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Social insurance with competitive insurance markets and risk misperception[1]

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Abstract

We examine the role of uniform and non-uniform social insurance to supplement a general income tax when neither public nor private insurers can observe individual risk, which is positively correlated with wages (e.g., for old age dependency). In the (private market) Rothschild and Stiglitz (1976) equilibrium low-wage/low-risk individuals are not fully insured. While social insurance provided to the poor has a negative incentive effect, it also increases their otherwise insufficient insurance coverage. Social insurance to the rich produces exactly the opposite effects. Whichever of these effects dominates, some social insurance is *always* desirable irrespective of the pattern of correlation. Finally, we introduce risk misperception which exacerbates the failure of private markets. Rather surprisingly, this does not necessarily strengthen the case for public insurance.

***JEL*-classification:** H21, H51, D82

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1 Introduction

A significant part of government intervention in the economy is, or can be, justified by redistributive considerations. In modern welfare states a large variety of instruments is used, including taxation, transfers, price subsidies, in-kind transfers, pension benefits, and more generally social insurance. From an economic perspective this raises many questions, for instance, which policies ought to be used, or how should they be designed and financed?

A starting point for addressing these questions is the so-called Atkinson and Stiglitz (1976) theorem which states, roughly speaking, that when preferences are separable between labor supply and goods, any (incentive compatible) Pareto-efficient allocation can be implemented by using only a general income tax. When the income tax is designed in an optimal way *given the information that is available,* an extra instrument is valuable only if it provides "better" information. In the Mirrleesian world considered by Atkinson and Stiglitz (AS), where individuals differ only in productivity, the separability of preferences implies that all these extra instruments do *not* provide any additional, pertinent information.

If we take the AS theorem at its face value, the welfare state should be downscaled dramatically and replaced by a "simple" well-designed tax and (cash) transfer policy. However, it is by now well known that one of the major limitations of this result is that it relies on the assumption that individuals differ only in a single (non observable) dimension, namely productivity. When there are other factors of individual heterogeneity, including preferences or "risk", the result no longer applies.

In this paper we focus on one of the both most prominent and most debated instrument, namely social insurance. Private insurance redistributes *ex post* between states of nature and premiums reflect individual risk. But only social insurance (or a suitable regulated private system) can effectively redistribute between *ex ante* heterogenous risk types. Put differently, social insurance can provide insurance against the "risk of being a bad risk", which private insurance cannot. This can be achieved through uniform premiums or, more generally, by a rate schedule where premium differences do not fully reflect risk differences.

Rochet (1991) and Cremer and Pestieau (1996) have shown that social insurance is desirable to supplement an optimal income tax, even when private insurance is actuarially fair, as long as productivity and risk are negatively correlated, that is when less productive individuals face the higher risk. This assumption is empirically supported for many health risks, but it is questionable for some risks, particularly those related to longevity. For instance, for annuity contracts, less productive individuals are the "better" risks because their life expectancy is shorter. This also applies to the need for long-term care when dependency is due to cognitive affection like Alzheimer's disease. The incidence of cognitive disorders increases significantly with age, and longevity is positively correlated to productivity (as well as education and wealth).[1] Not that higher income individuals are more likely *per se* to be affected by a cognitive disease, but lower income individuals are more likely to die of other courses before they reach the relevant age group. For these kinds of risks a positive correlation between risk and productivity can no longer be ruled out.[2] As shown by Cremer and Pestieau (1996), with fair private insurance markets, social insurance is then no longer desirable at least as long as the incentive constraint binds from high- to low-wage individuals.[3] Intuitively, the willingness to pay for insurance is then higher for the high- than for the low-wage individuals and providing social insurance further reinforces an already binding incentive constraint.

Rochet's (1991) argument is quite powerful for the negative correlation case. When social insurance is desirable even with perfect private markets, it will certainly have a role to play when there are market failures in private markets. With positive correlation this reasoning does not apply anymore and one can argue that social insurance does not come out as desirable, because private insurance is given an "unfair" advantage. More precisely, the fair insurance assumption implies that private insurance has better information on individuals' risks than the government (or whatever administration is in charge of taxes and transfers). And the difference is drastic, because private insurers perfectly observe risk while public authorities do not observe it at all. This assumption is hard to defend and our analysis shows that it has a significant effect on the results: once adverse selection in the private market is accounted for, social insurance is *always* desirable, even under positive correlation.

In this paper we shall concentrate on the case of a positive correlation between productivity and risk and revisit the role of social insurance when neither public nor private insurers can observe the risk type. Instead, the private insurance market suffers from asymmetric information, and we assume that a Rothschild and Stiglitz (RS) equilibrium emerges; see Rothschild and Stiglitz (1976). It is well-known that low-risk individuals will then be only partly insured. Consequently, we have a partial failure of the private insurance market. While public insurers do not have any superior knowledge of risk, they do have two advantages. First, they observe

[1] See Viscusi (1994), Gerdtham and Johannesson (2000), and Cristia (2009).

[2] See Cremer and Roeder (2013) for a detailed discussion of this issue.

[3] With a utilitarian social welfare function, this will be true unless the risk differential become so large that it outweighs the wage differential and the incentive constraint goes in the other direction.

income levels and can implement a means-tested social insurance scheme. Second, they can make insurance mandatory. We show that this is sufficient to make a case for social insurance. In a second step, we introduce the added feature that some individuals may misperceive their risk type and be overconfident. One would expect that this exacerbates the failure of the private markets and makes the role of public insurance even more compelling.

Methodologically this problem is quite challenging, even in an otherwise simple two-type setting. To keep it tractable we represent individuals' risk preferences by using Yaari's (1987) dual theory.[4] On the practical policy design side, our model is meant to apply in particular to the dependency risk and the associated market for LTC insurance.[5] As explained above, a positive correlation is likely to apply for old-age dependency. And myopia appears to be a pervasive phenomenon when it comes to severe forms of dependency.[6] Furthermore, in reality private insurance markets fail to a large extent to provide an appropriate coverage for this risk. Two of the major factors quoted to explain the thinness of private markets are precisely, adverse selection and myopia (risk misperception).[7]

We start by revisiting the fair insurance case within our dual theory setting; see Section 3. While this mainly rediscovers Rochet's (1991) results, it constitutes an interesting benchmark for the remainder of the analysis. Then, the introduction of adverse selection in the private market brings us to the RS setting; see Section 4. We consider both uniform and non-uniform (but self-selecting) social insurance. Our main result is that social insurance is always desirable irrespective of the pattern of correlation. Specifically, we show that in either case social insurance coverage provided to the low-wage (low-risk) individuals has two effects. First, an extra insurance effect because it increases coverage of otherwise underinsured individuals. Second, there is an incentive effect which is interpreted as in the fair private insurance case. We show that irrespective of the relative strength of these effects, some social insurance is always desirable. However, unless insurance is restricted in an *ad hoc* way to be uniform, only one of the types needs to receive social coverage. When the insurance effect outweighs the incentive effect (starting from an equilibrium without social insurance), social insurance is designed for the low-wage individuals, who may or may not end up fully insured. Otherwise, and quite surprisingly, social insurance is targeted toward the high-wage individuals. Though exacerbating

[4]This allows us to derive a closed-form solution for the RS equilibrium.

[5]We do admittedly neglect some important features of LTC and particularly the role played by informal care, which currently represents a significant part of total care.

[6]See Zhou-Richter, Browne and Gründl (2010), and Cremer and Roeder (2013) for a detailed discussion.

[7]See, for instance, Brown and Finkelstein (2009).

the insufficient coverage of the poor this policy relaxes an otherwise binding incentive constraint so that redistribution through the income tax is enhanced.

Finally, in Section 5 we introduce risk misperception, while continuing to consider an RS equilibrium in the private market. We consider a case where some high-risk individuals are overconfident and think they have a low risk. In equilibrium low-risk and overconfident individuals are pooled. Now, the low-risk individuals are not only underinsured, but also pay a higher than fair rate (effectively paying for the overconfident individuals who buy the same contract). Here we concentrate for technical reasons on uniform social insurance. Results are to some extent in line with expectations: there is an insurance term and an incentive term. The insurance term now reflects the combined failure brought about by adverse selection and overconfidence. However, it also turns out that this does not necessarily strengthen the case for public insurance. Quite surprising, overconfident individuals are of no *direct* relevance when it comes to the desirability of uniform social insurance. Overconfidence comes in *indirectly* though, because it increases the cost of private insurance which, in turn, affects the insurance term.

This paper builds on and relates to two strands of the literature on optimal (social) insurance. One theoretical literature that has considered optimal insurance and government redistribution problems jointly. This includes the already mentioned papers by Rochet (1991) and Cremer and Pestieau (1996) who assume that the private insurance market offers fair contracts. In other words, these authors assume that private insurers have better information than the government agencies who provide social insurance. This is a debatable assumption and it implies that the relative merits of private and social insurance are not assessed on a "level playing field". Our analysis shows that this has a rather dramatic effect on the results. It effectively reverses the conventional wisdom established by Rochet's seminal work according to which social insurance supplementing an optimal income tax is desirable if (and only if) risk and productivity are negatively correlated.[8] We show that under adverse selection in the private market, some social insurance is *always* welfare improving, irrespective of the pattern of correlation between the unobservable characteristics.

Chetty and Saez (2010), like us, assume incomplete private insurance, but they consider only one dimension of heterogeneity and restrict instruments, including taxation, to be "linear" (or, more precisely, affine since they include a uniform lump-sum transfer). Consequently, they

[8]Cremer and Pestieau (1996) have already shown that this result may be reversed when risk is the dominant source of heterogeneity so that under a positive correlation, incentive constraints bind from low-productivity (low-risk) individuals to high-productivity (high-risk) individuals. However, this is a rather extreme case and its practical relevance appears to be limited.

cannot address the question whether or not social insurance is a valuable extra instrument in a Mirrleesian world. Instead, they focus on the derivation of simple policy rules (based on elasticities) that can be tested empirically.[9]

The closest predecessors to our paper are Boadway *et al.* (2006) and Nishimura (2009), who introduce adverse selection in the private market. However, these authors also restrict their attention to uniform social insurance and do not consider misperception. They show that the welfare impact of social insurance under positive correlation is ambiguous. While we show that non-uniform social insurance is always desirable, we also explain why the uniformity assumption makes the result ambiguous.

Second, there are a few papers who study the effect of misperception in private markets and most notably Sandroni and Squintani (2007).[10] They focus on the characterization of the equilibrium and our private market solution is similar to theirs. They also look at some policies like mandatory *private* insurance (which does not redistribute amongst risk types). However, they do not allow for income taxation, nor do they consider redistributive social insurance. On the policy side, the most significant and rather surprising result is that misperception does *not* strengthen the case for mandatory insurance. Quite the opposite; while mandatory insurance is (roughly speaking) always desirable when individuals are rational (because it eliminates adverse selection) this is no longer true when some individuals misperceive their risk. This is effectively mirrored by our results which imply that misperception does not necessarily strengthen the case for social insurance. Intuitively, one would expect that an extra market failure increases the benefits of public intervention. However, in our paper like in Sandroni and Squintani (2007) the two market failures do not simply add up; their interaction is more complex. Because of the wider set of instruments we consider, the interaction becomes even more complicated in our setting. On top of the private insurance effects we have the incentive effect pertaining to the optimal taxation problem and, interestingly, we show that because of the interplay between private and public policy, none of these terms is directly affected by misperception.[11]

[9] Bastani *et al.* also combine Mirrleesian and RS problems. However, in their setting the adverse selection is in the labor market for which they consider a RS equilibrium

[10] Spinnewijn (2015) considers the design of unemployment insurance when individuals are overly optimistic about their job market prospects. He concentrates on moral hazard issues (intensity of search) and shows that they are reinforced by misperception, which in turn may reduce the appropriate level of unemployment insurance.

[11] Sandroni and Squintani (2007) have only the private insurance market incentive constraint; their policy involves pooling and is thus by definition incentive compatible. We have the government incentive constraint on top of the private market constraint. Interestingly, it turns out that the private market constraint rules out the possibility that social insurance can make the misperceiving individuals better off.

2 The model

Individuals supply labor ℓ which comes with a (monetary) labor disutility $v(\ell)$. The ability to generate income differs among individuals, *i.e.*, $w \in \{w_r, w_p\}$ with $0 < w_p < w_r$. Population size is normalized to one and the fraction of low- and high-productivity individuals is ν_p and ν_r respectively. Individuals face a health risk; the monetary value of the potential loss is L. In addition to labor productivity, agents differ in their probability of incurring this loss $\pi \in \{\pi_\ell, \pi_h\}$ with $0 < \pi_\ell < \pi_h < 1$. Productivity and risk are perfectly correlated. In other words, each level of w is associated with a unique level of π. So, we have either $\pi_p \equiv \pi_h > \pi_r \equiv \pi_\ell$ implying that the correlation between risk and productivity is negative, or $\pi_r \equiv \pi_h > \pi_p \equiv \pi_\ell$ implying that it is positive. A private insurance market offers insurance against this health risk. Additionally, a social insurance scheme which is financed by income taxation may exist.

We model individuals' risk preferences using Yaari's (1987) dual theory. Consider an individual with productivity w incurring a damage L with the probability π, and an insurance contract (P, I), where P is the premium, while I is the level of coverage. Let $T \lesseqgtr 0$ denote the income tax and $D \geq 0$ the social insurance benefits. This individual faces the lottery $X = (w\ell - v(\ell) - P - T, 1 - \pi; w\ell - v(\ell) - P - T - L + I + D, \pi)$. The utility associated with this lottery is given by

$$\begin{aligned} V(P, I; w, \pi) =& (1 - \phi(\pi))(w\ell - v(\ell) - T - P) + \phi(\pi)(w\ell - v(\ell) - T - P - L + I + D) \\ =& w\ell - v(\ell) - T - P + \phi(\pi)(-L + I + D), \end{aligned} \tag{1}$$

where $\phi(0) = 0$ and $\phi(1) = 1$. Risk aversion is represented by $\phi(\pi) > \pi$. Observe that the dual theory is based on the assumption that lotteries can be ranked, which here means that the individual is not better off when dependent than when in good health. In other words, (1) is only valid as long as $I + D \leq L$, that is as long as there is no overinsurance. Overinsurance cannot occur in equilibrium in our model. However, it can arise for some individual deviations. Specifically, we will show that mimicking individuals in the government incentive constraint may be overinsured. To deal with this in the simplest possible way, we assume that insurers will never pay out more than the effective loss $(L - D)$ to individuals. Formally, we rewrite (1) as

$$V(P, I; w, \pi) = w\ell - v(\ell) - T - P + \min[\phi(\pi)(-L + I + D); 0]. \tag{2}$$

To simplify notation we use the min-operator only where necessary and stick to (1) otherwise.[12]

[12] If insurers would pay out the full claim in case of overinsurance, preferences would be represented by

$$w\ell - v(\ell) - P - T - L + I + (1 - \phi(1 - \pi))(-L + I + D),$$

The information structure is in line with Mirrleesian optimal tax models. We assume that gross income $y = w\ell$ is publicly observable and can be taxed according to a nonlinear function. Individual wages, w, labor supply, ℓ and loss probabilities π are not publicly observable, nor are private insurance contracts (P, I). However, the realization of an individual's risk is observable and social insurance benefits are paid only in that event. Income taxation is optimized, but the design of the tax schedule is not our main focus. Instead, we are interested in the desirability and design of social insurance *given* that income taxation is also optimized. To do so, we study the implied mechanism design problem where individuals are offered vectors $\Omega_i = (y_i, T_i, D_i)$ for $i = p, r$ specifying the before tax income, the income tax and the social insurance benefits. We study the optimal feasible (balanced budget) and incentive compatible mechanism.[13]

2.1 Timing of the game

Let us specify the timing of the mechanism design game. In a first stage, the government announces a mechanism which consists of two vectors $\Omega_p = (y_p, T_p, D_p)$ and $\Omega_r = (y_r, T_r, D_r)$. In the second stage, individuals *ex ante* (before the realization of the health risk) choose one of these vectors. Finally, in stage 3, individuals buy insurance coverage in the private market. The operation of this market depends on the information available to insurance companies. We first assume that they observe an individual's risk type. In a second step, we consider the RS equilibrium of the private insurance market. Then, the insurers know that there are two types characterized by (Ω_p, π_p) and (Ω_r, π_r) respectively (both vectors being given at this point), but they do not observe who is who. In this scenario two premium-benefit contracts $(\pi_p I_p, I_p)$ and $(\pi_r I_r, I_r)$ are offered.

3 Full information in the private insurance market

When insurance companies observe an individual's risk type, they offer any insurance coverage at a fair price corresponding to each individual's health risk. Individuals choose their level of coverage to achieve full insurance, that is $I_i^* = L - D_i$ for $i = p, r$. With the considered information structure in the private market and the public sector, feasible allocations must

given that being in good health is now the "bad" state of nature. Using this specification, rather than (2) would complicate the expressions, but not change the results.

[13] In practice this mechanism would have to be implemented by using tax *cum* social insurance schedules which are specified as functions of the observable variable y. We do not examine this problem because we are mainly interested in the levels of D provided to the different types and this is part of the optimal mechanism.

satisfy the following incentive constraint

$$y_r - v\left(\frac{y_r}{w_r}\right) - T_r - \pi_r I_r^* + \phi(\pi_r)(-L + I_r^* + D_r) \geq$$
$$y_p - v\left(\frac{y_p}{w_r}\right) - T_p - \pi_r I_{rp}^* + \phi(\pi_r)(-L + I_{rp}^* + D_p). \quad (3)$$

That is the rich must be prevented from mimicking the poor. I_{rp}^* thereby denotes the insurance coverage of the rich when they mimick the poor; it is given by $I_{rp}^* = I_p^* = L - D_p$. Additionally, the resource constraint $\sum_i \nu_i(T_i - \pi_i D_i) = 0$ must hold. To incorporate redistributive concerns, we study the optimal allocation that maximizes the sum of a strictly concave transformation of individual utilities, that is $\nu_p \Psi(V_p) + \nu_r \Psi(V_r)$ with $\Psi' > 0$ and $\Psi'' < 0$. The formal problem is stated and the first-order conditions (FOCs) are derived in Appendix A.1. The Lagrangean expression of the problem is denoted by $\mathcal{L}$ while λ and μ represent the multipliers associated with respect to the government's incentive and resource constraint.

Assuming an interior solution for T_r and inserting it into the FOC of D_r yields

$$\frac{\partial \mathcal{L}}{\partial D_r} = \nu_r \Psi_r' \pi_r + \lambda \pi_r - \nu_r \Psi_r' \pi_r - \lambda \pi_r \equiv 0. \quad (4)$$

In words, irrespective of the correlation between income and risk, the same level of welfare is achieved for any level of social insurance for the rich, $D_r^* \in [0, L]$. Intuitively, D_r does not allow to relax the incentive constraint because it is irrelevant for the mimicking individual. Since individuals are fully insured, D_r is then essentially a redundant instrument as long as T_r is used in an appropriate way.

Lets turn to the FOCs of the poor. Assuming an interior solution for T_p and inserting it into the FOC of D_p amounts to

$$\frac{\partial \mathcal{L}}{\partial D_p} = \lambda(\pi_p - \pi_r) \lesseqgtr 0. \quad (5)$$

In words, when income and risk are positively correlated, *i.e.*, $\pi_p < \pi_r$ then $D_p^* = 0$. Otherwise, we have $D_p^* = L$. Note that when social insurance is restricted to be uniform that is $D_p = D_r = D$, then we have

$$\frac{\partial \mathcal{L}}{\partial D} = \frac{\partial \mathcal{L}}{\partial D_r} + \frac{\partial \mathcal{L}}{\partial D_p} = 0 + \lambda(\pi_p - \pi_r) \lesseqgtr 0, \quad (6)$$

so that $D^* = L$ in case of a negative correlation and $D^* = 0$ otherwise.

So far, we have been essentially confirming Rochet's results thereby showing that they continue to hold when individuals' risk preferences are represented using Yaari's (1987) dual theory. Full social insurance is desirable if and only if loss probabilities differ across individuals and are negatively correlated. In that case private insurance is completely crowded out by social insurance. On the other hand, when there is a positive correlation, social insurance has an adverse effect on the incentive constraint because it redistributes in the wrong direction, namely from the poor low-risk individuals to the rich high-risk ones.

Proposition 1 *When insurance companies offer coverage at an actuarial fair rate, then*

(i) full social insurance coverage for everyone is optimal when risk and productivity are negatively correlated,

(ii) no social insurance is desirable when risk and productivity are positively correlated.

While these results are interesting, they rely on the rather restrictive assumption that private insurers have better information about individuals' types than the tax or social insurance administration. This is at best debatable, and it would be more consistent to assume that private insurers and the administration have the same information concerning individuals' characteristics. This is precisely what we will do in the remainder of the paper. It is of course plain that the result for the negative correlation case won't change. If social insurance is beneficial when private markets are fair, it will certainly continue to be beneficial when private markets are imperfect. Formally, one can see this by noting that the optimal solution under fair markets remains available when private markets are also affected by adverse selection. Since full social insurance crowds out these markets anyway, private market imperfections become irrelevant and we return to the fair market case.

The interesting question arises for the case of positive correlation on which we focus in the remainder of the paper. Specifically, one may expect that the imperfections of private markets strengthen the case for social insurance and imply that (at least some) of it is desirable even under positive correlation.

4 Adverse Selection in the private insurance market

Assume from now on that risk and productivity are positively correlated so that $\pi_r \equiv \pi_h > \pi_p \equiv \pi_\ell$. To solve our problem we proceed by backward induction and start by characterizing the last stage, namely the private insurance market RS equilibrium induced by the tax and social insurance policy. We assume throughout our analysis that a RS equilibrium exists.

4.1 Private market equilibrium

Insurers offer two contracts. One is designed for the high risks with full insurance at a fair rate $(\pi_r I_r^*, I_r^*)$, where $I_r^* = L - D_r$, and one is designed for low risks who also pay a fair price, but who *may* receive only partial insurance. Formally, I_p^* is the largest level of I_p which satisfies the incentive constraint on the private insurance market

$$y_r - v\left(\frac{y_r}{w_r}\right) - T_r - \pi_r I_r^* + \phi(\pi_r)(-L + I_r^* + D_r) \geq$$
$$y_r - v\left(\frac{y_r}{w_r}\right) - T_r - \pi_p I_p + \phi(\pi_r)(-L + I_p + D_r) \tag{7}$$

and the condition $I_p \leq L - D_p$. The solution to (7) when it is binding is given by

$$I_p^* = \frac{\phi(\pi_r) - \pi_r}{\phi(\pi_r) - \pi_p}(L - D_r), \tag{8}$$

which defines the insurance coverage for the poor I_p^* as long as it is $I_p \leq L - D_p$; otherwise we have $I_p^* = L - D_p$. Formally, I_p^* is defined by

$$I_p^* = \min\left[\frac{\phi(\pi_r) - \pi_r}{\phi(\pi_r) - \pi_p}(L - D_r); L - D_p\right]. \tag{9}$$

Defining the maximum level of D_p for which the incentive constraint in the private market is binding as

$$\widetilde{D}_p \equiv L - \frac{\phi(\pi_r) - \pi_r}{\phi(\pi_r) - \pi_p}(L - D_r), \tag{10}$$

and noting that $\widetilde{D}_p > 0$ for any D_r, because

$$a \equiv \frac{\phi(\pi_r) - \pi_r}{\phi(\pi_r) - \pi_p} < 1, \tag{11}$$

we can write (10) as

$$\widetilde{D}_p = L(1 - a) + aD_r, \tag{12}$$

which implies $\widetilde{D}_p \geq D_r$. When $D_p > \widetilde{D}_p$, the level of I_p that yields full insurance (for the low-risk type) is sufficiently small not to be attractive to the high-risk type and equation (7) is not binding.

Having characterized the insurance market equilibrium, we can now turn to the determination of the optimal tax and social insurance policy. Formally, we have to determine the "best" consumption bundles $\Omega_i = (y_i, T_i, D_i)$ for $i = p, r$ which satisfy the resource constraint and are incentive compatible so that all individuals are at least as well off by choosing the bundle designed for them rather than by mimicking the other individuals. As usual we assume that only the downward constraint from r to p is binding. Before stating this problem we must have a closer look at this incentive constraint. The existence of private markets along with the assumption that private contracts are not publicly observable brings about one extra difficulty. The RS equilibrium is by definition incentive compatible so that *in equilibrium* all individuals will choose the bundle $\Omega_i = (y_i, T_i, D_i)$ and the contract $(\pi_i I_i, I_i)$ designed for them. However, this does *not* tell us which private contract an individual r mimicking one of type p will choose. This is the question we examine in the next subsection.

4.2 Incentive constraint of the government

Assume that the incentive constraint in the private market is binding. The mimicking individual chooses I_r^* if (and only if)

$$y_p - v\left(\frac{y_p}{w_r}\right) - T_p - \pi_r I_r^* + \min[\phi(\pi_r)(-L + I_r^* + D_p); 0] \geq$$
$$y_p - v\left(\frac{y_p}{w_r}\right) - T_p - \pi_p I_p^* + \min[\phi(\pi_r)(-L + I_p^* + D_p); 0], \tag{13}$$

which after simplification is implied by the insurance market incentive constraint (see equation 7) as long as $D_p \leq D_r$, so that he is not overinsured in any of the cases. The binding constraint implies that the mimicking type-r individual is effectively indifferent between the two contracts $(\pi_r I_r^*, I_r^*)$ and $(\pi_p I_p^*, I_p^*)$.

Assume now that $D_p > D_r$. Then, the last term on the LHS of (13) is equal to zero since $I_r^* = L - D_r$ while the last term on the RHS is still negative (if the private market incentive constraint is binding). Consequently, condition (13) can be rewritten as

$$-\pi_r I_r^* \geq -\pi_p I_p^* + \phi(\pi_r)(-L + I_p^* + D_p), \tag{14}$$

while the incentive constraint in the private market can be written as

$$-\pi_r I_r^* = -\pi_p I_p^* + \phi(\pi_r)(-L + I_p^* + D_r). \tag{15}$$

Combining these two expressions yields $D_r \geq D_p$. That is, when $D_p > D_r$ condition (13) is not satisfied and mimickers prefers I_p^* over I_r^*, while they are indifferent between the two contracts when $D_p \leq D_r$. We can thus write the government's problem as if the mimicking individuals would always choose I_p^*. The statement of the problem is then valid for both cases.[14]

[14] All this is true as long as the incentive constraint in the private market is binding. If not, that is when $D_p > \widetilde{D}_p$ is sufficiently large, we can no longer rule out the case where the mimicker chooses his "own" contract $(\pi_r I_r^*, I_r^*)$. We neglect this for the time being but reintroduce it when it will be relevant; see footnote 16.

4.3 Optimal policy: problem and first-order conditions

We are now in a position to state the problem determining the optimal allocation. Again, the government maximizes $\nu_p\Psi(V_p) + \nu_r\Psi(V_r)$ subject to the resource constraint $\sum_i \nu_i(T_i - \pi D_i)$ and the following incentive constraint

$$\begin{aligned} & y_r - v\left(\frac{y_r}{w_r}\right) - T_r - \pi_r I_r^* + \phi(\pi_r)(-L + I_r^* + D_r) \geq \\ & y_p - v\left(\frac{y_p}{w_r}\right) - T_p - \pi_p I_p^* + \phi(\pi_r)(-L + I_p^* + D_p). \end{aligned} \tag{16}$$

The Lagrangean function and the FOCs are stated in Appendix A.2. Assuming an interior solution for T_r and T_p and inserting the FOCs into the FOCs of D_r and D_p yields

$$\frac{\partial \mathcal{L}}{\partial D_r} = \left[\nu_p\Psi_p'\left(\phi(\pi_p) - \pi_p\right) - \lambda\left(\phi(\pi_r) - \pi_p\right)\right]\frac{\partial I_p^*}{\partial D_r}, \tag{17}$$

$$\frac{\partial \mathcal{L}}{\partial D_p} = \nu_p\Psi_p'\left(\phi(\pi_p) - \pi_p\right) - \lambda\left(\phi(\pi_r) - \pi_p\right). \tag{18}$$

The first term in (18) is positive and shows the benefit of providing more insurance to individual p; recall that these individuals are only partially insured by the private market. The second term measures the incentive costs; D_p provides larger benefits to the mimicker (type-r individuals) than to the mimicked individuals. Since the terms are of opposite sign the expression evaluated at zero is ambiguous. Whether or not we obtain $D_p^* > 0$ depends on the relative magnitude of the insurance and incentive term. Recall that with fair insurance markets, social insurance was never optimal because of the negative incentive effect. When there is asymmetric information in the private market, there continues to be a negative incentive effect. However, there is also a positive insurance effect for type-p individuals which corrects for the market failure. Observe that

$$\frac{\partial \mathcal{L}}{\partial D_r} = -a\frac{\partial \mathcal{L}}{\partial D_p}, \tag{19}$$

where $a = -\partial I_p^*/\partial D_r$ is defined by equation (11). Consequently, the interpretation of (17) is similar to that of (18) except that the signs of the two effects are reversed.

Before proceeding with the characterization of D_p^* and D_r^* let us consider the case where we impose the *ad hoc* assumption that D is required to be uniform so that $D_p = D_r = D$. We then have from (19)

$$\frac{\partial \mathcal{L}}{\partial D} = \frac{\partial \mathcal{L}}{\partial D_p} + \frac{\partial \mathcal{L}}{\partial D_r} = \frac{\partial \mathcal{L}}{\partial D_p}(1-a).$$

Substituting from (18) and rearranging successively yields

$$\frac{\partial \mathcal{L}}{\partial D} = (1-a)\nu_p\Psi_p'[\phi(\pi_p) - \pi_p] - \lambda[\pi_r - \pi_p]. \tag{20}$$

Interestingly, the incentive term is now exactly the same as in the fair private market case. We again have an insurance term similar to that in (18) but mitigated by the concomitant increase in D for the rich. Either way, we obtain that even simple uniform social insurance may be welfare improving due to private insurance market imperfections. And this is the case even though its incentive effect is negative.

Let us now return to the case where social insurance is not restricted to be uniform. Expressions (17) and (18) are valid as long as I_p^* is given by (8) that is as long as $D_p \leq \widetilde{D}_p$; otherwise we have $\partial I_p^*/\partial D_r = 0$ and $\partial I_p^*/\partial D_p = -1$ so that equation (17) reduces to

$$\frac{\partial \mathcal{L}}{\partial D_r} \equiv 0, \tag{21}$$

while (18) is replaced by[15]

$$\frac{\partial \mathcal{L}}{\partial D_p} = \nu_p \Psi'_p \pi_p - \lambda \pi_p - \mu \nu_p \pi_p = -\pi_p \frac{\partial \mathcal{L}}{\partial T_p}, \tag{22}$$

which is equal to zero as long as there is an interior solution for T_p. When both individuals (as well as the mimicker in the incentive constraint) are fully insured, D_p is equivalent to a transfer to individual p (a negative tax) and the instrument is redundant. Consequently, nothing can be gained by setting $D_p > \widetilde{D}_p$.[16]

4.4 Optimal policy: no redistributive concerns

The expressions presented in the previous subsection and their interpretations have shown that both redistributive and efficiency considerations are relevant. In other words, in addition to its redistributive role studied in Section 3, social insurance now corrects the partial insurance market failure brought about by adverse selection. Section 3 has shown that when there is no market failure so that redistribution is the only relevant concern, social insurance is *not* desirable under positive correlation.

Before studying the general solution, let us consider the "opposite" case namely where the government does not care about redistribution, *i.e.*, $\Psi' = 1$, but when there is adverse selection

[15] See Appendix A.2.

[16] As mentioned above, when $D_p > \widetilde{D}_p$ and D_p is sufficiently large, the mimicking individual may prefer to buy I_r^* in the private market. In that case, (21) and (22) are replaced by

$$\frac{\partial \mathcal{L}}{\partial D_r} = \lambda(\phi(\pi_r) - \pi_p) > 0,$$
$$\frac{\partial \mathcal{L}}{\partial D_p} = -\lambda(\phi(\pi_r) - \pi_p) < 0.$$

Consequently, we continue to find that nothing can be gained by setting $D_p > \widetilde{D}_p$.

in the private insurance market. Recall that without any government intervention, the low-risk agents have only partial insurance coverage $I_p^* < L$ in the RS equilibrium. This is, given that individuals are risk averse, *i.e.*, $\phi(\pi_i) > \pi_i$, an inefficient allocation. Intuitively, one would expect that social insurance at the very least ought to complete the poor's coverage to achieve full insurance. To see how this is brought out by the FOCs observe first that absent of redistributive concerns it is plain that the solution implies $\lambda = 0$. Since taxes and transfers have no impact on social welfare, they can always be set so that the self-selection constraint is satisfied at no welfare cost. In other words, a first-best outcome can be implemented. For instance, one can set $D_r^* = 0$ (the rich are fully insured anyway) and provide social insurance $D_p^* = L - I_p^*$ to the poor "financed" by a tax of $\pi_r D_p^*$ levied on the poor. Then, both individuals are fully insured and the rich individuals' incentives to mimick are nil, since they would pay the same price for social insurance as they do for private insurance. And it is obvious that the poor do not want to mimick the rich for otherwise, they would have to pay $\pi_r L$ to get full insurance. With this policy there is a budgetary surplus of $\nu_p D_p^*(\pi_r - \pi_p)$. It can be redistributed through a uniform transfer which has no impact on the incentive constraint yielding $T_p^* = \pi_r D_p^* - \nu_p D_p^*(\pi_r - \pi_p)$ and $T_r^* = -\nu_p D_p^*(\pi_r - \pi_p)$.[17]

Observe that $D_p^* = L - I_p^*$ characterizes the minimal level of D_p which implements a first-best solution. Higher levels of social insurance leave welfare unaffected, but the same allocation could also be implemented in a "trivial" way by providing uniform full social insurance $D^* = L$; see equation (20).

4.5 Optimal policy: general solution

We are now in a position to characterize the solution for the general case when redistributive concerns matter. To do this in the simplest possible way Figure 1 is useful.

The figure projects our problems into the (D_r, D_p)-space. Clearly, neither of these variables will exceed L so we can restrict our attention to the domain $[0, L] \times [0, L]$. The lines B, C, E, F represent some randomly selected level curves of $\mathcal{L}$. To represent them we use (19), which implies that these are all straight lines with slope of a. In addition, A represents equation (12) defining $\widetilde{D}_p$ as function of D_r. It is also a straight line, with slope a and thus parallel to the level curves. We can neglect the area above A because we already know from the previous subsection that

[17]To show that this solution indeed satisfies the FOCs note that with $\lambda = 0$ and $\Psi' = 1$ we automatically have $\mu = 1$. It then follows from equations (18), (21) and (22) that $\partial\mathcal{L}/\partial D_p > 0$ up to $D_p^* = L - I_p^*$ and $\partial\mathcal{L}/\partial D_p = 0$ when $D_p^* = L - I_p^*$.

Figure 1: Welfare level curves in the (D_r,D_p)-space.

nothing can be gained by setting $D_p > \widetilde{D}_p$. Concentrating on the area below A at, for instance, point c with both insurance benefits being strictly positive yields the same level of welfare as point d, on the vertical axis so that $D_r = 0$. Similarly, any point on E can be duplicated by the intersection of this line with the horizontal axis.

This figure shows that one of the two instruments is always redundant. Nothing can be gained by choosing a point at the interior of the square. Assuming that redundant instruments are not used it is plain from the graphical representation that the solution is then given by $D_r^* = 0$ and some $D_p^* \in (0,(1-a)L]$, when $\partial\mathcal{L}(0,0)/\partial D_p > 0$, that is when (18) evaluated at $D_r = D_p = 0$ is strictly positive. Note that from (19) this automatically implies that $\partial\mathcal{L}(0,0)/\partial D_r < 0$. On the other hand, when $\partial\mathcal{L}(0,0)/\partial D_r > 0$ so that $\partial\mathcal{L}(0,0)/\partial D_p < 0$ we can set $D_p^* = 0$ and determine the best $D_r^* \in (0,L]$.

Intuitively, when $\partial\mathcal{L}(0,0)/\partial D_p > 0$, we know from the discussion in the previous subsection that the positive insurance effect of D_p outweighs the negative incentive effect. The opposite is true for D_r. Consequently, we can set $D_r^* = 0$ and the solution of D_p is then either given by an "interior" solution in the interval $D_p^* \in (0,(1-a)L)$ such that (18) is zero. Alternatively, if welfare continues to increase up to $(1-a)L$, we obtain $D_p^* = (1-a)L$, that is $D_p^* = \widetilde{D}_p$ and social insurance for p is increased until the incentive constraint in the private market is no longer binding so that poor individuals are fully insured. This outcome is very much "as expected" but one has to keep in mind that it may or may not be the optimal policy.

The opposite case where $\partial\mathcal{L}(0,0)/\partial D_p < 0$ is more surprising. We can then set $D_p^* = 0$ and $D_r^* \in (0,L]$. This means that while p is already underinsured, while r is fully insured anyway, it proves beneficial to provide social insurance to the rich. This insurance, in turn will decrease the private insurance available to the poor, via the private market incentive constraint. So, even though the policy is aimed at redistributing to p, we give social insurance to r, leaving p underinsured. This is beneficial here because the incentive effect outweighs the insurance effect. In words, the policy relaxes the incentive constraint, which in turn makes it possible to redistribute more via income taxation.

Summing up, it turns out that unless $\partial\mathcal{L}(0,0)/\partial D_p = 0$, which is not generically true, the solution always implies that some social insurance is provided to one type of individuals. This is in sharp contrast with the fair insurance market case where no social insurance was desirable under positive correlation. Observe also that in either case the solution is obtained by balancing the positive effect of correcting a market failure with the adverse redistributive impact that social insurance has under a positive correlation between risk and ability. In that sense the general solution is roughly speaking a "convex combination" of the outcomes achieved in the two extreme cases presented in Section 3 (no market failure) and in Subsection 4.4 (no redistribution).

Proposition 2 *When the private insurance market is characterized by the Rothschild-Stiglitz equilibrium and the correlation between income and risk is positive, then*

(i) if $\Psi' = 1$ *so that there is no concern for redistribution, a first-best outcome with full insurance for all individuals can be implemented. The minimal amount of social insurance to achieve this is given by* $D_p^* = L - I_p^*$ *and* $D_r^* = 0$*. In other words, social insurance merely completes the market coverage provided to the poor.*

(ii) if Ψ' *is strictly concave so that redistribution matters, the solution is second-best and*

(a) the rich are always fully insured (be it private or public) while the poor may or may not be fully insured.

(b) it is always desirable to provide some social insurance.

(c) depending on the strength of the insurance and incentive effect, social insurance is always redundant for one type of individuals. It is positive for the poor if the insurance effect outweighs the incentive effect and it is positive for the rich otherwise.

5 Misperception and social insurance

We now introduce an additional source of imperfection in private insurance markets, namely the fact that some high-risk individuals may be overconfident and misleadingly think that they have a low health risk. As argued above this appears to be the case in reality, in particular for the dependency risk associated with long-term care needs. One can expect this to further strengthen the case for social insurance.

Formally, we then have three types of individuals indexed by r, o and p and in (strictly positive) proportions ν_r, ν_o and ν_p. Types r and p are the same as before and we have $w_r > w_p$ and $\pi_r \equiv \pi_h > \pi_p \equiv \pi_\ell$. Type o individuals are the same as type r except that they are overconfident and think that their risk is π_ℓ while in reality it is π_h.[18] And it is this perceived risk which determines their demand in the insurance market. Consequently, insurance companies

[18] Due to our perfect correlation assumption, this implies that individuals are not aware of the true distribution of types.

cannot screen between overconfident and low-risk agents. That is, individuals are separated on the basis of their beliefs.

The insurance market equilibrium is then as follows. For high-risk individuals the presence of overconfident individuals changes nothing; they get full insurance at an actuarial fair price. Low-risk agents, by contrast, get a different contract which insures self-selection. We assume that social insurance is uniform so that $D_p = D_r = D_o = D$. This assumption is necessary to keep the derivation of the private insurance market equilibrium tractable.[19] The average probability of a damage for overconfident and low-risk agents is given by $\pi_{po} \equiv (\nu_o \pi_r + \nu_p \pi_p)/(\nu_o + \nu_p)$. Specifically, the coverage, I^*_{po}, low-risk and overconfident agents get is determined by the following self-selection constraint

$$
\begin{aligned}
&y_r - v\left(\frac{y_r}{w_r}\right) - T_r - \pi_r I^*_r + \phi(\pi_r)(-L + I^*_r + D) \geq \\
&y_r - v\left(\frac{y_r}{w_r}\right) - T_r - \pi_{po} I_{po} + \phi(\pi_r)(-L + I_{po} + D),
\end{aligned}
\tag{23}
$$

where $I^*_r = L - D$. Formally, I^*_{po} is then defined by

$$
I^*_{po} = \frac{\phi(\pi_r) - \pi_r}{\phi(\pi_r) - \pi_{po}}(L - D). \tag{24}
$$

Note that I^*_{po} is the same for overconfident and low-risk individuals. We define

$$
b \equiv \frac{\phi(\pi_r) - \pi_r}{\phi(\pi_r) - \pi_{po}} < 1, \tag{25}
$$

which will be helpful in the following analysis. Turning to the incentive constraints in the government's problem, we consider three constraints. The first of these constraints prevents r from mimicking p, the second r from mimicking o, and the third o from mimicking p; see equations (A.1)–(A.3) in Appendix A.3. These constraints assume that mimicking individuals choose their "own" contract in the private market. With uniform social insurance this is necessarily true and follows directly from (23). Combining the incentive constraints shows that when the constraints from r to o and from o to p are satisfied, the constraint from r to p is necessarily also satisfied. There is thus no need to impose the latter as separate constraint.

In Appendix A.3 we show that the FOC with respect to D of the government's problem is

$$
\frac{\partial \mathcal{L}}{\partial D} = \nu_p \Psi'_p \left[\pi_{po} b + \phi(\pi_p)(1 - b) - \pi_p\right] - \lambda_2(\pi_r - \pi_p), \tag{26}
$$

[19] When social insurance is nonuniform we can no longer rule out the possibility that the private insurers can offer a menu of contracts that screens for the overconfident individuals. Since type-o individuals think that they have the same probability of incurring the loss L, they cannot be separated when their residual loss $L - D$ is the same. But when the D's differ the type space is too large to characterize the RS equilibrium for any combination of D's (and since D's are endogenous, we cannot restrict them to keep the private insurance equilibrium tractable.

where λ_2 denotes the Lagrange multiplier with respect to the incentive constraint when o mimicks p. Equation (26) includes once again an insurance term and an incentive term. The incentive term is negative given $\pi_r > \pi_p$, and its interpretation is exactly the same as in Section 3. When the mimicker has a higher risk than the mimicked individual, uniform social insurance reinforces the incentive constraint. Interestingly, only individuals r and p matter for this term; all effects pertaining to individuals o cancel out. This is because in the welfare function the utility of type-o agents is evaluated according to their true risk-type (π_h instead of π_ℓ) and it is not possible to reform the (y_o, T_o, D) bundle in a way that affects differently the r-type agents behaving as mimickers and the o-type agents. Consequently, a marginal variation in D associated with an adjustment in T_o that leaves the utility of type-o agents unaffected cannot relax the constraint associated with λ_1.[20]

Turning to the insurance term, it is positive because the first two terms in brackets represent a convex combination of two terms which are both larger than π_p. Recall that $b = -\partial I^*_{po}/\partial D$ is defined by (25) and satisfies $0 < b < 1$. To interpret this term, first observe that given the way we combined the FOCs, we are effectively considering a variation $\mathrm{d}D$ which is financed by an increase in T_i so that $\mathrm{d}T_i = \pi_i \mathrm{d}D$, where $\pi_o = \pi_r$. In words, individuals face a tax increase which equals the expected cost of the extra social insurance they receive. The first term in (26) measures the impact of this variation on the contribution to social welfare of p. To see this, note that the first term in brackets measures the premium savings in the private market. The second term measures the benefit of the extra insurance protection; it accounts for the fact that when D increases by 1, I^*_{po} decreases by b. Individuals thus receive a net extra benefit of $(1-b)$, which is weighted by $\phi(\pi_p)$ since it occurs in the bad state of nature. The third term represents the cost of the "compensating" tax increase. Finally, the term in brackets is multiplied by $\nu_p \Psi'_p$ to convert individual utility into contributions to social welfare.

It may be surprising at first, that only the impact on p appears in (26). To understand this, note that the counterpart of this term for r obviously vanishes because these individuals are already fully insured. Consequently, public insurance only crowds out private coverage at the same cost. For type-o individuals the corresponding bracketed term would be $[\pi_{po} b + \phi(\pi_r)(1-b) - \pi_r]$, but $\pi_{po} b + \phi(\pi_r)(1-b) = \pi_r$ so that this expression vanishes, too. Recall also that this equation is obtained by substituting for b, that is effectively by making use of the private market incentive constraint. Intuitively, this makes sense. Individuals r and o are alike as far as

[20] We thank the reviewer for suggesting this interpretation.

their real preferences are concerned and also with the considered variation $\mathrm{d}T_r = \mathrm{d}T_o = \pi_r \mathrm{d}D$. Consequently, if o would become better off by the considered variation, r mimicking o would be better off too and this would violate the IC constraint. In other words, the result arises because I^*_{po} adjusts to satisfy the private market IC constraint.

It thus turns out that individuals o are of no *direct* relevance when it comes to the desirability of uniform social insurance. Overconfidence comes in *indirectly* though because it increases the cost of private insurance for π_p; they now pay insurance protection at a price of $\pi_{po} > \pi_p$ which is larger than the fair rate (because they are pooled with individuals o).

To sum up, uniform social insurance is desirable if (and only if) the insurance term (evaluated at $D = 0$) outweighs the incentive term. Observe that $\nu_p \Psi'_p$ will decrease with D as the poor become better off.[21] Consequently, when the insurance term is positive for $D = 0$, we may get an interior solution at some level D^* for which (26) vanishes or full insurance with $D^* = L$.

Proposition 3 *Assume that some of the high-risk individuals are overconfident concerning their health risk while there continues to be adverse selection in the private insurance market and productivity and risk are positively correlated, then*

(i) uniform social insurance continues to have a positive insurance and a negative incentive effect. The incentive effect is the same as in the absence of overconfidence, while the insurance term has a different structure.

(ii) overconfident individuals have no direct relevance when it comes to the desirability of uniform social insurance. Any attempt to redistribute towards this group would be undone by the private market incentive constraints. However, overconfidence comes in indirectly though, because it increases the cost of private insurance for the low-productivity individuals.

6 Conclusion

This paper has revisited the role of social insurance to supplement a general income tax. We have assumed that neither public nor private insurers can observe an individual's risk type. Instead, the private insurance market suffers from asymmetric information, a RS equilibrium emerges and low-risk individuals are only partly insured. We have concentrated on the case of positive correlation between loss probability and productivity, which is relevant for the old-age dependency risk. This is the interesting case from our perspective, because it implies that social insurance, whether uniform or not, is not desirable when private insurance markets are fair.

[21] Recall that we know from (24) that they will never be fully insured except when $D = L$.

We have shown that with adverse selection in the insurance market social insurance does have a role to play. When there is no concern for redistribution it can achieve a first-best outcome by completing the market insurance coverage provided to the poor. In the general case, when redistribution is accounted for by adopting a strictly concave welfare function, the solution is second-best. Extending benefits to the poor, or on a universal basis, does have adverse incentive effects, but it also corrects a market failure and enhances insurance coverage of the previously underinsured. Uniform coverage was shown to be desirable only when the insurance benefits outweigh the incentive cost. A properly designed non-uniform insurance schedule, on the other hand, is *always* desirable irrespective of the pattern of correlation between productivity and risk. Under positive correlation, insurance benefits need to be targeted to one of the types only, and quite surprisingly this may be the productive individuals.

Finally, we have examined how the desirability of social insurance and its design are affected by overconfidence concerning the health risk of high-productivity agents. Intuitively, one would expect that an extra market failure increases the benefits of public intervention. However, the two market failures do not simply add up; their interaction is more complex. On top of the private insurance effects we have the incentive effect pertaining to the optimal taxation problem and, interestingly, we show that because of the interplay between private and public policy, none of these terms is directly affected by misperception. The existence of overconfident individuals is of no *direct* relevance for the desirability of uniform social insurance. Overconfidence comes in *indirectly* though, because it increases the cost of private insurance for the low-productivity individuals which enhances the benefits they receive from social insurance.

Our results are based on the assumption that overconfident individuals misperceive only their risk. In principle, individuals could also misperceive their productivity. Given the timing we consider in the model, the simultaneous misperception is not possible. Individuals start by working and earning income and they are paid at their marginal productivity. The labor market is perfect and technologies are linear as usual in a Mirrleesian world. Even if they initially misjudge their true productivity, they will be corrected when they go on the market. Consequently, when they buy insurance protection, they can no longer misperceive their productivity. This is of course due to the sequential nature of our game. Alternatively, one could consider a simultaneous specification. This would be more complicated but the results are "predictable". In particular, the optimal tax would then have a Pigouvian element to correct for the misperception. However, since there are no income effects in our model, this would have no impact on the private insurance market. Consequently, one can expect our results to go through. But this is admittedly a conjecture; specifying and solving the simultaneous model is complex and would require a different paper.

Additionally, we concentrated on a perfect correlation between risk and productivity. This reduced the possible number of types from four to two. A four-type specification could be dealt with in the optimal taxation (OT) stage of our model. The results would then depend on the pattern of binding incentive constraints. The main difficulty, however, would have be the determination of the induced insurance market equilibrium. Most of the insurance literature considers RS equilibria only for two types.[22] With four types we would still have only two probabilities, but (unless there is pooling) four different levels of social insurance coverage, D. The difficulties that this involves are shown by Wambach (2000) (our D plays a role similar to wealth in his model). The literature makes it clear that the RS equilibrium can be characterized only for a restricted set of D's. The difficulty is that we consider a sequential game so that the D's, which are given at the private insurance stage, are effectively endogenous in the OT stage. Consequently, to solve the OT stage we would have to determine the induced equilibrium for any possible pattern of D's or, in other words, since the D's are endogenous, we cannot restrict them in an *ad hoc* way.

[22]There are a few papers which consider more types, but the main message that emerges from these is that severe restrictions have to be imposed to obtain existence of an equilibrium.

References

[1] **Atkinson, A.B. and J.E. Stiglitz**, 1976, "The design of tax structure: direct versus indirect taxation," *Journal of Public Economics*, 6 (1), 55–75.

[2] **Bastani, S., T. Blumkin and L. Micheletto**, 2015, "Optimal wage redistribution in the presence of adverse selection in the labor market," *Journal of Public Economics*, 131, 41–57.

[3] **Boadway, R., M. Leite-Monteiro, M. Marchand, and P. Pestieau**, "Social insurance and redistribution with moral hazard and adverse selection," *Scandinavian Journal of Economics*, 2006, *108* (2), 279–298.

[4] **Brown, J.R. and A. Finkelstein**, "The private market for long-term care insurance in the united states: a review of the evidence," *Journal of Risk and Insurance*, 2009, *76*, 5–29.

[5] **Chetty, R. and E. Saez**, "Optimal taxation and social insurance with endogenous private insurance," *American Economic Journal: Economic Policy*, 2010, *2* (2), 85–116.

[6] **Cremer, H. and P. Pestieau**, "Redistributive taxation and social insurance," *International Tax and Public Finance*, 1996, *3*, 281–295.

[7] **Cremer, H. and K. Roeder**, "Long-term care policy, myopia and redistribution," *Journal of Public Economics*, 2013, *108*, 33–43.

[8] **Cristia, J.P.**, "Rising mortality and life expectancy differentials by lifetime earnings in the United States," *Journal of Health Economics*, 2009, *28*, 984–995.

[9] **Gerdtham, U.-G. and M. Johannesson**, "Income-related inequality in life-years and quality-adjusted life-years," *Journal of Health Economics*, 2000, *19* (6), 1007–1026.

[10] **Nishimura, Y.**, "Redistributive taxation and social insurance under adverse selection in the insurance market," *International Tax and Finance*, 2009, *16*, 176–197.

[11] **Rochet, J.-C.**, "Incentives, redistribution and social insurance," *Geneva Papers on Risk and Insurance Theory*, 1991, *16* (2), 143–165.

[12] **Rothschild, M. and J. Stiglitz,** "Equilibrium in competitive insurance markets: an essay on the economics of imperfect information," *Quarterly Journal of Economics,* 1976, *90*, 630–649.

[13] **Sandroni, A. and F. Squintani,** "Overconfidence, insurance, and paternalism," *The American Economic Review,* 2007, *97* (5), 1994–2004.

[14] **Spinnewijn, J.** "Unemployed but optimistic: Optimal insurance design with biased beliefs," *Journal of the European Economic Association,* 2015, *13* (1), 130–167.

[15] **Viscusi, W.K.**, "Mortality effects of regulatory costs and policy evaluation criteria," *RAND Journal of Economics*, 1994, *25*, 94–109.

[16] **Wambach, A.**, "Introducing heterogeneity in the Rothschild-Stiglitz model," *The Journal of Risk and Insurance*, 2000, *67* (4), 579–591.

[17] **Yaari, M.E.**, "The dual theory of choice under risk," *Econometrica*, 1987, *55*, 95–115.

[18] **Zhou-Richter, T., M. Browne and H. Gründl**, "Don't they care? Or, are they just unaware? Risk perceptions and the demand for long-term care insurance," *Journal of Risk and Insurance*, 2010, *77* (4), 715–747.

Appendix

A.1 Full information in the private insurance market: problem and FOCs

The government's problem can be written as

$$
\begin{aligned}
\mathcal{L} &= \nu_r \Psi \left(y_r - v\left(\frac{y_r}{w_r}\right) - T_r - \pi_r I_r^* + \phi(\pi_r)(-L + I_r^* + D_r) \right) \\
&+ \nu_p \Psi \left(y_p - v\left(\frac{y_p}{w_p}\right) - T_p - \pi_p I_p^* + \phi(\pi_p)(-L + I_p^* + D_p) \right) \\
&+ \lambda \left(y_r - v\left(\frac{y_r}{w_r}\right) - T_r - \pi_r I_r^* + \ phi(\pi_r)(-L + I_r^* + D_r) \right. \\
&\left. - y_p + v\left(\frac{y_p}{w_r}\right) + T_p + \pi_r I_{rp}^* - \phi(\pi_r)(-L + I_{rp}^* + D_p) \right) + \mu \left(\sum_i \nu_i (T_i - \pi_i D_i) \right),
\end{aligned}
$$

where $I_{rp}^* = I_p^* = L - D_p$, while $I_r^* = L - D_r$. Assuming an interior solution for T_r and T_p and noting that $\partial I_i^* / \partial D_i = -1$, the FOCs for the rich and poor can be written as

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial T_r} &= -\nu_r \Psi_r' - \lambda + \mu \nu_r = 0, \\
\frac{\partial \mathcal{L}}{\partial D_r} &= \nu_r \Psi_r' \pi_r + \lambda \pi_r - \mu \nu_r \pi_r, \\
\frac{\partial \mathcal{L}}{\partial T_p} &= -\nu_p \Psi_p' + \lambda + \mu \nu_p = 0, \\
\frac{\partial \mathcal{L}}{\partial D_p} &= \nu_p \Psi_p' \pi_p - \lambda \pi_r - \mu \nu_p \pi_p.
\end{aligned}
$$

A.2 Adverse selection in the private insurance market: problem and FOCs

The Lagrangean function associated with this problem is

$$
\begin{aligned}
\mathcal{L} &= \nu_r \Psi \left(y_r - v\left(\frac{y_r}{w_r}\right) - T_r - \pi_r I_r^* + \phi(\pi_r)(-L + I_r^* + D_r) \right) \\
&+ \nu_p \Psi \left(y_p - v\left(\frac{y_p}{w_p}\right) - T_p - \pi_p I_p^* + \phi(\pi_p)(-L + I_p^* + D_p) \right) \\
&+ \lambda \left(y_r - v\left(\frac{y_r}{w_r}\right) - T_r - \pi_r I_r^* + \phi(\pi_r)(-L + I_r^* + D_r) \right. \\
&\left. - y_p + v\left(\frac{y_p}{w_r}\right) + T_p + \pi_p I_p^* - \phi(\pi_r)(-L + I_p^* + D_p) \right) + \mu \left(\sum_i \nu_i (T_i - \pi_i D_i) \right).
\end{aligned}
$$

Differentiating with respect to D_r and T_r, and assuming that there is an interior solution for T_r yields

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial D_r} =& \nu_r \Psi_r' \pi_r + \nu_p \Psi_p' \left(-\pi_p + \phi(\pi_p)\right) \frac{\partial I_p^*}{\partial D_r} \\ & - \lambda \left(\phi(\pi_r) - \pi_p\right) \frac{\partial I_p^*}{\partial D_r} + \lambda \pi_r - \mu \nu_r \pi_r, \\ \frac{\partial \mathcal{L}}{\partial T_r} =& - \nu_r \Psi_r' - \lambda + \mu \nu_r = 0.\end{aligned}$$

combining these two conditions and using (8) yields

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial D_r} &= \pi_r \nu_r \Psi_r' + \left[\nu_p \Psi_p' \left(\phi(\pi_p) - \pi_p\right) - \lambda \left(\phi(\pi_r) - \pi_p\right)\right] \frac{\partial I_p^*}{\partial D_r} + \lambda \pi_r - \mu \nu_r \pi_r \\ &= \left[\nu_p \Psi_p' \left(\phi(\pi_p) - \pi_p\right) - \lambda \left(\phi(\pi_r) - \pi_p\right)\right] \frac{\partial I_p^*}{\partial D_r} \\ &= -a \left[\nu_p \Psi_p' \left(\phi(\pi_p) - \pi_p\right) - \lambda \left(\phi(\pi_r) - \pi_p\right)\right],\end{aligned}$$

where a is defined in equation (11). Differentiating the Lagrangean expression with respect to D_p and T_p yields

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial D_p} =& \nu_p \Psi_p' \left(-\pi_p \frac{\partial I_p^*}{\partial D_p} + \phi(\pi_p) \left(\frac{\partial I_p^*}{\partial D_p} + 1\right)\right) + \lambda \left(\pi_p \frac{\partial I_p^*}{\partial D_p} - \phi(\pi_r) \left(\frac{\partial I_p^*}{\partial D_p} + 1\right)\right) - \mu \nu_p \pi_p, \\ \frac{\partial \mathcal{L}}{\partial T_p} =& - \nu_p \Psi_p' + \lambda + \mu \nu_p = 0.\end{aligned}$$

Combining these two equations and noticing that $\partial I_p^*/D_p = 0$ from equation (9), we have expression (18) in the main text.

When, by contrast, $D_p > \tilde{D}_p$ then $\partial I_p^*/D_p = -1$ and the above equation reduces to equation (22) in the main text.

A.3 Misperception and social insurance: problem and FOCs

The incentive constraints of this problem are given by

$$\begin{aligned} & y_r - v\left(\frac{y_r}{w_r}\right) - T_r - \pi_r I_r^* + \phi(\pi_r)(-L + I_r^* + D) \geq \\ & y_p - v\left(\frac{y_p}{w_r}\right) - T_p - \pi_r I_r^* + \phi(\pi_r)(-L + I_r^* + D), \end{aligned} \quad (A.1)$$

$$\begin{aligned} & y_r - v\left(\frac{y_r}{w_r}\right) - T_r - \pi_r I_r^* + \phi(\pi_r)(-L + I_r^* + D) \geq \\ & y_o - v\left(\frac{y_o}{w_r}\right) - T_o - \pi_r I_r^* + \phi(\pi_r)(-L + I_r^* + D), \end{aligned} \quad (A.2)$$

$$\begin{aligned} & y_o - v\left(\frac{y_o}{w_r}\right) - T_o - \pi_{po} I_{po}^* + \phi(\pi_p)(-L + I_{po}^* + D) \geq \\ & y_p - v\left(\frac{y_p}{w_r}\right) - T_p - \pi_{po} I_{po}^* + \phi(\pi_p)(-L + I_{po}^* + D). \end{aligned} \quad (A.3)$$

The Lagrangean of the government can be written as

$$
\begin{aligned}
\mathcal{L} = \nu_r \Psi &\left(y_r - v\left(\frac{y_r}{w_r}\right) - T_r - \pi_r I_r^* + \phi(\pi_r)(-L + I_r^* + D) \right) \\
&+ \nu_p \Psi \left(y_p - v\left(\frac{y_p}{w_p}\right) - T_p - \pi_{po} I_{po}^* + \phi(\pi_p)(-L + I_{po}^* + D) \right) \\
&+ \nu_o \Psi \left(y_o - v\left(\frac{y_o}{w_r}\right) - T_o - \pi_{po} I_{po}^* + \phi(\pi_r)(-L + I_{po}^* + D) \right) \\
&+ \lambda_1 \left(y_r - v\left(\frac{y_r}{w_r}\right) - T_r - \pi_r I_r^* + \phi(\pi_r)(-L + I_r^* + D) \right. \\
&\left. - y_o + v\left(\frac{y_o}{w_r}\right) + T_o + \pi_r I_r^* - \phi(\pi_r)(-L + I_r^* + D) \right) \\
&+ \lambda_2 \left(y_o - v\left(\frac{y_o}{w_r}\right) - T_o - \pi_{po} I_{po}^* + \phi(\pi_p)(-L + I_{po}^* + D) \right. \\
&\left. - y_p + v\left(\frac{y_p}{w_r}\right) + T_p + \pi_{po} I_{po}^* - \phi(\pi_p)(-L + I_{po}^* + D) \right) \\
&+ \mu \left(\sum_i \nu_i T_i - (\nu_r \pi_r + \nu_p \pi_p + \nu_o \pi_r) D \right).
\end{aligned}
$$

The FOCs are given by

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial D} =& \nu_r \Psi_r' \pi_r + \nu_p \Psi_p' \left[-\pi_{po} \frac{\partial I_{po}^*}{\partial D} + \phi(\pi_p)\left(1 + \frac{\partial I_{po}^*}{\partial D}\right) \right] \\
&+ \nu_o \Psi_o' \left[-\pi_{po} \frac{\partial I_{po}^*}{\partial D} + \phi(\pi_r)\left(1 + \frac{\partial I_{po}^*}{\partial D}\right) \right] - \mu(\nu_r \pi_r + \nu_p \pi_p + \nu_o \pi_r), \qquad \text{(A.4)} \\
\frac{\partial \mathcal{L}}{\partial T_r} =& -\nu_r \Psi_r' - \lambda_1 + \mu \nu_r = 0, \\
\frac{\partial \mathcal{L}}{\partial T_p} =& -\nu_p \Psi_p' + \lambda_2 + \mu \nu_p = 0, \\
\frac{\partial \mathcal{L}}{\partial T_o} =& -\nu_o \Psi_o' + \lambda_1 - \lambda_2 + \mu \nu_o = 0.
\end{aligned}
$$

Using (24) and (25), equation (A.4) can be rewritten as

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial D} =& \nu_r \Psi_r' \pi_r + \nu_p \Psi_p' \left[\pi_{po} b + \phi(\pi_p)(1-b) \right] + \nu_o \Psi_o' [\pi_{po} b + \phi(\pi_r)(1-b)] \\
&- \mu(\nu_r \pi_r + \nu_p \pi_p + \nu_o \pi_r).
\end{aligned}
$$

Observe that $\pi_{po} b + \phi(\pi_r)(1-b) = b[\pi_{po} - \phi(\pi_r)] + \phi(\pi_r) = \pi_r$. Substituting, we have

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial D} =& \nu_r \Psi_r' \pi_r + \nu_o \Psi_o' \pi_r + \nu_p \Psi_p' \pi_p - \mu(\nu_r \pi_r \\
&+ \nu_p \pi_p + \nu_o \pi_r) + \nu_p \Psi_p' \left[\pi_{po} b + \phi(\pi_p)(1-b) - \pi_p \right] \\
=& -\lambda_1 \pi_r + \lambda_2 \pi_p + (\lambda_1 - \lambda_2) \pi_r + \nu_p \Psi_p' \left[\pi_{po} b + \phi(\pi_p)(1-b) - \pi_p \right] \\
=& \nu_p \Psi_p' \left[\pi_{po} b + \phi(\pi_p)(1-b) - \pi_p \right] - \lambda_2 (\pi_r - \pi_p).
\end{aligned}
$$

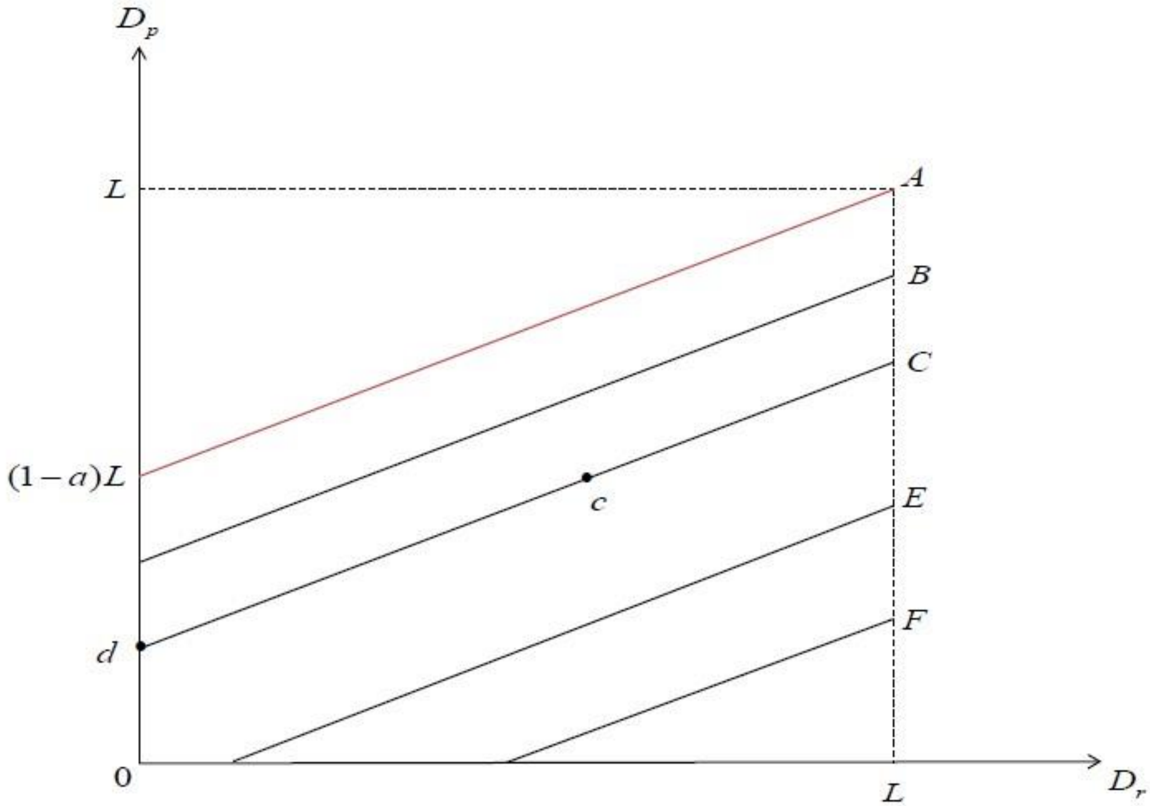

Figure 1: Welfare level curves in the (D_r, D_p)-space.

Highlights

- We study the role of social insurance to supplement income taxation when individuals differ in risk and ability
- Private insurance markets suffer from adverse selection
- Social insurance for the poor (rich) has negative (positive) incentive effects but increases (decreases) coverage
- Social insurance either for the rich or for the poor is always desirable
- Risk misperception may or may not strengthen the need for social insurance