Physica A 512 (2018) 280–292

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Multifractal analysis of the Chinese stock, bond and fund markets

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HIGHLIGHTS

- Auto-correlations and cross-correlations of Chinese security markets are verified.
- Existence of the multifractality of the three return series is confirmed.
- Sources of the multifractality are explored.
- Dynamic behaviors of the cross-correlations among the three markets are investigated.

ARTICLE INFO

Article history:
Received 28 August 2017
Received in revised form 18 March 2018
Available online xxxx

Keywords:
Stock market
Bond market
Fund market
Cross-correlation
Multiscale analysis

ABSTRACT

The stock, bond and fund markets are three important components of a financial market, and the volatility of the markets and correlations between the markets have been paid extensive attention by researchers and investors. In this paper, we devote our efforts to studying the Shanghai financial market, while the return series of Shanghai Composite Index, Shanghai Bond Index and Shanghai Fund Index are considered. Statistical tests are used to detect the nonlinear auto-correlated structures and long-range cross-correlations of the three time series. The multifractal detrended fluctuation analysis and multifractal spectrum analysis methods are applied, by which the existence of multifractality in these three return series are revealed and the sources of multifractality are explored. In particular, the multiscale multifractal detrended cross-correlation analysis method is employed for the first time to generate the Hurst surfaces, which can be used to visualize the dynamic behaviors of cross-correlations among the markets. Empirical results show that the cross-correlations among the markets present different fractal features at different time scales. Further, our study finds that the correlation between the stock and fund markets is stronger than that of the other two groups, and the correlation between the stock and bond markets is unstable. These findings can help to better understand the dynamic mechanisms that govern the volatility of security markets and aid in performing better financial risk assessment and management.

1. Introduction

In recent years, the dynamic features of financial markets have attracted much attention of many researchers. Especially, the dynamic relationships between the stock market and other financial markets have become hot topics in financial economics. For instance, Fang et al. [1] used the GARCH (1,1) model to investigate the relationship between the stock and

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https://doi.org/10.1016/j.physa.2018.08.067

bond markets in the United States, Britain, Japan and Germany during 1998–2004, and verified the existence of volatility transmission in these markets. Chuliá and Torró [2] utilized the multivariate GARCH model to analyze the European stock and bond markets, and found that there was a two-way volatility spillover effect between them. Chordia et al. [3] studied the liquidity of stock and bond markets by using the vector autoregressive model and pointed out that the volatility and liquidity of the two markets were significantly correlated, meaning that the liquidity and volatility of both the stock and bond markets were co-driven by some factors. Besides, Goetzmann and Massa [4] discussed the relationship between the index fund and the return series of S&P500 index, and showed that there was a simultaneous correlation between them. Connolly and Stivers [5] considered the influences of the treasury yields and the stock market volatility on market pricing. They showed that the uncertainty of stock market played an important role in cross-market pricing. Recently, Cenedese and Mallucci [6] investigated the dynamic correlation between the international stock and bond markets. They found that the international bonds flowing into emerging markets were more sensitive to the impact of interest rates than the stock market was. Kolluri et al. [7] used the multivariate cointegration test to examine the dynamics of India's stock and bond markets, and their interrelationships with five foreign equity markets. Kim et al. [8] utilized the Granger causality test to investigate the relationships between the Korean stock and fund markets, and showed that the information created from the fund market affected the return and volatility of the stock market. Li et al. in [9] proposed an asset pricing model for stock and bond markets and analyzed the dynamic relationship between the two markets from the perspective of portfolios.

Statistical methods including the GARCH-class and vector auto-regression models were mainly applied to investigate the volatility, risk measurement and volatility spillover of stock market, bond market or fund market in the aforementioned studies. They focused more on the linear correlation of the price fluctuation factors in different markets. However, many empirical studies have shown that financial markets are complex nonlinear dynamic systems with fractal and chaotic structures [10–14]. Financial time series are usually non-stationary because of their complexity and speculative natures. Therefore, using nonlinear statistical analysis methods to study financial time series has become a popular approach in financial market analysis.

During the last decades, several models and methods of fractal analysis, such as the rescaled range analysis (R/S analysis) [15], detrended fluctuation analysis (DFA) [16] and multifractal detrended fluctuation analysis (MF-DFA) [17], have been established and developed to detect the long-range auto-correlation, to examine the market efficiency, to explore the market volatility and to describe the nonlinear characteristics of markets, etc. [18–24]. In order to detect the long-range cross-correlation between two non-stationary time series, Podobnik and Stanley [25] proposed a new method based on detrended covariance, called detrended cross-correlation analysis (DCCA). Lately, a DCCA cross-correlation coefficient was introduced [26,27] based on the DFA and DCCA methods to quantify power-law cross-correlations in non-stationary time series. The DCCA method and the DCCA cross-correlation coefficient have been used to investigate the cross-correlations between two financial series in some papers [28,29]. Moreover, the DCCA method was extended to its multifractal version [30], named MF-DCCA or MF-DXA, which can reveal the multifractal features of two cross-correlated non-stationary series effectively. Recently, Shi and Shang et al. [31] generalized MF-DCCA to multiscale multifractal detrended cross-correlation analysis (MM-DCCA), which may provide much richer information than MF-DCCA does in the discussion of multifractal structures for two cross-correlated time series. Now, the fractal analysis methods have been widely used in the study of many fields, such as financial markets, energy markets, air pollution and geological survey, etc. [32–43].

On January 2, 2003, the issuance of Shanghai bond index marked the establishment of the index system of China's security markets involving the stock, bond and fund markets. In recent years, with the rapid development of China's economy and the gradual improvement of the financial market systems, the co-movements among different financial markets in China have also been greatly enhanced. The volatility, risk assessment and mutual relationships of China's security markets have attracted wide attention from researchers and investors.

In this paper the MF-DFA and multifractal spectrum analysis methods are firstly used to empirically analyze the multifractal volatility characteristics of the Shanghai stock, bond and fund markets, and to investigate the possible sources of multifractality. The MM-DCCA method is then employed to produce the Hurst surfaces for visualizing the dynamic cross-correlations between the markets at different time scales. Compared with some previous publications that mainly discussed the relationships between two markets, this paper considers the three indispensable components of a financial market namely the stock, bond and fund markets. The contributions of our study are threefold. First, we show that the return series of the considered three security markets are not Gaussian series, but fractal series with nonlinear auto-correlation structures and multifractal features. Second, we explore the sources of multifractality for the three return series, and find that both the long-range correlations and fat-tailed distributions are common reasons of the multifractality, while the fat-tailed distributions contribute more to the multifractality. Third, the MM-DCCA method is employed to investigate the cross-correlations among the three security markets in China, and the dynamic behaviors of multifractal cross-correlations are analyzed and compared at different time scales. To the best of our knowledge, this paper is the first one to visualize the cross-correlations among Chinese security markets from a three-dimensional perspective.

The rest of the paper is organized as follows. Section 2 introduces the methodology. Section 3 presents the data description and the basic statistical tests. In Section 4, the multifractal analysis of the three return series are performed based on the MF-DFA and multifractal spectrum analysis. The multifractal auto-correlation are discussed and the sources of multifractality are explored. The cross-correlations of three return series are empirically studied in Section 5 based on the MM-DCCA. Finally, the conclusions are drawn in Section 6.

2. Methodology

2.1. Multifractal analysis methods

The MM-DCCA method developed from the classical MF-DCCA method can analyze the multiscale behaviors of cross-correlation for non-stationary time series, and has important application in the analysis of financial time series [31]. Its algorithmic steps can be summarized as follows.

Let $x(t)$ and $y(t)$, $t = 1, 2, \ldots, N$, be two time series of equal length N.

Step 1. Calculate the profile of each series:

$$X(t) = \sum_{k=1}^{t}(x(k) - \bar{x}), \quad Y(t) = \sum_{k=1}^{t}(y(k) - \bar{y}), \quad t = 1, 2, \ldots, N, \tag{1}$$

where $\bar{x} = \frac{1}{N}\sum_{t=1}^{N} x(t)$ and $\bar{y} = \frac{1}{N}\sum_{t=1}^{N} y(t)$.

Step 2. Divide each of the series $\{X(t)\}$ and $\{Y(t)\}$ into $N_s = \mathrm{int}(N/s)$ non-overlapping segments of equal length s. Since the length N is usually not a multiple of the considered time scales s , a short part at the end of each profile will remain in most cases. In order not to disregard this part of the series, the same procedure is repeated starting from the opposite end. Therefore, $2N_s$ segments are obtained altogether.

Step 3. Calculate the local trends for each of the $2N_s$ segments v, $v = 1, 2, \ldots, 2N_s$ using least-square fit. Then calculate the covariance of each series:

$$F^2(s, v) = \frac{1}{s}\sum_{t=1}^{s} |X((v-1)s + t) - P^v(t)| \cdot |Y((v-1)s + t) - T^v(t)| \tag{2}$$

for $v = 1, 2, \ldots, N_s$ and

$$F^2(s, v) = \frac{1}{s}\sum_{t=1}^{s} |X(N - (v-1)s + t) - P^v(t)| \cdot |Y(N - (v-1)s + t) - T^v(t)| \tag{3}$$

for $v = N_s + 1, N_s + 2, \ldots, 2N_s$. Here $P^v(t)$ and $T^v(t)$ denote the fitting polynomials with order k for each profile in the segment v (conventionally called MF-DCCA-k).

Step 4. Average all segments to obtain the q-order fluctuation functions.

$$F_q(s) = \left\{\frac{1}{2N_s}\sum_{v=1}^{2N_s}[F^2(s, v)]^{q/2}\right\}^{1/q}, \quad q \neq 0, \tag{4}$$

$$F_0(s) = \exp\left\{\frac{1}{4N_s}\sum_{v=1}^{2N_s}\ln[F^2(s, v)]\right\}, \quad q = 0, \tag{5}$$

where q is a real parameter. For $q = 2$, the standard DCCA procedure is retrieved [25].

Step 5. Determine the scaling behaviors of the fluctuation functions through analyzing the log–log plots of $F_q(s)$ versus s for each fixed q. If the long-range cross-correlation exists in two series $\{x(t)\}$ and $\{y(t)\}$, then the following power-law relation holds for large values of s,

$$F_q(s) \sim s^{h_{xy}(q)}, \tag{6}$$

where $h_{xy}(q)$ is known as the generalized cross-correlation exponent, which may be used to characterize the power-law cross-correlation between two time series.

The above five steps define the classical MF-DCCA algorithm. In particular, if the time series $\{x(t)\}$ is identical to $\{y(t)\}$, MF-DCCA is equivalent to MF-DFA. In this case, the generalized cross-correlation exponent $h_{xy}(q)$ is just the generalized Hurst exponent $h(q)$. Next, to better characterize the multifractality of two cross-correlated time series at different time scales, the following step proceeds.

Step 6. Use a moving fitting window sweeping through all the range of scales s to obtain a sequence of overlapped windows. Then calculate the q-order fluctuation functions $F_q(s)$ at each window to get a quasi-continuous change of $h_{xy}(q)$ with respect to the scale s. Visualize this relationship by the Hurst surface $h_{xy}(q, s)$, and the points on the surface represent the dependence (q, s). Note that in the process of calculating $h_{xy}(q, s)$, the window center and the window itself are varying with the scale s. To better visualize the Hurst surface in a three-dimensional space, the scale axis is calibrated with the center of the fitting window. That is, the window $s \in [a, b]$ is represented by the center $s = (a + b)/2$. By using MF-DCCA at each fitting window, a multiscale multifractal analysis of time series can be performed without any initial assumptions about time scale.

The bivariate cross-correlation exponent $h_{xy}(q, s)$ has similar properties and interpretations as the univariate Hurst exponent $h(q)$. If $h_{xy}(q, s)$ is independent of q, the cross-correlations between two time series are monofractal. If $h_{xy}(q, s)$

varies with the change of q, the cross-correlations between two time series are multifractal. When $h_{xy}(q,s) > 0.5$, the cross-correlations are long-range persistent. When $h_{xy}(q,s) < 0.5$, the cross-correlations are anti-persistent. However, if $h_{xy}(q,s) = 0.5$, the cross-correlations between two time series do not exist or only exist in a very short-range.

The multifractal properties of time series are also studied by multifractal spectrum analysis, which is based on the following analytical relationship between the generalized Hurst exponents $h(q)$ and the Renyi exponent $\tau(q)$ [17]:

$$\tau(q) = qh(q) - 1. \tag{7}$$

If $\tau(q)$ is a nonlinear function of q, the time series has multifractal natures. The singularity exponent α and its multifractal spectrum $f(\alpha)$ can be obtained through Legendre transform:

$$\alpha = h(q) + qh'(q), \quad f(\alpha) = q[\alpha - h(q)] + 1. \tag{8}$$

In the multifractal case, the graph of $f(\alpha) \sim \alpha$ is the shape of a single peak bell. The multifractal spectrum width $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$ is usually used to quantify the multifractality degree of time series. The larger the $\Delta\alpha$, the more uneven the distribution of time series and thus the stronger the multifractality.

2.2. Cross-correlation test method

A new test method proposed by Podobnik et al. [44] can be utilized to test the cross-correlation between two time series at a certain significance level. For two stochastic time series $\{x_i\}$ and $\{y_i\}$ with the same length N, the cross-correlation function and the cross-correlation test statistic are defined as follows, respectively,

$$C_i = \frac{\sum_{k=i+1}^{N} x_k y_{k-i}}{\sqrt{\sum_{k=1}^{N} x_k^2 \sum_{k=1}^{N} y_k^2}}, \quad Q_{cc}(m) = N^2 \sum_{i=1}^{m} \frac{C_i^2}{N-i}. \tag{9}$$

Generally, the cross-correlation test statistic $Q_{cc}(m)$ approximately follows the $\chi^2(m)$ distribution with m degrees of freedom. If there is no cross-correlation between two time series, the cross-correlation test agrees with the $\chi^2(m)$ distribution. If the cross-correlation test exceeds the critical value of the $\chi^2(m)$ distribution, then the two series are significantly cross-correlated at a certain significance level.

3. Data description and statistical test

3.1. Data description

The Shanghai Stock Exchange, established in December 1990, is one of the two stock exchanges in mainland China. Up to February 17, 2017, the Shanghai Stock Exchange had 1234 listed companies, 9518 listed securities, with a total market capitalization of more than 30 trillion Yuan. The number of listed shares, the total market values and the turnover of treasury bonds all rank first in mainland China. Therefore, the daily closing data of the stock index, bond index and fund index in the Shanghai Stock Exchange can basically reflect the overall development situations of these three markets in mainland China.

The data used for this study are the time series of the Shanghai Composite Index (SHCI), Shanghai Bond Index (SHBI) and Shanghai Fund Index (SHFI). Because the opening dates of the indexes are different, we unify the original sample data interval from February 24, 2003 to February 17, 2017 (a total of 3399 trading days) to ensure that the sample time series of different markets are synchronous (data obtained from http://money.163.com/). Fig. 1 shows the fluctuation shapes of the daily closing data of SHCI, SHBI and SHFI.

To eliminate the possible heteroscedasticity of a time series, a logarithmic return series of the original data is usually used in practical analysis. Denoting the closing index at time t by $x(t)$, the logarithmic daily return is defined by $r(t) = \log(x(t)) - \log(x(t-1))$. The daily logarithmic returns of SHCI, SHBI and SHFI are presented in Fig. 2, respectively.

In Fig. 2, from the perspective of return rates, the earning rates of SHCI, SHBI and SHFI always hover around 0 over the whole time, without significant deviation from the trend, indicating that there is no long-term profit trend in the three markets. From the viewpoint of volatility, the violent fluctuations and volatility clustering exist in all three markets and the volatility of both the stock and fund markets is significantly greater than that of the bond market. This is because the bond is a financial contract issued to investors when the governments, financial institutions and industrial and commercial enterprises directly finance or borrow money from the society. Besides, the bond is also a debt certification that promises to pay interest at a certain rate and repayment of principal in accordance with the agreed terms. Different from the stock and fund, the bond has the characteristics of low risk and low profit. Since the bond investment has interest return regularly and is payable at maturity, the return rate of bond is more stable and its volatility is also smaller.

3.2. Basic statistical characteristics and cross-correlation test

In this subsection, classical statistical methods are first used to test the statistical characteristics of the return series of the stock, bond and fund markets. The mean, maximum, minimum, standard deviation, skewness, kurtosis, and Jarque–Bera statistics of the three return series are presented in Table 1, respectively.

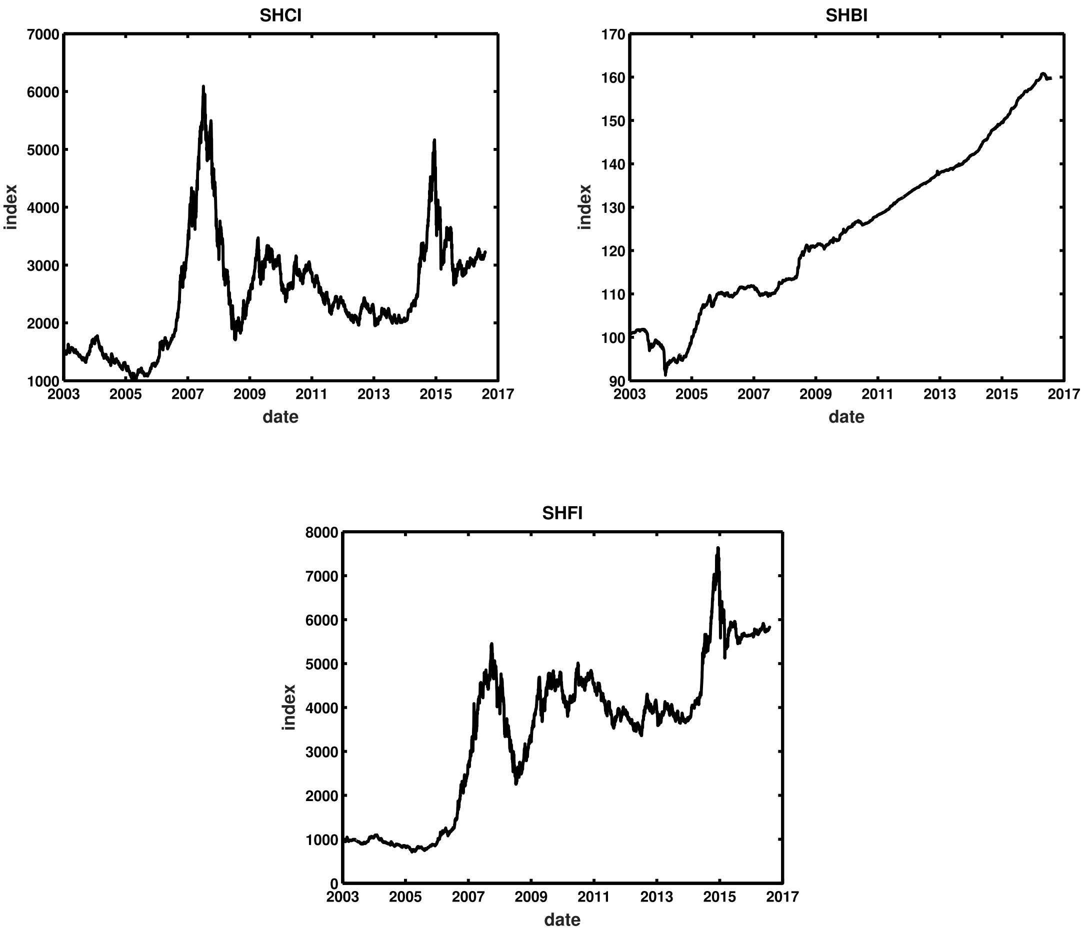

Fig. 1. Daily closing data of SHCI, SHBI and SHFI.

Table 1
Descriptive statistics for the returns of SHCI, SHBI and SHFI.

	Mean	Max	Min	S.D	Ske	Kur	J–B
SHCI	−0.0000985	0.040199	−0.03924	0.00725	0.50453	7.070572	2489.4***
SHBI	−0.0000590	0.006022	−0.00441	0.00043	1.90401	39.32784	188846.8***
SHFI	−0.000228	0.040578	−0.04087	0.00646	−0.0069	8.677341	4562.226***

***Denote 1% significance level.

Table 1 shows that the average returns of the stock, bond and fund markets are -0.00985%, −0.00590% and -0.0228%, respectively. All of them are close to 0, meaning that the three return series have the function of returning to balance. The fluctuations of both SHCI and SHFI are significantly greater than those of SHBI according to their standard deviations, which is in accordance with the results in Fig. 2. The skewnesses of the three return series are not equal to zero. SHCI and SHBI appear skewed to the right, while SHFI to the left. All kurtosises are greater than 3, and the J–B statistics indicate that the normality assumption of the three return series is rejected at 1% significance level, implying that all the three series present the features of sharp peaks and fat-tailed distributions.

Next, we perform the Ljung–Box auto-correlation test on the three return series, and the modified Q statistics with lags are given in Table 2.

As seen in Table 2, all the modified Q statistics of the three return series are significant at 1% significance level, indicating that the null hypothesis of white noise is rejected and showing the existence of auto-correlation. In order to further explore whether there are nonlinear structures in the three return series, we perform the BDS test. After eliminating the short-term linear factors in the series, according to the AIC criterion, the AR (4), AR (1) and AR (4) models are respectively applied to the fitting of the three return series, so as to obtain three systems of linearly independent residuals used in the BDS test. The BDS statistics from two-dimension to six-dimension are displayed in Table 3, from which it can be seen that the

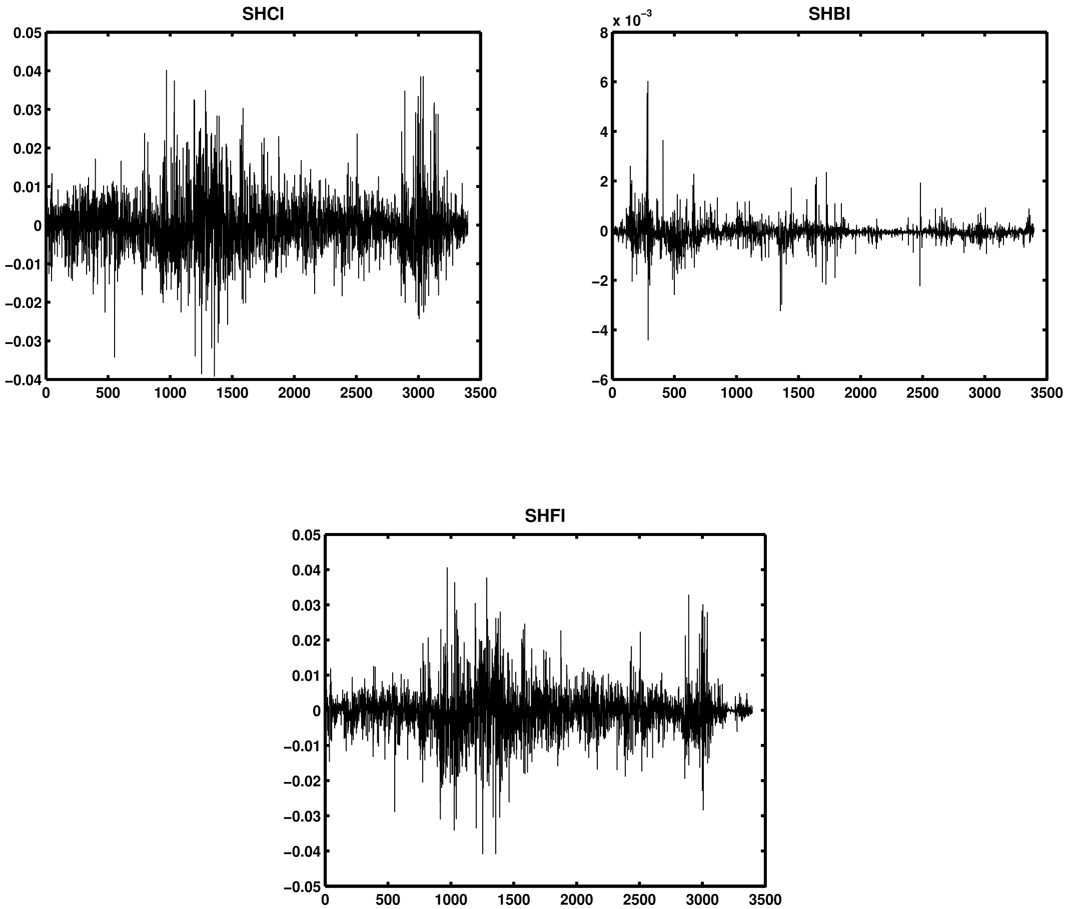

Fig. 2. Logarithmic returns of SHCI, SHBI and SHFI.

Table 2
Ljung–Box test: the modified Q statistics with lags.

Lags	5	10	20	30	40	50
SHCI	21.194***	36.964***	68.027***	78.536***	105.77***	124.88***
SHBI	329.33***	461.82***	497.71***	516.40***	566.11***	582.49***
SHFI	27.132***	51.547***	78.391***	92.805***	118.07***	154.12***

***Denote 1% significance level.

Table 3
BDS statistics at different dimensions.

Dimensions	2	3	4	5	6
SHCI	0.015039	0.035039	0.0495	0.058145	0.062229
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
SHBI	0.047257	0.091001	0.119943	0.138064	0.147062
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
SHFI	0.024691	0.054572	0.074273	0.087175	0.093354
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Note: (0.00) denote the values of probability.

probability values are all zero, meaning that the null hypothesis of the series being independent is rejected and showing that the nonlinear structures exist in the three series.

Fig. 3 presents the log–log plots of cross-correlation test statistics $Q_{cc}(m)$ for the three return series. The number of degrees of freedom varies from 1 to 1000. The log–log plots of the critical values of the $\chi^2(m)$ distribution at 5% significance level are also shown in Fig. 3 for comparison. We can find that the cross-correlation statistics of the three systems of series are all

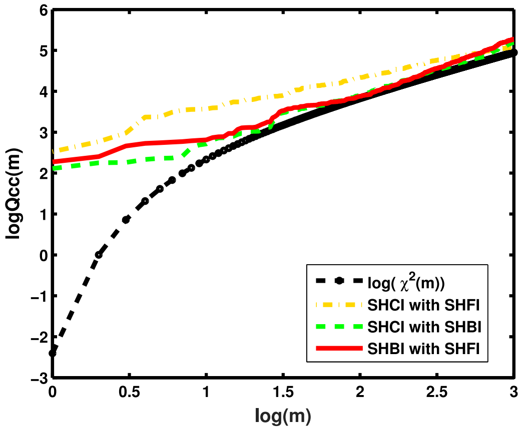

Fig. 3. Log–log plots of $Q_{cc}(m)$ versus m.

greater than the critical values at 5% significance level. Therefore, the null hypothesis of no cross-correlations can be rejected, indicating that the long-range cross-correlations exist in the three series.

4. Multifractal analysis of the auto-correlations

In Section 3, the basic characteristic statistics and the cross-correlation test have shown that nonlinear auto-correlated structures and long-range cross-correlations exist in the return series of SHCI, SHBI and SHFI. In this section, the multifractal analysis methods are used to quantitatively describe the complexity of the auto-correlated structures. We confirm that the auto-correlations have multifractality and explore the sources of multifractality for the three return series.

4.1. Confirmation of the multifractality

Firstly, MF-DFA method is applied to verify the existence of multifractality. Using 3-order fitting polynomial to eliminate the local trends of the series, we calculate the q-order fluctuation functions for different q from -5 to 5, and obtain the q-order generalized Hurst exponent $h(q)$. Fig. 4 shows the curves of the generalized Hurst exponents $h(q)$ versus q for the three return series.

It can be seen from Fig. 4 that these generalized Hurst exponents are obviously not constant but monotonically decrease with the increase of q from -5 to 5, meaning that the auto-correlations of the three return series are multifractal and also indicating that linear or monofractal analysis methods are irrelevant for the analysis. Moreover, Fig. 4 shows that none of the generalized Hurst exponents are identical to 0.5, indicating that the three markets are not stochastic but fractal markets. Specifically, the values of $h(q)$ of both SHBI and SHFI are greater than 0.5, implying that the fluctuations of the bond and fund markets are long-range persistent. That is to say, the returns of markets at next period are likely positive if the returns at the current period are positive. But, the values of $h(q)$ of SHCI are less than 0.5, indicating that the fluctuations of the stock market are anti-persistent, that is, the return is likely negative at the next period if it is positive at the current period and vice versa. Therefore, the multifractal analysis of the security markets can provide some references for investors to predict the short-term trends of markets.

Fig. 5 provides the plots of the Renyi exponents $\tau(q)$ versus q and the multifractal spectra $f(\alpha)$ versus α for the three return series. As seen from Fig. 5(a), the Renyi exponents of the three return series vary with q from -5 to 5 and all curves are concave. This shows that the multifractalities exist in auto-correlations of the three return series. Besides, the three multifractal spectra exhibit a bell shape as seen in Fig. 5(b). Further, it is seen that the multifractal spectra of both SHCI and SHFI are with right hooks, i.e., $\Delta f = f(\alpha_{\min}) - f(\alpha_{\max}) < 0$. This shows that the opportunity of the index loss is greater than that of the index profit. Similarly, the multifractal spectrum of SHBI is with left hook, i.e., $\Delta f = f(\alpha_{\min}) - f(\alpha_{\max}) > 0$. This implies that the chance of the index loss is lower than that of the profit. According to the results, it can be suggested that bond investment may be a better choice for risk-averse investors.

4.2. Sources of the multifractality

Previous analysis has confirmed that the return series of SHCI, SHBI and SHFI have obvious multifractal features. Generally, there are two major factors contributing to the multifractality, that is, the different long-range correlations for small and large fluctuations and the fat-tailed probability distribution in fluctuations. To explore the sources of multifractalities of the three return series, we need to shuffle and phase-randomize the original series so as to obtain the shuffled series and

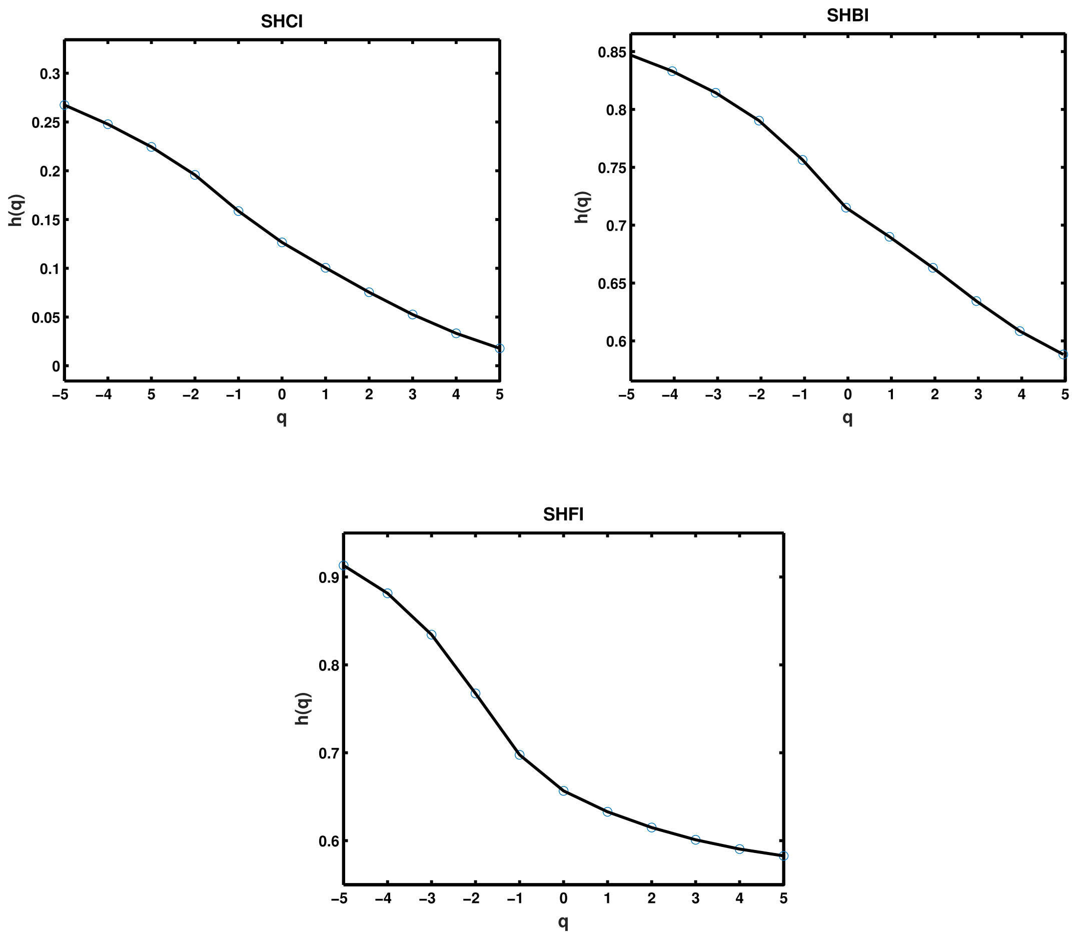

Fig. 4. Generalized Hurst exponents of the three return series.

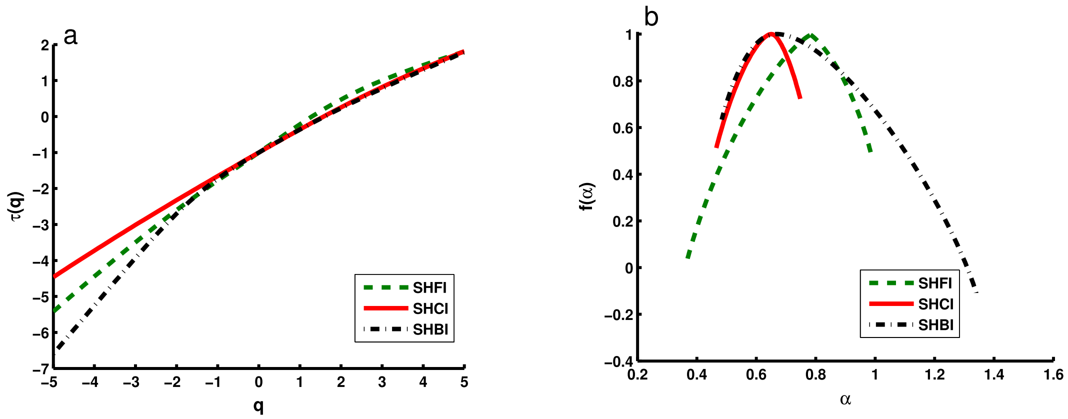

Fig. 5. Renyi exponents and multifractal spectra of SHCI, SHBI and SHFI.

the surrogated series, respectively. Note that the shuffling procedure can destroy the auto-correlation of original series but preserve the distributions of fluctuations, while the phase-randomization process can weaken the non-Gaussianity in series. Thus, we calculate the generalized Hurst exponents $h(q)$ for the original, shuffled and surrogated series and the corresponding multifractality degrees $\Delta h = h(q)_{\max} - h(q)_{\min}$ to find the sources of multifractality.

The generalized Hurst exponents and the multifractal spectra of the original, shuffled and surrogated series for the three return series are plotted in Figs. 6 and 7, respectively.

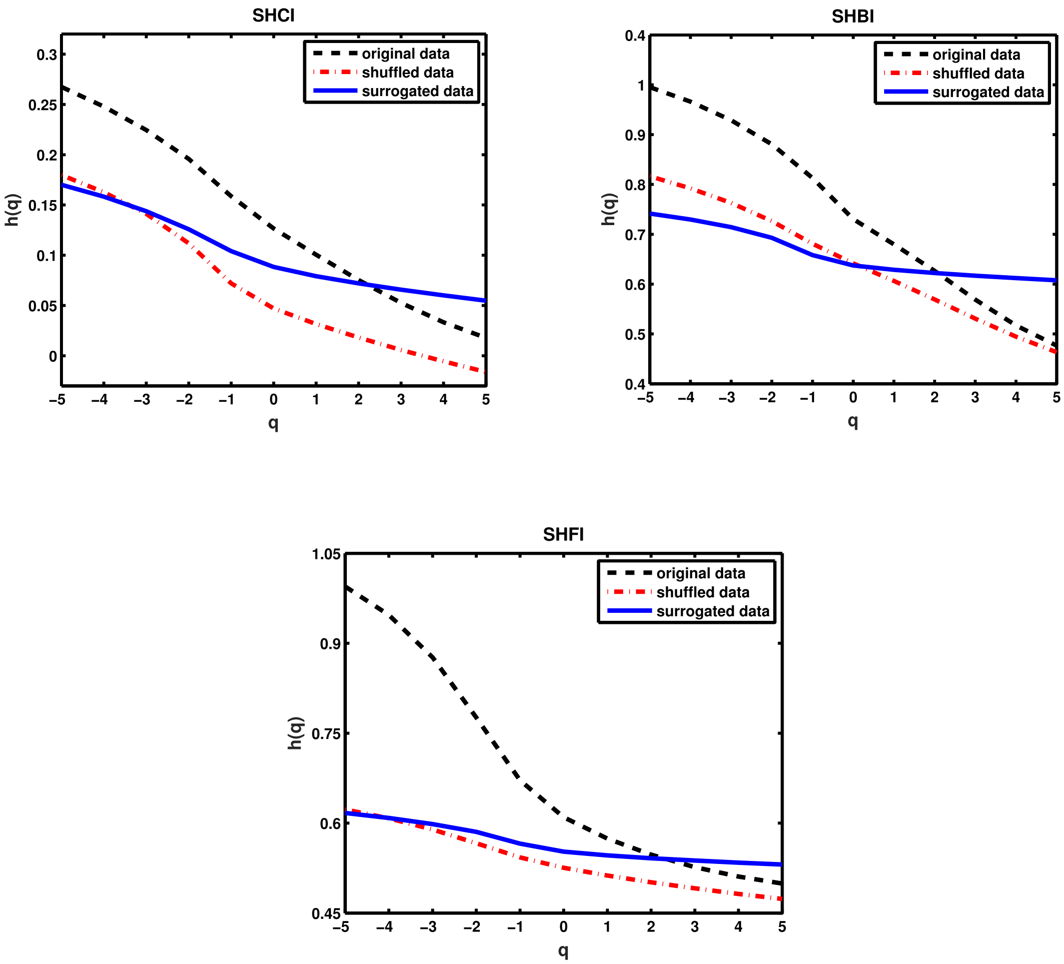

Fig. 6. Generalized Hurst exponents of the original, shuffled and surrogated series.

According to Fig. 6, the generalized Hurst exponents of both the shuffled and surrogated series vary with the values of q, and the multifractality degrees of the two series are obviously weaker than that of the original series, which shows that both long-range correlations and fat-tailed distributions contribute to the multifractality of the original series. Moreover, we note that the multifractality degrees of the surrogated series are weaker than those of the corresponding shuffled series, which implies that fat-tailed distributions in the three series have a dominant influence on their multifractality while long-range correlations are less important.

From Fig. 7, we can see that the spectra widths $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$ of all shuffled and surrogated series are smaller than those of the corresponding original series, confirming that the long-range correlations and fat-tailed distributions are the common causes of multifractality. Further, the fat-tailed distributions have more contributions to the multifractality than the long-range correlations because of the narrower spectra widths of the surrogated series. This conclusion agrees with the previous analysis from Fig. 6.

5. Multiscale multifractal analysis of the cross-correlations

The cross-correlation test based on the statistic $Q_{cc}(m)$ can only test the existence of cross-correlation between two time series qualitatively. In order to quantitatively study the cross-correlations between the return series of Chinese security markets, the MM-DCCA method is used to calculate the Hurst surfaces $h_{xy}(q, s)$ at different time scales s to visualize the cross-correlations.

Fig. 8 shows the log–log plots of the fluctuation functions $F_q(s)$ versus s between two series in the three return series. It can be seen that these log–log curves present a linear trend with q varying from -5 to 5, indicating that there are power-law cross-correlations in the three series.

We use a sliding fitting window sweeping through all time scales s to observe the changes of the generalized cross-correlation exponents $h_{xy}(q)$ at small scales $s \in (10, 50)$ and large scales $s \in (120, 600)$, respectively. By applying MF-DCCA

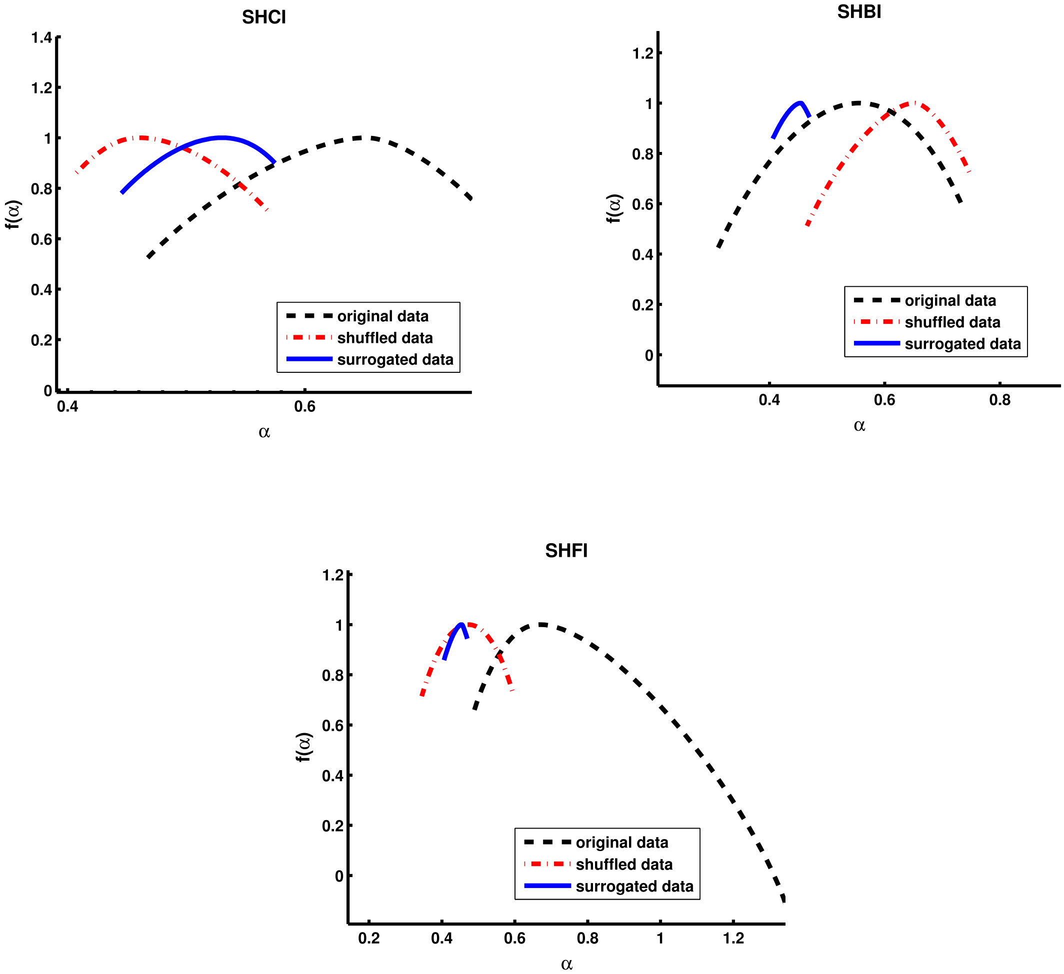

Fig. 7. Multifractal spectra of the original, shuffled and surrogated series.

to each fitting window, we obtain three Hurst surfaces $h_{xy}(q, s)$ as shown in Fig. 9, where the s-scale axis has been calibrated by the center of the fitting window with $s = 30$ (the center of (10, 50)) at the beginning and $s = 360$ (the center of (120, 600)) at the end. Obviously, the shapes of the three Hurst surfaces vary with (q, s), and the fluctuations at small and large scales are different. Moreover, all three surfaces show that the generalized cross-correlation exponents between any two series are greater than 0.5, indicating that there exist long-range cross-correlations among the three series.

From Fig. 9, the values of $h_{xy}(q, s)$ for SHCI with SHFI (Fig. 9(a)) are larger than those for SHCI with SHBI and SHBI with SHFI (Fig. 9(b) and (c)), which means that the cross-correlation between the stock and fund markets is the strongest among the three cross-correlations. This can be explained as in China's fund market, the funds are usually divided into stock funds, bond funds, money market funds and hybrid funds according to the classification of fund investment. The stock funds invest primarily in stocks. Especially, since a new policy regarding the stock position of stock funds was implemented in Chinese security markets on August 8, 2015, the stock position rose to no less than 80% in stock funds, which directly leads to the increase of stock proportion in the fund investment and the enhancement of impact on the stock market. For the bond funds, more than 80% of the fund assets are required to invest in the bond market, and the rest can be invested in the stock market, while the hybrid funds also have a certain proportion of stock investment. As a large amount of money in the fund market is invested in the stock market, the cross-correlation between the fund and stock markets is relatively larger than that of the other two. Thus, it also implies that there exist greater risks in the portfolio of the stocks and funds. In addition, the Hurst surface of SHCI with SHFI has obvious fluctuations at small scales, while the surface is close to flat at large scales. That is to say, the cross-correlations have a monofractal feature in the range of large scales. It also shows that the cross-correlations described at the small scales can exhibit much richer multifractal properties of the series.

The Hurst surface of SHCI with SHBI in Fig. 9(b) is different from those shown in Fig. 9(a) and (c). It presents an obviously downward slope at small scales, which suggests that the cross-correlation between the stock and bond markets is not very stable. The possible reasons are that the operation mechanisms for China's stock and bond markets are different, and the

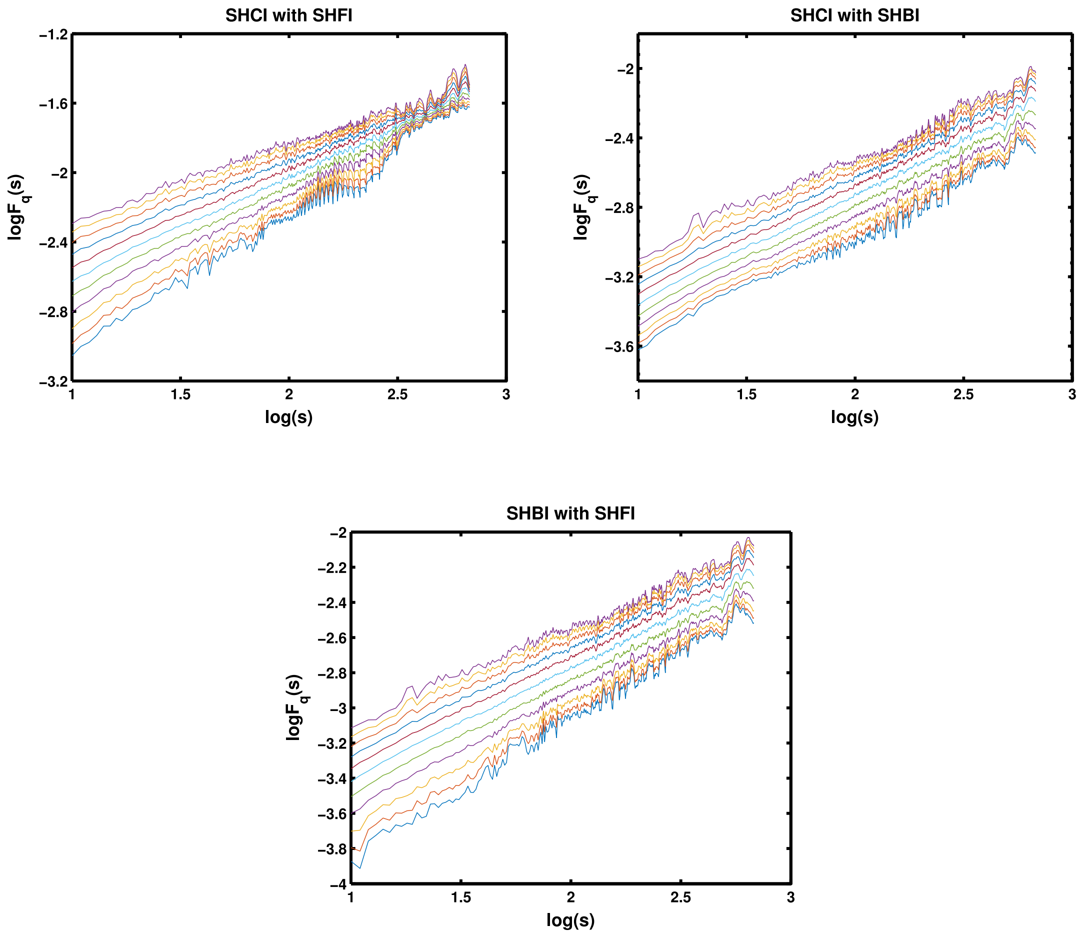

Fig. 8. Log–log plots of $F_q(s)$ versus s between the return series.

participants of markets are significantly distinct. There are plenty of retail investors in China's stock market, as well as many institutional investors such as funds, insurance, securities traders and private equity. However, the bond market is mainly composed of the exchange market and the inter-bank bond market, while the latter is the major component and its participants are mainly various commercial banks. These commercial banks account for the majority of transactions and are not allowed to involve in the stock market. Besides, China's bond market is also largely influenced by the government's intervention and control, as well as the macroeconomic situation and policies. In contrast, the stock market is relatively open and free, and the trends of the market are mainly influenced by market supply and demand. From the perspective of investors, risk seeker tends to invest in the stock market, expecting to get higher yields, while low-risk bond investments are often favored by risk averter. As a result, the investors' different risk preferences and investment decisions have partially contributed to the instability of the interaction. All these lead to a conclusion that it is difficult to establish a relatively stable relationship between the stock and bond markets.

Compared with the former two patterns, Fig. 9(c) exhibits a slight change from small scales to large scales with a relatively flat surface. This shows that there is a weaker cross-correlation between the bond and fund markets. This is because the main trading bodies of China's bond market are the financial institutions dominated by banks, while the investors in the fund market are relatively diversified. Moreover, the existence of securities type index fund, whose index computation is mainly based on the stock funds, makes the fund index relies largely on the float of the stock index. Therefore, the cross-correlation between the fund and bond indexes is weak.

A comprehensive comparison of Fig. 9(a), (b) and (c) leads to two findings. On one hand, from the perspective of multifractal analysis, the three Hurst surfaces exhibit relatively large fluctuations at small scales but relatively flat at large scales. This indicates that using small time scales to analyze the volatility of time series can reveal some hidden information that the series has not been detected in larger scale processing. It is also a reminder to researchers that when analyzing a shorter time series, the results obtained may be meaningless if the time scale is large and the time window is too wide. On the other hand, from the viewpoint of risk diversification, whether it is the stock market or the fund market, the results of

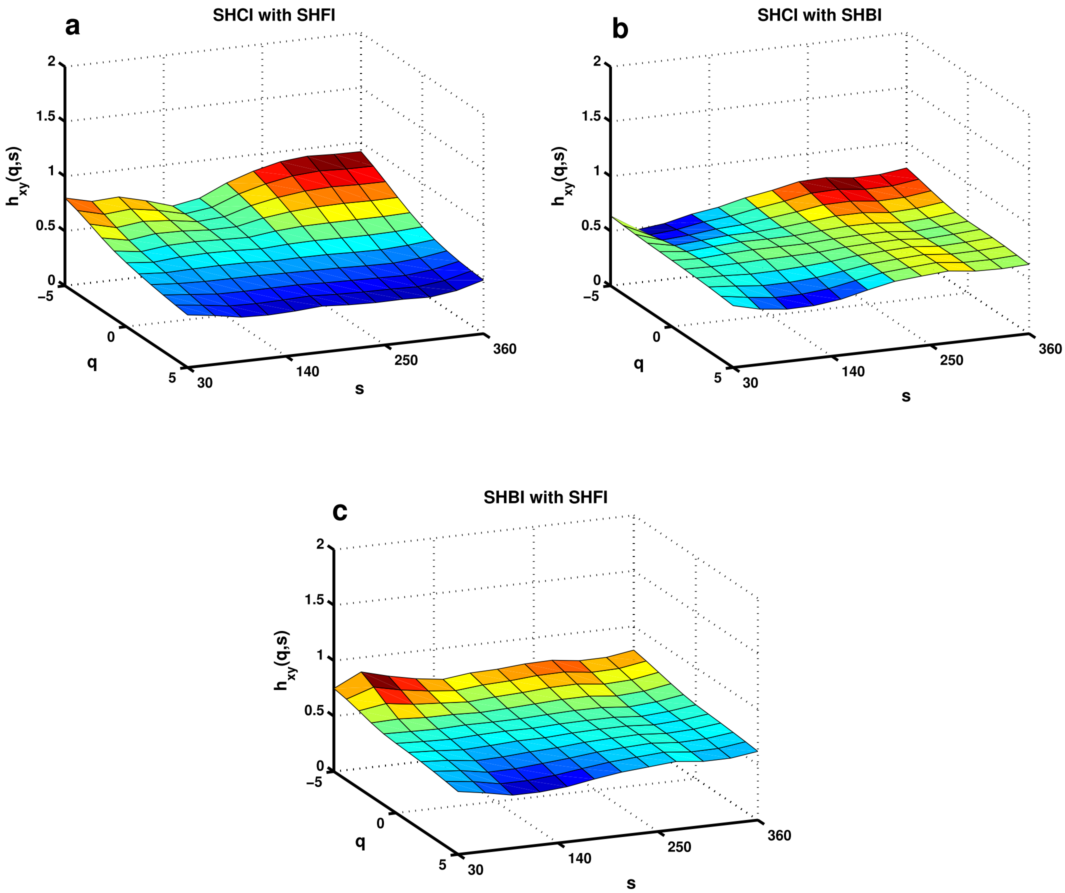

Fig. 9. Hurst surfaces of the return series for SHCI, SHBI and SHFI.

MM-DCCA indicate that the cross-correlations between the bond market and the above two markets are weak. Therefore, for the investors in the security markets, it is a good idea to try out a portfolio of bonds and stocks or bonds and funds, which can reduce the risk of investment while maximizing the profits.

6. Conclusions

In this paper, multifractal analysis methods were mainly applied to study the multifractal properties of China's stock, bond and fund markets. Different from some published papers, such as references [1–3,6–9], this paper studied the correlations among three financial markets instead of two, and focused mainly on their multifractal auto-correlations and cross-correlations, and emphasized more on different dynamic behaviors of the three markets at different time scales. Our findings can be summarized as follows.

Firstly, the descriptive statistics showed that the return series of the three markets disobey the Gaussian distribution. That is, the distributions of the three return series are non-normal due to their sharp peaks and fat-tailed statistical features. Moreover, the Ljung–Box and cross-correlation tests indicate that there exist nonlinear auto-correlated structures and long-range cross-correlations in the three series.

Then, the multifractal characteristics of three return series were analyzed by using the MF-DFA and multifractal spectrum analysis methods. The results showed that the auto-correlations of the three return series have multifractality. In addition, the sources of multifractality were explored. We found that both the long-range correlations and fat-tailed distributions are common causes for the multifractality, and fat-tailed distributions have major effect on the multifractality.

Finally, the multifractal cross-correlations between the return series were investigated based on MM-DCCA. By using a sliding scale window technology to obtain the Hurst surfaces visualizing the cross-correlations, we found that the three cross-correlations exhibit different multifractal features at small and large scales, and the cross-correlation between the stock and fund markets is stronger than that of the other two groups, while the cross-correlation between the stock and bond markets is unstable.

It is well known that the multifractal degree of interrelationship between two financial markets not only reflects the complexity of cross-correlations, but also reveals the level of the portfolio risk in the markets. According to the empirical results, some suggestions were offered to investors from the perspective of risk diversification in this paper. Therefore, this research has great reference value for the trading decision and risk management of security markets.

Acknowledgments

We thank the main editor H.E. Stanley and the anonymous reviewers for their helpful comments and suggestions. We are grateful for the financial support offered by the Humanities and Social Science Foundation from the Ministry of Education of China (grant no. 12YJAZH020).

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