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A physical supply-use table framework for energy analysis on the energy conversion chain

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HIGHLIGHTS

- Today, energy analysis addresses topics all along the energy conversion chain.
- The field of energy analysis would benefit from a common analysis framework.
- In response, a physical supply-use table framework is presented.
- Real-world examples demonstrate the range of applicability of the framework.
- Benefits include data structure uniformity and methodological consistency.

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ABSTRACT

In response to the oil crises of the 1970s, energy accounting experienced a revolution and became the much broader field of energy analysis, in part by expanding along the energy conversion chain from primary and final energy to useful energy and energy services, which satisfy human needs. After evolution and specialization, the field of energy analysis today addresses topics along the entire energy conversion chain, including energy conversion systems, energy resources, carbon emissions, and the role of energy services in promoting human well-being and development. And the expanded field would benefit from a common analysis framework that provides data structure uniformity and methodological consistency.

Building upon recent advances in related fields, we propose a physical supply-use table energy analysis framework consisting of four matrices from which the input-output structure of an energy conversion chain can be determined and the effects of changes in final demand can be estimated. Real-world examples demonstrate the physical supply-use table framework via investigation of energy analysis questions for a United Kingdom energy conversion chain.

The physical supply use table framework has two key methodological advances over the building blocks that precede it, namely extending a common energy analysis framework through to energy services and application of physical supply-use tables to both energy and exergy analysis. The methodological advances enable the following first-time contributions to the literature: (1) performing energy and exergy analyses on an energy conversion chain using physical supply-use table matrices comprised of disaggregated products in physical units when the last stage is any of final energy, useful energy, or energy services; (2) performing structural path analysis on an energy conversion chain; and (3) developing and utilizing a matrix approach to inhomogeneous units. The framework spans the entire energy conversion chain and is suitable for many sub-fields of energy analysis, including net energy analysis, societal energy analysis, human needs and well-being, and structural path analysis, all of which are explored in this paper.

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1. Introduction

1.1. A recent history of energy analysis: expansion through revolution and evolution

The modern field of energy analysis is rooted in energy accounting, which emerged in the 1950s from Leontief's input-output (IO) methods [1] and Barnett's energy balance tables [2]. With studies of the U.S. economy by Schurr and Netschert [3] and Morrison and Readling [4], the field remained closely aligned to energy accounting methods through the 1960s (see Berndt [5] for an overview of the early history of energy analysis).

The 1970s oil crises caused a revolution in the field: its focus expanded from merely accounting for production and sale of primary and final energy carriers to many other aspects of energy in society and the economy. Reistad [6, p. 429] said, "In this period of concern for our energy resources and the environment, it is imperative to consider the manner in which our energy resources are consumed." The study of technical energy efficiency became prominent, illustrated by a 1973 conference presentation by Hatsopoulos [7] and the 1975 American Institute of Physics reports on second-law efficiency [8], automobiles [9], and industrial processes [10]. At an economy-wide level, studies of net energy [11], useful energy [6], and energy services [12] were conducted. Furthermore, new studies of interactions between energy and the economy appeared, covering topics such as the energy impact of consumption decisions [13], the entropic nature of economic processes [14], energy and "potential" GDP [15], and questioning the value of the concept of energy intensity [16]. In 1978, Roberts [17, p. 200] noted that the term "energy *analysis*" was now preferred to "energy *accounting*," the name change signifying that the revolution was underway.

Following the 1970s, evolution and specialization led to the creation of several energy analysis sub-fields. Net energy analysis evolved from the study of single fossil fuel sources (e.g., oil, coal, gas) [18] to renewables [19,20] and to the consideration of economy-wide issues such as the minimum energy return on (energy) invested (EROI) required for a functioning society [21], the implications of declining EROI [22], energy expenditure and economic growth [23], and input-output methods to determine national-level EROI [24]. World-wide issues also received attention, including detailed studies of oil and gas production [25], correlations between EROI and oil prices [26], and social implications [27]. The empirical study of energy efficiency and rebound [28] specialized into evaluation of direct [29], indirect [30], and sectoral and economy-wide rebound for energy in the UK [31] and for energy intensity as opposed to energy efficiency [32]. A new sub-field, societal exergy analysis, emerged. Building on the earlier work of Reistad [6], Wall [33], and Kümmel et al. [34], Ayres and co-authors made significant advances on the role of physical resources flows in endogenous growth models [35], the role of physical work in economic growth [36], efficiencies of specific energy and economic sectors [37], and the impact of natural resource consumption and technological change on economic growth [38]. Recent work has standardized allocation of final energy to useful exergy categories [39], improved estimates of exergetic efficiencies [40], and explored theoretical efficiency limits of end-use devices [41]. Another new sub-field (energy decomposition analysis) expanded greatly largely due to the efforts of Ang who developed log-mean divisia index (LMDI) methods [42], compared them against other decomposition approaches [43], applied them to monitoring energy intensity [44], and provided a practical guide for implementation [45]. Further specialization of energy analysis occurred as researchers considered the role of energy in economic growth in terms of energy constraints [46], primary energy sources [47], empirical evidence from many countries [48], and causality directions and substitution possibilities via time-series analysis [49]. The benefits [50] and limitations [51] of the metaphor "the economy is society's metabolism" were explored by several authors, and the magnitude of the industrial energetic and material metabolism has been estimated for the EU [52] and the world [53]. Others have explored the role that energy plays in satisfying human needs [54] across various nations [55], have studied how energy enables well-being [56], and have developed a sufficiency framework for decoupling human well-being from energy consumption [57]. Lastly, analysis of long-run energy transitions has received much recent attention, with researchers studying countries (the UK [58], the U.S. [59], and Sweden [60]), causes (energy cost share [61] and policy [62]), and policy needs for a transition away from oil [26] and toward a sustainable future [63].

1.2. The energy conversion chain (ECC)

A notable feature of this history is an expanding analysis boundary. In the 1960s, energy accountants were focused on primary energy sources and final energy carriers. Today, energy analysts also consider the consumption of useful energy produced by consumer-owned devices [39] to generate energy services [64,65] that satisfy human needs and enable human well-being and development [57]. The expanded boundary covers the entire *energy conversion chain* (ECC), a term (to our knowledge) introduced by Crowe [66, p. 3] to describe energy conversion processes in diesel generators and fuel cells. We find the phrase to be apt for all types of energy analysis, so we define it more broadly to be a set of energy carriers, energy transformation devices, and energy services within spatial and temporal boundaries of interest. In this paper, we focus on economy-spanning ECCs comprised of primary, final, and useful energy carriers as well as the energy services they enable.

Fig. 1 shows an example ECC with two pathways: Natural gas (NG) to Residential end use and Crude to Transport end use. Activities in the

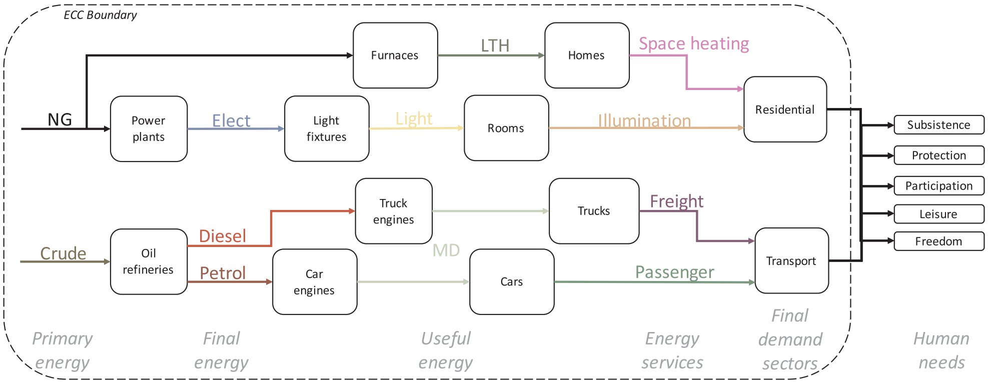


Fig. 1. Energy conversion chain (ECC) example. NG is Natural gas. LTH is Low-temperature heat. MD is Mechanical drive. Line colors indicate products and match Figs. 3, 7, 11 and B.1.

Residential and Transport final demand sectors, made possible by the ECC, partially satisfy Human needs, some of which are shown. For simplicity, Fig. 1 ignores interactions between the two pathways (e.g., electricity to operate an oil refinery), self-consumption (e.g., of electricity by power plants), and distribution (of electricity and fuels by the grid and transport systems, respectively). Real-world examples in Section 3 incorporate these complexities.

The expanding analysis boundary was accompanied by an increase in the number of questions addressed by energy analysis. Note that emissions concerns trace upstream to Primary energy at the far left of Fig. 1, but satisfaction of human needs in the Residential and Transport sectors is downstream at the far right. And there is a growing realization that focusing on a single part of the ECC yields an incomplete analysis. Mayumi and Giampietro [67, p. 65] say "[w]e should not study in isolation either patterns of production or patterns of consumption of energy carriers. Any metabolic system works by integrating the two sides (production and consumption of energy carriers) in an organic whole capable of expressing a desirable set of functions".

Indeed, climate-altering emissions and the role of energy in human development are just two aspects of contemporary energy analysis. Four questions that represent important topics in energy analysis subfields today are:

- Net energy analysis: What are the energy return ratios (ERRs) for energy production devices?
- Societal energy analysis: Where are the key energy saving opportunities in an economy?
- Human needs and well-being: How much primary energy is required to provide energy services?
- Structural path analysis: What are the key supply-chain paths through the ECC for delivering energy services?

These questions span the entire ECC from primary energy to energy services, they encompass issues relevant to many energy analysis subfields, and they require significant empirical data and interdisciplinary knowledge to address. We tackle these questions using a real-world ECC in Section 3.

1.3. The benefits and building blocks of an energy analysis framework for the ECC

In our opinion, efforts to address today's energy analysis questions would benefit from a data structure and associated analytical methods —an *energy analysis framework*— that (a) spans the entire ECC and (b) is suitable for many energy analysis sub-fields. Such a framework could organize and streamline questions to be asked, data to be gathered, analyses to be performed, and results to be reported.

We believe that an energy analysis framework with these benefits is possible, taking a physical supply-use table (PSUT) approach. In fact, several research communities have been developing techniques that provide the building blocks for such a framework. We identify five important developments below.

First, IO researchers have developed methods that employ supply-use tables to overcome problems of co-production (one industry makes more than one product) [68], to deal with wastes [69], to perform decomposition analysis [70], to analyze environmental impacts [71,72], and to combine decomposition and impact analysis [73]. Importantly for this study, with supply-use tables a single energy conversion device (e.g., an Oil refinery) can produce multiple outputs (e.g., Petrol and Diesel).

Second, Pauliuk and co-authors have developed SUT-based techniques for accounting physical [74] resource flows [75,76], drawing on waste accounting frameworks [77] that employed physical IO tables [78]. These physical approaches have been employed to study wood and paper flows [79], among other commodities. Others investigate international flows of embodied energy using matrix-based [80] and network [81] methods. These advances demonstrate that physical flows (including, in our case, energy carriers and energy services) can be accommodated in an SUT analysis framework.

Third, life-cycle analysis practitioners have overcome methodological issues to demonstrate material balances [82] in physically-extended economic SUT frameworks [83]. This development gives confidence that an energy analysis framework that obeys the first and second laws of thermodynamics can be developed. Others have developed matrix-based methods for determining energy return ratios [84], giving confidence that matrix-based analysis of the entire ECC will be successful.

Fourth, Rocco [85], Guevara [86], and their respective co-authors have developed advanced, mixed-units, matrix-based SUT and IO techniques. These techniques have been applied to the broader economy for life cycle assessment of electricity production in waste-to-energy technology [87], for determining the primary exergy cost of goods and services [88], for understanding the energy metabolism of the world [89], for decomposition of primary energy use [90], and for decoupling of exergy use from economic growth [91]. Their work gives confidence that techniques developed over decades of economic IO and SUT research can be applied to energy flows and energy services in an ECC.

Finally, Chong et al. [92] obtain primary-to-final "energy quantity conversion factors" via Leontief inverse of an IO table comprised of aggregated physical quantities. To our knowledge, their work is the first example of obtaining ECC efficiencies via IO techniques, albeit in support of the narrow objective of performing LMDI decomposition analysis of final energy consumption in Guangdong Province, China. However, it shows that application of IO techniques with quantities expressed in purely physical, not monetary, units is both feasible and beneficial for energy analysis.

1.4. Aim, originality, and scope of paper

The aim of this paper, then, is to build upon these recent advances to develop and demonstrate a PSUT-based energy analysis framework (the "PSUT framework" for short) that spans the entire ECC and is pertinent to many energy analysis questions. Such a PSUT framework should have two important characteristics, namely (a) applicability to the entire ECC (i.e., primary energy to energy services) and (b) applicability to both energy and exergy analysis. The representative contemporary energy analysis questions posed in Section 1.2 provide a context for demonstrating the PSUT framework. Although answers to these energy analysis questions can inform policy debates, we consider policy to be beyond the scope of this paper. Table 1 provides a summary of differences between recent work and this study.

Table 1
Differences among the previous works of Guevara et al. [86,90,91] and Rocco et al. [85,87,88], Chong et al. [92], and this study.

	Guevara/Rocco	Chong	This study
Units	Mixed physical and financial units	Physical units exclusively	Physical units exclusively
Energy quantification	Exergy	Energy	Energy or exergy
Last stage in ECC	Useful exergy	Final energy	Energy services
Structure	SUT	Input-output	SUT
Product aggregation	None	Extensive	None
Rest of economy	Energy system embedded within rest of economy	Absent	Absent

The paper proceeds as follows: Section 2 describes the PSUT framework. Section 3 gives real-world examples and answers the questions posed in Section 1.2. A discussion and conclusions (Sections 4 and 5) follow. Detailed appendices are provided for the interested reader.

2. The PSUT framework

2.1. Introduction to PSUT framework

Our energy analysis framework is a physical framework (the *P* in PSUT), because all values are quantified in physical units (e.g., ktoe, TJ, or passenger-km), not monetary units (e.g., $ or £). The framework accommodates industries with multiple inputs and multiple outputs, because it is based on supply-use table methods (the *SUT* in PSUT). The PSUT framework is applicable to analyses conducted in either energy or exergy terms, although we write simply "energy" where possible to avoid the awkward phrase "energy or exergy".

The structure of the PSUT framework comprises four matrices. The first three are typical of supply-use table (SUT) formulations of IO analyses, and we refer to them as the *PSUT matrices*: $\underset{p\times i}{\mathbf{U}}$ (a product-by-industry "use" matrix), $\underset{i\times p}{\mathbf{V}}$ (an industry-by-product "supply" or "make" matrix), and $\underset{p\times s}{\mathbf{Y}}$ (a product-by-sector "final demand" matrix). A fourth matrix is an auxiliary product-by-unit summation matrix ($\underset{p\times u}{\mathbf{S}_{units}}$) which identifies the physical units in which products are measured. To indicate whether industries (i), products (p), final demand sectors (s), or units (u) appear in rows or columns of matrices, we adopt the notation shown in Table 2. When a matrix is introduced (and when needed for clarity thereafter), this notation is typeset beneath matrix symbols. We mostly follow the Eurostat nomenclature for matrix symbols and the categories of products, industries, and final demand sectors [93]. Table 3 provides a mapping between Eurostat categories and energy and services concepts in the PSUT framework. See Appendix A for a comprehensive table of nomenclature.

The $\mathbf{U}$, $\mathbf{V}$, and $\mathbf{Y}$ matrices can be arranged spatially as shown in Fig. 2. Entries in the use matrix ($\mathbf{U}$) give the consumption of energy carriers and services (in rows) by energy transformation devices (in columns). Entries in the make matrix ($\mathbf{V}$) indicate the production of energy carriers and services (in columns) by energy transformation devices (in rows). Entries in the final demand matrix ($\mathbf{Y}$) specify consumption of energy carriers and services (in rows) by final demand sectors (in columns). (Non-energy uses of energy carriers or services appear in a column of $\mathbf{Y}$.) Entries in the units summation matrix ($\mathbf{S}_{units}$) specify the physical units (in columns) by which energy carriers and services (in rows) are measured. Note that not all final demand sectors in columns of $\mathbf{Y}$ correspond to final demand sectors in systems of national accounts. For example, Transport is an intermediate sector in systems of national accounts but a final demand sector here.

Table 2
Matrix dimension notation.

Notation	Meaning
p×p	Products in both rows and columns (e.g., $\underset{p\times p}{\mathbf{L}}$)
i×i	Industries in both rows and columns (e.g., $\hat{\mathbf{g}}$)
p×i	Products in rows and industries in columns (e.g., $\mathbf{U}$)
i×p	Industries in rows and products in columns (e.g., $\mathbf{V}$)
p×s	Products in rows and final demand sectors in columns (e.g., $\mathbf{Y}$)
p×u	Products in rows and units of products in columns (e.g., $\mathbf{S}_{units}$)

Table 3
Eurostat categories.

Eurostat category	ECC analogue
Products	Energy carriers (e.g., Oil, Electricity, Mechanical drive) Energy services (e.g., Passenger transport, Illumination)
Industries	Energy imports Energy extraction devices (e.g., Mines, Oil fields) Energy conversion devices (e.g., Power plants, Furnaces) Passive devices (e.g., Cars, Homes)
Final demand	Energy exports Energy storage (e.g., Bunkers, Stocks) Economic sectors (e.g., Residential, Transport)

2.2. Building and manipulating the PSUT matrices

Building and manipulating the PSUT matrices involves deciding an analytical approach, constructing and verifying the PSUT matrices, formulating the IO structure of the ECC, and estimating the effect of changes in final demand on the ECC. Each activity is described in subsections below.

2.2.1. Analytical approach

Before constructing the PSUT matrices introduced in Section 2.1, an analytical approach must be decided, i.e. a set of decisions must be made about analysis choices that is sufficient to allow construction of the PSUT matrices. Analysis choices include, but are not limited to, (a) the country, device, or process of interest (spatial boundary); (b) the time period over which the analysis applies (temporal boundary); (c) the method of accounting for primary energy corresponding to renewable energy production (partial substitution method, physical content method, or resource content method [94]); (d) whether to include non-energy uses of energy carriers in PSUT matrices; (e) whether entries in PSUT matrices represent energy or exergy quantities; and (f) whether the last stage of analysis will be final energy, useful energy, or energy services.

2.2.2. PSUT matrix construction

The PSUT matrices are populated with energy and energy services data gathered from sources including, but not limited to, (a) the International Energy Agency (IEA) [95] (for primary and final energy data), (b) estimates of final-to-useful transformation device efficiencies [96,97,40,98] (for calculating useful energy), (c) exergy/energy ratios (ϕ) [99] (for calculating exergy content from energy values), and (d) national statistical datasets [100, Table 2] (for energy services data). Note that all primary-to-final, final-to-useful, and useful-to-services transformation devices are included as "industries" in the $\mathbf{U}$, $\mathbf{V}$, and $\mathbf{Y}$ matrices. All entries in the PSUT matrices should be non-negative numbers, and all energy entries must be in the same units, typically TJ/year or ktoe/year for a large economy.

2.2.3. Thermodynamic verification

Regardless of analytical approach, PSUT matrices populated with energy carriers and energy services are verified by the first law of thermodynamics (see Appendix B for discussion of exergy and the second law of thermodynamics). Two fundamental input-output calculations are needed for first law verification: value added and aggregation. A value added matrix ($\underset{p\times i}{\mathbf{W}}$) is given by

$$\mathbf{W} = \mathbf{V}^{\mathrm{T}} - \mathbf{U}. \tag{1}$$

Aggregations are row, column, or matrix sums, and several are given in Table 4.

With rare exception, the column sums of the value added matrix ($\mathbf{i}^{\mathrm{T}}\mathbf{W}$) are positive in *financial* SUT analyses, because finished products are more valuable (in a monetary sense) than the raw materials from which they are made. (And industries with negative value added don't survive for long!). However, in the PSUT framework, column sums of the value added matrix ($\mathbf{i}^{\mathrm{T}}\mathbf{W}$) are often negative, because energy transformation devices produce less useable energy than they consume

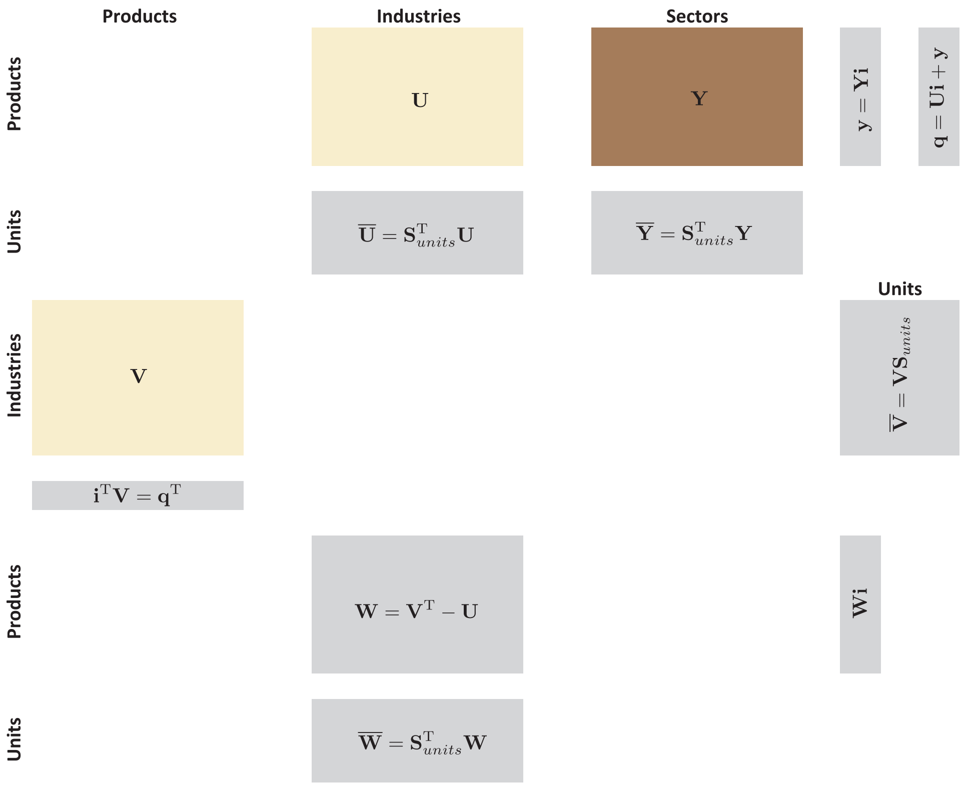


Fig. 2. PSUT structure. See Table A.1 for matrix and vector definitions. Note that $\mathbf{y}$, $\mathbf{q}$, and $\mathbf{Wi}$ are column vectors. All others structures are matrices.

due to inefficiencies and wastes. For example, coal-fired power plants produce about 1/3 as much electrical energy as they consume in coal energy, the difference being waste heat. Indeed, $\mathbf{i}^{T}\mathbf{W}$ will contain positive entries for extractive industries (free gifts from nature) and negative entries for ECC transformation devices (due to wastes and waste heat).

Unless the PSUT matrices conform to the first law of thermodynamics, all further calculations will be wrong. With the aggregations of Table 4 and the value added matrix of Eq. (1) in hand, energy and services balances should be verified across products and across industries. To evaluate the first law across products, the following equation applies:

$$\mathbf{Wi}-\mathbf{y} = \mathbf{0}. \tag{2}$$

Across industries, inputs must equal the sum of valuable products (outputs) and wastes. Thus, the first law can be expressed as

$$\text{outputs} + \text{wastes}-\text{inputs} = 0. \tag{3}$$

Assuming homogeneous units in the $\mathbf{U}$, $\mathbf{V}$, and $\mathbf{W}$ matrices, total output by industry is $\mathbf{g}$, waste by industry is $-\mathbf{W}^{T}\mathbf{i}$ (wastes are negative value added), and input by industry is $\mathbf{U}^{T}\mathbf{i}$. Substituting into Eq. (3) yields

$$\mathbf{g}-\mathbf{W}^{T}\mathbf{i}-\mathbf{U}^{T}\mathbf{i} = \mathbf{0}. \tag{4}$$

For ECCs with inhomogeneous units in the $\mathbf{U}$, $\mathbf{V}$, and $\mathbf{W}$ matrices, we substitute $\overline{\mathbf{V}}$ for outputs, $-\overline{\mathbf{W}}^{T}$ for wastes, and $\overline{\mathbf{U}}^{T}$ for inputs in Eq. (3) to obtain

$$\overline{\mathbf{V}}-\overline{\mathbf{W}}^{T}-\overline{\mathbf{U}}^{T} = \mathbf{0}. \tag{5}$$

Note that Eqs. (4) and (5) are helpful identities for checking calculations. See Appendix D for a short proof of Eq. (4). See Appendix E for details of the shift from Eq. (4) to Eq. (5).

Table 4
PSUT framework aggregations. See Table A.1 for matrix and vector nomenclature and Appendix C for a summary of relevant matrix and vector mathematics.

Equation	Note
$\mathbf{y} = \mathbf{Yi}$	Final demand by product
$\mathbf{i}^{T}\mathbf{Y}$	Final demand by sector (for homogeneous units)
$\underset{u\times s}{\overline{\mathbf{Y}}} = \mathbf{S}^{T}_{units}\mathbf{Y}$	Final demand by sector (for inhomogeneous units)
$\mathbf{g} = \mathbf{Vi}$	Total industry output (for homogeneous units)
$\underset{i\times u}{\overline{\mathbf{V}}} = \mathbf{VS}_{units}$	Total industry output (for inhomogeneous units)
$\mathbf{q} = \mathbf{Ui} + \mathbf{y}$	Total product output
$\mathbf{q}^{T} = \mathbf{i}^{T}\mathbf{V}$	Total product output
$\mathbf{i}^{T}\mathbf{U}$	Consumption by industry (for homogeneous units)
$\underset{u\times i}{\overline{\mathbf{U}}} = \mathbf{S}^{T}_{units}\mathbf{U}$	Consumption by industry (for inhomogeneous units)
$\mathbf{i}^{T}\mathbf{W}$	Value added by industry (for homogeneous units)
$\underset{u\times i}{\overline{\mathbf{W}}} = \mathbf{S}^{T}_{units}\mathbf{W}$	Value added by industry (for inhomogeneous units)
$E_p = (\mathbf{s}_p^{T}\mathbf{V}^{T}-\mathbf{s}_p^{T}\mathbf{Y})\mathbf{i}$	Total primary energy supply
$E_f = (\mathbf{s}_f^{T}\mathbf{Y})\mathbf{i}$	Final energy demand

2.2.4. Input-output structure

After construction (Section 2.2.2) and verification (Section 2.2.3), the complete IO structure of the ECC can be formulated. The IO structure of the ECC is represented by the set of matrices shown in Table 5.

We employ Eurostat Model B (the industry technology assumption) wherein each industry has its own specific way of production, regardless of its product mix [93, p. 349]. This model is appropriate for analyzing ECCs, because each energy transformation device produces its products in its own way. For example, coal-fired and gas-fired power plants must be able to produce electricity, each with its own mix of energy inputs. Other Eurostat models, which employ different assumptions (in particular, Model A, which assumes that “each product is produced in its own specific way, irrespective of the industry where it is produced” [93, p. 347]), are inappropriate for the PSUT framework.

Table 5
Calculations for IO structure.

Equation	Note
$\underset{p\times i}{\mathbf{Z}} = \mathbf{U}\hat{\mathbf{g}}^{-1}$	Input requirements for products per unit of output of an industry
$\underset{p\times i}{\mathbf{C}} = \mathbf{V}^{\mathrm{T}}\hat{\mathbf{g}}^{-1}$	Product mix matrix
$\underset{i\times p}{\mathbf{D}} = \mathbf{V}\hat{\mathbf{q}}^{-1}$	Market shares matrix
$\underset{p\times p}{\mathbf{A}} = \mathbf{Z}\mathbf{D}$	Input coefficients for intermediates
$\underset{p\times p}{\mathbf{L}} = (\mathbf{I}{-}\mathbf{A})^{-1}$	Product-by-product Leontief matrix
$\underset{i\times p}{\mathbf{L}} = \mathbf{D}(\mathbf{I}{-}\mathbf{A})^{-1}$	Industry-by-product Leontief matrix
$\underset{i\times p}{\mathbf{G}} = \underset{p\times p}{\mathbf{L}}\,\hat{\mathbf{y}}$	Assists “footprinting” in PSUT framework

The calculations in Table 5 can be verified by

$$\underset{i\times p}{\mathbf{L}}\,\mathbf{y} = \mathbf{g}, \tag{6}$$

and

$$\underset{p\times p}{\mathbf{L}}\,\mathbf{y} = \mathbf{q}. \tag{7}$$

Note that the description of IO structure in Table 5 requires that coproducts of any industry exhibit unit homogeneity. Specifically, rows of $\overline{\mathbf{V}}$ must contain exactly one nonzero element. If an industry has coproducts with inhomogeneous units (e.g., Airlines make both Passenger transport [passenger-km/yr] and Freight transport [tonne-km/yr]), the industry should be split and inputs allocated as appropriate for each industry. (E.g., “Airlines” becomes “Passenger airlines” and “Freight airlines” with inputs to “Airlines” allocated between “Passenger airlines” and “Freight airlines.”)

2.2.5. *Effect of changes to final demand*

After the IO structure of an ECC has been characterized by the matrices of Table 5, an important question may be answered with respect to final demand: “What would be the effect on the ECC of a change to final demand?” Calculations proceed as shown in Table 6 to perform an “upstream swim” from the adjusted final demand matrix ($\mathbf{Y}'$) to resource extraction, thereby creating a second set of PSUT matrices (including $\mathbf{U}'$ and $\mathbf{V}'$) that describe an adjusted ECC associated with $\mathbf{Y}'$. After the calculations in Table 6 are accomplished, the adjusted PSUT matrices ($\mathbf{U}'$, $\mathbf{V}'$, and $\mathbf{Y}'$) can be (a) analyzed using Eq. (1) and Tables 4 and 5 and (b) verified using Eqs. (2) and (4)–(7).

3. Results: Demonstrating the PSUT framework for real applications

With the PSUT framework now established (Section 2), we provide results for one real-world example for each of the four contemporary energy questions in Section 1.2, thereby illustrating application across a range of energy analysis sub-fields, including net energy analysis (Section 3.1), societal energy analysis (Section 3.2), human needs and well-being (Section 3.3), and structural path analysis (Section 3.4). The real-world examples are at the economy-wide level, although the PSUT framework could be applied at any level: device, firm, sector, economy-wide, or global.

Table 6
Estimating the effect of changes to final demand (“upstream swim”).

Equation	Note
$\mathbf{y}' = \mathbf{Y}'\mathbf{i}$	New final demand by product for $\mathbf{Y}'$
$\mathbf{g}' = \underset{i\times p}{\mathbf{L}}\,\mathbf{y}'$	Industry output for $\mathbf{Y}'$
$\mathbf{q}' = \underset{p\times p}{\mathbf{L}}\,\mathbf{y}'$	Product output for $\mathbf{Y}'$
$\mathbf{U}' = \mathbf{Z}\widehat{\mathbf{g}'}$	Use matrix for $\mathbf{Y}'$
$\mathbf{V}' = \mathbf{D}\widehat{\mathbf{q}'}$	Make matrix for $\mathbf{Y}'$

A real-world ECC (based on the two-path ECC of Fig. 1) illustrates the four numerical examples. The ECC is revealed sequentially as needed, the last stage extending from final energy (Section 3.1) to useful energy (Section 3.2) to energy services (Sections 3.3 and 3.4). All energy values are in ktoe/year, while energy services are expressed in differing physical units, e.g. passenger-km/year. All ECCs are constructed with energy quantification for energy carriers but could just as well have been constructed with exergy quantification by multiplying each energy flow by the appropriate exergy-to-energy ratio (ϕ, see Serrenho [99, Table 2]). See Appendix B for an ECC constructed from exergy flows. Data and calculations for all ECCs can be found in the data repository for this paper [101].

The real-world ECC is based on a portion of the UK’s ECC in 2000, and energy and services data have been rounded to 1–2 significant figures. Thus, numerical results should be interpreted with caution. Data from any combination of country and year would suffice for this paper, because the real-world ECC is used for demonstration purposes only. Primary and final energy data come from IEA energy statistics [95]. Brockway et al. [40] provide useful energy. Energy services data have been obtained from several sources. Passenger and Freight transport data are from the UK Department for Transport, Tables TSGB0702 and TSGB0401, respectively [102]. Illumination data are from Fouquet and Pearson [103]. We estimate residential Space heating service for 25 million homes, each with representative 100 m^2 floor space, 3 m ceiling height, and average 10 K temperature difference between heated space and ambient.

3.1. *Net energy analysis: What are the energy return ratios (ERRs) for energy production devices?*

Within the sub-field of net energy analysis, an important question is *What are the energy return ratios (ERRs) for energy production devices?* A common ERR is energy return on (energy) invested (EROI), a metric first explored by Hall [104], and utilized extensively in subsequent years by Murphy and Hall [105,106], Heun and de Wit [26], Lambert et al. [107], Brand-Correa et al. [24], and many others.

Significance: Large ERRs indicate an effective energy-producing industry that provides a large rate of energy to society for small rate of energy investment.

We adopt the nomenclature of Brandt et al. [108] in which GER_γ and NER_γ indicate the gross and net energy return ratios, respectively, for an energy production device. The subscript γ denotes an ERR analysis boundary that accounts for multiple interacting energy pathways (e.g., Oil fields that consume Electricity). Inspired by Brandt [109], we include the net-to-gross energy ratio (r_γ) as well. Larger values of all ERRs indicate an energy system that is more effective at providing energy to society with less energy consumed (see Appendix F for derivations of relationships among the three ERRs).

In the context of the PSUT framework, all of GER_γ, NER_γ, and r_γ become industry column vectors ($\mathbf{ger}_\gamma$, $\mathbf{ner}_\gamma$, and $\mathbf{r}_\gamma$) given by Eqs. (8)–(10).

$$\mathbf{ger}_\gamma = (\widehat{\mathbf{U}_{EIOU}^{\mathrm{T}}\mathbf{i}})^{-1}\mathbf{g} \tag{8}$$

$$\mathbf{ner}_\gamma = \mathbf{ger}_\gamma{-}\mathbf{i} \tag{9}$$

$$\mathbf{r}_\gamma = (\widehat{\mathbf{ger}_\gamma})^{-1}\mathbf{ner}_\gamma \tag{10}$$

To demonstrate, we calculate ERRs for each device of Fig. 3, the first version of our real-world ECC, wherein final energy is consumed by both (a) intermediate industries (Gas wells, Oil fields, Natural gas and Crude distribution, Power plants, and Oil refineries) and (b) final

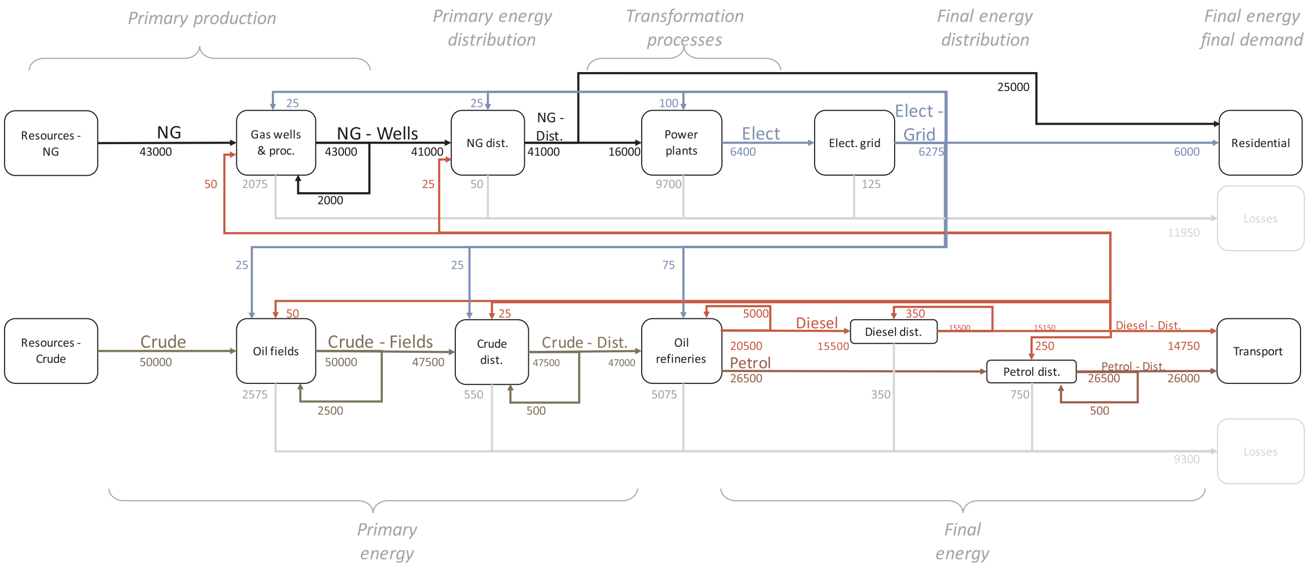


Fig. 3. A real-world ECC covering primary and final energy. All energy flows in units of ktoe/year. NG is Natural gas. Line colors indicate products.

demand. In comparison to Fig. 1, interacting flows, detailed self-consumption flows (energy industry own use), and distribution sectors are now included. The PSUT matrices associated with Fig. 3 are shown in Fig. 4. Fig. 5 shows the energy industry own use matrix ($\mathbf{U}_{EIOU}$) for the ECC of Fig. 3.

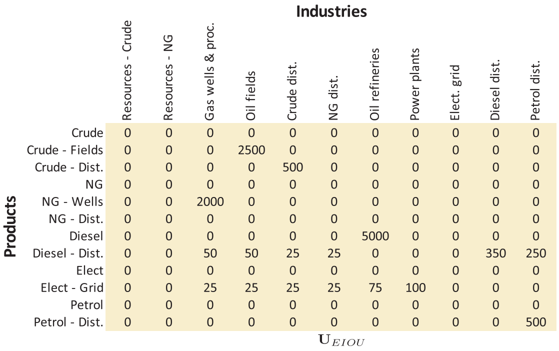
Fig. 6 shows ERR vectors for the ECC of Fig. 3. ERRs are most relevant for production stages of the ECC (Gas wells and Oil fields in this example), although (8)–(10) provide ERRs for all industries in the ECC. In Fig. 6, ERRs for Resources are ∞, because the energy to extract Resources is accounted in Gas wells and Oil fields. The ERRs for Elect grid are ∞, because there is no energy apart from Elect supplied to the Elect grid in Fig. 3.

Benefit of the PSUT framework: This real-world example shows that organizing ECC data in the PSUT framework allows computation of any ERR for all ECC devices with straightforward matrix mathematics.

3.2. Societal energy analysis: Where are the key energy saving opportunities in an economy?

Within the sub-field of societal energy analysis, an important question is *What are the device and sector energy efficiencies along an ECC?*

Significance: Answers to this question identify key energy saving opportunities in an economy, which is important because “[t]he efficient provision of energy services not only reduces the required amounts of primary energy but in general also reduces adverse environmental impacts” [109, p. 421].



Industries

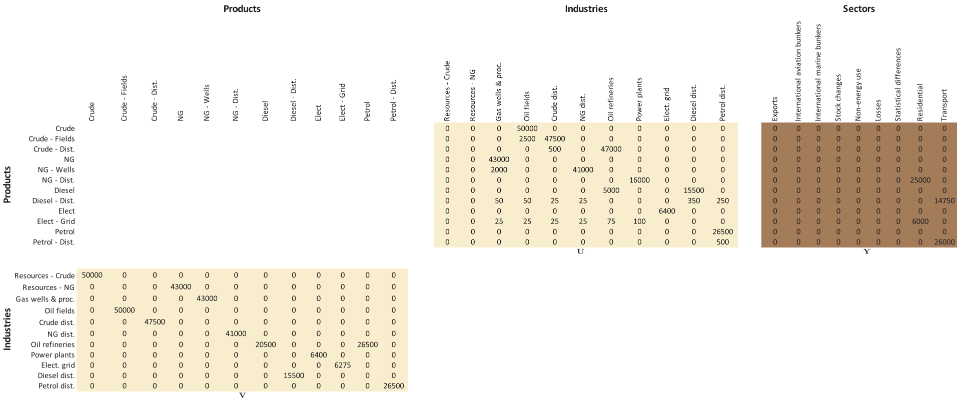
Products

	Resources - Crude	Resources - NG	Gas wells & proc.	Oil fields	Crude dist.	NG dist.	Oil refineries	Power plants	Elect. grid	Diesel dist.	Petrol dist.
Crude	0	0	0	0	0	0	0	0	0	0	0
Crude - Fields	0	0	0	2500	0	0	0	0	0	0	0
Crude - Dist.	0	0	0	0	500	0	0	0	0	0	0
NG	0	0	0	0	0	0	0	0	0	0	0
NG - Wells	0	0	2000	0	0	0	0	0	0	0	0
NG - Dist.	0	0	0	0	0	0	0	0	0	0	0
Diesel	0	0	0	0	0	0	5000	0	0	0	0
Diesel - Dist.	0	0	50	50	25	25	0	0	0	350	250
Elect	0	0	0	0	0	0	0	0	0	0	0
Elect - Grid	0	0	25	25	25	25	75	100	0	0	0
Petrol	0	0	0	0	0	0	0	0	0	0	0
Petrol - Dist.	0	0	0	0	0	0	0	0	0	0	500

$\mathbf{U}_{EIOU}$

Fig. 5. Energy industry own use (EIOU) for the ECC of Fig. 3. All numbers in units of ktoe/year.

Fig. 7 extends the last stage of analysis in our real-world ECC from final energy to useful energy such that final demand includes Low-temperature heat (LTH), Light, and Mechanical drive (MD). Some intermediate industries now also consume useful energy (e.g., the distribution industries consume MD–Truck engines, whereas in Fig. 3 they consumed Diesel, a final energy carrier). And for simplicity, self-consumption flows are internalized (e.g., self-consumption of 5000 ktoe of Diesel by Oil refineries in Fig. 3 is now internal to Oil refineries, thereby providing net Diesel output of 15,500 ktoe in Fig. 7). Given the ECC



Industries

Products

	Resources - Crude	Resources - NG	Gas wells & proc.	Oil fields	Crude dist.	NG dist.	Oil refineries	Power plants	Elect. grid	Diesel dist.	Petrol dist.
Crude	0	0	0	50000	0	0	0	0	0	0	0
Crude - Fields	0	0	0	2500	47500	0	0	0	0	0	0
Crude - Dist.	0	0	0	0	500	0	47000	0	0	0	0
NG	0	0	43000	0	0	0	0	0	0	0	0
NG - Wells	0	0	2000	0	0	41000	0	0	0	0	0
NG - Dist.	0	0	0	0	0	0	0	16000	0	0	0
Diesel	0	0	0	0	0	0	5000	0	0	15500	0
Diesel - Dist.	0	0	50	50	25	25	0	0	0	350	250
Elect	0	0	0	0	0	0	0	0	6400	0	0
Elect - Grid	0	0	25	25	25	25	75	100	0	0	0
Petrol	0	0	0	0	0	0	0	0	0	0	26500
Petrol - Dist.	0	0	0	0	0	0	0	0	0	0	500

$\mathbf{U}$

Sectors

Products

	Exports	International aviation bunkers	International marine bunkers	Stock changes	Non-energy use	Losses	Statistical differences	Residential	Transport
Crude	0	0	0	0	0	0	0	0	0
Crude - Fields	0	0	0	0	0	0	0	0	0
Crude - Dist.	0	0	0	0	0	0	0	0	0
NG	0	0	0	0	0	0	0	0	0
NG - Wells	0	0	0	0	0	0	0	0	0
NG - Dist.	0	0	0	0	0	0	0	25000	0
Diesel	0	0	0	0	0	0	0	0	0
Diesel - Dist.	0	0	0	0	0	0	0	0	14750
Elect	0	0	0	0	0	0	0	0	0
Elect - Grid	0	0	0	0	0	0	0	6000	0
Petrol	0	0	0	0	0	0	0	0	0
Petrol - Dist.	0	0	0	0	0	0	0	0	26000

$\mathbf{Y}$

Products

Industries

	Crude	Crude - Fields	Crude - Dist.	NG	NG - Wells	NG - Dist.	Diesel	Diesel - Dist.	Elect	Elect - Grid	Petrol	Petrol - Dist.
Resources - Crude	50000	0	0	0	0	0	0	0	0	0	0	0
Resources - NG	0	0	0	43000	0	0	0	0	0	0	0	0
Gas wells & proc.	0	0	0	0	43000	0	0	0	0	0	0	0
Oil fields	0	50000	0	0	0	0	0	0	0	0	0	0
Crude dist.	0	0	47500	0	0	0	0	0	0	0	0	0
NG dist.	0	0	0	0	0	41000	0	0	0	0	0	0
Oil refineries	0	0	0	0	0	0	20500	0	0	0	26500	0
Power plants	0	0	0	0	0	0	0	0	6400	0	0	0
Elect. grid	0	0	0	0	0	0	0	0	0	6275	0	0
Diesel dist.	0	0	0	0	0	0	0	15500	0	0	0	0
Petrol dist.	0	0	0	0	0	0	0	0	0	0	0	26500

$\mathbf{V}$

Fig. 4. PSUT matrices for the real-world ECC in Fig. 3. All numbers in units of ktoe/year.

Industries	$\mathbf{g}$	$\mathbf{U}^{\mathrm{T}}_{EIOU}\mathbf{i}$	$\mathbf{ger}_\gamma$	$\mathbf{ner}_\gamma$	$\mathbf{r}_\gamma$
Resources - Crude	50000	0	∞	∞	∞
Resources - NG	43000	0	∞	∞	∞
Gas wells & proc.	43000	2075	20.72	19.723	0.9517
Oil fields	50000	2575	19.42	18.417	0.9485
Crude dist.	47500	550	86.36	85.364	0.9884
NG dist.	41000	50	820	819	0.9988
Oil refineries	47000	5075	9.261	8.2611	0.892
Power plants	6400	100	64	63	0.9844
Elect grid	6275	0	∞	∞	∞
Diesel dist.	15500	350	44.29	43.286	0.9774
Petrol dist.	26500	750	35.33	34.333	0.9717

Fig. 6. Energy return ratios (ERRs) for the ECC of Fig. 3. $\mathbf{g}$ and $\mathbf{U}^{\mathrm{T}}_{EIOU}\mathbf{i}$ in ktoe/year. $\mathbf{ger}_\gamma$, $\mathbf{ner}_\gamma$, and $\mathbf{r}_\gamma$ are unitless.

shown in Fig. 7, PSUT matrices can be constructed as shown in Fig. 8.

A vector of ECC industry efficiencies ($\boldsymbol{\eta}_{E,\gamma}$) can be calculated by

$$\boldsymbol{\eta}_{E,\gamma} = (\widehat{\mathbf{U}^{\mathrm{T}}\mathbf{i}})^{-1}\mathbf{g}. \tag{11}$$

Fig. 9 shows device efficiencies ($\boldsymbol{\eta}_{E,\gamma}$) for the ECC shown in Figs. 7 and 8. Again, these are energy efficiency values for the γ system boundary, because they account for industry consumption of energy from other branches of the ECC (see Brandt et al. [108]). The $\boldsymbol{\eta}_{E,\gamma}$ vector in Fig. 9 shows that Power plants, Car and Truck engines, and Light fixtures have much lower efficiencies than other devices.

Beyond the last energy stage, all energy transformations are accomplished within final demand sectors (in this ECC, Residential and Transport). The efficiency of a final demand sector can be evaluated by comparing two final demand matrices ($\mathbf{Y}$). For example, Fig. 4 gives final demand by sector for final energy ($\mathbf{Y}_f$) and Fig. 8 gives final demand by sector for useful energy ($\mathbf{Y}_u$). A vector of efficiencies by which final demand sectors convert final energy to useful energy ($\boldsymbol{\eta}_{E,fu}$) can be calculated by

$$\boldsymbol{\eta}_{E,fu} = (\widehat{\mathbf{Y}^{\mathrm{T}}_{f}\mathbf{i}})^{-1}\mathbf{Y}^{\mathrm{T}}_{u}\mathbf{i}. \tag{12}$$

Fig. 10 shows the final-to-useful energy conversion efficiencies for final demand sectors of the ECCs shown in Figs. 3 and 7.

Benefit of the PSUT framework: These real-world examples demonstrate that the PSUT framework allows calculation of efficiencies for all ECC industries and final demand sectors with convenient matrix operations.

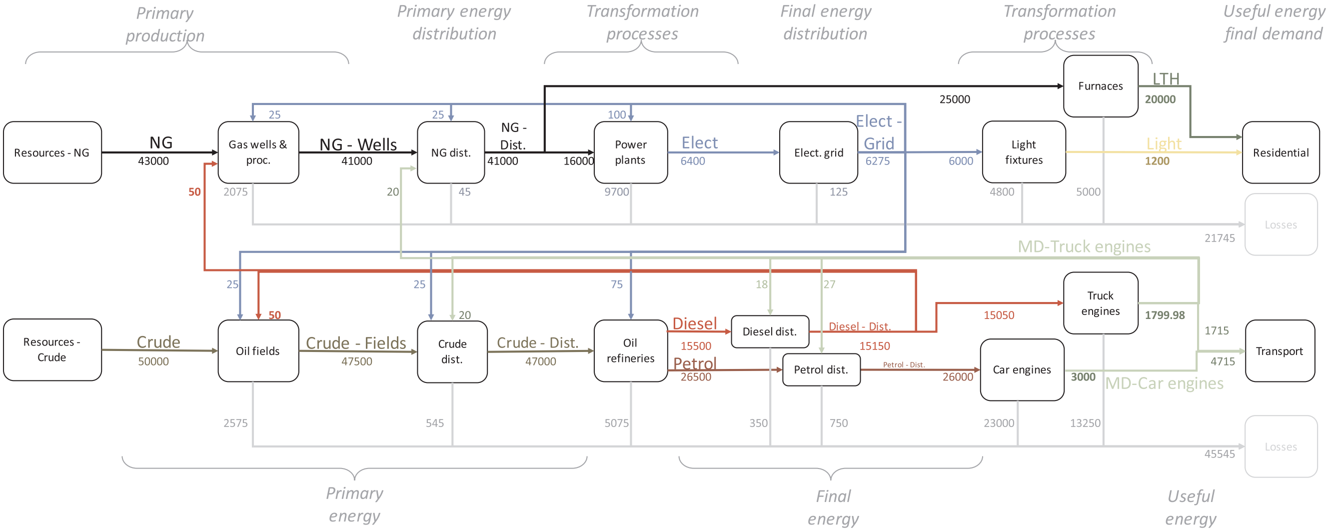
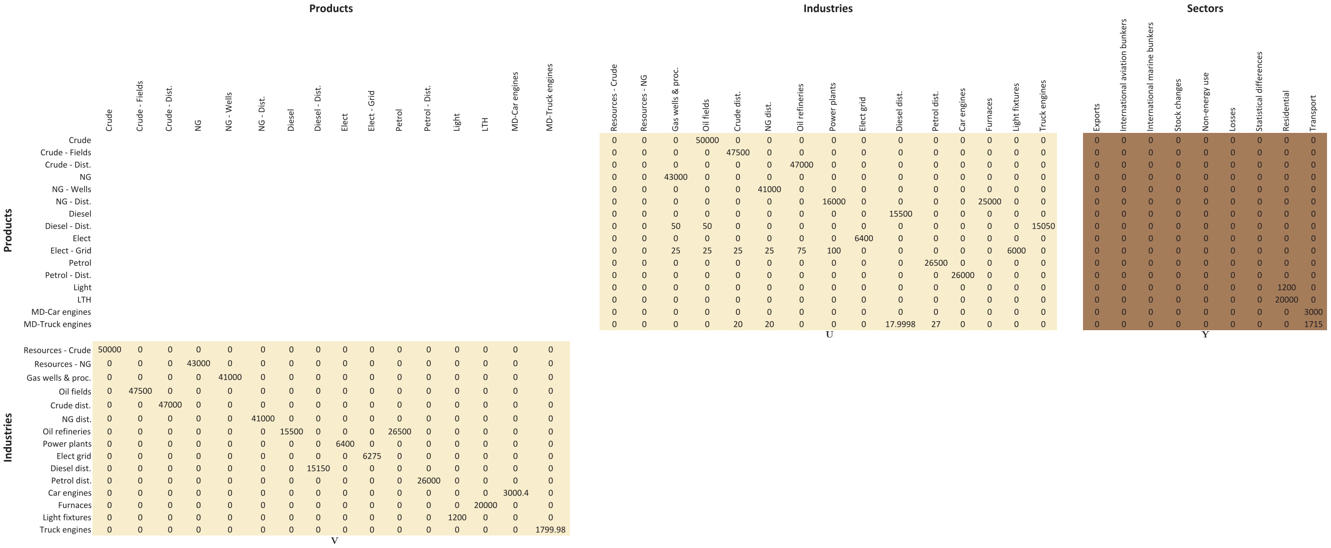


Fig. 7. A real-world ECC covering primary to useful energy. All energy flows in units of ktoe/year. NG is Natural gas. LTH is Low-temperature heat. MD is Mechanical drive. Line colors indicate products.



U

	Resources - Crude	Resources - NG	Gas wells & proc.	Oil fields	Crude dist.	NG dist.	Oil refineries	Power plants	Elect grid	Diesel dist.	Petrol dist.	Car engines	Furnaces	Light fixtures	Truck engines
Crude	0	0	0	50000	0	0	0	0	0	0	0	0	0	0	0
Crude - Fields	0	0	0	0	47500	0	0	0	0	0	0	0	0	0	0
Crude - Dist.	0	0	0	0	0	0	47000	0	0	0	0	0	0	0	0
NG	0	0	43000	0	0	0	0	0	0	0	0	0	0	0	0
NG - Wells	0	0	0	0	0	41000	0	0	0	0	0	0	0	0	0
NG - Dist.	0	0	0	0	0	0	0	16000	0	0	0	0	25000	0	0
Diesel	0	0	0	0	0	0	0	0	0	15500	0	0	0	0	0
Diesel - Dist.	0	0	50	50	0	0	0	0	0	0	0	0	0	0	15050
Elect	0	0	0	0	0	0	0	0	6400	0	0	0	0	0	0
Elect - Grid	0	0	25	25	25	25	75	100	0	0	0	0	0	6000	0
Petrol	0	0	0	0	0	0	0	0	0	0	26500	0	0	0	0
Petrol - Dist.	0	0	0	0	0	0	0	0	0	0	0	26000	0	0	0
Light	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LTH	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
MD-Car engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
MD-Truck engines	0	0	0	0	20	20	0	0	0	17.9998	27	0	0	0	0

Y

	Exports	International aviation bunkers	International marine bunkers	Stock changes	Non-energy use	Losses	Statistical differences	Residential	Transport
Crude	0	0	0	0	0	0	0	0	0
Crude - Fields	0	0	0	0	0	0	0	0	0
Crude - Dist.	0	0	0	0	0	0	0	0	0
NG	0	0	0	0	0	0	0	0	0
NG - Wells	0	0	0	0	0	0	0	0	0
NG - Dist.	0	0	0	0	0	0	0	0	0
Diesel	0	0	0	0	0	0	0	0	0
Diesel - Dist.	0	0	0	0	0	0	0	0	0
Elect	0	0	0	0	0	0	0	0	0
Elect - Grid	0	0	0	0	0	0	0	0	0
Petrol	0	0	0	0	0	0	0	0	0
Petrol - Dist.	0	0	0	0	0	0	0	0	0
Light	0	0	0	0	0	0	0	1200	0
LTH	0	0	0	0	0	0	0	20000	0
MD-Car engines	0	0	0	0	0	0	0	0	3000
MD-Truck engines	0	0	0	0	0	0	0	0	1715

V

	Crude	Crude - Fields	Crude - Dist.	NG	NG - Wells	NG - Dist.	Diesel	Diesel - Dist.	Elect	Elect - Grid	Petrol	Petrol - Dist.	Light	LTH	MD-Car engines	MD-Truck engines
Resources - Crude	50000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Resources - NG	0	0	0	43000	0	0	0	0	0	0	0	0	0	0	0	0
Gas wells & proc.	0	0	0	0	41000	0	0	0	0	0	0	0	0	0	0	0
Oil fields	0	47500	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Crude dist.	0	0	47000	0	0	0	0	0	0	0	0	0	0	0	0	0
NG dist.	0	0	0	0	0	41000	0	0	0	0	0	0	0	0	0	0
Oil refineries	0	0	0	0	0	0	15500	0	0	0	26500	0	0	0	0	0
Power plants	0	0	0	0	0	0	0	0	6400	0	0	0	0	0	0	0
Elect grid	0	0	0	0	0	0	0	0	0	6275	0	0	0	0	0	0
Diesel dist.	0	0	0	0	0	0	0	15150	0	0	0	0	0	0	0	0
Petrol dist.	0	0	0	0	0	0	0	0	0	0	0	26000	0	0	0	0
Car engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3000.4	0
Furnaces	0	0	0	0	0	0	0	0	0	0	0	0	0	20000	0	0
Light fixtures	0	0	0	0	0	0	0	0	0	0	0	0	1200	0	0	0
Truck engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1799.98

Fig. 8. PSUT matrices for the real-world ECC in Fig. 7. All numbers in ktoe/year.

Industries	Resources - Crude	50000	0	∞
	Resources - NG	43000	0	∞
	Gas wells & proc.	41000	43075	0.952
	Oil fields	47500	50075	0.949
	Crude dist.	47000	47545	0.989
	NG dist.	41000	41045	0.999
	Oil refineries	42000	47075	0.892
	Power plants	6400	16100	0.398
	Elect grid	6275	6400	0.98
	Diesel dist.	15150	15518	0.976
	Petrol dist.	26000	26527	0.98
	Car engines	3000.4	26000	0.115
	Furnaces	20000	25000	0.8
	Light fixtures	1200	6000	0.2
	Truck engines	1800	15050	0.12
		$\mathbf{g}$	$\mathbf{U}^{\mathrm{T}}\mathbf{i}$	$\eta_{E,\gamma}$

Fig. 9. Device efficiencies for the ECC of Fig. 7. $\mathbf{g}$ and $\mathbf{U}^{\mathrm{T}}\mathbf{i}$ in ktoe/year. $\eta_{E,\gamma}$ is unitless.

Sectors	Exports	0	0	N/A
	International aviation bunkers	0	0	N/A
	International marine bunkers	0	0	N/A
	Stock changes	0	0	N/A
	Non-energy use	0	0	N/A
	Losses	0	0	N/A
	Statistical differences	0	0	N/A
	Residential	21200	31000	0.684
	Transport	4715.4	40750	0.116
		$\mathbf{Y}_u^{\mathrm{T}}\mathbf{i}$	$\mathbf{Y}_f^{\mathrm{T}}\mathbf{i}$	$\eta_{E,fu}$

Fig. 10. Final-to-useful final demand sector efficiencies for the ECC of Fig. 7. $\mathbf{Y}_u^{\mathrm{T}}\mathbf{i}$ and $\mathbf{Y}_f^{\mathrm{T}}\mathbf{i}$ in ktoe/year. $\eta_{E,fu}$ is unitless.

3.3. Human needs and well-being: How much primary energy is required to provide energy services?

In general, purchasers of final energy are not interested in energy, per se. Rather, they are interested in the services that useful energy (when combined with infrastructure) provides. Indeed, energy services are desired because they contribute to human well-being by satisfying human needs such as subsistence, protection, participation, leisure, and freedom (see Fig. 1).

But final demand (whether expressed as final energy, useful energy, or energy services) contains "embodied" primary energy: the sum of all primary energy consumed and wasted throughout the ECC in the process of satisfying that final demand. And the ratio of final demand level to embodied primary energy is the consumption-based energy efficiency of meeting that final demand. When the ECC is extended through to energy services, this efficiency is important, because (a) primary-to-services efficiency is a factor in determining the primary energy requirements of providing energy services and (b) primary energy consumption is a proxy for environmental degradation and resource depletion. So an important question in sub-fields of human well-being and development is *What is the consumption-based primary-to-services efficiency of providing energy services?*

Significance: If we want to provide human well-being with minimal environmental impact, consumption-based primary-to-services energy efficiency is an important metric to monitor.

To illustrate the utility of the PSUT framework to comprehensively address these issues, we extend the ECC of Fig. 7 through to services, including final demand for Space heating, Illumination, Passenger transport, and Freight transport. Several intermediate industries now consume energy services rather than useful energy as they did in Fig. 7 (e.g., distribution industries consume Freight transport instead of MD–Truck engines). (Note that the ECC of Fig. 11 is reproduced in Fig. B.1 with exergy quantification of energy carriers.)

To assess the efficiency of providing an energy service, the embodied primary energy of that energy service is needed. The embodied primary energy of a final demand service is similar to a CO_2 [111] or material [112] "footprint." Just as a material footprint is the quantity of material consumed by all industries in the production chain to make a good (e.g., automobiles), so also embodied primary energy is the primary energy consumed by all industries in the ECC to provide an energy service (e.g., Passenger transport). And just as footprinting analysis is conducted with environmentally-extended input-output (EEIO) techniques developed for supply chains quantified in monetary units [113,114], calculation of embodied primary energy within the PSUT framework applies EEIO techniques to ECCs quantified in physical units.

In EEIO analysis, a diagonal matrix ($\hat{\mathbf{e}}$) is formed from a per-unit-output vector of industry ancillary products ($\underset{i\times 1}{\mathbf{e}}$) and pre-multiplied into $\mathbf{G}$ to obtain a "footprint" matrix ($\underset{i\times p}{\mathbf{Q}}$).

$$\mathbf{Q} = \hat{\mathbf{e}}\mathbf{G} = \hat{\mathbf{e}}\mathbf{D}(\mathbf{I}-\mathbf{A})^{-1}\hat{\mathbf{y}} \tag{13}$$

Extending EEIO analysis from supply chains in monetary units to ECCs in energy and energy services units in the context of the PSUT framework, we see that the choice of $\mathbf{e}$ determines the embodied product (energy carrier or service) obtained from Eq. (13).

The starting point for forming any number of $\mathbf{e}$ vectors is the value added matrix ($\mathbf{W}$), because its entries give the production (positive values) and consumption (negative values) of energy carriers and services by industry within the ECC. Matrix $\underset{p\times i}{\mathbf{E}}$ is formed from $\mathbf{W}$, and its rows give energy carriers or services produced (positive values) or consumed (negative values) per unit output by industries (in columns).

$$\mathbf{E} = (\mathbf{W} + \mathbf{U}_{EIOU})\hat{\mathbf{g}}^{-1} \tag{14}$$

Thus, any product row P of $\mathbf{E}$ ($\mathbf{e}_P^{\mathrm{T}}$) can serve as an appropriate $\mathbf{e}$ vector for Eq. (13):

$$\mathbf{Q}_P = \widehat{\mathbf{e}_P^{\mathrm{T}}}\mathbf{G}. \tag{15}$$

The matrix $\mathbf{Q}_P$ contains positive and/or negative entries. Positive entries in $\mathbf{Q}_P$ give the "footprint" of P embodied in the product of the j^{th} column of $\mathbf{Q}_P$ produced by the industry of the i^{th} row of $\mathbf{Q}_P$. Negative entries in $\mathbf{Q}_P$ show the consumption of P embodied in the product of the j^{th} column of $\mathbf{Q}_P$ by the industry of the i^{th} row of $\mathbf{Q}_P$.

Fig. 13 shows $\mathbf{Q}_{Crude}$ and $\mathbf{Q}_{NG}$ matrices for the ECC of Fig. 11. Using $\mathbf{Q}_{Crude}$ as an example, we see that the embodied Crude oil in Passenger transport is 31,998 ktoe/year. Because the entry is in the top row of $\mathbf{Q}_{Crude}$, we know that the embodied Crude was produced by the Resources–Crude industry. Lesser, but still nonzero, amounts of Crude oil are embodied in Freight transport (17,736 ktoe/year), Illumination (102.2 ktoe/year), and Space heating (164.3 ktoe/year), due to interactions among sectors of the ECC. The amount of Crude oil embodied in *all* final demand products (the sum of all positive entries in $\mathbf{Q}_{Crude}$) is 50,000 ktoe/year, the direct production of Crude by Resources–Crude (see Fig. 11).

Any product created in the ECC, not just primary energy carriers, can be analyzed like Crude oil and Natural gas above. Another interesting example for the ECC of Fig. 11 is Freight transport, for which $\mathbf{Q}_{Freight}$ is shown in Fig. 14. There is some amount of Freight transport created by Trucks embodied in all energy services (bottom row of Fig. 14), again because of interactions among the industries in the ECC. Space heating, for example, embodies 1×10^9 tonne-km/year of Freight transport that was produced by Trucks. The sum of the bottom row in $\mathbf{Q}_{Freight}$ is 1.5×10^{11} tonne-km/year, the gross production of Freight transport.

With embodied primary energy in hand, the consumption-based primary-to-services efficiency of providing an energy service can be determined by dividing the magnitude of an energy service by the embodied primary energy for that energy service (summed across all primary energy carriers). For the example of Passenger transport, we obtain

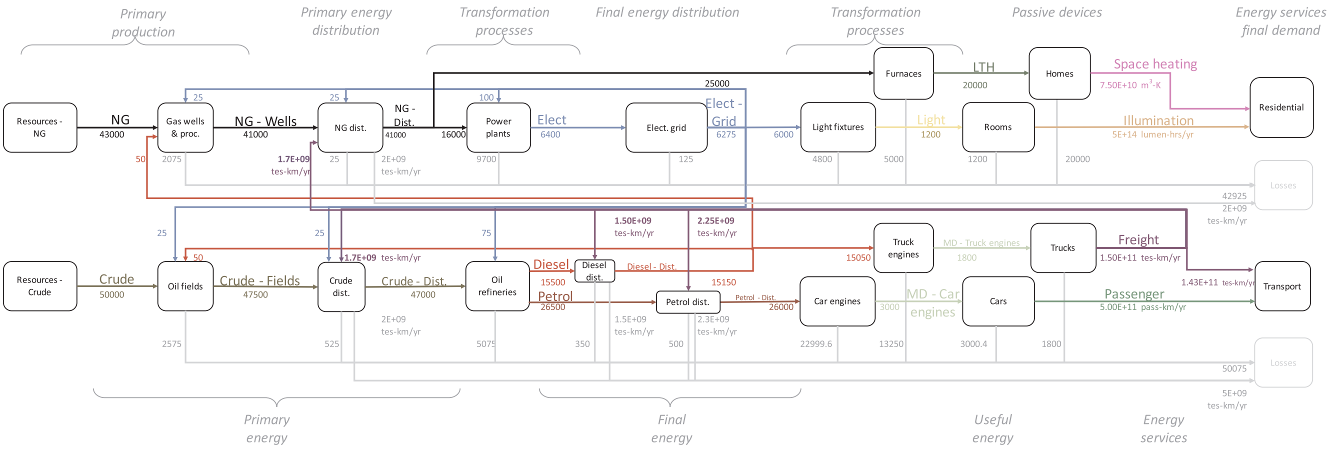
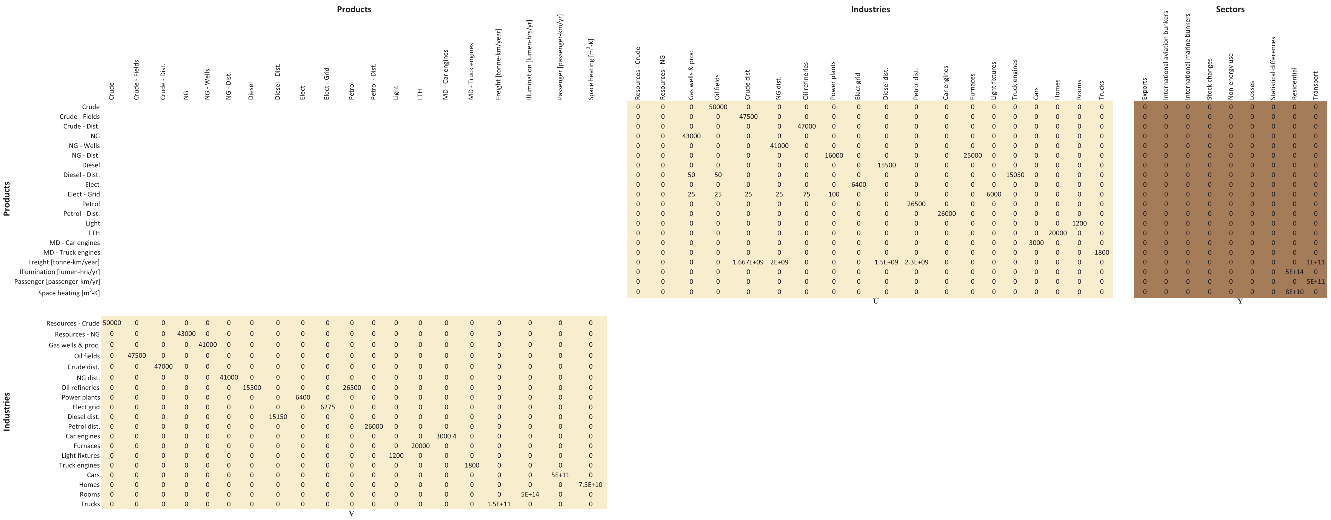


Fig. 11. A real-world ECC covering primary energy to energy services. All energy flows in units of ktoe/year; energy services in units shown. NG is Natural gas. LTH is Low-temperature heat. MD is Mechanical drive. “tes” is an abbreviation for metric tonnes. Line colors indicate products.



Products

Industries

Sectors

	Resources - Crude	Resources - NG	Gas wells & proc.	Oil fields	Crude dist.	NG dist.	Oil refineries	Power plants	Elect grid	Diesel dist.	Petrol dist.	Car engines	Furnaces	Light fixtures	Truck engines	Cars	Homes	Rooms	Trucks
Crude	0	0	0	50000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Crude - Fields	0	0	0	0	47500	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Crude - Dist.	0	0	0	0	0	0	47000	0	0	0	0	0	0	0	0	0	0	0	0
NG	0	0	43000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NG - Wells	0	0	0	0	0	41000	0	0	0	0	0	0	0	0	0	0	0	0	0
NG - Dist.	0	0	0	0	0	0	0	16000	0	0	0	0	25000	0	0	0	0	0	0
Diesel	0	0	0	0	0	0	0	0	0	15500	0	0	0	0	0	0	0	0	0
Diesel - Dist.	0	0	50	50	0	0	0	0	0	0	0	0	0	0	15050	0	0	0	0
Elect	0	0	0	0	0	0	0	0	6400	0	0	0	0	0	0	0	0	0	0
Elect - Grid	0	0	25	25	25	25	75	100	0	0	0	0	0	6000	0	0	0	0	0
Petrol	0	0	0	0	0	0	0	0	0	0	26500	0	0	0	0	0	0	0	0
Petrol - Dist.	0	0	0	0	0	0	0	0	0	0	0	26000	0	0	0	0	0	0	0
Light	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1200	0
LTH	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	20000	0	0
MD - Car engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3000	0	0	0
MD - Truck engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1800
Freight [tonne-km/year]	0	0	0	0	1.667E+09	2E+09	0	0	0	1.5E+09	2.3E+09	0	0	0	0	0	0	0	0
Illumination [lumen-hrs/yr]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Passenger [passenger-km/yr]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Space heating [m^3-K]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

U

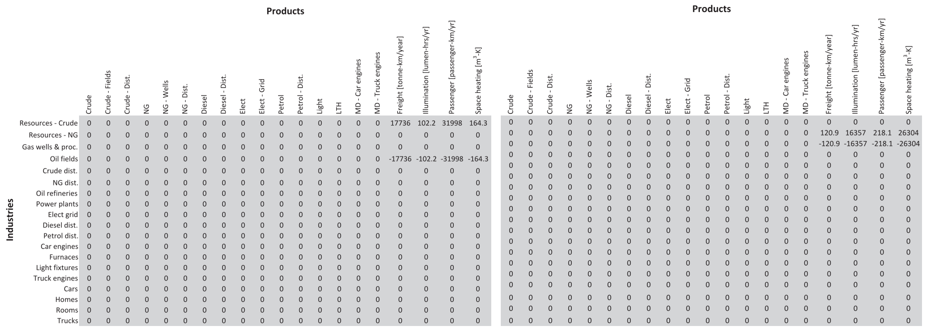
	Exports	International aviation bunkers	International marine bunkers	Stock changes	Non-energy use	Losses	Statistical differences	Residential	Transport
Crude	0	0	0	0	0	0	0	0	0
Crude - Fields	0	0	0	0	0	0	0	0	0
Crude - Dist.	0	0	0	0	0	0	0	0	0
NG	0	0	0	0	0	0	0	0	0
NG - Wells	0	0	0	0	0	0	0	0	0
NG - Dist.	0	0	0	0	0	0	0	0	0
Diesel	0	0	0	0	0	0	0	0	0
Diesel - Dist.	0	0	0	0	0	0	0	0	0
Elect	0	0	0	0	0	0	0	0	0
Elect - Grid	0	0	0	0	0	0	0	0	0
Petrol	0	0	0	0	0	0	0	0	0
Petrol - Dist.	0	0	0	0	0	0	0	0	0
Light	0	0	0	0	0	0	0	0	0
LTH	0	0	0	0	0	0	0	0	0
MD - Car engines	0	0	0	0	0	0	0	0	0
MD - Truck engines	0	0	0	0	0	0	0	0	0
Freight [tonne-km/year]	0	0	0	0	0	0	0	0	1E+11
Illumination [lumen-hrs/yr]	0	0	0	0	0	0	0	5E+14	0
Passenger [passenger-km/yr]	0	0	0	0	0	0	0	0	5E+11
Space heating [m^3-K]	0	0	0	0	0	0	0	8E+10	0

Y

	Crude	Crude - Fields	Crude - Dist.	NG	NG - Wells	NG - Dist.	Diesel	Diesel - Dist.	Elect	Elect - Grid	Petrol	Petrol - Dist.	Light	LTH	MD - Car engines	MD - Truck engines	Freight [tonne-km/year]	Illumination [lumen-hrs/yr]	Passenger [passenger-km/yr]	Space heating [m^3-K]
Resources - Crude	50000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Resources - NG	0	0	0	43000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Gas wells & proc.	0	0	0	0	41000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Oil fields	0	47500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Crude dist.	0	0	47000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NG dist.	0	0	0	0	0	41000	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Oil refineries	0	0	0	0	0	0	15500	0	0	0	26500	0	0	0	0	0	0	0	0	0
Power plants	0	0	0	0	0	0	0	0	6400	0	0	0	0	0	0	0	0	0	0	0
Elect grid	0	0	0	0	0	0	0	0	0	6275	0	0	0	0	0	0	0	0	0	0
Diesel dist.	0	0	0	0	0	0	0	15150	0	0	0	0	0	0	0	0	0	0	0	0
Petrol dist.	0	0	0	0	0	0	0	0	0	0	0	26000	0	0	0	0	0	0	0	0
Car engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3000.4	0	0	0	0	0
Furnaces	0	0	0	0	0	0	0	0	0	0	0	0	0	20000	0	0	0	0	0	0
Light fixtures	0	0	0	0	0	0	0	0	0	0	0	0	1200	0	0	0	0	0	0	0
Truck engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1800	0	0	0	0
Cars	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5E+11	0
Homes	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7.5E+10
Rooms	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5E+14	0	0
Trucks	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.5E+11	0	0	0

V

Fig. 12. PSUT matrices for the real-world ECC in Fig. 11. All energy flows in units of ktoe/year; energy services in units shown.



Products

Industries

	Crude	Crude - Fields	Crude - Dist.	NG	NG - Wells	NG - Dist.	Diesel	Diesel - Dist.	Elect	Elect - Grid	Petrol	Petrol - Dist.	Light	LTH	MD - Car engines	MD - Truck engines	Freight [tonne-km/year]	Illumination [lumen-hrs/yr]	Passenger [passenger-km/yr]	Space heating [m^3-K]
Resources - Crude	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	17736	102.2	31998	164.3
Resources - NG	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Gas wells & proc.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Oil fields	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-17736	-102.2	-31998	-164.3
Crude dist.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NG dist.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Oil refineries	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Power plants	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Elect grid	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Diesel dist.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Petrol dist.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Car engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Furnaces	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Light fixtures	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Truck engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Cars	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Homes	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Rooms	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Trucks	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(a) $\mathbf{Q}_{Crude}$

Products

	Crude	Crude - Fields	Crude - Dist.	NG	NG - Wells	NG - Dist.	Diesel	Diesel - Dist.	Elect	Elect - Grid	Petrol	Petrol - Dist.	Light	LTH	MD - Car engines	MD - Truck engines	Freight [tonne-km/year]	Illumination [lumen-hrs/yr]	Passenger [passenger-km/yr]	Space heating [m^3-K]
Resources - Crude	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Resources - NG	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	120.9	16357	218.1	26304
Gas wells & proc.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-120.9	-16357	-218.1	-26304
Oil fields	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Crude dist.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NG dist.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Oil refineries	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Power plants	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Elect grid	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Diesel dist.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Petrol dist.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Car engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Furnaces	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Light fixtures	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Truck engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Cars	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Homes	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Rooms	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Trucks	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(b) $\mathbf{Q}_{NG}$

Fig. 13. **Q** matrices for embodied Crude oil and Natural gas (NG) for the ECC in Fig. 11.

Products

Industries	Crude	Crude - Fields	Crude - Dist.	NG	NG - Wells	NG - Dist.	Diesel	Diesel - Dist.	Elect	Elect - Grid	Petrol	Petrol - Dist.	Light	LTH	MD - Car engines	MD - Truck engines	Freight [tonne-km/year]	Illumination [lumen-hrs/yr]	Passenger [passenger-km/yr]	Space heating [m^3-K]
Resources - Crude	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Resources - NG	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Gas wells & proc.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Oil fields	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Crude dist.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-6E+08	-3E+06	-1E+09	-5E+06
NG dist.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-5E+06	-6E+08	-8E+06	-1E+09
Oil refineries	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Power plants	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Elect grid	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Diesel dist.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1E+09	-8E+06	-4E+07	-1E+07
Petrol dist.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-7E-08	-2E+09	0
Car engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Furnaces	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Light fixtures	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Truck engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Cars	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Homes	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Rooms	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Trucks	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1E+11	6E+08	3E+09	1E+09

Fig. 14. $\mathbf{Q}_{Freight}$ matrix for embodied Freight transport for the ECC in Fig. 11.

$$\eta_{E,ps} = \frac{5 \times 10^{11}\,\text{pass-km/year}}{31,\,998\,\text{ktoe/year} + 218.07\,\text{ktoe/year}} = 1.55 \times 10^{7}\,\text{pass-km/ktoe}. \tag{16}$$

When each primary energy carrier is produced by a single Resources industry (as in Fig. 11), nonzero entries in the $\widehat{\mathbf{e}_P}$ and $\mathbf{D}$ matrices will be 1 (see Appendix G), and a vector of consumption-based primary-to-services energy efficiencies ($\boldsymbol{\eta}_{E,ps}$) can be obtained directly with

$$\boldsymbol{\eta}_{E,ps} = (\widehat{\mathbf{G}^{\mathrm{T}}\mathbf{s}_r})^{-1}\mathbf{y}. \tag{17}$$

Fig. 15 shows consumption-based primary-to-services energy efficiencies ($\boldsymbol{\eta}_{E,ps}$) for the real-world ECC in Fig. 11.

We note that consumption-based primary-to-services exergetic efficiencies (as shown in Fig. 15) account for all direct and indirect primary energy demanded by each service (the embodied energy of the service). The provision of Passenger transport to final demand provides an illustrative example.

For a narrow analysis boundary around automobiles and the service they provide, the final-to-services energy efficiency of Passenger transport is

$$\eta_{E,fs} = \frac{5 \times 10^{11}\,\text{passenger-km/yr}}{26,\,000\,\text{ktoe/yr}} = 1.92 \times 10^{7}\,\text{passenger-km/ktoe}, \tag{18}$$

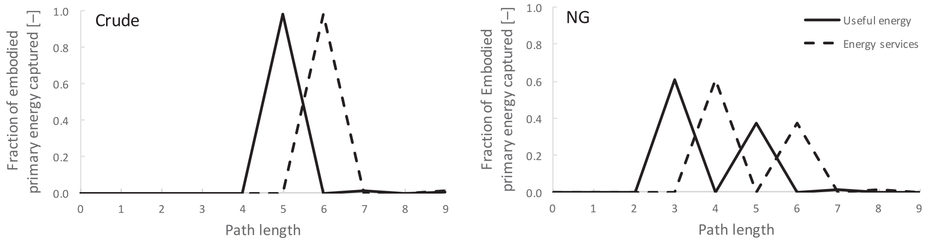
where the numerator is the service level provided by Cars (Passenger transport) and the denominator is the final energy consumed by Car engines (Petrol). However, the consumption-based primary-to-services energy efficiency of Passenger transport in Fig. 15 was obtained from an expanded analysis boundary (made possible by the PSUT framework), which accounts for all energy consumption in the ECC to provide Passenger transport by cars, including (a) Crude required to supply self-consumption of Petrol and Diesel and (b) Natural gas required to make electricity. With the wider analysis boundary, we find the consumption-based primary-to-services energy efficiency of providing Passenger transport to be 19% less: 1.55×10^{7} passenger-km/ktoe, as shown in Fig. 15.

The final-to-services energy efficiency of Passenger transport is, essentially, an expression of the fleet-average fuel efficiency of automobiles. We can cast the above results into familiar fuel economy units (miles per U.S. gallon) by assuming 0.13176 GJ of energy per U.S. gallon of Petrol and 1.5 passenger-miles per car-mile. Doing so, we obtain 25.1 car-miles/U.S. gallon as the average fuel economy of the UK automobile fleet circa 2000 for the narrow analysis boundary. (Bonilla [114, Fig. 2a] shows fleet average fuel economy of about 10 litres/100 km or 23.5 miles/U.S. gallon, indicating that the rounded data in our real-world ECC are close to reality and that our estimate of 1.5 passenger-miles/car-mile is reasonable.) Accounting for all indirect energy consumption along the ECC (expressed in Petrol gallon equivalents), we obtain 20.2 car-miles/U.S. gallon for the expanded analysis boundary, again 19% less than the fuel economy obtained from the narrow boundary.

As discussed above, the ECCs of Figs. 3, 7, and 11 are meant to be representative, as they contain only two of the many energy pathways in the UK economy in 2000. Each additional energy pathway included in this analysis will further reduce the final-to-services energy efficiency of Passenger transport. Expanding the analysis boundary to include the

Products		Resources - Crude	Resources - NG			
Crude	0	0	0	0	N/A	
Crude - Fields	0	0	0	0	N/A	
Crude - Dist.	0	0	0	0	N/A	
NG	0	0	0	0	N/A	
NG - Wells	0	0	0	0	N/A	
NG - Dist.	0	0	0	0	N/A	
Diesel	0	0	0	0	N/A	
Diesel - Dist.	0	0	0	0	N/A	
Elect	0	0	0	0	N/A	
Elect - Grid	0	0	0	0	N/A	
Petrol	0	0	0	0	N/A	
Petrol - Dist.	0	0	0	0	N/A	
Light	0	0	0	0	N/A	
LTH	0	0	0	0	N/A	
MD - Car engines	0	0	0	0	N/A	
MD - Truck engines	0	0	0	0	N/A	
Freight [tonne-km/yr]	1.43E+11	17736	120.87	17857	8.00E+06	tonnes-km/ktoe
Illumination [lumen-hrs/yr]	5E+14	102.2	16357	16460	3.04E+10	lumen-hrs/ktoe
Passenger [passenger-km/yr]	5E+11	31998	218.07	32216	1.55E+07	pass-km/ktoe
Space heating [m^3-K]	7.5E+10	164.35	26304	26468	2.83E+06	m^3-K/(ktoe/yr)
	$\mathbf{y}$	$\mathbf{G}^{\mathrm{T}}$		$\mathbf{G}^{\mathrm{T}}\mathbf{s}_r$	$\boldsymbol{\eta}_{E,ps}$	

Fig. 15. Consumption-based primary-to-services energetic efficiencies ($\boldsymbol{\eta}_{E,ps}$) for the ECC in Figs. 11 and 12. For brevity, this figure shows only the Resources industries of $\mathbf{G}$.



(a) Crude paths. (b) Natural gas paths.

Fig. 16. Fraction of embodied Crude and Natural gas energy captured by paths of varying lengths in the useful energy ECC of Fig. 7 (solid line) and the energy services ECC of Fig. 11 (dashed line).

embodied energy of materials in the automobile (the Ω boundary of Brandt et al. [108]) will both (a) further increase the embodied energy content of cars and (b) further reduce $\eta_{E,fs}$ and $\eta_{E,ps}$.

Benefit of the PSUT framework: This passenger transport example demonstrates that when data are organized into PSUT matrices, a picture of the embodied primary energy of an energy service (exclusive of the embodied energy of materials) can be obtained quickly and easily.

3.4. Structural path analysis: What are the key supply-chain paths through the ECC for delivering energy services?

We showed that embodied primary energy of final demand can be determined within the PSUT framework in Section 3.3. But as we evaluate strategies for reducing embodied energy, an important question emerges: *What are the critical supply chains involved in energy and services delivery to final demand?*

Significance: One approach to reducing environmental impacts of economic activity is to minimize the embodied primary energy of final demand as energy moves through the ECC.

To address this question, one needs to trace the large number of pathways for delivering energy through an ECC. Structural path analysis (SPA) [116,117] is an established IO technique that uses the Taylor series expansion [118] to "unravel" the Leontief inverse ($\underset{p\times p}{\mathbf{L}}$) and identify and quantify individual paths through a supply chain. SPA can be used within the PSUT framework to assess paths from resource extraction to final demand expressed in any form, including final energy (Fig. 3), useful energy (Fig. 7), or energy services (Fig. 11).

SPA provides two important results within the PSUT framework: (a) the lengths of paths from primary resources through the ECC to final demand and (b) the embodied primary energy of each path. The length of an ECC path is defined as the number of ECC industries through which energy or an energy service flows before reaching final demand. A zero length path is one where energy flows directly from resource extraction to final demand; a path of length 1 has a single industry between resources and final demand; etc. For simple supply chains, path lengths can be determined by inspection, but complex supply chains in real-world ECCs have far too many paths for each to be identified visually. Although the ECCs in this paper are increasingly complex (compare Figs. 3 and 11), it is obvious by inspection that there are no paths of length 0 or 1. For example, the *shortest* energy service delivery path in Fig. 11 has length 4, traversing from Natural gas through (1) Gas wells and processing to (2) Natural Gas distribution to (3) Furnaces to (4) Homes and, ultimately, to the Residential sector of final demand.

The magnitude of an ECC path is defined as the embodied primary energy of the service delivered by the path. For the real-world ECCs in this paper, ECC path magnitudes are measured in ktoe/year.

Calculations of path lengths rely on the Taylor series expansion of the Leontief inverse matrix. For the symmetric Leontief inverse matrix $\underset{p\times p}{\mathbf{L}}$, it can be shown [118] that

$$\underset{p\times p}{\mathbf{L}} = (\mathbf{I}-\mathbf{A})^{-1} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \mathbf{A}^4 + \cdots + \mathbf{A}^n + \cdots, \tag{19}$$

where n is the number of terms retained for a finite approximation to the infinite sum.

If the right side of Eq. (19) represents the ECC (instead of $\underset{p\times p}{\mathbf{L}}$ or $(\mathbf{I}-\mathbf{A})^{-1}$), paths of various lengths are found in matrices with corresponding powers of $\mathbf{A}$. For example, zero length paths are associated with $\mathbf{I}$, and the shortest path in Fig. 11 (length 4) would be associated with the $\mathbf{A}^4$ term of Eq. (19). (Additional details are provided in Appendix G.)

To demonstrate SPA within the PSUT framework, we perform five separate analyses, one for each combination of primary energy resource (Crude and Natural Gas) and ECC (Figs. 7 and 11) and one for Freight transport in the ECC of Fig. 11. (Note that Figs. 7 and 11 involve different final demand matrices ($\mathbf{Y}$): the final demand matrix of Fig. 7 is comprised of useful energy and the final demand matrix of Fig. 11 is comprised of energy services. SPA works with both types of final demand matrices within the PSUT framework. An SPA could also be performed with the ECC of Fig. 3, but we focus on Figs. 7 and 11 for simplicity.) All paths in all five analyses are evaluated for both length (the number of steps) and magnitude (embodied primary energy).

We first aggregate the magnitudes of all same-length paths originating at primary energy carriers to create Fig. 16, which shows aggregated magnitudes (as a fraction of total embodied primary energy) on the vertical axis and path lengths (from 0 to 9) on the horizontal axis. Nearly all (98%) of embodied Crude energy takes five steps to reach final demand expressed as useful energy (solid line in Fig. 16a). To reach final demand expressed as energy service, nearly all the embodied Crude energy takes six steps (dashed line in Fig. 16a). A review of Figs. 7 and 11 confirms that the energy service ECC (Fig. 11) has one additional stage compared to the useful energy ECC (Fig. 7). Indeed, inspection of Fig. 7 shows that the simplest path from Crude to final demand expressed as useful energy takes five steps: from Resources-Crude to (1) Oil fields to (2) Crude dist. to (3) Oil refineries to (4) Diesel or Petrol dist. to (5) Truck or Car engines to Transport. And inspection of Fig. 11 shows that the simplest path from Crude to final demand expressed as energy services takes six steps: from Resources-Crude to (1) Oil fields to (2) Crude dist. to (3) Oil refineries to (4) Diesel or Petrol dist. to (5) Truck or Car engines to (6) Trucks or Cars to Freight or Passenger Transport. Appendix G provides additional details of the path from Crude to Freight transport.

Paths from Natural gas extraction are slightly more complex. 61% of the embodied Natural gas takes three steps to reach final demand expressed as useful energy with a further 37% taking five steps (solid line in Fig. 16b). Again, the paths to final demand expressed as energy services (dashed line in Fig. 16b) are one step longer (four and six steps).

With the help of Figs. 7 and 11, it is possible to use Fig. 16 to interpret the primary paths through the real-world ECC. However, the embodied energy of more complex paths (e.g., paths which include

Table 7
Top 10 largest magnitude paths from Natural gas to final demand expressed as energy services in Fig. 11.

Size rank	Step 0	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6	Step 7	Step 8	Step 9	Step 10	Path magnitude [ktoe/yr]
1	Resources - NG	Gas wells & proc.	NG dist.	Furnaces	Homes							26,220
2	Resources - NG	Gas wells & proc.	NG dist.	Power plants	Elect. grid	Light fixtures	Rooms - Illum					16,045
3	Resources - NG	Gas wells & proc.	NG dist.	Power plants	Elect. grid	Power plants	Elect grid	Light fixtures	Rooms - Illum			256
4	Resources - NG	Gas wells & proc.	NG dist.	Power plants	Elect. grid	Oil refineries	Petrol dist.	Car engines	Cars - Pass trnsp			127
5	Resources - NG	Gas wells & proc.	NG dist.	Power plants	Elect. grid	Oil refineries	Diesel dist.	Truck engines	Trucks - Freight			70
6	Resources - NG	Gas wells & proc.	NG dist.	Power plants	Elect. grid	Oil fields	Crude dist.	Oil refineries	Petrol dist.	Car engines	Cars - Pass trnsp	42
7	Resources - NG	Gas wells & proc.	NG dist.	Power plants	Elect. grid	Crude dist.	Oil refineries	Petrol dist.	Car engines	Cars - Pass trnsp		42
8	Resources - NG	Gas wells & proc.	NG dist.	Power plants	Elect. grid	NG dist.	Furnaces	Homes - Space htg				41
9	Resources - NG	Gas wells & proc.	NG dist.	Power plants	Elect. grid	Gas wells & proc.	NG dist.	Furnaces	Homes - Space htg			41
10	Resources - NG	Gas wells & proc.	NG dist.	Power plants	Elect. grid	NG dist.	Power plants	Elect grid	Light fixtures	Rooms - Illum		25

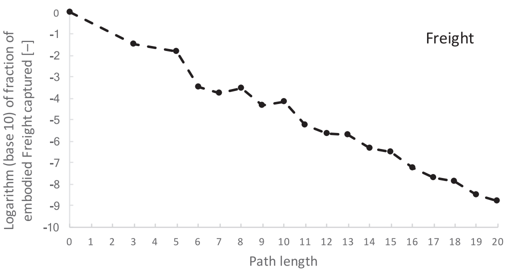


Fig. 17. Fraction of embodied Freight captured by paths of varying lengths in the energy services ECC of Fig. 11, expressed in $\log_{10}$ such that, e.g., −2 on the vertical axis is 10^{-2} or 1% of all embodied Freight transport.

Electricity inputs to Gas wells & proc.) cannot be identified by inspection.

SPA provides an additional method to investigate details of specific paths within the ECCs. To do so, the Leontief inverse ($\underset{p\times p}{\mathbf{L}}$) in Eq. (19) is expanded again such that the path from industry i to industry j via industry k is described by matrix elements A_{ik} and A_{kj}. With the doubly-expanded form of the Leontief inverse, it is possible to identify and rank the most important (largest magnitude) paths through the ECC (see Appendix G for additional details).

To illustrate the capability of SPA to identify paths within an ECC, we show results for the most interesting combination of resource and ECC, namely Natural gas through to energy services in Fig. 11 (the dashed line in Fig. 16b). The 10 paths of largest magnitude are shown in Table 7, comprising 99.8% of all embodied Natural gas in the ECC of Fig. 11. The largest magnitude path has length 4: from Resources-NG to (1) Gas wells & proc. to (2) NG dist. to (3) Furnaces to (4) Homes to Residential final demand. The second-largest path is a six-step path and the shortest path that provides Illumination: from Resources-NG to (1) Gas wells & proc. to (2) NG dist. to (3) Power plants to (4) Elect. grid to (5) Light fixtures to (6) Rooms to Residential final demand. The results of Table 7 confirm that the embodied energy captured in 4 and 6 steps (shown by the dashed line in Fig. 16b) comprises two large-magnitude paths only.

However, there are several more-complex routes from Natural gas to final demand through the ECC of Fig. 11. For example, the routes of paths with size rank 2–10 in Table 7 go through Power plants and the Elect. grid to make Electricity available to other portions of the ECC, some of which flows to industries that serve end uses other than Illumination. It would be impossible to find all paths by inspection from Figs. 11 and 16b, and this detailed SPA method is likely the only way to identify the length and magnitude of paths longer than, say, 6 steps.

In addition to primary energy carriers created at the upstream end of an ECC, SPA can be performed on any product created anywhere in the ECC. In the ECC of Fig. 11, Freight transport is created by Trucks and delivered to Transport final demand as well as distribution industries within the ECC. Most (95%) embodied Freight transport reaches Transport final demand directly (0 steps), but some Freight transport is provided to the distribution industries within the ECC. The shortest indirect path through the distribution industries to Transport final demand has length 3: from Trucks to (1) Diesel dist. to (2) Truck engines to (3) Trucks to Transport final demand. (A similar path with length 3 goes through Petrol dist.) Exponentially-decreasing amounts of embodied Freight transport complete this cycle twice, then three times, then four times, etc., each much smaller in magnitude than the last. The semi-log plot in Fig. 17 is the Freight transport version of Fig. 16, and it shows exponentially-decreasing embodied Freight as a (nearly) log-linear descending function of path length.

Benefit of the PSUT framework: This extended example demonstrates that when ECC data are arranged in a PSUT format, quantification of the magnitude of embodied product flows through every route of an ECC can be accomplished using SPA techniques.

4. Discussion

In this discussion, we briefly discuss the originality of this work (Section 4.1), explain limitations of the PSUT framework (Section 4.2), identify several additional applications (Section 4.3), and suggest future work (Section 4.4).

4.1. Originality

To our knowledge, the following elements of this paper are novel advances that appear in the literature for the first time:

(1) We performed energy analysis on an ECC using PSUT matrices comprised of disaggregated products with physical units only. (In Section 3, energy terms are in ktoe/year, energy services terms are in various units such as passenger-km/year. Previous papers mixed financial and physical units or performed analyses with aggregated products in physical units only in IO, not SUT, matrices.)
(2) We demonstrated that (1) could be accomplished with either energy or exergy entries in the PSUT matrices. (Section 3 utilizes energy entries, and Appendix B utilizes exergy entries.)
(3) We showed that the PSUT energy analysis framework could be used anywhere along an ECC. In particular, we showed that the framework could be used when energy services are the last stage of an ECC (Sections 3.3 and 3.4).
(4) We illustrated that changing the last stage of an ECC (from final energy to useful energy to energy services) can provide insights into ECC characteristics (Sections 3.2 and 3.4).
(5) We performed SPA on an ECC (Section 3.4 and Appendix G).
(6) We developed and utilized the $\mathbf{S}_{units}$ matrix to aggregate products with inhomogeneous units in the PSUT matrices (Appendix E).
(7) We derived relationships among the three ERRs: GER, NER, and r (Appendix F).

4.2. Limitations

We suggest two limitations of the PSUT framework. The first arises from the fact that the accuracy and level of detail of analyses performed with the PSUT framework are a function of the accuracy and availability of ECC data. At the primary and final energy stages of an ECC, data are readily available from the IEA [95] and national energy agencies, but they must be applied correctly [119], are they not without measurement errors and inaccuracies [120].

On the other hand, data availability at the useful energy or energy services stages are a challenge. At the useful stage, energy flows must be calculated from (a) estimates of allocation of final energy to end-use devices and (b) estimates of final-to-useful end-use device efficiencies (η_{fu}). Many challenges arise when estimating allocations and device efficiencies. Progress is being made on allocation of IEA final energy data [97,40], and probabilistic models are under development to quantify effects of allocation uncertainty [121]. Estimating time series for final-to-useful device efficiencies (η_{fu}) is time consuming, because economy-wide efficiencies are a function of many factors, including diffusion rates of new technologies, statistical distributions of device vintage, maintenance schedules, etc. All of these factors must be evaluated per device for each economy when estimating time series of device efficiencies (η_{fu}). Fortunately, here, too, progress is being made, and many countries have been analyzed, including the U.S. [40,122], the UK [40,123], the EU-15 [99], China [98], Mexico [124], and Portugal [39].

When pushing through to energy services, some data are readily available (e.g., Freight and Passenger transport) while other data are less available (e.g., Lighting).

However, we note that data limitations are not unique to the PSUT framework: all energy analyses on the ECC face similar challenges. Indeed, the PSUT framework is not a means to obtain or generate final energy or energy services data. Rather, it is a way to organize available data and streamline analyses of that data.

The second limitation arises from the inherent linearity of IO and SUT analysis methods, which are often and rightly criticized for their inability to represent non-linear effects and dynamics related to changes in final demand (see Section 2.2.5). We note here that non-linear effects exist in both (a) the physical realm (e.g., larger buildings are more efficient because heat loss scales with surface area but space heating service scales with volume) and (b) the economic realm (e.g., “economies of scale”). For the purposes of energy analysis, we believe that physical SUT techniques are less problematic than economic SUT techniques, because PSUT techniques avoid purely-economic non-linearities.

That said, we recognize that, at the economy-wide scale, the physical realm and the economic realm may interact in unexpected ways to produce non-linear effects. For example, if demand for electricity decreases, markets may prefer to mothball inefficient plants, thereby increasing the aggregate efficiency of electricity production. The methods of Section 2.2.5 would not predict such efficiency improvements and would instead assume that efficiency remains constant as final demand shifts. (To capture these non-linearities in a predictive sense, dynamic energy-economy models are needed.) However, when annual data for the entire ECC are available (e.g., IEA world energy statistics [95] as discussed in Appendix H), each year can be analyzed independently, and the PSUT framework will correctly observe and calculate year-to-year physical changes in an ECC (e.g., increasing efficiency of electricity production), regardless of their root cause (e.g., economic structural changes or technological efficiency changes). Section 4.3 discusses structural decomposition analysis (SDA) which can be applied to determine the dominant drivers of temporal trends.

4.3. Additional applications

There are many additional applications for the PSUT framework described in Section 2 and demonstrated in Section 3. Most of the additional applications are enabled by the supply-use table structure of the framework. For each additional application discussed below, we include questions that, taken together, illustrate the breadth of applicability of the PSUT framework. Due to space constraints, we do not provide real-world examples.

The PSUT framework could be used to study the question *Which country can provide energy services most efficiently?* To answer this question, a multi-regional PSUT (MR-PSUT) would need to be constructed. A MR-PSUT model would enable the calculation of embodied energy content of energy services consumed anywhere in the world, taking into account global supply chains that cover any energy conversion process in any country.

Further development of the MR-PSUT could involve producing annual tables, allowing the following question to be addressed: *What are the most important drivers of difference of the embodied energy of final demand between (a) two countries at a given time? and (b) for a given country between two times?* This question can be analyzed within the PSUT framework by the application of Structural Decomposition Analysis (SDA). SDA is an “analysis of …change by means of a set of comparative static changes in key parameters in an input-output table” [124, p. 3]. An SDA would be able to determine the importance of the following factors in contributing to country-by-country or year-by-year differences: (a) larger final demand for the energy service (b) the structure of the ECC involved in the delivery of the energy service, and (c) increasing or decreasing waste energy at various stages of the ECC.

Regarding energy services, an important question is *Are energy services being provided more efficiently over time?* Steps to answer this question using the PSUT framework would comprise: (a) gathering ECC time series data (through to energy services), (b) organizing time series data into PSUT matrices, with one set of $\mathbf{U}$, $\mathbf{V}$, and $\mathbf{Y}$ matrices for each time period (typically, one year) as shown in Appendix H, and (c) repeating the analysis of Section 3.3 for each year to obtain a time series of consumption-based primary-to-services energy efficiencies. The evolution of energy service efficiencies will then be obvious when graphed against time.

Turning to economics-related questions, one might want to know *What are the consumption-based energy intensities (in GJ/$) of economic sectors as defined by the system of national accounts?* (This question hearkens back forty years to the works of Bullard et al. [13], Costanza [126], and Roberts, whose 1978 definition of energy analysis was “a systematic way of tracing the flows of energy through an industrial system, resulting in the apportioning of a fraction of the primary energy inputs into the system to each of the outputs of that system” [17, p. 200].) To answer this question, the PSUT matrices should be embedded within a larger mixed units energy-economy SUT analysis that includes financial flows for non-energy sectors. Recent work by Guevara and co-authors [86,90,91] and Rocco and co-authors [85,87,88] has pursued this line of inquiry (see Table 1).

In addition, we speculate that analyses typically performed on individual energy conversion devices in the sub-field of exergoeconomics [127,128] could be applied to the economy-wide ECC boundary. A core question would be: *What is the optimum design of an economy-wide ECC to minimize its costs or its exergy destruction?* To answer this question, analysts would need to (a) obtain or generate efficiency vs. cost relationships for each device in an economy-wide ECC and (b) apply exergoeconomic techniques. Estimates of the cost of exergy destruction by each device in the ECC would be generated, and optimization of the ECC could be pursued for various objective functions, including minimizing exergy destruction or minimizing cost of energy service delivery. An optimal mix of exergy conversion devices could be determined for each objective.

Finally, we note that energy carriers and services change form through the ECC: all primary energy is completely consumed by transformation processes on the way to providing energy services. The PSUT framework is able to track all of these changes of form, even when the transformation is so complete that the quantities involved no longer exist as useable energy but rather as services only.

Of course, energy is not the only resource whose primary resources are “used up” to provide services measured in different units. Thus, we speculate that a version of the PSUT energy analysis framework could be applied to other service delivery networks involving other resource flows. For example, materials of all types (paper and wood [79], steel [129], water [130], the entire economy [112,131]) provide *material services* to society [132,133] and are (at least partially) “used up” in the process. A key question for a materials application of the PSUT framework would be *What is the consumption-based efficiency of providing material services to society?*

4.4. Future work

There are several areas available for future work on this topic. First, because the PSUT framework will be applied to real ECCs (see Hardt et al. [134] for application with LMDI decomposition analysis), efforts to improve the availability and accuracy of data along the ECC are needed, particularly regarding useful energy (see Section 4.2). If analyses are to reach energy services, robust data on services will be required. Therefore, additional work is encouraged on developing a common method to estimate useful energy and publishing databases of useful energy and energy services statistics. (These new directions should build on related efforts to develop consistent societal exergy accounting methods [94,135], to assess the effect of allocation uncertainties in societal exergy accounting [121], and to understand the basic driver of the energy system, end use [136].)

Second, further development of the additional PSUT framework applications described in Section 4.3 should be undertaken and demonstrated via real-world examples, similar to those in Section 3.

Third, development of a generalized mathematical approach for unit inhomogeneity of sector co-products would advance the PSUT framework by removing the requirement of co-product unit homogeneity and the need to split industries whose co-products are unit-inhomogeneous (see Section 2.2.4).

5. Conclusions

In this paper, we have built upon prior work in related fields to develop and demonstrate, via four real-world examples that address contemporary energy analysis questions, a new physical supply-use table energy analysis framework. The framework is applicable to all parts of the energy conversion chain and provides several important benefits to the field of energy analysis.

First, because physical supply-use table matrices can be asymmetric (i.e., non-square), the physical supply-use table framework allows analysis of energy conversion chains that include co-producing industries with disaggregated products. (In the energy conversion chain of Figs. 3, 7, and 11, Oil refineries co-produce Petrol and Diesel. Real refineries make dozens of products.) This characteristic overcomes a limitation of input-output-based methods that require symmetric (i.e., square) matrices, which, in turn, necessitate aggregations that discard energy conversion chain information.

Second, because the physical supply-use table framework allows analysis on the entire energy conversion chain, including co-producing industries, it can overcome communication challenges that may arise when different analysis techniques or different terms are used by different research communities who study different portions of the energy conversion chain.

Next, two advantages arise from units and product quantification. Because the physical supply-use table framework utilizes physical units exclusively, it overcomes a limitation of financial input-output and supply-use table methods in which monetary flows are proxies for physical flows, thereby introducing distortions into what otherwise should be purely physical (energy) analysis. Indeed, one of the main challenges with year-by-year economic, rather than physical, input-output or supply-use table energy decompositions is that effects of inflation must be removed before performing the analysis. If not, the importance of temporal changes in final demand is often exaggerated due to inflationary price increases. The physical supply-use table framework has a significant advantage over economic input-output and supply-use table analyses, because it uses data in physical units rather than economic spending information in monetary units. And because the physical supply-use table framework allows analyses in either energy or exergy quantifications of energy carriers, it can assist answering energy analysis questions posed in either energy or exergy terms.

Finally, we note that the physical supply-use table framework is useable by many sub-fields of energy analysis because it both (a) allows analysis anywhere along the energy conversion chain and (b) is flexible regarding energy quantification. For example, emissions footprinting is conducted with energy quantification and is concerned with extracted fossil fuels (primary energy) at the upstream end of the energy conversion chain, while societal exergy analysis is conducted with exergy quantification and is often concerned with useful exergy and exergy services at the downstream end of the energy conversion chain.

We believe that these advantages commend the physical supply-use table framework to the field of energy analysis. It can provide data structure uniformity and methodological consistency for many sub-fields. (For example, physical supply-use table matrices could complement the energy balance format currently employed by national and international energy agencies.) And, being a common framework, it could organize and streamline questions to be asked, data to be gathered, analyses to be performed, and results to be reported.

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Data Repository

A complete set of input and results datasets for this paper have been deposited at the University of Leeds Data Repository at https://doi.org/10.5518/393.

Appendix A. Nomenclature

We employ several symbol conventions in this paper. Boldface capital letters (e.g., $\mathbf{U}$) represent matrices. Boldface lowercase letters (e.g., $\mathbf{g}$) identify column vectors. (All vectors are assumed to be column vectors.) Symbols for PSUT matrices and vectors mostly follow Eurostat naming conventions [93, pp. 349–350]. Table A.1 lists the nomenclature for this paper.

Table A.1
Nomenclature.

Symbol	Description
E	Energy quantities
m	Summation index for infinite series
n	Number of terms to be retained in an infinite series
r	Net-to-gross energy ratio
X	Exergy quantities
Acronyms/abbreviations	
ECC	Energy conversion chain
EEIO	Environmentally-extended input-output
EIOU	Energy industry own use
EROI	Energy return on energy investment
ERR	Energy return ratio
EU	European Union
GER	Gross energy ratio
IO	Input-output
LMDI	Log-mean divisia index
LTH	Low-temperature heat
MD	Mechanical drive
NER	Net energy ratio
NG	Natural gas (primarily methane, CH_4)
PSUT	Physical supply-use table
SDA	Structural decomposition analysis
SPA	Structural path analysis
SUT	Supply-use table
UK	United Kingdom
U.S.	United States
Greek	
γ	The γ system boundary of Brandt et al. [108]
Ω	The Ω system boundary of Brandt et al. [108]
ϕ	Exergy-to-energy ratio at a point in the ECC
η	Efficiency
Subscripts	
Crude	Pertains to Crude oil, a primary energy carrier
E	Pertains to energy
EROI	Energy return on energy investment
f	Pertains to final stage of the ECC
Freight	Pertains to Freight transport, an energy service
fu	Pertains to final-to-useful conversion devices
i	Matrix row or column index; also step along an ECC path
j	Matrix row or column index
k	Matrix row or column index
NG	Pertains to Natural gas, a primary energy carrier
Oil	Pertains to Oil and oil products
P	Pertains to a product; also a row index for $\mathbf{E}$
p	Pertains to primary stage of the ECC
pf	Pertains to primary-to-final conversion devices
pu	Spans the primary-to-useful stages of the ECC
r	Pertains to Resources industries
s	Pertains to energy services stage of the ECC
u	Pertains to useful stage of the ECC
us	Pertains to useful-to-services passive devices
X	Pertains to exergy
$-$	Pertains to negative elements

(*continued on next page*)

Table A.1 (*continued*)

Symbol	Description
$+$	Pertains to positive elements
γ	Pertains to the γ energy return ratio system boundary
Superscripts	
-1	Denotes square matrix inverse
T	Denotes transpose of a vector or matrix
$'$	Denotes a new version of a vector or matrix
Subannotations	
i	Denotes industries (Table 2)
p	Denotes products (Table 2)
s	Denotes final demand sectors (Table 2)
u	Denotes units of products (Table 2)
Superannotations	
$\hat{\mathbf{v}}$	Denotes a square diagonal matrix formed by placing the elements of $\mathbf{v}$ on the diagonal of $\mathbf{I}$
$\overline{\mathbf{M}}$	Denotes collapse by summation over like units in $\mathbf{M}$ (E)
Column vectors	
$\mathbf{e}$	Vector formed from a single row of $\mathbf{E}$ (i×1)
$\mathbf{g}$	Total industry output (i×1)
$\mathbf{ger}_\gamma$	Gross energy ratios for the γ system boundary (i×1)
$\mathbf{i}$	Identity column vector ($\mathbf{i}^\mathrm{T}$ is the identity row vector)
$\mathbf{ner}_\gamma$	Net energy ratios for the γ system boundary (i×1)
$\mathbf{q}$	Total product output (p×1)
$\mathbf{r}_\gamma$	Net-to-gross energy ratios for the γ system boundary (i×1)
$\mathbf{y}$	Row sums of $\mathbf{Y}$ (p×1)
$\mathbf{0}$	Zero vector
$\boldsymbol{\eta}$	Efficiencies
Summation vectors	
$\mathbf{s}_f$	Logical inverse of $\mathbf{s}_p$: 0's for primary industries, 1's elsewhere
$\mathbf{s}_p$	1's for primary industries (Resources, Imports, Exports International aviation and marine bunkers, and Stock changes) 0's elsewhere
$\mathbf{s}_r$	1's for resource industries, 0's elsewhere
$\mathbf{s}_-$	1's for negative elements, 0's elsewhere
$\mathbf{s}_+$	1's for positive elements, 0's elsewhere
Matrices	
$\mathbf{A}$	Input coefficients for intermediate products (p×p)
$\mathbf{C}$	Product mix matrix (p×i)
$\mathbf{D}$	Market shares matrix (i×p)
$\mathbf{E}$	Waste per unit of industry output (p×i)
$\mathbf{G}$	Industry output requirements for final demand (i×p)
$\mathbf{I}$	Identity matrix (1's on diagonal, 0's elsewhere)
$\underset{\mathrm{i\times p}}{\mathbf{L}}$	Industry-by-product Leontief inverse matrix (i×p)
$\underset{\mathrm{p\times p}}{\mathbf{L}}$	Product-by-product Leontief inverse matrix (p×p)
$\mathbf{Q}$	Footprint matrix (i×p)
$\mathbf{U}$	Use matrix (p×i)
$\mathbf{U}_{EIOU}$	Energy industry own use portion of the $\mathbf{U}$ matrix
$\mathbf{V}$	Make matrix (i×p)
$\mathbf{W}$	Value added matrix (p×i)
$\mathbf{Y}$	Final demand matrix (p×s)
$\mathbf{Z}$	Input requirements per unit of industry output (p×i)
$\mathbf{0}$	Zero matrix
Summation matrices	
$\mathbf{S}_{units}$	Summation matrix for unit manipulation (p×u)

Appendix B. Exergy quantification in the PSUT framework

As discussed in Section 1.1, the sub-field of societal exergy analysis has informed discussions of the role of energy in society and the economy in recent years [85,96,137]. Analyses within the sub-field of societal exergy analysis are conducted with exergy quantification for energy carriers. Exergy is an alternative quantification of energy that gives the maximum useful work that could be generated by bringing a system into equilibrium with its surroundings. The purpose of this appendix is to demonstrate that the analyses conducted with energy quantification in Section 3 can also be conducted with exergy quantification.

When exergy quantifications are used for energy carriers, the equations of Sections 2 and 3 are unchanged, but nonzero energy entries in the PSUT matrices are different by the exergy-to-energy ratio (ϕ, see Serrenho [99, Table 2]). We assume that wastes and waste heat from each industry represent exergy destroyed by the industry, accounted by the second law of thermodynamics. We re-present the key results of Sections 3.3 and 3.4 in exergy terms below.

To begin, we convert all energy flows in Fig. 11 to exergy flows via multiplication by the exergy-to-energy ratio (ϕ), thereby obtaining Figs. B.1 and B.2.

Calculating the consumption-based primary-to-services *exergetic* efficiencies proceeds as discussed in Section 3.3 using the PSUT matrices of Fig. B.2. $\mathbf{Q}_{Crude}$ and $\mathbf{Q}_{NG}$ are shown in Fig. B.3. The vector of consumption-based primary-to-services efficiencies ($\eta_{X,ps}$) is shown in Fig. B.4.

The final-to-services exergetic efficiency for Passenger transport is given by

$$\eta_{X,fs} = \frac{5 \times 10^{11}\,\text{passenger-km/yr}}{27820\,\text{ktoe/yr}} = 1.8 \times 10^{7}\,\text{passenger-km/ktoe}, \tag{B.1}$$

slightly less than 1.92×10^{7} passenger-km/ktoe reported in Eq. (18) due to exergy quantification in the denominator of Eq. (B.1) and energy quantification in the denominator of Eq. (18). The denominators are different by the exergy-to-efficiency ratio for oil and oil products ($\phi_{Oil} = 1.07$).

Similarly, structural path analysis can be conducted using exergy quantification of energy carriers. Calculations proceed as discussed in Sections 3.4 and Appendix G using the PSUT matrices whose entries are now quantified as exergy (Fig. B.2). The fraction of embodied primary *exergy* captured by paths of varying lengths is shown in Fig. B.5, and largest magnitude paths for delivery of exergy services from natural gas are shown in Table B.1.

We note that Figs. 14 and 17 are unchanged for the exergy ECC of Fig. B.1, because the Freight transport quantities are unchanged between Figs. 11 and B.1.

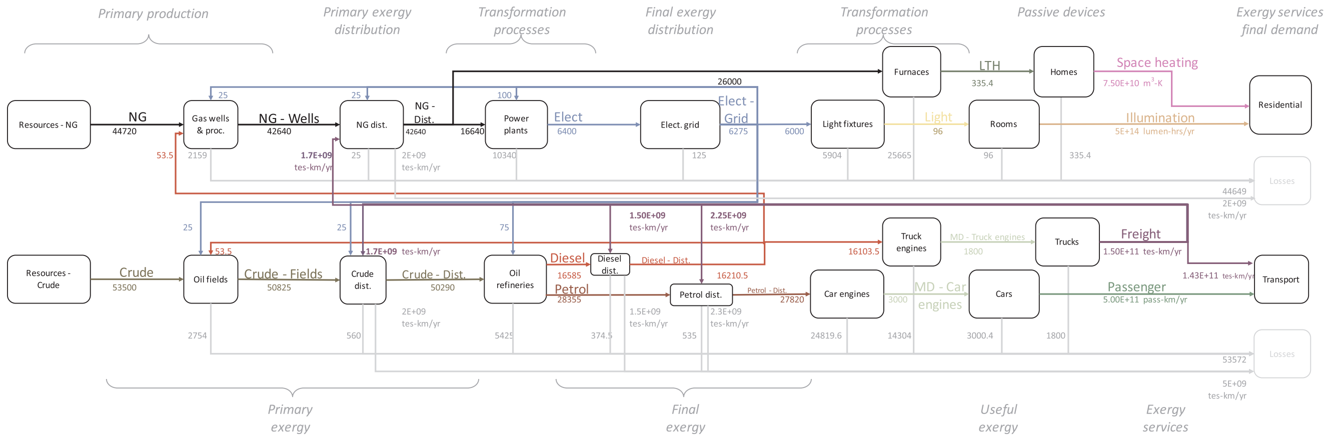
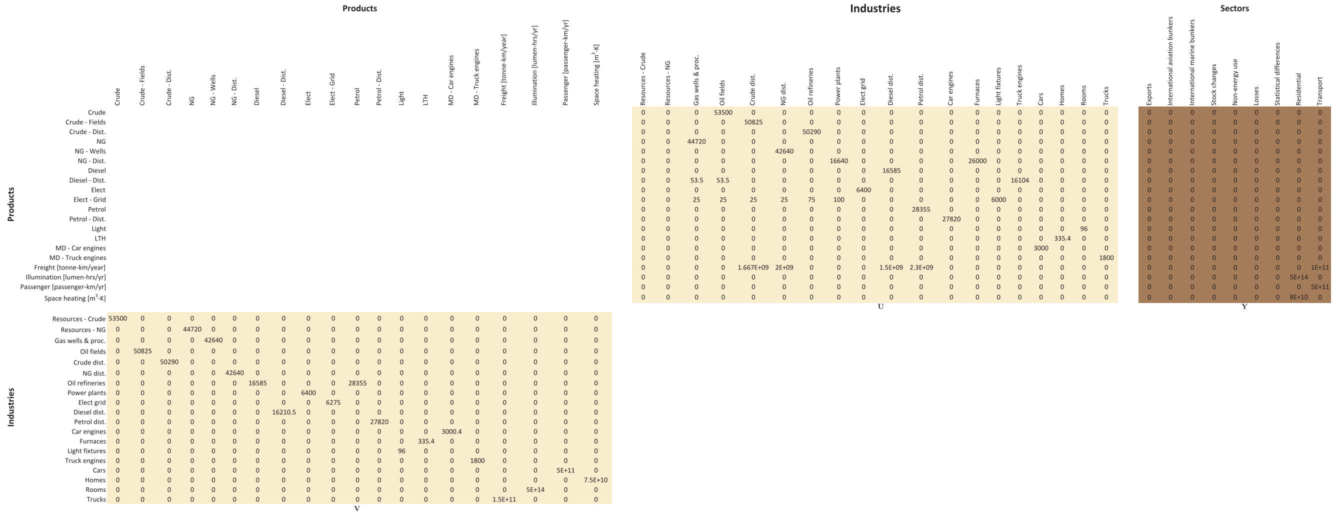


Fig. B.1. A real-world ECC covering primary exergy to exergy services. All exergy flows in units of ktoe/year; exergy services in units shown. NG is Natural gas. LTH is Low-temperature heat. MD is Mechanical drive. Line colors indicate products. "tes" is an abbreviation for metric tonnes. This figure is the exergy version of Fig. 11.



Industries

Products	Resources - Crude	Resources - NG	Gas wells & proc.	Oil fields	Crude dist.	NG dist.	Oil refineries	Power plants	Elect grid	Diesel dist.	Petrol dist.	Car engines	Furnaces	Light fixtures	Truck engines	Cars	Homes	Rooms	Trucks
Crude	0	0	0	53500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Crude - Fields	0	0	0	0	50825	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Crude - Dist.	0	0	0	0	0	0	50290	0	0	0	0	0	0	0	0	0	0	0	0
NG	0	0	44720	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NG - Wells	0	0	0	0	0	42640	0	0	0	0	0	0	0	0	0	0	0	0	0
NG - Dist.	0	0	0	0	0	0	0	16640	0	0	0	0	26000	0	0	0	0	0	0
Diesel	0	0	0	0	0	0	0	0	0	16585	0	0	0	0	0	0	0	0	0
Diesel - Dist.	0	0	53.5	53.5	0	0	0	0	0	0	0	0	0	0	16104	0	0	0	0
Elect	0	0	0	0	0	0	0	0	6400	0	0	0	0	0	0	0	0	0	0
Elect - Grid	0	0	25	25	25	25	75	100	0	0	0	0	0	6000	0	0	0	0	0
Petrol	0	0	0	0	0	0	0	0	0	0	28355	0	0	0	0	0	0	0	0
Petrol - Dist.	0	0	0	0	0	0	0	0	0	0	0	27820	0	0	0	0	0	0	0
Light	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	96	0
LTH	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	335.4	0	0
MD - Car engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3000	0	0	0
MD - Truck engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1800
Freight [tonne-km/year]	0	0	0	0	1.667E+09	2E+09	0	0	0	1.5E+09	2.3E+09	0	0	0	0	0	0	0	0
Illumination [lumen-hrs/yr]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Passenger [passenger-km/yr]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Space heating [m^3-K]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

U

Sectors

Products	Exports	International aviation bunkers	International marine bunkers	Stock changes	Non-energy use	Losses	Statistical differences	Residential	Transport
Crude	0	0	0	0	0	0	0	0	0
Crude - Fields	0	0	0	0	0	0	0	0	0
Crude - Dist.	0	0	0	0	0	0	0	0	0
NG	0	0	0	0	0	0	0	0	0
NG - Wells	0	0	0	0	0	0	0	0	0
NG - Dist.	0	0	0	0	0	0	0	0	0
Diesel	0	0	0	0	0	0	0	0	0
Diesel - Dist.	0	0	0	0	0	0	0	0	0
Elect	0	0	0	0	0	0	0	0	0
Elect - Grid	0	0	0	0	0	0	0	0	0
Petrol	0	0	0	0	0	0	0	0	0
Petrol - Dist.	0	0	0	0	0	0	0	0	0
Light	0	0	0	0	0	0	0	0	0
LTH	0	0	0	0	0	0	0	0	0
MD - Car engines	0	0	0	0	0	0	0	0	0
MD - Truck engines	0	0	0	0	0	0	0	0	0
Freight [tonne-km/year]	0	0	0	0	0	0	0	0	1E+11
Illumination [lumen-hrs/yr]	0	0	0	0	0	0	0	5E+14	0
Passenger [passenger-km/yr]	0	0	0	0	0	0	0	0	5E+11
Space heating [m^3-K]	0	0	0	0	0	0	0	8E+10	0

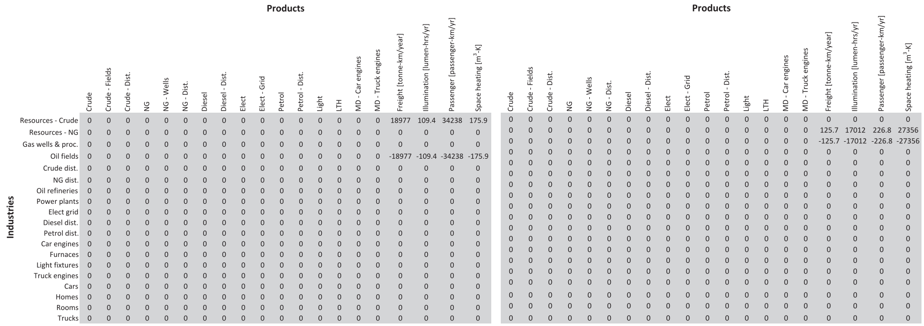
Y

Products

Industries	Crude	Crude - Fields	Crude - Dist.	NG	NG - Wells	NG - Dist.	Diesel	Diesel - Dist.	Elect	Elect - Grid	Petrol	Petrol - Dist.	Light	LTH	MD - Car engines	MD - Truck engines	Freight [tonne-km/year]	Illumination [lumen-hrs/yr]	Passenger [passenger-km/yr]	Space heating [m^3-K]
Resources - Crude	53500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Resources - NG	0	0	0	44720	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Gas wells & proc.	0	0	0	0	42640	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Oil fields	0	50825	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Crude dist.	0	0	50290	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NG dist.	0	0	0	0	0	42640	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Oil refineries	0	0	0	0	0	0	16585	0	0	0	28355	0	0	0	0	0	0	0	0	0
Power plants	0	0	0	0	0	0	0	0	6400	0	0	0	0	0	0	0	0	0	0	0
Elect grid	0	0	0	0	0	0	0	0	0	6275	0	0	0	0	0	0	0	0	0	0
Diesel dist.	0	0	0	0	0	0	0	16210.5	0	0	0	0	0	0	0	0	0	0	0	0
Petrol dist.	0	0	0	0	0	0	0	0	0	0	0	27820	0	0	0	0	0	0	0	0
Car engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3000.4	0	0	0	0	0
Furnaces	0	0	0	0	0	0	0	0	0	0	0	0	0	335.4	0	0	0	0	0	0
Light fixtures	0	0	0	0	0	0	0	0	0	0	0	0	96	0	0	0	0	0	0	0
Truck engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1800	0	0	0	0
Cars	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5E+11	0
Homes	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7.5E+10
Rooms	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5E+14	0	0
Trucks	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.5E+11	0	0	0

V

Fig. B.2. PSUT matrices for the ECC in Fig. B.1. All exergy flows in units of ktoe/year; exergy services in units shown. This figure is the exergy version of Fig. 12.



Products

Industries	Crude	Crude - Fields	Crude - Dist.	NG	NG - Wells	NG - Dist.	Diesel	Diesel - Dist.	Elect	Elect - Grid	Petrol	Petrol - Dist.	Light	LTH	MD - Car engines	MD - Truck engines	Freight [tonne-km/year]	Illumination [lumen-hrs/yr]	Passenger [passenger-km/yr]	Space heating [m^3-K]
Resources - Crude	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	18977	109.4	34238	175.9
Resources - NG	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Gas wells & proc.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Oil fields	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-18977	-109.4	-34238	-175.9
Crude dist.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NG dist.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Oil refineries	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Power plants	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Elect grid	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Diesel dist.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Petrol dist.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Car engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Furnaces	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Light fixtures	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Truck engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Cars	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Homes	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Rooms	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Trucks	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

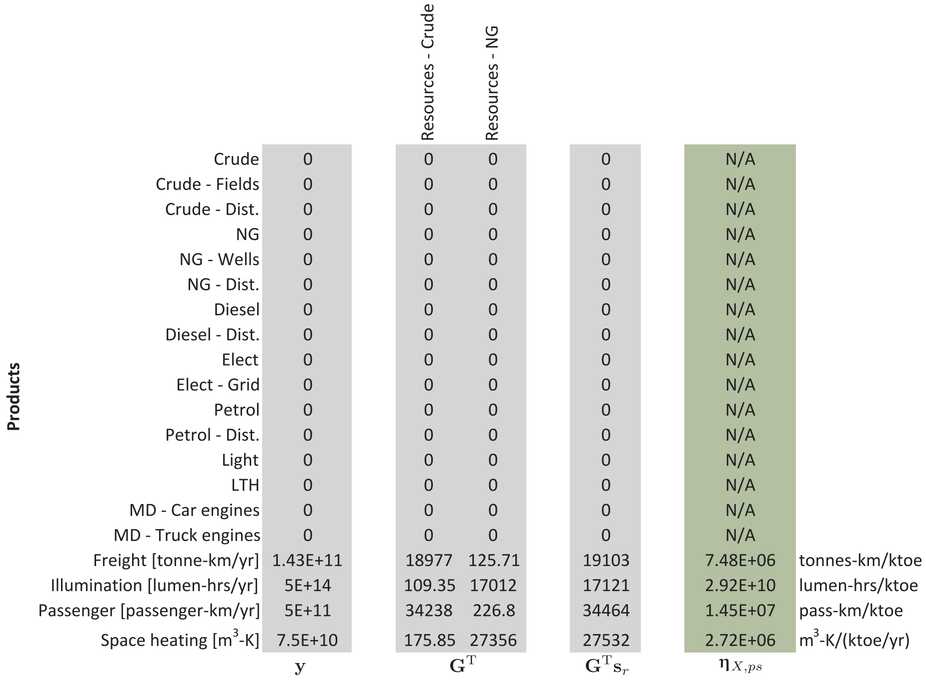
(a) $\mathbf{Q}_{Crude}$

Products

Crude	Crude - Fields	Crude - Dist.	NG	NG - Wells	NG - Dist.	Diesel	Diesel - Dist.	Elect	Elect - Grid	Petrol	Petrol - Dist.	Light	LTH	MD - Car engines	MD - Truck engines	Freight [tonne-km/year]	Illumination [lumen-hrs/yr]	Passenger [passenger-km/yr]	Space heating [m^3-K]
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	125.7	17012	226.8	27356
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-125.7	-17012	-226.8	-27356
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

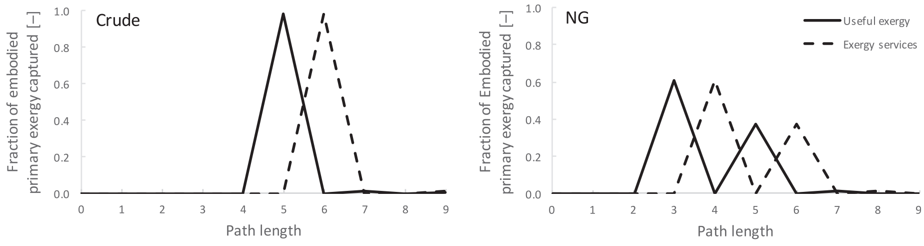
(b) $\mathbf{Q}_{NG}$

Fig. B.3. **Q** matrices for embodied Crude oil and Natural gas (NG) for the ECC in Fig. B.1. This figure is the exergy version of Fig. 13.



Products		Resources - Crude	Resources - NG			
Crude	0	0	0	0	N/A	
Crude - Fields	0	0	0	0	N/A	
Crude - Dist.	0	0	0	0	N/A	
NG	0	0	0	0	N/A	
NG - Wells	0	0	0	0	N/A	
NG - Dist.	0	0	0	0	N/A	
Diesel	0	0	0	0	N/A	
Diesel - Dist.	0	0	0	0	N/A	
Elect	0	0	0	0	N/A	
Elect - Grid	0	0	0	0	N/A	
Petrol	0	0	0	0	N/A	
Petrol - Dist.	0	0	0	0	N/A	
Light	0	0	0	0	N/A	
LTH	0	0	0	0	N/A	
MD - Car engines	0	0	0	0	N/A	
MD - Truck engines	0	0	0	0	N/A	
Freight [tonne-km/yr]	1.43E+11	18977	125.71	19103	7.48E+06	tonnes-km/ktoe
Illumination [lumen-hrs/yr]	5E+14	109.35	17012	17121	2.92E+10	lumen-hrs/ktoe
Passenger [passenger-km/yr]	5E+11	34238	226.8	34464	1.45E+07	pass-km/ktoe
Space heating [m^3-K]	7.5E+10	175.85	27356	27532	2.72E+06	m^3-K/(ktoe/yr)
	$\mathbf{y}$	$\mathbf{G}^{\mathrm{T}}$		$\mathbf{G}^{\mathrm{T}}\mathbf{s}_r$	$\boldsymbol{\eta}_{X,ps}$	

Fig. B.4. Consumption-based primary-to-services exergetic efficiencies ($\eta_{X,ps}$) for the ECC in Figs. B.1 and B.2. For brevity, this figure shows only the Resources industries of **G**. This figure is the exergy version of Fig. 15.



(a) Crude paths.

(b) Natural gas paths.

Fig. B.5. Fraction of embodied Crude and Natural gas exergy captured by paths of varying lengths in the exergy version of the useful energy ECC of Fig. 7 (solid line) and the exergy services ECC of Fig. 11 (dashed line). This figure is the exergy version of Fig. 16.

Table B.1

Top 10 largest magnitude paths from Natural gas to final demand expressed as exergy services in Fig. B.1. This table is the exergy version of Table 7.

Size rank	Step 0	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6	Step 7	Step 8	Step 9	Step 10	Path magnitude [ktoe/yr]
1	Resources - NG	Gas wells & proc.	NG dist.	Furnaces	Homes							27,268
2	Resources - NG	Gas wells & proc.	NG dist.	Power plants	Elect. grid	Light fixtures	Rooms - Illum					16,687
3	Resources - NG	Gas wells & proc.	NG dist.	Power plants	Elect. grid	Power plants	Elect grid	Light fixtures	Rooms - Illum			266
4	Resources - NG	Gas wells & proc.	NG dist.	Power plants	Elect. grid	Oil refineries	Petrol dist.	Car engines	Cars - Pass trnsp			132
5	Resources - NG	Gas wells & proc.	NG dist.	Power plants	Elect. grid	Oil refineries	Diesel dist.	Truck engines	Trucks - Freight			73
6	Resources - NG	Gas wells & proc.	NG dist.	Power plants	Elect. grid	Oil fields	Crude dist.	Oil refineries	Petrol dist.	Car engines	Cars - Pass trnsp	44
7	Resources - NG	Gas wells & proc.	NG dist.	Power plants	Elect. grid	Crude dist.	Oil refineries	Petrol dist.	Car engines	Cars - Pass trnsp		44
8	Resources - NG	Gas wells & proc.	NG dist.	Power plants	Elect. grid	NG dist.	Furnaces	Homes - Space htg				42
9	Resources - NG	Gas wells & proc.	NG dist.	Power plants	Elect. grid	Gas wells & proc.	NG dist.	Furnaces	Homes - Space htg			42
10	Resources - NG	Gas wells & proc.	NG dist.	Power plants	Elect. grid	NG dist.	Power plants	Elect grid	Light fixtures	Rooms - Illum		26

Appendix C. Matrix and vector algebra relationships

In this appendix, we present some relationships from matrix and vector algebra that may assist the reader.

First, column sums and row sums are conveniently calculated with identity vectors. For example, post-multiplying a matrix (**M**) by the identity column vector (**i**) gives row sums in a column vector.

$$\mathbf{Mi} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = \begin{Bmatrix} M_{11} + M_{12} + M_{13} \\ M_{21} + M_{22} + M_{23} \\ M_{31} + M_{32} + M_{33} \end{Bmatrix} \tag{C.1}$$

Pre-multiplying **M** by the transpose of the identity vector ($\mathbf{i}^{\mathrm{T}}$) gives column sums in a row vector.

$$\begin{aligned} \mathbf{i}^{\mathrm{T}}\mathbf{M} &= \{1 \;\; 1 \;\; 1\} \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \\ &= \{M_{11} + M_{21} + M_{31} \;\; M_{12} + M_{22} + M_{32} \;\; M_{13} + M_{23} + M_{33}\} \end{aligned} \tag{C.2}$$

Second, given a matrix **M** and identity vector **i**, row sums of transposed **M** are the same as column sums of **M** transposed:

$$\mathbf{M}^{\mathrm{T}}\mathbf{i} = (\mathbf{i}^{\mathrm{T}}\mathbf{M})^{\mathrm{T}} \tag{C.3}$$

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}^{\mathrm{T}} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = \left(\{1 \;\; 1 \;\; 1\} \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \right)^{\mathrm{T}}$$

$$\begin{bmatrix} M_{11} & M_{21} & M_{31} \\ M_{12} & M_{22} & M_{32} \\ M_{13} & M_{23} & M_{33} \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = \{M_{11} + M_{21} + M_{31} \;\; M_{12} + M_{22} + M_{32} \;\; M_{13} + M_{23} + M_{33}\}^{\mathrm{T}}$$

$$\begin{Bmatrix} M_{11} + M_{21} + M_{31} \\ M_{12} + M_{22} + M_{32} \\ M_{13} + M_{23} + M_{33} \end{Bmatrix} = \begin{Bmatrix} M_{11} + M_{21} + M_{31} \\ M_{12} + M_{22} + M_{32} \\ M_{13} + M_{23} + M_{33} \end{Bmatrix}.$$

Third, Section 3 includes several terms of the form $\hat{\mathbf{b}}^{-1}\mathbf{a}$, which is the matrix algebra notation for an element-wise quotient of two same-length column vectors:

$$\begin{aligned} \hat{\mathbf{b}}^{-1}\mathbf{a} &= \begin{bmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{bmatrix}^{-1} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} \\ &= \begin{bmatrix} 1/b_1 & 0 & 0 \\ 0 & 1/b_2 & 0 \\ 0 & 0 & 1/b_3 \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix} \\ &= \begin{Bmatrix} a_1/b_1 \\ a_2/b_2 \\ a_3/b_3 \end{Bmatrix}. \end{aligned} \tag{C.4}$$

Finally, we point out that Tables 5 and 6 contain several terms of the form $\mathbf{M}\hat{\mathbf{v}}^{-1}$, which is equivalent to dividing each *column* of matrix **M** by the associated entry in column vector **v**.

$$\begin{aligned} \mathbf{M}\hat{\mathbf{v}}^{-1} &= \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} 1/v_1 & 0 & 0 \\ 0 & 1/v_2 & 0 \\ 0 & 0 & 1/v_3 \end{bmatrix} \\ &= \begin{bmatrix} M_{11}/v_1 & M_{12}/v_2 & M_{13}/v_3 \\ M_{21}/v_1 & M_{22}/v_2 & M_{23}/v_3 \\ M_{31}/v_1 & M_{32}/v_2 & M_{33}/v_3 \end{bmatrix} \end{aligned} \tag{C.5}$$

Reversing the order of multiplication divides each *row* of **M** by the corresponding element of **v**.

$$\begin{aligned} \hat{\mathbf{v}}^{-1}\mathbf{M} &= \begin{bmatrix} 1/v_1 & 0 & 0 \\ 0 & 1/v_2 & 0 \\ 0 & 0 & 1/v_3 \end{bmatrix} \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \\ &= \begin{bmatrix} M_{11}/v_1 & M_{12}/v_1 & M_{13}/v_1 \\ M_{21}/v_2 & M_{22}/v_2 & M_{23}/v_2 \\ M_{31}/v_3 & M_{32}/v_3 & M_{33}/v_3 \end{bmatrix} \end{aligned} \tag{C.6}$$

Appendix D. Proof that Eq. (4) is an identity

We begin with a restatement of Eq. (4).

$$\mathbf{g}^{\mathrm{T}} - \mathbf{W}^{\mathrm{T}}\mathbf{i} - \mathbf{U}^{\mathrm{T}}\mathbf{i} = \mathbf{0} \tag{4}$$

Next, we substitute definitions for **g** and **W** from Table 4 and Eq. (1).

$$\mathbf{Vi}-(\mathbf{V}^{\mathrm{T}}-\mathbf{U})^{\mathrm{T}}\mathbf{i}-\mathbf{U}^{\mathrm{T}}\mathbf{i}=\mathbf{0} \tag{D.1}$$

Simplification gives

$$\mathbf{Vi}-\mathbf{Vi}+\mathbf{U}^{\mathrm{T}}\mathbf{i}-\mathbf{U}^{\mathrm{T}}\mathbf{i}=\mathbf{0}\,, \tag{D.2}$$

and

$$\mathbf{0}=\mathbf{0}. \tag{D.3}$$

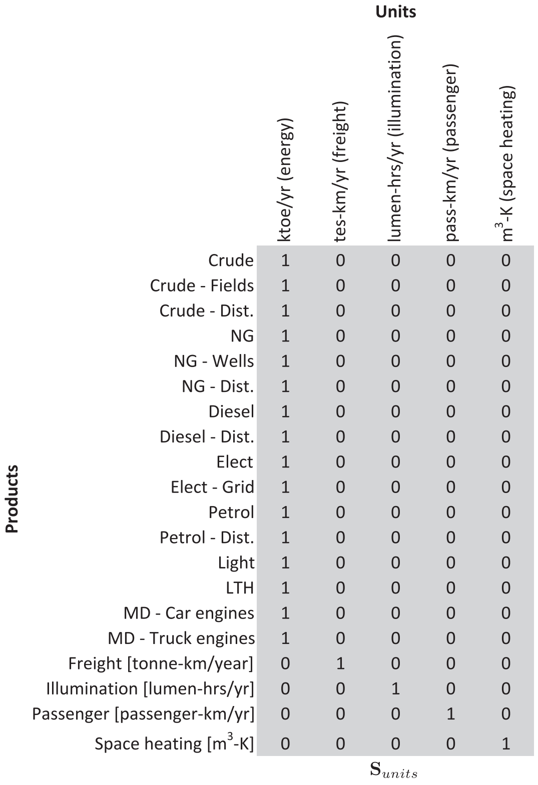
Appendix E. Aggregation across products with inhomogeneous units

When inhomogeneous units are present along the product dimension of any of the **U**, **V**, **W**, or **Y** matrices, care must be taken to obtain appropriate row and column sums for energy and energy services balances. Inhomogeneous product units are likely when any of the **U**, **V**, **W**, or **Y** matrices contain energy services on their product dimensions. Under such circumstances, aggregation across products must be done in a unit-aware manner, as shown in Eq. (5).

A units summation matrix ($\mathbf{S}_{units}$) facilitates such aggregations. $\mathbf{S}_{units}$ is products × units and is formed by placing a “1” to indicate the units of any product. See Fig. E.1 for an example $\mathbf{S}_{units}$ matrix for the ECC of Fig. 11. Note that if the **U**, **V**, **W**, and **Y** matrices are unit-homogeneous, $\mathbf{S}_{units}$ simplifies to an identity vector that provides simple row sums ($\mathbf{i}$) or column sums ($\mathbf{i}^{\mathrm{T}}$).

Post-multiplying **V** by $\mathbf{S}_{units}$ or pre-multiplying **U**, **W**, or **Y** by $\mathbf{S}_{units}^{\mathrm{T}}$ reduces the size of the product dimension from the number of products to the number of unique product units, aggregating all products of like units. We use an over-bar applied to a matrix symbol (e.g., $\overline{\mathbf{V}}$) to indicate that summation across products of the same units has occurred. Aggregation equations are given in Table E.1.

An example is instructive. Fig. E.2 shows the make matrix (**V**) from Fig. 11. The first 16 columns contain energy quantities in units of ktoe/year. The last 4 columns contain energy services with varying units. Applying the second equation from Table E.1 to the make matrix of Fig. E.2 with the unit summation matrix of Fig. E.1 performs row sums by unit to give the result shown in Fig. E.3.



	ktoe/yr (energy)	tes-km/yr (freight)	lumen-hrs/yr (illumination)	pass-km/yr (passenger)	m^3-K (space heating)
Crude	1	0	0	0	0
Crude - Fields	1	0	0	0	0
Crude - Dist.	1	0	0	0	0
NG	1	0	0	0	0
NG - Wells	1	0	0	0	0
NG - Dist.	1	0	0	0	0
Diesel	1	0	0	0	0
Diesel - Dist.	1	0	0	0	0
Elect	1	0	0	0	0
Elect - Grid	1	0	0	0	0
Petrol	1	0	0	0	0
Petrol - Dist.	1	0	0	0	0
Light	1	0	0	0	0
LTH	1	0	0	0	0
MD - Car engines	1	0	0	0	0
MD - Truck engines	1	0	0	0	0
Freight [tonne-km/year]	0	1	0	0	0
Illumination [lumen-hrs/yr]	0	0	1	0	0
Passenger [passenger-km/yr]	0	0	0	1	0
Space heating [m^3-K]	0	0	0	0	1

Fig. E.1. Example $\mathbf{S}_{units}$ matrix. “tes” is an abbreviation for metric tonnes.

Table E.1
Aggregation by units across products.

Equation	Meaning
$\overline{\mathbf{U}}=\mathbf{S}_{units}^{\mathrm{T}}\mathbf{U}$	Column sums of **U** by unit
$\overline{\mathbf{V}}=\mathbf{V}\mathbf{S}_{units}$	Row sums of **V** by unit
$\overline{\mathbf{W}}=\mathbf{S}_{units}^{\mathrm{T}}\mathbf{W}$	Column sums of **W** by unit
$\overline{\mathbf{Y}}=\mathbf{S}_{units}^{\mathrm{T}}\mathbf{Y}$	Column sums of **Y** by unit

Products

Industries	Crude	Crude - Fields	Crude - Dist.	NG	NG - Wells	NG - Dist.	Diesel	Diesel - Dist.	Elect	Elect - Grid	Petrol	Petrol - Dist.	Light	LTH	MD - Car engines	MD - Truck engines	Freight [tonne-km/year]	Illumination [lumen-hrs/yr]	Passenger [passenger-km/yr]	Space heating [m^3-K]
Resources - Crude	50000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Resources - NG	0	0	0	43000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Gas wells & proc.	0	0	0	0	41000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Oil fields	0	47500	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Crude dist.	0	0	47000	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
NG dist.	0	0	0	0	0	41000	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Oil refineries	0	0	0	0	0	0	15500	0	0	0	26500	0	0	0	0	0	0	0	0	0
Power plants	0	0	0	0	0	0	0	0	6400	0	0	0	0	0	0	0	0	0	0	0
Elect grid	0	0	0	0	0	0	0	0	0	6275	0	0	0	0	0	0	0	0	0	0
Diesel dist.	0	0	0	0	0	0	0	15150	0	0	0	0	0	0	0	0	0	0	0	0
Petrol dist.	0	0	0	0	0	0	0	0	0	0	0	26000	0	0	0	0	0	0	0	0
Car engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3000.4	0	0	0	0	0
Furnaces	0	0	0	0	0	0	0	0	0	0	0	0	0	20000	0	0	0	0	0	0
Light fixtures	0	0	0	0	0	0	0	0	0	0	0	0	1200	0	0	0	0	0	0	0
Truck engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1800	0	0	0	0
Cars	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5E+11	0
Homes	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7.5E+10
Rooms	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5E+14	0	0
Trucks	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1.5E+11	0	0	0

$\mathbf{V}$

Fig. E.2. Make matrix ($\mathbf{V}$) for the ECC in Fig. 11.

Units

Industries	ktoe/yr (energy)	tes-km/yr (freight)	lumen-hrs/yr (illumination)	pass-km/yr (passenger)	m3-K (space heating)
Resources - Crude	50000	0	0	0	0
Resources - NG	43000	0	0	0	0
Gas wells & proc.	41000	0	0	0	0
Oil fields	47500	0	0	0	0
Crude dist.	47000	0	0	0	0
NG dist.	41000	0	0	0	0
Oil refineries	42000	0	0	0	0
Power plants	6400	0	0	0	0
Elect grid	6275	0	0	0	0
Diesel dist.	15150	0	0	0	0
Petrol dist.	26000	0	0	0	0
Car engines	3000	0	0	0	0
Furnaces	20000	0	0	0	0
Light fixtures	1200	0	0	0	0
Truck engines	1800	0	0	0	0
Cars	0	0	0	5E+11	0
Homes	0	0	0	0	8E+10
Rooms	0	0	5E+14	0	0
Trucks	0	2E+11	0	0	0

$\overline{\mathbf{V}}$

Fig. E.3. Example $\overline{\mathbf{V}}$ matrix. for the ECC in Fig. 11. "tes" is an abbreviation for metric tonnes.

Appendix F. Relationships among energy return ratios

This appendix demonstrates relationships among the three energy return ratios (ERRs) discussed in Section 3.1 (*GER*, *NER*, and *r*) and proves that any two can be expressed in terms of the third. We begin with the definitions of net energy (E_{net}), gross energy ratio (*GER*), net energy ratio (*NER*), and net-to-gross energy ratio (*r*).

$$E_{net} \equiv E_{gross} - E_{consumed} \tag{F.1}$$

$$GER \equiv \frac{E_{gross}}{E_{consumed}} \tag{F.2}$$

$$NER \equiv \frac{E_{net}}{E_{consumed}} \tag{F.3}$$

$$r \equiv \frac{E_{net}}{E_{gross}} \tag{F.4}$$

We substitute Eq. (F.1) into Eq. (F.3) to obtain

$$NER = \frac{E_{gross} - E_{consumed}}{E_{consumed}}, \tag{F.5}$$

which simplifies to

$$NER = GER - 1, \tag{F.6}$$

a scalar version of Eq. (9).

Dividing numerator and denominator of Eq. (F.4) by $E_{consumed}$ yields

$$r = \frac{NER}{GER}, \tag{F.7}$$

which is a scalar version of Eq. (10). For completeness, we note that substituting Eq. (F.6) into Eq. (F.7) gives

$$r = 1 - \frac{1}{GER}. \tag{F.8}$$

Eqs. (F.6) and (F.8) show that *NER* and *r* can be expressed in terms of *GER*.

To show that *GER* and *r* can be expressed in terms of *NER*, we solve Eq. (F.6) for *GER* to obtain

$$GER = NER + 1. \tag{F.9}$$

Substituting Eq. (F.9) into Eq. (F.7) yields

$$r = \frac{1}{1 + \frac{1}{NER}}, \tag{F.10}$$

thereby demonstrating that *GER* and *r* can be expressed in terms of *NER*.

Finally, solving Eq. (F.8) for *GER* gives

$$GER = \frac{1}{1 - r}, \tag{F.11}$$

and solving Eq. (F.10) for *NER* gives

$$NER = \frac{1}{\frac{1}{r} - 1}, \tag{F.12}$$

showing that *GER* and *NER* can be expressed in terms of *r* and completing the proof that any ERR can be expressed in terms of the other two and that any two ERRs can be expressed in terms of the third. Table F.1 summarizes these results.

Table F.1
Summary of relationships among ERRs. Rows show that any ERR (row title) can be expressed in terms of the other two (column titles). Columns show that any two ERRs (row titles) can be expressed in terms of the third (column title).

	GER	*NER*	*r*
GER	—	$GER = NER + 1$	$GER = \frac{1}{1-r}$
NER	$NER = GER - 1$	—	$NER = \frac{1}{\frac{1}{r} - 1}$
r	$r = 1 - \frac{1}{GER}$	$r = \frac{1}{1 + \frac{1}{NER}}$	—

Appendix G. Details of structural path analysis

This appendix provides additional details of the calculations that identify the key supply-chain paths through the ECC. The technique, known as Structural Path Analysis (SPA), can be used to decompose the embodied primary energy associated with final demand to the sum of an infinite number of production chains, called paths. Wood and Lenzen [115, p. 371] describe this process as “unraveling the Leontief inverse using its series expansion.” SPA was developed initially for use with symmetric input-output tables in monetary units [138,139] rather than the SUT format employed by the PSUT framework discussed in Section 2. We adapt SPA for the PSUT framework here.

SPA proceeds by substituting the right side of Eq. (19) for the $(\mathbf{I} - \mathbf{A})^{-1}$ term in Eq. (13) to obtain

$$\mathbf{Q} = \hat{\mathbf{e}}\mathbf{D}\mathbf{I}\hat{\mathbf{y}} + \hat{\mathbf{e}}\mathbf{D}\mathbf{A}\hat{\mathbf{y}} + \hat{\mathbf{e}}\mathbf{D}\mathbf{A}^2\hat{\mathbf{y}} + \hat{\mathbf{e}}\mathbf{D}\mathbf{A}^3\hat{\mathbf{y}} + \cdots + \hat{\mathbf{e}}\mathbf{D}\mathbf{A}^n\hat{\mathbf{y}} + \cdots \tag{G.1}$$

which can be simplified to give

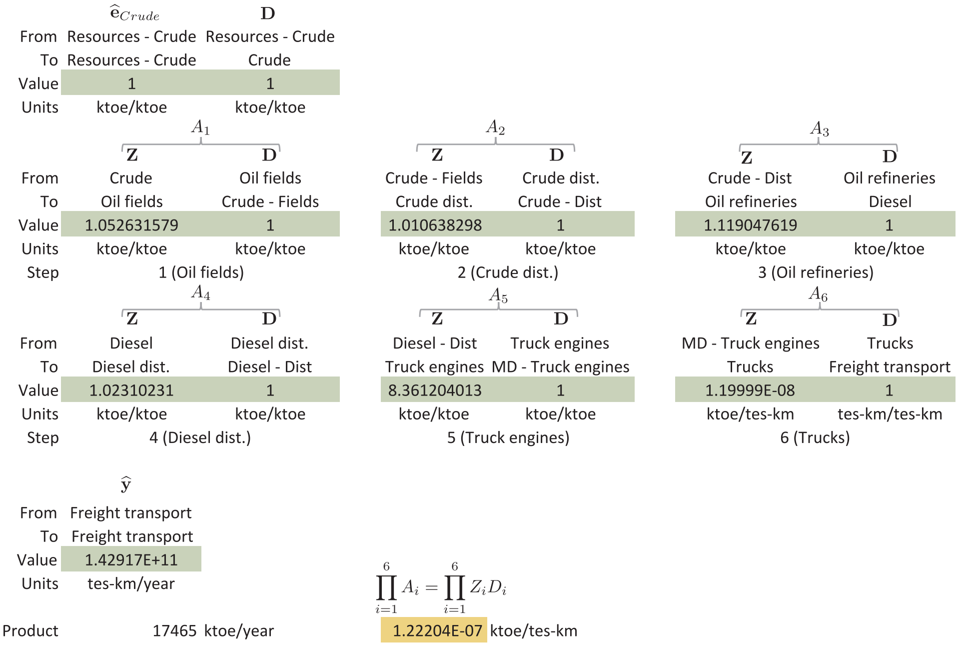
$$\mathbf{Q} \approx \sum_{m=0}^{n} \hat{\mathbf{e}}\,\mathbf{D}\mathbf{A}^{m}\hat{\mathbf{y}}. \tag{G.2}$$

In practice, Eq. (G.2) is implemented as the product of a series of entries in the $\hat{\mathbf{e}}$, $\mathbf{D}$, $\mathbf{Z}$, and $\hat{\mathbf{y}}$ matrices for paths through the ECC found by a search algorithm (see Table 5 for definitions of vectors and matrices in the PSUT framework). The provision of Freight transport in the ECC of Fig. 11 provides an example.

Fig. G.1 shows the calculation of the embodied Crude in Freight transport for a 6-step path. (There are other, longer, paths from Crude to Freight transport that are not captured by this calculation.) We start with the appropriate entries in the $\hat{\mathbf{e}}$ and $\mathbf{D}$ matrices, followed by a series of 6 entries in the $\mathbf{Z}$ and $\mathbf{D}$ matrices, representing the 6 steps of the shortest path from Crude to Freight transport. Finally, the Freight transport entry in $\hat{\mathbf{y}}$ is shown.

The product of all values in Fig. G.1 is 17,465 ktoe/year, the embodied primary energy of the 6-step path from Crude to Freight transport. Note that 17,465 ktoe/year is 98.5% of *all* embodied Crude in Freight transport (17,736 ktoe/year in Figs. 15 and 13a), the difference being Crude embodied in paths that take more than 6 steps to reach Freight transport.

In Fig. G.1, the product of the 12 values that comprise the $\mathbf{A}$ matrix (entries in $\mathbf{Z}$ and $\mathbf{D}$ at each step in the path) is 1.22204×10^{-7} ktoe/tonne-km. Fig. G.2 shows the $\mathbf{A}^6$ matrix, which contains the sum of magnitudes of all length-6 paths from products in rows to products in columns of the ECC of Fig. 11. The only length-6 path from Crude to Freight transport appears in the Crude row and the Freight transport column of the $\mathbf{A}^6$ matrix, and its value is the same as the product of all $\mathbf{A}$ entries in Fig. G.1.



	$\widehat{\mathbf{e}}_{Crude}$	$\mathbf{D}$
From	Resources - Crude	Resources - Crude
To	Resources - Crude	Crude
Value	1	1
Units	ktoe/ktoe	ktoe/ktoe

	A_1		A_2		A_3	
	$\mathbf{Z}$	$\mathbf{D}$	$\mathbf{Z}$	$\mathbf{D}$	$\mathbf{Z}$	$\mathbf{D}$
From	Crude	Oil fields	Crude - Fields	Crude dist.	Crude - Dist	Oil refineries
To	Oil fields	Crude - Fields	Crude dist.	Crude - Dist	Oil refineries	Diesel
Value	1.052631579	1	1.010638298	1	1.119047619	1
Units	ktoe/ktoe	ktoe/ktoe	ktoe/ktoe	ktoe/ktoe	ktoe/ktoe	ktoe/ktoe
Step	1 (Oil fields)		2 (Crude dist.)		3 (Oil refineries)	

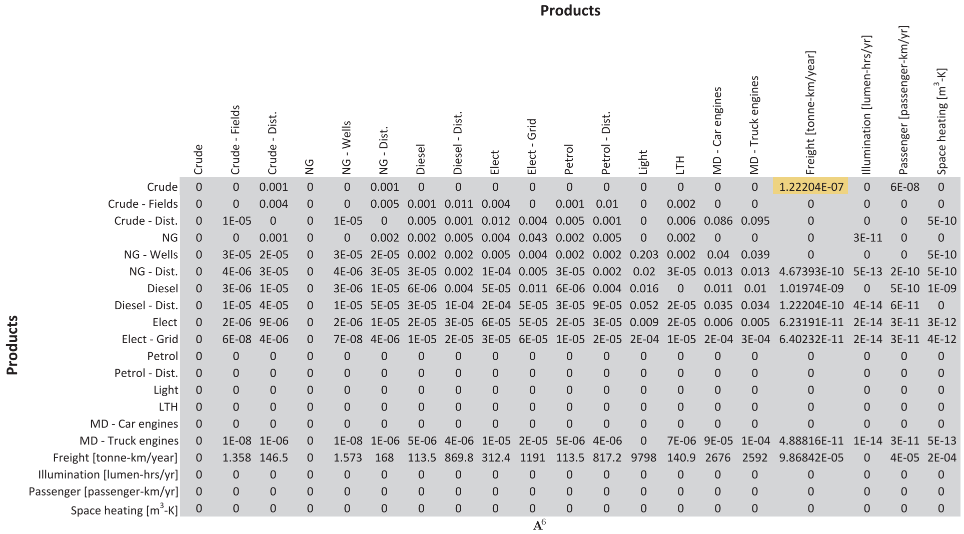
	A_4		A_5		A_6	
	$\mathbf{Z}$	$\mathbf{D}$	$\mathbf{Z}$	$\mathbf{D}$	$\mathbf{Z}$	$\mathbf{D}$
From	Diesel	Diesel dist.	Diesel - Dist	Truck engines	MD - Truck engines	Trucks
To	Diesel dist.	Diesel - Dist	Truck engines	MD - Truck engines	Trucks	Freight transport
Value	1.02310231	1	8.361204013	1	1.19999E-08	1
Units	ktoe/ktoe	ktoe/ktoe	ktoe/ktoe	ktoe/ktoe	ktoe/tes-km	tes-km/tes-km
Step	4 (Diesel dist.)		5 (Truck engines)		6 (Trucks)	

	$\widehat{\mathbf{y}}$
From	Freight transport
To	Freight transport
Value	1.42917E+11
Units	tes-km/year

$$\prod_{i=1}^{6} A_i = \prod_{i=1}^{6} Z_i D_i$$

Product	17465 ktoe/year	1.22204E-07	ktoe/tes-km

Fig. G.1. The path from Resources–Crude to Freight transport through the **A** matrix for the ECC in Fig. 11. "tes" is an abbreviation for metric tonnes.



Products

Products	Crude	Crude - Fields	Crude - Dist.	NG	NG - Wells	NG - Dist.	Diesel	Diesel - Dist.	Elect	Elect - Grid	Petrol	Petrol - Dist.	Light	LTH	MD - Car engines	MD - Truck engines	Freight [tonne-km/year]	Illumination [lumen-hrs/yr]	Passenger [passenger-km/yr]	Space heating [m^3-K]
Crude	0	0	0.001	0	0	0.001	0	0	0	0	0	0	0	0	0	0	1.22204E-07	0	6E-08	0
Crude - Fields	0	0	0.004	0	0	0.005	0.001	0.011	0.004	0	0.001	0.01	0	0.002	0	0	0	0	0	0
Crude - Dist.	0	1E-05	0	0	1E-05	0	0.005	0.001	0.012	0.004	0.005	0.001	0	0.006	0.086	0.095	0	0	0	5E-10
NG	0	0	0.001	0	0	0.002	0.002	0.005	0.004	0.043	0.002	0.005	0	0.002	0	0	0	3E-11	0	0
NG - Wells	0	3E-05	2E-05	0	3E-05	2E-05	0.002	0.002	0.005	0.004	0.002	0.002	0.203	0.002	0.04	0.039	0	0	0	5E-10
NG - Dist.	0	4E-06	3E-05	0	4E-06	3E-05	3E-05	0.002	1E-04	0.005	3E-05	0.002	0.02	3E-05	0.013	0.013	4.67393E-10	5E-13	2E-10	5E-10
Diesel	0	3E-06	1E-05	0	3E-06	1E-05	6E-06	0.004	5E-05	0.011	6E-06	0.004	0.016	0	0.011	0.01	1.01974E-09	0	5E-10	1E-09
Diesel - Dist.	0	1E-05	4E-05	0	1E-05	5E-05	3E-05	1E-04	2E-04	5E-05	3E-05	9E-05	0.052	2E-05	0.035	0.034	1.22204E-10	4E-14	6E-11	0
Elect	0	2E-06	9E-06	0	2E-06	1E-05	2E-05	3E-05	6E-05	5E-05	2E-05	3E-05	0.009	2E-05	0.006	0.005	6.23191E-11	2E-14	3E-11	3E-12
Elect - Grid	0	6E-08	4E-06	0	7E-08	4E-06	1E-05	2E-05	3E-05	6E-05	1E-05	2E-05	2E-04	1E-05	2E-04	3E-04	6.40232E-11	2E-14	3E-11	4E-12
Petrol	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Petrol - Dist.	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Light	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
LTH	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
MD - Car engines	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
MD - Truck engines	0	1E-08	1E-06	0	1E-08	1E-06	5E-06	4E-06	1E-05	2E-05	5E-06	4E-06	0	7E-06	9E-05	1E-04	4.88816E-11	1E-14	3E-11	5E-13
Freight [tonne-km/year]	0	1.358	146.5	0	1.573	168	113.5	869.8	312.4	1191	113.5	817.2	9798	140.9	2676	2592	9.86842E-05	0	4E-05	2E-04
Illumination [lumen-hrs/yr]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Passenger [passenger-km/yr]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Space heating [m^3-K]	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$\mathbf{A}^6$

Fig. G.2. $\mathbf{A}^6$ matrix for the ECC in Figs. 11 and 12.

Appendix H. Constructing PSUT matrices from IEA world energy statistics

This appendix gives rules for populating the PSUT matrices (**U**, **V**, and **Y**) with primary and final energy data from the IEA [95], thereby providing an example for how to construct PSUT matrices from published country-level energy data. Similar rules for constructing PSUT matrices could be generated for data from other sources.

The broadest categorization of the IEA data is Supply and Consumption. Supply comprises domestic Production, Imports, Exports, International marine bunkers, International aviation bunkers, Stock changes, and Transfers. Statistical differences, Transformation processes, and Energy industry own use are the remaining categories before Consumption. Consumption in the IEA data is final demand in the PSUT framework (expressed as final energy) and is organized by Industry, Transport, Other (Residential, Commercial and public services, Agriculture/forestry, Fishing, Non-specified industry), and Non-energy use. Table H.1 gives rules for constructing PSUT matrices from IEA data.

Table H.1

Rules for constructing PSUT matrices from IEA World Energy Statistics [95]. The sign of IEA entries affects the PSUT matrix in which data should be entered, as indicated by (+) and (−). Before placing negative numbers into the PSUT matrices, the absolute value should be taken, as indicated by “as +” below. The IEA energy statistics data contain Flow and Product columns whose entries become the names of rows or columns in the PSUT matrices, as indicated by (row×column) notation.

IEA Category (IEA sign)	PSUT matrix (row×column)
Production (+)	**V** (Flow×Product)
Imports (+)	**V** (Flow×Product)
Exports (−)	**Y** (Product×Flow, as +)
International marine bunkers (+)	**V** (Flow×Product)
International marine bunkers (−)	**Y** (Product×Flow, as +)
International aviation bunkers (+)	**V** (Flow×Product)
International aviation bunkers (−)	**Y** (Product×Flow, as +)
Stock changes (+)	**V** (Flow×Product)
Stock changes (−)	**Y** (Product×Flow, as +)
Transfers (+)	**V** (Flow×Product)
Transfers (−)	**U** (Product×Flow, as +)
Statistical differences (+)	**V** (Flow×Product)
Statistical differences (−)	**Y** (Product×Flow, as +)
Transformation processes (+)	**V** (Flow×Product)
Transformation processes (−)	**U** (Product×Flow, as +)
Energy industry own use (−)	**U** (Product×Flow, as +)
Industry (+)	**Y** (Product×Flow)
Transport (+)	**Y** (Product×Flow)
Other (+)	**Y** (Product×Flow)
Non-energy use (+)	**Y** (Product×Flow)

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