ARTICLE IN PRESS

Engineering Structures xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Analysis of tuned liquid column damper nonlinearities

K. Dziedziech[a], A. Ghosh[b], J. Iwaniec[a], B. Basu[c], W.J. Staszewski[a,*], T. Uhl[a]

[a] Department of Robotics and Mechatronics, AGH University of Science and Technology, Al. Mickiewicza 30, 30-059 Kraków, Poland
[b] Department of Civil Engineering, Indian Institute of Engineering Science and Technology, Shibpur, Howrah 711103, India
[c] Department of Civil, Structural and Environmental Engineering, Trinity College Dublin, School of Engineering, Dublin 2, Ireland

ARTICLE INFO

Keywords:
Tuned liquid column damper
Output-only vibration analysis
Nonlinear damping identification
Wavelet transform

ABSTRACT

Broadband environmental excitations from wind, ocean wave and earthquakes are especially dangerous for flexible tall structures such as wind turbines, towers or cable-stayed bridges. Liquid Dampers have been studied for almost thirty years in terms of their capability for suppression of vibration in such structures. The work presented in this paper focuses on the Tuned Liquid Column Damper, both open and sealed, and the identification of its time-varying properties of nonlinear damping, frequency and air pressure identification. Experimental tests are conducted on a full scale model of the damper which is subjected to both white noise and harmonic excitation by means of a hydraulic shaker. Exponential decay of the displacement of the liquid column was measured and analysed. The identification procedure was conducted step-wise, first, mode separation with the use of Continuous Wavelet Transform was carried out and then identification of the instantaneous damping ratio for the first mode of vibration was performed. Results indicate that the damping ratio is nonlinear, time-varying and depends on the level of vibration. The air pressure data in the sealed TLCD was also recorded and analysed to reveal the nonlinear nature of the pressure change and the presence of higher odd harmonics.

1. Introduction

Liquid dampers have been successfully utilized in practise to supress amplitudes of structural vibration under wind, wave and earthquake excitation. The concept of using fluid to stabilize the rolling of ships was developed in the 1860s [1]. This damping technology is known by the name of anti-roll-tanks and widely used in naval architecture [2]. As compared to sloshing [3] or rectangular dampers [4], Tuned Liquid Column Dampers (TLCDs), which dissipate energy by the movement of an oscillatory column of liquid through orifice(s) provided in the cross section of a U-shaped container, have attracted special attention due to their high volumetric efficiency with respect to given amount of liquid, consistent behaviour across a wide range of excitation levels and a damping mechanism that can be quantified in a definite manner. The theoretical concept of the TLCD was developed in [5] and experimentally verified in [6]. Also its nonlinear mathematical description was provided and the relationship between head-loss coefficient and liquid damping validated. Since then, considerable research work has been carried out on the characterization and performance of TLCDs [7–14] including investigations on applicability to control of short period structures, wind turbines and on the impact of soil-structure interaction.

Since a TLCD is described by a relatively simple mathematical model, it is amenable for semi-active and active control. Some studies on TLCD, acting as an active vibration damper, were carried out in [15,16]. Experimental study for calculating the effectiveness of semi-active LCD for wind-induced vibration was carried out in [17]. Effectiveness of the semi-active LCD system and the hybrid viscous damper LCD control system for the suppression of wind-induced motion of high-rise buildings was examined in [18]. Attempts to achieve enhanced TLCD performance by using Electro-Rheological fluid (ER fluid) or Magneto-Rheological fluid (MR fluid) were presented in [19–21]. Experimental and theoretical investigations on the equivalent viscous damping of structures with TLCD having MR-fluids was carried out in [14,22]. Some researchers considered the vertical limbs of the TLCD to be sealed and utilized the air-spring effects in the sealed U-tube to extend the applicability of the control device to the high frequency range [23]. Sealed TLCDs were also studied for the control of wind induced multi-modal lateral and torsional vibration of long span cable stayed bridges by [24] and for the seismic vibration control of steel jacket platforms by [25].

Previous research studies in the field are related mainly to the design and control aspects of TLCDs. Identification of vibration/modal parameters has attracted much less attention. This is particularly

* Corresponding author.
E-mail addresses: dziedzie@agh.edu.pl (K. Dziedziech), aparna@civil.iiests.ac.in (A. Ghosh), jiwaniec@agh.edu.pl (J. Iwaniec), basub@tcd.ie (B. Basu), w.j.staszewski@agh.edu.pl (W.J. Staszewski), tuhl@agh.edu.pl (T. Uhl).

https://doi.org/10.1016/j.engstruct.2018.01.015

Please cite this article as: Dziedziech, K., Engineering Structures (2018), https://doi.org/10.1016/j.engstruct.2018.01.015

relevant to the identification and characterisation of the time-varying behaviour of these parameters. The present paper aims to model and experimentally identify nonlinear, time-varying damping in conventional TLCDs and nonlinear behaviour of air pressure in sealed TLCDs. The structure of the paper is as follows. Two simple mathematical models associated with the analysed nonlinear behaviour of the TLCD are presented in Section 2. The identification procedure, based on wavelet analysis, is briefly described in Section 3. Experimental work and results are presented in Section 4. Finally, the paper is concluded in Section 5.

2. Modelling of TLCD

This section briefly describes two nonlinear models associated with the TLCD performance. Firstly, the displacement of the liquid column in the base-excited TLCD is analysed to obtain the relevant equation of motion and to demonstrate the nonlinear, time-variant nature of damping in this equation. Then the air pressure in the limbs above the liquid column in a sealed TLCD is modelled to exhibit its nonlinear behaviour.

2.1. Displacement of liquid column in TLCD

Following the work in [26] this section presents a simple mathematical model that describes the behaviour of the first vibration mode of the analysed TLCD shown in Fig. 1.

The TLCD considered is composed of a tube-like container of horizontal dimension B and cross-sectional area A. The total effective length of the liquid column in the damper, denoted by L, is the sole parameter that controls the natural frequency of liquid oscillation in the conventional TLCD. This parameter is used for tuning the natural frequency of the TLCD to the structural frequency for the purpose of vibration control. The static height of liquid in the vertical limbs of the U-tube is given by h. It contains liquid of mass density ρ. Orifice(s) are installed in the horizontal portion of the tube. The coefficient of head loss, controlled by the opening ratio of the orifice(s), is denoted by ξ. The dynamic equilibrium of the horizontal forces acting on the liquid in the horizontal portion of the TLCD leads to the equation of motion of the liquid mass oscillating in the U-tube, when excited at the base. Let the TLCD be subjected to a horizontal base acceleration $\ddot{z}(t)$. The change in elevation of the liquid column is denoted by $x(t)$. The inertia force of the liquid in the horizontal portion of the TLCD can be expressed as

$$F_1 = \rho AB(\ddot{x} + \ddot{z}) \tag{1}$$

where the overdots represent differentiation with respect to time. The force due to head-loss caused by the orifice is given by

$$F_2 = \frac{1}{2}\rho A\xi\left|\dot{x}\right|\dot{x} \tag{2}$$

The hydrostatic force from the left limb of the damper on the horizontal portion may be written as

$$F_3 = -\rho A(h-x)(g-\ddot{x}) \tag{3}$$

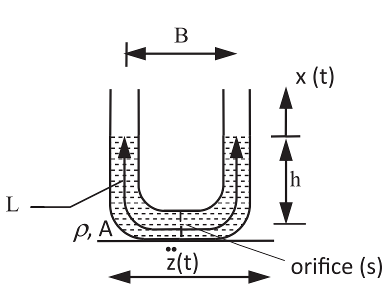

Fig. 1. Schematic diagram of the analysed TLCD system.

where g is the acceleration due to gravity. The hydrostatic force from the right limb of the damper on the horizontal portion can be expressed as

$$F_4 = \rho A(h + x)(g + \ddot{x}) \tag{4}$$

Total effective length L of the liquid column is given as

$$L = B + 2h \tag{5}$$

The algebraic summation of all these forces leads to the equation of motion

$$\rho AL\ddot{x} + \frac{1}{2}\rho A\xi\left|\dot{x}\right|\dot{x} + 2\rho Agx = -\rho AB\ddot{z} \tag{6}$$

A quadratic damping model is clearly visible in this equation. Though in past studies on the TLCD the liquid motion considered is the oscillation of the liquid column in the U-tube container, it is possible that the liquid in the vertical limbs may be subjected to sloshing. This has been earlier studied in [27] in the case of the Liquid Column Vibration Absorber (LCVA) in which the cross-sectional area of the vertical limbs is greater than that of the horizontal portion of the damper. Furthermore, as also noted in [28], higher value of the ratio of the horizontal length B to the total effective length L of the liquid column is desirable for higher damper efficiency. This leads to shallower depth of liquid in the vertical limbs of the damper. As a result, the propensity of the liquid in the limbs to sloshing is greater. For a TLCD with a relatively larger column width, the sloshing of the liquid in the limbs can represent a higher mode of vibration of the liquid motion.

2.2. Air pressure in sealed TLCD

Once both limbs are sealed, the air pressure above the liquid level can be analysed. Let P_0, V_0 be the initial pressure and volume in each sealed limb above the liquid level. As the liquid column oscillates, let $x(t)$ be the change in liquid level, due to which the changes in pressure P, and volume V, are denoted by $\delta P(t)$ and $\delta V(t)$ respectively. Considering the polytropic relation $PV^n = c$, where n and c denote the polytropic index and constant respectively, the following equations for the two sealed limbs are obtained.

$$(P_0 + \delta P(t))(V_0 - \delta V(t))^n = c \tag{7}$$

$$(P_0 - \delta P(t))(V_0 + \delta V(t))^n = c \tag{8}$$

By the generalized binomial theorem, Eq. (7) can be expanded as

$$P_0 + \delta P(t) = c'\left(1-\frac{\delta V(t)}{V_0}\right)^{-n} = c'\left(1-\alpha_1\frac{\delta V(t)}{V_0} + \alpha_2\left(\frac{\delta V(t)}{V_0}\right)^2 - \alpha_3\left(\frac{\delta V(t)}{V_0}\right)^3 \ldots\right) \tag{9}$$

where c' and $\alpha_i, i = 1,2,\ldots$ represent constants.

Similarly, Eq. (8) can be expanded as

$$P_0 - \delta P(t) = c'\left(1 + \frac{\delta V(t)}{V_0}\right)^{-n} = c'\left(1 + \alpha_1\frac{\delta V(t)}{V_0} + \alpha_2\left(\frac{\delta V(t)}{V_0}\right)^2 + \alpha_3\left(\frac{\delta V(t)}{V_0}\right)^3 \ldots\right) \tag{10}$$

Eqs. (9) and (10) leads to

$$2\delta P(t) = 2c'\left(-\alpha_1\frac{\delta V(t)}{V_0} - \alpha_3\left(\frac{\delta V(t)}{V_0}\right)^3 - \alpha_5\left(\frac{\delta V(t)}{V_0}\right)^5 \ldots\right) \tag{11}$$

Since $\delta V(t) = Ax(t)$, Eq. (11) can be written as

$$\delta P(t) = \beta_1 x(t) + \beta_2 x^3(t) + \beta_3 x^5(t)\ldots \tag{12}$$

and if $x(t)$ is harmonic, then Eq. (12) becomes

$$\delta P(t) = \beta_1' sin(\omega t) + \beta_2' sin^3(\omega t) + \beta_3' sin^5(\omega t)... \quad (13)$$

where β_i and $\beta_i', i = 1,2,...$ are scalar constants. This clearly shows that the pressure change is nonlinear and odd harmonics are exhibited in the above model.

3. Identification procedure

This section introduces the identification procedure for natural frequency and nonlinear damping ratio. The procedure is based on the Continuous Wavelet Transform. The concept of wavelet ridges along with the Crazy Climbers algorithm, utilized for identification of natural frequency, is presented. Finally, the logarithmic decrement is introduced for damping identification.

3.1. Continuous wavelet transform

Continuous Wavelet Transform is particularly known for its ability to analyse of time-varying systems, but it is also very successful in the analysis of nonlinear systems [29]. For a time domain signal $x(t)$ representing vibration response, the Continuous Wavelet Transform can be defined as

$$X(a, b) = \frac{1}{\sqrt{|a|}} \int_{\infty}^{\infty} x(t)\psi\left(\frac{t-b}{a}\right)dt \quad (14)$$

where b is a translation indicating localization in the time domain, a is a scale parameter providing localization in the frequency domain and $\psi(t)$ is a continuous function in both, i.e. the time and frequency domain, called the mother wavelet. Following the above definition, the dynamic response wavelet auto-power spectrum can be defined as

$$G_{XX}(a, b) = X(a, b)X^*(a, b) \quad (15)$$

where the superscript "$*$" indicates a complex conjugate. This function shows how the power in a signal is distributed over the combined time-frequency plane and thus will be used for identification of natural frequencies in the analysed time-varying system.

3.2. Wavelet function

It is well known that when the time-frequency analysis is used, the proper selection of the mother wavelet function is important, as discussed in [30]. The compact support in the time and frequency domain and complex nature are often desirable features of the wavelet function. The need for compact support is required for the combined time-frequency localisation. The ability of wavelet analysis to capture instantaneous time-variant behaviour is essential when identification is performed. Complex, rather than real, wavelet functions are needed to avoid amplitude zero crossings in the time and frequency domain. A number of different wavelet functions that fulfil these requirements can be used in practice. The work presented in this paper utilises the complex Morlet wavelet function defined as

$$\psi(t) = e^{-\frac{|t|^2}{2}} e^{j\omega_0 t} \quad (16)$$

This function has been selected following numerous successful applications related to the analysis of time-varying and nonlinear dynamic systems [31–33]. Fig. 2 illustrates the complex Morlet wavelet function in the time and frequency domain.

3.3. Damping estimation

Damping estimation can be conducted on the basis of decay of dynamic responses. It is well known for the linear time-invariant (LTI) systems that the amplitude decay of dynamic response can be mathematically described as

$$y(t) = Ae^{-\zeta\omega t} sin(\sqrt{1-\zeta^2}\omega t + \theta) \quad (17)$$

where $Ae^{-\zeta\omega t}$ is the envelope of the impulse response, ζ is the damping ratio and θ is the phase. Then, the damping ratio may be evaluated with the use of curve fitting techniques, as explained in [31]. It is clear that for LTI systems that the natural frequency and damping ratio characteristics are independent of time. For nonlinear time-varying systems, other methods are required, as demonstrated in [34,35]. The work presented in this paper utilises two different approaches. The first approach is based on the modified version of the logarithmic decrement [36]. The logarithmic decrement method uses the natural logarithm of the ratio of the amplitudes of two successive displacement peaks

$$\delta(t) = ln\left(\frac{x(t)}{x(t+T)}\right) \quad (18)$$

where $x(t)$ is the amplitude at time t and $x(t+T)$ is the amplitude of the peak one period away. The damping ratio can be then found from the logarithmic decrement as

$$\zeta(t) = \frac{1}{\sqrt{1 + \left(\frac{2\pi}{\delta(t)}\right)^2}} \quad (19)$$

The instantaneous damping ratio will be obtained in the first approach following the above procedure. The second approach that will be used originates from the classical damping ratio estimation procedure based on the envelope function. It is well known that for time-varying systems the general form of the response envelope can be written as

$$env(t) = Ae^{-\zeta(t)\omega(t)t} \quad (20)$$

where the exponential decay is controlled by the instantaneous damping and frequency. In practical applications curve fitting is often used to obtain a smooth envelope. Once the envelope is obtained, the logarithm is calculated to give

$$ln(env(t)) = ln(A) - \zeta(t)\omega(t)t \quad (21)$$

Up to this point, all above mentioned derivations were analytical. Further considerations will be done numerically, the $env(t)$ is the measured envelope of the system impulse response, the $ln(A)$ is a constant value of amplitude of the system impulse response, the $\omega(t)$ is the ridge of the instantaneous frequency, the $\zeta(t)$ is the nonlinear damping ratio. It is important to note that both, i.e. instantaneous frequency and nonlinear damping ratio of the system impulse response, are slowly varying functions. If these functions were not slowly varying, it could cause loss of the numerical stability of the solution. The ridge of the instantaneous frequency can be used to separate damping ratio and natural frequency [31,37], i.e.

$$-\frac{ln(env(t)) - ln(A)}{\omega(t)} = \zeta(t)t \quad (22)$$

Then numerical differentiation can be applied to obtain the damping ratio as

$$\zeta(t) = diff\left(-\frac{ln(env(t)) - ln(A)}{\omega(t)}\right) \quad (23)$$

It is important to note that all approaches based on curve fitting techniques, although yield smooth results, are highly dependent on the goodness of the fit.

4. Experimental identification of TLCD

A simple experiment was conducted to obtain vibration response data from the full scale TLCD shown in Fig. 3. The TLCD device was made from 3 mm thick acrylic pipes of inner diameter 0.104 m. Water was used as the damper liquid. The centre to centre horizontal dimension of the damper tube (B) was 1.151 m. and the total length of the liquid column was 1.751 m. This leads to the theoretical fundamental frequency of the damper as 0.533 Hz. The horizontal tube was fitted

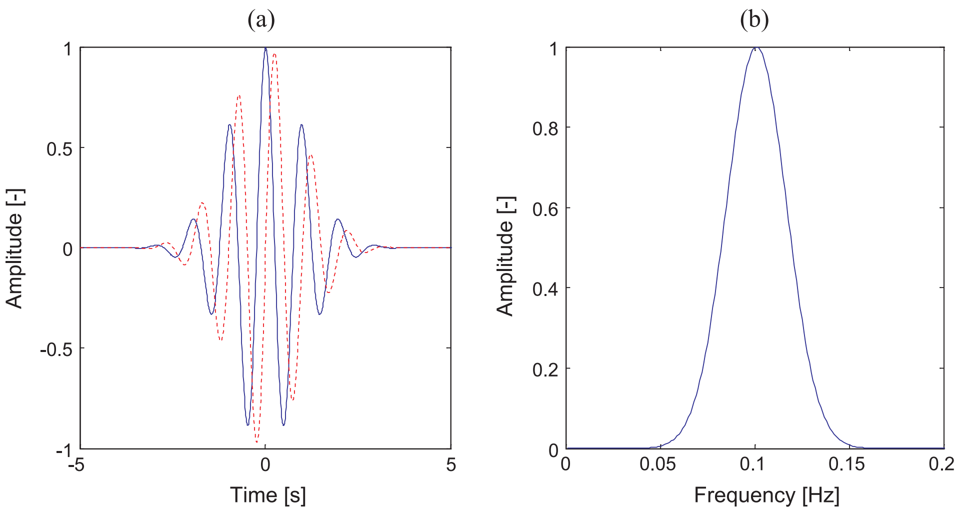

Fig. 2. Complex Morlet wavelet function: (a) time domain; (a) frequency domain.

Fig. 3. Tuned Liquid Column Damper used in the experimental analysis.

with a circular orifice plate with the orifice opening ratio of 74.89%. Each vertical limb of the U-tube was fitted with a wave gauge with a maximum measurement capacity of 0.3 m to measure the liquid displacement. The TLCD was attached to a steel frame which was excited using a servo-hydraulic loading apparatus, consisting of a Series 111 MTS accumulator and a high speed linear hydraulic actuator with a 150 kN capacity, 0.25 m (± 0.125 m) stroke and a mass of 0.5 tonnes.

4.1. Liquid column response analysis

A 0–5 Hz band-limited white noise excitation was used as the input in the experimental analysis. The analysed TLCD was excited for 120 s. After that time the excitation was turned off to obtain the decaying vibration response. The response data was captured using a water level gauge. Fig. 4 indicates the captured vibration response. It is important to note that the identification analysis was limited only to the decaying part of the captured response, indicated by the red frame in Fig. 4a and zoomed in Fig. 4b.

Wavelet analysis was used for the system identification by filtering. Fig. 5a provides the extracted natural frequency for the analysed first vibration mode which is close to the theoretically calculated expected frequency of 0.533 Hz. The curve-fitted response signal, shown in Fig. 5b, was used for damping estimation. The envelope function was calculated and damping ratio estimated following the procedure described in Section 3. The result is given in Fig. 5c, based on the logarithmic decrement is presented. These characteristics clearly displays the nonlinear nature of damping, confirming the theoretical model described in Section 2.1. One should note that in contrast to Fig. 4b – where the response includes all analysed modes – Fig. 5b exhibits only the extracted mode of interest (after the wavelet-based filtering).

The results show that the first vibration mode is quadratically damped and the relevant dynamics of the system can be described by the model represented by Eq. (6).

4.2. Air pressure analysis

In the second part of the experimental study the sealed TLCD system was excited using a single harmonic excitation. The frequency of this excitation corresponded to the natural frequency of the first vibration mode of the TLCD. The change of air pressure, in both sealed limbs of the TLCD, was monitored above the water level. The change in pressure over time is given in Fig. 6. The frequency analysis and combined time-frequency analysis was applied to reveal the nonlinear nature of the analysed pressure signal. The combined time-frequency domain has been performed using the Continuous Wavelet Transform, as described in Section 3.1. Higher odd harmonics are clearly visible in Fig. 6b and c. These results show that the model given in Section 2.2 can be used to describe the nonlinear behaviour of the pressure change in the analysed sealed TLCD.

5. Conclusions

The dynamics of a TLCD was investigated. The major focus was on identification of frequency of the oscillating liquid column, nonlinear damping estimation and air pressure characterization. The response of the first vibration mode associated with the – liquid column of the TLCD was considered to obtain the dynamic equation of motion with a quadratic damping model. The air pressure above the water level in case of a sealed TLCD was also modelled to reveal its nonlinear nature. Experimental study was performed to identify and validate the proposed models.

The experimental results show that the proposed quadratic damping model can be used to describe the dissipative behaviour associated with the first vibration mode of the analysed TLCD. The time-varying damping was effectively identified using the combined envelope analysis, curve fitting and logarithmic decrement. Further, in case of the sealed TLCD, the air pressure above the liquid level can be modelled

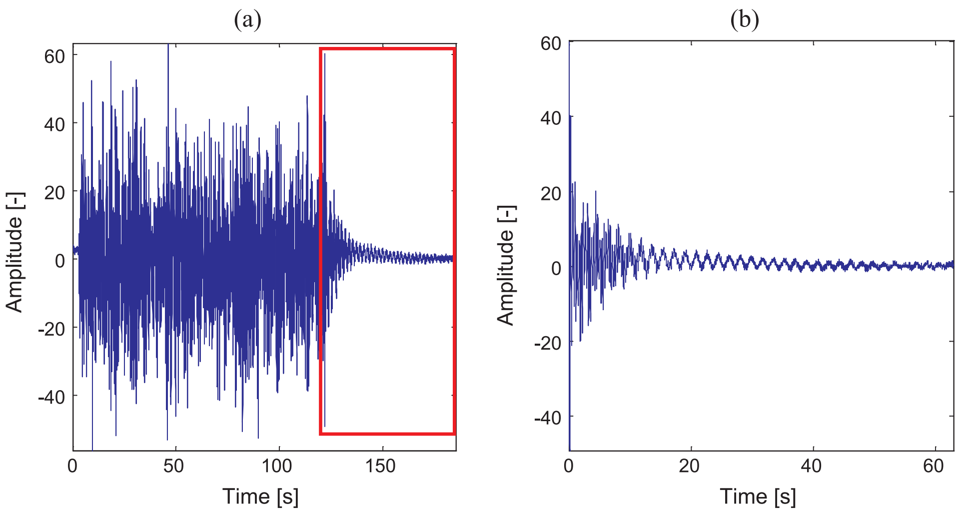

Fig. 4. Vibration response from the analysed TLCD: (a) total response (the decaying part of the signal is indicated by the red rectangular frame); (b) zoomed decaying part. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

using the constant pressure-volume polytropic relation. This model leads to a nonlinear behaviour of the pressure change and indicates the presence of higher odd harmonics. The experimental work and time-frequency analysis of the air pressure confirmed the nonlinear behaviour predicted by the model.

Acknowledgements

The work presented in this paper was supported by funding from the WELCOME research project No. 2010-3/2, sponsored by the Foundation for Polish Science (Innovative Economy, National Cohesion Programme, EU).

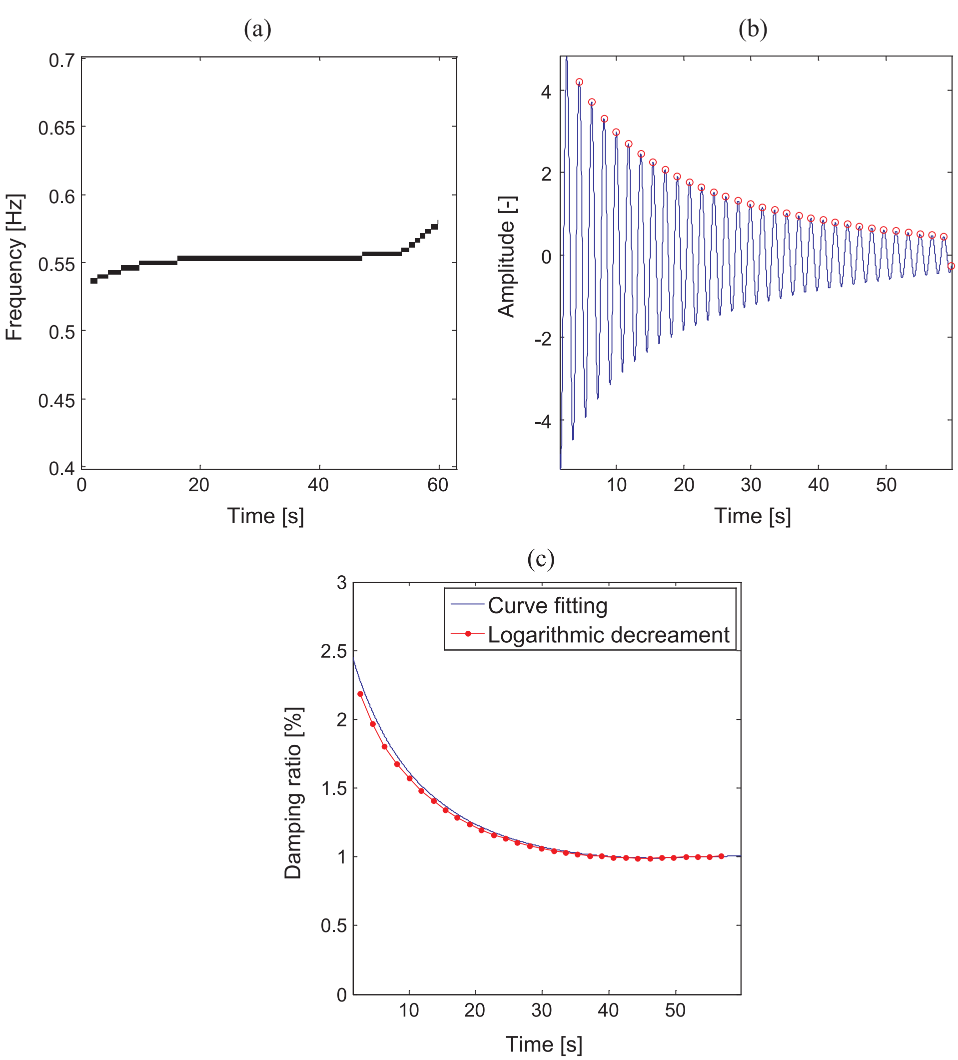

Fig. 5. Identification results for the 1st mode of vibration for the analysed TLCD: (a) extracted natural frequency; (b) extracted vibration response; (c) estimated damping ratio.

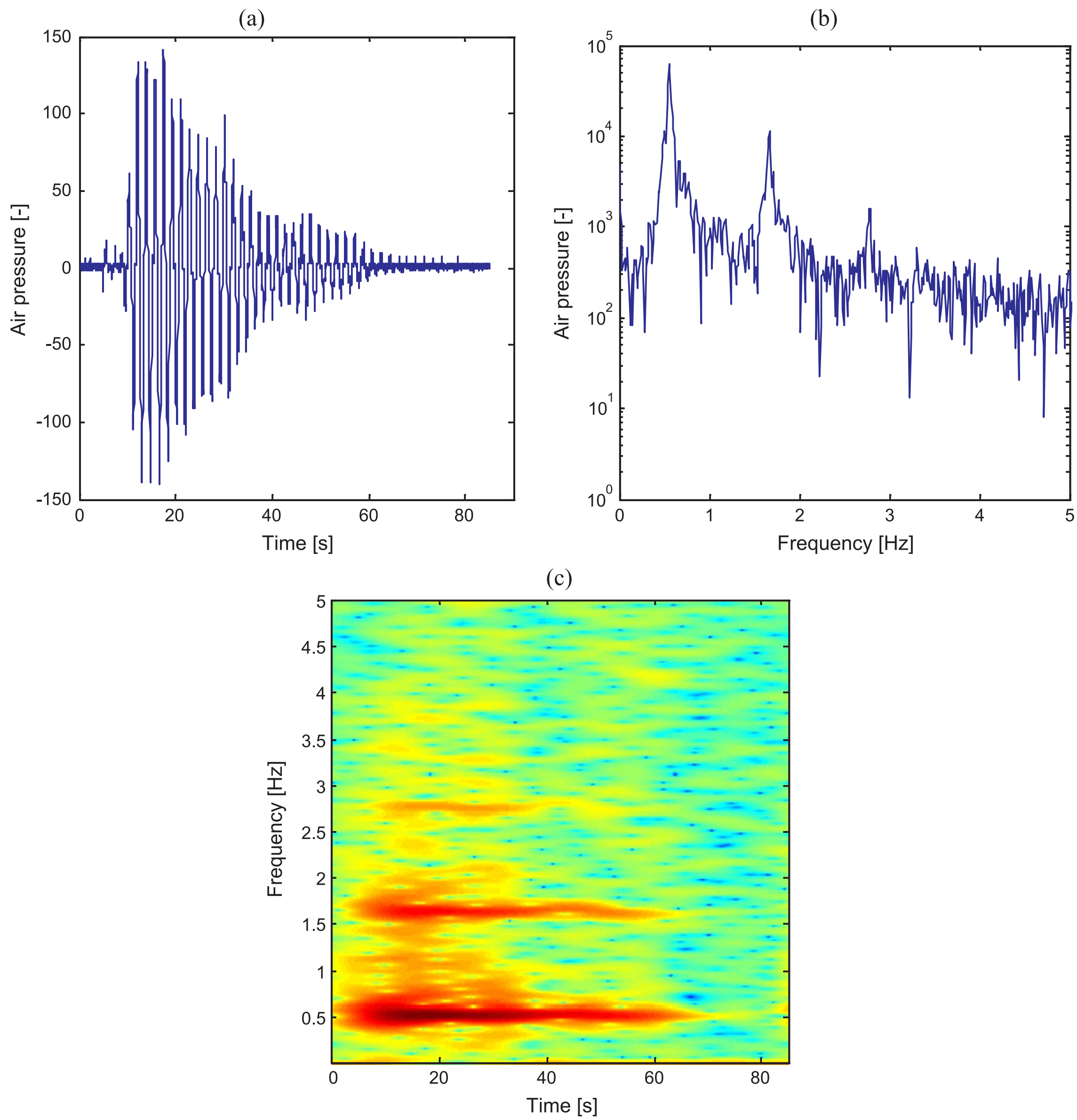

Fig. 6. Identification results for the 1st mode of vibration for the analysed TLCD for air pressure: (a) time domain; (b) frequency domain and (c) combined time-frequency domain.

References

[1] Froude W. On the rolling of ships. Trans Inst Naval Arch 1861;2(1861):180–227.
[2] Gawad AA, Nayfeh AH, Ragab SA, Mook DT. Roll stabilization by anti-roll passive tanks. Ocean Eng 2001;28(5):457–69.
[3] El-Sayad MA, Hanna SN, Ibrahim RA. Parametric excitation of nonlinear elastic systems involving hydrodynamic sloshing impact. Nonlinear Dyn 1999;18(1):25–50.
[4] Ikeda T. Nonlinear parametric vibrations of an elastic structure with a rectangular liquid tank. Nonlinear Dyn 2003;33(1):43–70.
[5] Saoka Y, Sakai F, Takaeda S, Tamaki T. On the suppression of vibrations by tuned liquid column dampers. In: Annual meeting of JSCE, JSCE, Tokyo; 1988.
[6] Sakai F, Takaeda S, Tamaki T. Tuned liquid column damper-new type device for suppression of building vibration. In: International conference on high-rise building; 1989. p. 926–31.
[7] Balendra T, Wang CM, Cheong HF. Effectiveness of tuned liquid column dampers for vibration control of towers. Eng Struct 1995;17(9):668–75.
[8] Won AYJ, Pires JA, Haroun MA. Stochastic seismic performance evaluation of tuned liquid column dampers. Earthq Eng Struct Dyn 1996;25(11):1259–74.
[9] Ghosh A, Basu B. Seismic vibration control of short period structures using the liquid column damper. Eng Struct 2004;26(13):1905–13.
[10] Ghosh A, Basu B. Effect of soil interaction on the performance of liquid column dampers for seismic applications. Earthq Eng Struct Dyn 2005;34(11):1375–89.
[11] Ghosh A, Basu B. Alternative approach to optimal tuning parameter of liquid column damper for seismic applications. J Struct Eng 2007;133(12):1848.
[12] Ghosh A, Basu B. Seismic vibration control of nonlinear structures using the liquid column damper. J Struct Eng 2008;134(1):146–53.
[13] Wu JC, Chang CH, Lin YY. Optimal designs for non-uniform tuned liquid column dampers in horizontal motion. J Sound Vib 2009;326(1–2):104–22.
[14] Colwell S, Basu B. Tuned liquid column dampers in offshore wind turbines for structural control. Eng Struct 2009;31(2):358–68.
[15] Balendra T, Wang CM, Yan N. Control of wind-excited towers by active tuned liquid column damper. Eng Struct 2001;23(9):1054–67.
[16] De Souza SLT, Caldas IL, Viana RL, Balthazar JM, Brasil RMLRF. Dynamics of vibrating systems with tuned liquid column dampers and limited power supply. J Sound Vib 2006;289(4–5):987–98.
[17] Yalla Swaroop K, Kareem Ahsan. Semiactive tuned liquid column dampers: experimental study. J Struct Eng 2003;129(7):960–71.
[18] Kim H, Adeli H. Wind-induced motion control of 76-story benchmark building using the hybrid damper-TLCD system. J Struct Eng 2011;131:1794–802. May 2011.
[19] Sakamoto D, Oshima N, Fukuda T. Tuned sloshing damper using electro-rheological fluid. Smart Mater Struct 2001;10(5):963–9.
[20] Ni YQ, Ying ZG, Wang JY, Ko JM, Spencer BF. Stochastic optimal control of wind-excited tall buildings using semi-active MR-TLCDs. Probabil Eng Mech 2004;19(3):269–77.
[21] Wang JY, Ni YQ, Ko JM, Spencer BF. Magneto-rheological tuned liquid column dampers (MR-TLCDs) for vibration mitigation of tall buildings: modelling and analysis of open-loop control. Comput Struct 2005;83(25–26):2023–34.
[22] Colwell S, Basu B. Experimental and theoretical investigations of equivalent viscous damping of structures with TLCD for different fluids. J Struct Eng 2008;134(1):154–63.
[23] Reiterer M, Ziegler F. Bi-axial seismic activation of civil engineering structures equipped with tuned liquid column dampers. J Seismol Earthq Eng

2005;7(1):45–60.
[24] Shum KM, Xu YL, Guo WH. Wind-induced vibration control of long span cable-stayed bridges using multiple pressurized tuned liquid column dampers. J Wind Eng Ind Aerodyn 2008;96(2):166–92.
[25] Mousavi SA, Bargi K, Zahrai SM. Optimum parameters of tuned liquid column–gas damper for mitigation of seismic-induced vibrations of offshore jacket platforms. Struct Control Heal Monit 2013;20(3):422–44.
[26] Ghosh A. A study on liquid dampers for aseismic design of structures. India: Jadavpur University; 2003.
[27] Konar T, Ghosh A. Bimodal vibration control of seismically excited structures by the liquid column vibration absorber. J Vib Control 2013;19(3):385–94.
[28] Konar T, Ghosh A. Passive control of seismically excited structures by the liquid column vibration absorber. Struct Eng Mech 2010;36(5):561–73.
[29] Yang Y, Peng ZK, Dong XJ, Zhang WM, Meng G. Nonlinear time-varying vibration system identification using parametric time–frequency transform with spline kernel. Nonlinear Dyn Aug. 2016;85(3):1679–94.
[30] Staszewski WJ. Wavelets for mechanical and structural damage identification. In: Polish Academy of Sciences, IMP PAN; 2000.
[31] Staszewski WJ. Identification of damping in Mdof systems using time-scale decomposition. J Sound Vib 1997;203(2):283–305.
[32] Staszewski WJ. Identification of non-linear systems using multi-scale ridges and skeletons of the wavelet transform. J Sound Vib 1998;214(4):639–58.
[33] Staszewski WJ, Wallace DM. Wavelet-based frequency response function for time-variant systems – an exploratory study. Mech Syst Signal Process 2014;47(1–2):35–49.
[34] Feldman M. Non-linear system vibration analysis using Hilbert transform–I. Free vibration analysis method ‘Freevib’. Mech Syst Signal Process 1994;8(2):119–27.
[35] Feldman M. Hilbert transform applications in mechanical vibration vol. 25. John Wiley & Sons; 2011.
[36] Inman DJ. Engineering vibration, vol. 3. Prentice Hall Upper Saddle River; 2001.
[37] Ruzzene M, Fasana A, Garibaldi L, Piombo B. Natural frequencies and dampings identification using wavelet transform: application to real data. Mech Syst Signal Process 1997;11(2):207–18.