TORSIONAL BEHAVIOR OF CRACKED REINFORCED CONCRETE PILES IN SAND

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ABSTRACT

A finite element numerical procedure for the analysis of torsional behavior of cracked reinforced concrete pile is proposed. A trilinear torque-twist model is used to represent the torsional response of the reinforced concrete pile element with maximum torsional stresses. The remaining part of the pile is modeled as a linear elastic torsional behavior. The soil torsional resistance along pile shaft is represented by the nonlinear springs, modeled as the hyperbolic function, acting at the midpoint of each pile element. The proposed hyperbolic function is compared against available experimental results and showed to be very satisfactorily. Both linear and nonlinear analyses were performed and showed the importance of taking into account the changing of pile stiffness once concrete has cracked.

I. INTRODUCTION

Structures may be subjected to significant loads due to wind, wave or earthquake actions. These loads may induce torsional forces on piles due to eccentricity of lateral loadings. Therefore, increasing attention is being focused on the torsional behavior of piles during the past few decades [3-6, 8-12]. Most of the available researches on pile torsional behavior are emphasizing on the effect of linear or nonlinear soil behavior on pile structure performance. In addition, most proposed analytical methods of these researches considered pile structure as linear behavior. However, the possibility of concrete cracking on reinforced concrete pile may significantly affect the structural performance under torque. An ongoing research is conducted at the University of Florida, USA, sponsored by the US Transportation Research Board, to study the possible concrete cracking on torsional behavior of reinforced concrete piles [1]. Therefore, a study, considering nonlinearity properties on both soil and pile, is another formidable problem in the analysis of pile foundations.

In this paper, a numerical procedure for the analysis of reinforced concrete piles under torque is proposed. A trilinear torque-twist model is used to represent the torsional response of the reinforced concrete pile element with maximum torsional stresses. This model considers variation of the pile torsional stiffness due to concrete cracking or steel

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yielding. When an embedded pile is subjected to a torsional load, failure will occur at a point, where the torsional stresses will be a maximum. Once failure occurs at that level, there will be no mechanism to induce further failure at a location of the pile which is situated within the soil medium, where a linear elastic torsional behavior is assumed. Hence, the trilinear torque-twist model is usually applied on the element including pile head only, where the torque is applied on it. The soil torsional resistance along the pile shaft is represented by the nonlinear springs, modeled as the hyperbolic function, acting at the midpoint of each pile element. The proposed hyperbolic function is compared against existing experimental data. The comparison between predicted and experimental results appear to be very satisfactorily. A linear analysis result of the proposed approach is also verified by comparison with the available solutions by Chow [3]. Very good agreement is observed. A study is further extended to predict the torsional behavior of cracked reinforced concrete piles. The study showed the importance of taking into account the changing of pile stiffness once concrete has cracked in the analysis.

II. MATERIAL MODELING

Most of the available experimental results on reinforced concrete beams' torsional behavior are for rectangular sections. The test results showed the torque-twist relationship is nonlinear, which was also often simplified as a trilinear function, and has been used quite successfully [2, 7]. As explained by Hsu [7], a circular section reinforced concrete beam's torsional behavior is similar to that of a rectangular section's performance. Hence, in the following, a trilinear material property representing reinforced concrete pile's torsional behavior, proposed by Hsu [7], is introduced first. The elastic response, concrete cracking in torsion, post-cracking concrete stiffness, and yielding of reinforcements are included in the model.

Subsequently, the hyperbolic function, originally used by Dutt [4] only to evaluate the ultimate shear stress along pile-soil interface under torque, is extended by the authors to represent the nonlinear soil shear stress-strain relationship due to torque. The torque-twist relating to the hyperbolic function is also described in detail.

Finally, the pile base stiffness, considered as a rigid circular footing on the surface of a homogeneous elastic half-space, is introduced.

1. Material model for the pile element

A trilinear model is used to represent the torsional response of the reinforced concrete pile element with maximum stresses. Fig. 1 shows a typical torque-twist response for pure torsion of a circular reinforced concrete beam.

Two distinct points are required to define the shape of the curve. They are: 1) the torque at first cracking, T_{cr}, and the corresponding α_{cr}; 2) the torque at first full yielding of all the reinforcement, T_{yp}, and the corresponding α_{yp}.

(1) Elastic response and cracking in torsion

Excluding the contribution from the steel reinforcement, the torsional stiffness $(GJ)_0$ for an uncracked section can be represented by

$$(GJ)_0 = G \cdot \left(\frac{\pi r^4}{2}\right), \tag{1}$$

where G=shear modulus of the pile; J=polar moment of inertia of the pile; r=radius of the pile. The maximum shear stress is

$$\tau_{\max} = \frac{2T}{\pi r^3}. \tag{2}$$

For pure torsional moment on the beam, the maximum tensile stress is numerically equal to the maximum shear stress, $\tau_{\max}$. Consequently, the torsional moment existing in the concrete beam at first cracking is

$$\tau_{cr} = \frac{\pi r^3}{2} f_t', \tag{3}$$

where f_t' is the uniaxial tensile strength of the concrete. The corresponding twist at the cracking torque is

$$\alpha_{cr} = \frac{T_{cr}}{(GJ)_0}. \tag{4}$$

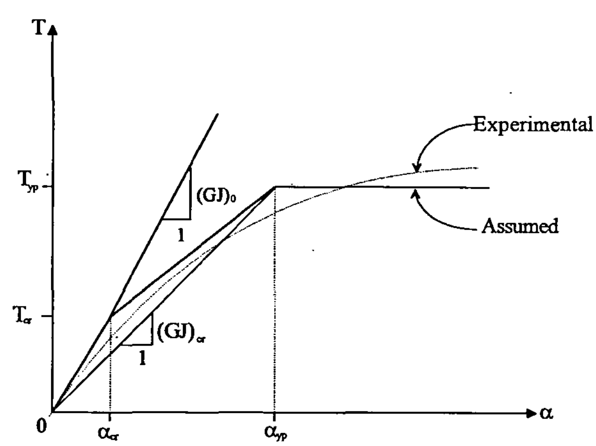

Fig. 1. Typical torque-twist relationship for a circular reinforced concrete beam under torsion.

(2) Post-cracking stiffness and yielding

Based on the study by Hsu [7], the post-cracking stiffness $(GJ)_{cr}$ can be expressed as

$$(GJ)_{cr} = \frac{E_s \pi d_1^2 d^2}{16\left(\frac{1}{\rho_l} + \frac{1}{\rho_h}\right)}, \tag{5}$$

where $\rho_l = \frac{A_l}{A_c}$; $\rho_h = \frac{A_h u}{A_c s}$; $A_c = \frac{\pi d^2}{4}$; $u = \pi d_1$, and d= diameter of the concrete cross section; d_1=diameter of the circle formed by the center line of a hoop bar; A_l=total cross-section area of longitudinal steel; A_h=cross-section area of one hoop; s=spacing of hoop reinforcement; E_s=Young's modulus of the steel; and E_c=Young's modulus of the concrete.

The torque required to cause full yielding of the longitudinal and the hoop steel reinforcement [7], assuming an elastic-plastic material for the reinforcement, is

$$T_{yp} = 2\pi r^2 \frac{A_h f_{hy}}{s \cdot \tan\psi}, \tag{6}$$

where ψ is the crack inclination and

$$\tan\psi = \sqrt{\frac{As}{2\pi r} \cdot \frac{s}{A_h} \cdot \frac{f_{sy}}{f_{hy}}}, \tag{7}$$

where A_s=total area of the longitudinal steel; f_{sy}= yield strength of the longitudinal steel reinforcement; f_{hy}=yield strength of the hoop reinforcement.

The deformation corresponding to the condition of first full yielding is

$$\alpha_{yp} = \frac{T_{yp}}{(GJ)_{cr}}. \tag{8}$$

2. Modulus of subgrade reaction of soil

(1) Pile shaft

Both Tucker [12] and Dutt [4] used power function to represent the soil stress-strain characteristics in torsional shear. However, it was also found that the power function can not appropriately describe the stress-strain behavior at higher torsional strain. The hyperbolic load transfer function, which was used by Dutt [4] only to evaluate the ultimate shear stress, is extended in the study to represent soil shear stress-strain relationship under torque. The hyperbolic function (see Fig. 2) can be expressed as:

$$\frac{\gamma_s}{\tau_s} = \alpha + b\gamma_s, \tag{9}$$

where γ_s is the soil shear stain; τ_s is the soil shear

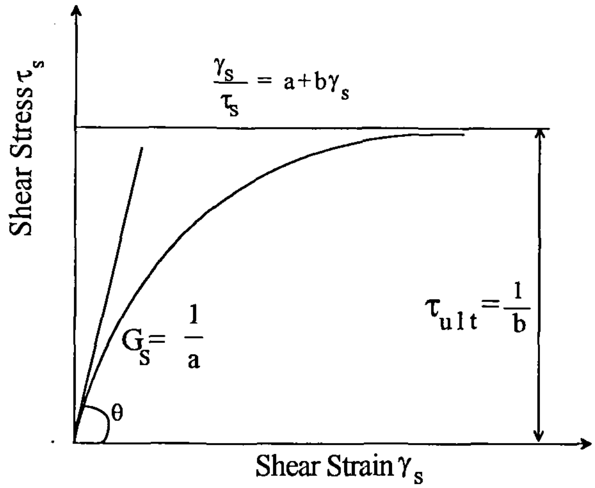

Fig. 2. Hyperbolic function for soil torsional stress-strain relationship.

stress while a and b are constants. Very little information is presently available on criteria describing torsional load transfer from the pile to the soil. The solution of the general model for the soil-pile interaction due to torsional loading, based on the premise by Ha [5], that the individual torque-twist properties of the soil and pile are known. Hence, following the procedures described by Ha [5], the torsional resistance of the soil, T, over a length H of a rigid cylinder in terms of soil shear stress-strain properties can be expressed as

$$T = \frac{\pi d^2 H \tau_s}{2} \quad \text{and} \tag{10}$$

$$\alpha = \frac{\gamma_s}{2},$$

where d is the pile diameter; α is the angle of rotation at the surface of the rotating cylinder.

Based on Eqs. (9) and (10), the soil response to torsional loading is expressed as:

$$T = \pi d^2 H \left(\frac{a}{a + 2b\alpha}\right). \tag{11}$$

The subgrade reaction modulus of the soil is obtained from Eq. (11) as

$$k_{s(shaft)} = \frac{dT}{d\alpha} = \frac{\pi a H d^2}{(a + 2b\alpha)^2}. \tag{12}$$

(2) Pile base

Based on Chow [3], a rigid circular footing on the surface of a homogeneous elastic half-space is assumed for the stiffness of the soil at the pile base. The stiffness can be expressed as

$$k_{s(base)} = \frac{16}{3} G_s r^3, \tag{13}$$

where G_s is the shear modulus of soil.

III. FINITE ELEMENT FORMULATION

Following the procedure proposed by Chan [2], the element stiffness matrix for a pile with uniform section and material properties can be expressed as

$$k_p = \frac{GJ}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \qquad (14)$$

where L is length of the considered pile element.

The soil torsional resistance along the pile shaft is represented by the nonlinear springs acting at the midpoint of each pile element. Following the procedure of Chow [3], the soil element stiffness matrix is (see Fig. 3)

$$k_s = \int_0^L k_{s(shaft)} \begin{Bmatrix} 1-\frac{z}{L} \\ \frac{z}{L} \end{Bmatrix} \left\{1-\frac{z}{L}\ \frac{z}{L}\right\} dz. \qquad (15)$$

Since sandy soil is often with stiffness increasing linearly with depth, the modulus of subgrade reaction along the element (Fig. 3) is given by

$$k_{s(shaft)} = k^0_{s(shaft)} + k^*_{s(shaft)}\, z, \qquad (16)$$

where $k^0_{s(shaft)}$=subgrade reaction modulus at node 1 and $k^*_{s(shaft)}$=rate of increase subgrade reaction modulus with depth, z. Hence,

$$k_s = k^0_{s(shaft)} L \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix} + k^*_{s(shaft)} L^2 \begin{bmatrix} 1/4 & 1/12 \\ 1/12 & 1/4 \end{bmatrix}. \qquad (17)$$

The stiffness at pile base, $k_{s(base)}$, is only considered at the pile base element.

The global tangential stiffness matrix K is formulated from Eqs. (14) and (17) as

$$K=k_p+k_s \qquad (18)$$

and the following incremental form of nonlinear algebraic equations is obtained

$$K\Delta\alpha=\Delta T, \qquad (19)$$

where ΔT is the incremental applied torque. The Newton-Raphson iteration method is used for this nonlinear analysis.

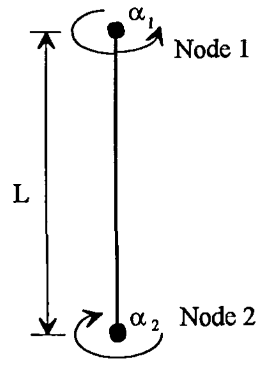

Fig. 3. Typical torsional pile element.

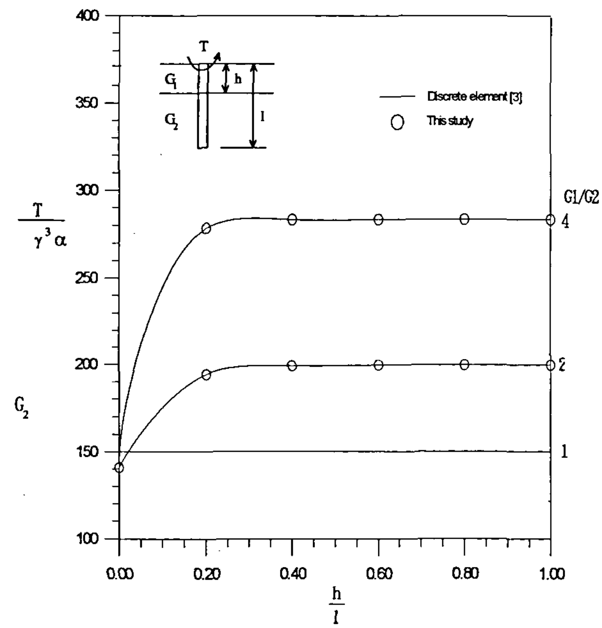

Fig. 4. Pile twist in two-layer soil. (l/r=50, G_{pile}/G_2=1000)

IV. NUMERICAL STUDIES

1. Linear analysis examples

The computer code has been verified by comparison with solutions by Chow [3], for piles embedded in homogeneous and nonhomogeneous soils, as shown in Fig. 4, which shows the torsional response of a pile embedded in a two-layer soil system. In the figure, G_1 and G_2 represent the soil shear modulus at soil layer h and soil layer l, respectively. When h is equal to zero, the soil becomes homogeneous. G_{pile} and r represent the shear modulus and the radius of the pile, respectively. Very good agreement between the solutions is observed. The results show the ratio of $T/(G_2r^3\alpha)$ approaches to a constant value when h/l is larger than 0.2, regarding a constant G_1/G_2 soil shear modulus ratio of 4 or 2.

2. Nonlinear analysis example

A series of pile torsional tests were conducted by Dutt [4] for a 1.9 inch (4.826 cm) diameter circular pile and a 2 inch (5.08 cm) outside dimension

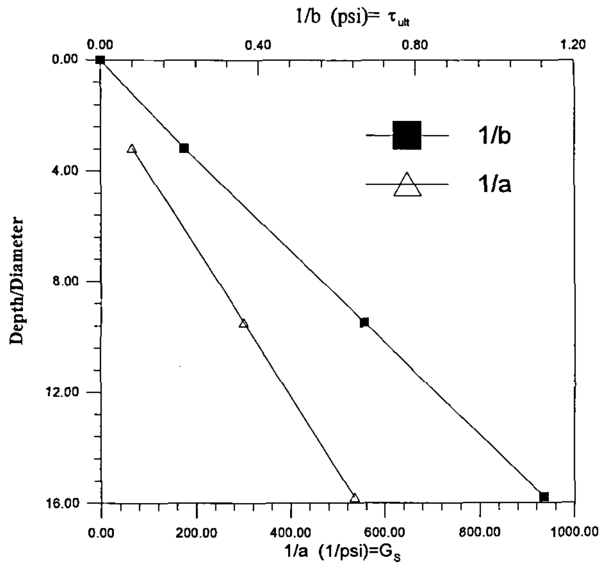

Fig. 5. Variation of a and b two constants with depth (1psi= 6.89Kpa).

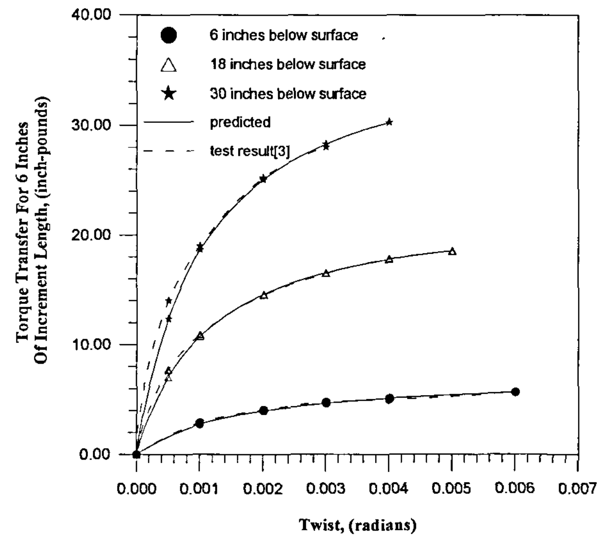

Fig. 7. Comparison of predicted and experimental torque transter curves for circular pile embedded in dense sand (1in= 2.54cm; 1 inch-pound=11.303 N-cm).

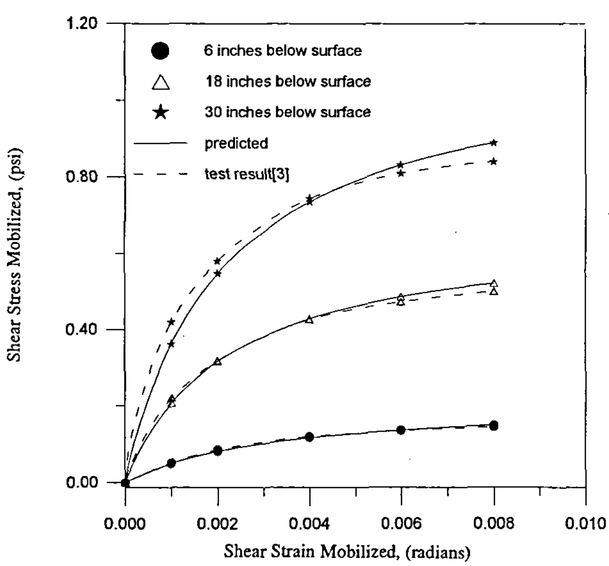

Fig. 6. Shear-strain mobilized in soil for circular pile embedded in dense sand (1 inch=2.54cm; 1 psi=6.89 Kpa).

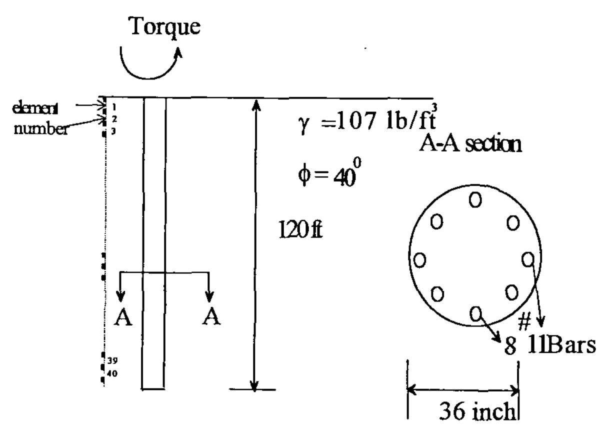

Fig. 8. Properties of the analytical pile and the surrounding soil (1 inch=2.54 cm; 1 lb=0.454 kg).

square pile embedded in loose or dense sand. For the circular pile embedded in dense sand, the experimental results showed the ultimate soil shear stress, τ_{ult}, and the soil shear modulus, G_s, are increasing linearly along the embedded depth of the pile as shown in Fig. 5. Utilizing the results of Fig. 5, the values of two constants, a and b, can be obtained and substituted into Eq. (9) to compute shear stress-strain curves at various depths below the surface of the sand as shown in Fig. 6. Similarly, using Eq. (11), the torque transfer versus twist curves, which are expressed as shear stress and strain mobilized at the soil-pile interface, can also be generated as shown in Fig. 7. Comparison of the measured and predicted results of Figs. 6 and 7 for sand surrounding torsionally loaded piles indicates that the overall correlation between the two is good.

A study is conducted to predict the behavior of cracked reinforced concrete piles under torque. A reinforced concrete pile, 120 feet (36.576 m) long and 36 inches (91.44 cm) in diameter, as shown in Fig. 8, is considered for the analysis. This pile is designed to have 8 number 11 reinforcing bars and have 0.11 square inch (0.71 cm^2) for each hoop reinforcing bar spaced at every 6 inches (15.24 cm). In addition, the pile is assumed to be embedded in the same dense sand soil condition of the Dutt [4] test pile, as shown in Fig. 4. Since the torque is

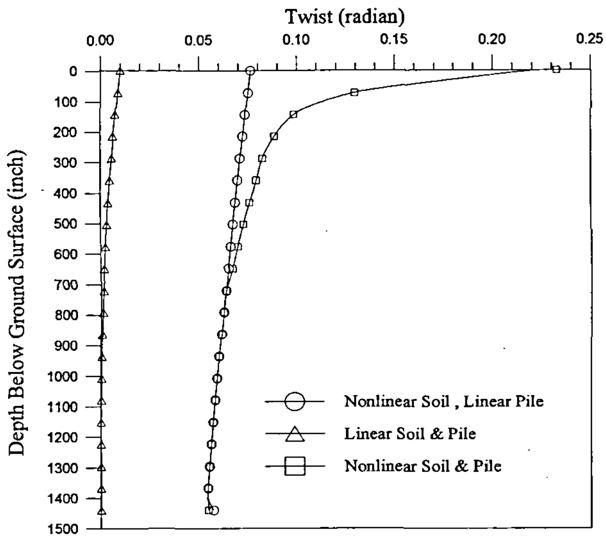

Fig. 9. Analytical results of twist with depth under torque=0.4×10^7 in-lbf (1 inch=2.54cm; 1in-lbf=11.303 N-cm).

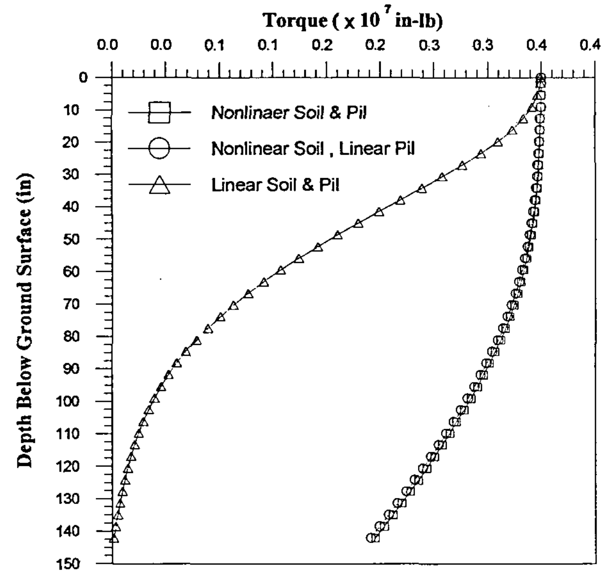

Fig. 10. Analytical results of twist transfer with depth under torque=0.4×10^7 in-lbf (1 inch=2.54cm; 1 in-lbf=11.303 N-cm).

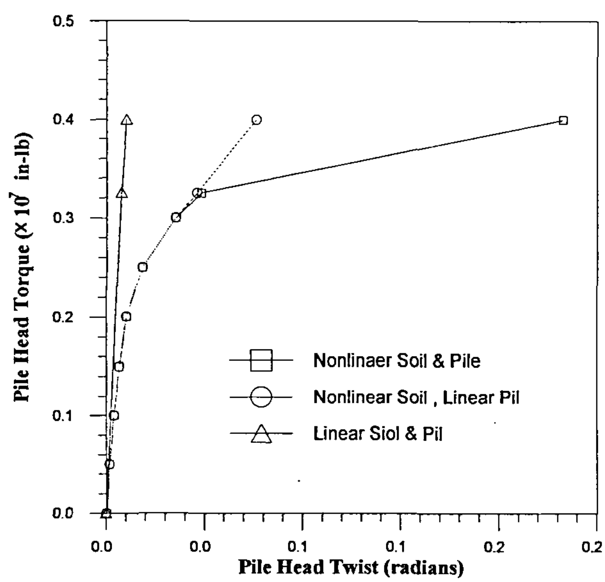

Fig. 11. Pile head torque-twist curves at various loading conditions (1 in-lbf=11.303 N-cm).

applied at the pile head, the maximum torsional stresses will occur at the element including the pile head. Hence, only the pile element number one considered as nonlinear behavior. As explained in the above, once failure occurs at the pile head level, there will be no mechanism to induce further failure at a location of the pile which is situated within the soil medium, the remaining pile elements. Element 2 to element 40 (see Fig. 8), are considered as linear behavior. Three different analyses were considered. The first case assumed both soil and pile with nonlinear material properties. The second analysis assumed nonlinear soil, but with linear pile material properties. Both soil and pile were considered to be linear properties for the third analysis.

The variation of twist with depth of the analytical pile under torque, 4×10^7 in-lbf (45.2 N-cm), is shown in Fig. 9. As shown in the figure, since concrete has cracked in the first analytical case, which has induced large rotation on the pile head element, the assumption of linear pile condition is shown to have underestimated the twist induced from torque at cracked pile sections. When both soil and pile are assumed to behave with linear characteristics, the least pile twist was predicted since the linear analysis can not reflect the variation of soil and pile stiffnesses during analysis. The effect of concrete cracking does not influence the torque distribution along the depth even if a linear pile behavior was assumed, which represents the distribution relating to soil failure only and has nothing to do with concrete cracking effect, as shown in Fig. 10. In addition, the third analysis showed a rapid decreasing rate of torque distribution towarding the pile base. The pile head torque-twist curves at various loading conditions are shown in Fig. 11. Similarly, the effect of concrete cracking significantly increases the pile head twist angles.

V. CONCLUSIONS

A finite element procedure is presented to analyze the performance of cracked reinforced concrete piles under torque. The main findings were the following:

(1) The assumption that the along pile shaft soil torsional stress-strain curve is of the form of the

hyperbolic function is valid.

(2) In the reinforced concrete pile torsional behavior study, the general linear analysis significantly underestimates the distribution of pile twist and torque with depth.

(3) If linear pile embedded in nonlinear soil is assumed for reinforced concrete pile torsional analysis, this will underestimate the pile twist behavior, but not torque distribution at cracked concrete pile sections.

(4) An analytical method which is able to reflect the variation of soil and pile stiffness during analysis is necessary for a torsional behavior study of reinforced concrete piles.

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NOMENCLATURE

A_h	cross-section area of one steel hoop
A_i	total cross-section area of longitudinal steel
A_s	total area of the longitudinal steel
d	diameter of the concrete pile cross section
d_1	diameter of the circle formed by the center line of a hoop bar
E_c	Young's modulus of the concrete
E_s	Young's modulus of the steel
f_{hy}	yield strength of the hoop steel
f_{sy}	yield strength of the longitudinal steel
f'_t	uniaxial tensile strength of the concrete
G	shear modulus of the pile
G_s	shear modulus of the soil
J	polar moment of inertia of the pile
$k_{s(base)}$	subgrade reaction modulus of soil at pile base
$k_{s(shaft)}$	subgrade reaction modulus of soil along pile shaft
r	radius of the pile
s	spacing of hoop steel
T	torsional resistance of the soil
T_{cr}	torque at first concrete cracking
T_{yp}	torque at first full yielding of all the reinforcements

Greek symbols

α	twist at the surface of the rotating pile shaft
α_{cr}	twist at first concrete cracking
α_{yp}	twist at first full yielding of all the steel
τ	concrete shear stress
τ_s	soil shear stress
γ_s	soil shear strain

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含裂縫鋼筋混凝土基樁於砂土中受扭力之行爲

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摘　要

本文提出一應用有限元素法分析鋼筋混凝土基樁受扭力至混凝土產生裂縫後之表現行表。採用三直線扭力—扭轉關係式表示鋼筋混凝土基樁於最大扭轉應力元素之扭轉行爲。樁身其餘部份則以線性彈性模式表示。沿樁身土壤抗扭轉阻力則以雙曲線模式之非線性彈簧模擬。土壤反力彈簧作用在基樁元素之中點。提出之雙曲線函數與實驗結果比較後顯出此函數之適用性。基樁之線性及非線性行爲均予分析，結果顯示混凝土裂開後，其對樁身勁度之影響必需於分析過程中加以考慮。

關鍵詞：鋼筋混凝土樁，扭轉行爲，雙曲線模式。