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Fair framework for multiple criteria decision making

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ABSTRACT

As the determination of criteria weights is important for multiple criteria decision making, a number of attempts have been made to assign weights to criteria. However, whether criterion weight assignment is fair to each criterion and to each alternative is rarely taken into account. To address this issue, in this paper, we propose a fair framework in the context of the evidential reasoning approach, which is a type of multiple criteria utility function method. In the fair framework, two strategies are prepared for a decision maker to choose, which are the superior strategy and the inferior strategy. To achieve the objective in line with the selected strategy, two levels of fairness including the fairness among criteria and the fairness among alternatives are defined based on the performances of alternatives on each criterion. By following the two levels of fairness defined, two optimization models are constructed successively to generate possible sets of fair criterion weights. With a view to making all possible sets of fair criterion weights treated in generating a solution, they are incorporated into another optimization model constructed to generate the minimum and maximum expected utilities of each alternative, by which the solution is made with a decision rule preferred by the decision maker. A supplier evaluation problem is analyzed to demonstrate the applicability and validity of the fair framework.

1. Introduction

In real life people usually take into account a number of factors when making decisions. Different factors may conflict with each other and people have to balance conflicting factors in their choice of action, such as cost and quality of products. When one of the factors is understood as the term "criterion", action as alternative, and one person as a decision maker, such a situation is regarded as the context of multiple criteria decision making (MCDM). In MCDM, all criteria must be measurable, be capable of measuring different aspects of the problem considered, and distinguish between alternatives. As evaluating one alternative on multiple criteria is easier than evaluating the alternative in a comprehensive way, MCDM is beneficial for a decision maker to make a choice of alternatives when multiple criteria decision methods are used. The use of multiple criteria decision methods can help a decision maker relieve the cognitive overload caused by the large volume of information needed to be combined to create solutions to complex issues (Brownlow & Watson, 1987).

To conduct MCDM, many different types of multiple criteria decision methods have been proposed, including multiple criteria utility function (MCUF) methods (Butler, Jia, & Dyer, 1997; Butler, Morrice, & Mullarkey, 2001; Keeney & Raiffa, 1993; Wakker, Jansen, & Stiggelbout, 2004), multiple criteria value function methods (Belton and Stewart, 2002; Chin, Fu, & Wang, 2015; Fischer, 1995; Fu & Xu, 2016; Fu, Xu, & Yang, 2016; Keeney, 2002; Lan, Chen, Ning, & Wang, 2015; Sari, 2017; Yan, Zhang, & Li, 2017; Zhang, Wang, Li, & Chen, 2017), distance based methods such as the extensions of TOPSIS method (Baykasoğlu & Gölcük, 2015; Wang, Liu, Li, & Niu, 2016) and VIKOR method (Madjid, Reza, Francisco, & Elahe, 2016; Qin, Liu, & Pedrycz, 2015), and outranking methods such as PROMETHEE methods (Chen, 2014a; Miłosz & Krzysztof, 2016) and ELECTRE methods (Chen, 2014b; Corrente, Greco, & Słowiński, 2016). Although there are different principles in different types of multiple criteria decision methods for using individual performances of each alternative on each criterion to generate solutions, criterion weights are always taken into account in the process of generating solutions. The weight of a criterion generally reflects the impact of the performance of the criterion on the overall performance (Butler et al., 2001). The weights of all criteria can accomplish the weighted trade-off between the performances of alternative on each criterion, which means that different sets of criterion weights may usually result in different solutions to the same decision problem considered. As a result, determining criterion weights is a key step of multiple criteria decision methods.

To address the determination of criterion weights in MCDM, a large

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amount of research has been conducted in literature. When a decision maker is capable of providing subjective preferences for criterion weight assignment, many methods have been proposed to use the subjective preferences to determine criterion weights. Representative methods include point allocation method (Doyle, Green, & Bottomley, 1997; Roberts & Goodwin, 2002), direct rating method (Bottomley & Doyle, 2001; Roberts & Goodwin, 2002), eigenvector method (Saaty, 1977; Takeda, Cogger, & Yu, 1987), Delphi method (Hwang & Yoon, 1981), linear programming model (Horowitz & Zappe, 1995), and goal programming model (Shirland, Jesse, Thompson, & Iacovou, 2003). If there is flexibility in criterion weights in nature and a decision maker cannot provide reliable or credible subjective preferences for criterion weights, the performances of alternatives assessed on each criterion are used to objectively determine criterion weights. In the method of handling such a situation, criterion weights reflect the amount of information or discriminating power contained in criteria or the contribution of performances of alternatives on each criterion to some special goal, such as high solution reliability (Fu & Xu, 2016). Representative methods include entropy method (Chen & Li, 2010, 2011; Deng, Yeh, & Willis, 2000; He, Guo, Jin, & Ren, 2016), standard deviation (SD) method (Deng et al., 2000), correlation coefficient and standard deviation integrated (CCSD) method (Wang & Luo, 2010), criteria importance through intercriteria correlation (CRITIC) method (Diakoulaki, Mavrotas, & Papayannakis, 1995), deviation maximization method (Şahin & Liu, 2016; Wang, 1998), and multiple objective programming model (Choo & Wedley, 1985). In a compromised situation where a decision maker can only provide incomplete information about criterion weights, such information is generally considered as constraints to be incorporated into the methods of using performances of alternatives on each criterion to objectively determine criterion weights (Chin et al., 2015; Fan, Ma, & Zhang, 2002; Fu & Wang, 2015; Fu & Xu, 2016; Ma, Fan, & Huang, 1999; Pei, 2013; Rao, Patel, & Parnichkun, 2011; Wang & Parkan, 2006).

To address a real decision problem, using subjective preferences of a decision maker for criterion weight assignment is beneficial for generating criterion weights that are satisfactory to the decision maker. Different methods of assigning criterion weights by using subjective preferences, however, may elicit different criterion weights. There is no single method that can guarantee more accurate criterion weights than others (Barron & Barrett, 1996; Deng et al., 2000; Diakoulaki et al., 1995). This brings the decision maker into difficulty in choosing an appropriate method. When subjective preferences of the decision maker are fully or partially unavailable, the amount of information or discriminating power contained in criteria is generally used to objectively determine criterion weights. Although Fu and Xu (2016) proposed to objectively determine criterion weights by measuring and using the contribution of performances of alternatives on each criterion to high solution reliability, there are few similar studies of depending on some special goal to carry out criterion weight assignment objectively. In particular, under this condition, how to assign weights to criteria with a view to guaranteeing that the weights are fair to each criterion and to each alternative is rarely taken into account in existing studies.

In this paper, we investigate the assignment of weights to criteria to achieve two levels of fairness including the fairness among criteria and the fairness among alternatives as defined in Section 3.2. A fair framework in the context of the evidential reasoning (ER) approach which is a type of MAUF method (Fu, Yang, & Yang, 2015; Wang, Yang, Xu, & Chin, 2006; Yang, 2001; Yang, Wang, Xu, & Chin, 2006), is proposed to make solutions to MCDM problems by treating all possible criterion weights that are generated by following the two levels of fairness. In the fair framework, a decision maker is allowed to choose one from two strategies including the superior strategy and the inferior strategy. After the choice, the fairness among criteria and the fairness among alternatives are defined to achieve the objective in line with the selected strategy. By following the fairness among criteria defined, an optimization model is constructed to generate a set of fair criterion weights for each alternative to help the alternative attain the objective in line with the selected strategy to the maximum extent. Based on the set of fair criterion weights for each alternative, the other optimization model is constructed by following the fairness among alternatives to generate possible sets of fair criterion weights. To treat all possible sets of fair criterion weights, another optimization model is constructed to determine the minimum and maximum expected utilities of each alternative, in which the ER algorithm (Wang, Yang, & Xu, 2006) is used to combine individual assessments of alternatives on each criterion to generate the overall assessments of alternatives. A decision rule consistent with the preferences of the decision maker is then used to generate a solution to the decision problem considered based on the resulting expected utilities of each alternative.

The rest of the present paper is organized as follows. Section 2 presents the ER distributed modeling framework for MCDM problems. Section 3 introduces the fair framework. In Section 4, a supplier evaluation problem is investigated to demonstrate the applicability and validity of the fair framework, which is strengthened by comparing the fair framework with five existing methods. Section 5 provides a further analysis of the applicability of the fair framework by simulation experiments. Finally, this paper is concluded in Section 6.

2. ER distributed modeling framework for MCDM problems

Suppose a MCDM problem includes M alternatives denoted by a_l ($l = 1, \ldots, M$) and L criteria denoted by e_i ($i = 1, \ldots, L$). Relative weights of the L criteria are denoted by $w = (w_1, w_2, \ldots, w_L)$ such that $0 \le w_i \le 1$ and $\sum_{i=1}^{L} w_i = 1$.

Suppose $\Omega = \{H_1, H_2, \ldots, H_N\}$ denotes a set of grades and the utilities of grades $u(H_n)$ ($n = 1, \ldots, N$) satisfy the constraint $0 = u(H_1) < u(H_2) < \ldots < u(H_N) = 1$ in the ER context. The M alternatives are assessed at the L criteria by using H_n ($n = 1, \ldots, N$). Let $\beta_{n,i}(a_l)$ denote the belief degree assigned to grade H_n when a decision maker assesses alternative a_l on criterion e_i. Then the assessment can be profiled by a belief distribution $B(e_i(a_l)) = \{(H_n, \beta_{n,i}(a_l)),\ n = 1, \ldots, N; (\Omega, \beta_{\Omega,i}(a_l))\}$, where $\beta_{n,i}(a_l) \ge 0$, $\sum_{n=1}^{N} \beta_{n,i}(a_l) \le 1$, and $\beta_{\Omega,i}(a_l) = 1 - \sum_{n=1}^{N} \beta_{n,i}(a_l)$ represents the degree of global ignorance (Fu & Wang, 2015; Xu, 2012; Yang & Xu, 2013). If $\beta_{\Omega,i}(a_l) = 0$, the assessment is complete; otherwise, it is incomplete. When belief distribution of each alternative on each criterion is given, a belief decision matrix $S_{L\times M}$ is formed. Note that because the degree of global ignorance could be assigned to any grades, its impact needs to be analyzed in MCDM.

In the ER approach, the individual assessments $B(e_i(a_l))$ ($i = 1, \ldots, L$) together with criterion weights are combined to generate the overall assessment $B(y(a_l)) = \{(H_n, \beta_n(a_l)),\ n = 1, \ldots, N; (\Omega, \beta_\Omega(a_l))\}$, where $\beta_\Omega(a_l)$ represents the degree of aggregated global ignorance. Based on the overall assessment, the utilities of grades $u(H_n)$ ($n = 1, \ldots, N$) are used to produce the minimum and maximum expected utilities of alternative a_l, i.e., $u^-(a_l) = \sum_{n=2}^{N} \beta_n(a_l) \cdot u(H_n) + (\beta_1(a_l) + \beta_\Omega(a_l)) \cdot u(H_1)$ and $u^+(a_l) = \sum_{n=1}^{N-1} \beta_n(a_l) \cdot u(H_n) + (\beta_N(a_l) + \beta_\Omega(a_l)) \cdot u(H_N)$ with $0 \le u^-(a_l) \le 1$ and $0 \le u^+(a_l) \le 1$. The expected utilities are then used to compare alternatives with the help of a decision rule consistent with the preferences of the decision maker.

3. Fair framework

In MCDM, criterion weights play an important role in characterizing how the performance on each criterion affects the overall performance in comparison with other criteria, i.e., making trade-offs among all criteria (Butler et al., 1997). On the assumption that there is flexibility in criterion weight assignment, we present the fair framework for making solutions to MCDM problems in the ER context.

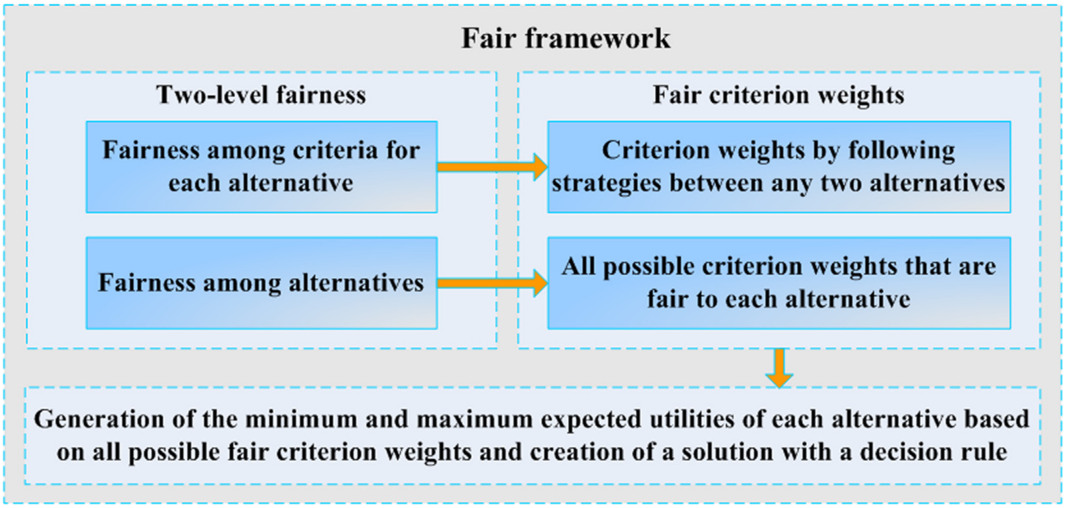

Fig. 1. Profile of the fair framework.

3.1. Illustration of the fair framework

To make clear the idea of the fair framework, we give its profile in Fig. 1. In the following we demonstrate the fair framework in accordance with Fig. 1.

Fig. 1 shows that there are two levels of fairness in the framework, which are the fairness among criteria and the fairness among alternatives. Two strategies are available in the framework, one of which is the superior strategy and the other is the inferior strategy. When one strategy is adopted by a decision maker, the integrated fair criterion weights of each alternative are generated by following the first level of fairness, which are then used to produce all possible sets of criterion weights fair to each alternative by following the second level of fairness. From the possible sets of criterion weights, the minimum and maximum expected utilities of each alternative are obtained and used to make solutions with the help of a decision rule consistent with the preferences of the decision maker.

3.2. Determination of possible fair criterion weights

In accordance with the profile of the fair framework shown in Fig. 1, we introduce how to determine possible sets of fair criterion weights when the superior or inferior strategy is adopted by a decision maker. As a common basis, the minimum and maximum expected utilities of each alternative on each criterion are firstly presented.

Similar to the fact that the aggregated assessment $B(y(a_l))$ can be combined with the utilities of grades $u(H_n)$ $(n = 1, \ldots, N)$ to generate the minimum and maximum expected utilities of alternative a_l, the minimum and maximum expected utilities of alternative a_l on criterion e_i can be calculated as

$$u^{-}(e_i(a_l)) = \sum_{n=2}^{N} \beta_{n,i}(a_l)\cdot u(H_n) + (\beta_{1,i}(a_l) + \beta_{\Omega,i}(a_l))\cdot u(H_1) \quad \text{and} \tag{1}$$

$$u^{+}(e_i(a_l)) = \sum_{n=1}^{N-1} \beta_{n,i}(a_l)\cdot u(H_n) + (\beta_{N,i}(a_l) + \beta_{\Omega,i}(a_l))\cdot u(H_N). \tag{2}$$

Here, $u^{-}(e_i(a_l))$ and $u^{+}(e_i(a_l))$ are also limited to [0, 1], similar to $u^{-}(a_l)$ and $u^{+}(a_l)$. Based on $[u^{-}(e_i(a_l)), u^{+}(e_i(a_l))]$, we firstly discuss the generation of possible sets of fair criterion weights by following the two levels of fairness on the assumption that the superior strategy is adopted by a decision maker.

Definition 1. Suppose that the minimum and maximum expected utilities of alternatives a_l and a_m on each criterion, $[u^{-}(e_i(a_l)), u^{+}(e_i(a_l))]$ and $[u^{-}(e_i(a_m)), u^{+}(e_i(a_m))]$ $(i = 1, \ldots, L)$ are obtained. To accomplish the objective to maximize the degree to which alternative a_l is superior to a_m, the fairness among criteria is defined as the fair contribution of the difference between the expected utilities of the two alternatives on each criterion to the objective.

Following the fairness among criteria shown in Definition 1, we calculate a set of fair superior criterion weights between alternatives a_l and a_m denoted by $\overline{w}_i(a_{lm})$ from $[u^{-}(e_i(a_l)), u^{+}(e_i(a_l))]$ and $[u^{-}(e_i(a_m)), u^{+}(e_i(a_m))]$, which is

$$\overline{w}_i(a_{lm}) = \frac{(u^{-}(e_i(a_l)) - u^{+}(e_i(a_m)) - (-1))/2}{\sum_{j=1}^{L} (u^{-}(e_j(a_l)) - u^{+}(e_j(a_m)) - (-1))/2}, \quad i = 1, \ldots, L. \tag{3}$$

Here, $(u^{-}(e_i(a_l)) - u^{+}(e_i(a_m)) - (-1))/2$ is a normalization of $u^{-}(e_i(a_l)) - u^{+}(e_i(a_m))$ because $0 \le u^{-}(e_i(a_l)) \le 1$ and $0 \le u^{+}(e_i(a_m)) \le 1$. With the consideration of generating $[u^{-}(e_i(a_l)), u^{+}(e_i(a_l))]$ and $[u^{-}(e_i(a_m)), u^{+}(e_i(a_m))]$, the obtainment of $\overline{w}_i(a_{lm})$ requires $O((N + 1)\cdot L)$ computations. Through using Eq. (3) for any other alternatives different from alternative a_l, $(M - 1)$ sets of fair superior criterion weights by following the fairness among criteria can be obtained. This means that alternative a_l is compared with the other $(M - 1)$ alternatives on each criterion and the number of comparisons between alternatives is $(M - 1)\cdot L$. When M alternatives are considered, the total number of comparisons between alternatives is $M\cdot(M - 1)\cdot L$.

Choosing the quantity $u^{-}(a_l) - \max_{m \neq l}\{u^{+}(a_m)\}$ as an indicator to judge to what extent alternative a_l is superior to all others, we construct the following optimization model to find an integrated set of fair superior criterion weights from the convex combinations of the $(M - 1)$ sets of fair superior criterion weights $\overline{w}_i(a_{lm})$ $(m \neq l)$.

$$\text{MAX} \quad \overline{F}(a_l) = u^{-}(a_l) - \max_{m \neq l}\{u^{+}(a_m)\} \quad (l = 1, \ldots, M) \tag{4}$$

$$\text{s.t.} \quad \overline{w}_i(a_l) = \sum_{m=1, m\neq l}^{M} \overline{\theta}_m^{l*}\cdot \overline{w}_i(a_{lm}), \quad i = 1, \ldots, L, \tag{5}$$

$$\overline{w}_i(a_{lm}) = \frac{(u^{-}(e_i(a_l)) - u^{+}(e_i(a_m)) - (-1))/2}{\sum_{j=1}^{L} (u^{-}(e_j(a_l)) - u^{+}(e_j(a_m)) - (-1))/2}, \tag{6}$$

$$0 \leqslant \overline{\theta}_m^{l*} \leqslant 1, \quad m \neq l, \tag{7}$$

$$\sum_{m=1, m\neq l}^{M} \overline{\theta}_m^{l*} = 1. \tag{8}$$

In this model, $\overline{\theta}_m^{l*}$ $(m \neq l)$ represents decision variables used to accomplish the convex combination of $\overline{w}_i(a_{lm})$. Note that the process of generating the aggregated assessments $B(y(a_l))$ $(l = 1, \ldots, M)$ from individual assessments $B(e_i(a_l))$ $(i = 1, \ldots, L)$ and $\overline{w}_i(a_l)$ by using the ER algorithm (Wang et al., 2006) is implicitly included in the model, which is intended for calculating $[u^-(a_l), u^+(a_l)]$ $(l = 1, \ldots, M)$ and then forming the objective of the model. Implementing the ER algorithm requires $O(L{\cdot}N + 2(N + 2))$ computations, as presented in (Wang et al., 2006), so the complexity of maximizing $\overline{F}(a_l)$ is $O(((N + 1){\cdot}L{\cdot}(M - 1) + (M - 1)){\cdot}L + (L{\cdot}N + 2(N + 2) + N){\cdot}M)$. As is shown in Eq. (5), $\overline{w}_i(a_l)$ is constructed from $\overline{w}_i(a_{lm})$ $(m \neq l)$. The calculation of $\overline{w}_i(a_{lm})$ $(i = 1, \ldots, L,\ m \neq l)$ requires $(M - 1){\cdot}L$ times of comparisons between alternatives. As a result, the number of comparisons between alternatives involved in the model is $(M - 1){\cdot}L$. For the optimization models associated with M alternatives, the total number of comparisons between alternatives is $M{\cdot}(M - 1){\cdot}L$.

From finding solutions to the model, the optimal $\overline{F}(a_l)$ denoted by $\overline{F}^*(a_l)$ may be generated with many sets of $\overline{\theta}_m^{l*}$. This indicates that the solutions to the model may not be unique. To cover all possible sets of optimal $\overline{\theta}_m^{l*}$, the post-optimal solution space of the model is defined as $\overline{F}(a_l) \geq \overline{F}^*(a_l) - \varepsilon$ where ε is a very small positive number such as 0.001 to compensate computational error incurred in generating $\overline{F}^*(a_l)$.

Definition 2. Suppose that the integrated set of fair superior criterion weights for alternative a_l $(l = 1, \ldots, M)$ denoted by $\overline{w}(a_l) = (\overline{w}_1(a_l), \ldots, \overline{w}_i(a_l), \ldots, \overline{w}_L(a_l))$ is obtained from solving the optimization model shown in Eqs. (4)–(8). Assume that the convex combination of $\overline{w}(a_l)$ is denoted by $\overline{w} = (\overline{w}_1, \ldots, \overline{w}_i, \ldots, \overline{w}_L) = (\sum_{l=1}^{M} \overline{\theta}_l^*{\cdot}\overline{w}_1(a_l), \ldots, \sum_{l=1}^{M} \overline{\theta}_l^*{\cdot}\overline{w}_i(a_l), \ldots, \sum_{l=1}^{M} \overline{\theta}_l^*{\cdot}\overline{w}_L(a_l))$. Then, the fairness among alternatives is defined as the fair contribution of $\overline{w}(a_l)$ to $\overline{w}$.

By following the fairness among alternatives shown in Definition 2 to calculate $\overline{w}_i$ from $\overline{w}_i(a_l)$ $(l = 1, \ldots, M)$, each alternative is compared with the other $(M - 1)$ alternatives on each criterion. As a result, the number of comparisons between alternatives is $M{\cdot}(M - 1){\cdot}L$. In accordance with the fairness among alternatives, the distance between $\overline{w}(a_l)$ and $\overline{w}$ is required to be nearly equal to each other. For this purpose, the Euclidean distance between $\overline{w}(a_l)$ and $\overline{w}$ is calculated as

$$d(\overline{w}, \overline{w}(a_l)) = \sqrt{\sum_{i=1}^{L} (\overline{w}_i - \overline{w}_i(a_l))^2}, \quad l = 1, \ldots, M. \tag{9}$$

Then, following the principle of maximum entropy we construct an optimization model to carry out the fairness among alternatives shown in Definition 2 within the intersection of post-optimal solution spaces $\overline{F}(a_l) \geq \overline{F}^*(a_l) - \varepsilon$ $(l = 1, \ldots, M)$.

$$\text{MAX} \quad \overline{G} = -\sum_{l=1}^{M} \frac{d(\overline{w}, \overline{w}(a_l))}{\sum_{m=1}^{M} d(\overline{w}, \overline{w}(a_m))} \cdot \ln \frac{d(\overline{w}, \overline{w}(a_l))}{\sum_{m=1}^{M} d(\overline{w}, \overline{w}(a_m))} \tag{10}$$

$$\text{s.t.} \quad d(\overline{w}, \overline{w}(a_l)) = \sqrt{\sum_{i=1}^{L} (\overline{w}_i - \overline{w}_i(a_l))^2}, \quad l = 1, \ldots, M, \tag{11}$$

$$0 \leqslant \overline{\theta}_m^{l*} \leqslant 1, \quad m \neq l, \tag{12}$$

$$\sum_{m=1, m\neq l}^{M} \overline{\theta}_m^{l*} = 1, \tag{13}$$

$$0 \leqslant \overline{\theta}_l^* \leqslant 1, \quad l = 1, \ldots, M, \tag{14}$$

$$\sum_{l=1}^{M} \overline{\theta}_l^* = 1, \tag{15}$$

$$\overline{F}(a_l) \geqslant \overline{F}^*(a_l) - \varepsilon, \quad l = 1, \ldots, M. \tag{16}$$

In the above model, $\overline{\theta}_l^*$ is also decision variable in addition to $\overline{\theta}_m^{l*}$. Note that Eqs. (4)–(6) are not explicitly included in the model to avoid repetition. Based on the complexity of implementing the ER algorithm and the complexity of maximizing $\overline{F}(a_l)$, the complexity of maximizing $\overline{G}$ is $O((((N + 1){\cdot}L{\cdot}(M - 1) + (M - 1)){\cdot}L + (L{\cdot}N + 2(N + 2) + N){\cdot}M){\cdot}M + L{\cdot}M + 2{\cdot}M)$. Both the calculation of $d(\overline{w}, \overline{w}(a_l))$ $(l = 1, \ldots, M)$ and the calculation of $\overline{F}(a_l)$ $(l = 1, \ldots, M)$ depend on $\overline{w}_i(a_l)$. This means that each alternative is compared with the other $(M - 1)$ alternatives on each criterion and the number of comparisons between alternatives is $M{\cdot}(M - 1){\cdot}L$.

Similar to the post-optimal solution space $\overline{F}(a_l) \geq \overline{F}^*(a_l) - \varepsilon$, to cover all possible sets of optimal $\overline{\theta}_m^{l*}$ and $\overline{\theta}_l^*$, the post-optimal solution space $\overline{G} \geq \overline{G}^* - \varepsilon$ is defined to compensate computational error incurred in generating $\overline{G}^*$. The set of optimal criterion weights denoted by $\hat{\overline{w}} = (\hat{\overline{w}}_1, \ldots, \hat{\overline{w}}_i, \ldots, \hat{\overline{w}}_L)$ corresponding to the optimal $\overline{\theta}_m^{l*}$ and $\overline{\theta}_l^*$ is called the set of fair superior criterion weights achieving both the fairness among criteria shown in Definition 1 and the fairness among alternatives shown in Definition 2. All possible sets of fair superior criterion weights are covered by the post-optimal solution spaces $\overline{F}(a_l) \geq \overline{F}^*(a_l) - \varepsilon$ and $\overline{G} \geq \overline{G}^* - \varepsilon$.

Different from what has been discussed above, in the following we focus on the generation of possible sets of fair criterion weights on the assumption that the inferior strategy is adopted by a decision maker.

Definition 3. Suppose that the minimum and maximum expected utilities of alternatives a_l and a_m on each criterion, $[u^-(e_i(a_l)), u^+(e_i(a_l))]$ and $[u^-(e_i(a_m)), u^+(e_i(a_m))]$ $(i = 1, \ldots, L)$ are obtained. To accomplish the objective to maximize the degree to which alternative a_l is inferior to a_m, the fairness among criteria is defined as the fair contribution of the difference between the expected utilities of the two alternatives on each criterion to the objective.

Following the fairness among criteria shown in Definition 3, we calculate a set of fair inferior criterion weights between alternatives a_l and a_m, $\underline{w}_i(a_{lm})$ as

$$\underline{w}_i(a_{lm}) = \frac{(u^-(e_i(a_m)) - u^+(e_i(a_l)) - (-1))/2}{\sum_{j=1}^{L} (u^-(e_j(a_m)) - u^+(e_j(a_l)) - (-1))/2}, \quad i = 1, \ldots, L. \tag{17}$$

The obtainment of $\underline{w}_i(a_{lm})$ also requires $O((N + 1){\cdot}L)$ computations. The number of comparisons between alternatives is still $(M - 1){\cdot}L$. When M alternatives are considered, the total number of comparisons between alternatives is $M{\cdot}(M - 1){\cdot}L$.

Similar to the situation where the superior strategy is adopted, we construct the following optimization model to find an integrated set of fair inferior criterion weights from the convex combinations of the $(M - 1)$ sets of fair inferior criterion weights $\underline{w}_i(a_{lm})$ $(m \neq l)$ calculated by using Eq. (17).

$$\text{MAX} \quad \underline{F}(a_l) = \min_{m\neq l}\{u^-(a_m)\} - u^+(a_l) \quad (l = 1, \ldots, M) \tag{18}$$

$$\text{s.t.} \quad \underline{w}_i(a_l) = \sum_{m=1, m\neq l}^{M} \underline{\theta}_m^{l*}{\cdot}\underline{w}_i(a_{lm}), \quad i = 1, \ldots, L, \tag{19}$$

$$\underline{w}_i(a_{lm}) = \frac{(u^-(e_i(a_m)) - u^+(e_i(a_l)) - (-1))/2}{\sum_{j=1}^{L} (u^-(e_j(a_m)) - u^+(e_j(a_l)) - (-1))/2}, \tag{20}$$

$$0 \leqslant \underline{\theta}_m^{l*} \leqslant 1, \quad m \neq l, \tag{21}$$

$$\sum_{m=1, m\neq l}^{M} \underline{\theta}_m^{l*} = 1. \tag{22}$$

The complexity of maximizing $\underline{F}(a_l)$ is also $O(((N + 1){\cdot}L{\cdot}(M - 1) + (M - 1)){\cdot}L + (L{\cdot}N + 2(N + 2) + N){\cdot}M)$. Similar to the optimization model shown in Eqs. (4)–(8), the number of comparisons between alternatives involved in this model is $(M - 1){\cdot}L$. The total number of comparisons between alternatives involved in the models associated with M alternatives is $M{\cdot}(M - 1){\cdot}L$.

Definition 4. Suppose that the integrated set of fair inferior criterion weights for alternative a_l $(l = 1, \ldots, M)$ denoted by $\underline{w}(a_l) = (\underline{w}_1(a_l), \ldots, \underline{w}_i(a_l), \ldots, \underline{w}_L(a_l))$ is obtained from solving the optimization model in Eqs. (18)–(22). Assume that the convex combination of $\underline{w}(a_l)$ is denoted by

$\underline{w} = (\underline{w}_1, \ldots, \underline{w}_i, \ldots, \underline{w}_L) = (\sum_{l=1}^{M} \underline{\theta}_l^* \cdot \underline{w}_1(a_l), \ldots, \sum_{l=1}^{M} \underline{\theta}_l^* \cdot \underline{w}_i(a_l), \ldots, \sum_{l=1}^{M} \underline{\theta}_l^* \cdot \underline{w}_L(a_l))$. Then, the fairness among alternatives is defined as the fair contribution of $\underline{w}(a_l)$ to $\underline{w}$.

Similar to Definition 2, each alternative is compared with the other $(M-1)$ alternatives on each criterion and the number of comparisons between alternatives is $M\cdot(M-1)\cdot L$. Within the intersection of post-optimal solution spaces $\underline{F}(a_l) \geq \underline{F}^*(a_l) - \varepsilon$ $(l = 1, \ldots, M)$, we construct the following optimization model to achieve the fairness among alternatives shown in Definition 4.

$$\text{MAX} \quad \underline{G} = -\sum_{l=1}^{M} \frac{d(\underline{w}, \underline{w}(a_l))}{\sum_{m=1}^{M} d(\underline{w}, \underline{w}(a_m))} \cdot \ln \frac{d(\underline{w}, \underline{w}(a_l))}{\sum_{m=1}^{M} d(\underline{w}, \underline{w}(a_m))} \tag{23}$$

$$\text{s.t.} \quad d(\underline{w}, \underline{w}(a_l)) = \sqrt{\sum_{i=1}^{L} (\underline{w}_i - \underline{w}_i(a_l))^2}, \quad l = 1, \ldots, M, \tag{24}$$

$$0 \leqslant \underline{\theta}_m^{l*} \leqslant 1, \quad m \neq l, \tag{25}$$

$$\sum_{m=1, m\neq l}^{M} \underline{\theta}_m^{l*} = 1, \tag{26}$$

$$0 \leqslant \underline{\theta}_l^* \leqslant 1, \quad l = 1, \ldots, M, \tag{27}$$

$$\sum_{l=1}^{M} \underline{\theta}_l^* = 1, \tag{28}$$

$$\underline{F}(a_l) \geqslant \underline{F}^*(a_l) - \varepsilon, \quad l = 1, \ldots, M. \tag{29}$$

The complexity of maximizing $\underline{G}$ is also $O((((N+1)\cdot L\cdot(M-1) + (M-1))\cdot L + (L\cdot N + 2(N+2) + N)\cdot M)\cdot M + L\cdot M + 2\cdot M)$. Similar to the optimization model shown in Eqs. (10)–(16), each alternative is compared with the other $(M-1)$ alternatives on each criterion in this model and the number of comparisons between alternatives is $M\cdot(M-1)\cdot L$. All possible sets of fair inferior criterion weights denoted by $\widehat{\underline{w}} = (\widehat{\underline{w}}_1, \ldots, \widehat{\underline{w}}_i, \ldots, \widehat{\underline{w}}_L)$ are covered by the post-optimal solution spaces $\underline{F}(a_l) \geq \underline{F}^*(a_l) - \varepsilon$ and $\underline{G} \geq \underline{G}^* - \varepsilon$. Next, we discuss how to generate solutions to MCDM problems.

3.3. Generation of solution

With the consideration of all possible sets of fair superior or inferior criterion weights, the minimum and maximum expected utilities of each alternative are determined and used to generate solutions with the help of a decision rule consistent with the preferences of a decision maker.

On the assumption that the superior strategy is adopted by a decision maker, within the post-optimal solution spaces $\overline{F}(a_l) \geq \overline{F}^*(a_l) - \varepsilon$ and $\overline{G} \geq \overline{G}^* - \varepsilon$, we construct the following optimization model to determine the minimum and maximum expected utilities of each alternative.

$$\text{MIN} \quad \overline{u}^-(a_l) \tag{30}$$

$$\text{s.t.} \quad 0 \leqslant \overline{\theta}_m^{l*} \leqslant 1, \quad m \neq l, \tag{31}$$

$$\sum_{m=1, m\neq l}^{M} \overline{\theta}_m^{l*} = 1, \tag{32}$$

$$0 \leqslant \overline{\theta}_l^* \leqslant 1, \quad l = 1, \ldots, M, \tag{33}$$

$$\sum_{l=1}^{M} \overline{\theta}_l^* = 1, \tag{34}$$

$$\overline{F}(a_l) \geqslant \overline{F}^*(a_l) - \varepsilon, \quad l = 1, \ldots, M, \tag{35}$$

$$\overline{G} \geqslant \overline{G}^* - \varepsilon. \tag{36}$$

Solving this model generates the optimal $\overline{u}^-(a_l)$. When the objective of this model is changed to "MAX $\overline{u}^+(a_l)$", the optimal $\overline{u}^+(a_l)$ can be obtained. Due to the similarity between the model shown in Eqs. (30)–(36) and the model shown in Eqs. (10)–(16) and the fact that $\overline{u}^-(a_l)$ is calculated in the latter, the complexity of minimizing $\overline{u}^-(a_l)$ is the same as the complexity of maximizing $\overline{G}$, which is $O((((N+1)\cdot L\cdot(M-1) + (M-1))\cdot L + (L\cdot N + 2(N+2) + N)\cdot M)\cdot M + L\cdot M + 2\cdot M)$. Because the calculation of $\overline{F}(a_l)$ $(l = 1, \ldots, M)$ and the calculation of $\overline{G}$ depend on $\overline{w}_i(a_l)$, the number of comparisons between alternatives in this model is still $M\cdot(M-1)\cdot L$. If the inferior strategy is adopted by the decision maker, the optimal $[\underline{u}^-(a_l), \underline{u}^+(a_l)]$ $(l = 1, \ldots, M)$ can be similarly obtained within the post-optimal solution spaces $\underline{F}(a_l) \geq \underline{F}^*(a_l) - \varepsilon$ and $\underline{G} \geq \underline{G}^* - \varepsilon$. The relevant optimization model is omitted to avoid repetition.

Until now, three main steps are involved in the process of generating the optimal $[\overline{u}^-(a_l), \overline{u}^+(a_l)]$ from a belief decision matrix when the superior strategy is adopted. The first step is to solve the optimization model shown in Eqs. (4)–(8) to obtain the optimal $\overline{F}(a_l)$ $(l = 1, \ldots, M)$, the second is to solve the optimization model shown in Eqs. (10)–(16) to obtain the optimal $\overline{G}$, and the third is to solve the optimization model shown in Eqs. (30)–(36) to obtain the optimal $[\overline{u}^-(a_l), \overline{u}^+(a_l)]$ $(l = 1, \ldots, M)$. There are $M\cdot(M-1)\cdot L$ times of comparisons between alternatives in each step. As a whole, the number of comparisons between alternatives in the process of generating the optimal $[\overline{u}^-(a_l), \overline{u}^+(a_l)]$ is $3M\cdot(M-1)\cdot L$. Similarly, there are $3M\cdot(M-1)\cdot L$ times of comparisons between alternatives in the process of generating the optimal $[\underline{u}^-(a_l), \underline{u}^+(a_l)]$.

Based on the optimal $[\overline{u}^-(a_l), \overline{u}^+(a_l)]$ or $[\underline{u}^-(a_l), \underline{u}^+(a_l)]$, a decision rule consistent with the preferences of the decision maker can be used to generate solutions. For example, if the minimax regret rule (Wang et al., 2006) is preferred by the decision maker, $[\underline{u}^-(a_l), \underline{u}^+(a_l)]$ will be used to generate solutions. This is because there is great similarity between the comparison among alternatives in their worst scenarios by using the minimax regret rule and the process of generating the set of fair inferior criterion weights. In this situation, the maximum loss of $[\underline{u}^-(a_l), \underline{u}^+(a_l)]$ is calculated first, and then the alternative with smaller maximum loss will be preferred. On the other hand, maximin decision rule, maximax decision rule, and Hurwicz rule may also be used to rank alternatives. What decision rule should be applied depends on the decision maker's preferences and behaviors.

As maximin and maximax decision rules are two special cases of Hurwicz rule, we discuss how to use Hurwicz rule to generate solutions given $[\overline{u}^-(a_l), \overline{u}^+(a_l)]$ or $[\underline{u}^-(a_l), \underline{u}^+(a_l)]$. Optimism degree γ is needed for the application of Hurwicz rule. The decision maker is considered optimistic when $0.5 \leq \gamma \leq 1$ and pessimistic when $0 \leq \gamma \leq 0.5$. It is not difficult to find the correlation between the optimism of the decision maker and the superior strategy and the correlation between the pessimism of the decision maker and the inferior strategy. If such correlation is accepted, $[\underline{u}^-(a_l), \underline{u}^+(a_l)]$ will be used together with γ to compare alternatives when $0 \leq \gamma \leq 0.5$ and $[\overline{u}^-(a_l), \overline{u}^+(a_l)]$ will be used together with γ to compare alternatives when $0.5 \leq \gamma \leq 1$. Although this is feasible in theory, it is not easy in practice because it may be a burden on the decision maker to provide a precise optimism degree. To handle this situation, when the pessimism is preferred by the decision maker for the decision problem considered, $[\underline{u}^-(a_l), \underline{u}^+(a_l)]$ together with all values of γ limited to $[0, 0.5]$ will be used to compare alternatives; while $[\overline{u}^-(a_l), \overline{u}^+(a_l)]$ together with all values of γ limited to $[0.5, 1]$ will be used to compare alternatives when the optimism is preferred by the decision maker. In detail, suppose that

$$\begin{aligned} E([\underline{u}^-(a_l), \underline{u}^+(a_l)]) &= \int_0^{0.5} \gamma\cdot\underline{u}^+(a_l) + (1-\gamma)\cdot\underline{u}^-(a_l)d\gamma \\ &= \frac{\gamma^2}{2}\cdot\underline{u}^+(a_l) + \left(\gamma - \frac{\gamma^2}{2}\right)\cdot\underline{u}^-(a_l)\Big|_0^{0.5} \\ &= 0.125\cdot\underline{u}^+(a_l) + 0.375\cdot\underline{u}^-(a_l) \quad \text{and} \end{aligned} \tag{37}$$

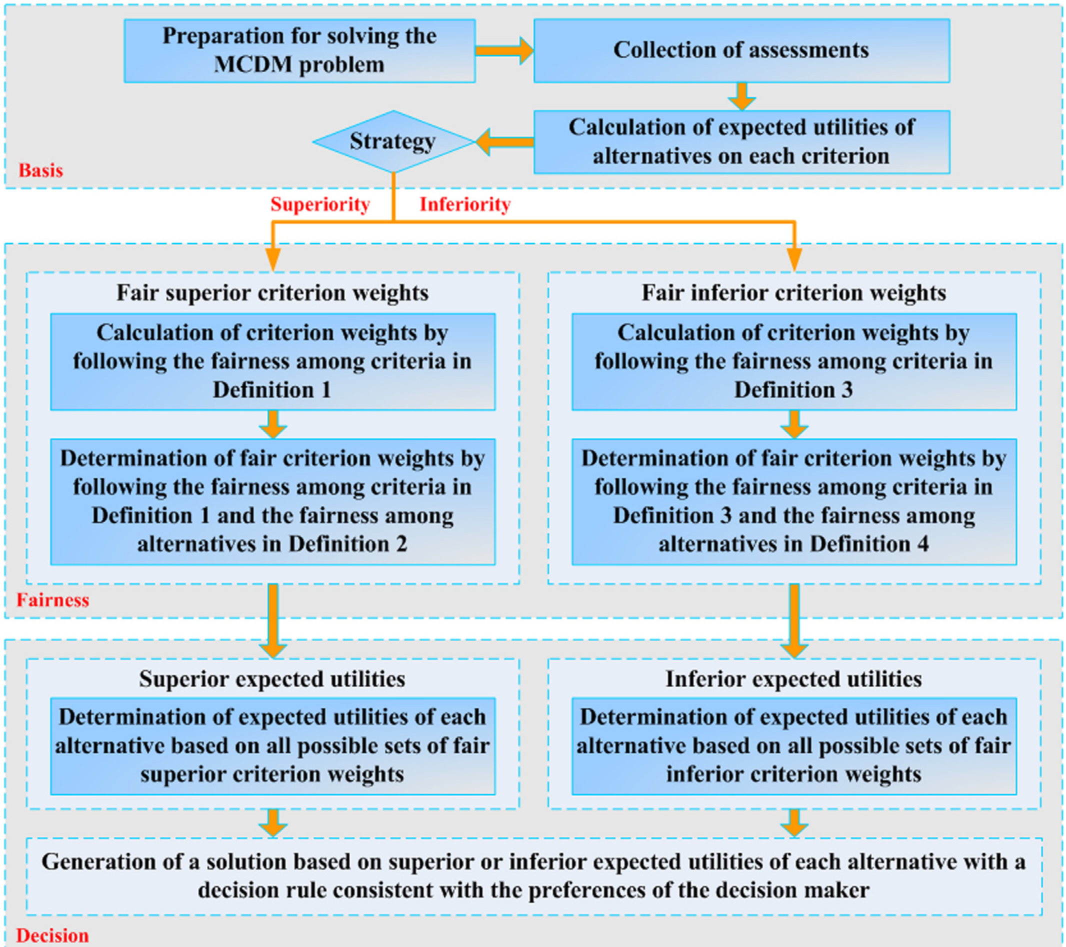

Fig. 2. Process of the fair framework.

$$E([\bar{u}^{-}(a_l), \bar{u}^{+}(a_l)]) = \int_{0.5}^{1} \gamma \cdot \bar{u}^{+}(a_l) + (1-\gamma)\cdot \bar{u}^{-}(a_l) d\gamma$$
$$= \frac{\gamma^2}{2}\cdot \bar{u}^{+}(a_l) + \left(\gamma - \frac{\gamma^2}{2}\right)\cdot \bar{u}^{-}(a_l) \Big|_{0.5}^{1}$$
$$= 0.375\cdot \bar{u}^{+}(a_l) + 0.125\cdot \bar{u}^{-}(a_l) \quad (38)$$

represent the integrated utilities of alternative a_l when the inferior and superior strategies are adopted, respectively. Then, $E([\underline{u}^{-}(a_l), \underline{u}^{+}(a_l)])$ is used to compare alternatives when the decision maker is pessimistic about the decision problem under consideration, while $E([\bar{u}^{-}(a_l), \bar{u}^{+}(a_l)])$ is used to compare alternatives when the decision maker is optimistic about the decision problem.

3.4. Process of the fair framework

Considering what has been discussed in Sections 3.2 and 3.3, we develop the process of the fair framework, which is shown in Fig. 2.

Fig. 2 shows that there are three hierarchies in the fair framework, which are basis, fairness, and decision hierarchies. In the basis hierarchy, individual assessments of each alternative are collected from a decision maker and the expected utilities of each alternative on each criterion are calculated as the foundations of the fair framework. More importantly, the superior or inferior strategy is selected by the decision maker to decide what solution will be generated. In the fairness hierarchy, all possible sets of fair criterion weights following the selected strategy are produced. Finally, the possible sets of fair criterion weights are used to generate a solution in the decision hierarchy.

Evaluating alternatives on each criterion is very important for generating a satisfactory solution to a MCDM problem. If the individual assessments of alternatives provided cannot characterize the real and complete preferences of the decision maker, the decision result generated by using the assessments will not be satisfactory to the decision maker. Based on the provided individual assessments of alternatives, their minimum and maximum expected utilities on each criterion are calculated by using Eqs. (1) and (2).

In general, the purpose of MCDM is to generate what a decision maker anticipates. For this purpose, the interaction between the decision maker and a MCDM method or model is important and necessary. If such interaction is not allowed or not implemented in an effective way, the decision result generated may not be accepted by the decision maker. In the fair framework, the superior strategy and the inferior strategy are available to the decision maker. It is the first step to produce all possible sets of fair criterion weights and further generate a solution. When the decision maker selects the superior or inferior strategy in accordance with his or her own preferences for a decision problem, the obtained possible sets of fair criterion weights and the generated solution can be considered to be consistent with the preferences of the decision maker. Along with the change in the preferences of the decision maker as to the problem, a different strategy may be selected by the decision maker, which may result in a different solution to the problem. This will be verified by the supplier evaluation problem in Section 4. Selecting the superior strategy means that the decision maker is willing to take into account the best scenarios of each alternative and thus pay more attention to gain. In other words, the decision maker prefers to an alternative that can create more profits although the failings of the alternative may cause a large amount of loss. On the contrary, the decision maker prefers to take into account the worst scenarios of each alternative and thus pay more attention to loss when he or she selects the inferior strategy. These analyses indicate that the choice of the superior or inferior strategy provides the thorough interaction between the fair framework and a decision maker and guarantees that the solution generated is consistent with the preferences of the decision maker.

When the superior strategy is adopted by the decision maker, the two levels of fairness shown in Definitions 1 and 2 are followed to obtain possible sets of fair criterion weights. On the contrary, the decision maker follows the two levels of fairness shown in Definitions 3 and 4 to obtain possible sets of fair criterion weights. Definitions 1 and 3 guarantee that each criterion is fairly treated to contribute to the objective determined by the strategy taken in. It is assured from Definitions 2 and 4 that each alternative is fairly treated to contribute to the objective determined by the selected strategy.

Within the feasible region formed by all possible sets of fair criterion weights following the selected strategy, the optimal expected utilities of each alternative are obtained and used to rank alternatives. Because the expected utilities are intervals instead of precise values, the decision maker is encouraged to select a decision rule in accordance with his or her preferences to compare alternatives by using their expected utilities. The choice of decision rules provides the interaction between the fair framework and the decision maker and contributes to the generation of a solution consistent with the preferences of the decision maker. In general, the selected decision rule or the specified parameters associated with the rule should be consistent with the selected strategy, as demonstrated in Section 3.3. Or else, there is incompatibility between the choices of the decision maker in different hierarchies of the fair framework and it cannot be guaranteed that the decision made in this situation is what the decision maker anticipates. In addition, when Hurwicz rule is selected by the decision maker, using Eq. (37) or Eq. (38) to compare alternatives can relieve the burden on the decision maker to specify a precise optimism degree.

As a whole, the proposed fair framework shown in Fig. 2 supports the evaluation of each alternative in accordance with the real and complete preferences of a decision maker, provides the interaction between the framework and the decision maker, and examines the compatibility between the choices of the decision maker in different hierarchies of the framework. This facilitates generating what the decision maker anticipates.

The above analyses show that the interaction between the decision maker and the fair framework is allowed. This does not, however, indicate that the fair framework can be considered as an interactive MCDM method completely. The similarity between the fair framework and an interactive MCDM method is that the decision maker is encouraged to provide preferences for parameters and alternatives when facing a decision problem. It is preferred that the minimum number of preferences are required from the decision maker to relieve his or her burden (Almeida, Almeida, Costa, & Almeida-Filho, 2016; Özpeynirci, Özpeynirci, & Kaya, 2017), which is another common point of the fair framework and an interactive MCDM method. More importantly, it is common for the fair framework and an interactive MCDM method to generate what the decision maker anticipates. Although so, there are some differences between the fair framework and an interactive MCDM method. Interactive MCDM methods usually depend on iterative interactions to achieve the preferences of the decision maker for the parameters (Almeida et al., 2016) and the pairwise comparisons between alternatives (Özpeynirci et al., 2017) in the process of MCDM. The iterative process ends when the decision maker cannot give additional preferences (Almeida et al., 2016). Differently, in the fair framework the decision maker directly evaluates each alternative on each criterion, selects the superior or inferior strategy, chooses a decision rule, and specifies possible parameters associated with the decision rule. Meanwhile, the decision maker is required to guarantee the compatibility between the selected strategy and the chosen decision rule with the specified parameters. An iterative process is not required for these in the fair framework.

Note that although the proposed fair framework is based on the ER approach, it provides an exploration of determining criterion weights in MCDM by considering fair contributions of criteria and alternatives to the solution generated. Such an idea can be applied in other situations where individual assessments in MCDM are profiled by other expressions, such as interval numbers (Wu, Xu, Xu, & Chen, 2016), intuitionistic fuzzy numbers (Wang & Chen, 2017), hesitant fuzzy linguistic numbers (Zhang, Ju, & Liu, 2017), and type-2 fuzzy numbers (Chen, 2014a; Qin et al., 2015). The key challenge of applying the fair framework in these situations is how to calculate the difference between individual assessments profiled by the above expressions and how to combine individual assessments to generate the collective assessment.

3.5. Comparison with the fairness in other methods

The generation of weights fair to support all alternatives is an important issue in MCDM. Meanwhile, it is also a hot topic in other methods, such as in data envelopment analysis (DEA) method (Banker, Charnes, & Cooper, 1984; Charnes, Cooper, & Rhodes, 1978).

Many extensions of DEA focus on finding the weights of inputs and outputs (or indicators) in favor of all decision-making units (DMUs). To guarantee the best attainable efficiency of each DMU simultaneously, Kao and Hung (2005) constructed several optimization models to minimize the total difference between the best attainable efficiency of each DMU and its efficiency derived from a set of common weights. This idea was extended in the situation where the efficiency of each DMU in multiple periods needs to be evaluated (Kao, 2010). Given a fixed input, Avellar, Milioni, and Rabello (2007) proposed a DEA model to distribute the input to all DMUs by satisfying the requirement that all DMUs are efficient. On the condition that all DMUs are efficient, a set of common weights associated with inputs and outputs is derived from the optimization model constructed to maximize the average efficiency of all DMUs. With the aim of making the efficiency of each DMU close to its ideal efficiency in a fair way, Cook and Zhu (2007) proposed a method for determining a set of common weights of inputs and outputs that minimizes the penalty imposed on the most disadvantaged DMU. To compare efficient DMUs, Liu and Peng (2008) proposed a method for determining a set of common weights that is the most favorable for determining the absolute efficiency of efficient DMUs. With a view to making a fair comparison among DMUs, Hatefi and Torabi (2010) proposed a two-step method. The first step is to construct an optimization model for each DMU to generate a set of best weights with respect to indicators that maximizes its efficiency, and the second is to construct another optimization model to generate a set of common weights with respect to indicators that maximizes the efficiency of each DMU simultaneously. To produce the smallest total difference between the relative performance of the DMU and that of a virtual DMU composed of the mean of all DMUs, Yang et al. (2017) constructed an optimization model to determine a set of common weights associated with the indicators of DMUs.

The above analyses concerning the fairness in relevant DEA methods (or models) show that a common objective of all DMUs is designed to be reached to determine a set of common weights associated with inputs and outputs on the condition that each DMU is fairly treated to be close to its ideal efficiency. Different DMUs are not directly compared in pairs. This is different from the proposed fair framework. In the fair framework, pairwise comparison between alternatives provides a basis for the obtainment of a set of fair criterion weights when the superior strategy or the inferior strategy is chosen. Two levels of fairness are involved in the fair framework, which are the fairness among criteria and the fairness among alternatives. Pairwise comparison between alternatives is involved in the two levels of fairness. Meanwhile, the fairness among criteria can effectively avoid extreme weights that are zero-valued or very close to 0. However, extreme weights are usually not avoided by the extensions of DEA focusing on fairness. On the other hand, there is no common objective for all alternatives in the fair framework. In fact, except specific requirements, in MCDM all alternatives are usually not associated with a common objective that is expected to be reached.

4. Illustrative example

In this section, a supplier evaluation problem is examined by the fair framework to demonstrate the determination of fair criterion weights and the process of finding solutions to MCDM problems. In the problem, the decision maker from an enterprise of producing railway rolling stock equipment located in Changzhou is to evaluate five suppliers in order to find the most appropriate three ones to purchase products such as transmission systems and their parts, and metal rubber ring. To facilitate analyzing the supplier evaluation problem, a solution system is developed in the MATLAB environment.

4.1. Description of the supplier evaluation problem

The rapid development of society and economy in China creates striking opportunities to prompt the leap-forward development of transportation. As one of the most important transportation means, high-speed train makes people's trip outstandingly convenient. Under the conditions, producing railway rolling stock equipment has gradually become an important national strategy in the field of manufacturing propelled by "Made in China 2025 Strategy" and the relevant industry attracted much attention. Similar to other manufacturing industries, evaluating suppliers is an important task for the industry of producing railway rolling stock equipment. In particular, it is also a challenge that enterprises have to face due to the strict requirements of supplier performance in such an industry.

An enterprise located in Changzhou of Jiangsu Province of China that mainly produces key parts of high-speed train is a representative one facing the challenge of evaluating suppliers in accordance with strict requirements. Five qualified suppliers are identified from those cooperating with the enterprise. Three suppliers are located in Changzhou, Zhuzhou, and Shanghai, and the remaining two in Guangzhou. They provide bearings, transmission systems and their parts, metal rubber ring, general rubber sealing ring, and scrap steel for the enterprise. To satisfy the requirements of production capacity of the enterprise in current situation with the consideration of production capacity of the five suppliers, the board of directors of the enterprise intends to select the most appropriate three suppliers through evaluating the five ones. The manager of the purchasing department acts as the decision maker to take charge of the selection process. Six factors (or criteria) are identified to evaluate the five suppliers, which include quality management system, physical quality, after-sales service, price, delivery performance, and environmental security. Five experts from the departments of production, quality management, research and development, sales, and finance are invited to help the decision maker to evaluate the five suppliers. The assessment of each supplier on any of the criteria is assumed not to depend on the assessments on other criteria, so the assessments on criteria can be considered independent of each other.

Suppose that the five suppliers are represented by S_l ($l = 1, \ldots, 5$), and the six criteria by e_i ($i = 1, \ldots, 6$). The five suppliers are assessed on each criterion using the following set of grades: *Poor* (*P*), *Average* (*A*), *Good* (*G*), *VeryGood* (*V*), and *Excellent* (*E*), i.e., $\Omega = \{H_n, n = 1, \ldots, 5\} = \{Poor, Average, Good, VeryGood, Excellent\} = \{P, A, G, V, E\}$.

The utility of each grade $u(H_n)$ ($n = 1, \ldots, 5$) is set to be $(0, 0.25, 0.5, 0.75, 1)$ by using a probability assignment approach (Winston, 2011). Due to the strict requirements of supplier performance and the outstanding importance of evaluating suppliers, the decision maker is not willing to assign weights to criteria by using subjective methods, but to determine criterion weights by giving fair treatment to each criterion and further to each alternative. As a result, the fair framework is applied to determine fair criterion weights and generate a solution to the supplier evaluation problem. The parameter ε to help form the post-optimal solution space of variables $\overline{\theta}_m^{l*}$ and $\overline{\theta}_l^{*}$ (or $\underline{\theta}_m^{l*}$ and $\underline{\theta}_l^{*}$) is specified as 0.001 to compensate the computational error.

4.2. Determination of fair criterion weights

With the assistance of the five experts, the assessments of each of the five suppliers are generated. As the decision maker is from the purchasing department, he is very familiar with the criteria of price and delivery performance, familiar with the criterion of after-sales service, and not familiar with the criteria of quality management system, physical quality, and environmental security. When taking into account different types of criteria, the decision maker adopts different ways to provide the assessments of the five suppliers. Suppose that $B^k(e_i(a_l))$ ($k = 1, \ldots, 5$) represents the assessments of the five experts denoted by t_k ($k = 1, \ldots, 5$). Facing the unfamiliar criteria, the decision maker decides to directly accept the average of the assessments of the five experts, i.e., $B^k(e_i(a_l))/5$ as the assessment $B(e_i(a_l))$. While facing the familiar criteria, the decision maker intends to give his assessment denoted by $B^D(e_i(a_l))$ and then use the weighted combination of $B^k(e_i(a_l))/5$ with $B^D(e_i(a_l))$, i.e.,

$$\delta_i \cdot B^k(e_i(a_l))/5 + (1-\delta_i) \cdot B^D(e_i(a_l)) \tag{39}$$

as the assessment $B(e_i(a_l))$. Note that the parameter δ_i is set by the decision maker in accordance with his recognition of criterion e_i. When he is very familiar with criterion e_i, he generally sets δ_i by a value less than 0.5. While he sets that $\delta_i = 0.5$ for his familiar criterion e_i. In particular, δ_i can be considered as 1 for his unfamiliar criterion e_i. By following this principle, five experts give their assessments and the decision maker gives his assessments and δ_i on the six criteria, as presented in Table 1. By using the data in Table 1 and Eq. (39), the assessments $B(e_i(a_l))$ are generated and presented in Table 2. A belief decision matrix $S_{6\times 5}$ is formed.

Based on the data in Table 2, $u^-(e_i(S_l))$ ($i = 1, \ldots, 6$, $l = 1, \ldots, 5$) and $u^+(e_i(S_l))$ are calculated by using Eqs. (1) and (2). They are presented in Table A.1 of Section A.1 in Appendix A of the supplementary material. As the enterprise must satisfy the strict requirements of producing key parts of high-speed train, it is prudent for the decision maker to take into consideration the worst scenarios of each supplier. For the consideration, the decision maker decides to select the inferior strategy for the supplier evaluation problem.

On the condition that the inferior strategy is adopted, fair inferior criterion weights between any two suppliers are calculated by using Eq. (17), as presented in Table A.2 of Section A.1. The fairness among criteria is accomplished. Based on the fair inferior criterion weights in Table A.2, the optimization model shown in Eqs. (18)–(22) for any supplier S_l is solved to obtain $\underline{F}^*(S_l)$ $(l = 1, \ldots, 5) = (-0.0999, 0.1437, -0.0616, -0.0639, -0.09)$. The post-optimal solution space $\underline{F}(S_l) \geq \underline{F}^*(S_l)-\varepsilon$ is then formed, in which S_l is guaranteed to be inferior to all the others to the maximum extent. From solving the optimization model shown in Eqs. (23)–(29) within the intersection of the spaces $\underline{F}(S_l) \geq \underline{F}^*(S_l)-\varepsilon$ ($l = 1, \ldots, 5$), $\underline{G}^*$ is obtained as 1.6051 to form the post-optimal solution space $\underline{G} \geq \underline{G}^*-\varepsilon$ and the set of fair inferior criterion weights achieving both the fairness among criteria shown in Definition 3 and the fairness among alternatives shown in Definition 4 is obtained as (0.1532, 0.1568, 0.148, 0.1969, 0.1287, 0.2164). The integrated set of fair inferior criterion weights for each supplier is also obtained, as presented in Table 3.

4.3. Generation of the solution to the supplier evaluation problem

Within the intersection of the post-optimal solution spaces $\underline{F}(S_l) \geq \underline{F}^*(S_l)-\varepsilon$ ($l = 1, \ldots, 5$) and $\underline{G} \geq \underline{G}^*-\varepsilon$, the optimization model

Table 1
Original assessment data from the five experts and the decision maker for the five suppliers where t_k denotes the kth expert and DM denotes the decision maker.

Attributes	S_1	S_2	S_3	S_4	S_5
e_1 $(\delta_1, 1-\delta_1) = (1, 0)$	t_1: {(A, 1)}; t_2: {(A, 1)}; t_3: {(G, 1)}; t_4: {(A, 1)}; t_5: {(Ω, 1)}	t_1: {(V, 1)}; t_2: {(V, 1)}; t_3: {(E, 1)}; t_4: {(G, 1)}; t_5: {(E, 0.5), (Ω, 0.5)}	t_1: {(A, 1)}; t_2: {(A, 1)}; t_3: {(A, 1)}; t_4: {(G, 1)}; t_5: {(G, 0.5), (Ω, 0.5)}	t_1: {(V, 1)}; t_2: {(V, 1)}; t_3: {(V, 1)}; t_4: {(E, 1)}; t_5: {(V, 0.5), (Ω, 0.5)}	t_1: {(G, 1)}; t_2: {(V, 1)}; t_3: {(V, 1)}; t_4: {(V, 1)}; t_5: {(Ω, 1)}
e_2 $(\delta_2, 1-\delta_2) = (1, 0)$	t_1: {(G, 1)}; t_2: {(G, 1)}; t_3: {(V, 1)}; t_4: {(V, 1)}; t_5: {(G, 0.5), (Ω, 0.5)}	t_1: {(G, 1)}; t_2: {(G, 1)}; t_3: {(V, 1)}; t_4: {(V, 1)}; t_5: {(Ω, 1)}	t_1: {(G, 1)}; t_2: {(G, 1)}; t_3: {(G, 1)}; t_4: {(G, 1)}; t_5: {(G, 0.5), (Ω, 0.5)}	t_1: {(A, 1)}; t_2: {(A, 1)}; t_3: {(P, 1)}; t_4: {(P, 0.5), (A, 0.5)}; t_5: {(G, 0.5), (Ω, 0.5)}	t_1: {(P, 1)}; t_2: {(P, 1)}; t_3: {(P, 1)}; t_4: {(P, 1)}; t_5: {(Ω, 1)}
e_3 $(\delta_3, 1-\delta_3) = (0.5, 0.5)$	t_1: {(V, 1)}; t_2: {(G, 1)}; t_3: {(V, 1)}; t_4: {(V, 1)}; t_5: {(G, 1)}; DM: {(V, 0.2), (E, 0.8)}	t_1: {(A, 1)}; t_2: {(A, 1)}; t_3: {(A, 1)}; t_4: {(A, 1)}; t_5: {(P, 1)}; DM: {(P, 0.8), (Ω, 0.2)}	t_1: {(G, 1)}; t_2: {(V, 1)}; t_3: {(V, 1)}; t_4: {(G, 1)}; t_5: {(E, 1)}; DM: {(V, 0.8), (Ω, 0.2)}	t_1: {(V, 1)}; t_2: {(V, 1)}; t_3: {(E, 1)}; t_4: {(V, 1)}; t_5: {(Ω, 1)}; DM: {(E, 1)}	t_1: {(E, 1)}; t_2: {(V, 1)}; t_3: {(V, 1)}; t_4: {(E, 1)}; t_5: {(Ω, 1)}; DM: {(V, 0.6), (Ω, 0.4)}
e_4 $(\delta_4, 1-\delta_4) = (0.4, 0.6)$	t_1: {(G, 1)}; t_2: {(V, 1)}; t_3: {(Ω, 1)}; t_4: {(V, 1)}; t_5: {(G, 1)}; DM: {(G, 0.4), (V, 0.6)}	t_1: {(A, 1)}; t_2: {(P, 1)}; t_3: {(A, 1)}; t_4: {(P, 1)}; t_5: {(A, 1)}; DM: {(P, 0.4), (A, 0.6)}	t_1: {(G, 1)}; t_2: {(V, 1)}; t_3: {(G, 1)}; t_4: {(V, 1)}; t_5: {(V, 1)}; DM: {(G, 1)}	t_1: {(P, 1)}; t_2: {(G, 1)}; t_3: {(A, 0.5), (Ω, 0.5)}; t_4: {(G, 1)}; t_5: {(A, 1)}; DM: {(A, 0.6), (Ω, 0.4)}	t_1: {(Ω, 1)}; t_2: {(G, 1)}; t_3: {(Ω, 1)}; t_4: {(V, 1)}; t_5: {(G, 1)}; DM: {(G, 0.6), (V, 0.4)}
e_5 $(\delta_5, 1-\delta_5) = (0.4, 0.6)$	t_1: {(V, 1)}; t_2: {(V, 1)}; t_3: {(E, 1)}; t_4: {(E, 1)}; t_5: {(E, 1)}; DM: {(V, 0.2), (E, 0.8)}	t_1: {(P, 1)}; t_2: {(P, 1)}; t_3: {(A, 1)}; t_4: {(A, 1)}; t_5: {(A, 1)}; DM: {(A, 0.7), (Ω, 0.3)}	t_1: {(V, 1)}; t_2: {(V, 1)}; t_3: {(V, 0.5), (Ω, 0.5)}; t_4: { (V, 1)}; t_5: {(V, 1)}; DM: {(V, 0.8), (Ω, 0.2)}	t_1: {(V, 1)}; t_2: {(E, 1)}; t_3: {(V, 0.5), (Ω, 0.5)}; t_4: {(V, 1)}; t_5: {(E, 1)}; DM: {(V, 0.7), (Ω, 0.3)}	t_1: {(A, 1)}; t_2: {(A, 1)}; t_3: {(Ω, 1)}; t_4: {(G, 0.5), (Ω, 0.5)}; t_5: {(G, 1)}; DM: {(A, 0.4), (G, 0.6)}
e_6 $(\delta_6, 1-\delta_6) = (1, 0)$	t_1: {(P, 1)}; t_2: {(P, 1)}; t_3: {(A, 1)}; t_4: {(A, 1)}; t_5: {(P, 0.5), (Ω, 0.5)}	t_1: {(G, 1)}; t_2: {(G, 1)}; t_3: {(G, 1)}; t_4: {(V, 1)}; t_5: {(G, 0.5), (E, 0.5)}	t_1: {(A, 1)}; t_2: {(A, 1)}; t_3: {(A, 1)}; t_4: {(A, 1)}; t_5: {(Ω, 1)}	t_1: {(A, 1)}; t_2: {(A, 1)}; t_3: {(A, 1)}; t_4: {(A, 1)}; t_5: {(A, 0.5), (G, 0.5)}	t_1: {(G, 1)}; t_2: {(G, 1)}; t_3: {(G, 1)}; t_4: {(G, 0.5), (V, 0.5)} t_5: {(V, 1)}

shown in Eqs. (30)–(36) with the objectives of "MIN $\underline{u}^{-}(S_l)$" and "MAX $\underline{u}^{+}(S_l)$" are solved to find the minimum and maximum expected utilities of the five suppliers, i.e., $[\underline{u}^{-}(S_l), \underline{u}^{+}(S_l)]$ $(l = 1, \ldots, 5)$ = ([0.4949, 0.5879], [0.3763, 0.4655], [0.4472, 0.5528], [0.4228, 0.5698], [0.4532, 0.5964]). As stated in Section 3.3, there is the similarity between the comparison among alternatives in their worst scenarios by using the minimax regret rule and the process of generating the set of fair inferior criterion weights. For this reason, the minimax regret rule is chosen to compare alternatives. From $[\underline{u}^{-}(S_l), \underline{u}^{+}(S_l)]$, the maximum loss of each supplier is calculated as (0.1015, 0.2201, 0.1492, 0.1736, 0.1347), which results in a ranking order of the five suppliers, i.e., $S_1 \succ S_5 \succ S_3 \succ S_4 \succ S_2$ where the notation '$\succ$' denotes 'superior to'.

Table 3
Integrated set of fair inferior criterion weights for each supplier.

Suppliers	$\underline{w}_i(S_l)$ $(i = 1, \ldots, 6)$
S_1	(0.2632, 0.1563, 0.0923, 0.1243, 0.0611, 0.3029)
S_2	(0.1305, 0.0522, 0.2393, 0.235, 0.1732, 0.1697)
S_3	(0.262, 0.1625, 0.1312, 0.1222, 0.1138, 0.2083)
S_4	(0.1757, 0.2607, 0.0572, 0.1592, 0.0845, 0.2627)
S_5	(0.1931, 0.2466, 0.0939, 0.1059, 0.1765, 0.184)

The ranking order of the five suppliers indicates that S_1, S_5, and S_3 are the most appropriate three options with the preferential order of $S_1 \succ S_5 \succ S_3$. This is the solution to the supplier evaluation problem. The

Table 2
Assessment data for the five suppliers generated from the data in Table 1.

Attributes	S_1	S_2	S_3	S_4	S_5
e_1	{(A, 0.6), (G, 0.2), (Ω, 0.2)}	{(G, 0.2), (V, 0.4), (E, 0.3), (Ω, 0.1)}	{(A, 0.6), (G, 0.3), (Ω, 0.1)}	{(V, 0.7), (E, 0.2), (Ω, 0.1)}	{(G, 0.2), (V, 0.6), (Ω, 0.2)}
e_2	{(G, 0.5), (V, 0.4), (Ω, 0.1)}	{(G, 0.4), (V, 0.4), (Ω, 0.2)}	{(G, 0.9), (Ω, 0.1)}	{(P, 0.3), (A, 0.5), (G, 0.1), (Ω, 0.1)}	{(P, 0.8), (Ω, 0.2)}
e_3	{(G, 0.2), (V, 0.4), (E, 0.4)}	{(P, 0.5), (A, 0.4), (Ω, 0.1)}	{(G, 0.2), (V, 0.6), (E, 0.1), (Ω, 0.1)}	{(V, 0.3), (E, 0.6), (Ω, 0.1)}	{(V, 0.5), (E, 0.2), (Ω, 0.3)}
e_4	{(G, 0.4), (V, 0.52), (Ω, 0.08)}	{(P, 0.4), (A, 0.6)}	{(G, 0.76), (V, 0.24)}	{(P, 0.08), (A, 0.48), (G, 0.16), (Ω, 0.28)}	{(G, 0.52), (V, 0.32), (Ω, 0.16)}
e_5	{(V, 0.28), (E, 0.72)}	{(P, 0.16), (A, 0.66), (Ω, 0.18)}	{(V, 0.84), (Ω, 0.16)}	{(V, 0.62), (E, 0.16), (Ω, 0.22)}	{(A, 0.4), (G, 0.48), (Ω, 0.12)}
e_6	{(P, 0.5), (A, 0.4), (Ω, 0.1)}	{(G, 0.7), (V, 0.2), (E, 0.1)}	{(A, 0.8), (Ω, 0.2)}	{(A, 0.9), (G, 0.1)}	{(G, 0.7), (V, 0.3)}

above analysis shows that S_1, S_5, and S_3 are appropriate to supply products to the enterprise in current situation. In the future, if the situation of the enterprise has changed, other suppliers may become the most appropriate choices for the enterprise.

In the situation where the inferior strategy is adopted, if the correlation between the pessimism and the inferior strategy is accepted by the decision maker, Hurwicz rule with the optimism degree γ limited to [0, 0.5] can be applied based on $[\underline{u}^-(S_l), \underline{u}^+(S_l)]$ to compare alternatives. To relieve the burden on the decision maker to provide a precise optimism degree, it is calculated using Eq. (37) that $E([\underline{u}^-(S_l), \underline{u}^+(S_l)])$ $(l = 1, \ldots, 5) = (0.2591, 0.1993, 0.2368, 0.2298, 0.2445)$. The new ranking order of the five suppliers is derived from $E([\underline{u}^-(S_l), \underline{u}^+(S_l)])$, which is $S_1 \succ S_5 \succ S_3 \succ S_4 \succ S_2$. The ranking order is clearly the same as the one generated by using the minimax regret rule.

To examine whether the rankings of the five suppliers alter with different optimism degrees, it is assumed that the optimism degree γ moves from 0 to 1 with a step of 0.05, then the expected utility of each supplier, i.e., $(1-\gamma)\cdot\underline{u}^-(S_l) + \gamma\cdot\underline{u}^+(S_l)$ is calculated and presented in Table A.3 of Section A.1. For each value of γ, the rankings of the five suppliers can be obtained from the data in Table A.3. The movement of the rankings of the five suppliers with the increase in γ is plotted in Fig. 3. It can be found from Fig. 3 that the rankings of the five suppliers remain unchanged with the movement of γ from 0 to 0.55, S_4 varies from the fourth to the third when γ increases to 0.6, and S_5 varies from the second to the first when γ increases to 0.85. Given different values of γ, different ranking orders of the five suppliers may be generated. In addition, $E([\underline{u}^-(S_l), \underline{u}^+(S_l)])$ characterizes the average situation of each supplier. The ranking order generated by $E([\underline{u}^-(S_l), \underline{u}^+(S_l)])$ can be verified qualitatively by Fig. 3.

4.4. Influence of the superior strategy on solutions

What has been analyzed above is on the condition that the inferior strategy is adopted. It is an interesting question to find out what will happen if the superior strategy is adopted. Answering this question helps to highlight the significant influence of the superior strategy on solutions to the supplier evaluation problem made by using the fair framework.

To examine the influence of the superior strategy on solutions to the supplier evaluation problem, we find the solution to the supplier evaluation problem under the conditions where the superior strategy is adopted and compare it with the solutions generated in Section 4.3. By using Eq. (3), fair superior criterion weights between any two suppliers are calculated, as presented in Table A.4 of Section A.2 in Appendix A.

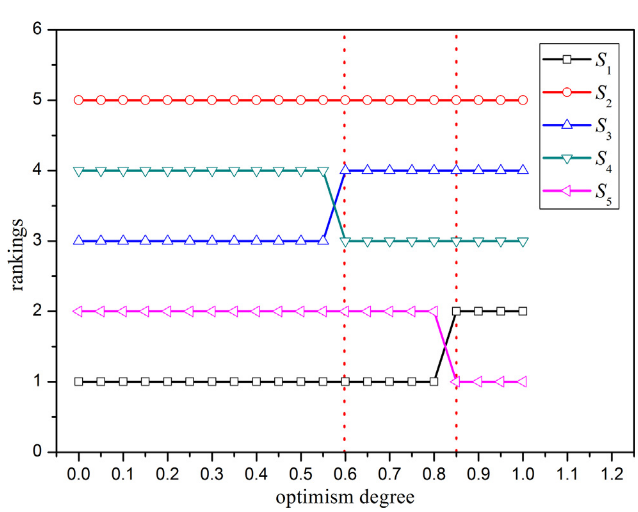

Fig. 3. Movement of the rankings of the five suppliers with the increase in the optimism degree from 0 to 1 when the inferior strategy is adopted.

Table 4
Integrated set of fair superior criterion weights for each supplier.

Suppliers	$\overline{w_i}(S_l)$ $(i = 1, \ldots, 6)$
S_1	(0.0828, 0.2075, 0.1866, 0.1803, 0.2548, 0.0881)
S_2	(0.1951, 0.2716, 0.0409, 0.1369, 0.0902, 0.2654)
S_3	(0.164, 0.156, 0.1697, 0.177, 0.1403, 0.193)
S_4	(0.2334, 0.1318, 0.1966, 0.1128, 0.1693, 0.1562)
S_5	(0.1635, 0.1348, 0.1385, 0.2026, 0.0986, 0.262)

From solving the optimization model shown in Eqs. (4)–(8) for any supplier S_l on the basis of fair superior criterion weights between any two suppliers, it is obtained that $\overline{F}^*(S_l)$ $(l = 1, \ldots, 5) = (0.0356, -0.026, -0.1347, -0.0736, -0.0602)$. Within the intersection of the spaces $\overline{F}(S_l) \geq \overline{F}^*(S_l) - \varepsilon$ $(l = 1, \ldots, 5)$, the optimization model shown in Eqs. (10)–(16) is solved to obtain $\overline{G}^*$ as 1.599 and the set of fair superior criterion weights achieving both the fairness among criteria shown in Definition 1 and the fairness among alternatives shown in Definition 2 as (0.1399, 0.2401, 0.1125, 0.1582, 0.171, 0.1783). The integrated set of fair superior criterion weights for each supplier is also obtained, as presented in Table 4.

Within the spaces of $\overline{F}(S_l) \geq \overline{F}^*(S_l) - \varepsilon$ $(l = 1, \ldots, 5)$ and $\overline{G} \geq \overline{G}^* - \varepsilon$, the optimization model shown in Eqs. (30)–(36) with the objectives of "MIN $\overline{u}^-(S_l)$" and "MAX $\overline{u}^+(S_l)$" are solved to find the minimum and maximum expected utilities of the five suppliers, i.e., $[\overline{u}^-(S_l), \overline{u}^+(S_l)]$ $(l = 1, \ldots, 5) = ([0.5288, 0.6196], [0.3869, 0.495], [0.4575, 0.5649], [0.4068, 0.5461], [0.3924, 0.5311])$. Assume that the correlation between the optimism and the superior strategy is accepted by the decision maker, Hurwicz rule with the optimism degree γ limited to [0.5, 1] is applied based on $[\overline{u}^-(S_l), \overline{u}^+(S_l)]$ to compare alternatives. By using Eq. (38), $E([\overline{u}^-(S_l), \overline{u}^+(S_l)])$ $(l = 1, \ldots, 5)$ is calculated as (0.2985, 0.234, 0.269, 0.2556, 0.2482), which results in the ranking order of the five suppliers, i.e., $S_1 \succ S_3 \succ S_4 \succ S_5 \succ S_2$. This indicates that S_1, S_3, and S_4 are the most appropriate three options with the preferential order of $S_1 \succ S_3 \succ S_4$. This solution is clearly different from the solutions generated by using the minimax regret rule and Hurwicz rule when the inferior strategy is adopted because S_3, S_4, and S_5 have changed from the third, fourth, and second best choices to the second, third, and fourth best choices, respectively. The observation indicates the significant influence of the superior strategy on the ranking order of the five suppliers and the solution to the supplier evaluation problem.

Similar to the situation where the inferior strategy is adopted, the movement of the rankings of the five suppliers with the increase in the optimism degree γ is conducted to analyze the stability of the ranking order $S_1 \succ S_3 \succ S_4 \succ S_5 \succ S_2$. Suppose that the optimism degree γ is increased from 0 to 1 with a step of 0.05. On this assumption, the expected utility of each supplier, i.e., $(1-\gamma)\cdot\overline{u}^-(S_l) + \gamma\cdot\overline{u}^+(S_l)$ is calculated and presented in Table A.5 of Section A.2. From the data in Table A.5, the movement of the rankings of the five suppliers with the increase in γ is plotted in Fig. 4, which justifies the ranking order generated by $E([\overline{u}^-(S_l), \overline{u}^+(S_l)])$ in a qualitative way. Through comparing Fig. 4 with Fig. 3, it can be found that the rankings of the five suppliers associated with the superior strategy remain unchanged with the increase in γ and thus are more stable than those associated with the inferior strategy. This observation is made for the supplier evaluation problem and further highlights the influence of the superior and inferior strategies on the ranking order of the five suppliers.

4.5. Comparison with existing methods

To validate the proposed fair framework, it is compared with five existing methods for determining criterion weights by addressing the supplier evaluation problem. The methods to be compared include the entropy method (Deng et al., 2000), the SD method (Deng et al., 2000), the CRITIC method (Diakoulaki et al., 1995), the CCSD method (Wang & Luo, 2010), and the method developed by Chin et al. (2015). The

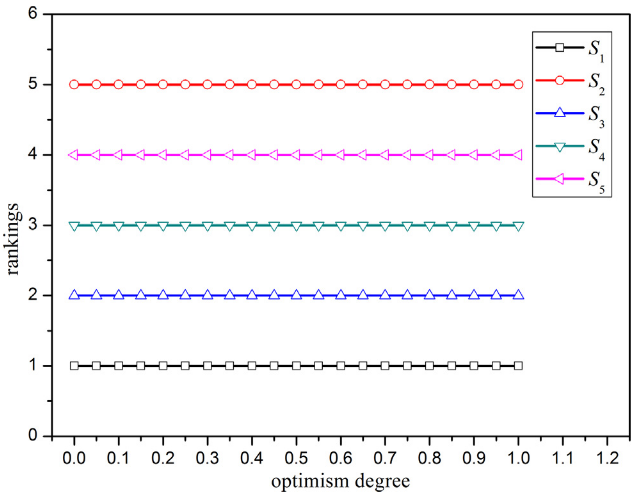

Fig. 4. Movement of the rankings of the five suppliers with the increase in the optimism degree from 0 to 1 when the superior strategy is adopted.

former four methods are classical ones, which are extended by Chin et al. (2015) to handle belief distributions in the ER approach. Meanwhile, Chin et al.'s method is developed based on the ER approach. For this reason, the five methods are selected to be compared with the fair framework. Details of the five methods can be found in Chin et al. (2015), which are omitted here to save space.

To address the supplier evaluation problem with the data shown in Table 2 and the settings presented in Section 4.1, the five methods are used to determine five sets of criterion weights, as presented in Table 5. Based on the five sets of criterion weights, five sets of the overall assessments of each supplier are obtained by using the ER algorithm (Wang et al., 2006) and then combined with $u(H_n)$ $(n = 1, \ldots, 5)$ to generate the expected utilities of each supplier $[u^-(S_l),\ u^+(S_l)]$ $(l = 1, \ldots, 5)$, which are also presented in Table 5. From the resulting expected utilities of the five suppliers, their integrated utilities are calculated by using Eqs. (37) and (38) on the assumption that the inferior and superior strategies are adopted. The ranking orders of the five suppliers are then derived from their integrated utilities. Note that in Table 5 "S" and "I" in the columns of "Integrated utilities" and "Ranking order" mean that the superior and inferior strategies are adopted, respectively.

Table 5 shows that criterion weights obtained by using the entropy method are significantly different from those obtained by using the methods of SD, CRITIC, and CCSD. Meanwhile, Chin et al.'s method creates a set of well-balanced criterion weights, as demonstrated in Chin et al. (2015). Different from the five methods, the fair framework does not intend to create a unique set of criterion weights but to consider all possible sets of criterion weights that follow the fairness among criteria and the fairness among alternatives. The two levels of fairness guarantee the fair contributions of the six criteria and the five suppliers to the solution made. Extreme criterion weights that are beneficial to any of the five suppliers are effectively avoided. Or else, some criteria may contribute almost nothing to the solution, which may not be what the decision maker anticipates in general. Compared with using one set of criterion weights to generate a solution, considering all possible sets of criterion weights following the two levels of fairness to do it seems more reliable, convinced, and fair.

The difference between the criterion weights derived from the entropy and those derived from the methods of SD, CRITIC, and CCSD causes the difference between the expected utilities of the five suppliers associated with the entropy method and those associated with the methods of SD, CRITIC, and CCSD, as presented in Table 5. Although so, it is interesting to find that the ranking orders of the five suppliers associated with the four methods are the same no matter whether the

Table 5
Criterion weights, the expected utilities of the five suppliers, and their integrated utilities and ranking orders by using the five methods.

Methods	w_i $(i = 1, \ldots, 6)$	$[u^-(S_l), u^+(S_l)]$ $(l = 1, \ldots, 5)$	Integrated utilities	Ranking order
Entropy	(0.1147, 0.1614, 0.255, 0.0872, 0.2803, 0.1013)	{[0.6789, 0.724], [0.2935, 0.4021], [0.5416, 0.6448], [0.5655, 0.6967], [0.4208, 0.5801]}	S: (0.3564, 0.1875, 0.3095, 0.332, 0.2701); I: (0.3451, 0.1603, 0.2837, 0.2991, 0.2303)	S: $S_1 > S_4 > S_3 > S_5 > S_2$ I: $S_1 > S_4 > S_3 > S_5 > S_2$
SD	(0.1467, 0.1589, 0.1958, 0.136, 0.209, 0.1535)	{[0.5909, 0.6532], [0.345, 0.4381], [0.495, 0.5947], [0.5038, 0.6343], [0.4381, 0.5827]}	S: (0.3188, 0.2074, 0.2849, 0.3008, 0.2733); I: (0.3032, 0.1841, 0.26, 0.2682, 0.2371)	S: $S_1 > S_4 > S_3 > S_5 > S_2$ I: $S_1 > S_4 > S_3 > S_5 > S_2$
CRITIC	(0.1453, 0.1713, 0.1823, 0.1142, 0.2163, 0.1706)	{[0.5809, 0.6444], [0.3602, 0.4557], [0.4859, 0.5902], [0.4951, 0.623], [0.4298, 0.5701]}	S: (0.3143, 0.2159, 0.2821, 0.2955, 0.2675); I: (0.2984, 0.192, 0.256, 0.2635, 0.2324)	S: $S_1 > S_4 > S_3 > S_5 > S_2$ I: $S_1 > S_4 > S_3 > S_5 > S_2$
CCSD	(0.1436, 0.1792, 0.1783, 0.1038, 0.2212, 0.1739)	{[0.5805, 0.644], [0.3647, 0.4622], [0.484, 0.5901], [0.4928, 0.6197], [0.424, 0.5636]}	S: (0.3141, 0.2189, 0.2818, 0.294, 0.2643); I: (0.2982, 0.1945, 0.2553, 0.2623, 0.2294)	S: $S_1 > S_4 > S_3 > S_5 > S_2$ I: $S_1 > S_4 > S_3 > S_5 > S_2$
Chin et al.	(0.156, 0.1744, 0.1562, 0.1553, 0.1797, 0.1784)	{[0.5487, 0.6193], [0.3738, 0.4615], [0.4717, 0.5701], [0.463, 0.5931], [0.4349, 0.5716]}	S: (0.3008, 0.2198, 0.2728, 0.2803, 0.2687); I: (0.2832, 0.1979, 0.2482, 0.2478, 0.2345)	S: $S_1 > S_4 > S_3 > S_5 > S_2$ I: $S_1 > S_3 > S_4 > S_5 > S_2$

superior strategy or the inferior strategy is adopted. This is because $u^-(S_l) > u^-(S_m)$ and $u^+(S_l) > u^+(S_m)$ $(l \neq m)$ always hold for the expected utilities of any two suppliers associated with the four methods. It is not the case for the expected utilities of the five suppliers associated with Chin et al.'s method due to the well-balanced criterion weights derived from the method. Two different ranking orders of the five suppliers are generated when the superior and inferior strategies are adopted, respectively.

When the methods of entropy, SD, CRITIC, and CCSD are used to determine criterion weights, the most appropriate three suppliers are S_1, S_4, and S_3 with the preferential order of $S_1 \succ S_4 \succ S_3$. The same solution can be obtained when Chin et al.'s method is applied to determine criterion weights and the superior strategy is adopted. However, a different solution is obtained when the inferior strategy is adopted, in which the rankings of S_3 and S_4 alter. On the other hand, from Sections 4.3 and 4.4 we can find that two different solutions are obtained by using the fair framework when the superior and inferior strategies are adopted, respectively. They are S_1, S_3, S_4 with the preferential order of $S_1 \succ S_3 \succ S_4$ and S_1, S_5, and S_3 with the preferential order of $S_1 \succ S_5 \succ S_3$. For the supplier evaluation problem, the decision maker attempts to take into account the worst scenarios of each supplier because the products and materials provided by the suppliers must satisfy the strict requirements of producing key parts of high-speed train. In accordance with the willingness of the decision maker, the inferior strategy is selected. Under the conditions, the solution generated by the fair framework, i.e., S_1, S_5, and S_3 with the preferential order of $S_1 \succ S_5 \succ S_3$, is clearly different from those associated with the five methods. When the superior strategy is adopted, the solution generated by the fair framework is also different from those associated with the five methods. The reason why the solution generated by the fair framework is considered as what the decision maker anticipates is that the strategy selected by the decision maker is involved in the process of obtaining all possible sets of criterion weights that follow the two levels of fairness and generating the expected utilities of the five suppliers. The selection of strategy to determine fair criterion weights reflects the interaction between the fair framework and the decision maker and facilitates making the obtained solution consistent with the real preferences of the decision maker. Relevant theoretical analysis can be found in Section 3.4. The willingness of the decision maker, however, is not considered in the obtainment of criterion weights by using the five methods.

5. Simulation experiment

In the supplier evaluation problem presented in Section 4, the five suppliers are evaluated on six criteria. The problem is not a large-scale one. It is interesting to find out whether the fair framework is applicable to the decision problem with a number of alternatives and criteria. To examine this, 10 combinations of alternatives and criteria are selected, which are presented in Table 6. For each combination, a random belief decision matrix is generated to do simulation experiments. Both the superior strategy and the inferior strategy are considered in the experiments.

The simulation experiments are implemented on a personal computer with processor of Intel Core i7-6700 CPU 3.4 GHz, RAM of 16 GB, and operating system of 64 Bits Windows 7. The solution system, which is developed in the MATLAB environment and used to analyze the supplier evaluation problem in Section 4, is adapted to carry out the simulation experiments. Solution time for random decision problems with the ten combinations of alternatives and criteria when the superior and inferior strategies are adopted is presented in Table 6.

Table 6 shows that solution time has grown rapidly with the increase in the number of alternatives and the number of criteria. It is the similar case when the superior and inferior strategies are adopted. This highlights the difficulty in using the fair framework to solve MCDM problems. If a decision maker does not care solution time very much but pay more attention to decision results, the fair framework is applicable for large-scale decision problems. Or else, different high-performance computing systems such as high-performance computer (e.g., high-end system and cluster system) and distributed multiprocessor system (e.g., cloud computing) can help to achieve the computation results of the fair framework for large-scale decision problems within acceptable time frames.

Table 6
Solution time of the fair framework for random decision matrices with different numbers of alternatives and criteria when the superior and inferior strategies are adopted.

No	Number of alternatives	Number of criteria	Strategy	Solution time (s)
1	6	8	Superior	71.34
1	6	8	Inferior	151.97
2	7	9	Superior	279.06
2	7	9	Inferior	223.99
3	8	10	Superior	328.97
3	8	10	Inferior	464.08
4	9	11	Superior	897.42
4	9	11	Inferior	2353.37
5	10	12	Superior	2196.11
5	10	12	Inferior	2924.04
6	11	13	Superior	4280.43
6	11	13	Inferior	6826.5
7	11	14	Superior	6974.17
7	11	14	Inferior	7225.55
8	12	14	Superior	7691.06
8	12	14	Inferior	9234.94
9	13	15	Superior	12744.75
9	13	15	Inferior	21894.01
10	14	16	Superior	32125.64
10	14	16	Inferior	24982.07

When advanced high-performance computing systems are not available, there are other feasible ways to efficiently solve large-scale decision problems by using the fair framework, which are associated with the characteristics of the problems. For a decision problem with a large number of alternatives and limited criteria, the problem can be divided into several subproblems, in which each subproblem should be comparable to each other as much as possible or almost identical. Each subproblem is solved by using the fair framework and then the solutions to all subproblems are combined to generate the solution to the decision problem by using the fair framework again. For a decision problem with limited alternatives and a large number of criteria, through clustering analysis or other techniques a multi-hierarchical criterion tree is constructed based on all criteria. Iteratively applying the fair framework on each hierarchy in accordance with the relationship between criteria on adjacent hierarchies can generate the solution to the decision problem. Finally, a combination of the ways to solve the above two types of decision problems can help to solve a decision problem with a large number of alternatives and criteria.

6. Conclusions

The weight of a criterion is an important concept in MCDM that reflects the impact of the individual assessment of the criterion on the overall assessment. Appropriate or sound criterion weight assignment is crucial to making rational trade-offs among all criteria in MCDM methods. To focus on the assignment of weights to criteria, a large amount of research has been conducted. Although so, whether the criterion weight assignment is fair to each criterion and to each alternative is rarely taken into account in existing studies. To address this problem, a fair framework in the context of the ER approach was explored in this paper, which mainly includes two levels of fairness, i.e., the fairness among criteria and the fairness among alternatives. In the fair framework, two strategies were firstly provided for the decision maker to choose with the consideration of the decision problem, which are the superior strategy and the inferior strategy. After the choice, the fairness among criteria was defined and used to construct an optimization model to help each alternative achieve the objective in line with the selected strategy to the maximum extent. Based on the results derived from solving the optimization model, the fairness among alternatives was defined and used to construct the other optimization model to produce fair criterion weights in line with the selected strategy. By following the idea of treating all possible sets of fair criterion weights in line with the selected strategy, another optimization model was constructed to generate the minimum and maximum expected utilities of each alternative, which were used to generate a solution to the decision problem considered by exploring the decision maker's behaviors or what decision rule is preferred by the decision maker.

What we investigate in this paper is a new attempt to explore the determination of criterion weights accomplishing both the fairness among criteria and the fairness among alternatives, which are not considered in existing studies on how to determine criterion weights from a decision matrix. While the concept of fairness is taken into account in the process of determining criterion weights which are used to generate a solution, as reported in this paper, a decision maker's behaviors are not integrated with the fairness, which drives more research to be conducted. In the meantime, how to handle the preferences of the decision maker for balancing the two levels of fairness is an interesting issue, which will be investigated in the next step. In addition, many interesting issues will be found when the idea of following the two levels of fairness is extended to other MCDM methods in which the preferences of a decision maker are expressed in different ways, as indicated at the end of Section 3.4.

Not only that, it can be perceived that we live in an era of informalization and intelligence. Under the conditions artificial intelligence is a trend in most fields, in which decision making field is obviously included. Many interesting issues will be inspired by applying artificial intelligence techniques, such as multi-agent system and machine learning, to uncertain MCDM, including how to quickly find effective or acceptable solutions to large-scale MCDM problems, how to learn the preferences of a decision maker for MCDM problems in different situations, and how to coordinate the interactions between the method and the decision maker in the process of MCDM. Also, the combination of MCDM and artificial intelligence makes it possible to extend the application field of MCDM. With the consideration of artificial intelligence techniques, MCDM can be applied in specific fields in which individual assessments are not provided by a decision maker (usually people) but by other sources, such as online data, industrial equipment, and healthy equipment.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.cie.2018.07.039.

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