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Improved stochastic linearization technique for structures with nonlinear viscous dampers

Dario De Domenico*, Giuseppe Ricciardi

Dept. of Engineering, University of Messina, Contrada Di Dio, Sant'Agata, 98166 Messina, Italy

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ABSTRACT

The stochastic linearization technique (SLT), in its conventional force-based Gaussian version (FB-G-SLT), is widely adopted in the literature to handle the nonlinear power-law constitutive behavior of fluid viscous dampers (FVDs), either for evaluating the system response, or within optimal design strategies. In this note, the authors prove that, for the kind of nonlinearity induced by FVDs, this version of FB-G-SLT is not the best choice, especially if the velocity-related response is of major importance in an optimal design process. As an example, the energy dissipated by the devices depends upon the square of the velocity at the ends of the dampers. Among six different formulations of SLT examined in this note, an equal-energy non-Gaussian SLT (EE-NG-SLT) is found to be superior to the FB-G-SLT since it more satisfactorily captures the nonlinear response of the structure as predicted by Monte Carlo simulations. This alternative EE-NG-SLT, without requiring higher computational effort than the FB-G-SLT, is therefore recommended, especially when energy-based (or, in general, velocity-related) objective functions are used in the optimization problem of nonlinear viscous dampers.

1. Introduction

Besides seismic base isolation [1,2] and other strengthening techniques [3], the use of fluid viscous dampers (FVDs) as energy dissipation devices has rapidly increased in the last few years [4] for seismic retrofitting of existing civil engineering structures. Their appealing properties include the large energy dissipation capability, the low maintenance required, and the generation of forces that are out of phase with the elastic forces, thereby not increasing the stress in the structure. Experimental evidence [5–9] reveals that the constitutive behavior of FVDs can be described by a fractional velocity power law

$$f_{\mathrm{d}}^{NL} = c_{\mathrm{d}}|\dot{u}|^{\alpha}\,\mathrm{sgn}(\dot{u}) \tag{1}$$

where c_{d} is the damping coefficient, α is a velocity exponent, $\dot{u}$ represents the relative velocity at the ends of the device and $\mathrm{sgn}(\cdot)$ is the signum function. The exponent α is responsible for the nonlinear damping of FVDs and depends upon the hydraulic circuit employed. Typically, α ranges from 0.10 to 0.50 for seismic applications.

As a result of the nonlinear constitutive behavior of FVDs, linear methods of analysis, e.g., the response spectrum method, are no longer applicable. In this regard, attempts have been made to determine an equivalent viscous damping ratio due to the added nonlinear FVDs [10], or an equivalent damping coefficient of energy-equivalent FVDs associated with the same energy dissipation as the nonlinear FVDs [11]. The above-mentioned relationships, extremely useful for preliminary design purposes, are amplitude-dependent due to the nonlinear nature of FVDs. Furthermore, they are strictly valid under the hypothesis of harmonic motion. Since the earthquake-induced motion is not really harmonic but is generally modelled as a random process, the estimation of the equivalent damping coefficient can be alternatively performed within the framework of the stochastic linearization technique (SLT) [12]. The SLT was applied in a number of research papers and to a general class of nonlinear behaviors, not just confined to the one shown in (1), e.g.: in the context of tuned liquid column damper optimization [13,14], characterized by quadratic-times-signum-like damping; in the field of nonlinear energy sinks optimization [15], featured by a cubic stiffness; for optimizing the performance of hysteretic dampers [16], represented by a Bouc-Wen model. The SLT has also been successfully applied to derive equivalent linear properties of bilinear systems [17,18] in the framework of response spectrum analysis [19,20].

Coming back to the nonlinearity of FVDs given by (1), which is of interest to the present note, in the literature the SLT has been applied by introducing the stochastic nature of the earthquake input via a power-spectral-density (PSD) function $S_{\ddot{u}_{\mathrm{g}}}(\omega)$, for example the Kanai-Tajimi PSD [21–23], or a particular spectrum-compatible PSD function [24,25]. All these quoted papers used a popular force-based Gaussian SLT (F-G-SLT). In this note, it is demonstrated that this variant of SLT is not the best option for the kind of nonlinearity induced by the FVDs.

* Corresponding author.
E-mail address: dario.dedomenico@unime.it (D. De Domenico).

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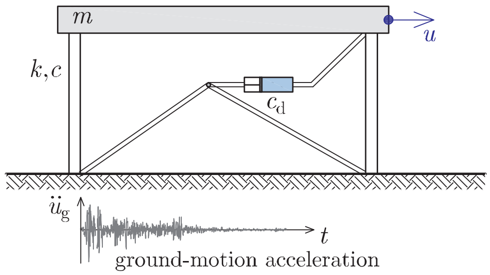

Fig. 1. One-story building with added FVD subject to a ground motion acceleration.

After a critical assessment of six alternative formulations of SLT, we identify a so-called equal-energy non-Gaussian SLT (EE-NG-SLT) that, without increased computational effort, is better able to capture the nonlinear response of the system.

2. Stochastic analysis of SDOF system with nonlinear FVD

For the sake of simplicity, reference is made to a single-degree-of-freedom (SDOF) system with added FVD as sketched in Fig. 1.

The equations of motion of this one-story building subject to a base acceleration $\ddot{u}_g$ are

$$m\ddot{u} + c\dot{u} + ku + c_d|\dot{u}|^\alpha \operatorname{sgn}(\dot{u}) = -m\,\ddot{u}_g \tag{2}$$

where m, c, k are the mass, inherent damping, and stiffness coefficients, respectively, and the nonlinear force-velocity law of the FVD (1) is considered. The ground motion acceleration $\ddot{u}_g$ is assumed as the realization of a stationary zero-mean Gaussian random process described by a PSD function, e.g. the Kanai-Tajimi function modified by Clough and Penzien

$$S_{\ddot{u}_g}(\omega) = \frac{\omega_g^4 + 4\zeta_g^2\omega_g^2\omega^2}{(\omega_g^2-\omega^2)^2 + 4\zeta_g^2\omega_g^2\omega^2}\,\frac{\omega^4}{(\omega_f^2-\omega^2)^2 + 4\zeta_f^2\omega_f^2\omega^2} S_w \tag{3}$$

where ω_g, ζ_g, ω_f, ζ_f are filter parameters that affect the earthquake frequency content and can be associated with different soil characteristics, while the white-noise spectral level S_w can be related to the peak ground acceleration (PGA) $\ddot{u}_{g0}$ according to [26]

$$S_w = \frac{0.141\zeta_g\ddot{u}_{g0}^2}{\omega_g\sqrt{1+4\zeta_g^2}}. \tag{4}$$

The probabilistic characterization of the response process is completely accomplished through the knowledge of the probability density function (PDF). For a linear system (i.e., for $c_d = 0$ or $\alpha = 1$ in (2)) excited by a zero-mean Gaussian random process, both displacement and velocity are zero-mean Gaussian processes too. Unfortunately, due to the nonlinearity induced by the FVD, the response is in general non-Gaussian, and is markedly non-Gaussian for severe nonlinearity.

The SLT mainly consists in the following three steps that involve three levels of approximation:

1) replace the nonlinear system with an equivalent linearized system;
2) compute the linearization coefficients through some "equivalence criterion";
3) assume the PDF of the system response a priori to explicitly determine expressions of the linearization coefficients.

For the specific problem at hand, step 1) typically involves the replacement of the nonlinear FVD force in (2) with the following linearized viscous damping force $f_d^L = c_{d,eq}\dot{u}$

$$m\ddot{u} + (c + c_{d,eq})\,\dot{u} + ku = -m\,\ddot{u}_g\,. \tag{5}$$

In (5) $c_{d,eq}$ denotes the linearization coefficient that has to be identified in step 2). To this aim, the most popular choice in the literature is to minimize, in a least square sense, the error/difference e between the nonlinear and linearized force [12]

$$\begin{aligned} &e = f_d^{NL} - f_d^L = c_d|\dot{u}|^\alpha \operatorname{sgn}(\dot{u}) - c_{d,eq}\dot{u} \quad (\text{force-based error}) \\ &\frac{\partial}{\partial c_{d,eq}} E[e^2] = 0 \to c_{d,eq}^{FB} = \frac{E[\dot{u}\,f_d^{NL}]}{E[\dot{u}^2]} = c_d\frac{E[|\dot{u}|^{\alpha+1}]}{E[\dot{u}^2]}. \end{aligned} \tag{6}$$

where $E[\cdot]$ is the expectation operator and the superscript *FB* stands for "force-based" SLT.

Instead of the difference of forces as per (6), two alternative equivalence criteria were proposed by Elishakoff and Zhang [27] (see in this regard also the more recent paper [28]) and are here adapted to the kind of nonlinearity induced by the FVDs. These two strategies are based on the functions expressing the energy dissipated in the nonlinear and linearized system, respectively

$$\begin{aligned} &\mathcal{E}_d^{NL}(\dot{u}) = \int_0^{\dot{u}} f_d^{NL}\,\mathrm{d}\dot{x} = \int_0^{\dot{u}} c_d|\dot{x}|^\alpha \operatorname{sgn}(\dot{x})\mathrm{d}\dot{x} = c_d\frac{|\dot{u}|^{1+\alpha}}{1+\alpha} \\ &\mathcal{E}_d^L = \int_0^{\dot{u}} f_d^L\,\mathrm{d}\dot{x} = \int_0^{\dot{u}} c_{d,eq}\dot{x}\,\mathrm{d}\dot{x} = \frac{1}{2}c_{d,eq}\dot{u}^2\,. \end{aligned} \tag{7}$$

In the first version, the linearization coefficient $c_{d,eq}$ is found by minimizing the mean square error/difference e of dissipated energy in the nonlinear and linearized system

$$\begin{aligned} &e = \mathcal{E}_d^{NL} - \mathcal{E}_d^L = c_d\frac{|\dot{u}|^{1+\alpha}}{1+\alpha} - \frac{1}{2}c_{d,eq}\dot{u}^2 \quad \left(\text{energy-based error}\right) \\ &\frac{\partial}{\partial c_{d,eq}}\mathrm{E}[e^2] = 0 \to c_{d,eq}^{EB} = \frac{2E[\dot{u}^2\,\mathcal{E}_d^{NL}]}{E[\dot{u}^4]} = \frac{c_d}{1+\alpha}\frac{2E[|\dot{u}|^{3+\alpha}]}{E[\dot{u}^4]} \end{aligned} \tag{8}$$

with the superscript *EB* standing for "energy-based" SLT. The second SLT assumes that the mean-square values of the dissipated energies are equal

$$\mathrm{E}[(\mathcal{E}_d^{NL})^2] = \mathrm{E}[(\mathcal{E}_d^L)^2] \to c_{d,eq}^{EE} = 2\sqrt{\frac{\mathrm{E}[(\mathcal{E}_d^{NL})^2]}{\mathrm{E}[\dot{u}^4]}} = \frac{2c_d}{1+\alpha}\sqrt{\frac{\mathrm{E}[|\dot{u}|^{2+2\alpha}]}{\mathrm{E}[\dot{u}^4]}} \tag{9}$$

with *EE* standing for "equal-energy" SLT.

The three sets of linearization coefficients $c_{d,eq}^{FB}$, $c_{d,eq}^{EB}$, $c_{d,eq}^{EE}$ represent the outcome of the step 2). However, in order to compute the averages appearing in (6), (8), (9), some hypothesis should be made in step 3) regarding the PDF $p_{\dot{u}}(\dot{u})$. The most popular choice is to assume that $p_{\dot{u}}(\dot{u})$ is a zero-mean Gaussian PDF, which is however valid only for Gaussian excitation and linear behavior of the system. This leads to the following expressions

$$\begin{aligned} &c_{d,eq}^{FB-G} = c_d\left[\frac{2^{(1+\alpha)/2}\Gamma(1+\alpha/2)}{\sqrt{\pi}}\right]\sigma_{\dot{u}}^{\alpha-1} \quad \left(\text{force-based Gaussian SLT}\right) \\ &c_{d,eq}^{EB-G} = c_d\left[\frac{2^{(5+\alpha)/2}\Gamma(2+\alpha/2)}{3\sqrt{\pi}(1+\alpha)}\right]\sigma_{\dot{u}}^{\alpha-1} \quad \left(\text{energy-based Gaussian SLT}\right) \\ &c_{d,eq}^{EE-G} = c_d\left[\frac{2^{(3+\alpha)/2}\sqrt{\Gamma(\alpha+3/2)}}{\sqrt{3}\,\pi^{1/4}(1+\alpha)}\right]\sigma_{\dot{u}}^{\alpha-1} \quad \left(\text{equal-energy Gaussian SLT}\right) \end{aligned} \tag{10}$$

where $\Gamma(\cdot)$ is the gamma function and $\sigma_{\dot{u}}$ the standard deviation of the velocity. Alternatively, for reasons that will be clarified below, it is assumed that $p_{\dot{u}}(\dot{u})$ is a zero-mean non-Gaussian PDF with an exponential shape

$$p_{\dot{u}}^{NG}(\dot{u}) = \frac{1}{\sqrt{2}\sigma_{\dot{u}}}\exp\left[-\frac{\sqrt{2}}{\sigma_{\dot{u}}}|\,\dot{u}\,|\right] \tag{11}$$

that, similarly to the Gaussian PDF, is an even function complying with the normalization condition and having variance $\sigma_{\dot{u}}^2$. Exploiting this non-Gaussian PDF in (6), (8), (9) leads to

$$c_{d,\mathrm{eq}}^{FB-NG} = c_{\mathrm{d}}[2^{-(1+\alpha)/2}\Gamma(2+\alpha)]\sigma_{\dot{u}}^{\alpha-1} \quad \text{(force–based non–Gaussian SLT)}$$
$$c_{d,\mathrm{eq}}^{EB-NG} = c_{\mathrm{d}}\left[\frac{2^{-(3+\alpha)/2}\Gamma(4+\alpha)}{3(1+\alpha)}\right]\sigma_{\dot{u}}^{\alpha-1} \quad \left(\text{energy–based non–Gaussian SLT}\right)$$
$$c_{d,\mathrm{eq}}^{EE-NG} = c_{\mathrm{d}}\left[\frac{2^{-\alpha/2}\sqrt{\Gamma(3+2\alpha)}}{\sqrt{3}\,(1+\alpha)}\right]\sigma_{\dot{u}}^{\alpha-1} \quad \left(\text{equal–energy non–Gaussian SLT}\right) \tag{12}$$

Note that, apart from the terms within square brackets, these six versions of SLT share the same mathematical structure, thus they are associated with an identical computational effort. Basically, these six versions of SLT arise from three different equivalence criteria in step 2) and two different PDF assumptions in step 3). Since the linearization coefficients depend on $\sigma_{\dot{u}}$ that is unknown and is implicitly related to $c_{d,\mathrm{eq}}$, typical input-output relationships in the frequency domain must be applied iteratively [12].

3. Applications and remarks

Some applications to a one-story building model are presented in order to critically assess and compare the accuracy of the six SLTs. The nonlinear response of the system is obtained through Monte Carlo simulations (MCS) with 2×10^6 samples generated from the Kanai-Tajimi PSD function, and fourth-order Runge-Kutta integration.

As a first example, the building period $T = 2\pi\sqrt{k/m} = 1$ s and the damping ratio $\zeta = c/2\sqrt{m\,k} = 0.05$. Considering firm soil conditions, the PSD function (3) is defined by $\omega_{\mathrm{g}} = 15 rad/s$, $\zeta_{\mathrm{g}} = 0.6$, $\omega_{\mathrm{f}} = 1.5 rad/s$, $\zeta_{\mathrm{f}} = 0.6$, PGA $\ddot{u}_{g0} = 0.3$ g. Assuming $c_{\mathrm{d}}/m = 2.0$, the "true" PDF of the velocity response (as predicted by MCS) for $\alpha = 0.3$ (softening damping), $\alpha = 1$ (linear case – Gaussian response), and $\alpha = 3.0$ (hardening damping) is depicted in Fig. 2. It is noted that for the kind of nonlinearity induced by FVDs (softening damping $\alpha < 1$), the PDF of the velocity is sharper than the Gaussian in the near-zero region, whereas in the hardening case is smoother. This is well known in the literature for power-law stiffness, but probably less known for power-law damping. This observation (confirmed in a variety of other configurations here not reported for brevity) motivates the introduction of an exponential-like shape of the PDF (11) for computing the averages.

Since the resulting linearization coefficients $c_{d,\mathrm{eq}}$ affect not only the velocity response, but also the displacement response, it is expected that this exponential-shape PDF assumption leads to more accurate results in general. To prove this, the relative errors of the SLT computed as

$$\mathrm{err}_x = \left|\frac{\sigma_x^{\mathrm{SLT}} - \sigma_x^{\mathrm{MCS}}}{\sigma_x^{\mathrm{MCS}}}\right| \times 100 \quad \left(\text{with } x = u,\, \dot{u}\right) \tag{13}$$

are reported in Table 1. It can be noticed that the energy-based versions of SLT with the Gaussian assumption do not provide accurate estimates of the nonlinear response. Conversely, introducing the exponential-like PDF significantly lowers the errors to more acceptable values. The FB

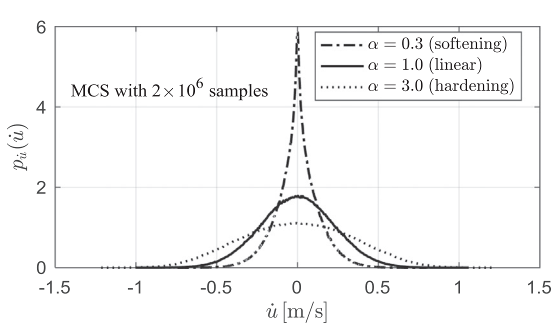

Fig. 2. Exact velocity response PDF of SDOF system with linear and nonlinear FVD.

Table 1
Relative errors of different SLT formulations against MCS for $T = 1$ s, $\zeta = 0.05$, $\alpha = 0.3$.

SLT formulation	$c_{\mathrm{d}}/m = 2$		$c_{\mathrm{d}}/m = 3$	
	Displacement err_u [%]	Velocity $\mathrm{err}_{\dot{u}}$ [%]	Displacement err_u [%]	Velocity $\mathrm{err}_{\dot{u}}$ [%]
FB-G-SLT	5.2	4.3	10.9	8.0
EB-G-SLT	19.5	18.4	29.4	27.0
EE-G-SLT	22.5	21.5	33.3	31.0
FB-NG-SLT	4.2	4.9	1.4	4.7
EB-NG-SLT	6.7	7.3	4.7	8.0
EE-NG-SLT	0.9	1.6	3.0	0.2

and EB versions of NG-SLT give comparable errors to the conventional FB-G-SLT in the case $c_{\mathrm{d}}/m = 2$, and lower errors in the case $c_{\mathrm{d}}/m = 3$. On the other hand, the EE-NG-SLT offers notable improvements, reducing the errors of around 4–5 times as compared to the FB-G-SLT for $c_{\mathrm{d}}/m = 2$, and even more for $c_{\mathrm{d}}/m = 3$.

To examine the performance of these two versions of SLT in a broader range of dynamic parameters of the system, in Fig. 3 both the standard deviations of displacement and velocity are reported by varying the building period T for two different values of the c_{d}/m ratio. Excellent results are obtained with the proposed EE-NG-SLT, much better than the FB-G-SLT, not only in terms of velocity, but also in terms of displacements. The improved accuracy of the EE-NG-SLT in capturing the exact standard deviations of the nonlinear response has important design implications because it is also related to a better prediction of the maximum values of the response (through the peak factor). Indeed, apart from the peak factor, the graphs of Fig. 3 are related to the displacement response spectrum, and to the velocity response spectrum.

To examine the performance of these two versions of SLT in a broader range of nonlinear parameters of the FVD, in Fig. 4 the standard deviation of the velocity response (which is more directly influenced by the nonlinear constitutive behavior of the FVD than the displacement response) is reported by varying the c_{d}/m ratio and the α exponent. The improvements of the proposed NG-SLT are really evident for the range of parameters investigated. The better performance of the EE-NG-SLT over the FB-G-SLT is more manifest in the case of more severe nonlinearities, i.e., for larger values of the c_{d}/m ratio and for smaller values of the α exponent. Indeed, in these cases the non-Gaussian nature of the PDF of the response is more marked, therefore the underlying assumption of a Gaussian response through which the expression $(10)_1$ is derived is not really appropriate. In these cases, the new exponential PDF introduced in (11) seems to be more convenient and leads to results that are closer to the MCS. It emerges that the proposed EE-NG-SLT turns out to be particularly suitable (and certainly preferable to the conventional FB-G-SLT) for marked nonlinearities of the FVDs. On the contrary, as the α exponent increases, the system would behave more linearly and, as a result, the Gaussian approximation could still be a reasonable and acceptable assumption.

As a final remark, the authors would like to emphasize the importance of the velocity-related response within optimal design strategies of nonlinear FVDs. Indeed, since the primary aim of FVDs is to dissipate the largest possible amount of energy from the earthquake excitation, a possible objective function (OF) to maximize (within a constrained optimization problem) could be related to the energy dissipated by FVDs for unit time e_{d} (or dissipated power). For the FVD of the analyzed SDOF system, this function reads

$$e_{\mathrm{d}}^{NL} = c_{\mathrm{d}}\,|\dot{u}|^{1+\alpha} \quad \text{(energy dissipated for unit time–nonlinear FVD)} \tag{14}$$

Once the SLT is applied, the linearized function approximating (14) turns out to be

$$e_{\mathrm{d}}^{L} = c_{d,\mathrm{eq}}\,\dot{u}^2 \quad \text{(energy dissipated for unit time–linearized system)} \tag{15}$$

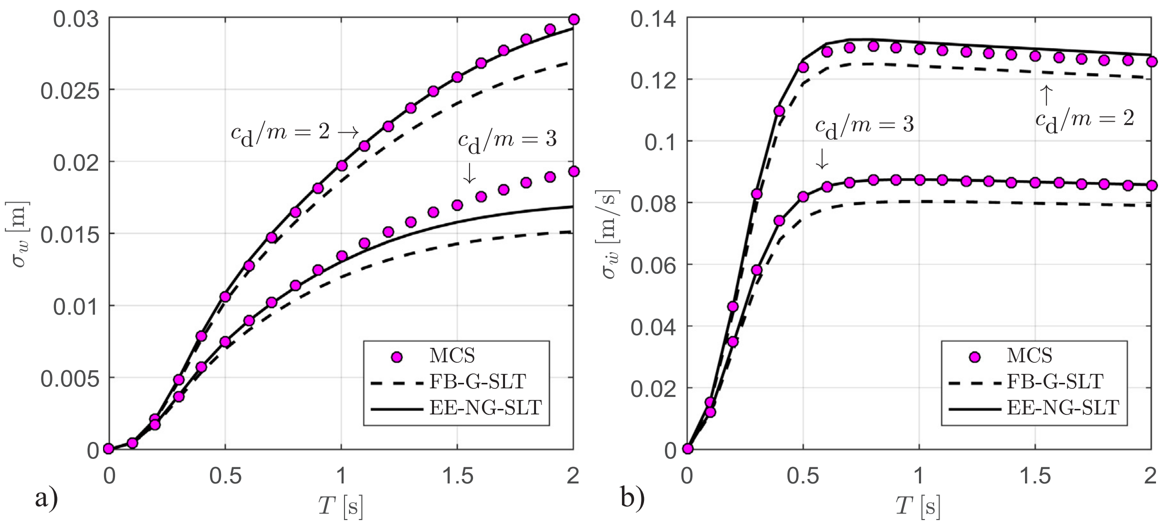

Fig. 3. Accuracy of SLT for $\alpha = 0.3$ and different periods of the SDOF system: a) standard deviation of displacement; b) standard deviation of velocity.

The expected value of (15) has been considered as OF in some recent papers by the authors [29,30] and other authors [31–33] in the context of tuned-mass-damper optimization, and for optimizing the performance of hysteretic dampers placed between adjacent buildings [16]. In order to achieve the most accurate maximization of the dissipated energy, the OF (15) should reflect as close as possible the true dissipated energy by the nonlinear FVD (14). The alternative SLTs are then critically scrutinized in order to assess how accurately they predict the dissipated energy of the nonlinear FVD. Since Eq. (15) represents a function $y = f(\dot{u})$ of the random variable $\dot{u}$, first the PDF of $e_{\rm d}^L$ is determined as [34]

$$p_y(y) = \frac{1}{c_{d,\rm eq}\sqrt{y/c_{d,\rm eq}}} p_{\dot{u}}\left(\sqrt{\frac{y}{c_{d,\rm eq}}}\right)\mathcal{H}(y) \quad \left(\text{with} y = e_{\rm d}^L\right) \tag{16}$$

where $\mathcal{H}(\cdot)$ is the Heaviside step function and $p_{\dot{u}}(\dot{u})$ may be either a Gaussian or the exponential-like PDF (11), depending on the choice of the SLT. In both cases, the expected value of $y \equiv e_{\rm d}^L$, representing the OF to maximize according to what stated above, is

$$\mathrm{E}[y] = \mathrm{E}[e_{\rm d}^L] = \int_{-\infty}^{\infty} y\, p_y(y)\, dy = c_{d,\rm eq}\sigma_{\dot{u}}^2 \tag{17}$$

whereby the variance of the velocity appears. The accurate prediction of the velocity-related response is therefore of great importance when addressing energy-based indicators like the energy dissipated by FVDs.

In Fig. 5 the PDF of $y \equiv e_{\rm d}$ computed by MCS is reported against the function (16). Note that in both cases of Gaussian and exponential-like shape of $p_{\dot{u}}(\dot{u})$, the resulting $p_y(y)$ is an exponential function having only positive values due to the mathematical structure of (16). Although both the versions of SLT capture the qualitative shape of the distribution obtained by MCS, significant improvements are achieved by the EE-NG-SLT (more clearly illustrated in a semi-logarithmic scale plot). The errors on the mean value $\mathrm{E}[y] = \mathrm{E}[e_{\rm d}]$ are 5.5% for the FB-G-SLT and 1.1% for the EE-NG-SLT.

4. Conclusions

Response evaluation of structures equipped with FVDs can be complex due to their nonlinear power-law constitutive behavior. In the framework of SLT, the nonlinear FVDs can be replaced by equivalent linear FVDs, so that linear methods of analysis (e.g., the response spectrum method), or linear random vibration theory are still applicable.

Based on the outcome of this study, it is recommended that the equivalent damping coefficients of nonlinear FVDs be calculated through a developed equal-energy non-Gaussian stochastic linearization technique. The proposed method offers improved accuracy and significant advantages over the force-based Gaussian stochastic linearization technique, widely adopted in the literature. It has been demonstrated that it is better able to capture the nonlinear response of the

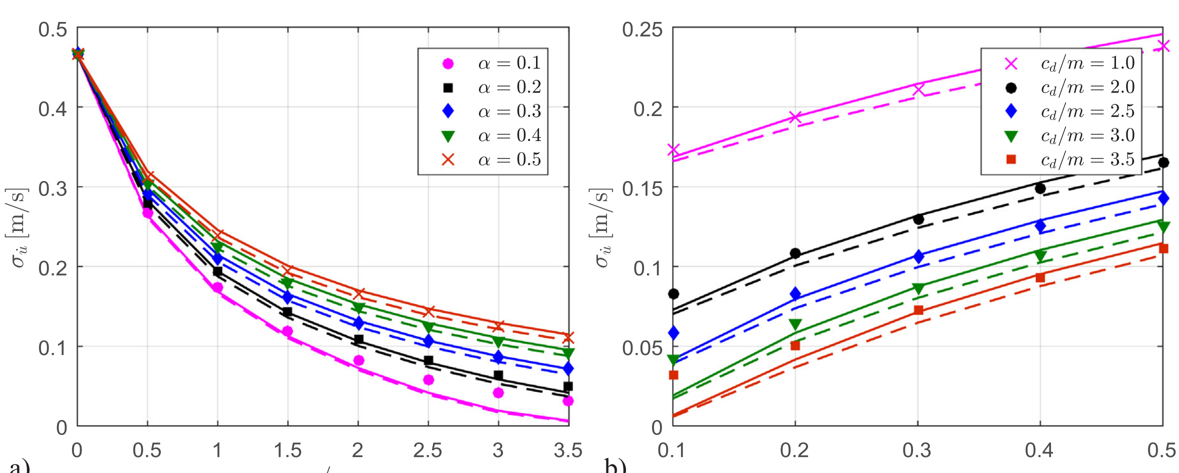

Fig. 4. Standard deviation of the velocity for $T = 1$ s and different constitutive parameters of the FVD: comparison between MCS (markers), FB-G-SLT (dashed lines) and EE-NG-SLT (solid lines).

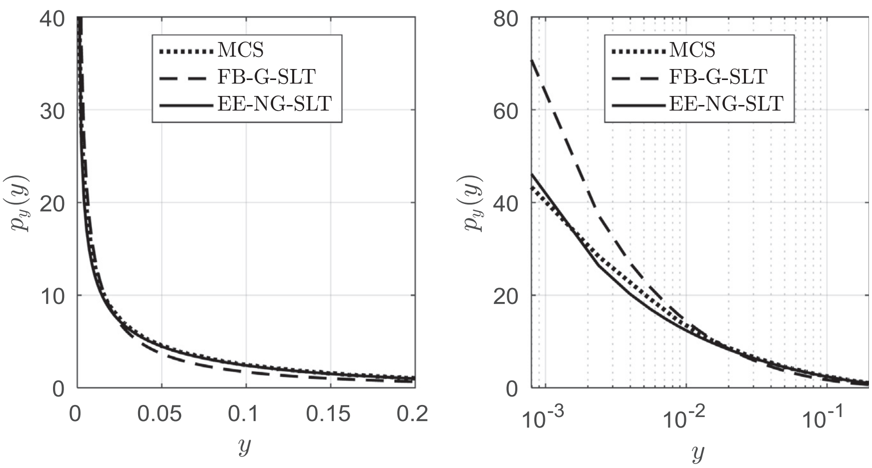

Fig. 5. PDF of dissipated energy for unit time $y \equiv e_d$ in a linear and semi-logarithmic scale plot for $c_d/m = 3.0$ and $\alpha = 0.3$.

system in terms of displacement, velocity, and dissipated energy of FVDs, especially for more severe nonlinearities of the FVDs. Furthermore, the proposed expression does not entail any additional computational effort in comparison with the conventional formula, which is important for design purposes.

Two major assumptions are made in this note and should be acknowledged here. The first one is the stationarity of both the input and output processes. The results reported in this note refer strictly to a stationary response analysis. In reality, earthquake ground motions are non-stationary in nature: strictly speaking, the variances of the response process entering the SLT expressions would be functions of the time. However, it is meant that ground motion records are sufficiently long so that there exists a central part of the response, usually the strong motion phase that is of higher importance for design purposes, in which the stationary assumption could be at least acceptable. The second assumption is that the analysis has been restricted to SDOF systems. However, in multi-degree-of-freedom systems with multiple dampers, one-to-one relationships between equivalent damping coefficients and relative velocity variances are involved. Consequently, the developed expressions for SDOF systems are still applicable by substituting $\sigma_{\dot{u}}$ with $\sigma_{\Delta\dot{u}_j}$, where $\Delta\dot{u}_j$ is the relative velocity at the ends of the j^{th} device.

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