Environ Earth Sci (2013) 68:2359–2363
DOI 10.1007/s12665-012-1917-3

ORIGINAL ARTICLE

# Trend predictions in water resources using rescaled range (R/S) analysis

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Received: 19 May 2011 / Accepted: 16 August 2012 / Published online: 1 September 2012


**Abstract** Based on historical and observational data of wet-and-low water resource changes, this article used the rescaled range (R/S) analysis principle and method to calculate the $H$ index and establish the relation formula of $R(i)/S(i)$ and $i$. Based on $\{x_i\}$, and by using the least squares method, a new time series calculation method was proposed which endows the Brownian motion equation with forecasting abilities. This is a new attempt to forecast trend changes of water resources. Utilizing the time series data of water resources in Jinhua City, China, and the Brownian motion equation, aforecast was made of future trends in wet-and-low water resource changes. Satisfactory validation results were obtained, which indicate that this is an effective method for forecasting water resource changes.

**Keywords** Water resources · Future trend · $H$ index · R/S analysis

## Introduction

Jinhua City is located in the middle of Zhejiang Province, China. In recent years, with the aim of becoming the hub of mid-west Zhejiang, Jinhua has experienced significant economic growth. However, the spatial–temporal patterns of precipitation in Jinhua are not uniformly distributed. This is further exacerbated by the polluted state of the Jinhua River and the lack of water storage projects, making the impending 'water shortage' a major challenge (Shao et al. 2009). In fact, with the growing population and rapid economic development, man's most common problem today is insufficient water resources (Hajkowicz and Higgins 2008). Water resources are limited, and if people continue to overuse them, the ecosystem will deteriorate further (Cazurra 2008), bringing about grave consequences (Alsharif et al. 2008; Feng et al. 2011).

There are two types of statistical analyses used to forecast trend changes in water resources: multivariate and time series. The former makes forecasts based on statistical relations between water resources and associated factors, while the latter makes forecasts according to the self-evolvement rule of water resources. Discrimination analysis, cluster analysis and regression analysis are all forms of multivariate analyses, of which the regression analysis is the most widely used method. Periodicity analysis, autoregressive analysis, Markov analysis and rescaled range (R/S) analysis are all forms of time series analyses.

Periodicity refers to the recurring phenomenon of objects by a certain time interval. Periodicity analysis is the identification of recurrent patterns in a time series data set (Gan et al. 1991). Autoregressive analysis is about making linear models, under the assumption that the time series is stable, and, by means of extrapolation, predicting future trends (Weiss et al. 2012). Markov analysis is a theory studying the state of natural phenomena, as well as the law of the state transition, which determines the trend of natural phenomena, according to the transition probability and the status probability, for the purpose of making forecasts (Thyer and Kuczera 2003). These methods have their respective advantages, and can be used for dynamic simulation of time series.

R/S analysis was developed by Hurst when summing up the multi-year hydrological observation data of the Nile River (Banks et al. 2010), and was later reinforced and improved by Mandelbrot and Wallis making it a fractal

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theory for time series research (Liang and Gong 2009). The most important advantage of R/S analysis is that it makes no assumption of the distribution within the time series data set. Regardless of a normal or non-normal distribution, the consistency of the analyzed result will not be affected. The method can differentiate random and non-random time series from fractal time series, and identify a statistical structure from a random or confirmation structure—a fractal structure (Falconer 1990).

The R/S analysis method can obtain relatively ideal time series results, thus it has been successfully applied in many fields (Gou et al. 2009; Salomao et al. 2009). Miranda and Andrade (1999) applied an R/S analysis to a precipitation time series for twenty-one stations in northeast Brazil, and showed that drought occurrence does not alter the persistence of the rainfall distribution. To avoid the influence of the trend and the sinusoidal component on the result, Oliver and Ballester (1996) removed both of these features from the original time series and the value obtained for the Hurst exponent (0.717 ± 0.002) suggested that the non-periodic component was a correlated random process. Horvath and Schiller (2003) discussed the corrosion potential noise using an R/S analysis, and the differences between the R/S patterns of general and pitting corrosion were found to be particularly distinctive. Kikuchi and Tsutsumi (2001) found that radial distributions of the Hurst exponent and the Mann–Whitney statistic became flatter as circulating solid mass flux increased. This result indicated that the gas–solid flow structure became more uniform and deterministic with increasing circulating solid mass flux. Matos et al. (2004) applied R/S and DF analyses to backscattered ultrasonic signals obtained from three different cast iron samples in order to investigate the fractal nature of the microstructure of these materials. The results showed a scenario with two distinct regions, the calculated parameters of which could be used to estimate their fractal dimensions. However, at present, nobody has studied the prediction of future variation trends of water resources using R/S analysis.

As economies rapidly develop, many nations have faced shortages in water resources, especially those in areas prone to droughts and with medium to large metropolises (Kolomyts and Surova 2010). This has resulted in a significant problem in the coordination of economic development with the usage of water resources. As a result, the R/S analysis method and principles are used to discuss future wet-and-low water resource changes in Jinhua City based on historical and observational data (Huang and Li 1990; Mandelbrot 1968).

## Principles and methods of R/S analysis

French scientist Mandelbrot (1967) proposed a new theory, fractal geometry, which was based on the research of hierarchical structures of many complex phenomena in nature. In recent years, the fractal theory has been widely applied in fields such as geomorphologic features, seismic activities and cell reproduction (Hong and Hong 1988; Li and Wang 1993).

By measuring the lengths of coastlines in the United Kingdom (UK), Mandelbrot discovered something that few people would have been aware of in the past: the smaller the measurement scale, the longer the coastline. In other words, the length of a coastline changes with the measuring scale. As a result, Mandelbrot raised a simple but profound question: how long is the coastline of the UK? In fact, many phenomena exist in nature, such as coastlines, trees, blood vessels and lightning, whose integer structures do not change even when the geometry scale zooms in, or out, for they all have a self-similar structure. Based on this feature of self-similarity, Mandelbrot gave the name “fractal” to any natural phenomena whose components are similar, in certain ways, to the overall structure.

Fractal is described by fractal dimension. Suppose the sub-period of one research period is $r$ (scale), and the number of sub-periods occurring naturally in the research period is $N(r)$, if $r$ and $N(r)$ satisfy:

$$N(r) = Cr^{-D} \tag{1}$$

then this natural phenomenon has a time fractal structure, and $D$ is its time fractal dimension ($C$ is a constant).

The wet-and-low changes in water resources are expressed as discontinuous points on the time axis, which is an irregular Cantor set. The fractional Brownian motion model is used to predict the wet-and-low changes in water resources. Suppose there is a “granular” flowing randomly on the $x$ axis, after each interval $\tau$, it will move $x$ leftwards or rightwards, then the distribution density function of $x$ is:

$$f(x,\tau) = \frac{1}{\sqrt{4\pi G\tau}} \exp\left(-\frac{x^2}{4G\tau}\right) \tag{2}$$

where $G$ is the diffusion coefficient. Suppose $\{x_1, x_2, \ldots x_n\}$ is a step series and an independently distributed time series. After $n$ steps, the position of “granular” on the $x$ axis is:

$$B(t = n\tau) = \sum_{i=1}^{n} x_i. \tag{3}$$

Equation (3) is the Brownian function. $\gamma(t)$ is a related function as follows:

$$\gamma(t) = \frac{E\{[B_H(0) - B_H(-t)]\,[B_H(t) - B_H(0)]\}}{\sqrt{E[B_H(0) - B_H(-t)]^2}\sqrt{E[B_H(t) - B_H(0)]^2}} \tag{4}$$

where $E$ is the mathematical expectation of a random variable. After a simplified calculation:

$$\gamma(t) = 2^{2H-1} - 1 \quad (5)$$

in which $H$ is the Hurst index.

From (5) it is known that when $H = 1/2$, $\gamma(t) = 0$, this is an ordinary Brownian motion; when $H \neq 1/2$, $\gamma(t) \neq 0$, this is a fractional Brownian motion. Mandelbrot extended the $H$ index as $0 < H < 1$, and obtained:

$$R(i)/S(i) = (ai)^H \quad (6)$$

where $a$ is a constant, $R(i)$ is range, and $S(i)$ is standard deviation:

$$R(i) = \max_{1 \le i \le n} x_i - \min_{1 \le i \le n} x_i \quad (7)$$

$$S(i) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2} \quad (8)$$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad (9)$$

From Eq. (6) authors can obtain:

$$\ln[R(i)/S(i)] = H \ln i + H \ln a \quad (10)$$

Based on time series $\{x_i\}$ and using the least squares method, a linear regression equation of Eq. (10) can be obtained. Therefore know that the linear slope is $H$ index.

In order to forecast future trends in wet-and-low changes $x_{n+1}$, from Eq. (9) authors can obtain:

$$x_{n+1} = (n+1)\bar{x} - (x_1 + x_2 + \cdots + x_n) \quad (11)$$

Let $R(n+1)/S(n+1) = [a(n+1)]^H = K$, and substitute Eqs. (7)–(8) into it, then:

$$\frac{(x_{n+1} - x_1)}{\sqrt{\frac{1}{n+1}\left[(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2 + (x_{n+1} - \bar{x})^2\right]}} = K$$
$$(x_1 < x_2 < \cdots < x_n < x_{n+1})$$

Substituting Eq. (11) into the above obtains:

$$\bar{x} = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \quad (12)$$

in which:

$$\begin{aligned} A &= (n+1)\left[(n+1)^2 - nK^2\right] \\ B &= -2(n+1)\left[(n+1)(x_1 + u) - K^2 u\right] \\ C &= (n+1)(x_1 + u)^2 - K^2(u^2 + v) \\ u &= x_1 + x_2 + \cdots + x_n \\ v &= x_1^2 + x_2^2 + \cdots + x_n^2 \end{aligned}$$

Finally authors substitute $\bar{x}$ into Eq. (11), and obtain the future trend $x_{n+1}$ in wet-and-low water resource changes.

## Application example

Jinhua is located in a humid region of China. Since the 1990s, Jinhua has been experiencing rapid economic development. In 2011, Jinhua's GDP reached USD 8,100. However, Jinhua has suffered from significant water insufficiency problems in recent years (Feng et al. 2005). The average volume of water resources in Jinhua has been 9.2 billion m$^3$ for many years, but the per capita water resource is only 1,660 m$^3$, which is far lower than the international warning standard of 2,000 m$^3$. The great drought in 1996 made the human and livestock populations (160,000 and 80,000, respectively) suffer from water insufficiency, resulting in a direct economic loss of USD 25 million. The summer of 2003 was unusually hot. Such high temperatures have occurred only once in the last 50 years. The summer drought of 2003 was followed by an autumn drought, with rainfall from July to October amounting to only 124 mm, about 30 % of that of a normal year. The drought continued in 2004 (Feng and Luo 2011). Water shortage has become the key limiting factor for Jinhua to develop into a wealthy society by 2020. To ensure the healthy and stable development of the Jinhua economy, it will be necessary to analyze the future trend changes of the water resources in this area.

Based on the rainfall data from May to September collected from different regions, the "Atlas of the Drought/Flood Category for the Last 500 Years in China" (1470–1979) was used to classify drought/flood into five categories (Academy of Meteorological Sciences, China Meteorological Administration 1981). From the view point of water resources, the five categories include: Level 1, an especially abundant year; Level 2, a relatively abundant year; Level 3, an average year; Level 4, a relatively low year; and Level 5, an especially low year. In order to obtain a reliable and complete time series of water resources, first details were verified from local chronicles, including "Jinhua County Chronicle" and "Quzhou County Chronicle". The period of change in Jinhua water resources was extended to 2011, in accordance with the rainfall data from May to September. In this article, authors grouped especially abundant years and relatively abundant years as 'abundant years', and classified the ten-year periods which contained more than three abundant years as 'abundant periods'. Since 1600, Jinhua has experienced 15 abundant periods (Table 1). If the year 1600 is set as the zero point, and 1621–1630 as 2 (other years follow in the same sequence), then obtain the time series (1991–2000 is for validation only):

$\{x_i\} = \{2, 3, 4, 5, 8, 13, 16, 18, 19, 20, 26, 28, 32, 34\}$.

By calculation, authors obtain Hurst index $H = 0.1623$, constant $a = 75.0193$, relative function $\gamma(15) = -0.3739 \neq 0$ (belongs to fractional Brownian motion), then:

$$R(i)\,/\,S(i) = (75.0193i)^{0.1623}. \tag{13}$$

Using Eqs. (11)–(12), obtain the next abundant period in Jinhua: $x_{15} = 38.10 \approx 39$, after resetting the zero point, it should be 1991–2000, which was correctly predicted.

Next, substitute 1991–2000 as new information into $\{x_\mathrm{i}\}$, and obtain a new time series:

$\{x_\mathrm{i}\} = \{2, 3, 4, 5, 8, 13, 16, 18, 19, 20, 26, 28, 32, 34, 39\}$.

After calculation, obtain $H = 0.1645$, $a = 69.2391$, $\gamma(16) = -0.3719 \neq 0$ (still belongs to fractional Brownian motion), then the future abundant period of Jinhua is $x_{16} = 42.44 \approx 43$; after resetting the zero period, it should be 2031–2040. From this know that in the next 20 years or more, Jinhua will be in a period consisting mainly of 'low' years. The outlook of water resources in Jinhua is rather gloomy, and preparations for long-term water-saving are required.

Future changes of wet-and-low water resources have always been the focus of various studies. In the above derivation, which obtained satisfactory validation results, the authors propose a new time series calculation method which endows the Brownian motion equation with a predictive function. This method could be used as a tool to anticipate future changes in wet-and-low water resources.

## Conclusion

As an absolutely necessary requirement for living and production, water resources are the key determinative factors for sustainable social and economic development. It is extremely important for us to predict future trends in wet-and-low water resource changes and implement corresponding programmes of action. In this article, the R/S analysis principle and method was used to calculate the $H$ index and establish the relation formula of $R(i)/S(i)$ and $i$. Based on $\{x_i\}$ and using the least squares method, the authors proposed a new time series calculation method which endowed the Brownian motion equation with forecasting abilities. It is a new attempt to establish forecast patterns of the changing trends in water resources. Using time series data of the water resources in Jinhua City, China, and the Brownian motion equation, a forecast on the future trends in wet-and-low water resource changes was made and satisfactory validation results were obtained. The example calculated by authors proved that the use of Eq. (11) to forecast future trends in wet-and-low water

**Table 1** Abundant periods and their $R(i)/S(i)$ since 1600

| $i$ | Years | Abundant years/10a | Time series | $R(i)$ | $S(i)$ | $R(i)/S(i)$ |
|---|---|---|---|---|---|---|
| 1 | 1621–1630 | 5 | 2 | | | |
| 2 | 1631–1640 | 4 | 3 | 1 | 0.5000 | 2.0000 |
| 3 | 1641–1650 | 5 | 4 | 2 | 0.8165 | 2.4495 |
| 4 | 1651–1660 | 4 | 5 | 3 | 1.1180 | 2.6833 |
| 5 | 1681–1690 | 5 | 8 | 6 | 2.0591 | 2.9139 |
| 6 | 1731–1740 | 5 | 13 | 11 | 3.7156 | 2.9605 |
| 7 | 1761–1770 | 4 | 16 | 14 | 4.9487 | 2.8290 |
| 8 | 1781–1790 | 4 | 18 | 16 | 5.8296 | 2.7446 |
| 9 | 1791–1800 | 5 | 19 | 17 | 6.3906 | 2.6602 |
| 10 | 1801–1810 | 4 | 20 | 18 | 6.7941 | 2.6494 |
| 11 | 1861–1870 | 6 | 26 | 24 | 7.8140 | 3.0714 |
| 12 | 1881–1890 | 4 | 28 | 26 | 8.6651 | 3.0006 |
| 13 | 1921–1930 | 4 | 32 | 30 | 9.6752 | 3.1007 |
| 14 | 1941–1950 | 7 | 34 | 32 | 10.5386 | 3.0365 |
| 15 | 1991–2000 | 5 | 39 | 37 | 11.6516 | 3.1755 |

changes is feasible. Today, when it is difficult to understand the future trends in wet-and-low water changes, making statistical forecasts using time series data sets can be an effective method to help anticipate such changes. There is potential for combining this method with other methods, to further enhance the forecast accuracy of changing trends in water resources.

The sustainable usage of future water resources involves many social and economic aspects, which is a complex systemic engineering. At the same time, water resources have become the new focus and bottleneck of the 21st century. It is only when the importance of sustainable water resources is realized that the goal of sustainable economic development can be achieved. A good understanding of future changing trends in water resources will not only provide important information for scientific management and macro decision-making, but will also offer a reliable basis for policy-makers to work out their policies.

**Acknowledgments** This work was supported by National Natural Science Foundation of China (No. 41171430).

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