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Uncertainties in road vehicle suspensions

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Abstract

Road vehicles are subject to random excitation by the unevenness of the road. For a dynamical analysis, vehicle models of the vertical vibrations as well as guideway models of the road unevenness are required. The fundamental dynamics of vehicle suspensions can be already modeled by a quarter car featuring the decoupling of the car body motion and the wheel motion. This suspension model is characterized by five design parameters where two of them, the shock absorber and the tire spring, are highly uncertain due to wear and poor maintenance. For the assessment of the vehicles performance three criteria have to be used: ride comfort, driving safety and suspension travel. These criteria depend on all the five design parameters resulting in a conflict or a pareto-optimal problem, respectively. In this paper, the uncertainties of the parameters are projected into a criteria space in order to support the decision to be made on the basis of a pareto-optimal problem. Simulations with uncertainties support the robust suspension design. It is shown that controlled suspension parameters remain uncertain due to the unpredictable decisions made by the driver.

Keywords: Road vehicle suspensions; random excitation; parameter uncertainties; assessment criteria; conflict problem; robust design.

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1. Introduction

Road vehicle suspensions generating propulsion and guidance forces are essential for driving safety. These forces are mainly controlled by the driver accelerating, braking and steering the vehicle. Statistics of road fatalities since 2001 in Europe[1] show a strong reduction from 54.900 to 31.500 in 2010 and 26.000 in 2013, see Fig. 1. The main reason for this encouraging development is the introduction of occupant protection systems. In 1998 the federal legislation made airbags mandatory in the United States, and since 2006 the wearing of seatbelts is compulsory in all vehicles throughout the European Union. On the other hand, a survey[2] on the cause of accidents with physical injury from 2012 shows that three-fourths of them are due to driver malfunction, while still one-fourth has other reasons, including vehicle malfunction by uncertain design parameters.

In this paper, the vehicle and guideway modeling with uncertain parameters is considered and the assessment criteria for vehicle-guideway-systems are presented and analyzed by the covariance analysis. Multicriteria optimization

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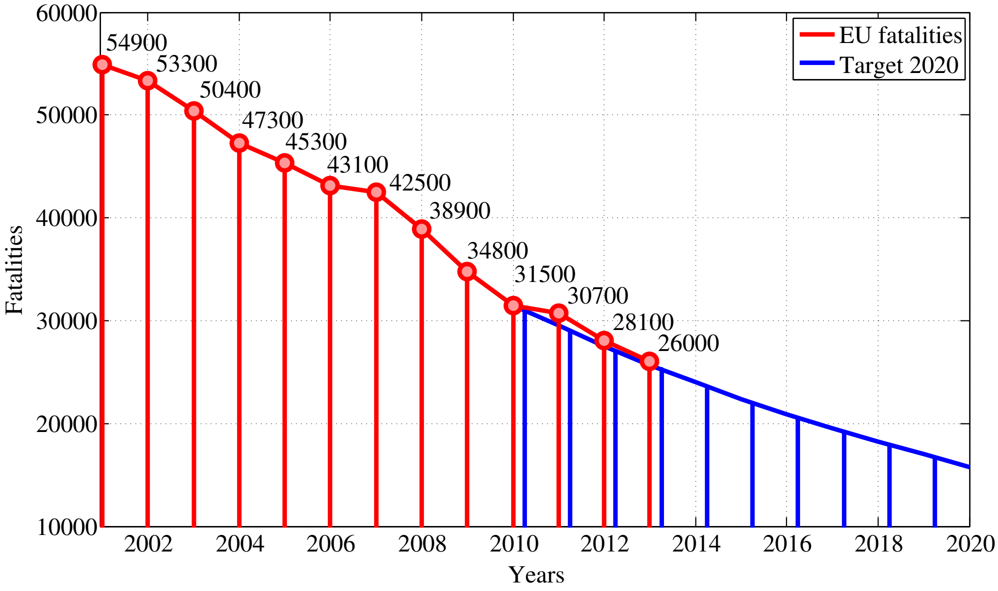


Fig. 1: Road fatalities in the EU since 2001[2].

and uncertainty analysis are performed with the software FAMOUS based on fuzzy arithmetic and introduced to multibody dynamics by Walz and Hanss[3].

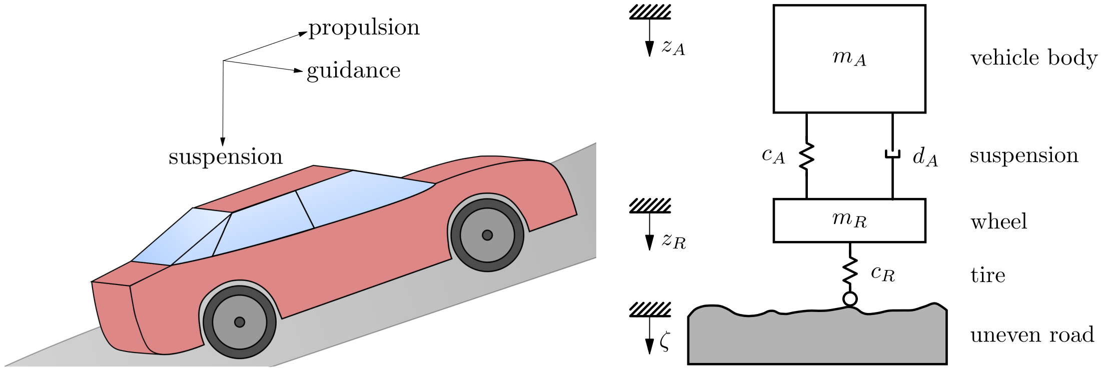
2. Vehicle modeling and uncertain parameters

Multibody system dynamics is a standard tool for modeling all kind of vehicles, see Ref.[4]. The complexity of the required model depends on the engineering task. For the suspension analysis of a passenger car, Fig. 2a, there may be chosen, e.g., a full car 3D model with 19 degrees of freedom (dof), a half car 2D model with 8 dof or a 1D quarter car model with 2 dof, respectively. These different models can be validated against each other using benchmarking as shown, e.g., in Ref.[5].

For fundamental research, more simple models are preferable if they represent the basic dynamical phenomena under consideration. In the case of suspension systems, the principle of frequency decoupling between the high frequency wheel motion following the unevenness of the road, and the low frequency car body motion providing a comfortable ride for the passengers is already constituted by a quarter car model, Fig. 2b. The uneven road is characterized by the profile $\zeta(t)$, the vertical motion of the wheel is described by $z_R(t)$, and the body motion by $z_A(t)$, also called heave. The quarter car model has five parameters, the nominal values of which read for a standard middle-class passenger car as follows: body mass $m_A = 1200$ kg, body spring $c_A = 30000$ N/m, shock absorber $d_A = 4800$ Ns/m, wheel mass $m_R = 80$ kg and tire spring $c_R = 320000$ N/m. In addition, the three uncertain epistemic parameters have to be represented by fuzzy numbers with the lower and upper boundaries: $1200 < m_A < 2000$ kg, $1200 < d_A < 6000$ Ns/m, and $160000 < c_R < 320000$ N/m. The body mass is restricted by the empty weight and the maximal payload, the shock absorber may be worn out and slipping, or blocked and sticky. The tire stiffness depends on the air pressure, which is often too low since the driver does check it rarely.

3. Guideway modeling and vehicle guideway system

The unevenness of the road is generally uncertain. However, it is an aleatoric variable which can be measured and represented to be a stochastic process with random variables $\zeta(x)$ where x means the position on the guideway, see[4].



(a) Main engineering tasks of a passenger car. (b) Quarter car model.

Fig. 2: Modelling of a vehicle.

The measurements of the stochastic road unevenness are usually documented by the spectral density $\Phi_\zeta(\Omega)$ depending on a spatial frequency Ω with limited frequency range and waviness $w = 2$,

$$\Phi_\zeta(\Omega) = \Phi_0 \left(\frac{\Omega_0}{\Omega}\right)^w, \quad 0 < \Omega_I \leq \Omega \leq \Omega_{II} < \infty. \tag{1}$$

From the presented model of the guideway unevenness depending on the spatial coordinate x, a corresponding model for the vehicle excitation depending on time t can be found. Due to the waviness $w = 2$, a white noise excitation process $\dot{\zeta}(t)$ representing the vertical unevenness velocity with constant spectral density is obtained

$$\tilde{\Phi}_{\dot{\zeta}}(\omega) = \omega^2 \upsilon\, \tilde{\Phi}_0 \left(\frac{\Omega_0}{\omega}\right)^2 = \upsilon\, \tilde{\Phi}_0 \Omega_0^2 = \text{const.} \tag{2}$$

where ω is the frequency in the time domain and υ means the vehicle speed.

The guideway vehicle system is now composed by the vehicle body, the suspension devices, the guideway and its unevenness with white noise velocity excitation $\dot{\zeta}(t) \sim (0, q)$ characterized by zero mean and intensity q. Therefore, the linear equations of motion of the quarter car model with respect to its equilibrium condition are differentiated with respect to time reading as

$$\begin{bmatrix} m_A & 0 \\ 0 & m_R \end{bmatrix} \begin{bmatrix} \dddot{z}_A \\ \dddot{z}_R \end{bmatrix} + \begin{bmatrix} d_A & -d_A \\ -d_A & d_A \end{bmatrix} \begin{bmatrix} \ddot{z}_A \\ \ddot{z}_R \end{bmatrix} + \begin{bmatrix} c_A & -c_A \\ -c_A & c_A + c_R \end{bmatrix} \begin{bmatrix} \dot{z}_A \\ \dot{z}_R \end{bmatrix} = \begin{bmatrix} 0 \\ c_R \end{bmatrix} \dot{\zeta}(t). \tag{3}$$

Then, the equations of motion (3) are rewritten in state-space form where the state vector $\dot{\boldsymbol{x}}(t)$, the white noise excitation vector $\boldsymbol{w}(t)$, the system matrix $\boldsymbol{A}$ and the input matrix $\boldsymbol{B}$ appear,

$$\ddot{\boldsymbol{x}} = \boldsymbol{A}\dot{\boldsymbol{x}} + \boldsymbol{B}\boldsymbol{w}(t). \tag{4}$$

4. Assessment criteria and covariance analysis

The main tasks of a vehicle suspension are threefold:

- Carry the vehicle body on constant height with small vertical vibrations providing high driving comfort. Human perception of vibration depends on acceleration.
- Let the vehicle wheels follow the uneven road with small load variations resulting in high driving safety. Propulsion and guidance forces depend on wheel load.

- Restrict the suspension travel between wheel and vehicle body by rubber stops operating only on bumpy roads.

Based on these tasks the two essential criteria are described mathematically as follows:

- Criterion 1: the vertical body acceleration characterizing the driving comfort $a_A = \ddot{z}_A$ has to be minimal.
- Criterion 2: the dynamical wheel load affecting the driving safety $f = c_R(\zeta - z_R) = m_A\ddot{z}_A + m_R\ddot{z}_R$ has to be minimal.

This means that a multicriteria optimization problem is given. Due to the random excitation of the vehicle guideway system, the criteria are also random variables characterized by their standard deviations or variances, respectively.

The response of the state equation of the stochastic vehicle guideway system (4) is characterized by the covariance matrix $\boldsymbol{P}$ of the standard deviations of its state variables, following from the Lyapunov matrix equation as

$$\boldsymbol{AP} + \boldsymbol{PA}^{\mathrm{T}} + \boldsymbol{BqB}^{\mathrm{T}} = \boldsymbol{0}. \tag{5}$$

An explicit solution is available with the matrix polynomial

$$\boldsymbol{P} = \frac{1}{2\det\boldsymbol{H}}\sum_{k=0}^{3} H_{k+1,1}\sum_{m=0}^{2k}(-1)^m\boldsymbol{A}_m\boldsymbol{Q}\boldsymbol{A}_{2k-m}^{\mathrm{T}}, \qquad \boldsymbol{Q} = q\boldsymbol{BB}^{\mathrm{T}}. \tag{6}$$

Therefore, the standard deviations of both assessment criteria can be evaluated explicitly as

$$\begin{aligned}\sigma_a^2 &= \frac{q}{2}\left[\frac{c_R d_A}{m_A{}^2} + \frac{c_A{}^2(m_A + m_R)}{d_A m_A{}^2}\right],\\ \sigma_f^2 &= \frac{q}{2}\left[\left(1 + \frac{m_R}{m_A}\right)^3\frac{c_A{}^2 m_A}{d_A} + \left(1 + \frac{m_R}{m_A}\right)^2 c_R d_A - 2\left(1 + \frac{m_R}{m_A}\right)\frac{c_A c_R m_R}{d_A} + \frac{c_R{}^2 m_R}{d_A}\right].\end{aligned} \tag{7}$$

It turns out that the two criteria are linearly depending on the unevenness of the road but strongly nonlinear on the vehicle parameters. In the next section, this issue will be investigated in more detail using simulations.

5. Multicriteria optimization and uncertainty

For design optimization of road vehicle suspensions based on a quarter-car model five design variables are available. It is known from vehicle dynamics, see Mitschke and Wallentowitz[6], that suspension problems result in a conflict due to contradictions between assessment criteria. Therefore, by means of an uncertainty analysis additional recommendations for robust design may be found.

Due to the long history of automotive engineering, it turns out that the nominal parameters are close to the optimum. The shock absorber is a nearly unrestricted design variable but uncertain due to wear and tear. First of all, the criteria space is shown only for the uncertain damping, Fig. 3. The nominal damping parameter d_A = 4800 Ns/m looks fine, but low and high damping both comfort and safety are downgraded where low damping is comparatively more dangerous than high damping. The pareto-optimal damping parameters are presented in Fig. 4 and they read as d_A = 1890 Ns/m for the best comfort and d_A = 4580 Ns/m for the highest safety. It is pointed out that wear and tear reduce regularly the damping parameter, therefore drivers feel more comfortable in a used car, forgetting about safety. On the other hand, new cars are delivered with a nominal damping parameter which is even higher than the pareto-optimal one.

Fig. 5 shows a fuzzy conflict diagram including the uncertain tire stiffness with the boundary parameter given in Chapter 2. For selected damper values, a softer tire stiffness c_R improves both comfort and safety, but its durability will be strongly reduced for low tire air pressure. Fig. 6 indicates that the vehicle payload m_A affects mainly the drive comfort by changing the eigenfrequency of the vehicle body and its related acceleration.

In the case of three uncertain parameters, fuzzy arithmetic techniques are required. Based on the book by Hanss[7], the software package FAMOUS[8] (Fuzzy Arithmetic Modeling Of Uncertain Systems) was developed. This software

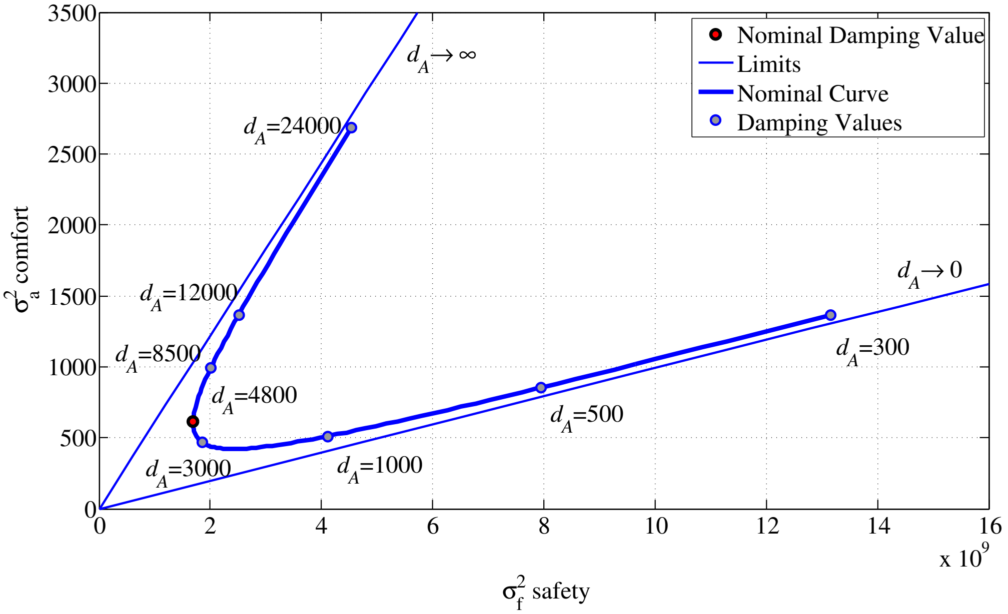


Fig. 3: Conflict diagram of vehicle suspensions for variable damping d_A.

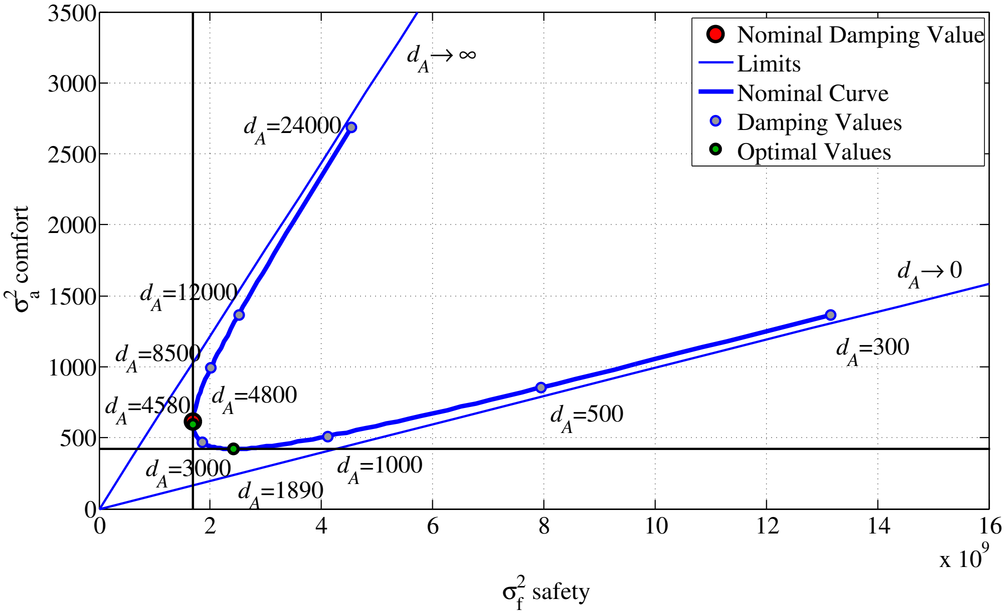


Fig. 4: Pareto-optimal points for damping d_A.

is applied to the road vehicle suspensions using the three uncertain parameters damping d_A, tire stiffness c_R and body mass m_A. The result is depicted in Fig. 7 and shows that the uncertainty area related to $d_A = 2500$ Ns/m is found between the pareto-optimal damping parameters. Considering the damping values $d_A = 1250$ Ns/m and

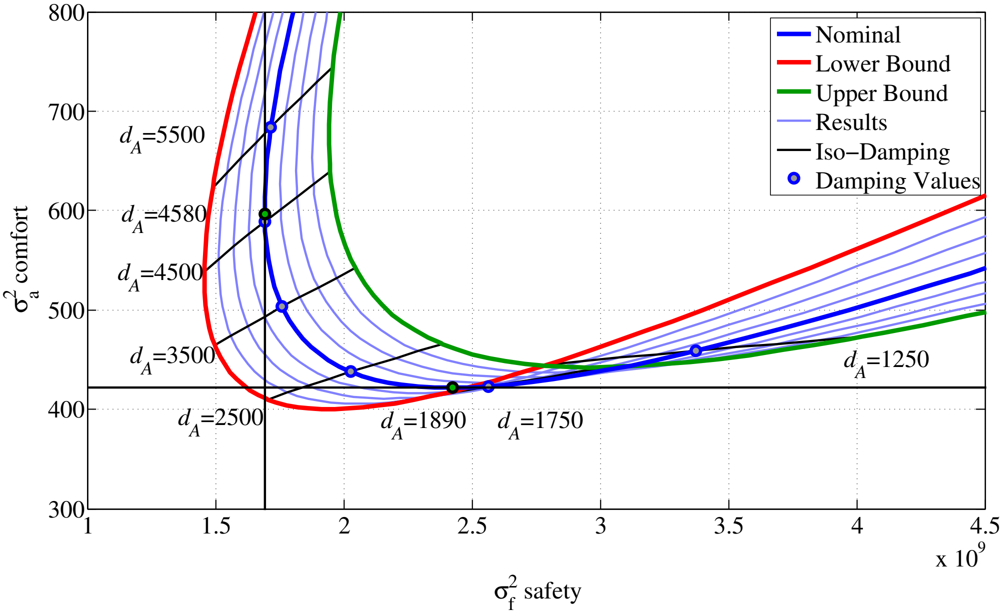


Fig. 5: Fuzzy conflict diagram for additional variable tire stiffness c_R.

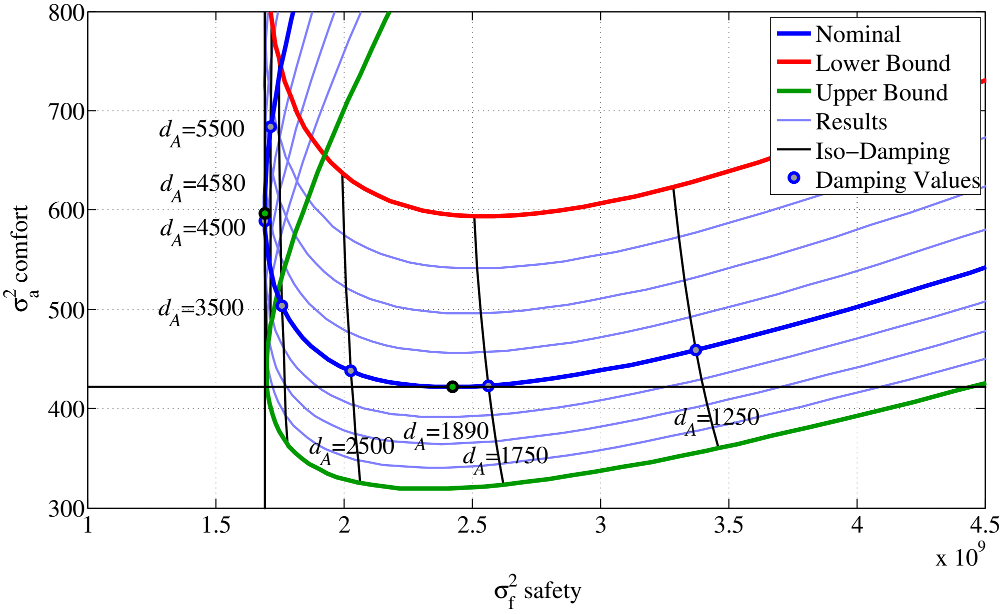


Fig. 6: Fuzzy conflict diagram for additional variable body mass m_A.

d_A = 6000 Ns/m, once again, the finding that low damping is more dangerous and high damping less comfortable is confirmed.

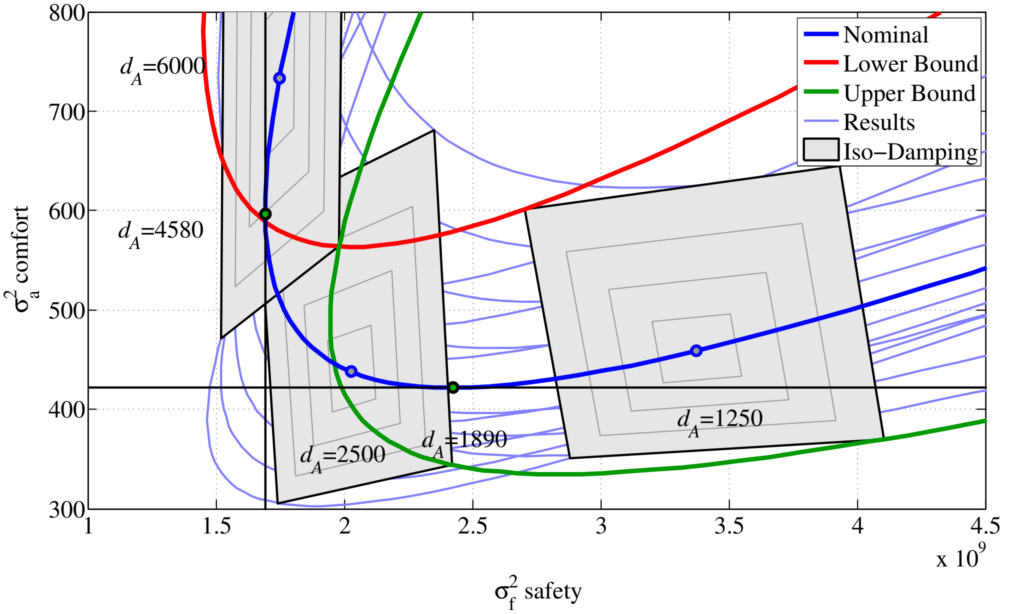


Fig. 7: Fuzzy conflict diagram for variable uncertain parameters d_A, c_R and m_A.

6. Controlling uncertain suspension parameters

In new standard passenger cars, suspension parameters are chosen once according to the nominal values discussed in Chapter 2. They are codified in the vehicle documents for maintenance and replacement at least during the warranty period. However, drivers may have different expectations on their cars being more comfortable or more sportive. Therefore, car manufacturers offer vehicle with controllable adaptive shock absorbers allowing the driver to change the characteristics of the car during driving. Electronically adjustable dampers are available, e.g., from ThyssenKrupp Bilstein and applied by BMW as electronic damper control (EDC) or Chrysler as outstanding ride, handling and capability, respectively. Mercedes-Benz uses controlled suspensions with its Active Body Control (ABC). Their statements read as follows.

ThyssenKrupp Bilstein[9]: The design and fine-tuning of suspension and damping systems always represent a compromise between ride comfort on the one hand and driving safety and agility on the other. The experts at ThyssenKrupp Bilstein purposefully work at developing semi-active damping systems designed to resolve this conflict further.

BMW[10]: Electronic Damper Control (EDC) automatically or manually adjusts each damper to suit the driving conditions meaning you enjoy outstanding comfort along with the best in BMW on-road safety. EDC reduces variations in wheel load, ensures tires having excellent traction and counteracts bodyshell movement regardless of the weight your automobile may be carrying or the state of the road's surface. The Driving Experience Control switch with ECO PRO let the driver choose between various programs (like COMFORT, NORMAL, SPORT or SPORT+) and adjust the suspension to suit his individual needs.

Chrysler[11]: The Grand Cherokee SRT rides on a short- and long-arm (SLA) independent front suspension with coil springs, Bilstein adaptive damping suspension (ADS), upper- and lower-control arms ("A" arms), and a stabilizer bar. The rear suspension is a multi-link design with coil spring, Bilstein ADS, aluminum lower control arm, independent upper links (tension and camber), plus a separate toe link, and a stabilizer bar. Software improvements to the retuned Selec-Track system further distinguish the five dynamic modes: Auto, Sport, Tow, Track, and Snow, enabling drivers to choose a vehicle setting that more ideally meets their requirements and ambient conditions.

Mercedes-Benz[12]: Active Body Control (ABC) with crosswind stabilization allows superior driving dynamics without compromising on comfort through a combination of active suspension and passive damping. ABC features all-round self-leveling with the same functions as the air suspension AIRMATIC. Four computer-controlled spring struts with plunger cylinders virtually eliminate pitching, rolling and vertical movements. The engine's pressure supply system provides the servohydraulic valves with hydraulic pressure of up to 200 bar. The constantly available system pressure and the employed accumulators enable Active Body Control (ABC) to respond to initial signs of body movement in a fraction of a second.

With electronically controllable dampers, the driver has to make his personal choice weighing comfort against safety. However, the problem of uncertainty remains due to the individual choice of a set of parameters by the driver. Even if a smartphone app for adaptive dampers is introduced as suggested by O'Brien[13], the conflict and uncertainty problem is not solvable.

Another uncertain parameter is the air pressure in the tire. Although tire stiffness cannot be controlled during traveling on the road, it can be at least monitored. Continental is offering a tire pressure monitoring system (TPMS) with the following features.

Continental[14]: Our comfortable system TMPS for tire pressure monitoring detects even small pressure fluctuations, locates the affected tires and informs the driver with warnings of varying urgency. Function: A co-rotating wheel module with an integrated valve measures tire pressure and temperature and transmits these data as an HF radio signal. Future generations of systems networked with TPMS, DDS and ESP will make important contributions to active accident avoidance, such as ESP control dependent of tire pressure and load-dependent tire pressure recommendation. A sensor-transponder integrated in the tire without a battery will supply pressure and temperature data as well as information about the tire itself.

As a matter of fact, monitoring a parameter does not change its uncertainty as long as any kind of feedback control is missing, as it is for an inflated tire rolling on the road.

7. Conclusions

Road vehicle suspensions are generating vertical forces in the contact patches of the tires on the road providing horizontal contact forces for vehicle propulsion and guidance. The static contact forces are complemented by the dynamic loads due to suspension vibrations excited by the guideway unevenness, which is represented by aleatoric uncertainties. In addition, epistemic uncertain design parameters are identified for road vehicle suspensions. These many uncertainties contribute more and more to road fatalities since the accidents due to driver misconduct are decreasing thanks to the development of more reliable occupant protection systems networking, too, with driver assistant systems.

Vehicle systems and their design parameters are discussed for models of different complexity. The most simple quarter car model describes already all required basic dynamic phenomena. This model features five design parameters with three of them epistemic uncertain to be represented by fuzzy sets. The stochastic process of the aleatoric guideway unevenness is approximated by white velocity noise and added to the vehicle's equations of motion, resulting in the state equations of the vehicle guideway system.

From the main tasks of road vehicle suspension systems, the assessment criteria driving comfort and driving safety are derived and mathematically defined by state variables. Due to the random excitation of the vehicle, both criteria are also random variables and characterized by their standard deviation or variance, respectively. Covariance analysis is introduced and explicit algebraical equations for the criteria are found depending nonlinearly on the suspension parameters.

The assessment criteria comfort and safety are with respect to the shock absorber in conflict with each other requiring a multicriteria optimization resulting only in pareto-optimal parameters. Simulation results are presented considering the shock absorber, the tire and the payload of the vehicle. While the uncertainty of one or two parameters are visualized by 2D graphs, for three uncertain parameters the software package FAMOUS has to be used. Finally, recently developed devices for controlling the uncertain parameter of shock absorbers are presented and their industrial applications are reviewed. In principle, the engineering uncertainties are replaced by the drivers preferences which are uncertain, again. Thus, the inclusion of uncertainties even in advanced systems is a great challenge for multibody and vehicle dynamics.

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