



## Fatigue properties of crumb rubber asphalt mixtures used in railways

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### HIGHLIGHTS

- Monotonic and cyclic fatigue tests were made on railway asphalt mixtures.
- Crumb rubber modification of railway mixtures considerably increases fatigue life.
- Air void have more influences on railway mixtures compared to highway ones.
- Crumb rubber modification improves railway asphalt mixes more than highway mixtures.
- Findings obtained improve current understanding of low-vibration-railways behavior.

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### ABSTRACT

The use of asphalt mixtures in railway tracks provides a positive contribution to the bearing capacity, stability, durability and more importantly damping properties of the railway structure particularly for the new generation of low-vibration-railway track systems. However, one of the main concerns in the use of asphalt mixtures in railways is their fatigue cracking caused by the repeated traffic loading. The present study focused on the fatigue damage behavior of asphalt mixtures in the railways through using the viscoelastic continuum damage theory. Three experiments of creep compliance, constant crosshead rate and cyclic fatigue tests were carried out on some asphalt mixtures with different air void contents and aging conditions. Based on the results obtained, the crumb rubber modification of railways asphalt mixtures at low stress levels increases the fatigue life of asphalt up to 7.2 times while this increase is at the most 3.6 for the highway asphalt mixture. Furthermore, reducing the air void content from 4% to 2% for the crumb rubber modified asphalt mixture increases the fatigue life by 9.4 times at high stress levels and 18.2 times at low stress levels, which is a great improvement. It was shown that the lowest fatigue life is obtained in the aged crumb rubber modified mixture (i.e., 20 percent of unmodified asphalt mixture).

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## 1. Introduction

Railway transportation system is usually preferred over other modes of transportations because of its higher speed, better safety, and less environmental negative impacts. In recent years, with the development of high-speed tracks, this transport system has been given more attention. High-speed tracks require a safer and more reliable railway structure with lower maintenance operations [1,2]. Furthermore, transmission of vibrations to surrounding structures in the urban railways is a serious concern. A very effective solution that improves both the stability and the durability of

the structure, contributes to the reduction of need for maintenance operations, and reduces the induced railway vibrations is the use of asphalt mixture in the railway construction [2–6]. Moreover, the use of asphalt mixture makes the possibility of reducing the railway height, and in turn provides great advantages in the construction of tunnels with smaller diameter or lighter bridges. Asphalt mixtures can be used as the ballast layer (Fig. 1a) or the sub-ballast layer (Fig. 1b) in the railway system. According to the literature, the behavior of the overall structure can be improved by using the asphalt mixture as a sub-ballast layer in the track-bed [3]. Also replacing the ballast layer by an asphalt layer can cause more elasticity of the track support system as well as easier construction and maintenance (i.e., easier geometry deviations corrections) [3]. Asphalt underlayment design and construction standards for railways typically follow the recommendations set forth by the Asphalt Institute [7].

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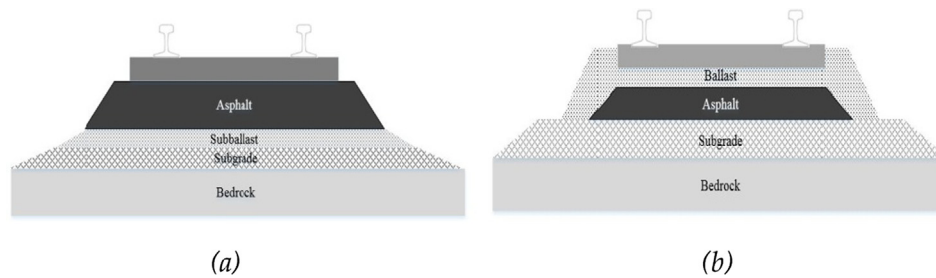


Fig. 1. Schematic views of track-bed using asphalt mixture as (a) ballast layer; (b) sub-ballast layer.

It is generally agreed that fatigue cracking, rutting, and low-temperature cracking are the three principal types of distress considered in the flexible pavement design [8]. However, rutting of the asphalt mixture is not a serious concern in the track-bed, since the pressures are applied through the ballast over a wide area [9] and also the temperatures are not sufficiently high to promote rutting [7]. Bleeding is also of little concern since there is not a direct contact between the wheels and the asphalt layer [9]. However, cracking caused by the fatigue due to repetitive nature of train loads is important in asphalt pavements of railways [10]. This is noteworthy mentioning that the vertical stress levels on the top of the hot mix asphalt (HMA) layer in the railway track-bed are much less than (about one-third of) the one in the highway asphalt layer even with the heavy tonnage of railroad loadings [3,11]. This is due to the wider load distribution of the sleepers on the underneath layer [3]. In 2007, the Design Standard for Railway Structures used by Japanese railway organizations was revised in order to consider the fatigue life of the railway track pavement as it is greatly influenced by the number of passing trains [3,9,12]. The track-bed HMA is specifically designed to have a railway support system with medium modulus, high flexibility, low voids, and fatigue resistance with the ability of bearing high tensile strains without cracking [13]. Recent studies have also indicated that coarse aggregate has an important effect on the fatigue cracking of asphalt mixtures [14]. The typical asphalt mixture used in railway applications is a dense-graded highway base mixture with a maximum aggregate size of 25–37.5 mm [3,15]. In addition, in order to have a low to medium modulus mixture in railway structure, it is usually tried to increase the asphalt binder content by 0.5% over the considered optimum for highway applications. This usually results in an asphalt mixture with a lower air void of 1%–3% [7,15]. This slight modification to the typical highway mixture can impart the ideal properties of the track structure. The positive effect of the air void reduction on the fatigue properties of asphalt mixtures used in highway applications have been evaluated by some researchers. A 1% decrease in air void content was reported to improve the fatigue properties of asphalt mixtures between 8.2 and 43.8% and the rutting resistance by 7.3–66.3% [16]. Harvey et al. showed that a 1% increase in air void content of an asphalt mixture with 5% asphalt content and 5% air voids, causes a 30-percent reduction in the fatigue life [17]. Tao et al. also showed that increasing the air voids from 4% to 6%, 8%, 10% and 12% reduces the fatigue life by 5%, 25%, 30% and 50%, respectively [18]. A review of the available literature indicates that the fatigue properties of asphalt mixtures of railway applications at very low air void contents have not been sufficiently investigated and need further investigations.

The use of special additives or polymer-modified binders can also offer the possibility of complying with specific requirements (heavy duty, lower temperatures, and noise/vibration reduction) for the mixture or the construction [3,15]. In the past few years, various additives, including hydrated lime, cement, amines and polymers were used to improve the physical properties of

asphalt mixtures used in the highways. In addition, with the environmental problems of waste tires and the low costs of crumb rubber production in one hand, and the necessity of using vibrations-damping-materials as a base (ballast) in the railway, on the other hand, an investigation of using this material (as an additive) to improve the properties of railway asphalt binder (and mixture) has become an important need in the railway field. Although various studies have been made on the performance characteristics and modeling the behavior of crumb rubber modified asphalt mixtures, crumb rubber mixtures designed and used in the railway track-bed layers have not been sufficiently investigated.

In recent years, many researchers have focused on mathematical and experimental modeling of asphalt fatigue properties. One of these models was based on the fundamental theory of viscoelastic continuum damage, which was first used in 1990 by Kim and Little [19]. They applied the Schapery's nonlinear viscoelastic constitutive theory for the materials with distributed damage in order to describe the sand asphalt mixture behavior under controlled strain cyclic loading [19]. Later on (in 1998), it was shown that the theory of VECD can properly describe the behavior of asphalt concrete under both controlled stress and controlled strain cyclic loadings [20]. In 2001, Daniel showed that the damage characteristics of asphalt materials were independent of the mode of loading and it could be determined using simpler tests [21]. Chehab and Kim (2002) validated the time-temperature superposition principle at high levels of damage [22]. These two findings significantly reduced the required testing protocol while simultaneously extending the scope of application of this model. Later on, Underwood (2006) used polymer modified asphalt and demonstrated that the time-temperature superposition principle was also acceptable for modified bitumen [23]. On the same line, Kutay and Gibson (2008) used a push-pull (tension-compression) test to develop a simplified model for computation of damage in cyclic tests [24]. Haddadi and Ameri (2015) validated the simplified viscoelastic continuum damage model for flexural mode of loading [25]. More recently, Wei and Kim (2016) illustrated the viscoelastic-viscoplastic coupling phenomenon in asphalt concrete using results obtained from experiments [26].

## 2. Research objective and methodology

As indicated above, despite various studies made on the performance characteristics and modeling of asphalt mixtures, the asphalt mixture (in particular the crumb rubber modified mixture) used in the railway track-bed layers has not been sufficiently investigated. Due to the widespread application of viscoelastic continuum damage (VECD) theory in modeling the fatigue properties of asphalt mixtures in one hand, and the necessity of using fatigue life and fatigue damage models in the design of asphalt mixtures used in the track-bed on the other hand, the VECD model was used

in this research for the evaluation of the fatigue properties of asphalt mixtures designed and used in the railway systems.

Asphalt railway tracks are made of asphalt mixtures which contain larger aggregate size and higher content of asphalt binder compared to the regular asphalt mixtures used in roads and highways. These differences cause a lower air void of around 2% in the railway asphalt mixture which considerably influences the fatigue properties of railway asphalt mixtures [27,28]. Furthermore, various studies have shown that stress level on the top of the HMA layer in the track-bed structure is much less than that of the highway pavements [3,15]. These differences in the loading characteristics of railways and highways make the necessity of new independent research on the fatigue properties of railway asphalt mixtures.

In response to this need, asphalt mixtures containing higher binder contents and a lower air void of about 2% were made in this research. The specific gradation of railway asphalt mixtures together with very fine crumb rubber powder were used to construct the railways asphalt mixtures. A mixture aging procedure was also applied in order to evaluate the effect of aging phenomenon. The non-damage-inducing creep compliance test was first used to obtain the linear viscoelastic behavior of asphalt mixtures. Then, different types of damage-inducing fatigue tests including some monotonic loading tests and a constant crosshead cyclic test were performed on all samples. The test results obtained were used to apply in the VECD model. The damage characteristics and the consequent fatigue life curves were derived and used for the phenomenological interpretation of different materials characteristics. The results obtained have led to drive new practical findings which can improve the current understanding of the behavior of the railway track asphalt. Note that limitations of the current understanding have been reported in the development of the low vibration railway track systems [28,29]. Addressing these limitations, the fatigue characteristics of railway asphalt mixture, the effects of various parameters (including air void) on asphalt fatigue life, and the effectiveness of the aged crumb rubber modified mixture in the improvement of railway asphalt mixture properties have been derived in this research.

### 3. Theory

As indicated above, viscoelastic continuum damage theory was utilized in this research to model the fatigue properties of different asphalt mixtures used in the railway structure. For this purpose, with the Schapery's extended elastic-viscoelastic "correspondence principle", which is applicable to both linear and nonlinear viscoelastic materials, the pseudo-strain parameter is first calculated using Eq. (1). Based on Schapery's theory, constitutive equations of a viscoelastic material can be expressed as an elastic material if the strains are expressed as pseudo variables in the form of convolution integral presented in Eq. (1) [30]:

$$\varepsilon^R = \frac{1}{E_R} \int_0^t E(t-\tau) \frac{\partial \varepsilon}{\partial \tau} d\tau \quad (1)$$

where  $E_R$  is a particular reference modulus,  $E(t)$  is the uniaxial relaxation modulus,  $\varepsilon(t)$  is the function of strain and  $\varepsilon^R$  represents the pseudo strain parameter. There is a linear relationship between stress and pseudo strain for the linear viscoelastic materials without damage. When the micro cracks appear, this relationship goes to nonlinear section causing the reduction of stiffness known as pseudo secant modulus or pseudo stiffness. Pseudo stiffness parameter represents the changing stiffness of the material due to growing damage. In continuum damage mechanics, it is assumed that the damage body can be viewed as a homogeneous continuum on

a macroscopic scale, and the influence of damage is typically reflected in terms of reduction in stiffness or strength of material. By eliminating the effects of anisotropy, nonlinear viscoelasticity and plasticity in the viscoelastic continuum damage theory, it is possible to assume that internal damage in materials is caused due to decrease in pseudo stiffness. In this case, equation of pseudo stiffness can be expressed as follows [31]:

$$C(S) = \frac{\sigma}{\varepsilon^R} \quad (2)$$

where  $C$  is the pseudo stiffness and  $S$  is the internal variable that initializes the material damage. It should be noted that when the body is intact and there is no damage, the value of  $C$  equals to 1 and  $S$  is zero. Schapery developed a series of viscoelastic damage models to obtain the  $S$  value. His models are based on his studies on mechanics of viscoelastic damage, micromechanical models and the theory of potential work. According to his investigations, the damage evolution law for a specific damage is described as follows [32]:

$$\dot{S} = \left( -\frac{\partial W^R}{\partial S} \right)^\alpha \quad (3)$$

In this equation,  $W^R$  is the pseudo strain energy density function,  $S$  represents the internal state of the damage state, and  $\alpha$  is a positive constant related only to the viscoelasticity of the material. Moreover, dot denotes the derivative of time. Researchers have provided several solutions to Eq. (3). Kim and Lee proposed a solution which utilized the chain rule where no assumption regarding  $\alpha$  was made [20,33]. This solution, which is presented by Eq. (4), was utilized in this research.

$$S_{i+1} = S_i + \left[ -\frac{1}{2} (C_i - C_{i-1}) (\varepsilon_i^R)^2 \right]^{\frac{\alpha}{1+\alpha}} \Delta t^{\frac{1}{1+\alpha}} \quad (4)$$

Some researchers have used a trial and error procedure to derive the  $\alpha$  value as a constant value, which generates a unique damage curve for various load amplitudes [25]. Others have suggested some relationships such as  $\alpha = \frac{1}{m}$  and  $\alpha = 1 + \frac{1}{m}$  where  $m$  is the maximum slope of relaxation modulus over time in a log-log scale. They have observed that if the material fracture energy and failure stress are constant, then  $\alpha$  value can be defined as  $+\frac{1}{m}$ . Also, if the fracture process zone size and fracture energy are constant, then the value of  $\alpha$  can be defined as  $\frac{1}{m}$  [20]. Furthermore, some researchers have suggested  $\alpha = 1 + \frac{1}{m}$  for monotonic or controlled strain tests and  $\alpha = \frac{1}{m}$  for the controlled stress tests in either the push-pull or the pull-pull configuration [34].

Finally, the damage characteristic curve ( $C$  versus  $S$ ) can be fitted to a power law form, as in Eq. (5):

$$C(S) = C_0 - C_1(S)^{C_2} \quad (5)$$

In the above equation,  $C_1$  and  $C_2$  are regression coefficients and  $C_0$  usually equals to 1. It is worth mentioning that damage was used to attribute the obtained and inherent properties of materials and, it would be independent from temperature, type of loading (monotonic or cyclic), loading mode (controlled stress or controlled strain) and other situations such as loading amplitude and loading rate while viscoelasticity is the dominant mechanism in the materials [31]. Once the  $C(S)$  curve has been obtained for the selected material in a given rate of loading or stress or strain level, it can be used to calculate the fatigue life of the material in a desired stress or strain level. The number of cycles to failure (fatigue life) can be predicted using the damage evolution law (Eq. (3)) and the damage characteristic relationship (Eq. (5)) in the controlled stress testing mode as described by Underwood et al. [34] as under:

$$N_{failure} = \frac{(f_{red})(2^{3\alpha})|E^*|^{2\alpha}}{[(\sigma_{0,pp})(\beta + 1)]^{2\alpha}(e_{0,ta}^R)^{2\alpha}K_1} \int_0^{\widehat{S}_{failure}} \left( \frac{(1 - \widehat{C}_1(\widehat{S})^{C_2})^\alpha}{\widehat{C}_1 C_2 \widehat{S}^{C_2-1}} \right) (d\widehat{S}) \tag{6-a}$$

In which:

$$K_1 = \frac{1}{t_{final} - t_{initial}} \int_{t_{initial}}^{t_{final}} (g(t))^{2\alpha} dt \tag{6-b}$$

$$t_{initial} = \frac{\cos^{-1}(\beta)}{2\pi f} \tag{6-c}$$

$$t_{final} = \frac{1}{f} - \frac{\cos^{-1}(\beta)}{2\pi f} \tag{6-d}$$

$$\beta_i = \frac{(\sigma_{peak})_i + (\sigma_{valley})_i}{|\sigma_{peak}|_i + |\sigma_{valley}|_i} \tag{6-e}$$

$$\widehat{S}_{failure} = \frac{S_{failure}}{|E^*|^{\frac{2\alpha}{\alpha+1}}} \tag{6-f}$$

$$\widehat{C}_1 = C_1 (|E^*|^{\frac{2\alpha}{\alpha+1}})^{C_2} \tag{6-g}$$

$\sigma_{peak}, \sigma_{valley}$  = maximum and minimum of stress in the tests,  
 $(\beta = 0$  for the central loading,  $\beta = 1$  for the only tension and  $\beta = -1$  for the only compression) (6-h)

In these equations,  $f_{red}$  is the reduced frequency,  $e_{0,ta}^R$  is pseudo-strain tension amplitude and  $K_1$  is loading shape factor defined in Eq. (6-b) and  $g(t)$  is the loading function which is a Haversine here. In this research  $S_{failure}$  was defined as the damage occurs in  $t$  number of cycles to reduce stiffness or modulus to 50% of its initial value. Finally, it is observed that by using Eq. 6 one can obtain the fatigue life of the asphalt mixture in the desired peak-to-peak stress level of  $\sigma_{0,pp}$ .

### 4. Experiments

#### 4.1. Materials

According to the railway published reports and manuals, the aggregates used in the asphalt mixture of rail pavement should have the same gradation of the road pavements base layer's [3,13,15]. Therefore, type 3 continuous gradation of Iran code of practice, presented in Table 1, was met in this research using three sources of aggregates: gravel, sand and filler. The aggregate's physical specifications are presented in Table 2. The XRF test results (shown in Table 3) for all three aggregate sources indicate that the amount of silica is more than calcium carbonate; thus, all the aggregates are categorized as silica materials.

The physical characteristics of the neat (PG 64–22) and crumb rubber modified asphalt are presented in Table 4. The crumb rubber powder used in this research was produced by Yazd Tire Company, using ambient grinding technology, with a mesh size of between #50 and #100. According to ASTM D8, standard rubber

**Table 2**  
Physical properties of the aggregates.

Aggregate Test	Result	Standard
Los Angeles abrasion test (%) (Gravel)	24	ASTM C131
Weight loss against Sodium-sulfate (%) (Gravel)	2	ASTM C88
Weight loss against Sodium-sulfate (%) (Sand)	4	ASTM C88
Elongated aggregate percent (%) (Gravel)	9	ASTM D4791
Sand equivalent (%) (Sand)	89	ASTM D2419
Fractured faces %)(Gravel)	100	ASTM D5821

modified binder is a compound of bitumen, crumb rubber powder, lubricant oils and special additives which should be mixed and reacted in an adequate time and temperature. In this research, 20% crumb rubber powder was used as the binder modifier. Crumb rubber asphalt mixtures can be prepared in different methods including the dry process, the original wet process and the Terminal Blend (TB) [35–37]. The original wet process method (i.e., the McDonald method) includes a high shear mixing vessel and a separated storage agitated tank to allow the digestion of rubber particles into the bitumen through a continuous low shear stirring [38]. The TB method is a different form of the wet process in which the crumb rubber particles are blended with asphalt binder at the refinery or at an asphalt binder distribution terminal. The final modified binder is then transported to the asphalt concrete mixing plant or job site to be used [37]. The main difference between these two methods is their producing temperatures. That is, the producing temperature for the TB rubberized asphalt is from 220 to 260 °C while it is usually in the range of 170–190 °C for the original wet process. Besides, the interaction time of TB rubberized asphalt (up to 6 h) is much longer than that of the wet-process (about 0.5–2 h) for allowing the rubber particles to fully digest into the asphalt binder [36]. However, in the CalTrans Manual (2006), two different definitions (wet process – no agitation and wet process – high viscosity) were made in which the latter usually refers to the original wet process and the former to the terminal blend [39], although there is not a meaningful correlation between these two classifications.

In this research, the crumb rubber particles were mixed with the asphalt binder in a high shear mixer at 185 °C for 60 min and with a rotational speed of 5000 rpm. The mixing conditions were adapted from the original wet process method and the procedures suggested in the available literature [40–44]. The main problem of conventional wet process is the lower storage stability of the rubber modified binders. However, the modified binders used in this research were immediately mixed with aggregates in order to make the asphalt mixtures and to avoid the aforementioned problem. Furthermore, the required agitation in the wet process is usually applied to the rubber modified binders with high viscosity of more than 1.5 Pa.s and at the temperature of 190 °C [39]. However, the modified binder produced in this research exhibits a much lower viscosity in this temperature (since the viscosity at lower temperature of 165 °C is about 1 Pa.s) and therefore no agitation was required. This seems to be the result of fine rubber particles used in this research. Finally, it is believed that the extreme production and curing conditions such as high temperature and long mixing time applied during the TB method can negatively influence the mechanical performance and elastic properties of crumb rubber modified binders [45–47]. These concerns have also led

**Table 1**  
Aggregates gradation used in this study.

Sieve size (mm)	25(1 in.)	19(3/4 in.)	9.5(3/8 in.)	4.75(#4)	2.38 (#8)	0.3(#50)	0.075 (#200)
Percent passing (%)	100	94	75	52	41	15	7
Iran Code Limits (%)	100	90–100	56–80	35–65	23–49	5–19	2–8

**Table 3**  
XRF analysis results of the aggregates.

	CO <sub>2</sub> %	K <sub>2</sub> O%	Na <sub>2</sub> O%	MgO%	CaO%	Fe <sub>2</sub> O <sub>3</sub> %	Al <sub>2</sub> O <sub>3</sub> %	SiO <sub>2</sub> %	Weight Loss on Ignition (L.o.I)
Gravel	10.41	0.96	2.16	2.2	15.5	6.4	12.5	48.36	12.23
Sand	12.75	0.94	1.71	1.86	18.48	5.8	8.67	48.11	14.42
Filler	12.11	1.03	2.1	2.5	16.92	5.2	11.05	47.07	14.13

**Table 4**  
Physical characteristics of the neat and CR modified binders.

Test	Method	Neat binder	CR modified binder
Penetration at 25 °C (0.1 mm)	ASTM D5	64.5	46
Softening point (°C)	ASTM D36	54	67
Specific gravity at 25 °C	ASTM D3289	1.018	1.18
PG grade	ASTM D6373	64–22	76–28
Viscosity at 135 °C (Pa.s)	ASTM D4402	0.32	2.94
Viscosity at 165 °C (Pa.s)	ASTM D4402	0.11	1.04
Mixing Temperature (°C)	–	150	174
Compaction Temperature (°C)	–	138	156

the authors to select the conventional wet processes method with lower mixing time and temperature compared to the TB method.

#### 4.2. Mix design

Using different mixing and compaction temperatures for the samples may influence the comparability of the results, however, selecting a specific mixing and compacting condition for all the mixtures would result in different coating degrees of aggregates and variability in void contents [48]. So, different mixing and compaction temperatures, based on the viscosity results presented in Table 4, were selected for unmodified and crumb rubber modified mixtures in this research. The mixing and compaction temperatures of unmodified asphalt mixture were computed based on the equiviscous method with a viscosity limit of  $0.17 \pm 0.02$  Pa.s for mixing and  $0.28 \pm 0.03$  Pa.s for compaction. West et al. (2010) reported that the asphalt pavement factories/companies usually increase the mixing and compaction temperature of modified binders up to 20 °C compared to unmodified binders [49]. However, a simplified zero shear viscosity method suggested in NCHRP-459 (and later in NCHRP-648) was used in this research to derive the mixing and compaction temperatures of crumb rubber modified mixtures [50]. In this method, a compaction viscosity of  $1.4 \pm 0.1$  Pa.s and a mixing viscosity of  $0.75 \pm 0.05$  Pa.s were selected instead of rate-dependency measurements of modified binders. The mixing and compaction temperatures of unmodified and modified mixtures are presented in Table 4. The asphalt mixtures in this study were made immediately after the crumb rubber modified binder was prepared in the high shear mixer at 185 °C and cooled in a couple of minutes to reach to the target temperature of 174 °C.

As previously indicated, for the asphalt mixture in the railway structures, a higher asphalt binder content is chosen compared to the considered optimum one for the highways. It causes a lower air void of 1%–3%. Based on the aforementioned concept, the fatigue properties of four mixture types were evaluated in this research using VECD theory. These mixtures include a control mix made of the neat binder and 2% air void content (Neat-2AV) and a crumb rubber modified mixture with the same air void content (CR-2AV). Also, a crumb rubber modified mixture with a higher air void content of 4% (CR-4AV) was selected to account for the highway pavement asphalt mixture. Finally, a crumb rubber modified mixture with the air void content of 2% but aged in the lab (CR-2AV-Aged) was added to the list to account for the effect of short term aging which occurs in the asphalt mixtures during

construction. For the latter purpose, the asphalt specimens were aged in an oven at 135 °C for 4 h based on AASHTO R30 procedure (i.e., mixture conditioning of hot mix asphalt) [51–53]. The asphalt mixtures were compacted using 6-inch mold Super-pave gyratory compactor [ASTM D6925] with the desired air void of 2 or 4 percent. All samples were then cored to a diameter of 4 in. and cut to the specific height required for each test.

#### 4.3. Test design

The tests performed in this research can be divided into two realms of damage-inducing and non-damage-inducing. The non-damage-inducing test consists of a creep compliance test, used to obtain the linear viscoelastic behavior of asphalt mixtures. Different small stress values (in the range of 20 to 200 kPa) were selected at different temperatures (–5, 10, 25 and 40 °C) for different materials in order to keep the strain values of the specimens in a linear range. The damage-inducing tests including the constant crosshead rate (or monotonic) test and the constant crosshead cyclic test were performed at 25 °C. It is assumed that at this temperature the effect of plasticity can be disregarded and the viscoelastic continuum damage (VECD) theory would be applicable. The monotonic test was performed in three constant rates of loading (0.0001, 0.0004 and 0.0016 1/s in this research) to account for the damage properties of the specimens with different loading rates. This test was performed for all four materials. The cyclic fatigue test was performed only for two mixtures of CR-2AV and CR-2AV-Aged, in the controlled stress mode, using a Haversine waveform with 10 Hz loading frequency (i.e., no rest time) and 250 kPa or 100 kPa stress level respectively for the mixtures. All the tests were conducted using Universal Testing Machine (UTM). Platens were designed to conduct the tests in the direct tension mode. Special care was required to be taken to eliminate any load eccentricity during the axial test, since it may origin non-uniform stress distribution causing a failure close to one of the platens. Therefore, a specialized platen gluing jig and a ball-jointed top platen were designed based on the research reported in [23,24] in order to eliminate any bending even in the non-perfectly parallel specimen ends. Prior to testing, the platens were glued to the specimen with Epoxy adhesive. The strain data was also gathered using three extensometers located in the middle portion of the specimens with a gage length of 75 mm.

## 5. Results and discussions

In this study the viscoelastic continuum damage model, in which the pseudo-strain function is a key parameter, was used. To compute the pseudo-strain (Eq. (1)), the relaxation modulus function, which determines the behavior of the materials in the linear region, is required. The relaxation modulus test is fairly cumbersome to perform due to the immediate increase in strain input which results in a large initial load response. However, the relaxation modulus can be predicted through the theory of linear viscoelasticity, using other parameters such as the creep compliance or the complex modulus, both being much easier to be derived in the laboratory [54]. In this research, the creep test was performed in four different temperatures and the master creep

compliance curve was then obtained using the Time-Temperature Superposition principle [55]. In order to fulfill the linear viscoelastic conditions, the applied stress in the creep test must be low enough in order to avoid any significant damage in specimen. The creep compliance curves for the selected materials were shown in Fig. 2. The effect of air void content on the creep compliance curves was presented in this figure. Based on this figure, it is observed that at all loading time ranges, the CR-4AV mixture exhibits the greatest compliance values. This indicates that increasing the air void content of the CR modified binder from 2% to 4% causes an increase in the compliance even more than that of changing the binder type from CR modified to the neat one. Furthermore, this shows the great influence of changing the air void content on the linear viscoelastic properties of asphalt mixtures. It can also be observed in Fig. 2 that the creep compliance was decreased with aging at all loading time ranges. Also, with the increase of loading times, the difference between the compliance of aged and unaged mixtures became larger. This trend is expected because as the temperature or the time increases, the modulus gradient between the asphalt binder and the aggregate particles becomes larger and consequently the effect of different binders in the mixtures becomes more prominent. Finally, it is observed in Fig. 2 that the Neat-2AV mixture revealed a limited range of variation in creep compliance compared to the CR-2AV modified mixture. The rate of variations for the Neat-2AV mixture is also limited only to the intermediate loading times. These can be further attributed to the lower time/temperature dependency of the unmodified binders or mixtures compared to the CR modified ones.

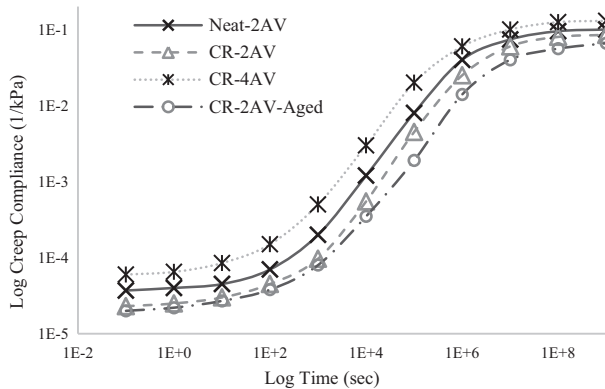


Fig. 2. Creep compliance curves for all mixtures.

The creep compliance test results were then used to derive the relaxation modulus function and the material constant  $\alpha$  defined as the maximum slope of the relaxation modulus curve in the logarithmic scale. To do so, the Prony series function was first fitted on the master creep compliances curve and then the inter-conversion between the creep compliance and the relaxation modulus was made using Eq. (7). The required mathematical relationships for converting the creep compliance data to the relaxation modulus master curve were generated in Appendix A. More details about the inter-conversion of these functions can be found elsewhere [54,56]. A pure power law function was then fitted on the transition region of the relaxation modulus curve to obtain the material constant  $\alpha$ . The creep compliance and relaxation modulus functions in the form of Prony series are presented in Table 5.

$$\int_0^t E(t - \tau) \frac{\partial D(\tau)}{\partial \tau} d\tau = 1 \tag{7}$$

In the monotonic loading tests, three different stain rates of 100, 400, and 1600 micro-strain per second were used as aforementioned. Fig. 3a displays the stress-strain curves for the CR-2AV-Aged specimens. As expected, for a material with high viscoelasticity, the response to a specified strain level was highly dependent on the strain rate, such that the specimen with the higher rate of loading bears a higher level of stress. Also, as shown in Fig. 3a, a nonlinear relationship is observed in the initial stages of loading. However, the nonlinearity observed in this zone is related only to the time effects of the material which is not in fact a nonlinearity effect. When these time effects were removed using pseudostrain concept, different loading rates appears on the same line at the lower stress levels, as illustrated in Fig. 3b. This indicates that damage dose not commence at the outset of loading. In fact, the damage does not begin until the stress level approaches around 200 kPa. This behavior implies that the “correspondence principle” can effectively eliminate the strain rate dependency of the material when the damage is negligible. However, if a significant level of damage is induced in the specimen, additional variable should be employed in a constitutive equation to represent the damage growth in the system. Similar results for the other materials were also obtained.

The cyclic loading tests were conducted in the stress controlled with a Haversine loading form. Typical hysteretic stress-strain curves for the CR-2AV specimen were presented in Fig. 4a at selected cycles. As expected, the stress-strain loops shifted to the right side with the increasing loading cycle and the dissipated

Table 5  
The creep compliance and relaxation modulus Prony series parameters.

			Parameter number (i or j)						
			0	1	2	3	4	5	6
Neat-2AV	Creep Compliance	Log $\tau_i$		-1.00	1.00	3.00	5.00	6.00	7.00
		Log $D_i$	-4.46	-5.45	-4.97	-4.09	-2.26	-1.34	-1.35
	Relaxation Modulus	Log $\tau_j$		-0.30	1.70	2.74	4.70	5.74	6.70
		Log $E_j$	1.02	3.52	4.04	4.12	2.02	0.28	0.92
CR-2AV	Creep Compliance	Log $\tau_i$		-1.00	1.00	3.00	5.00	6.00	7.00
		Log $D_i$	-4.67	-5.63	-4.91	-4.76	-2.75	-1.54	-1.29
	Relaxation Modulus	Log $\tau_j$		-0.30	1.70	2.70	4.70	5.74	6.70
		Log $E_j$	1.09	4.11	3.60	4.48	2.28	0.85	1.03
CR-4AV	Creep Compliance	Log $\tau_i$		-1.00	1.00	3.00	5.00	6.00	7.00
		Log $D_i$	-4.24	-5.64	-4.44	-3.69	-1.64	-1.32	-1.27
	Relaxation Modulus	Log $\tau_j$		-0.30	1.70	2.74	4.70	5.74	6.70
		Log $E_j$	0.90	3.57	3.97	3.66	1.57	0.49	0.83
CR-2AV-Aged	Creep Compliance	Log $\tau_i$		-1.00	1.00	3.00	5.00	6.00	7.00
		Log $D_i$	-4.74	-5.55	-5.04	-4.31	-3.23	-1.89	-1.34
	Relaxation Modulus	Log $\tau_j$		-0.30	1.70	2.74	4.70	5.74	6.70
		Log $E_j$	1.23	4.14	3.76	4.52	2.90	0.96	1.16

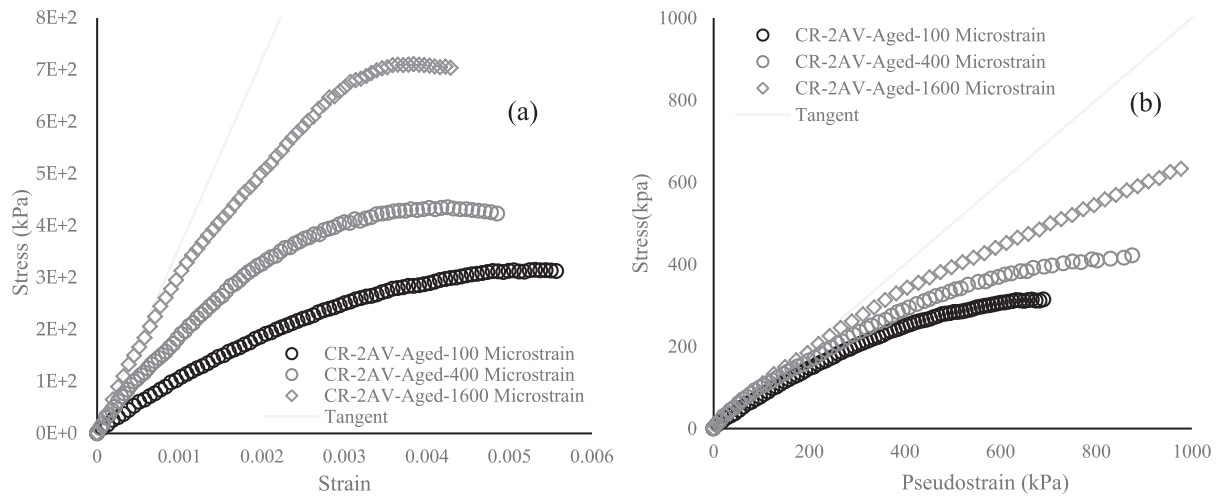


Fig. 3. (a) Stress-strain; (b) Stress-pseudo strain curves for CR-2AV-Aged specimen at different strain rates of loading.

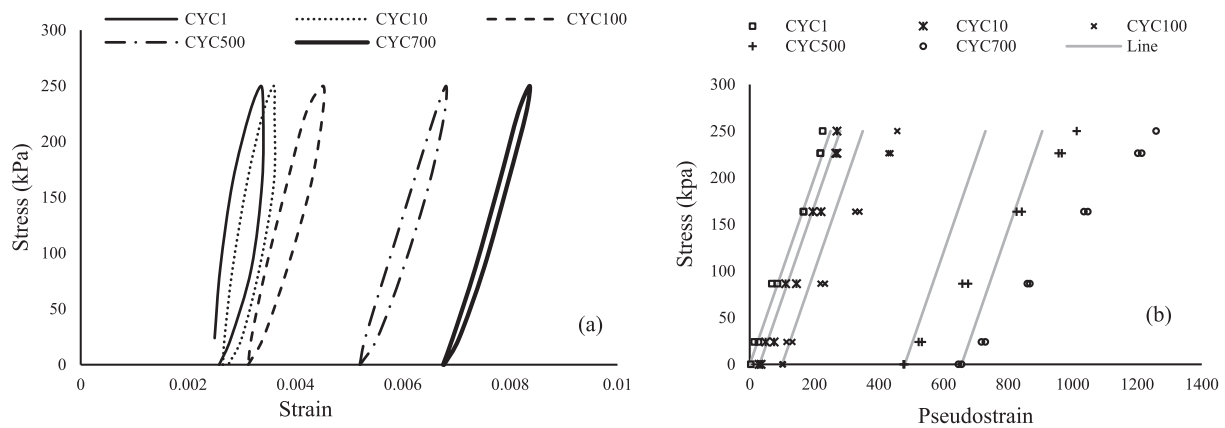


Fig. 4. (a) Stress-strain; (b) Stress-pseudo strain curves for CR-2AV-Aged specimen at different cycles of loading.

energy determined with the area inside the stress-strain loops was reduced. Fig. 4b depicts the plot of stress over pseudo strain calculated based on Eq. (1). Moreover, as expected, the curve of stress-pseudostrain for initial cycles is linear with unity slope; hence, by increasing the number of cycles of the load, these slopes, called pseudo-stiffness, starts to decrease. Similar results for the other materials were also obtained.

Based on the monotonic loading test and cyclic test results, the damage characteristic curves for all the asphalt mixtures were derived using Eqs. (1)(4). The curves for the four mixtures at different loading types are presented in Fig. 5. The cyclic test was only performed for the CR-2AV-Aged and the CR-2AV mixtures as previously mentioned. The damage characteristics curve is known to be a unique curve for an individual material, independent of the test conditions such as the loading type, loading frequency or the rate of loading. As illustrated in Fig. 5, some variations are observed among the curves for the same material, although they are negligible. Also, there is not an oriented pattern of variations among different types of tests for all asphalt mixtures. So, different loading types of monotonic and cyclic as well as different loading rates of the monotonic test are all assumed to show the same results. The small variations are due to the experimental errors in the lab. Based on the results of the three (or four) tests performed on each asphalt mixture type, a model function in form of Eq. (6) was fitted to the damage characteristic curves and the model coefficients were derived (Table 6).

A better comparison among the damage characteristic curves of all mixtures can be seen in Fig. 6. A desirable damage characteristic curve for a material is the one that bear the greatest damage level for a given pseudo stiffness value [57]. So, the most favorably positioned damage characteristics curves belongs to the aged crumb rubber modified mixture (CR-2AV-Aged), positioned over the other mixtures, on the top right side of the diagram. Crumb rubber modified (CR-2AV) and the unmodified (Neat-2AV) mixtures are positioned in the next levels and the crumb rubber mixture with 4% air void (CR-4AV) positioned the last. The results obtained indicate that, like the modulus response, the damage characteristics curves are strongly affected by 2% change in the air void content of the CRM mixtures. As can be seen in this figure the asphalt mixture is more damage resistant with smaller air void contents (CR-2AV) - compared to the higher air void contents (CR-4AV). This proves that the damage characteristics curve can well demonstrate the material behavior.

Fig. 6 also indicates that the unmodified asphalt mixture (Neat-2AV) shows a steeper slope in the damage curve, resulting in a high decrease of the pseudo-stiffness at higher damage levels. This indicates that the unmodified mixtures, compared to modified ones, are more sensitive to damage growth; on the other hand, the modified mixtures can maintain their total structure and resist against modulus loss.

It should finally be noted that the damage characteristic curve considers only the damage behavior of the material whilst the

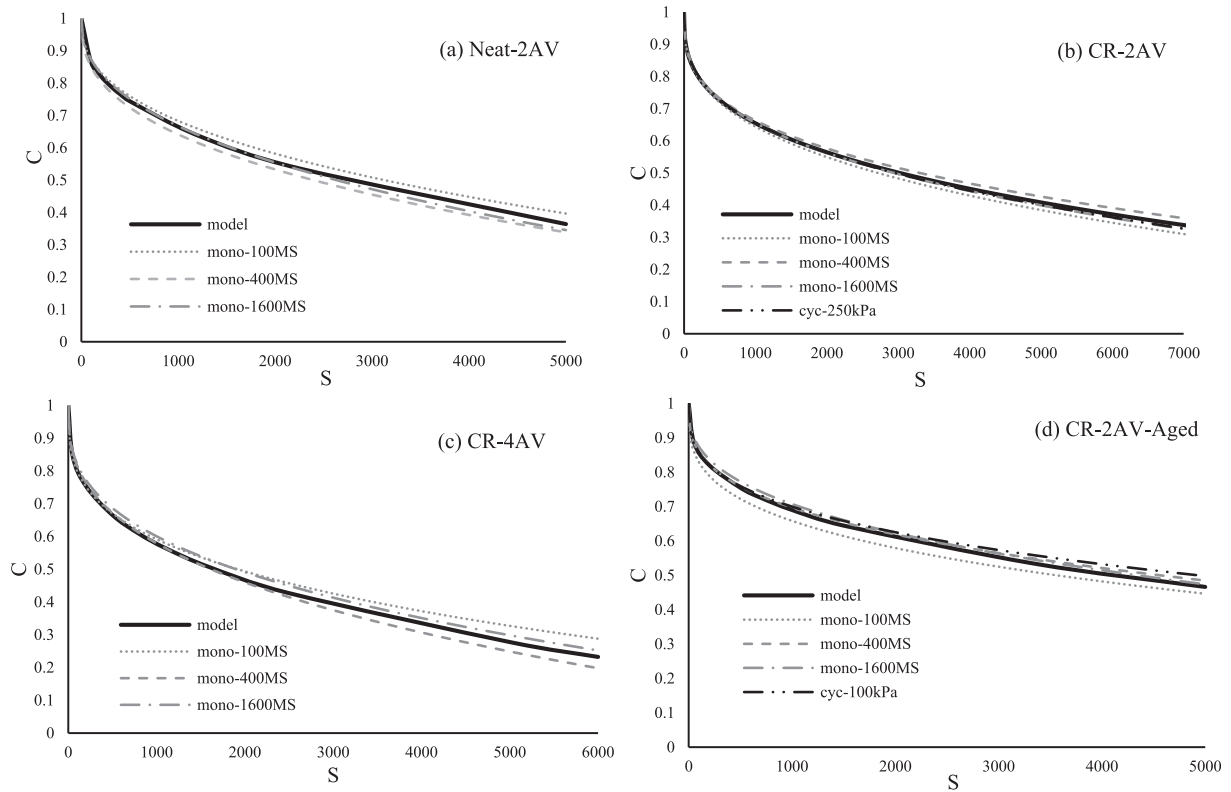


Fig. 5. (a) Neat-2AV; (b) CR-2AV; (c) CR-4AV and; (d) CR-2AV-Aged.

**Table 6**  
Model Coefficients for the damage characteristic curves for all asphalt mixtures.

Mixture Types	$C_0$	$C_1$	$C_2$
Neat-2AV	1.000	0.022	0.395
CR-2AV	1.000	0.034	0.344
CR-4AV	1.000	0.042	0.334
CR-2AV-Aged	1.000	0.030	0.338

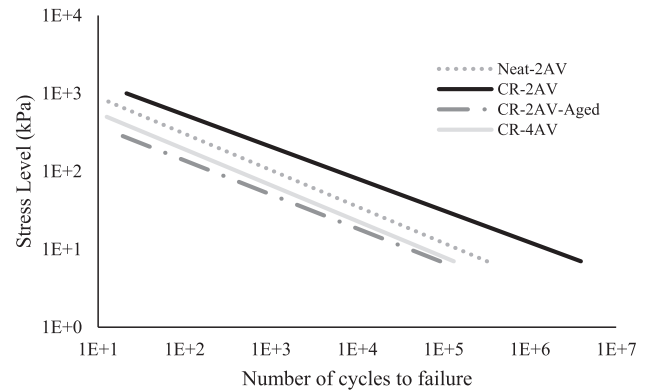


Fig. 7. Number of cycles to failure versus stress level.

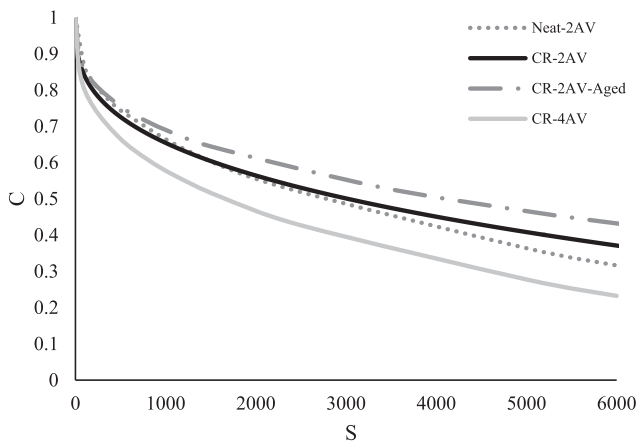


Fig. 6. Damage characteristic curves (C-S curves) of four mixtures.

fatigue properties of the materials should be interpreted based on both the damage characteristics and the stiffness modulus of the material. This assessment will be further performed introducing the fatigue life diagrams. Therefore, the excellence of aged crumb rubber modified mixture over the others in this diagram should not be misinterpreted to the better fatigue life of this mixture.

Having the coefficients of C-S curves known (Table 6), the fatigue life of all asphalt mixtures can be predicted as a function of different stress levels, using Eq. (6). The stress levels versus the number of load cycle to fail for all the mixtures are drawn in Fig. 7. The asphalt mixture with CR modifier and 2% air void (CR-2AV) exhibits the highest fatigue life for all stress levels. The unmodified mixture (Neat-2AV) has the second highest fatigue life. This comparison between the CR modified and unmodified asphalt mixtures indicates that the crumb rubber modification of asphalt mixture can greatly increase the fatigue life of rail track-bed asphalt materials. Besides, the difference between the fatigue lives of these two mixtures tends to increase in lower stress levels. Therefore, the superiority of crumb rubber modified mixtures, which have a higher fatigue life at lower stress levels, becomes more pronounced when they are used in railway structure.

It is also interesting to notice that the mixture with CR modifier and high air void content (CR-4AV) has a lower fatigue life



**Table 7**  
Fatigue life cycle and fatigue life ratio of different mixtures at two different stress levels.

Stress level	10 kPa			300 kPa			
	Mixture	Cycles to failure	Ratio to Neat-2AV	Ratio to CR-4AV	Cycles to failure	Ratio to Neat-2AV	Ratio to CR-4AV
CR-2AV		54,165	7.2	18.2	197	3.6	9.4
Neat-2AV		7483	1	2.5	55	1	2.6
CR-4AV		2983	0.4	1	21	0.4	1
CR-2AV-Aged		1637	0.2	0.5	9	0.2	0.4

compared to the CR modified mixture with lower air void content (CR-2AV). This indicates the great influence of air void content in the fatigue life of the final mixtures. This could be observed in the creep compliance master curves and the damage characteristic curves (Figs. 2 and 6). The obtained results also indicate that the air void content should be rather decreased by increasing the asphalt content when using this type of asphalt mixture in the railway track bed. It is also clear in Fig. 7 that the aged CR modified mixture (CR-2AV-Aged) curve is positioned below all other curves, indicating a lower fatigue life of the aged mixture compared to the others. It should be reminded that CR-2AV-Aged mixture was shown to have the most favorably positioned damage characteristic curve in Fig. 6. However, it could not guarantee the highest fatigue life of this mixture. This difference can attribute to the higher modulus (lower compliance) of the aged CR mixtures as shown in Fig. 2.

In order to present a quantitative evaluation of four types of asphalt mixtures, the fatigue life cycles at two different stress levels of 40 and 400 kPa were calculated from the data points in Fig. 7. The results are presented in Table 7. A small stress level of 40 kPa was chosen as the lower boundary of stress levels on the top of the asphalt mixtures to simulate the load of a light railway transit (LRT). This stress level can occur due to light or empty cars/trains passing the railway track [11]. It should be noted that stress levels of 90–120 kPa represent the stress level in the asphalt mixture in the regular heavy railways and 30–50 kPa in the light-weight railway tracks [11,15,58]. The asphalt stress level of 400 kPa was considered for the case of asphalt mixture under relatively light highway traffic load [8]. Comparisons of the results of CR-2AV and Neat-2AV samples (presented in Table 7) show that crumb rubber modification of asphalt mixture at high stress levels (as in highways) increases the fatigue life to about 3.6 times while this increase is about 7.2 times at lower stress levels (as in railways). So it seems that crumb rubber modification is much more effective in the railway application compared to the highway one. The second important comparison was made between CR-2AV and CR-4AV samples. The results indicate that decreasing the air void content of CR modified mixtures from 4% to 2% increases the fatigue life about 9.4 times at high stress levels (highway application) and about 18.2 times at low stress levels (railway application). This further implies that getting to the target air void content of 2% significantly increases the fatigue life in the case of railway applications. The results obtained indicate that the lowest fatigue life is achieved in the aged crumb rubber modified mixture (20 percent of unmodified asphalt mixture). The results also shows that the effect of aging does not vary when the stress levels are changed.

## 6. Summary and conclusions

The viscoelastic continuum damage theory was used in this research to interpret the fatigue properties of asphalt mixtures. Crumb rubber modified asphalt mixtures were prepared in two different air void contents to investigate the mechanical behavior of the railway asphalt mixtures and compare them with the regular road pavements asphalt mixtures. An unmodified mixture was also

used to evaluate the effectiveness of the CR modification, and an aged mixture was utilized to evaluate the fatigue performance of the aged asphalt mixtures in the railway applications.

The creep compliance test was carried out to investigate the linear viscoelastic behavior of asphalt mixtures. Two different loading forms of constant crosshead rate (or monotonic) test and the constant crosshead cyclic test were utilized to drive the damage-induced properties of the asphalt samples. The strain rate dependency of the mixtures during the monotonic test, when the damage was negligible, was shown to be correctly eliminated using the “correspondence principle” and the introduction of pseudo-strain concept. The damage characteristic curves for an individual material were shown to be unique independent of the loading type or the rate of loading and hence the viscoelastic continuum damage theory was shown to precisely work for these materials; and the testing errors among different samples were shown to be negligible. A model function was fitted to the characteristic curves of the mixtures and the obtained curves were compared.

It was observed that the aged crumb rubber modified mixture tolerated the greatest damage level for a given pseudo stiffness value. However, this mixture later showed the lowest fatigue life in all stress levels (20 percent of unmodified asphalt mixture), which can be attributed to the higher modulus (lower compliance) of the aged CR modified mixtures. The fatigue life of crumb rubber modified mixture was shown to be about 3.6 times of fatigue life of unmodified asphalt mixtures for the highway applications, while for the railway applications, this ratio approached to 7.2 times. Therefore, it can be concluded that the crumb rubber modification is much more effective in the railway practices in comparison to the highway ones. Due to the importance of the method of constructing mixtures, much more attention should be paid to the final crumb rubber modified mixtures made and compacted in the track-bed structure in the field. As it was shown in this research, reducing the air void content of the crumb rubber modified mixture from 4% to 2% increases the asphalt fatigue life by 9.4 times in highway applications and 18.2 times in railway applications, which is a great growth. So, decreasing the air void content via increasing the asphalt binder was shown to largely improve the fatigue life of in-track-bed asphalt mixtures.

The results and new findings obtained in this research provide a better understanding of the long term behavior of asphalt mixture in the railway which will pave a more accurate design of asphalt railway tracks (particularly the newly introduced low-vibration railway-asphalt-tracks).

## 7. Data availability

The raw/processed data required to reproduce these findings cannot be shared at this time as the data also forms part of an ongoing study.

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