the ABER performance of multi-hop parallel FSO cooperative communication system under moderate turbulence condition ($\sigma_R^2 = 1.35$) with different hops in each path. The

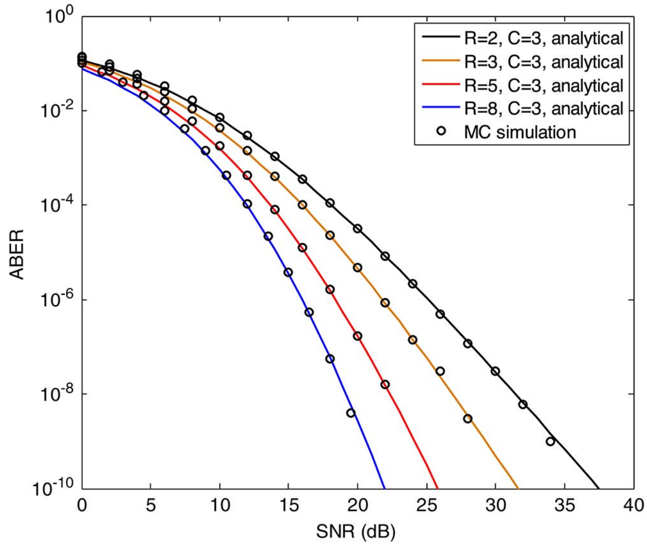

Fig. 4. ABER performance of multi-hop parallel FSO system with different numbers of cooperative path under moderate turbulence condition ($\sigma_R^2 = 1.35$). Link distance $L = 1225$ m, aperture size $D = 3$ mm, coherence radius $\rho_0 = 9.27$ mm, and $(\alpha, \beta, \eta) = (2.4, 0.95, 0.55)$.

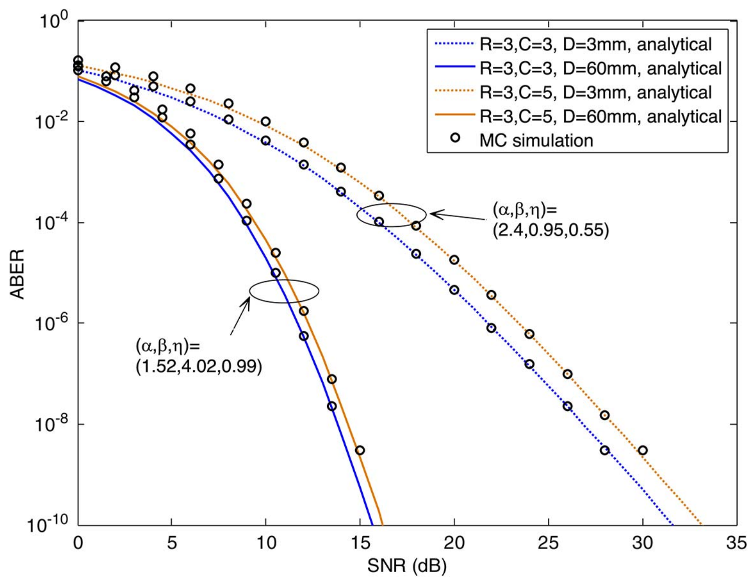

Fig. 5. ABER performance of multi-hop parallel FSO system with different hops in each path under moderate turbulence condition ($\sigma_R^2 = 1.35$).

parameters (α, β, η) of $D = 3$ mm are equal to (2.4, 0.95, 0.55) and those of $D = 60$ mm are equal to (1.52, 4.02, 0.99). It can be clear seen that the ABER values increase with the increasing hops for both apertures over EW fading channels. But this effect will be restrained by aperture averaging. That is, the degradation of the system performance by increasing the end-to-end distance can be mitigated by the aperture averaging technique. For example, to achieve the ABER value of 10^{-8}, the SNR difference of $R = 3$, $C = 3$ and $R = 3$, $C = 5$ is about 2 dB with a receiver of 3 mm aperture, but less than 1 dB when the receiver of 60 mm aperture is adopted under EW distribution.

Fig. 6 shows the outage probability of the present FSO system with $R = 3$, $C = 3$ at different SNRs. Two and three aperture sizes have been considered for weak and moderate turbulence

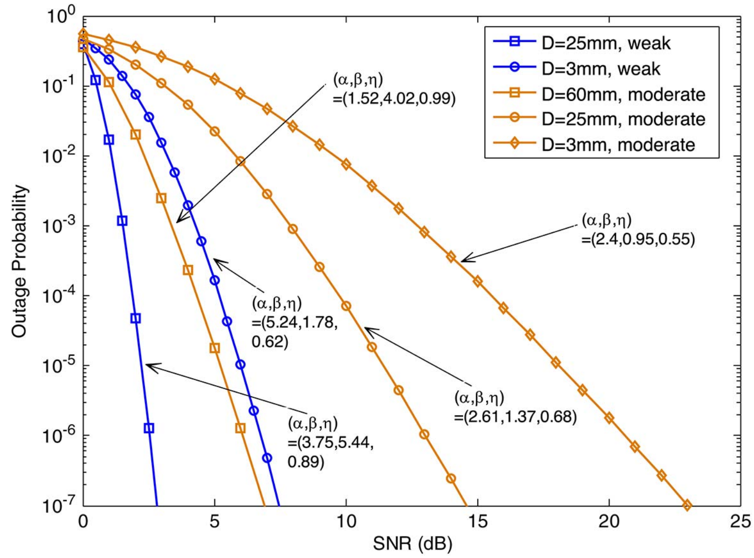

Fig. 6. Outage probability versus SNR for multi-hop parallel FSO system under weak and moderate conditions with $R = 3$, $C = 3$.

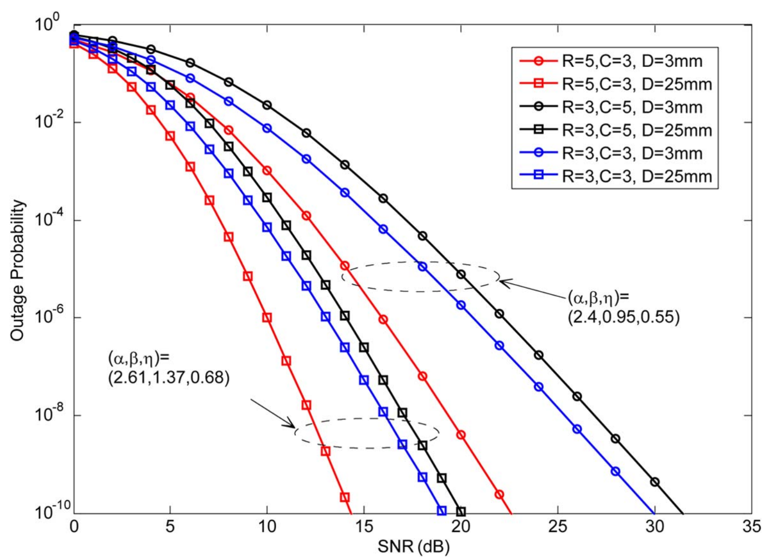

Fig. 7. Outage probability versus SNR for multi-hop parallel FSO system under moderate condition with different R and C.

conditions, respectively. It can be seen that the outage probability is not only determined by the SNR but also by the strength of the turbulence and the aperture size, which has been also found in the FSO PP link over EW fading channels [33]. The outage probability with different R and C under moderate turbulence ($\sigma_R^2 = 1.35$) condition has been given in Fig. 7. The aperture sizes of $D = 3$ mm and $D = 25$ mm have been selected. It can be found from this figure that the outage probability decreases with the increase of aperture size regardless of the selected R and C of the system. For example, when the SNR equals to 10 dB and the aperture size increases from 3 mm to 25 mm, the outage probability of $R = 5$, $C = 3$ decreases from 10^{-3} to 10^{-6}, the outage probability of $R = 3$, $C = 3$ decreases from 0.7×10^{-2} to 0.7×10^{-4} and that of $R = 3$, $C = 5$ decreases from 0.2×10^{-1} to 0.2×10^{-3}. In addition, when the aperture size is

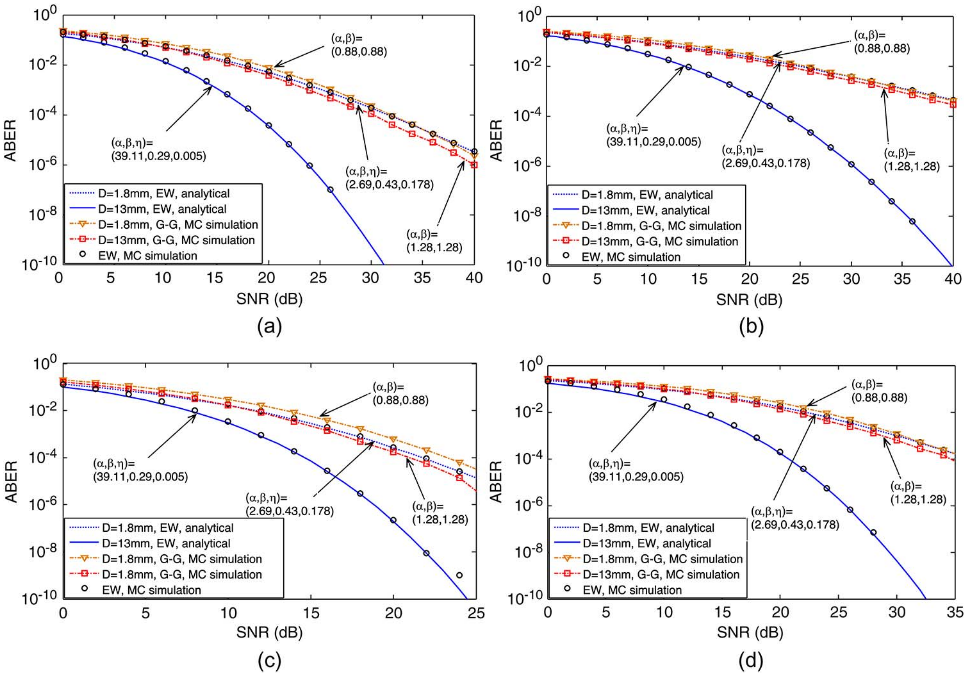

Fig. 8. ABER performance comparison of G-G fading channels with that of EW fading channels under strong turbulence condition. The structure of (a) is $R = 3$, $C = 3$. The structure of (b) is $R = 1$, $C = 3$. The structure of (c) is $R = 8$, $C = 3$. The structure of (d) is $R = 3$, $C = 6$.

fixed, the outage probability increases with the increasing hops. For instance, when the SNR equals to 20 dB with 3 mm receiver aperture ($D = 3$ mm), the outage probability of $R = 3$, $C = 3$ is about 0.2×10^{-5} while that of $R = 3$, $C = 5$ is about 0.8×10^{-5}. However, when the SNR still equals to 20 dB but with 25 mm receiver aperture ($D = 25$ mm), the outage probability of $R = 3$, $C = 5$ is about 10^{-10}, which is five orders magnitude lower than that of $R = 3$, $C = 3$ with 3 mm aperture condition. In a word, aperture averaging can substantially mitigate the outage performance deterioration caused by the increasing path distance for the current FSO system over EW fading channels.

The ABER performances of G-G and EW fading channels under strong turbulence condition ($\sigma_R^2 = 19.2$) with two aperture sizes ($D = 1.8$ mm and $D = 13$ mm) have been shown in Fig. 8. The parameters for both EW and G-G fading channels are selected from [29]. The results of G-G fading channels are based on the MC simulations. It can be found that when the number of hops ($C = 3$) in each path is fixed, the diversity order of the present system increases with the increase of R under both EW and G-G fading channels. When the number of cooperative paths is fixed ($R = 3$), the ABER of $C = 3$ is lower than that of $C = 6$ for both EW and G-G models. That is, the ABER values increase with the increasing hops for both apertures over EW and G-G distributions. However, it should be noted from Fig. 8 (a)–(d) that, with the increase of the aperture size from $D = 1.8$ mm to $D = 13$ mm for different structure parameters R and C, the ABER performance will be significantly improved by aperture averaging under EW fading channels while that of G-G channels do not enhance too much. This phenomenon shows that EW distribution is more efficient than G-G model for aperture averaging under strong turbulence channels regardless of the selected structure parameters R and C.

Fig. 9 presents the ABERs for BPSK modulation over EW and LN fading channels under weak turbulence condition ($\sigma_R^2 = 0.15$) with two aperture sizes ($D = 3$ mm and $D = 25$ mm). The parameters for both fading channels are selected from [32]. The results of LN fading channels are based on our Monte Carlo simulations. It can be seen from this figure that with the aperture size $D = 3$ mm, the ABER results of EW are a little bit larger than that of LN at settled

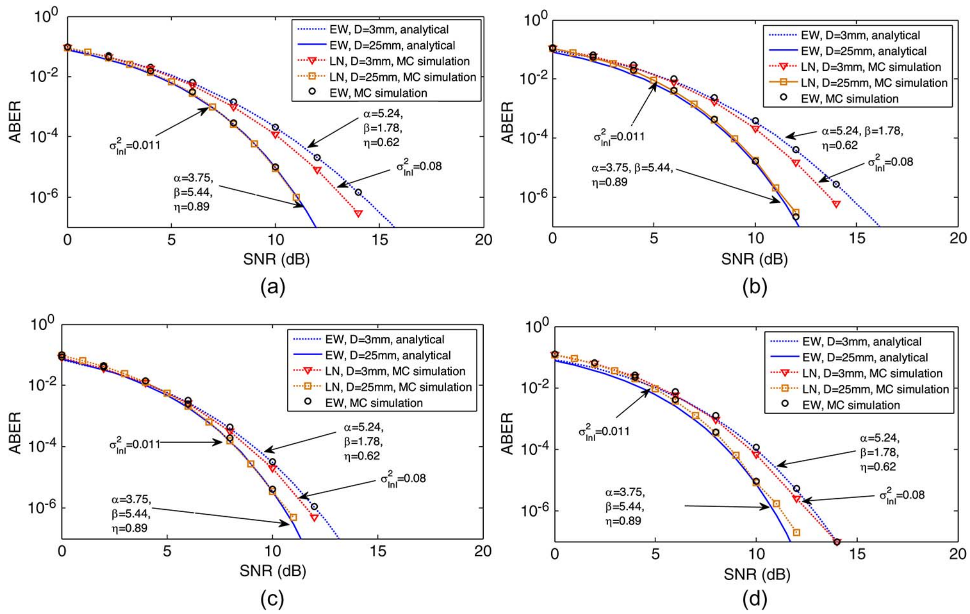

Fig. 9. ABER performance comparison of LN fading channels with that of EW fading channels under weak turbulence condition. The structure of (a) is $R = 1$, $C = 2$. The structure of (b) is $R = 1$, $C = 4$. The structure of (c) is $R = 3$, $C = 2$. The structure of (d) is $R = 3$, $C = 4$.

SNR values. However, when the number of hops (C) fixed, the difference of ABER results between LN and EW fading channels decreases with the increase of the number of parallel path R. Besides, the ABERs of the EW and LN are almost equal when the aperture size equals to 25 mm $(D = 25\ \text{mm})$ for the structure parameters R and C selected.

5. Conclusion

In this paper, the performance of a generalized multi-hop parallel DF based FSO cooperative communication system under EW fading channels have been studied analytically and numerically with the max-min criterion employed. The PDF and CDF of the max-min EW RV have been deduced and the corresponding analytical expressions of ABER and outage probability of BPSK modulation with i.i.d. links have been achieved, respectively. Then, the ABER for non-identically and independently distributed (non-i.i.d.) FSO system is derived based on the Gauss-Laguerre quadrature rule. Furthermore, the ABER performance of the considered systems are investigated systematically combined with MC simulations. The comparison between i.i.d. and non-i.i.d. systems over EW fading model demonstrates that the performance of both systems could be enhanced by large aperture sizes with the structure parameters R and C selected. This work will be of great help for the system design of multi-hop parallel FSO systems.

Appendix

ABER Performance Analysis of the Present System With Non-Identically and Independently Distributed Links

For the present communication system with non-i.i.d. links, the (10) can be rewritten as

$$\hat{F}_{\gamma_{\max,\min}}(\gamma) = \prod_{m=1}^{R}\left[1 - \prod_{n=1}^{C}\left(1 - \left\{1 - \exp\left[-\left(\frac{\gamma}{\bar{\gamma}_{m,n}\eta_{m,n}^{2}}\right)^{\frac{\beta_{m,n}}{2}}\right]\right\}^{\alpha_{m,n}}\right)\right]. \tag{25}$$

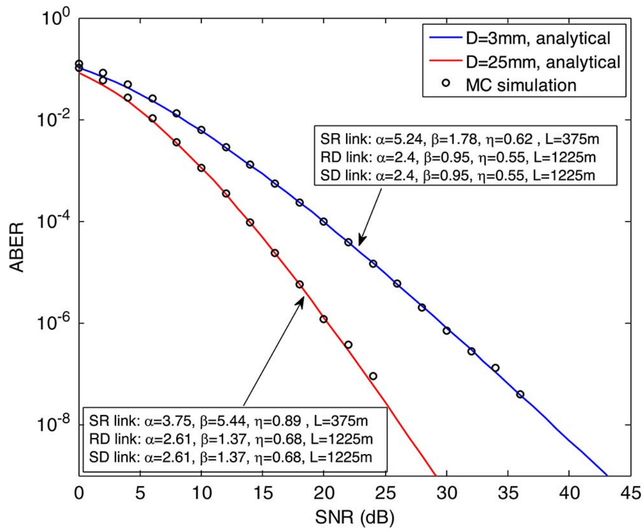

Fig. 10. ABER performance of multi-hop parallel FSO system with non-i.i.d. links over EW fading channels. The structure parameters R and C are 1 and 2, respectively.

Substituting (12) and (25) into (15), the CDF of random variable $\gamma_{\max}$ can be expressed as

$$\begin{aligned}\hat{F}_{\gamma_{\max}}(\gamma) &= \hat{F}_{\gamma_{\max,\min}}(\gamma) \times F_{\gamma_{s,d}}(\gamma)\\ &= \prod_{m=1}^{R}\left[1-\prod_{n=1}^{C}\left(1-\left\{1-\exp\left[-\left(\frac{\gamma}{\bar{\gamma}_{m,n}\eta_{m,n}^{2}}\right)^{\frac{\beta_{m,n}}{2}}\right]\right\}^{\alpha_{m,n}}\right)\right]\\ &\quad\times\left\{1-\exp\left[-\left(\frac{\gamma}{\bar{\gamma}_{s,d}\eta_{s,d}^{2}}\right)^{\frac{\beta_{s,d}}{2}}\right]\right\}^{\alpha_{s,d}}. \end{aligned} \tag{26}$$

The PDF of random variable $\gamma_{\max}$ of non-i.i.d. system can be obtained by differentiating (26) with regard to γ as follows:

$$\hat{f}_{\gamma_{\max}}(\gamma) = \frac{\mathrm{d}\hat{F}_{\gamma_{\max}}(\gamma)}{\mathrm{d}\gamma}. \tag{27}$$

However, it is hard to achieve an analytical expression of $\hat{f}_{\gamma_{\max}}(\gamma)$ based on (27).

In fact, the BER expression of BPSK modulation has been given in (17). Thus the ABER expression of non-i.i.d. scenario could be written as

$$\begin{aligned}\hat{P}_e &= \int_0^{\infty} P_e(\gamma)\times\hat{f}_{\gamma_{\max}}(\gamma)d\gamma = \int_0^{\infty} P_e(\gamma)d\hat{F}_{\gamma_{\max}}(\gamma) = P_e(\gamma)\hat{F}_{\gamma_{\max}}(\gamma)\Big|_0^{\infty} - \int_0^{\infty}\hat{F}_{\gamma_{\max}}(\gamma)dP_e(\gamma)\\ &= -\int_0^{\infty}\hat{F}_{\gamma_{\max}}(\gamma)dP_e(\gamma) = \frac{1}{2\sqrt{\pi}}\int_0^{\infty}\gamma^{-1/2}e^{-\gamma}\hat{F}_{\gamma_{\max}}(\gamma)\mathrm{d}\gamma. \end{aligned} \tag{28}$$

Here, (28) can be accurately approximated based on the generalized Gauss-Laguerre quadrature function as [38] mentioned, $\int_0^{\infty} x^{\beta}e^{-x}f(x)dx = \sum_{i=1}^{n} H_i f(x_i)$. Then (28) can be further expressed by a truncated series

$$\hat{P}_e \approx \frac{1}{2\sqrt{\pi}} \sum_{i=1}^{n} H_i \prod_{m=1}^{R} \left[1 - \prod_{n=1}^{C} \left(1 - \left\{ 1 - \exp\left[-\left(\frac{x_i}{\bar{\gamma}_{m,n}\eta_{m,n}^2} \right)^{\frac{\beta_{m,n}}{2}} \right] \right\}^{\alpha_{m,n}} \right) \right] \times \left\{ 1 - \exp\left[-\left(\frac{x_i}{\bar{\gamma}_{s,d}\eta_{s,d}^2} \right)^{\frac{\beta_{s,d}}{2}} \right] \right\}^{\alpha_{s,d}} \quad (29)$$

where x_i is the i-th root of the generalized Laguerre polynomial $L_n^{(-1/2)}(x)$ and the weight H_i can be calculated by (21).

The ABER performance of single relay FSO communication system ($R = 1$, $C = 2$) with non-i.i.d. links has been shown in Fig. 10. In it, the analytical results are obtained from (29) with two aperture sizes ($D = 3$ mm and $D = 25$ mm) considered. The length of source-to-relay (SR) link equals to 375 m. And the length of both relay-to-destination (RD) link and source-to-destination (SD) link is assumed to be 1225 m. As seen, the analytical results have good agreement with the MC simulations, which verifies the accuracy of our models. Besides, the ABER performance of this FSO system with non-i.i.d. links has been significantly improved by increasing the aperture size over EW distribution. For example, when SNR value is equal to 20 dB, the ABER of apertures size $D = 3$ mm is about 10^{-4} while that of $D = 25$ mm is approximate 10^{-6}. Compared with the Fig. 3 on the i.i.d. FSO system, it can be also known that the performance of both non-i.i.d. and i.i.d. FSO systems over EW fading channels could be improved with large aperture diameters adopted for the structure parameters R and C selected.

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