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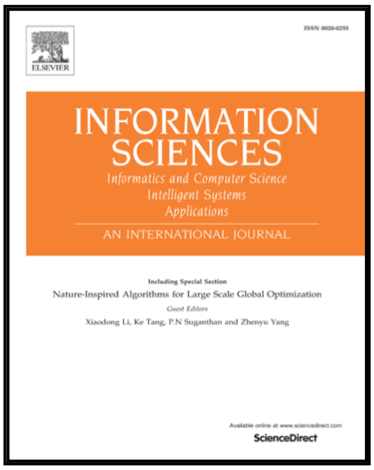

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Distributed Consensus Control for Multi-Agent Systems under Denial-of-Service

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Abstract

This paper investigates the problem of distributed consensus control for multi-agent systems under denial-of-service (DoS) attacks. Different from the existing results where DoS attacks on all the channels are same, in this paper, the adversaries compromise each channel independently. The objective is to design distributed controllers such that the consensus is still achieved in the presence of DoS attacks. Both state-feedback and observer-based controllers are considered. First, the decay rates under different attack modes are obtained by solving a class of linear matrix inequalities. Second, sufficient conditions on the duration of the DoS attacks, under which the consensus is still achieved, are proposed. The difficulty that there is no one-to-one match between the obtained decay rates and DoS duration limitations, is overcome by introducing the equivalent decay rates corresponding to channels. Moreover, the computational complexity is reduced greatly by introducing a novel scaling method. Finally, two examples are presented to illustrate the effectiveness of the proposed approaches.

Keywords: Multi-agent systems, Distributed consensus control, Observer-based controller, Linear matrix inequality, Denial-of-service attacks

1. Introduction

Cyber-physical systems (CPSs) has been intensively studied over the past few years, such as stability analysis [7, 18], sliding-mode observer [33], fault/attack detection [22], and control problems [21, 31], for its immense field of application, such as power grid systems, deep sea exploiting systems, and multi-agent systems (MASs).

Compared with the general computing systems where attacks limit their impact to the cyber realm, CPSs where attacks even can impact the physical world for the tight integra-

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tion of computation, networking, and physical process are more vulnerable [30]. Therefore, security problems for CPSs have attracted considerable attention: the performance degradation under stealthy integrity attacks [23], secure state estimation under sparse sensor attacks [19, 28], security control and distributed filtering under deception attacks [3, 4] and stability analysis under denial-of-service (DoS) attacks [27].

Especially, for the widespread applications of MASs [5], such as wireless sensor networks [1], spacecraft systems [2], service robots [12], and formation control of unmanned vehicles [15], the consensus of MASs has attracted intensive study, such as fault-tolerant consensus control [29], adaptive tracking control problem [17], distributed event-triggered control [13], and synchronization using observer [32]. While most of the previous results focus on leaderless consensus, leader following consensus is considered in [14, 16]. Besides, as a kind of CPSs, the security problems of MASs also have been intensively studied. For example, [26] considers a line consensus network with misbehaving agents which can bee seen as the sparse attacks considered in [19, 28], and [8] investigates distributed secure consensus for MASs under DoS attacks.

DoS attacks which is one of the most common attacks in CPSs, compromise the systems through rendering some or all components of a control system inaccessible. Especially, while the agents of MASs always communicate with each other individually, MASs are more vulnerable to DoS attacks since defending DoS attacks for all transmission channels is almost impossible. Although secure consensus for MASs under DoS attacks has been considered in [8, 9, 27], all the channels are seen as one channel from the angle of the adversaries which limits the application of the existing methods. Instead, studying multiple transmission channel case, where the adversaries can attack partial or all channels at any time with DoS duration limitation, is more practical, and this is the major motivation of this study.

Although CPSs with multiple channels under DoS have been considered in our previous result [20], the computational complexity has not been addressed well. Especially, while the number of transmission channels grows linearly with the size of common linear systems, as the size of MASs increases, the combinatorial transmission channels lead to explosive growth of the computational complexity. Thus, besides analyzing the stability, more efforts on addressing the computational complexity should be made for MASs.

This paper investigates the distributed secure consensus for MASs under DoS attacks. The considered DoS attacks are energy-limited, and on different channels, they are independent of each other. The main contributions of this work are summarized as follows:

(i) Both distributed state-feedback and observer-based controllers are proposed. To achieve secure consensus, the basic idea is to discard the information obtained from the attacked channels.

(ii) While the adversaries can attack partial or all channels at any time, various attack modes are considered. Based on the proposed two controllers, linear matrix inequality (LMI) technique is utilized to obtain the decay rates under different attack modes. Especially, based on the structure of the controller gain, a novel scaling method is introduced to reduce the computational complexity.

(iii) Based on the obtained decay rates corresponding to attack modes, by introducing a class of equivalent decay rates corresponding to communication channels, sufficient conditions on the duration of the DoS attacks, under which the consensus is still achieved, are provided in terms of inequalities.

The rest is organized as follows. In Section 2, the preliminaries are presented. Section 3 provides the decay rates under different attack modes. Section 4 analyzes the stability. Two examples are given in Section 5, and Section 6 concludes this paper.

Notation : For a matrix P, P^T denotes its transpose, $He(P) \triangleq P + P^T$, $P < 0$ denotes negative definiteness, and $\underline{\lambda}(P)$ denotes the smallest eigenvalue of P. Given vector $v_i \in \mathbb{R}^n$, $||v_i||$ is the Euclidean norm of v_i, and $col(v_1, \cdots, v_n) = [v_1^T, \cdots, v_n^T]^T$. $\mathbb{R}$ denotes the set of reals and $\mathbb{N}$ denotes the set of natural numbers. Given two sets Γ_1 and Γ_2, $\Gamma_1 \backslash \Gamma_2$ is the relative complement of Γ_2 in Γ_1, and $|\Gamma_1|$ is denoted as the cardinality of Γ_1. For interval $\mathcal{D}(t_1, t_2)$, $|\mathcal{D}(t_1, t_2)|$ is its length over $[t_1, t_2)$. $\mathbf{1}_n$ denotes the $n \times 1$ vector with all elements equal to 1, and I_n denotes the $n \times n$ identity matrix. I and 0 represent identity matrix and zero matrix with appropriate dimensions, respectively. $\otimes$ represents the Kronecker product.

2. Preliminaries

2.1. Algebraic Graph Theory

Consider a weighted undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ where $\mathcal{V} \in \{1, 2, \cdots, N\}$ and $\mathcal{E} \subseteq \mathcal{V}^2$ represent the agent set and edge set, respectively. $\mathcal{N}_i = \{j \in \mathcal{V} | (j, i) \in \mathcal{E}\}$ is the neighborhood set of agent i. If an edge ordered by $(i, j) \in \mathcal{E}$, agent j can be directly supplied with information from agent i, and each edge (i, j) is assigned a real-valued weight $a_{ji} > 0$. If there is no edge connecting node i and node j, one has $(i, j) \notin \mathcal{E}$, $a_{ji} = 0$ and $(j, i) \notin \mathcal{E}$, $a_{ij} = 0$. The Laplacian matrix of $\mathcal{G}$ is denoted by $\mathcal{L} = [l_{ij}]$, where

$$l_{ii} = \sum_{j=1}^{N} a_{ij}, \ l_{ij} = -a_{ij} \ for \ i \neq j. \tag{1}$$

Without loss of generality, $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$ are the eigenvalues of $\mathcal{L}$ and ν_i is the orthogonal eigenvector of $\mathcal{L}$ corresponding to λ_i. It is assumed that there is no self loop in the graph, and the considered undirected graph $\mathcal{G}$ is connected.

2.2. System description

Consider an MAS with N identical general linear dynamics of agents described by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) \tag{2}$$

where $x_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^{n_u}$ are the state and control input, respectively ($i = 1, 2, \cdots, N$). A and B are matrices of appropriate sizes. While A is not Hurwitz stable, it is assumed that (A, B) is stabilizable. Setting $x(t) = col(x_1(t), x_2(t), \cdots, x_N(t))$ and $u(t) = col(u_1(t), u_2(t), \cdots, u_N(t))$, then the whole MAS with agents described by (2) can be formulated as

$$\dot{x}(t) = (I_N \otimes A)x(t) + (I_N \otimes B)u(t). \tag{3}$$

2.3. Denial-of-Service Attack

As one of the most common attacks in CPSs, DoS attacks compromise the systems through rendering some or all components of a control system inaccessible. In this paper, different from [8, 9, 27] where all the transmission channels are assumed to be under the same DoS attack, it is assumed that DoS attacks on different channels (the edge $(i, j) \in \mathcal{E}$) are independent of each another. Besides, it is reasonable to assume that while channel (i, j) is attacked, channel (j, i) is also attacked.

As discussed in [8], it needs to terminate attack activities and shift to a sleep period to supply its energy for next attack. Then, similar to [20, 27], for each transmission channel ($(i, j) \in \mathcal{E}$), the following assumption on DoS duration (the time attack lasts) is given.

Assumption 1. *[27] (DoS Duration) There exist positive scalars ς_{ij} and $\mu_{ij} < 1$ such that*

$$|\mathcal{D}_{(i,j)}(s, t)| \leq \varsigma_{ij} + \mu_{ij}(t - s) \tag{4}$$

where $\mathcal{D}_{(i,j)}(s, t)$ is the union of DoS intervals of channel $(i, j) \in \mathcal{E}$ over $[s, t)$, $i < j$. μ_{ij} reflects the attack intensity: for edge (i, j), a maximum of $100\mu_{ij}\%$ of communication denials on the average are permitted.

Remark 1. *Assumption 1 is inspired by the concept of average dwell time [34], and larger μ_{ij} implies more intensive attacks. Since edges (i, j) and (j, i) are seen as one channel, only the edge (i, j) with $i < j$ is considered, and $\mathcal{D}_{(i,j)} = \mathcal{D}_{(j,i)}$. Besides, it should be also noted that while [8] only considers two cases (under DoS or not), in this paper, various attack modes (different channels $(i, j) \in \mathcal{E}$ are attacked: partial or all) should be considered which is a major difficulty in this study.*

Then, some discussions on the attack modes are given. Define

$$\Gamma(t) = \{(i,j) \in \mathcal{E} | t \in \mathcal{D}_{(i,j)}(0,\infty)\} \tag{5}$$

as the set of channels which are attacked at time t, and

$$\Xi_\Gamma(t_1,t_2) = (\cap_{(i,j)\in\Gamma}\mathcal{D}_{(i,j)}(t_1,t_2)) \cap (\cap_{(i,j)\notin\Gamma}\bar{\mathcal{D}}_{(i,j)}(t_1,t_2)) \tag{6}$$

as the union of the intervals where the channels indexed by the set $\Gamma \subseteq \mathcal{E}$ are attacked and the channels indexed by $\mathcal{E}\backslash\Gamma$ are not attacked ($t_1 < t_2$ and $\bar{\mathcal{D}}_{(i,j)}(t_1,t_2) = [t_1,t_2]\backslash\mathcal{D}_{(i,j)}(t_1,t_2)$).

Remark 2. *Actually, (5) provides an index for the attack modes. It is easy to see that $\Gamma(t) \subseteq \mathcal{E}$, and for any $(i,j) \in \Gamma(t)$, one has $(j,i) \in \Gamma(t)$. Thus, from $\Gamma(t) = \emptyset$ to $\Gamma(t) = \mathcal{E}$, there are $2^{\frac{|\mathcal{E}|}{2}}$ different attack modes. Then, (6) partitions the interval $[t_1,t_2]$ into $2^{\frac{|\mathcal{E}|}{2}}$ subintervals $\Xi_\Gamma(t_1,t_2)$ which will play an important role in the following study. It is easy to see that*

$$\cup_{\Gamma\subseteq\mathcal{E}}\Xi_\Gamma(t_1,t_2) = [t_1,t_2] \tag{7}$$

$$\mathcal{D}_{(i,j)}(t_1,t_2) = \cup_{\Gamma\subseteq\mathcal{E},(i,j)\in\Gamma}\Xi_\Gamma(t_1,t_2). \tag{8}$$

2.4. Control Objective

This paper considers a class of MASs under DoS attacks where adversaries compromise each channel independently. The goal is to develop a class of distributed control laws $u_i(t)$ for the system (2) such that the consensus is still achieved under DoS attacks satisfying Assumption 1:

$$\lim_{t\to\infty} ||x_i(t) - x_j(t)|| = 0,\ \forall i,j = 1,2,\cdots,N. \tag{9}$$

Remark 3. *To achieve such goal, the following two steps will be adopted: (i) designing a class of distributed control laws, (ii) analyzing the resilience to DoS attacks of the system (2) under the proposed control laws. However, as discussed in Remark 1, since the adversaries attack each channel independently, there are numerous cases (indexed by $\Gamma \subseteq \mathcal{E}$) to be considered. Although CPSs with multiple channels under DoS have been considered in our previous result [20], the computational complexity for analyzing the decay rates under different cases has not been addressed well. In this paper, while the number of channels $|\mathcal{E}|/2 \le (N^2-N)/2$ may be very large (as shown in Remark 2, the number of attack modes is $2^{\frac{|\mathcal{E}|}{2}}$), an important task is to provide an efficient way to analyze these decay rates.*

3. Distributed Control Laws

In this section, two secure distributed control laws are proposed, and the major objective is to analyze the decay rates for the closed-loop system under different attack modes indexed by (5).

3.1. Distributed State Feedback Controller

In this subsection, the following distributed state-feedback consensus controller is adopted:

$$u_i(t) = K \sum_{j \in \mathcal{N}_i, (j,i) \notin \Gamma(t)} a_{ij}(x_j(t) - x_i(t)) \tag{10}$$

where $K \in \mathbb{R}^{n \times n}$ is the controller gain to be designed. (10) is similar to the existing state-feedback controllers in [25, 29], and the difference is that the attack mode $\Gamma(t)$ is taken into account. The basic strategy for security is to discard the term $x_j - x_i$ while $(j,i) \in \Gamma(t)$.

Remark 4. *It should be noted that although the attack mode $\Gamma(t)$, which is actually unknown for each agent, is utilized in (10), not all the elements in $\Gamma(t)$ are necessary. It is easy to see that for agent i, if $j \in \mathcal{N}_i$, whether $(j,i) \in \Gamma(t)$ is known (while x_j is unavailable, $(j,i) \in \Gamma(t)$ is confirmed by agent i, and vice versa) which ensures that the proposed controller (10) is feasible.*

Then, substituting (10) into (3) yields

$$\dot{x}(t) = (I_N \otimes A - (\mathcal{L} - \mathcal{L}_{\Gamma(t)}) \otimes BK)x(t) \tag{11}$$

where $\mathcal{L}_{\Gamma(t)}$ is defined as $\mathcal{L}$ with a_{ij} (utilized in (1), $(j,i) \notin \Gamma(t)$) replaced by 0. Considering that the undirected graph $\mathcal{G}$ is connected, $\mathcal{L}$ is symmetric and positive semi-definite, and $\lambda_2 > 0$ (defined in Section 2.1). Setting $\Psi = [\mathbf{1}_N/\sqrt{N}\ \nu]$ and $\mathcal{M} = I_N - (\mathbf{1}_N \mathbf{1}_N^T)/N$ where $\nu = [\nu_2, \cdots, \nu_N]$, ν_i is defined in Section 2.1 ($\mathcal{L}\nu_i = \lambda_i \nu_i$), the following properties are obtained

$$\Psi^T \Psi = \Psi \Psi^T = I_N,\ \Psi^T \mathcal{L} \Psi = diag\{0, \lambda_2, \cdots, \lambda_N\} \tag{12}$$

$$\Psi^T \mathcal{L}_\Gamma \Psi = diag\{0, \nu^T \mathcal{L}_\Gamma \nu\},\ \mathcal{L} - \mathcal{L}_\Gamma \geq 0 \tag{13}$$

$$\mathcal{M}\mathcal{L} = \mathcal{L}\mathcal{M} = \mathcal{L},\ \mathcal{M}\mathcal{L}_\Gamma = \mathcal{L}_\Gamma \mathcal{M} = \mathcal{L}_\Gamma. \tag{14}$$

Defining an error vector $\delta_i = x_i - \bar{x}(t)$ where $\bar{x}(t) = \frac{1}{N}\sum_{i=1}^N x_i(t)$, one has

$$\delta(t) = (\mathcal{M} \otimes I_n)x(t) \tag{15}$$

where $\delta(t) = col(\delta_1(t), \cdots, \delta_N(t))$. Based on (11), (14) and (15), the time derivative of δ is obtained:

$$\dot{\delta}(t) = (I_N \otimes A - (\mathcal{L} - \mathcal{L}_{\Gamma(t)}) \otimes BK)\delta(t). \tag{16}$$

Next, the following theorem analyzes the decay rates of the system (11) with controller (10) under different attack modes.

Theorem 3.1. *For a connected undirected graph $\mathcal{G}$ with agents (2), given scalars α_Γ, if there exist positive symmetric definite matrices X and R such that*

$$I_{N-1} \otimes (He(AX) - \alpha_\Gamma X) - 2(\Lambda - \Lambda_\Gamma) \otimes BRB^T < 0 \tag{17}$$

where $\Lambda = diag\{\lambda_2, \cdots, \lambda_N\}$, $\Lambda_\Gamma = \nu^T \mathcal{L}_\Gamma \nu$, and $\Gamma \subseteq \mathcal{E}$, then the following inequalities are guaranteed

$$\dot{V}(t) \leq \alpha_\Gamma V(t),\ t \in \Xi_\Gamma(0, \infty) \tag{18}$$

where

$$V(t) = \delta^T(t)(I_N \otimes P)\delta(t) \tag{19}$$

and $P = X^{-1}$, under the distributed state-feedback consensus controller (10) with

$$K = RB^T P. \tag{20}$$

Proof. Choose $V(t)$ as a Lyapunov function candidate. Based on (5) and (6), for $t \in \Xi_{\Gamma(t)}$, it follows from (16) that

$$\dot{V}(t) = \delta^T (I_N \otimes He(PA) - 2(\mathcal{L} - \mathcal{L}_{\Gamma(t)}) \otimes PBK)\delta \tag{21}$$

where $\delta = \delta(t)$. Setting $\tilde{\delta} = (\Psi^T \otimes I_n)\delta = col(\tilde{\delta}_1, \cdots, \tilde{\delta}_N)$, one has $\tilde{\delta}_1 = (\mathbf{1}_N^T/\sqrt{N} \otimes I_n)(\mathcal{M} \otimes I_n)x(t) = 0$. Then, it follows from (21) that

$$\begin{aligned}
\dot{V}(t) &\overset{(a)}{=} \tilde{\delta}^T (I_N \otimes He(PA) - (\Psi^T(\mathcal{L} - \mathcal{L}_{\Gamma(t)})\Psi) \otimes 2PBK)\tilde{\delta} \\
&\overset{(b)}{=} \tilde{\delta}_{2:N}^T (I_{N-1} \otimes He(PA) - 2(\Lambda - \Lambda_{\Gamma(t)}) \otimes PBK)\tilde{\delta}_{2:N} \\
&\overset{(c)}{\leq} \alpha_{\Gamma(t)} \tilde{\delta}_{2:N}^T (I_{N-1} \otimes P)\tilde{\delta}_{2:N} \overset{(d)}{=} \alpha_{\Gamma(t)} V(t)
\end{aligned} \tag{22}$$

where $\tilde{\delta}_{2:N} = col(\tilde{\delta}_2, \cdots, \tilde{\delta}_N)$, (a) holds for that $\Psi^T \Psi = I_N$ (given in (12)), (b) holds for that $\tilde{\delta}_1 = 0$, (c) is obtained by pre- and post-multiplying (17) by $I_{N-1} \otimes P$ with the help of (20), and (d) holds for that $\tilde{\delta}_1 = 0$ and $\Psi\Psi^T = I_N$. While (22) implies (18), the proof is completed.

In Theorem 3.1, α_Γ should be chosen such that (17) is feasible for all $\Gamma \subseteq \mathcal{E}$, and as shown in (18), α_Γ are the decay rates. However, since there are $2^{\frac{|\mathcal{E}|}{2}}$ different Γ (as discussed in Remark 2), solving (17) with $\Gamma \subseteq \mathcal{E}$ is very hard for large systems. In the following, a corollary is given to reduce the computation requirement at the cost introducing of some conservatism.

Corollary 3.2. *For a connected undirected graph $\mathcal{G}$ with agents (2), given scalars α_0, $\alpha_\emptyset$, $\alpha_\mathcal{E}$ and $\alpha_{\{(i,j),(j,i)\}}$, if there exist positive symmetric definite matrices X and R such that*

$$He(AX) - \alpha_{\{(i,j),(j,i)\}}X - 2\underline{\lambda}_{(i,j)}BRB^T < 0 \tag{23}$$

$$He(AX) - \alpha_\emptyset X - 2\lambda_2 BRB^T < 0 \tag{24}$$

$$He(AX) - \alpha_\mathcal{E} X < 0 \tag{25}$$

$$\alpha_0 X - (He(AX) - 2\lambda_N BRB^T) < 0 \tag{26}$$

where $\underline{\lambda}_{(i,j)} = \underline{\lambda}(\Lambda - \Lambda_{\{(i,j),(j,i)\}})$, $i < j$ and $(i,j) \in \mathcal{E}$ (Λ is defined in Theorem 3.1), then (18) with given $\alpha_\emptyset$ and

$$\alpha_\Gamma = \min\{\alpha_0 - \sum_{i<j,(i,j)\in\Gamma}(\alpha_0 - \alpha_{\{(i,j),(j,i)\}}),\ \alpha_\mathcal{E}\},\ \Gamma \neq \emptyset \tag{27}$$

is guaranteed under the distributed state-feedback controller (10) with K defined in (20).

Proof. It follows from (23)-(25) that

$$\Omega - I_{N-1} \otimes \alpha_\Gamma X + 2\Lambda_\Gamma \otimes BRB^T < 0 \tag{28}$$

where $\Omega = I_{N-1} \otimes He(AX) - 2\Lambda \otimes BRB^T$, holds for $\Gamma = \emptyset$, $\mathcal{E}$ and $\{(i,j),(j,i)\}$.

Then, for $\alpha_\Gamma = \alpha_0 - \sum_{i<j,(i,j)\in\Gamma}(\alpha_0 - \alpha_{\{(i,j),(j,i)\}})$ ($\Gamma \neq \emptyset$), one can deduce that

$$\begin{aligned} &\Omega - I_{N-1} \otimes \alpha_\Gamma X + 2\Lambda_\Gamma \otimes BRB^T \\ &\overset{(a)}{\leq} \Omega - I_{N-1} \otimes \alpha_\Gamma X - \sum_{i<j,(i,j)\in\Gamma}(\Omega - I_{N-1} \otimes \alpha_{\{(i,j),(j,i)\}}X) \\ &\overset{(b)}{\leq} (\frac{|\Gamma|}{2} - 1)(I_{N-1} \otimes \alpha_0 X - \Omega) \overset{(c)}{<} 0 \end{aligned} \tag{29}$$

where (a) is obtained from (28) and the fact that $\Lambda_\Gamma = \sum_{i<j,(i,j)\in\Gamma}\Lambda_{\{(i,j),(j,i)\}}$, (b) holds for that $\alpha_\Gamma = -(|\Gamma|/2 - 1)\alpha_0 + \sum_{i<j,(i,j)\in\Gamma}\alpha_{\{(i,j),(j,i)\}}$, and (c) is finally obtained from (26).

Meanwhile, for $\alpha_\Gamma = \alpha_\mathcal{E}$ ($\Gamma \neq \emptyset$), considering that $\mathcal{L} - \mathcal{L}_\Gamma \geq 0$ (given in (13)), it is easy to see that

$$\Omega - I_{N-1} \otimes \alpha_\Gamma X + 2\Lambda_\Gamma \otimes BRB^T \leq I_{N-1} \otimes (He(AX) - \alpha_\mathcal{E} X) \leq 0 \tag{30}$$

where (25) is utilized. Finally, (28), (29) and (30) imply that (17) holds for all $\Gamma \subseteq \mathcal{E}$. Then, the proof is completed with the help of Theorem 3.1.

Remark 5. *Compared with Theorem 3.1 containing $2^{\frac{|\mathcal{E}|}{2}}$ LMIs (17) of order $(N-1)n$, Corollary 3.2 containing $\frac{|\mathcal{E}|}{2}+3$ LMIs of order n requires much less computing resources. However, it should be also noted that Corollary 3.2 is more conservative than Theorem 3.1, and such fact will be shown in Section 5-Example. Besides, since $\Gamma(t)$ is unknown, for the time-varying system (11) indexed by $\Gamma(t)$, a common state-feedback controller gain K is designed based on the common Lyapunov function (19) at the cost of introducing some conservatism. Meanwhile, (30) implies that (18) holds for all $\alpha_\Gamma = \alpha_\mathcal{E}$. Then, while $\alpha_\mathcal{E} < 0$ (A is Hurwitz stable), the agent group will achieve consensus under any DoS attacks. Thus, after (2), A is assumed to be not Hurwitz stable which implies that $\alpha_\mathcal{E} \geq 0$.*

3.2. Distributed Observer-based Controller

In this subsection, the following distributed observer-based consensus controller is adopted:

$$\dot{\hat{x}}^i(t) =(I_N \otimes A - \mathcal{L} \otimes BK)\hat{x}^i(t) + [F_i^0 \ F_i] \begin{bmatrix} I_N^i \otimes I \\ (\mathcal{A}^i - \mathcal{A}^i_{\Gamma(t)}) \otimes I \end{bmatrix} (\hat{x}^i(t) - x(t)) \tag{31}$$

$$u_i(t) =K \sum_{j\in\mathcal{N}_i,(j,i)\notin\Gamma(t)} a_{ij}(x_j(t) - x_i(t)) + K \sum_{j\in\mathcal{N}_i,(j,i)\in\Gamma(t)} a_{ij}(\hat{x}^i_j(t) - x_i(t)) \tag{32}$$

where $\hat{x}^i(t) = col(\hat{x}^i_1(t), \cdots, \hat{x}^i_N(t))$ is the state estimation available for the agent i ($\hat{x}^i_j$ is the estimation of x_j available for the agent i), K and $[F_i^0 \ F_i]$ are the controller and observer gains to be designed, respectively. Compared with the state-feedback controller (10), for each agent, an observer is proposed to estimate the global state with the help of local information, and the state estimation is utilized in (32) for faster convergence rate.

It follows from (32) that

$$(I_N \otimes B)u(t) = -(\mathcal{L} \otimes BK)x(t) + \sum_{j=1}^{N}((I_N^j)^T \mathcal{A}^j_{\Gamma(t)} \otimes BK)(x(t) - \hat{x}^j(t)) \tag{33}$$

where $u(t)$ is defined before (3), $\mathcal{A}^i_\Gamma$ and I^i_N are the ith row of $\mathcal{A}_\Gamma$ and I_N, respectively, $\mathcal{A}_\Gamma$ is defined as $\mathcal{L}_\Gamma$ (defined in (11)) with the diagonal elements replaced by 0. Substituting (33) into (3), one has

$$\dot{x}(t) = (I_N \otimes A - \mathcal{L} \otimes BK)x(t) + \sum_{j=1}^{N}((I_N^j)^T \mathcal{A}^j_{\Gamma(t)} \otimes BK)e^j(t) \tag{34}$$

where $e^i = x - \hat{x}^i$. Then, combining (31) and (34) yields

$$\begin{aligned}\dot{e}^i(t) =&(I_N \otimes A - \mathcal{L} \otimes BK + F_i^0(I_N^i \otimes I) + F_i((\mathcal{A}^i - \mathcal{A}^i_{\Gamma(t)}) \otimes I))e^i(t) \\ &+ \sum_{j=1}^{N}((I_N^j)^T \mathcal{A}^j_{\Gamma(t)} \otimes BK)e^j(t)\end{aligned}$$

where $\mathcal{A}^i$ is the ith row of $\mathcal{A}$, $\mathcal{A}$ is defined as $\mathcal{L}$ with l_{ii} replaced by 0. Next, the dynamics of the collective vector $e = col(e^1, \cdots, e^N)$ is expressed as

$$\dot{e}(t) =(\mathbb{A}_{1,\Gamma(t)} + \mathbb{A}_{2,\Gamma(t)})e(t) \tag{35}$$

where $\mathbb{A}_{1,\Gamma} = diag\{\mathbb{A}^1_{1,\Gamma}, \cdots, \mathbb{A}^N_{1,\Gamma}\}$, $\mathbb{A}^i_{1,\Gamma} = I_N \otimes A - (\mathcal{L} - (I_N^i)^T\mathcal{A}^i_\Gamma) \otimes BK + F_i^0(I_N^i \otimes I) + F_i((\mathcal{A}^i - \mathcal{A}^i_\Gamma) \otimes I)$, $\mathbb{A}_{2,\Gamma}$ is defined as $\mathbf{1}_N \otimes [(I_N^1)^T\mathcal{A}^1_\Gamma \otimes BK, \cdots, (I_N^N)^T\mathcal{A}^N_\Gamma \otimes BK]$ with the diagonal block $(I_N^i)^T\mathcal{A}^i_\Gamma \otimes BK$ replaced by 0. Now, based on (34) and (35), the dynamics of the augmented vector $col(\delta(t), e(t))$ (δ is defined in (15)) is expressed as

$$\begin{bmatrix} \dot{\delta}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} I_N \otimes A - \mathcal{L} \otimes BK & \mathbb{A}_{3,\Gamma(t)} \\ 0 & \mathbb{A}_{1,\Gamma(t)} + \mathbb{A}_{2,\Gamma(t)} \end{bmatrix} \begin{bmatrix} \delta(t) \\ e(t) \end{bmatrix} \tag{36}$$

where $\mathbb{A}_{3,\Gamma} = [(\mathcal{M}^1)^T \mathcal{A}_\Gamma^1 \otimes BK, \cdots, (\mathcal{M}^N)^T \mathcal{A}_\Gamma^N \otimes BK]$, $\mathcal{M}^i$ is the ith row of $\mathcal{M}$, and the first row block is obtained from (34) similar to (16).

First, while K is given (using Theorem 3.1 or Corollary 3.2), the following theorem provides a class of appropriate observer gains F_i^0 and F_i, and analyzes the decay rates.

Theorem 3.3. *For a connected undirected graph $\mathcal{G}$ with agents (2), given scalars α_Γ, if there exist positive symmetric definite matrices P and $\tilde{P}$, and matrices S_i^0 and S_i such that*

$$\begin{bmatrix} \mathbb{A}_{0,\Gamma} & (\nu^T \otimes I)\mathbb{A}_{3,\Gamma}^P \\ * & He(\mathbb{A}_{1,\Gamma}^{\bar{P}} + \bar{P}\mathbb{A}_{2,\Gamma}) - \alpha_\Gamma \bar{P} \end{bmatrix} < 0 \tag{37}$$

where $\bar{P} = I_N \otimes \tilde{P}$, $\mathbb{A}_{0,\Gamma} = He(I_{N-1} \otimes PA - \Lambda \otimes PBK) - \alpha_\Gamma I_{N-1} \otimes P$, $\mathbb{A}_{3,\Gamma}^P$ is defined as $\mathbb{A}_{3,\Gamma}$ with BK replaced by PBK, $\mathbb{A}_{1,\Gamma}^{\bar{P}}$ is defined as $\mathbb{A}_{1,\Gamma}$ with $\mathbb{A}_{1,\Gamma}^i$ replaced by $\tilde{P}(I_N \otimes A - (\mathcal{L} - (I_N^i)^T \mathcal{A}_\Gamma^i) \otimes BK) + S_i^0(I_N^i \otimes I) + S_i((\mathcal{A}^i - \mathcal{A}_\Gamma^i) \otimes I)$, and $\Gamma \subseteq \mathcal{E}$, then (18) with

$$V(t) = \delta^T(t)(I_N \otimes P)\delta(t) + e^T(t)\bar{P}e(t) \tag{38}$$

is guaranteed under the distributed observer-based consensus controller (32) with

$$F_i^0 = \tilde{P}^{-1}S_i^0, \; F_i = \tilde{P}^{-1}S_i. \tag{39}$$

Proof. Considering the Lyapunov function candidate (38), for $t \in \Xi_{\Gamma(t)}$, the time derivative of $V(t)$ is obtained from (36)

$$\dot{V}(t) = 2\begin{bmatrix} \delta(t) \\ e(t) \end{bmatrix}^T \begin{bmatrix} I_N \otimes PA - \mathcal{L} \otimes PBK & \mathbb{A}_{3,\Gamma(t)}^P \\ 0 & \bar{P}\mathbb{A}_{1,\Gamma(t)} + \bar{P}\mathbb{A}_{2,\Gamma(t)} \end{bmatrix} \begin{bmatrix} \delta(t) \\ e(t) \end{bmatrix}. \tag{40}$$

Similar to (22), it follows from (40) that

$$\begin{aligned} \dot{V}(t) &= 2\begin{bmatrix} \tilde{\delta}_{2:N} \\ e(t) \end{bmatrix}^T \begin{bmatrix} I_{N-1} \otimes PA - \Lambda \otimes PBK & (\nu^T \otimes I)\mathbb{A}_{3,\Gamma(t)}^P \\ 0 & \bar{P}\mathbb{A}_{1,\Gamma(t)} + \bar{P}\mathbb{A}_{2,\Gamma(t)} \end{bmatrix} \begin{bmatrix} \tilde{\delta}_{2:N} \\ e(t) \end{bmatrix} \\ &\le \alpha_{\Gamma(t)} V(t) \end{aligned} \tag{41}$$

where (37), (39) and the fact that $\tilde{\delta}_1 = 0$ are utilized ($\tilde{\delta}$ is defined before (22)). Thereby proving the theorem.

Similar to the discussions after Theorem 3.1, Theorem 3.3 is also not suitable for large systems. Then, by specifying

$$F_i^0 = (\mathcal{A}^i - \kappa I_N^i)^T \otimes BK, \; F_i = (I_N^i)^T \otimes BK \tag{42}$$

where κ is a positive scalar, $\mathbb{A}_{1,\Gamma}^i$ defined in (35) becomes

$$\mathbb{A}_{1,\Gamma}^i = I_N \otimes A - \tilde{\mathcal{L}}_i \otimes BK \tag{43}$$

where $\tilde{\mathcal{L}}_i = \mathcal{L} + \kappa(I_N^i)^T I_N^i - He((I_N^i)^T \mathcal{A}^i)$, which is independent of attack mode Γ. Then, the following corollary is provided.

Corollary 3.4. *For a connected undirected graph $\mathcal{G}$ with agents (2), given scalars κ, α_0, $\alpha_\emptyset$ and $\alpha_{\{(i,j),(j,i)\}}$, if there exist positive symmetric definite matrices X and R such that*

$$He(AX) - \alpha_{\{(i,j),(j,i)\}}X - 2\underline{\tilde{\lambda}}_{(i,j)}BRB^T < 0 \tag{44}$$

$$He(AX) - \alpha_\emptyset X - 2\lambda_m BRB^T < 0 \tag{45}$$

$$\alpha_0 X - (He(AX) - 2\lambda_M BRB^T) < 0 \tag{46}$$

where $\underline{\tilde{\lambda}}_{(i,j)} = \underline{\lambda}(diag\{\Lambda, \Lambda_1, \cdots, \Lambda_N\} - \tilde{\Psi}_{\{(i,j),(j,i)\}})$, *$i < j$ and $(i,j) \in \mathcal{E}$,* $\lambda_m = min(\lambda_2, \lambda_{11}, \cdots, \lambda_{N1})$, $\lambda_M = max(\lambda_N, \lambda_{1N}, \cdots, \lambda_{NN})$,

$$\tilde{\Psi}_\Gamma = \begin{bmatrix} 0 & \tilde{\Psi}_\Gamma^1 & \cdots & \tilde{\Psi}_\Gamma^N \\ * & \tilde{\Psi}_\Gamma^{11} & \cdots & \tilde{\Psi}_\Gamma^{1N} \\ * & * & \ddots & \vdots \\ * & * & * & \tilde{\Psi}_\Gamma^{NN} \end{bmatrix},$$

$\tilde{\Psi}_\Gamma^i = (\mathcal{M}^i\nu)^T \mathcal{A}_\Gamma^i \Psi_i$, $\tilde{\Psi}_\Gamma^{ij} = \Psi_i^T((I_N^j)^T \mathcal{A}_\Gamma^j + (\mathcal{A}_\Gamma^i)^T I_N^i)\Psi_j$ *($i \neq j$) and* $\tilde{\Psi}_{ii} = 0$, *Ψ_i is an orthonormal matrix such that* $\Psi_i^T \tilde{\mathcal{L}}_i \Psi_i = \Lambda_i = diag\{\lambda_{i1}, \cdots, \lambda_{iN}\}$, *and λ_{il} is the lth smallest eigenvalue of $\tilde{\mathcal{L}}_i$ ($i,j = 1, \cdots, N$), then (18) with $V(t)$ defined in (38) ($P = \tilde{P} = X^{-1}$), given $\alpha_\emptyset$ and*

$$\alpha_\Gamma = \alpha_0 - \sum_{i<j,(i,j)\in\Gamma} (\alpha_0 - \alpha_{\{(i,j),(j,i)\}}),\ \Gamma \neq \emptyset, \tag{47}$$

is guaranteed under the distributed observer-based consensus controller (32) with (20) and (42).

Proof. Considering (20), (43) and the facts that $\Psi_i^T\Psi_i = 1$ and $\Psi_i^T\tilde{\mathcal{L}}_i\Psi_i = \Lambda_i$, pre- and post-multiplying (37) with $P = \tilde{P} = X^{-1}$ by $diag\{I_{N-1} \otimes X, \Psi_1^T \otimes X, \cdots, \Psi_N^T \otimes X\}$ and its transpose, respectively, yields

$$diag\{\tilde{\mathbb{A}}, \tilde{\mathbb{A}}_1, \cdots, \tilde{\mathbb{A}}_N\} + \tilde{\Psi}_\Gamma \otimes BRB^T < 0 \tag{48}$$

where $\tilde{\mathbb{A}} = I_{N-1} \otimes (He(AX) - \alpha_\Gamma X) + \Lambda \otimes BRB^T$, $\tilde{\mathbb{A}}_i = I_N \otimes (He(AX) - \alpha_\Gamma X) + \Lambda_i \otimes BRB^T$. It is easy to see that (48) is equivalent to (37).

Finally, based on the fact that $\tilde{\Psi}_\Gamma = \sum_{i<j,(i,j)\in\Gamma} \tilde{\Psi}_{\{(i,j),(j,i)\}}$, the proof can be completed by following the exact same argument of Corollary 3.2.

Remark 6. *Compared with Theorem 3.3 containing $2^{\frac{|\mathcal{E}|}{2}}$ LMIs (37) of order $(N-1)n + N^2 n$, by specifying the observer gains in (42), Corollary 3.4 only contains $\frac{|\mathcal{E}|}{2} + 2$ LMIs of order n. Thus, adopting Corollary 3.4 reduces the computational complexity greatly. Besides, it should be noted that compared with the state-feedback controller (10), while the observer-based controller (32) is adopted, the Laplacian matrix $\mathcal{L}$ should be known for each agent. Meanwhile, since the number of estimator grows with the number of neighbors which may lead to more installation costs, the proposed controller (32) is not suitable for large systems. In Section 5-Example, it will be shown that the controller (32) provides faster convergence rate than (10).*

4. Stability Analysis

In Section 3, Theorems 3.1, 3.3 and two corollaries are provided to analyze the decay rates for the closed-loop system under different attack modes Γ ($\Gamma \subseteq \mathcal{E}$, and $(i,j) \in \Gamma$ and $(j,i) \in \Gamma$ hold simutaniously). Based on these obtained decay rates α_Γ, this section focuses on analyzing the stability of the closed-loop system under DoS attacks.

Theorem 4.1. *For a connected undirected graph $\mathcal{G}$ with agents (2), if there exist a Lyapunov function $V(t)$ satisfying (18) under controller (10) (or (32)) and scalars θ_1^{ij} and θ_2^{ij} such that*

$$\theta_1^{ij} - \theta_2^{ij} \geq 0 \tag{49}$$

$$\alpha_\Gamma - (\sum_{(i,j)\in\Gamma} \theta_1^{ij} + \sum_{(i,j)\in\mathcal{E}\backslash\Gamma} \theta_2^{ij}) \leq 0 \tag{50}$$

$$\bar{\mu} = \sum_{(i,j)\in\mathcal{E}} (\mu_{ij}\theta_1^{ij} + (1-\mu_{ij})\theta_2^{ij}) < 0 \tag{51}$$

where $(i,j) \in \mathcal{E}$ and $\Gamma \subseteq \mathcal{E}$, then the agent group achieves consensus (9) despite the DoS attacks satisfying Assumption 1.

Proof. Assume that ζ_k ($k \in \mathbb{N}, \zeta_0 = 0$) are the time instants where $\Gamma(t)$ changes (at least one DoS off/on (or on/off) transition occurs).

(i) For $t \in [\zeta_k, \zeta_{k+1})$, (18) implies

$$\begin{aligned} V(t) \leq & e^{\alpha_{\Gamma(\zeta_k)}(t-\zeta_k)} V(\zeta_k) \\ \leq & e^{\mathfrak{D}_k} V(\zeta_0) = e^{\mathfrak{D}(0,t)} V(0) \end{aligned} \tag{52}$$

where $\mathfrak{D}_k = \alpha_{\Gamma(\zeta_k)}(t-\zeta_k) + \sum_{m=1}^{k} \alpha_{\Gamma(\zeta_m)}(\zeta_m - \zeta_{m-1})$, $\mathfrak{D}(0,t) = \sum_{\Gamma\subseteq\mathcal{E}} \alpha_\Gamma |\Xi_\Gamma(0,t)|$.

(ii) It follows from (50) that

$$\begin{aligned} \mathfrak{D}(0,t) \leq & \sum_{\Gamma\subseteq\mathcal{E}} (\sum_{(i,j)\in\Gamma} \theta_1^{ij} + \sum_{(i,j)\in\mathcal{E}\backslash\Gamma} \theta_2^{ij}) \Xi_\Gamma(0,t) \\ = & \sum_{(i,j)\in\mathcal{E}} (\theta_1^{ij} \sum_{\Gamma\subseteq\mathcal{E},(i,j)\in\Gamma} |\Xi_\Gamma(0,t)| + \theta_2^{ij} \sum_{\Gamma\subseteq\mathcal{E},(i,j)\notin\Gamma} |\Xi_\Gamma(0,t)|) \\ \overset{(a)}{=} & \sum_{(i,j)\in\mathcal{E}} ((\theta_1^{ij} - \theta_2^{ij})|\mathcal{D}_{(i,j)}(0,t)| + \theta_2^{ij} t) \\ \overset{(b)}{\leq} & \bar{\mu} t + \bar{\varsigma} \end{aligned} \tag{53}$$

where $\bar{\varsigma} = \sum_{(i,j)\in\mathcal{E}} (\theta_1^{ij} - \theta_2^{ij}) \varsigma_{ij}$, (a) is obtained from (8) and the fact that $\sum_{\Gamma\subseteq\mathcal{E},(i,j)\notin\Gamma} |\Xi_\Gamma(0,t)| = |[0,t]/\mathcal{D}_{(i,j)}(0,t)| = t - |\mathcal{D}_{(i,j)}(0,t)|$.

(iii) Finally, it follows from (52) and (53) that $\lim_{t\to\infty} V(t) = 0$ which yields

$$\lim_{t\to\infty} ||\delta(t)|| = 0. \tag{54}$$

where the fact that $\underline{\lambda}(P)||\delta(t)||^2 \le V(t)$ holds for both $V(t)$ defined in (19) and (38) is utilized. While (54) implies (9), the agent group achieves consensus despite DoS attacks and the proof is completed.

Remark 7. *Although the decay rates α_Γ have been provided in Section 3, since α_Γ corresponding to attack modes Γ and $\mathcal{D}_{(i,j)}$ corresponding to channels (i,j) are unmatched which is the major difficulty for studying the systems with multiple channels under DoS attacks, the technique for switched systems with stable and unstable subsystems which has been adopted in [8] does not apply here. Thus, in Theorem 4.1, a class of equivalent decay rates θ_1^{ij} and θ_2^{ij} which are corresponding to the cases that the channel $(i,j) \in \mathcal{E}$ is and is not under DoS attacks, respectively, is introduced. Moreover, while the disturbance is taken into account, similar to Theorem 4.1 in our previous result [20], input-to-state stability is available.*

Remark 8. *It is easy to see that (49)-(51) can be verified by using LMI toolbox. However, since $\Gamma \subseteq \mathcal{E}$ which implies that the number of inequalities (50) is $2^{\frac{|\mathcal{E}|}{2}}$, solving (50) may be very time-consuming for large systems. Considering that for any sets $\Gamma_1 \subseteq \Gamma_2$, $\alpha_{\Gamma_2} + (\sum_{(i,j)\in\Gamma_2} \theta_1^{ij} + \sum_{(i,j)\in\mathcal{E}\setminus\Gamma_2} \theta_2^{ij}) \ge \min\{\alpha_{\Gamma_1}, \alpha_{\Gamma_2}\} + (\sum_{(i,j)\in\Gamma_1} \theta_1^{ij} + \sum_{(i,j)\in\mathcal{E}\setminus\Gamma_1} \theta_2^{ij})$ under the limitation (49), for given sets $\tilde{\Gamma}$ and $\tilde{\mathcal{E}}$ satisfying that $\Gamma \subseteq \tilde{\Gamma}$ and α_Γ approximates $\alpha_{\tilde{\Gamma}}$ for any $\Gamma \in \tilde{\mathcal{E}}$, inequalities (50) indexed by $\Gamma \in \tilde{\mathcal{E}}$ can be replaced by one inequality (50) with α_Γ and Γ replaced by $\max_{\Gamma\in\tilde{\mathcal{E}}}\{\alpha_\Gamma\}$ and $\tilde{\Gamma}$, respectively. Based on such idea, the computational complexity can be reduced.*

In the following, another theorem, which can be verified directly (without the help of LMI toolbox) but introduces more conservatism, is proposed.

Theorem 4.2. *For a connected undirected graph $\mathcal{G}$ with agents (2), given scalars α_0 (defined in Corollary 3.2 and Corollary 3.4) and α_Γ, if one of the following is true,*

$$\alpha_\emptyset + (\alpha_M - \alpha_\emptyset) \sum_{i<j,(i,j)\in\mathcal{E}} \mu_{ij} < 0 \tag{55}$$

$$\bar{\alpha}_0 - \sum_{i<j,(i,j)\in\mathcal{E}} \mu_{ij} \Upsilon_{(i,j)} < 0 \tag{56}$$

where $\alpha_M = \max\{\alpha_\Gamma\}$, $\Gamma \subseteq \mathcal{E}$, $\bar{\alpha}_0 = \max\{\alpha_0, \alpha_\emptyset\}$, $\Upsilon_{(i,j)} = \alpha_0 - \alpha_{\{(i,j),(j,i)\}}$, then the agent group achieves consensus (9) in the presence of the DoS attacks satisfying Assumption 1.

Proof. Based on the fact that $\alpha_\Gamma \le \alpha_M$, one can deduce that

$$\begin{aligned}\mathfrak{D}(0,t) \le& \alpha_\emptyset|\Xi_\emptyset(0,t)| + \alpha_M \sum_{\emptyset\subset\Gamma\subseteq\mathcal{E}} |\Xi_\Gamma(0,t)| \\ \overset{(a)}{=}& \alpha_\emptyset t + (\alpha_M - \alpha_\emptyset) \sum_{\emptyset\subset\Gamma\subseteq\mathcal{E}} |\Xi_\Gamma(0,t)| \\ \overset{(b)}{\le}& \bar{\mu} t + \bar{\varsigma}\end{aligned} \tag{57}$$

where $\bar{\mu} = \alpha_\emptyset + (\alpha_M - \alpha_\emptyset)\sum_{i<j,(i,j)\in\mathcal{E}} \mu_{ij}$ and $\bar{\varsigma} = \sum_{i<j,(i,j)\in\mathcal{E}} \varsigma_{ij}$, (7) is utilized in (a), and (b) is obtained from (4) and the facts that $\sum_{\emptyset\subset\Gamma\subseteq\mathcal{E}} |\Xi_\Gamma| \le \sum_{i<j,(i,j)\in\mathcal{E}} \mathcal{D}_{(i,j)}$ and $\alpha_M - \alpha_\emptyset > 0$.

Based on (27) and (47), it is easy to obtain that

$$\begin{aligned}\mathfrak{D}(0,t) \leq & \alpha_{\emptyset}|\Xi_{\emptyset}(0,t)| + \sum_{\emptyset\subset\Gamma\subseteq\mathcal{E}} (\alpha_0 - \sum_{i<j,(i,j)\in\Gamma} \Upsilon_{(i,j)})|\Xi_{\Gamma}(0,t)| \\ \leq & \bar{\alpha}_0 \sum_{\Gamma\subseteq\mathcal{E}} |\Xi_{\Gamma}(0,t)| - \sum_{i<j,(i,j)\in\mathcal{E}} \Upsilon_{(i,j)} \sum_{\Gamma\subseteq\mathcal{E},(i,j)\in\Gamma} |\Xi_{\Gamma}(0,t)| \\ \overset{(a)}{=} & \bar{\alpha}_0 - \sum_{i<j,(i,j)\in\mathcal{E}} \Upsilon_{(i,j)}|\mathcal{D}_{(i,j)}(0,t)| \\ \overset{(b)}{\leq} & \bar{\mu}t + \bar{\varsigma} \end{aligned} \tag{58}$$

where $\bar{\mu} = \bar{\alpha}_0 - \sum_{i<j,(i,j)\in\mathcal{E}} \mu_{ij}\Upsilon_{(i,j)}$, $\bar{\varsigma} = \sum_{i<j,(i,j)\in\mathcal{E}} \Upsilon_{(i,j)}\varsigma_{ij}$, (a) is obtained from (7) and (8), and (b) is obtained from Assumption 1 and the fact that $\Upsilon_{(i,j)} < 0$ (obtained by adding (23) and (26)).

From (57) and (58), both (55) and (56) imply that $\mathcal{D}(0,t) \leq \bar{\mu}t + \bar{\varsigma}$ holds with $\bar{\mu} < 0$. Then, the proof is completed by following the same argument of Theorem 4.1.

Remark 9. *Based on the obtained decay rates α_Γ, Theorems 4.1 and 4.2 are provided to analyze the biggest μ_{ij} defined in (4) such that the consensus is still achieved under DoS attacks satisfying Assumption 1. Besides, it should be noted that for any $\varsigma_{ij} < \infty$, (52) and (53) (or (57), (58)) imply that (54) is always achievable if $\bar{\mu} < 0$.*

Remark 10. *In this paper, for an undirected graph, a class of secure distributed consensus controller for multi-agent systems under DoS attacks has been provided. However, it should be noted that if the communication graph becomes directed [11], the proposed methods will fail to work. The major difficulty comes from that the symmetric property of Laplacian matrix, which is utilized in Section 3, is lost for directed graphs. Thus, how to extend the proposed method on fixed undirected graph to the directed one requires further study. Besides, while this paper considers a fixed graph, the proposed methods also fail to work for time-varying graphs, such as Markovian network topologies [10]. Because of the randomly changing edges, the resilient consensus control problem for time-varying graphs which is more challenging will be studied in the future work.*

5. Example

Example 1: In this example, a group of three agents in $\mathbb{R}^2$ is considered, and the dynamics of the agents are described by (2) with

$$A = \begin{bmatrix} 0 & 1 \\ 0.2 & -2 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Assuming that the considered communication graph is complete, and $a_{ij} = 1$ for $i \neq j$. The initial condition $x(0) = col(x_1(0), x_2(0), x_3(0)) = [5\ 4\ \ -4\ \ -5\ 3\ \ -3]^T$.

In the following, the proposed state-feedback and observer-based distributed consensus controllers (10) and (32) are applied to the considered MAS to show the effectiveness of the proposed methods.

Step 1. Theorems 3.1, 3.3 and Corollaries 3.2, 3.4 are adopted to provide the decay rates α_Γ where $\Gamma \subseteq \{(1,2),(2,1),(1,3),(3,1),(2,3),(3,2)\}$, and the results are shown in Table 1.

Table 1: Decay rates α_Γ and DoS duration μ_{ij}.

α_Γ	$\Gamma = \emptyset$	$\lvert\Gamma\rvert = 2$	$\lvert\Gamma\rvert = 4$	$\lvert\Gamma\rvert = 6$	μ_{ij}
Theorem 3.1	-4.1	-4.1	0.2	0.2	0.62
Corollary 3.2	-2.9	-0.8	0.22	0.22	0.58
Theorem 3.3	-3.2	-2.2	0.3	0.8	0.6
Corollary 3.4	-1.7	-0.07	1.8	1.8	0.32

Step 2. Based on the decay rates obtained in Step 1, Theorem 4.1 is adopted to analyze the stability, and the maximum allowable μ_{ij} are also provided in Table 1 ($\mu_{12} = \mu_{13} = \mu_{23}$). As discussed in Remark 1, μ_{ij} reflects the attack intensity. For example, in Table 1, $\mu_{ij} = 0.62$ implies that for each edge $(i,j) \in \mathcal{E}$, a maximum of 62% of communication denials on the average are permitted under the consensus controller (10) obtained by using Theorems 3.1. In other words, larger μ_{ij} implies better resilience to DoS attacks.

While Table 1 provides the decay rates α_Γ and maximum allowable DoS duration μ_{ij}, by adopting the controllers (10) and (32) obtained in Step 1 ($K = [1.0936\ 0.5199]$), the following two figures are provided. The first figure in Figure 2 shows the state responses under controller (10) without DoS. The lower two figures of Figure 2 show the state responses under the DoS attacks shown in Figure 1.

Based on the description above, under the proposed controllers, the consensus of the considered MAS is still achieved in the presence of DoS attacks. Besides, Figure 2 shows that the observer-based controller (32) provides faster convergence rate than the state-feedback one (10).

Example 2: In this example, the following LC oscillator network [24, 29] is considered. The dynamic of each agent is described as

$$\begin{bmatrix} \dot{v}_i \\ \dot{c}_i \end{bmatrix} = \begin{bmatrix} 0 & 1/C \\ -1/L & 0 \end{bmatrix} \begin{bmatrix} v_i \\ c_i \end{bmatrix} + Bu_i(t)$$

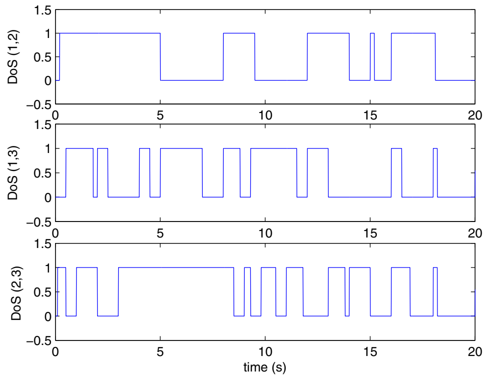

Figure 1: DoS signals (DoS $(i,j) = 1$ means that channel (i,j) is under attack).

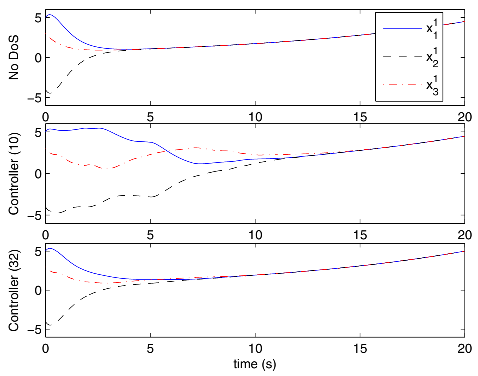

Figure 2: State responses (x_i^1 is the first element of x_i).

where v_i, c_i, $C = 0.5$ and $L = 1$ are the voltage, current, capacitor and inductance of each LC oscillator, respectively. The considered communication graph is shown in Figure 3, the Laplacian matrix $\mathcal{L} = EWE^T$ where E is given in (65) of [29] and $W = diag\{1.5, 2, 1, 2, 1.5, 1\}$, and the initial condition is $x(0) = col(x_1(0), x_2(0), x_3(0), x_4(0), x_5(0)) = [1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10]^T$.

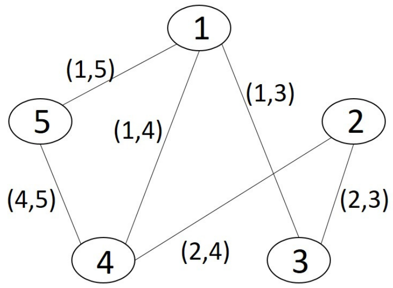

Figure 3: Communication graph.

While Example 1 illustrates the effectiveness of the proposed methods, this example focuses on discussing the computational complexity. Compared with Example 1 where only 8 different attack modes Γ should be considered, in this example, total $2^6 = 64$ different Γ should be considered ($\Gamma \subseteq \{(1,3),(3,1),(1,4),(4,1),\cdots,(4,5),(5,4)\}$).

First, to obtain the decay rates α_Γ, Theorems 3.1, 3.3, and Corollaries 3.2, 3.4 are performed by using MATLAB R2014a on a desktop equipped with an Intel Core i7-6700 processor operating at 3.4 GHz and 4 GB of memory. For controller (10), while solving LMIs in Theorem 3.1 costs $0.1015s$, solving LMIs in Corollary 3.2 only costs $0.0113s$. Besides, for controller (32), while solving LMIs in Theorem 3.3 costs $12.25s$, solving LMIs in Corollary 3.4 only costs $0.0074s$. These facts verify the discussions in Remarks 5 and 6. Meanwhile, it should be also noted that although Corollaries 3.2 and 3.4 reduce the computational complexity greatly, as shown in Figures 4 and 5, Theorems 3.1 and 3.3 provides better convergence performance (Table 1 also verifies such fact).

Second, based on the decay rates α_Γ obtained by using Theorem 3.1 ($\alpha_\emptyset = -3$, $\alpha_\Gamma = -1.5$ ($|\Gamma| = 2$), $\alpha_\Gamma = 3.1$ ($\Gamma = \mathcal{E}$)), Theorems 4.1 and 4.2 are utilized to analyze largest DoS duration μ_{ij} under which the consensus is achieved. For convenience, it is assumed that μ_{ij} are same for all $(i,j) \in \mathcal{E}$. Then, adopting Theorem 4.1 provides $\mu_{ij} = 0.1791$ at the cost of $0.1213s$, and it is easy to verify that (55) in Theorem 4.2 holds for $\mu_{ij} = 0.0819$ which is only half of 0.1791.

Based on the description above, although adopting Corollaries 3.2 and 3.4 to obtain the desired controllers requires much less computing resources, the controllers obtained by using

Theorems 3.1 and 3.3 provide better performance. Besides, based on the same decay rates α_Γ, while verifying (55) and (56) in Theorem 4.2 is very easy, LMI technique can be utilized to verify (49)-(51) in Theorem 4.1 with larger μ_{ij}, which reflects greater resilience to DoS attacks, derived.

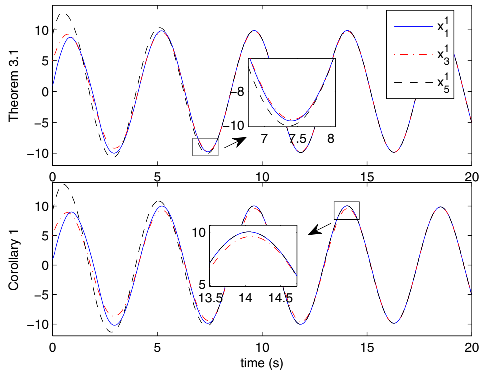

Figure 4: State responses under controller (10).

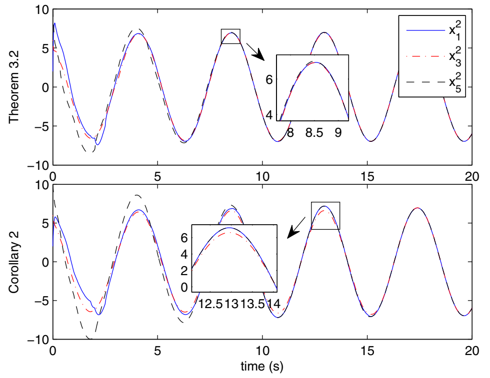

Figure 5: State responses under controller (32).

6. Conclusions

In this paper, the problem of distributed consensus control for MASs under DoS attacks has been investigated. While the adversaries compromise each channel independently, various attack modes have been considered. Based on the proposed distributed state-feedback and observer-based controllers, the decay rates under different attack modes are obtained by solving LMIs. Besides, a novel scaling method has been proposed to reduce the computational complexity at the cost of introducing some conservatism. Then, based on the obtained decay rates, sufficient conditions on the duration of DoS attacks, under which the stability is still guaranteed, have been proposed. It is shown that under the proposed distributed state-feedback and observer-based controller, the consensus is achieved despite the DoS attacks satisfying the proposed conditions. For future work, we can extend the proposed results for event-triggered consensus [6], and such problem is challenging for that whether the event is triggered may be unknown for the existence of DoS attacks.

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