A Validation Scheme for Intelligent and Effective Multiple Criteria Decision-Making

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Graphical abstract
Highlights

- A validation scheme for evaluating MCDM methods is developed.
- Comparisons among MCDM methods are conducted to show viability of the proposed scheme.
- Conventional MCDM including PLP, AHP, TOPSIS, VIKOR, and ELECTRE are examples for the test.
- A synthetic decision-maker with preference generation, alternative settings, and method-oriented parameter settings are key factors to evaluate the effectiveness.
- Absolute rank deviation and Kenall’s tau rank test are suggested to statistically analyze the effectiveness.

ABSTRACT

Multiple criteria decision-making (MCDM) methods have various practical applications. Decision-makers face MCDM problems with conflicting criteria daily. Hence, MCDM methods have been developed to enable decision-makers to enhance decision quality. MCDM methods use various calculation approaches to evaluate the rank of alternatives. However, little evidence supports the consistency between the alternative chosen by the MCDM method and the decision-maker’s intuitive ideal alternative. Therefore, the objective of this study is to develop an operational validation scheme to examine and compare the effectiveness of MCDM methods. In the validation scheme, control variables include the number of alternatives, number of criteria, data set distributions, and nondominated data set options (Pareto efficient frontier or complete data set). We also add three weight distributions, namely uniform weights, rank order centroid weights, and rank sum weights, to determine the effect of weights on the MCDM methods. We test linear, quadratic, Chebycheff, and prospect utility functions. In addition to the compensatory, noncompensatory, and partially compensatory utility functions, we use the prospect theory utility function. Mean absolute rank deviation and Kendall’s statistical rank test, are applied to examine the effectiveness of the methods. To show the viability, this study illustrates the proposed scheme by an evaluation process of numerical comparisons among common MCDM methods including technique for order preference by similarity to ideal solution (TOPSIS), VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR), elimination et choix traduisant la réalité (ELECTRE), the piecewise linear prospect (PLP) theory method, and Analytic Hierarchy Process (AHP). Moreover, method-oriented parameter settings such as normalization methods, distance functions, VIKOR’s v, and ELECTRE’s thresholds are examined. Through the aforementioned settings, we compare the MCDM methods’ ranks with the decision-maker’s ranks by using assumed preference utility functions. The results reveal that interactive MCDM methods such as PLP and AHP outperform the others in terms of rank consistency. However, the performance of the MCDM methods is affected by the percentage of existing efficient solutions. More investigations into the applicability of the utility functions in various situations are suggested.

Keywords: Piecewise linear prospect (PLP) theory method; analytic hierarchy process (AHP); technique for order preference by similarity to ideal solution (TOPSIS); VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR); elimination et choix traduisant la réalité (ELECTRE)

1 Introduction

The Industry 4.0 proposal has been driving the fourth technological revolution based on the concepts and technologies of cyber-physical systems and the Internet of things for developing the new German economic policy [1–3]. Among the key development directions, the more decentralized self-organization and adaptation to human needs have triggered developing methodologies, methods of modeling, and data models and exchange formats for smarter production [1]. Owing to the changes and decomposition of conventional production hierarchy and the broader coverage of responses to strategic objectives and customer preferences from the operational-level decisions, more effective multiple criteria decision-making and soft computing techniques and their integration are largely demanded for achieving autonomous and automatic decisional intelligence [4–6].
Although studies have developed numerous novel multiple criteria decision-making (MCDM) methods and integrations of existing ones for specific problems, their validation and verifications are mostly within the scope of efficiency comparisons, consistency tests, or cross examinations with real cases [7–13]. However, the essential questions of how to evaluate the selection of appropriate MCDM methods are not well addressed. In particular, identifying the true preferred alternative is the most difficult part of validating MCDM methods.

Validating MCDM methods through interaction with decision-makers is challenging. Discovering and applying suitable metrics help to clarify decision-makers' true preferences. Multiple criteria are particularly difficult to evaluate when decision-makers use different units. The normalization and aggregation process influences the stability of results and can lead to rank reversals (i.e., adding new alternatives or deleting alternatives may change the original ranks [14]), especially with multiple decision-makers. Self-inconsistency or cognitive bias during the preference elicitation process causes rank reversals [15,16]. In these cases, the decision quality is unsatisfactory and the optimal alternative may not be presented. Furthermore, inappropriate use of MCDM methods results in rank reversals. Therefore, understanding the influence of cognitive bias on various MCDM methods and debiasing on the basis of statistical rank tests are important.

In short, this study aims to develop a validation scheme that can be commonly used to compare effectiveness of various MCDM methods. To show the viability of the proposed validation scheme, this study illustrate the evaluation and comparisons among common MCDM methods. In the validation scheme, the impact of control variables, such as the existence of efficient solutions, normalization methods, aggregation methods, and degree of interaction with decision-makers, on the MCDM validity is also investigated.

The remainder of this paper is organized as follows. Section 2 presents a review of the literature on MCDM methods. Section 3 develops a validation scheme for examining effective of MCDM methods. Section 4 illustrates the proposed validation schedule with comparisons among common MCDM methods and discusses the implications. Section 5 concludes with discussions on future research directions.

## 2 Literature review on multiple criteria decision-making (MCDM) methods

MCDM has enabled the development of many tools and solutions for problems including choice, sorting, ranking, description, elimination, and design. According to Köksalan et al. [17], Stewart et al. [18], and Ishizaka and Nemery [19], over 15,000 articles and books have been published regarding conventional MCDM methods including multiattribute utility theory, utilités additive [20], goal programming (GP), analytic hierarchy process/analytic network process (AHP/ANP) [8,9], technique for order preference by similarity to ideal solution (TOPSIS), VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR), data envelopment analysis (DEA), elimination et choix traduisant la réalité (LECTRE), preference ranking organization method for enrichment evaluation (PROMETHEE), measuring attractiveness by a categorical-based evaluation technique, piecewise linear prospect (PLP) theory method [21], and evolutionary multiobjective optimization (EMO). The applications of MCDM methods are pervasive and include critical success factors [22,23], supply chain evaluation and analysis [24], quality cost models [25], inventory classification and evaluation [26–32], vendor evaluation [4,6], process management [33], order allocation [5], scheduling [34–37], resource allocation [38,39], technology strategy [40], product development [41], flow control [42], disaster and emergency management [43], and location analysis [44]. In short, MCDM methods convincingly solve semistructured problems for making smarter decisions. However, the validity of the MCDM methods determines the effectiveness of solutions. A careless choice of the method leads to unfavorable results.

MCDM methods have various categories. The decision context categorizes the MCDM methods into multiple-attribute decision-making (MADM) and multiple-objective decision-making (MODM) [45]. MADM focuses on evaluation problems when the number of alternatives is finite. Through a systematic evaluation process for the relative importance of attributes and relative scores of alternatives on attributes, MADM can assist decision-makers in ranking alternatives and finding the most preferred alternative. MODM focuses on planning where objectives, constraints, and decision variables are predetermined to generate the feasible region and then to find
the most preferred solution(s) based on decision-makers’ preferences. Weighted goal programming is applicable when the weights among all objectives can be determined before the solution process, whereas preemptive goal programming is suitable when the objective weights are not known but the order of objectives can be identified. Finally, when the objective preferences are not identifiable, multiobjective programming can be used for generating the set of efficient solutions.

MCDM methods can also be categorized into priori, interactive, and posteriori methods based on the timing of eliciting criteria weights [46,47]. Priori methods presume existing weights before solutions, interactive methods elicit weights through solution process, and posteriori methods find the efficient solution set, i.e. nondominated data set options or Pareto efficient frontier, prior to final evaluation and ranking. Therefore, a compromise is necessary when selecting methods. Priori methods require decision-makers to give criteria weights without knowing the complete solution sets; the cost of interactions with the decision-maker is usually very high, particularly for critical decisions; and the efficient solution set is usually computationally expensive to determine. In addition, Ishizaka and Nemery [19] classified MCDM tools according to the level of interaction. Well-known methods such as AHP/ANP, TOPSIS, and VIKOR belong to the American school, which employs compensatory criteria to enable decision-makers to aggregate weights and scores into a single objective for ranking. ELECTRE and PROMETHEE belong to the French school, which focuses on noncompensatory criteria and considers the situation when alternatives may not be sortable and only an outranking relation can be found. However, these MCDM method classifications are descriptive. A framework for answering questions about the effectiveness of methods has not been established.

MCDM methods are used to solve semistructured decision problems that fall between structured problems, which can be automated, and unstructured problems such as love. Semistructured problems have decision rules, but decision-makers must reveal their own preferences to make the judgment because of noncompensatory multiple criteria. Most MCDM methods were developed as normative models, in which basic assumptions are the foundations of prescriptive decisions. Other descriptive MCDM models help decision-makers to make practical decisions. During the provision of recommendations to decision-makers in real-world problems to enable them determine an optimal solution, normative and descriptive aspects (i.e. how human beings make decisions and what decisions they should make [48]) must be considered. Specifically, when MCDM methods are being compared, their assumptions must be challenged and exceptions addressed. As Saaty and Tran [7] highlighted,

*Validity is the goal in decision-making, not consistency, which can be successively improved by manipulating the judgments as the answer gets farther and farther from reality.*

An effective MCDM method should be able to suggest the most favorable alternative that meets with the decision-maker’s preferences.

Because MCDM decision problems are highly complex, decision-makers may not be able to determine the optimal solution. Subsequently, the result of the normative model could become the representative of the best alternative. For example, TPOSIS and VIKOR possess special computing characteristics that implicitly assume that decision-makers prefer compromise solutions [10] and compute the distance to the ideal point or the reference point for the judgment. Such an assumption may not be apparent to MCDM users who believe the results beyond doubt. Opricovic and Tzeng [11] compared well-known normative MCDM models including TOPSIS, VIKOR, ELECTRE, and PROMETHEE and discussed issues of reliability including stability and sensitivity. However, the results focused on the discussions of the best alternatives for a specific problem. The question as to which method is more appropriate in general remained unanswered. They suggested

*The validation procedures have to be developed, and application feasibility should be explored. The conceptual and operational validation of the application of a method in real world problems is needed. Researchers are challenged to provide a guide for choosing the method that is both theoretically well founded and practically operational to solve actual problems.*

By contrast, interactive methods such as AHP and PLP require interaction with decision-makers and have high implementation costs. Whether interactive methods are generally more suitable than TOPSIS or VIKOR has yet
to be determined. Thus, developing a validation scheme for testing the effectiveness of MCDM methods is crucial.

3 A validation scheme for testing the effectiveness of multiple criteria decision-making methods

We develop a validation scheme for testing the effectiveness of MCDM methods (Fig. 1). The independent variable of interest is the MCDM method in use, which means that the aim is to test whether the effectiveness of various MCDM methods differs significantly under different scenarios. Control variables are alternative settings, parameter settings, and preference generation. With a specific setting of alternatives and preferences, a synthetic decision-maker can make a decision and rank alternatives automatically. The synthetic decision-maker is the innovative design in this study which enables the simulation of the interactions following the interactive methods so as we are able to compare various MCDM across the priori, interactive, and posteriori categories. Because the lack of universal utility functions, the preference generations based on various utility functions from existing studies help develop synthetic decision-makers. On the other hand, the effectiveness metrics, absolute rank deviation and Kendall’s tau coefficient [49, 50] for rank test are applied.

This study developed the validation scheme for evaluating MCDM methods on the basis of the comparison template outlined by Lahdelma et al. [12], which considers only efficiency metrics. In addition to method comparison, synthetic data are generated according to three types of variable: alternative generations, preference generations, and the parametric settings of methods. Alternative generations include the number of alternatives, number of criteria, alternative distributions, and the existence of efficient solutions. The value of an alternative on criterion $j$ is denoted by $x_j$. The alternative generation distributions include sphere (1), simplex (2), and concave (3) distributions. The variable, existence of efficient solutions, is determined according to the percentage of synthetically generated efficient solutions. If all the alternatives are efficient, the data set is called true (nondom = TRUE); otherwise, it is called false and contains inefficient (dominated) solutions (nondom = FALSE). To control the alternative generation distributions, we set the false existence of efficient solutions parameter to 0.2 to indicate 20% efficiency and 80% inefficiency of the generated alternatives. Thus, if 10 alternatives are generated, 2 of them are guaranteed to be efficient. The detailed pseudo codes of alternative generation with sufficient solutions filtering were specified in a previous study [12]. The alternative distributions are illustrated in Fig. 2.

$$\{x \in \mathbb{R}^n | x \geq 0 \land \sum_j x_j^2 \leq 1\} \quad (1)$$
$$\{x \in \mathbb{R}^n | x \geq 0 \land \sum_j x_j \leq 1\} \quad (2)$$
$$\{x \in \mathbb{R}^n | x \geq 0 \land \sum_j \sqrt{x_j} \leq 1\} \quad (3)$$

The preference generations combine two factors, namely weight distributions ($w_j$) and utility functions ($u_i$). Weight distributions show the distributions of the relative importance of various criteria. Conventional weight distributions are adopted including namely uniform weights (4), rank order centroid (ROC) weights (5) [51], and rank sum (RS) weights (6). The different weight settings simulate decision-makers’ preferences over various criteria. For example, uniform weights simulate consistent weights among criteria, the ROC expresses significant differences between high and low weights, and RS shows linearly decreasing weights (Fig. 3).

$$w_j = \frac{1}{n}, \quad j = 1, 2, \ldots, n \quad (4)$$
\[ w_j = \frac{1}{n} \sum_{k=j}^{n} \frac{1}{k}, \quad j = 1, 2, \ldots, n \]  
\[ w_j = \frac{2(n+1-j)}{n(n+1)}, \quad j = 1, 2, \ldots, n \]  

Four common utility functions are tested, namely linear utility (7), Chebycheff utility (8), quadratic utility (9) [12], and prospect utility (10) [15]. Prospect utility considers the reference point and decision-maker's risk preference. We test the max, min, and median reference points and risk preference settings of \( P = 0.5, 1.0, \) and 2.0. The min/max/median mean that the reference point is taken based on the min/max/median values of criteria. The risk preference setting is equal to the ratio \( P = w_j^-/w_j^+ \), and the settings of \( P = 0.5, 1.0, \) and 2.0 indicate risk-seeking, risk-neutrality, and risk-aversion, respectively.

\[ u_i = \sum_{j=1}^{n} w_j x_{ij} \]  
\[ u_i = \min_{j} \left\{ \frac{x_{ij}}{w_j} \right\} \]  
\[ u_i = -\sum_{j=1}^{n} (x_{ij} - w_j)^2 \]  
\[ u_i = \sum_{j=1}^{n} w_j^+ (x_j - x_{rj})^+ + \sum_{j=1}^{n} w_j^- (x_j - x_{rj})^- \]  

To illustrate the proposed validation scheme, common MCDM methods such as TOPSIS, VIKOR, ELECTRE, PLP, and AHP are compared. The method-oriented parametric settings of methods comprise normalization methods, aggregation methods, distance functions, and the VIKOR parameter \( v \). We test three levels for the VIKOR parameter \( v \), namely 0.25, 0.5, and 0.75, to investigate the trade-offs between group maximal utility and individual mini-max regret. We sort the \( Q_t \) of VIKOR in ascending order to rank alternatives. In PLP, the utility functions are applied for pairwise comparisons. To obtain all alternative ranks, we iteratively implement PLP by selecting the optimal alternative in each iteration and withdrawing it in order to generate the alternative set for the following iteration until no alternatives remain.

4 Results and discussions

Following the experiment design, we consider three types of alternative distribution, four types of utility function, and 36 settings. Combined with the six MCDM methods, namely PLP (P), AHP (A), TOPSIS (T), VIKOR (V), revised TOPSIS (Tr), and ELECTRE II (Eii), and 30 replications for each method, a total of 77,760 instances are available for statistical analysis. For each instance, we use the mean absolute rank difference between the utility function-based true rank and the rank suggested by the method and settings. As expected, Table 1 shows that the difference increases with the number of criteria. The existence of efficient solutions is vital. With more efficient solutions, alternatives that meet with decision-makers' ranks (i.e., ranks using utility functions) are more difficult to rank. Therefore, when a solution set contains many competing alternatives, decision-makers must carefully select the appropriate MCDM method. The comparison reveals that the PLP outperforms the other methods followed by the AHP, whereas the VIKOR and ELECTRE II are the two most underperforming methods in terms of total mean absolute rank deviation (Table 2). PLP and AHP are interactive models that can elicit decision-makers' preferences and rank alternatives accordingly.
When \( P = 1.0 \) for the prospect utility function, the results coincide with the results of the linear utility function because their utility functions become identical in this case. In fact, the risk-neutral prospect utility function is equivalent to the linear utility function. The linear utility function case conforms most easily. The assumption of utility linearity in the conventional MCDM models is the root causes of this result. Therefore, MCDM methods can be developed on the basis of more appropriate utility functions resulting from advanced descriptive models. Notably, PLP and AHP result in zero difference in the linear utility case. PLP is a more efficient pairwise comparison if the goal is to determine the optimal alternative, because it iteratively reduces the alternative set after each pairwise comparison. AHP is a comprehensive pairwise comparison in which all alternatives are compared. However, if the full ranking of all alternatives is the goal, then AHP is a more intuitive method to adopt. With the prospect utility function, all methods perform less effectively when the decision-maker’s preference is not risk-neutral. In particular, the risk-aversion utility function leads to more inconsistent ranks than the synthetic decision-maker’s judgment does. This warns that existing MCDM methods do not incorporate with the decision-makers’ risk preferences.

Table 3 shows that Kendall’s rank test demonstrates a similar result. Specifically, with a quadratic utility function, AHP generates a statistically significant reverse rank, (i.e., Kendall’s \( \tau \) equals one and the \( p \)-value is extremely small). In addition, with a linear or Chebycheff utility function, AHP produces the same rank. The result shows that PLP and AHP outperform other methods with more consistent ranks conforming to synthetic decision-makers’ ranks. In summary, although PLP and AHP outperform other methods, different decision-makers’ utility functions influence the effectiveness of the methods applied. More studies should measure the true utility function of decision-makers under various situations rather than place excessive emphasis on the development of new methods without carefully measuring their effectiveness.

The results illustrate the process of examining common MCDM methods through the proposed validation scheme. Unlike existing studies which focus more on efficiency comparisons, consistency tests, or cross examinations with real cases, the proposed scheme helps researchers and practitioners to select appropriate MCDM methods, particularly when priori, interactive, and posteriori ones are all feasible to choose. The proposed scheme provides more scientific basis of effectiveness evaluation among MCDM methods in addition to conventional descriptive comparisons and guidance [52]. The proposed validation scheme also responds to the argument in Saaty [53] saying no indication of approval or disapproval to descriptive methods and no way of disproving normative or prescriptive “what ought to be” methods. For Industry 4.0, the effectiveness is the key for MCDM methods to be applied to smart production practice. The proposed scheme will help enlarge the use of MCDM methods to broader industry practice as performance of these methods become measurable and trustworthy.

5 Conclusion

This study develops an operational validation scheme. The scheme is illustrated by examining and comparing conventional MCDM methods, namely PLP, AHP TOPSIS, revised TOPSIS, VIKOR, and ELECTRE II, with mean absolute rank deviation and Kendall’s \( \tau \) serving as effectiveness metrics. The results show the effectiveness that the ranks of interactive methods are the closest to synthetic decision-makers' ranks. With more efficient solutions, decision-makers should more carefully select the appropriate MCDM method, because on the efficient solution frontier, preferred and not preferred alternatives are difficult to differentiate. Most existing methods assume linear utility functions. MCDM methods can be developed on the basis of appropriate utility functions resulting from advanced descriptive models. Finally, other than comparing existing methods, the proposed validation scheme can also serve as a template for examining newly developed methods.

Because numerous novel MCDM methods have been developed, discussing all of them is nearly impractical. Instead, this study examines basic MCDM methods with various extensions to investigate the fundamental problems of validation. However, this study neglects many key elements. Further study should discuss the effects of aggregation settings and the normalization of approaches as summarized and discussed in
[54,55]. Extensions of this study should also be done toward validation of group decision-making and integration of methods as many novel methods have been developed [44,56,57].

6 Acknowledgement
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REFERENCES


Fig. 1 A validation scheme for testing the effectiveness of multiple criteria decision-making methods.
Fig. 2 Alternative generation distribution illustration.

Fig. 3 Weight generation illustration.
### Table 1 Experiment results.

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### Table 2 Comparisons of methods and utility functions using absolute rank deviation.

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<td>1.59</td>
<td>1.36</td>
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</table>
Table 3 Comparisons of methods and utility functions using Kendall’s rank test.

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>A</th>
<th>T</th>
<th>Tr</th>
<th>V</th>
<th>Eii</th>
<th>Avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>tau</td>
<td>p</td>
<td>tau</td>
<td>p</td>
<td>tau</td>
<td>p</td>
<td>tau</td>
</tr>
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<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>0.76</td>
<td>0.03</td>
<td>0.76</td>
</tr>
<tr>
<td>QUADRATIC</td>
<td>0.81</td>
<td>0.05</td>
<td>-1.00</td>
<td>0.00</td>
<td>0.50</td>
<td>0.14</td>
<td>0.51</td>
</tr>
<tr>
<td>CHEBYCHEFF</td>
<td>0.81</td>
<td>0.04</td>
<td>1.00</td>
<td>0.00</td>
<td>0.37</td>
<td>0.23</td>
<td>0.35</td>
</tr>
<tr>
<td>P=1</td>
<td>1.00</td>
<td>0.00</td>
<td>1.00</td>
<td>0.00</td>
<td>0.76</td>
<td>0.03</td>
<td>0.76</td>
</tr>
<tr>
<td>P=0.5</td>
<td>0.84</td>
<td>0.02</td>
<td>0.82</td>
<td>0.03</td>
<td>0.70</td>
<td>0.05</td>
<td>0.70</td>
</tr>
<tr>
<td>max</td>
<td>0.78</td>
<td>0.03</td>
<td>0.80</td>
<td>0.02</td>
<td>0.67</td>
<td>0.06</td>
<td>0.66</td>
</tr>
<tr>
<td>min</td>
<td>0.91</td>
<td>0.00</td>
<td>0.84</td>
<td>0.03</td>
<td>0.76</td>
<td>0.03</td>
<td>0.76</td>
</tr>
<tr>
<td>median</td>
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<td>0.02</td>
<td>0.82</td>
<td>0.03</td>
<td>0.67</td>
<td>0.07</td>
<td>0.67</td>
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<tr>
<td>P=2</td>
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<td>0.15</td>
<td>0.55</td>
<td>0.13</td>
<td>0.63</td>
<td>0.08</td>
<td>0.63</td>
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<tr>
<td>max</td>
<td>0.50</td>
<td>0.14</td>
<td>0.58</td>
<td>0.09</td>
<td>0.54</td>
<td>0.13</td>
<td>0.55</td>
</tr>
<tr>
<td>min</td>
<td>0.57</td>
<td>0.13</td>
<td>0.53</td>
<td>0.15</td>
<td>0.76</td>
<td>0.03</td>
<td>0.76</td>
</tr>
<tr>
<td>median</td>
<td>0.50</td>
<td>0.17</td>
<td>0.53</td>
<td>0.14</td>
<td>0.60</td>
<td>0.10</td>
<td>0.59</td>
</tr>
<tr>
<td>Avg.</td>
<td>0.81</td>
<td>0.05</td>
<td>0.67</td>
<td>0.04</td>
<td>0.66</td>
<td>0.08</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Note 1: Tau is Kendall’s coefficient of concordance. Tau ranges from -1.0 to 1.0 where 1.0 means the resulting ranked sequence of a MCDM method is identical to the synthetic decision-maker’s ranked sequence and -1.0 means the reverse rank of the synthetic decision-maker’s.

Note 2: * shows the case when the statistical p-value is not greater than 0.05.