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Probabilistic analysis of ultimate seismic bearing capacity of strip foundations

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ABSTRACT

This paper presents a reliability analysis of the pseudo-static seismic bearing capacity of a strip foundation using the limit equilibrium theory. The first-order reliability method (FORM) is employed to calculate the reliability index. The response surface methodology (RSM) is used to assess the Hasofer–Lind reliability index and then it is optimized using a genetic algorithm (GA). The random variables used are the soil shear strength parameters and the seismic coefficients (k_h and k_v). Two assumptions (normal and non-normal distribution) are used for the random variables. The assumption of uncorrelated variables was found to be conservative in comparison to that of negatively correlated soil shear strength parameters. The assumption of non-normal distribution for the random variables can induce a negative effect on the reliability index of the practical range of the seismic bearing capacity.

1. Introduction

Uncertainty is an important issue in engineering design as geotechnical engineers can basically introduce uncertainty in the design when using a global safety factor. Reliability methods have therefore become promising when assessing the effect of uncertainty on geotechnical structure design. The designs using reliability assessment were applied to many geotechnical engineering projects (e.g. Mollon et al., 2009a,b, 2011, 2013; Griffiths and Fenton, 2001; Griffiths et al., 2002; Kulhawy and Phoon, 2002).

Many theories have also been used to study the seismic bearing capacity of a strip foundation (e.g. Budhu and AlKarni, 1993; Dormieux and Pecker, 1995; Soubra, 1997). Their results indicated that the value of the bearing capacity decreased with the increase of the seismic acceleration coefficient. Inertia forces in the soil mass decrease the bearing capacity of the soil and, as a result, the bearing capacity of the foundation decreases. In recent years, some researchers such as Zeng and Steedman (1998), Garnier and Pecker (1999), Askari and Farzaneh (2003), Gajan et al. (2005), Knappett et al. (2006), and Merlos and Romo (2006) have drawn the same conclusions by using the dynamic centrifuge tests. Using the characteristics method, Cascone and Casablanca (2016) evaluated the static and seismic bearing capacity factors for a shallow strip foundation by the pseudo-static approach. Other researchers such as Pane et al. (2016) numerically obtained the bearing capacity of soils under dynamic conditions. Shafiee and Jahanandish (2010) employed the finite element method to determine the seismic bearing capacity of strip foundations with various seismic coefficients and friction angles. They also presented curves relating the seismic bearing capacity factors to the seismic acceleration coefficient.

In this context, the homogeneous soils and seismic properties are used to analyze the seismic bearing capacity of strip foundations. The bearing capacity is calculated using a single deterministic set of parameters. Reliability analysis is then used to assess the combined effects of uncertainties and provide a logical framework for selecting the bearing capacity that is appropriate for a degree of uncertainty and the failure consequences. Thus, the reliability assessment useful for providing better engineering decisions is performed as an alternative to the deterministic assessment.

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Over the last fifteen years, the reliability analysis of shallow foundations subjected to a centered static vertical load has been studied by Fenton and Griffiths (2002, 2003), Sivakumar Babu et al. (2006), and Youssef Abdel Massih et al. (2008). However, the reliability analyses of shallow foundations subjected to inclined, eccentric or complex loads are rarely investigated (Ahmed and Soubra, 2014). Probabilistic approaches for seismic bearing capacity of shallow foundation are seldom elaborated in the literature (Youssef Abdel Massih et al., 2008; Baroth et al., 2011). Johari et al. (2017) used the slip lines method coupled with the random field theory to estimate the seismic bearing capacity of strip foundations. The bearing capacity factors N_i (N_c, N_q and N_γ) are assessed stochastically, with the values depending on friction angle.

In previous researches, different types of simulation approaches were used to assess the reliability of geotechnical systems, in which the response surface methodology (RSM) is basically used. Monte Carlo simulation (MCS) (Wang et al., 2010) and importance sampling (IS) (Mollon et al., 2009a) offered the implied estimates of the system failure probability (P_f). However, they are rather time-consuming (e.g. finite element method or finite difference method). Different types of RSMs such as classic RSM, artificial neural network (ANN) based RSM (Cho, 2009) and Kriging-based RSM (Zhang et al., 2013) have been proposed to overcome this disadvantage. However, they are all approximate methods which cannot provide precise estimates.

This paper presents a reliability analysis of the seismic bearing capacity of a strip foundation under pseudo-static seismic loading. The uncertain parameters are modeled by random variables. These variables are the soil shear strength parameters and the seismic coefficients (k_h and k_v). Only the punching failure mode of the ultimate limit states is studied. The deterministic model is based on the limit equilibrium theory (Budhu and Al-Karni, 1993). The Hasofer–Lind reliability index (β_HL) was adopted to calculate the reliability of the seismic bearing capacity. The RSM optimized by the genetic algorithm (GA) have been used to find the approximate performance function and derive β_HL. The RSM optimized by GA saves computation time compared with the conventional RSM methods (Hamrouni et al., 2017a,b, 2018). The influence of normal and non-normal parameters distribution as well as the correlation between soil shear strength parameters on the failure probability is studied.

2. Ellipsoid approach in reliability theory

The safety of geotechnical structures can be represented by its β_HL value which takes the inherent uncertainties as input parameters. The β_HL (Hasofer and Lind, 1974) is the most widely used indicator in the literature. Its matrix formulation is (Ditlevsen, 1981)

$$\beta_\mathrm{HL} = \min_{\boldsymbol{G}(\boldsymbol{x})=\boldsymbol{0}} \sqrt{(\boldsymbol{x}-\boldsymbol{\mu})^\mathrm{T}\boldsymbol{C}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})} \tag{1}$$

where $\boldsymbol{\mu}$ is a vector of mean values, $\boldsymbol{x}$ is a vector representing the n random variables and $\boldsymbol{C}$ is a matrix covariance.

The minimization of Eq. (1) is performed using the constraint $\boldsymbol{G}(\boldsymbol{x}) \le \boldsymbol{0}$ where the n-dimensional domain of the random variables is separated by the limit state performance ($\boldsymbol{G}(\boldsymbol{x}) = \boldsymbol{0}$) into two regions: an unsafe region F represented by $\boldsymbol{G}(\boldsymbol{x}) \le \boldsymbol{0}$ and a safe region given by $\boldsymbol{G}(\boldsymbol{x}) > \boldsymbol{0}$. Eq. (1) is used in a form of the classical method to calculate β_HL, which is based on the transformation of the performance limit state initially defined in the space of the physical variables. This state must be shown in the space of the normal random variables, centered, reduced and uncorrelated, which is also called standard space. The β_HL is the shortest distance between the origin of the space and the state boundary surface.

Low and Tang (2004) proposed an interpretation of β_HL. The concept of iso-probability ellipsoid leads to a simpler calculation method for β_HL in the original physical variables (see Fig. 1). Low and Tang (2004), Mollon et al. (2009b), Lü et al. (2011), Low (2014) and Hamrouni et al. (2017a,b, 2018) demonstrated that the ellipticity (ratio between the axes) of the critical dispersion ellipsoid corresponds to the value of β_HL, which is the smallest ellipsoid dispersion that just touches the limit state surface to the unit dispersion ellipsoid, i.e. the one obtained for $\beta_\mathrm{HL} = 1$ in Eq. (1) without minimization.

They also stated that the intersection point between the critical dispersion ellipsoid and the equivalent performance limit state surface is called the design point (see Fig. 1). In the case of non-normal random variables, the Hasofer–Lind method can be extended. A transformation of each non-normal random variable into an equivalent normal random variable with an average $\boldsymbol{\mu}^N$ and a standard deviation $\boldsymbol{\sigma}^N$ was proposed by Rackwitz and Flessler (1978). Using the above-mentioned procedure, the transformation makes it possible to estimate a solution in a reduced space. The equivalent parameters evaluated at the design point $\boldsymbol{X}_i^*$ are given by

$$\boldsymbol{\mu}_i^N = -\boldsymbol{\sigma}_i^N \boldsymbol{\Phi}^{-1}\left[\boldsymbol{F}_{\boldsymbol{X}_i}\left(\boldsymbol{X}_i^*\right)\right] + \boldsymbol{X}_i^* \tag{2}$$

$$\boldsymbol{\sigma}_i^N = \frac{\boldsymbol{\phi}\left\{\boldsymbol{\Phi}^{-1}\left[\boldsymbol{F}_{\boldsymbol{X}_i}\left(\boldsymbol{X}_i^*\right)\right]\right\}}{\boldsymbol{f}_{\boldsymbol{X}_i}\left(\boldsymbol{X}_i^*\right)} \tag{3}$$

where $\boldsymbol{\Phi}$ and $\boldsymbol{\phi}$ are the cumulative density function (CDF) and the probability density function (PDF) of the standard variables, respectively; $\boldsymbol{F}_{\boldsymbol{X}_i}$ and $\boldsymbol{f}_{\boldsymbol{X}_i}$ are the CDF and PDF of the original non-normal random variables, respectively. The CDFs and PDFs of the real variables and the equivalent normal variables identified at the design point on the performance state surface are assimilated after derivation of Eqs. (2) and (3).

Low and Tang (1997, 2004) implemented an inclined ellipsoid and an optimization algorithm to minimize the dispersion ellipsoid. Eq. (1) can then be rewritten as

$$\beta_\mathrm{HL} = \min_{x \in F} \sqrt{\left[\frac{\boldsymbol{x}-\boldsymbol{\mu}_{\boldsymbol{x}}^N}{\boldsymbol{\sigma}_{\boldsymbol{x}}^N}\right]^\mathrm{T} [\boldsymbol{R}]^{-1} \left[\frac{\boldsymbol{x}-\boldsymbol{\mu}_{\boldsymbol{x}}^N}{\boldsymbol{\sigma}_{\boldsymbol{x}}^N}\right]} \tag{4}$$

where $[\boldsymbol{R}]^{-1}$ is the inverse of the correlation matrix $[\boldsymbol{R}]$. The configuration of the ellipsoid can be presented by this equation.

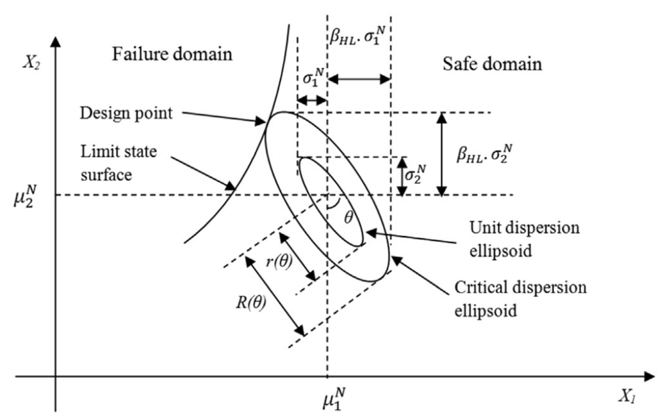

Fig. 1. Design point and equivalent normal dispersion ellipses in the space of two random variables.

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The probability of failure is approximated using the first-order reliability method (FORM) as follows:

$$\boldsymbol{P}_{\mathrm{f}} \approx \boldsymbol{\Phi}(-\beta_{\mathrm{HL}}) \tag{5}$$

2.1. RSM optimized by GA

If the objective function has a known analytical form, β_{HL} may be easily calculated. When using numerical calculations, it is impossible to obtain an explicit analytical form of the objective function. The RSM can then be used to approach this function by successive iterations to calculate β_{HL} and the design point. An algorithm based on the RSM proposed by Tandjiria et al. (2000) was used in this work consequently. This method approximates the function of performance by an explicit function of the random variables using an iterative process. The quadratic form (see Eq. (6)) of the approximate performance function is the most widely used form in the literature (second-order polynomial with squared terms):

$$G(x) = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n} b_i x_i^2 \tag{6}$$

where a_i and b_i are the coefficients to be determined, and x_i represents the random variable.

For a higher accuracy purpose, a more complex performance function (Eq. (7)) can be used which contains quadratic and crossed terms:

$$G(x) = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ij} x_i x_j \tag{7}$$

The parameters a_i and b_{ij} in Eq. (7) can be determined using the iterative method, but it seems to be rather time-consuming (Youssef Abdel Massih and Soubra, 2008; Mollon et al., 2009b). This method is used for a specific point of the limit state, thus it has to repeat this calculation for determination of other reliability index values. In this paper, the parameters a_i and b_{ij} will be calculated by an optimization using GA (Bouacha et al., 2014; Hamrouni et al., 2017a,b, 2018). The coefficients a_i and b_{ij} are determined from a number of deterministic calculations using values of the variables x_i, by the least-squares regression analysis.

In this study, the parameters of the optimization problem for parameters a_i and b_i are translated into chromosomes with a data string. To begin with the procedure of GA, an initial population is needed. The size of the initial population depends on the nature of the problem, and it usually contains several hundreds and thousands of possible solutions (in our study, a number of 50 was chosen). This population is generated randomly, covering the whole range of possible solutions, i.e. the research space (Tang et al., 1996).

The minimum square error (MSE) is represented by the fitness function in the GA approach to compare the results obtained with Eq. (7) and the deterministic results. This permits to determine the values of a_i and b_{ij}, on which no constraints occur.

Several possible solutions are obtained from the variables space and the physical conditions of these solutions are compared. If no solution is reached, a new population is created from the original (parent) chromosomes using "crossover" and "mutation" operations. From two random solutions (parents), the crossover forms a child (new solution) by the exchange of genes. Mutation is used to maintain population diversity by randomly switching a single variable into a chromosome, as the process converges towards a solution. The operation of the GA process is detailed in the flowchart shown in Fig. 2. The key advantages of GA are described as follows:

(1) It is a population-based approach and thus considers a wide range of possible solutions; and
(2) The mutation process restricts the solution to local minima that can occur in alternative solution techniques.

The stop criteria for GA search operation are crucial and probably difficult. Since GA is a stochastic global optimization technique, it is rather difficult to know when the algorithm has reached its optimum. With the measures of convergence and diversity, it is possible to compare the performances of the GA during operation state.

3. Deterministic model

Budhu and Al-Karni (1993) assumed the logarithmic failure surfaces as shown in Fig. 3 for determination of the seismic bearing capacity of soils. They modified the commonly used static bearing capacity equations of Meyerhof (1963) to obtain the dynamic bearing capacity as follows:

$$q_{\mathrm{ud}} = cN_c s_c d_c i_c e_c + \left(\gamma D_{\mathrm{f}}\right) N_q s_{cq} d_q i_q e_q + \frac{\gamma}{2} B N_\gamma s_\gamma d_\gamma i_\gamma e_\gamma \tag{8}$$

The bearing capacity factors in Eq. (8) are computed by the following equations:

$$N_c = (N_q - 1)\cot\varphi \tag{8a}$$

$$N_q = e^{\pi \tan\phi} \tan^2(\pi/4 + \phi/2) \tag{8b}$$

$$N_\gamma = (N_q - 1)\tan(1.4\varphi) \tag{8c}$$

where B is the width of the foundation, D_{f} is the depth of the foundation, φ is the friction angle, c is the cohesion of soil, γ is the unit weight, s is the shape factor, d is the depth factors, i is the inclination factor, and H is the depth of the failure zone from the ground surface.

The parameters e_c, e_q and e_γ are the seismic factors that are estimated using the following equations:

$$e_c = \exp\left(-4.3k_{\mathrm{h}}^{1+D}\right) \tag{8d}$$

$$e_q = (1 - k_{\mathrm{v}})\exp\left(-\frac{5.3k_{\mathrm{h}}^{1.2}}{1 - k_{\mathrm{v}}}\right) \tag{8e}$$

$$e_\gamma = \left(1 - \frac{2}{3}k_{\mathrm{v}}\right)\exp\left(-\frac{9k_{\mathrm{h}}^{1.2}}{1 - k_{\mathrm{v}}}\right) \tag{8f}$$

where k_{h} and k_{v} are the horizontal and vertical seismic coefficients, respectively; and D is given by the following equation:

$$D = \frac{\gamma H}{C} \frac{0.5B}{\cos\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)} \exp\left(\frac{\pi}{2}\tan\varphi\right) + D_{\mathrm{f}} \tag{9}$$

4. Reliability analysis of seismic bearing capacity

For the failure mechanisms of Budhu and Al-Karni (1993), in this paper, the deterministic results presented consider the case of a shallow strip foundation with a width of 2.5 m and a depth of 1 m. The unit weight of the soil is 18 kN/m^3. The values of the internal friction angle and cohesion are 30° and 20 kPa, respectively. By using Eq. (8), the ultimate seismic bearing capacity reaches only 729.51 kPa with k_{h} and k_{v} values of 0.2 and 0.06, respectively.

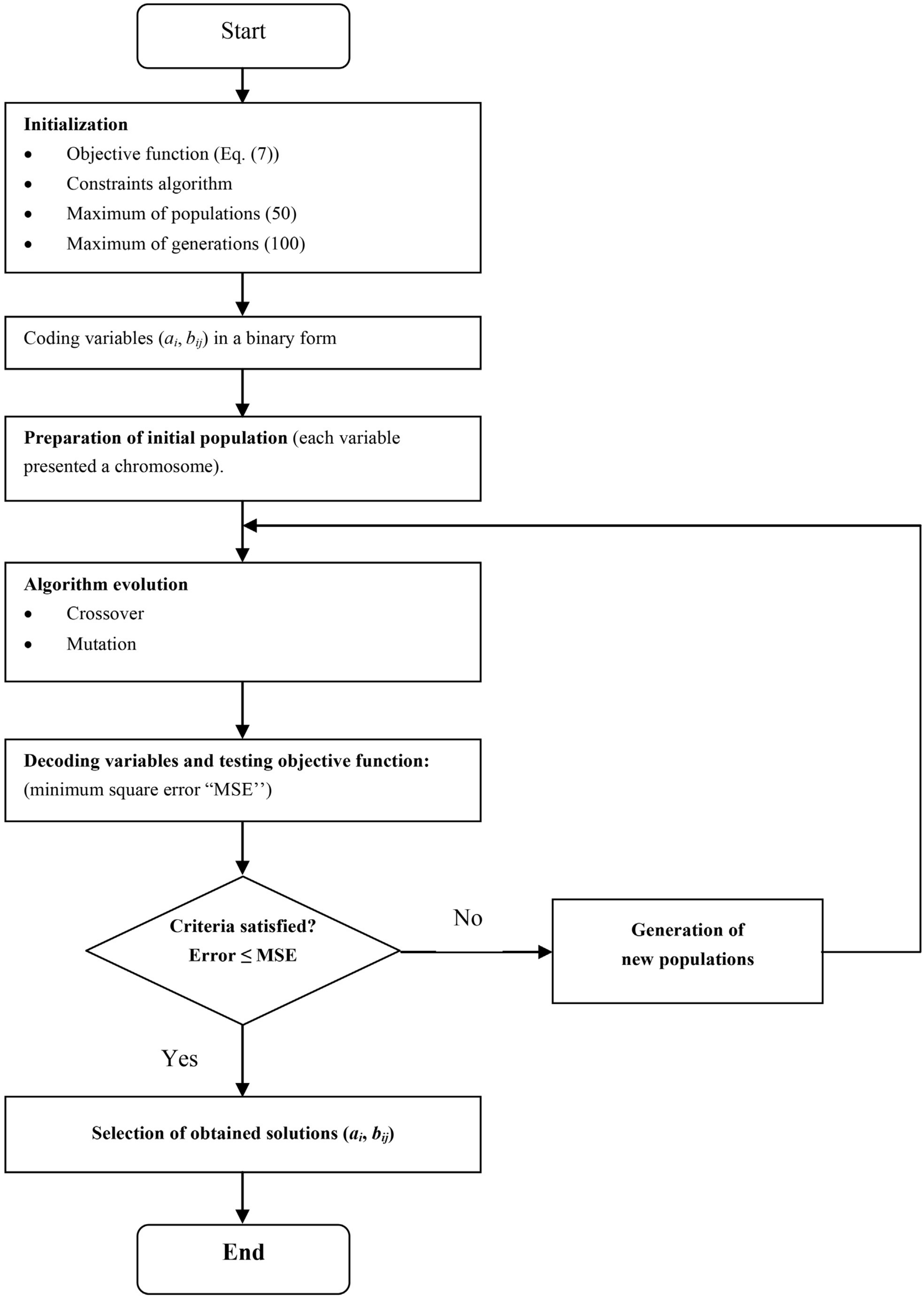

Fig. 2. Principle of optimization with GA.

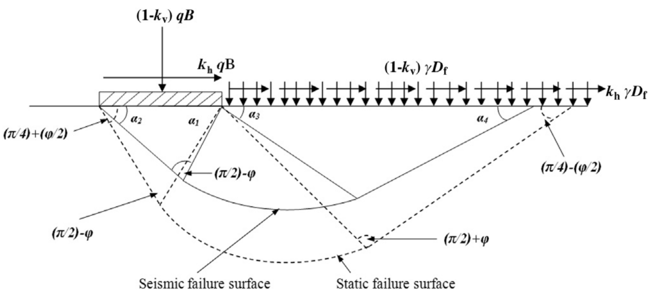

Fig. 3. Soil failure mechanism under static and seismic conditions assumed in the theory of Budhu and Al-Karni (1993).

The Algerian seismic regulation (RPA, 2003) recommended that, in Algeria, k_v equals $\pm 0.3k_h$ for several types of structures such as foundations, retaining walls and slopes. Our study takes this linear relationship between the two seismic coefficients, and will be discussed only with the parameter k_h.

4.1. Performance function

Three random variables used in this study are the soil shear strength parameters (c and φ) and the seismic coefficient (k_h). The values of the mean and the variation coefficient are chosen and presented in Table 1. Two cases are studied: random variables with normal and non-normal distribution, respectively. The parameters c and k_h are assumed to follow a lognormal distribution and φ is considered to follow a Beta distribution to better represent the friction angle (Fenton and Griffiths, 2003). The parameters of the Beta distribution are determined from the mean value and standard deviations of φ. A negative correlation between the variables c and φ ($\rho_{c,\varphi}$) are considered equal to -0.5.

The performance function used in this study is given by

$$G = q_{ud} - q_{u,min} \tag{10}$$

Failure will occur when the value seismic bearing capacity q_{ud} is larger than the threshold value $q_{u,min}$ (considered as deterministic). The numerical algorithm implementation of RSM optimized by GA was used:

(a) In this text, 100 sample points taken from the direct Monte Carlo method were used to calculate q_{ud} using the random variables.
(b) Coefficients (a_i) of the performance equation $G(\varphi, c, k_h)$ are optimized by GA using 100 sample points.

$$\begin{aligned} G(\varphi, c, k_h) = {} & a_1 + a_2\varphi + a_3\varphi^2 + a_4c + a_5c^2 + a_6k_h + a_7k_h^2 \\ & + a_8\varphi c + a_9\varphi k_h + a_{10}ck_h + a_{11}\varphi ck_h \end{aligned} \tag{11}$$

Table 1
Probabilistic model.

Variable	Mean value (μ)	Variation coefficient (σ)	Limitations of non-normal variables	Distribution type	
				Case 1: normal	Case 2: non-normal
φ (°)	30	10%	[0, 45°]	Normal	Beta
c (kPa)	20	15%	[0, +∞]	Normal	Lognormal
k_h	0.20	25%	[0, 1]	Normal	Lognormal

(c) The Matlab optimization tool (fmincon) is utilized to determine the minimum value of β_{HL} and the corresponding design point (φ^*, c^*, k_h^*) using the condition $\boldsymbol{G}(\boldsymbol{x}) \leq \mathbf{0}$ ($\boldsymbol{G}(\boldsymbol{x})$ is presented in step b).

The advantage of GA is that it moves in the search space with more possible solutions. The successful use of GA depends on how accurately and quickly it converges to the optimal solution, avoiding local minima to reduce the computation time. However, the major disadvantage of GA in case of a large number of variables is that it requires a significant computation time before the optimal solution is finally reached. In our case study, the number of variables is 11 and the computation time is less than 1 min, which makes it reasonable to choose this optimization method.

4.2. Numerical results

Eq. (11) is proposed to approximate the performance limit state. The case with normal uncorrelated variables is used to present the efficiency of GA approach. After 100 running of the GA, the best results are selected to illustrate the best combination that satisfies Eq. (11). The MSE is 1.9e^{-1} with $a_1 = 710.48405$, $a_2 = -12.1999$, $a_3 = 0.37112$, $a_4 = -18.54146$, $a_5 = -0.1532$, $a_6 = 184.02496$, $a_7 = 1575.78991$, $a_8 = 3.78538$, $a_9 = -103.37221$, $a_{10} = 132.45428$ and $a_{11} = -14.90744$.

For the case with normal uncorrelated variables, the β_{HL} values obtained after convergence are 3.981, 2.691 and 1.795 for $q_{u,min}$ values of 200 kPa, 300 kPa and 400 kPa, respectively. These indices correspond to the failure probabilities of 0.003%, 0.357% and 3.629%, respectively, using the FORM.

A good way to validate the convergence of the GA optimization approach is to consider the value provided by the model at the design point. In case of normal uncorrelated variables, q_{ud} values provided by the deterministic model and the quadratic polynomial are 200.02 kPa and 198.31 kPa, 301.5 kPa and 298.42 kPa, and 399.2 kPa and 399.90 kPa, respectively, which can be compared with the acceptable maximum efficacy of 200 kPa, 300 kPa and 400 kPa. In this paper, the use of a GA is very effective to optimize the unknown parameters of the performance function. Thus, a quadratic polynomial with crossed terms between parameters is used as the function of response surface that permits to obtain a good approximation of the performance limit state in previous analyses.

5. Reliability index, design point and partial safety factors

Table 2 shows the results of β_{HL}, design points (φ^*, c^* and k_h^*), and partial factors for different bearing capacity limit values. The calculations were performed for several cases: normal variables, non-normal variables, variables correlated or not. Note that the values of β_{HL} and partial factors increase with the decrease of the bearing capacity limit value. The results of β_{HL} are also shown in Fig. 4, with a negative correlation between the shear strength variables. It is the same as a lower extent when considering non-normal variables.

For example, for a bearing capacity limit value equal to 300 kPa, the comparison of the results between the correlated or uncorrelated variables shows that the value of β_{HL} corresponding to the negatively correlated variables is greater than that of the uncorrelated variables case. For the same bearing capacity limit values, the value of β_{HL} decreases by about 10% if one considers non-normal variables. We can conclude that the simplified assumption considering the uncorrelated normal variables is safer compared to

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Table 2
Indices of reliability, design points, and partial safety factors.

	$q_{u,min}$	β_{HL}	P_f (%)	φ^* (°)	c^* (kPa)	k_h^*	F_φ	F_c	F_{k_h}
Normal uncorrelated variables	200	3.981	0.003	21.648	13.997	0.301	1.455	1.429	1.506
	300	2.691	0.357	23.94	16.362	0.265	1.3	1.222	1.325
	400	1.795	3.629	25.796	17.868	0.243	1.195	1.119	1.217
	500	1.121	13.12	27.328	18.815	0.228	1.117	1.063	1.139
	600	0.592	27.685	28.561	19.484	0.215	1.061	1.026	1.076
	729.5	0	49.99	30	20	0.2	1	1	1
Non-normal uncorrelated variables	200	3.339	0.042	22.856	16.942	0.367	1.37	1.181	1.837
	300	2.5	0.622	23.791	17.458	0.278	1.31	1.146	1.39
	400	1.69	4.553	25.528	18.25	0.241	1.209	1.096	1.207
	500	1.077	14.086	27.081	18.841	0.221	1.129	1.062	1.107
	600	0.595	27.577	28.363	19.321	0.208	1.069	1.035	1.041
	729.5	0	48.94	29.94	19.82	0.198	1.002	1.009	0.994
Normal correlated variables (ρ_c, $\varphi = -0.5$)	200	4.812	0.000	22.412	18.387	0.376	1.4	1.088	1.881
	300	3.286	0.051	23.95	19.063	0.305	1.3	1.049	1.525
	400	2.144	1.602	25.585	20.056	0.266	1.206	0.997	1.331
	500	1.316	9.407	27.153	20.237	0.24	1.126	0.988	1.198
	600	0.673	25.037	28.457	20.311	0.22	1.065	0.985	1.099
	729.5	0	49.99	30	20	0.2	1	1	1
Non-normal correlated variables (ρ_c, $\varphi = -0.5$)	200	3.683	0.012	25.845	19.654	0.442	1.192	1.018	2.211
	300	2.84	0.225	24.974	19.586	0.334	1.24	1.021	1.672
	400	1.949	2.567	25.675	20.088	0.269	1.201	0.996	1.343
	500	1.242	10.72	26.979	20.138	0.234	1.134	0.993	1.17
	600	0.673	25.062	28.25	20.141	0.213	1.075	0.993	1.065
	729.5	0	48.91	29.98	19.94	0.197	1.0008	1.003	0.988

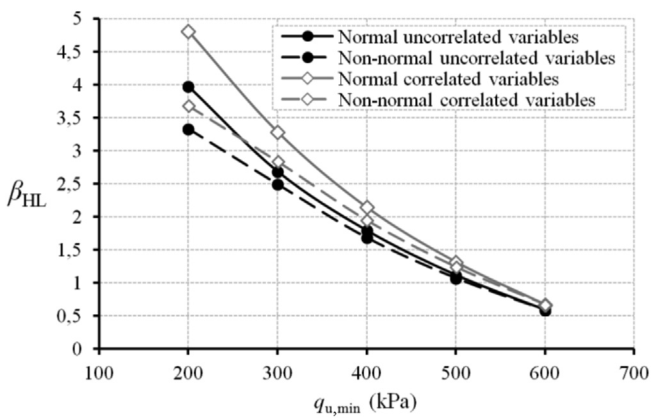

Fig. 4. Reliability index related to the performance limit.

more complex probabilistic models. This can therefore lead to uneconomic designs.

Engineers are interested in reliability index values of $2 < \beta_{HL} < 4$, suggesting that taking into account the non-normal variables has an influence on β_{HL}. This observation is related to the fact that the distribution functions of normal and non-normal variables differ in the zones of different design points obtained. The random variables (φ, c) and k_h have reverse effects on the behavior of the model. For example, a low bearing capacity is induced by the reduction of (φ, c) and the increase of (k_h).

The coordinates (φ^*, c^* and k_h^*) of the design points obtained for bearing capacity limit values can be used to calculate the partial factors F_φ, F_c and F_{k_h} as follows:

$$F_\varphi = \frac{\tan \mu_\varphi}{\tan \varphi^*} \tag{12}$$

$$F_c = \frac{\mu_c}{c^*} \tag{13}$$

$$F_{k_h} = \frac{k_h^*}{\mu_{k_h}} \tag{14}$$

where μ_φ, μ_c and μ_{k_h} represent the mean values of friction angle, cohesion of soil and horizontal seismic coefficient, respectively.

Factors for each bearing capacity limit value are also provided in Table 2. For uncorrelated variables, with the increase of the bearing capacity limit values, the partial factors are even lower, and almost are equal to 1 for the limiting case. In the case of negatively correlated variables, it is sometimes observed that the value of the design point c^* slightly exceeds the average value of c, which gives F_c less than 1 and large values of F_φ and F_{k_h}. This is due to the negative correlation between c and φ. This correlation implies that if the value of c is smaller than its mean, then the values of φ and k_h will probably be high. For this reason, a case with F_c less than 1 does not necessarily indicate failure, as long as the value of F_φ is high. This conclusion is similar to those of Youssef Abdel Massih and Soubra (2008), Mollon et al. (2009b), and Hamrouni et al. (2017a,b, 2018).

From the β_{HL} values obtained by RSM optimized by GA, the failure probability values are provided directly by the FORM approximation, as shown in Fig. 5. Taking into account a negative

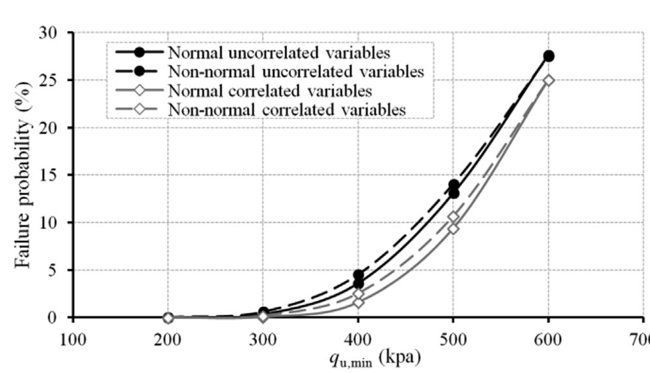

Fig. 5. Failure probability in relation to the performance limit.

correlation between c and φ and the use of bounded laws instead of the normal laws can significantly reduce the failure probability, when other parameters remain unchanged. The assumptions of normal and uncorrelated laws are quite acceptable. It is also observed that the failure probability is much more sensitive to the variations of φ and k_h than c.

6. Conclusions

An analysis based on the reliability of seismic bearing capacity of strip foundation subjected to a vertical load with pseudo-static seismic loading is presented in this paper. The main conclusions are drawn as follows:

(1) The use of GA is very effective in optimizing the unknown parameters of the performance limit function. A quadratic polynomial function with crossed terms between the parameters makes it possible to obtain a satisfactory approximation of the limit performance state in the previous analyses.
(2) Assumption of negatively correlated shear strength parameters (c, φ) was found conservative with respect to uncorrelated variables. For uncorrelated shear strength parameters values, the design point value c^* slightly exceeds the average value of c, which gives partial safety coefficients F_c less than 1 and large values of F_φ and F_{k_h}. For this reason, a case with F_c less than 1 does not necessarily indicate failure, as long as the value of F_φ is high.
(3) For the higher values of the minimum seismic bearing capacity, the reliability index β_{HL} is low and induces a very high failure probability value, indicating the vulnerability of this structure.
(4) The simplified assumption considering a normal variable is safer compared to more complex probabilistic models (non-normal variables). This can lead to uneconomical designs.

Conflicts of interest

The authors wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

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Appendix A. Supplementary data

Supplementary data related to this article can be found at https://doi.org/10.1016/j.jrmge.2018.01.009.

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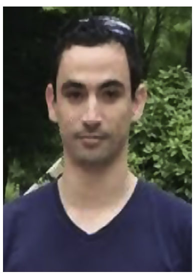

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