Journal of Economic Dynamics & Control 90 (2018) 118–137

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

Dynamic bankruptcy procedure with asymmetric information between insiders and outsiders☆

Michi Nishihara[a,*], Takashi Shibata[b]

[a] Graduate School of Economics, Osaka University, 1-7 Machikaneyama, Toyonaka, Osaka 560-0043, Japan
[b] Graduate School of Social Sciences, Tokyo Metropolitan University, 1-1 Minami-Osawa, Hachioji, Tokyo 192-0397, Japan

A R T I C L E I N F O

Article history:
Received 1 September 2017
Revised 14 December 2017
Accepted 5 February 2018
Available online 15 February 2018

JEL classification:
D82
G13
G33

Keywords:
Bankruptcy
Adverse selection
Accounting quality
Signaling game
Real options
M&A

A B S T R A C T

We develop a dynamic model in which a distressed firm optimizes the bankruptcy choice and its timing. When the distressed firm's shareholders sell the assets, they are better informed about the asset value than outsiders are. Most notably, we show that this asymmetric information can delay the asset sales to signal asset quality to outsiders. More debt and lower asset value can reduce the signaling cost and mitigate the asset sales delay. We also show that the firm changes the bankruptcy choice from selling out to liquidation bankruptcy when the signaling cost associated with selling out is high. This distortion in the bankruptcy choice greatly lowers the debt value, whereas it has a weak impact on the equity value.

1. Introduction

Since the seminal works by Black and Cox (1976), Leland et al. (1994), Mella-Barral and Perraudin (1997), and Fan and Sundaresan (2000), an increasing number of studies investigate corporate bankruptcy decisions in continuous-time models. In dynamic bankruptcy models, prior works examine bankruptcy timing, debt renegotiation, liquidation, agency conflicts between equity- and debt holders, and so on.[1] However, no study incorporates the stylized fact that a distressed firm has

☆ We thank the editor and anonymous referees, as well as Junichi Imai, Haejun Jeon, Mami Kobayashi, Yue Kuen Kwok, Yuki Sato, Ryuta Takashima, and Neng Wang for their helpful suggestions. We also thank the participants at the 18th Asia Pacific Management Conference, the Winter Workshop on Operations Research, Finance and Mathematics in 2017, the Spring National Conference of Operations Research Society of Japan in 2017, the 5th Asian Quantitative Finance Conference, and Nippon Finance Association 25th Annual Conference for helpful feedback. This work was supported by the JSPS KAKENHI (Grant number JP26350424, JP26242028, JP17K01254, JP16KK0083, JP17H02547) and the JSPS Postdoctoral Fellowships for Research Abroad. This paper was partially written when Michi Nishihara was a JSPS Postdoctoral Fellow for Research Abroad (Visiting Researcher at the Swiss Finance Institute, École polytechnique fédérale de Lausanne). The authors thank the Swiss Finance Institute (especially, Erwan Morellec) and the JSPS Postdoctoral Fellowships for Research Abroad.

* Corresponding author.
E-mail addresses: nishihara@econ.osaka-u.ac.jp (M. Nishihara), tshibata@tmu.ac.jp (T. Shibata).

[1] An incomplete list includes Mella-Barral (1999), Francois and Morellec (2004), Broadie et al. (2007), Lambrecht and Myers (2008), Gryglewicz (2011), Moraux and Silaghi (2014), Christensen et al. (2014), Correia and Poblacion (2015), and Nishihara and Shibata (2016).

https://doi.org/10.1016/j.jedc.2018.02.006
0165-1889/

difficulty selling assets due to asymmetric information about asset quality (e.g., Gilson et al., 2000; Hotchkiss and Mooradian, 1998, and Povel and Singh, 2006) into dynamic bankruptcy models.

To our knowledge, this is the first study that incorporates asymmetric information between insiders and outsiders in a dynamic bankruptcy model. In this novel framework, we address several questions. How can a distressed firm use asset sales timing as a signaling tool to resolve informational issues? How does asymmetric information affect bankruptcy timing and the procedure, as well as the debt and equity values?

Our model builds on the standard setup in Mella-Barral and Perraudin (1997). Shareholders of a distressed firm make a bankruptcy choice between selling out and default, as well as its timing.[2] The model does not distinguish between shareholders and managers, assuming that managers act in shareholders' interests. Selling out is a rather successful exit. Indeed, shareholders sell all assets and obtain the residual value, that is, the sales price minus the face value of debt, while debt holders are repaid the face value of debt. On the other hand, default is an unsuccessful exit. Shareholders stop coupon payments of debt, and the former debt holders take over the firm and can either instantly sell assets (called liquidation bankruptcy) or operate the firm (called operating concern bankruptcy). A fraction of the firm value is lost to bankruptcy costs associated with ownership change.

We add asymmetric information about asset quality to this standard setup. To be precise, the firm's shareholders are better informed than outsiders about whether the firm's running cost is high or low.[3] The asset value will be higher to outsiders as the running cost is lower. Shareholders cannot directly transmit information about whether the firm is a high- or low-cost type to outsiders. Although outsiders cannot directly observe the firm's type, they can guess the firm's type through the sales timing.

In the model, we derive a separating equilibrium where the low-cost firm can separate itself from the high-cost firm through its bankruptcy choice and timing, and the firm's type is perfectly revealed to outsiders. In equilibrium, the low-cost firm's bankruptcy choice and timing can change with asymmetric information, while the high-cost firm's bankruptcy choice and timing remain unchanged. In other words, the low-cost firms pay all costs due to asymmetric information.

Most notably, we show that asymmetric information can delay the low-cost firm's sales timing because the firm signals its asset quality to outsiders by delaying sales until the point at which the high-cost firm cannot imitate the low-cost firm's sales. In the delayed sales case, only shareholders pay signaling costs, and debt holders suffer no loss due to asymmetric information because they are retired the face value of debt. A number of studies have investigated distressed firms' asset sales with depressed prices (cf. fire sales in Pulvino, 1998; Shleifer and Vishny, 1992, and Eckbo and Thorburn, 2008). However, by developing the dynamic bankruptcy model with asymmetric information, we first show that a distressed firm can potentially avoid fire sales due to information issues by delaying asset sales.

We can see the empirical evidence of the key result. For instance, Mason (2005), who examines the liquidation procedure of failed banks, shows that they delay liquidation of assets with high asymmetric information and obtain higher prices. Marquardt and Zur (2015), who examine the effects of target firm accounting quality on merger and acquisitions (M&As), also show that lower target firm accounting quality tends to delay the sales procedure. These findings can be explained by the mechanism that firms delay asset sales timing to signal asset quality.

Although our result is novel in the context of liquidation timing, the signaling mechanism is consistent with that of the prior literature on dynamic trading between informed sellers and uninformed buyers (e.g., Fuchs and Skrzypacz, 2013; Janssen and Roy, 2002, and Fuchs et al., 2016). Actually, in dynamic models, unlike the static adverse selection models (cf. Akerlof, 1970), the sales timing becomes a signal of asset quality to outsiders. Our result also mirrors a key result of the prior literature on real options signaling games (e.g., Grenadier and Malenko, 2011; Morellec and Schürhoff, 2011, and Bustamante, 2012). They show that a firm can accelerate investment timing to signal quality to outsiders in real options models. They study call type options, where acceleration is a signal of a good type, whereas this paper focuses on the put type option of liquidation, where delay is a signal of a good type.

With respect to the delayed sales timing, we find two results that differ from those in the standard literature on dynamic liquidation timing models. One is the impact of existing debt on the sales timing. Under symmetric information, the sales timing is independent of debt because the face value of debt is retired to creditors (cf. Mella-Barral and Perraudin, 1997). In contrast to this standard result, we show that higher debt can accelerate sales. Indeed, higher debt decreases the residual value from selling out and hence decreases the incentive for the high-cost firm to imitate the low-cost firm's sales. Thus, higher debt can play a positive role in alleviating the delay in asset sales timing.

We also find a counter-intuitive impact of asset value on the sales timing. Under symmetric information, higher asset value straightforwardly accelerates asset sales. However, we show that higher asset value can delay asset sales under asymmetric information. This is because higher asset value increases the residual value from selling out and hence increases the incentive for the high-cost firm to imitate the low-cost firm's sales. Thus, higher asset value can play a negative role in intensifying the delay in asset sales timing.

Next, we show that the low-cost firm changes its bankruptcy choice from selling out to liquidation bankruptcy when the signaling cost by delaying sales is higher than the direct cost, that is, the asset value minus the face value of debt. This

[2] Although Mella-Barral and Perraudin (1997) also examine renegotiation between equity- and debt holders, we exclude the possibility of debt renegotiation to focus on asymmetric information between insiders and outsiders.

[3] For simplicity, we consider the case with two types. Our key findings remain unchanged, even if we consider a case with a continuum of types, following Grenadier and Malenko (2011).

result strongly contrasts prior findings that shareholders prefer to default if and only if the asset value is lower than the face value of debt. The failure to sell out lowers the firm value and sales price due to bankruptcy costs. Notably, we find that, in this liquidation bankruptcy case, unlike in the delayed sales case, debt holders suffer severe losses, although shareholders' loss is small. To our knowledge, this is the first study that reveals how equity and debt holders pay information costs due to asymmetric information in the bankruptcy procedure.

Several empirical findings are consistent with these results. For instance, Hotchkiss and Mooradian (1998), Stromberg (2000), and Thorburn (2000) show that distressed firms are more likely to be acquired by better informed firms, including former owners. Marquardt and Zur (2015), McNichols and Stubben (2015), and Cain et al. (2017) show that higher target firm accounting quality plays a role in increasing the success probability of M&As as well as the equity values of targets and bidders. These findings align with our result that the firm can more efficiently sell out under symmetric information than under asymmetric information. Many papers on fire sales of distressed firms (e.g., Pulvino (1998) and Acharya et al. (2007)) show that firms, especially creditors, suffer severe loss due to fire sales to industry outsiders during industry-wide distress. Stromberg (2000) shows that asset sales to less informed firms lowers sales prices, while Thorburn (2000) shows that creditors recover more when former owners buy back firms. These findings support our result that asymmetric information can lead to liquidation bankruptcy, where the firm sells assets at the depressed price due to bankruptcy costs, and debt holders suffer severe losses.

Our contribution to the literature is fourfold. First, we complement the literature on dynamic bankruptcy decisions (e.g., Leland et al. (1994), Mella-Barral and Perraudin (1997), Lambrecht and Myers (2008), and Gryglewicz (2011)) by showing several new results from asymmetric information between insiders and outsiders. Second, we complement the literature on the asset sales of distressed firms (e.g., Shleifer and Vishny (1992), Maksimovic and Phillips (1998), Stromberg (2000), and Eckbo and Thorburn (2008)) by showing the possibility that a distressed firm can avoid selling assets at depressed prices by delaying the asset sales timing. Third, we complement the literature on accounting quality in M&As (e.g., Raman et al. (2013), Marquardt and Zur (2015), and McNichols and Stubben (2015)) by developing the theoretical model to explain empirical findings about the effects of target firm accounting quality on M&As. Lastly, we complement the literature on dynamic signaling models (e.g., Janssen and Roy (2002), Grenadier and Malenko (2011), Daley and Green (2012), and Fuchs et al. (2016)) by showing that a similar mechanism holds in the context of dynamic liquidation timing problems.

The remainder of this paper is organized as follows. We introduce the model setup in Section 2. After Section 3.1 shows the model solution under symmetric information, Section 3.2 shows the equilibrium under asymmetric information. Section 3.3 explains alternative models and checks the robustness of our results. In Section 4, we demonstrate the economic implications along with numerical examples. Section 5 concludes the paper.

2. Model setup

2.1. Firm until bankruptcy

The model builds on the standard setup of Mella-Barral and Perraudin (1997) and Lambrecht and Myers (2008). Consider a firm with console debt with coupon C, that is, the firm continues to pay coupon C to debt holders until bankruptcy. The firm receives continuous streams of earnings before interest and taxes (EBIT) $X(t) - w_i$, where $X(t)$ follows a geometric Brownian motion

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}B(t) \ (t > 0), \quad X(0) = x,$$

where $B(t)$ denotes the standard Brownian motion defined in a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\})$ and μ, $\sigma(>0)$ and $x(>0)$ are constants. We assume that the initial value, $X(0) = x$, is sufficiently high to exclude the firm's bankruptcy at time 0. For convergence, we assume that $r > \mu$, where a positive constant r denotes the risk-free interest rate.

The running cost $w_i(\geq 0)$ can take two types: $w_i = w_L$ (low-cost type) and $w_i = w_H$ (high-cost type), where $w_L < w_H$. All agents know the prior probability of the low-cost type, $q \in (0, 1)$ as well as all information (e.g., $X(t)$ and C) except for the firm's type i. Under symmetric information, all agents know the firm's type i (or equivalently, shareholders can directly prove the firm's type to other agents at no cost), whereas under asymmetric information, only shareholders know the firm's type i and cannot directly prove the firm's type to other agents. We assume that managers act in the shareholders' interests, and hence we do not distinguish between shareholders and managers.

Even in the presence of financial reporting requirements, outsiders do not completely comprehend corporate earnings. Actually, a number of papers have shown the evidence of earnings management through accrual and real activities manipulation (e.g., Healy and Wahlen (1999) and Leuz et al. (2003)). For example, mangers tend to overstate earnings prior to season equity offers (e.g., Teoh et al. (1998b)), initial public offers (e.g., Teoh et al. (1998a)), and stock-financed acquisitions (e.g., Erickson and Wang (1999)) to influence short-term stock price performance. Burns and Kedia (2006) show that stock option compensation is positively related to earnings management. More recent studies, including Raman et al. (2013), Marquardt and Zur (2015), and McNichols and Stubben (2015), reveal the effects of target firm accounting quality on M&As. In particular, Cain et al. (2017) show the evidence of adverse selection in target-initiated transactions and argued that targets can use income increasing accounting changes before being acquired.

2.2. Bankruptcy choice between selling out and default

As in Mella-Barral and Perraudin (1997) and Lambrecht and Myers (2008), we examine the firm's bankruptcy choice between selling out (liquidation without default) and default. In the sales case (closing the business by selling assets to outsiders), the firm sells its total assets and receives the sales price from bidder(s). As in Lambrecht (2001), Lambrecht and Myers (2008), and Nishihara and Shibata (2016), we assume that the sales value is expressed as a linear function

$$\frac{aX(t)}{r-\mu} - \frac{bw_i}{r} + \theta \quad (i = L, H), \tag{1}$$

where $a \in [0, 1)$, $b \in (0, 1)$, and $\theta \geq 0$ are constants.[4] We assume that all agents know the parameter values of a, b, and θ. We can interpret the sales value (1) as follows. After scrapping partial assets at a fixed price θ, an acquirer can perpetually receive cash flows $aX(t) - bw_i$ from the remaining assets, where $a < 1$ and $b < 1$ mean that both the revenues and costs contract due to the decrease in assets. In the transaction, the acquirer retains assets which can be efficiently utilized and liquidates unnecessary assets. Then, the running cost decreases from w_i to bw_i by downsizing, though the profitability a/b might lower than 1 due to economies of scale. The parameter a may include synergies in acquisition, that is, an increase in the acquirer's cash flows in the related business. If we take account of the transaction costs and the acquirer's return, the proceeds decrease to $(1 - k_T - k_A)(aX(t)/(r - \mu) - bw_i/r + \theta)$, where $k_T(> 0)$ and $k_A(> 0)$, stand for the transaction costs and the acquirer's return, respectively.[5] In such a case, we have only to replace a, b, and θ with $a' = (1 - k_T - k_A)a, b' = (1 - k_T - k_A)b$, and $\theta' = (1 - k_T - k_A)\theta$. Following the absolute priority rule (APR) of debt, debt holders are repaid the face value of debt, which equals C/r in the case of the console debt. Shareholders receive the residual value, that is, (1) minus C/r.

On the other hand, in the default case, shareholders declare default and stop paying coupon C on the debt; thereafter, debt holders lose coupon payments. Following the APR, at the time of default, debt holders take over the firm, while shareholders receive nothing. Following the standard literature, including Leland et al. (1994), Goldstein et al. (2001), and Lambrecht and Myers (2008), a fraction $\alpha \in (0, 1)$ of the firm's asset value is lost to the bankruptcy costs (filing fees, attorney fees, etc.) associated with ownership changes at the time of default. Then, the firm value, which former debt holders take over, becomes $(1 - \alpha)U_i(X(t))$, where $U_i(X(t))$ denotes the unlevered firm value with running cost w_i. Former debt holders either operate the firm as a going concern or sell all assets instantly by $(1 - \alpha)\times$ (1).

3. Model solutions

3.1. Symmetric information

As a benchmark, we solve the problem under symmetric information. Outsiders observe the firm's cost w_i and evaluate the assets using (1) depending on the type i and state variable $X(t)$. When the firm chooses to sell out (we denote the sales case by the subscript 1), the equity value, as in Mella-Barral and Perraudin (1997), becomes the following value function:

$$\begin{aligned} E_{i,1}(x) &= \sup_T \mathbb{E}\left[\int_0^T e^{-rt}(X(t) - w_i - C)dt + e^{-rT}\left(\frac{aX(T)}{r-\mu} - \frac{bw_i}{r} + \theta - \frac{C}{r}\right)\right] \\ &= \frac{x}{r-\mu} - \frac{w_i + C}{r} + \sup_T \mathbb{E}\left[e^{-rT}\left(\frac{(a-1)X(T)}{r-\mu} + \frac{(1-b)w_i}{r} + \theta\right)\right] \\ &= \frac{x}{r-\mu} - \frac{w_i + C}{r} + \sup_{x_{i,1}}\left(\frac{x}{x_{i,1}}\right)^{\gamma}\left(\frac{(a-1)x_{i,1}}{r-\mu} + \frac{(1-b)w_i}{r} + \theta\right) \quad (i = L, H), \end{aligned} \tag{2}$$

where $\gamma = 1/2 - \mu/\sigma^2 - \sqrt{(\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2}(< 0)$, and sales time T (we optimize trigger $x_{i,1}$) runs over stopping times.[6]

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[4] Although Mella-Barral and Perraudin (1997) assume a rather simple form, that is, $a = 0$, we assume that the asset value can depend on the state variable $X(t)$. However, we do not directly model fire sales, as Shleifer and Vishny (1992) and Pulvino (1998), among others, discuss. They argue that a distressed firm sells assets at depressed prices because potential bidders in the same industry tend to be financially distressed as well.

[5] There are mixed findings about how firms are sold (e.g., Hotchkiss and Mooradian (1998), Eckbo and Thorburn (2008), Boone and Mulherin (2007), and Meier and Servaes (2015)). Although our model does not specify whether the firm is sold by auction or by negotiation, asymmetric information is stronger in auction cases. By negotiation, asymmetric information between the seller and the acquirer decreases, while the acquirer's return k_A can increase. It would be interesting for future researches to investigate the dynamic choice between auction and negotiation.

[6] We always have $aX(T)/(r - \mu) - bw_i/r + \theta - C/r \geq 0$ when the firm chooses to sell out rather than default. Accordingly, we do not need to impose a limited liability condition in problem (2).

When the firm chooses to default (we denote the default case by the subscript 2), the equity value, as in Leland et al. (1994) and Goldstein et al. (2001), becomes the following value function:

$$\begin{aligned} E_{i,2}(x) &= \sup_T \mathbb{E}\left[\int_0^T e^{-rt}(X(t)-w_i-C)dt\right] \\ &= \frac{x}{r-\mu} - \frac{w_i+C}{r} + \sup_T \mathbb{E}\left[e^{-rT}\left(-\frac{X(T)}{r-\mu}+\frac{w_i+C}{r}\right)\right] \\ &= \frac{x}{r-\mu} - \frac{w_i+C}{r} + \sup_{x_{i,2}} \left(\frac{x}{x_{i,2}}\right)^{\gamma}\left(-\frac{x_{i,2}}{r-\mu}+\frac{w_i+C}{r}\right) \quad (i=L,H), \end{aligned} \tag{3}$$

where default time T (we optimize trigger $x_{i,2}$) runs over stopping times.

As Décamps et al. (2006) show, $\max\{E_{i,1}(x), E_{i,2}(x)\}$ is equal to the result by the dynamic choice between selling out and default if $X(0)=x$ is higher than the sales trigger.[7] Then, by solving (2) and (3), and comparing $E_{i,1}(x)$ and $E_{i,2}(x)$, we obtain the optimal choice between selling out and default. For the proof of the proposition, see Appendix A.

Proposition 1 (Symmetric information). *For each type $i \in \{L,H\}$, we have the following results. If*

$$\left(\frac{1}{1-a}\right)^{\frac{-\gamma}{1-\gamma}} \geq \frac{w_i+C}{(1-b)w_i+r\theta} \tag{4}$$

holds, shareholders choose to sell out. The optimal sales trigger is

$$x_{i,1} = \frac{\gamma(r-\mu)}{(\gamma-1)r}\frac{(1-b)w_i+r\theta}{1-a}. \tag{5}$$

The equity value is

$$E_{i,1}(x) = \frac{x}{r-\mu} - \frac{w_i+C}{r} + \left(\frac{x}{x_{i,1}}\right)^{\gamma}\left(\frac{(a-1)x_{i,1}}{r-\mu}+\frac{(1-b)w_i}{r}+\theta\right). \tag{6}$$

The debt value is $D_{i,1}(x)=C/r$ (risk-less debt).

If (4) does not hold, shareholders choose to default. The optimal default trigger is

$$x_{i,2} = \frac{\gamma(r-\mu)(w_i+C)}{(\gamma-1)r}. \tag{7}$$

The equity value is

$$E_{i,2}(x) = \frac{x}{r-\mu} - \frac{w_i+C}{r} + \left(\frac{x}{x_{i,2}}\right)^{\gamma}\left(-\frac{x_{i,2}}{r-\mu}+\frac{w_i+C}{r}\right). \tag{8}$$

If

$$\left(\frac{1}{1-a}\right)^{\frac{-\gamma}{1-\gamma}} < \frac{w_i+C}{(1-b)w_i+r\theta} \leq \frac{1}{1-a} \tag{9}$$

holds, the debt value is

$$D_{i,2}(x) = \frac{C}{r} - \left(\frac{x}{x_{i,2}}\right)^{\gamma}\left(\frac{C}{r}-(1-\alpha)\left(\frac{ax_{i,2}}{r-\mu}-\frac{bw_i}{r}+\theta\right)\right). \tag{10}$$

If (9) does not hold, the debt value is

$$D_{i,2}(x) = \frac{C}{r} - \left(\frac{x}{x_{i,2}}\right)^{\gamma}\left(\frac{C}{r}-(1-\alpha)U_i(x_{i,2})\right), \tag{11}$$

where $U_i(X(t))$ is defined by (6) with $C=0$, that is, the unlevered firm value with running cost w_i.

The results for $a=0$ are essentially the same as those of Mella-Barral and Perraudin (1997); the results in the default case are essentially the same as those of Goldstein et al. (2001). Note that in (6) and (8), $x/(r-\mu)-(w_i+C)/r$ correspond to the expected value of infinite streams of cash flows $X(t)-(w_i+C)$. The third terms in (6) and (8) are the values of the option to sell out and default, respectively. In (10) and (11), the first term C/r is the face value of debt, that is, the risk-less debt value, while the second term indicates the discount due to default risk. As we show in Appendix B, $D_{i,2}(x)$ is below the face value due to default risk.

[7] When $X(0)=x$ is smaller than the sales trigger, the optimal policy can be the dynamic choice as follows. Shareholders choose to default when $X(t)$ drops to a lower threshold, while they choose to sell out when $X(t)$ rises to a higher threshold. For details, see Décamps et al. (2006).

For low C (compared to the asset value), (4) holds. In this case, shareholders directly sell the total assets to outsiders, and debt holders are retired the face value of debt. Although we refer to this case selling out, Mella-Barral and Perraudin (1997) call this case no bankruptcy because the debt is riskless. The existing debt imposes no efficiency loss, that is, the firm value $E_{i,1}(x) + C/r$ is equal to the maximum value, which is the unlevered firm value,[8] because the firm avoids incurring bankruptcy costs. In short, the firm is optimally liquidated with no costs.

For intermediate C, (9) holds, and $x_{i,2} \le x_{i,1}$ holds in this region. In this case, shareholders declare default at the default trigger $x_{i,2}$, and debt holders are not retired the face value of debt but instead take over the firm. Former debt holders sell the total assets as soon as $X(t)$ decreases below the sales trigger $x_{i,1}$.[9] Due to $x_{i,2} \le x_{i,1}$, they sell the total assets immediately after taking over the firm, although the asset value is less than the face value. Following Mella-Barral and Perraudin (1997), we call this case liquidation bankruptcy which can be related to Chapter 7 (liquidation) bankruptcy in the United States. The existing debt imposes an efficiency loss. Indeed, due to bankruptcy cost α and delayed liquidation, the firm value $E_{i,2}(x) + D_{i,2}(x)$ is discounted from the unlevered firm value. In short, the firm inefficiently liquidates by incurring additional costs.

For high C, (9) does not hold, and $x_{i,2} > x_{i,1}$ holds. In this case, shareholders declare default at the default trigger $x_{i,2}$, and debt holders are not retired at the face value of debt, but instead take over the firm, after which the former debt holders operate the firm as a going concern until $X(t)$ decreases below the sales trigger $x_{i,1}$. Following Mella-Barral and Perraudin (1997), we call this case operating concern bankruptcy. Operating concern bankruptcy can be related to Chapter 11 (reorganization) bankruptcy in the United States, although we do not model debt renegotiation and restructuring process.[10] The existing debt imposes an efficiency loss. In fact, due to bankruptcy cost α, the firm value $E_{i,2}(x) + D_{i,2}(x)$ is discounted from the unlevered firm value.

Proposition 1 shows that a firm with less debt and higher asset value tends to sell out without default. In addition, if the firm goes into bankruptcy, less debt and higher asset value tend to lead to liquidation bankruptcy rather than operating concern bankruptcy. These results are consistent with the stylized fact that smaller/younger firms with lower leverage ratios are more likely to go into Chapter 7 bankruptcy rather than Chapter 11 bankruptcy (e.g., Bris et al. (2006)). Another interesting result from Proposition 1 is the impact of cash flow volatility σ. Because $\partial\gamma/\partial\sigma > 0$, condition (4) is more likely to hold for lower σ. Intuitively, a higher σ increases the option value of default more than the option value of selling out because the convexity of the option to default is stronger than that for selling out.[11] This implies that a firm with higher cash flow volatility tends to fail to sell out. Although this is not seen in Mella-Barral and Perraudin (1997), who assume $a = 0$, this result is consistent with the stylized fact that higher cash flow volatility is more likely to lead to an unsuccessful bankruptcy.

3.2. Asymmetric information

Although Section 3.1 assumed that outsiders have perfect knowledge of the distressed firm's asset quality, this is not the case in the real world. Many firms have difficulty selling assets at a fair price, especially during financial distress. For example, refer to Gilson et al. (2000), Povel and Singh (2006), and Hotchkiss et al. (2008) on this matter. In particular, small and/or private firms, which have less transparency and disclosure, pay costs due to strong asymmetric information. Indeed, Marquardt and Zur (2015), McNichols and Stubben (2015), and Cain et al. (2017) show the effects of accounting quality on the sales procedure.

Now, suppose that asymmetric information exists between the firm's insiders and outsiders. Only shareholders (who are equal to managers in this paper) know the firm's type i, while outsiders do not observe the firm's type. Intuitively, shareholders of the high-cost firm may have an incentive to imitate the low-cost firm's sales timing and receive a higher asset value (1) with $i = L$, whereas shareholders of the low-cost firm have no incentive to imitate the high-cost firm. Although financial reporting requirements try to prevent managers from cheating, managers can potentially engage in earnings management through accrual and real activities manipulation (e.g., Healy and Wahlen (1999) and Leuz et al. (2003)).

Shareholders receive nothing when they declare default rather than selling out. Then, shareholders of the high-cost firm have no incentive to imitate the low-cost firm if the low-cost firm chooses to default. The default time (the default trigger $x_{i,2}$ defined by (7)), which is not affected by asymmetric information, reveals the firm's type to outsiders. This means that former debt holders, who take over the firm after default, have no concerns about asymmetric information. This reasoning leads to the following proposition.

[8] Following Mella-Barral and Perraudin (1997) and Lambrecht and Myers (2008), we omit the tax benefits of debt from the model. Then, the unlevered firm value agrees with the maximum firm value.

[9] Even after default, the sales trigger remains unchanged from $x_{i,1}$ defined by (5) because both cash flow and asset value are multiplied by $(1-\alpha)$.

[10] A number of papers, including Sundaresan and Wang (2007), Moraux and Silaghi (2014), Christensen et al. (2014), Shibata and Nishihara (2015), Nishihara and Shibata (2016), and Silaghi (2018) incorporate the debt renegotiation and restructuring process into dynamic bankruptcy timing models. It could be interesting for a future study to investigate the effects of asymmetric information on debt renegotiation and the restructuring process.

[11] By the same logic, Kort et al. (2010) show that higher volatility is more likely to lead to lumpy rather than stepwise investment.

Proposition 2 (Low-cost firm default case)**.**
Case I: *Suppose that*

$$\left(\frac{1}{1-a}\right)^{\frac{-\gamma}{1-\gamma}} < \frac{w_L + C}{(1-b)w_L + r\theta}, \tag{12}$$

that is, shareholders of the low-cost firm choose to default under symmetric information. The bankruptcy choice and timing, as well as the equity and debt values, of both types of firms are unchanged from those in the case of symmetric information.

When the face value of debt is high compared to asset value, (12) is likely to hold, and asymmetric information has no effects on the bankruptcy procedure. This is straightforward because asymmetric information matters only in the sales case. More interestingly, high volatility σ removes losses due to asymmetric information because (12) is more likely to hold for higher σ. Lambrecht and Myers (2008) and Nishihara and Shibata (2017) argue that risky debt mitigates manager-shareholder conflicts. Although they focus on manager-shareholder conflicts, Proposition 2 complements their results by showing that more debt and higher risks can mitigate insider-outsider conflicts.

Next, suppose that (12) does not hold. In general, there can be two types of equilibria, namely, separating (informative) equilibrium and pooling (uninformative) equilibrium. Following a majority of papers (e.g., Janssen and Roy (2002), Grenadier and Malenko (2011), and Adelino et al. (2017)),[12] we focus mainly on the least-cost separating equilibrium for the low-cost firm. In Appendix C, we present pooling equilibria and explain why all pooling equilibria are removed by the Intuitive Criterion proposed by Cho and Kreps (1987).

In the separating equilibrium, shareholders of the low-cost firm optimize the bankruptcy choice and timing within the policies that can separate the low-cost firm from the high-cost firm. When the low-cost firm prefers to sell out in the separating equilibrium, as in (2), the equity value of the low-cost firm is

$$\begin{aligned} E_L^s(x) &= \sup_T \mathbb{E}\left[\int_0^T e^{-rt}(X(t) - w_L - C)dt + e^{-rT}\left(\frac{aX(T)}{r-\mu} - \frac{bw_L}{r} + \theta - \frac{C}{r}\right)\right] \\ &= \frac{x}{r-\mu} - \frac{w_L + C}{r} + \sup_T \mathbb{E}\left[e^{-rT}\left(\frac{(a-1)X(T)}{r-\mu} + \frac{(1-b)w_L}{r} + \theta\right)\right] \\ &= \frac{x}{r-\mu} - \frac{w_L + C}{r} + \sup_{x_L^s}\left(\frac{x}{x_L^s}\right)^{\gamma}\left(\frac{(a-1)x_L^s}{r-\mu} + \frac{(1-b)w_L}{r} + \theta\right), \end{aligned} \tag{13}$$

where the sales trigger x_L^s is optimized subject to the following incentive compatibility condition (ICC)

$$\underbrace{\frac{x}{r-\mu} - \frac{w_H + C}{r} + \left(\frac{x}{x_L^s}\right)^{\gamma}\left(\frac{(a-1)x_L^s}{r-\mu} + \frac{w_H - bw_L}{r} + \theta\right)}_{\text{Value by imitation}} \le \underbrace{\max\{E_{H,1}(x), E_{H,2}(x)\}}_{\text{Value by truthful action}}. \tag{14}$$

Throughout the paper, the superscript s represents the separating equilibrium. The left-hand side of (14) is the expected value of the high-cost firm's shareholders who imitate the low-cost firm's sales timing and receive the higher asset value $ax_L^s/(r-\mu) - bw_L/r + \theta$, whereas the right-hand side of (14) is the expected value of the high-cost firm's shareholders who truthfully take the first-best policy derived in Proposition 1. Under ICC, shareholders of the high-cost firm are better off following the first-best policy as the high-cost type rather than imitating the low-cost firm's sales timing.[13] Thus, through the sales trigger x_L^s, outsiders verify the low-cost type and pay the higher asset value $ax_L^s/(r-\mu) - bw_L/r + \theta$ to the low-cost firm. Appendix D shows that the single crossing condition holds with respect to the sales timing.

Recall that shareholders of the low-cost firm can gain $E_{L,2}(x)$ by choosing default because the high-cost firm has no incentive to imitate the low-cost firm's default timing. Accordingly, shareholders of the low-cost firm gain $\max\{E_L^s(x), E_{L,2}(x)\}$. By solving problem (13) subject to (14) and comparing $E_L^s(x)$ with $E_{L,2}(x)$, we have the least-cost separating equilibrium as follows. For the proof, see Appendix E.

Proposition 3 (Separating equilibrium)**.** *Suppose that (12) does not hold.*
Case II: *Suppose that $x_{L,1}$ defined by (5) with $i = L$ satisfies ICC (14). The bankruptcy choices and timing, as well as the equity and debt values, of both types of firms do not change from those of the symmetric information case.*
Case III: *Suppose that $x_{L,1}$ dose not satisfy ICC (14). We define $x_L^s \in (0, x_{L,1})$ by equating ICC (14), that is, the solution to*

$$\begin{aligned} &\left(\frac{1}{x_L^s}\right)^{\gamma}\left(\frac{(a-1)x_L^s}{r-\mu} + \frac{w_H - bw_L}{r} + \theta\right) \\ &= \max\{\left(\frac{1}{x_{H,1}}\right)^{\gamma}\left(\frac{(a-1)x_{H,1}}{r-\mu} + \frac{(1-b)w_H}{r} + \theta\right), \left(\frac{1}{x_{H,2}}\right)^{\gamma}\left(-\frac{x_{H,2}}{r-\mu} + \frac{w_H + C}{r}\right)\}, \end{aligned} \tag{15}$$

[12] Important exceptions are Morellec and Schürhoff (2011) and Bustamante (2012) who closely examine both separating and pooling equilibria.

[13] We consider only ICC for the high-cost firm because we can easily check that the solution satisfies ICC for the low-cost firm, that is, the low-cost firm has no incentive to imitate the high-cost type.

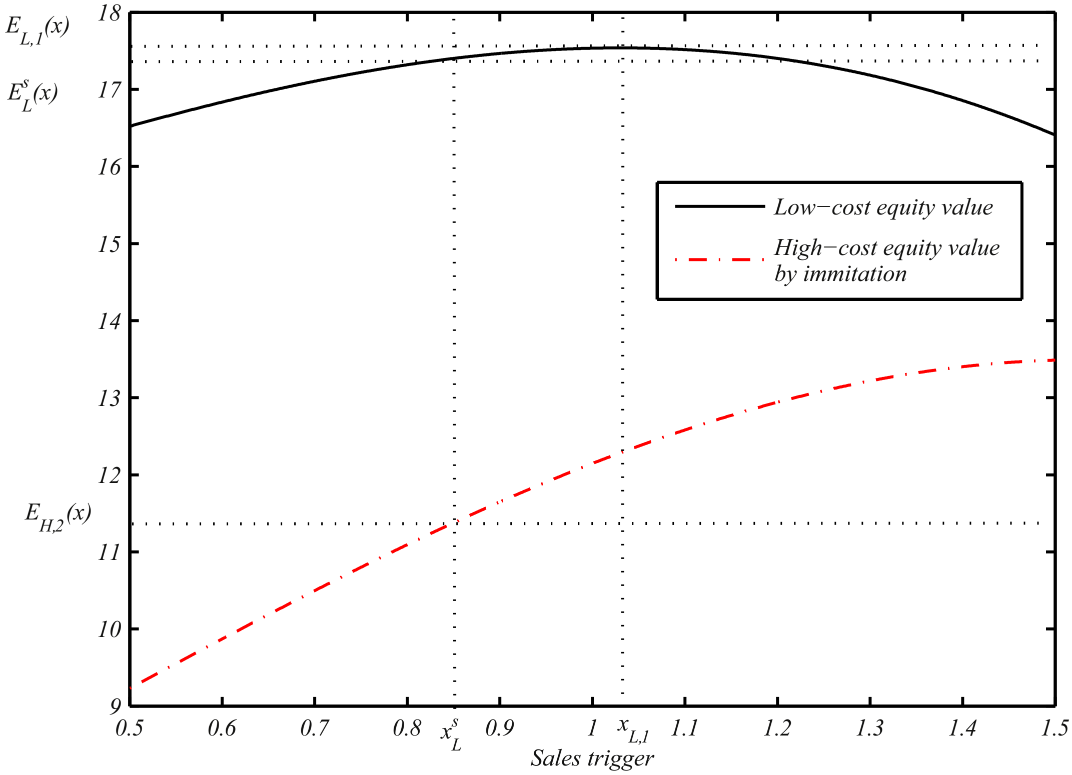

Fig. 1. The low-cost firm's equity value and high-cost firm's imitation value as functions of the sales trigger in Case (III) of Proposition 3. The figure illustrates how to determine the sales trigger x^s_L and the equity value $E^s_L(x)$ in the separating equilibrium. The parameter values are set in Table 1, where $\max\{E_{H,1}(x), E_{H,2}(x)\} = E_{H,2}(x)$ holds.

where $x_{H,1}$ and $x_{H,2}$ are defined by (5) and (7) with $i = H$, respectively. Suppose that

$$\left(\frac{1}{x^s_L}\right)^{\gamma}\left(\frac{(a-1)x^s_L}{r-\mu}+\frac{(1-b)w_L}{r}+\theta\right) \ge \left(\frac{1}{x_{L,2}}\right)^{\gamma}\left(-\frac{x_{L,2}}{r-\mu}+\frac{w_L+C}{r}\right), \tag{16}$$

where $x_{L,2}$ is defined by (7) with $i = L$. Shareholders of the low-cost firm choose to sell out. The sales trigger is x^s_L, and the equity value is

$$E^s_L(x) = \frac{x}{r-\mu} - \frac{w_L+C}{r} + \left(\frac{x}{x^s_L}\right)^{\gamma}\left(\frac{(a-1)x^s_L}{r-\mu}+\frac{(1-b)w_L}{r}+\theta\right). \tag{17}$$

The debt value remains unchanged from $D_{L,1}(x) = C/r$ (risk-less debt). The bankruptcy choice and timing, as well as the equity and debt values, of the high-cost firm do not change from those of the symmetric information case.[14]

Case IV: *Suppose that $x_{L,1}$ does not satisfy ICC (14) and that (16) does not hold. Shareholders of the low-cost firm choose to default. The default trigger is $x_{L,2}$, and the equity and debt values of the low-cost firm are $E_{L,2}(x)$ and $D_{L,2}(x)$, defined by (8) and (10) with $i = L$, respectively. The bankruptcy choice and timing, as well as the equity and debt values, of the high-cost firm are unchanged from those of the symmetric information case.*

In Case (II), the low-cost firm has no incentive to imitate the high-cost firm's first-best sales trigger $x_{L,1}$. This case occurs mainly because the firm's types are discrete. We do not have this case when we consider a continuum of types. We omit the explanation here because the results are trivial.[15]

In Case (III), the low-cost firm signals the firm's type to outsiders by decreasing the sales trigger. Fig. 1 illustrates the mechanism that determines the sales trigger x^s_L and equity value $E^s_L(x)$ in this case. The parameter values are set in Table 1. In the figure, the upper curve shows (13) as a function of x^s_L, while the lower curve shows the left-hand side of ICC (14) as a function of x^s_L. Without ICC (14), the equity value is the first-best value $E_{L,1}(x) = 17.538$ with the sales trigger $x_{L,1} =$

[14] In Case (III), outsiders' belief is given as follows: The firm is a low-cost type at probability one for any sales at a trigger in $(x, x^s_L]$, while the firm is a high-cost type at probability one for the other policies. In Cases (I), (II), and (IV), outsiders' belief is trivial.

[15] According to our computations, Case (II) can occur for low levels of a and b. This is because lower a and b decrease the imitation value, i.e., the left-hand side of ICC (14). For intermediate and high levels of a and b, ICC (14) becomes binding, which leads to Cases (III) or (IV).

Table 1
Baseline parameter values.

r	μ	σ	x	w_H	w_L	q	a	b	θ	α	C
0.06	0.01	0.2	2	1	0.5	0.5	0.5	0.5	13	0.3	1

1.03. However, the lower curve is above $\max\{E_{H,1}(x), E_{H,2}(x)\} = E_{H,2}(x) = 11.381$ for $x_{L,1} = 1.03$, and hence outsiders cannot distinguish between low- and high-cost firms. The low-cost firm decreases the sales trigger from $x_{L,1} = 1.03$ to $x_L^s = 0.851$, at which the lower curve crosses $\max\{E_{H,1}(x), E_{H,2}(x)\} = E_{H,2}(x) = 11.381$, to signal the firm's type to outsiders. Due to the second-best trigger $x_L^s = 0.851$, the low-cost firm's equity value $E_L^s(x) = 17.402$ is below $E_{L,1}(x) = 17.538$, but it is still above $E_{L,2}(x) = 17.296$.

This logic is in line with that of the literature on dynamic trading between informed sellers and uninformed buyers (e.g., Fuchs and Skrzypacz, 2013; Janssen and Roy, 2002, and Fuchs et al., 2016). The delay in sales makes it more costly for the high-cost firm to feign the low-cost type. The low-cost firm can reveal its type to outsiders by delaying the sales timing to the point at which the high-cost firm does not imitate the low-cost firm's policy. Recently, Grenadier and Malenko (2011), Morellec and Schürhoff (2011), and Bustamante (2012) have developed real options models under asymmetric information between insiders and outsiders. Although they examine call type options such as investment and financing options, we examines the put type option of selling out. Then, in our model delay is a signal of a good type, while acceleration is a signal of a good type in Morellec and Schürhoff (2011) and Bustamante (2012). This is a key difference between our model and the previous real options signaling models.

Although in Case (III), the signaling cost decreases the equity value from $E_{L,1}(x)$ to $E_L^s(x)$, shareholders of the low-cost firm are better off selling out because $E_L^s(x) > E_{L,2}(x)$. Note that lower C, higher θ, higher a, and lower b tend to lead to Case (III). In other words, when the face value of debt is lower than the asset value, the firm tends to adopt the policy of delayed sales.

A number of papers, including those by Shleifer and Vishny (1992), Hotchkiss and Mooradian (1998), Acharya et al. (2007), and Eckbo and Thorburn (2008), both theoretically and empirically investigate asset sales at discounted prices by firms in financial distress. Nevertheless, to our knowledge, no paper shows that a distressed firm can potentially delay asset sales to keep asset prices higher. Thus, our result complements the literature on distressed firms' fire sales by showing that a distressed firm can avoid fire sales by delaying the asset sales timing as a signaling tool under asymmetric information.

In Case (IV), the signaling cost in the sales decreases the equity value below the equity value in the default case, that is, $E_L^s(x)$ is lower than $E_{L,2}(x)$. In other words, the signaling cost is higher than the direct cost, that is, the asset value minus the face value of debt. Then, shareholders of the low-cost firm give up selling out and resort to a default. They do not change the default trigger from $x_{L,2}$ because the high-cost firm has no incentive to imitate the low-cost firm's default timing. Although the default trigger does not change from $x_{L,2}$, the equity value decreases from the first-best value $E_{L,1}(x)$ to $E_{L,2}(x)$ with the change in bankruptcy choice.

In Case (IV), liquidation bankruptcy occurs, in other words, debt holders sell the firm immediately after taking over the firm. Note that outsiders observe that the firm is the low-cost type at the default trigger $x_{L,2}$, but asymmetric information affects the debt value. In fact, the debt value changes from the first-best value $D_{L,1}(x) = C/r$ to $D_{L,2}(x)$, defined by (10) with $i = L$, with asymmetric information. Case (IV), where neither (16) nor (12) holds, tends to hold when C is balanced with levels of θ, a, and b. In other words, when the face value of debt is close to the asset value, under asymmetric information, the firm can change the bankruptcy choice from selling out to liquidation bankruptcy. This is in sharp contrast with the standard result that a firm goes into default if and only if the asset value is lower than the face value of debt (cf. Mella-Barral and Perraudin (1997)).

Next, we examine costs due to asymmetric information between insiders and outsiders. Note that the high-cost firm's equity and debt values do not change with asymmetric information. We denote by $E_L^*(x)$ and $D_L^*(x)$ the equity and debt values under symmetric information, respectively, and denote by $E_L^{**}(x)$ and $D_L^{**}(x)$ the equity and debt values under asymmetric information, respectively. Proposition 3 immediately yields the following corollary.

Corollary 1 (Informational cost).
Cases (I) and (II):

$$E_L^*(x) - E_L^{**}(x) = 0$$
$$D_L^*(x) - D_L^{**}(x) = 0$$

Case (III):

$$E_L^*(x) - E_L^{**}(x) = \left(\frac{x}{x_{L,1}}\right)^{\gamma}\left(\frac{(a-1)x_{L,1}}{r-\mu} + \frac{(1-b)w_L}{r} + \theta\right) - \left(\frac{x}{x_L^s}\right)^{\gamma}\left(\frac{(a-1)x_L^s}{r-\mu} + \frac{(1-b)w_L}{r} + \theta\right) (> 0)$$
$$D_L^*(x) - D_L^{**}(x) = 0$$

Case IV:

$$E_L^*(x) - E_L^{**}(x) = \left(\frac{x}{x_{L,1}}\right)^\gamma \left(\frac{(a-1)x_{L,1}}{r-\mu} + \frac{(1-b)w_L}{r} + \theta\right) - \left(\frac{x}{x_{L,2}}\right)^\gamma \left(-\frac{x_{L,2}}{r-\mu} + \frac{w_L + C}{r}\right)(>0)$$

$$D_L^*(x) - D_L^{**}(x) = \left(\frac{x}{x_{L,2}}\right)^\gamma \left(\frac{C}{r} - (1-\alpha)\left(\frac{ax_{L,2}}{r-\mu} - \frac{bw_L}{r} + \theta\right)\right)$$

In Cases (I) and (II), there is no informational cost because the low-cost firm conducts the first-best policy. In Case (III), the signaling cost equals the informational cost, and hence, shareholders of the low-cost firm pay the cost. Actually, the sales trigger decreases from $x_{L,1}$ to x_L^s, which lowers the equity value from $E_{L,1}(x)$ to $E_L^s(x)$. On the other hand, debt holders pay no cost because, as in the symmetric information case, they are retired the face value of debt. The delay in the sales timing does not affect the debt value.

Case (IV) is the most interesting case, where both equity and debt holders pay the informational cost. As we explained above, shareholders give up selling out and lose the residual value, that is, the asset value minus the face value of debt. Although we cannot mathematically prove that $C/r - (1-\alpha)(ax_{L,2}/(r-\mu) - bw_L/r + \theta)$ is positive, this term is always positive according to our numerical analysis.[16] This implies that debt holders cannot recover the face value of debt by selling assets. Thus, both equity and debt holders suffer losses due to asymmetric information. In Section 4.1, we will see how equity and debt holders share the informational cost in Case (IV).

3.3. Alternative models

This paper models asymmetric information about the quality of existing assets (i.e., running cost w_i) of the target firm because we focus on the nearly bankrupt firm. On the other hand, previous papers, such as Grenadier and Malenko (2011), Morellec and Schürhoff (2011), and Bustamante (2012), model asymmetric information about the quality of a new investment project (i.e., future revenue) because they focus on growth investment and financing problems. We can add asymmetric information of future revenue to our setup in several ways. For instance, the asset quality can also influence the revenue parameter a (say, $a_L > a_H$) through synergies in acquisition. When we consider a difference between a_L and a_H, the left-hand side of ICC (14) increases, which makes ICC more binding. Then, the low-cost firm's sales time in Case (III) is further delayed, and the firm is more likely to choose to default (i.e., Case (IV)).

Below, we explain that even in the absence of asymmetric information about running cost, other types of asymmetric information lead to the same results as those of Section 3.2. Now, we remove the running cost w_i from our model. First, asymmetric information about the firm's competitive advantage over its rivals may exist between insiders and outsiders. In this case, the firm's insiders are better informed about how long the existing product is likely to maintain market share. We can model this situation by assuming that revenue $X(t)$ drops to zero following the Poisson process with the arrival rate $\lambda_i = \lambda_L$ or λ_H, where $\lambda_L < \lambda_H$. For simplicity, we assume that the Poisson process is independent of the Brownian motion $B(t)$ of the dynamics of $X(t)$. Although the firm value to outsiders is equal to $aX(t)/(r + \lambda_i - \mu) + \theta$ depending on the firm's type i, they do not know whether $i = L$ or H. Only the firm's insiders know the arrival rate λ_i. The bad-type firm with λ_H has an incentive to sell the firm at a higher price by imitating the good-type firm with λ_L. However, the bad-type firm is less patient than the good-type firm because the bad-type's $X(t)$ is more likely to collapse. This corresponds to that in Section 3.2, the bad-type firm is less patient because of the higher running cost w_H. Taking account of the tradeoff, the separating equilibrium can occur, where the good-type firm can separate itself from the bad-type firm by decreasing the sales trigger. If the signaling cost is higher than the direct cost, that is, the asset value minus the face value of debt, the firm changes the bankruptcy choice from selling out to default.

Asymmetric information can also be modeled by filtering theory (see Chapter 9 of Liptser and Shiryaev (2001)). For example, suppose that only insiders know the expected growth rate of $X(t)$, $\mu_i = \mu_L$ or μ_H, where $\mu_L < \mu_H$, while all agents can observe cash flows $X(t)$. Although the firm value to outsiders is equal to $aX(t)/(r - \mu_i) + \theta$ depending on the firm's type i, they do not know whether $i = L$ or H. In this case, outsiders can estimate the value of μ_i through observations $X(t)$. By defining $Y(t) = \log X(t)$ and $\eta_i = \mu_i - \sigma^2/2$, we have

$$\mathrm{d}Y(t) = \eta_i \mathrm{d}t + \sigma \mathrm{d}B(t), \ \ Y(0) = \log x.$$

Outsiders' problem of estimating parameter η_i through observations $Y(t)$ can be treated as a standard filtering problem (e.g., Décamps et al., 2005 and Gryglewicz, 2011). By Theorem 9.1 of Liptser and Shiryaev (2001), outsiders' belief of $\eta_i = \eta_H$ at time t (i.e., the probability of the good type conditional on observations up to time t), denoted by $\pi(t)$, follows

$$\mathrm{d}\pi(t) = \frac{(\eta_H - \eta_L)\pi(t)(1 - \pi(t))}{\sigma}\mathrm{d}W(t), \ \ \pi(0) = q \in (0, 1) \tag{18}$$

[16] If the term is negative, we have the abnormal result that $D_L^{**}(x) > C/r$, which means that the debt value becomes the face value plus the expected gain due to default. Nishihara and Shibata (2017) show that the abnormal result can hold for plausible parameter values under asymmetric information between managers and shareholders. In this paper, however, we cannot find the abnormal result for any numerical example even when we set $\alpha = 0$.

Table 2
Baseline results.

$x_{L,1}$	x_L^s	$x_{H,2}$	$E_L^*(x)$	$E_L^{**}(x)$	$E_H^*(x)$	$D_L^*(x)$	$D_L^{**}(x)$	$D_H^*(x)$
1.03	0.851	1.28	17.538	17.402	11.381	16.667	16.667	14.404

where

$$dW(t) = \frac{dY(t) - \eta_L dt - (\eta_H - \eta_L)\pi(t)dt}{\sigma}, \quad W(0) = 0. \tag{19}$$

We can see from (18) and (19) that for lower volatility σ, outsiders update the belief $\pi(t)$ more quickly.

How does the update of estimate influence the signaling problem? The estimation process $\pi(t)$ does not influence the separating equilibrium, which is independent of the prior probability of the type. Indeed, Propositions 2 and 3 and Corollary 1 in Section 3.2 do not depend on the prior probability of the good type, q. Then, we have similar results to those of Section 3.2. The bad-type firm with μ_L has an incentive to sell the firm at a higher price by imitating the good-type firm with μ_H, while the bad-type firm is impatient due to the low growth rate μ_L. By this tradeoff, the separating equilibrium can occur, where the good-type firm decreases the sales trigger to signal asset quality to outsiders. If the signaling cost is higher than the direct cost, the firm prefers to default. On the other hand, the update of estimate influences the pooling equilibrium. Actually, we can see that the pooling equilibrium described in Appendix C depend on the prior probability q. In the filtering model, the probability $\pi(t)$ is updated by (18). As time passes, $\pi(t)$ approaches 1 if the real type is $i = H$. Then, the pooling equilibrium becomes less costly for the good-type firm. This suggests that the update of estimate may change from the separating equilibrium to the pooling equilibrium, although any pooling equilibrium cannot survive under the Intuitive Criterion in Cho and Kreps (1987) (see Appendix C).

4. Economic implications

4.1. Comparative statics

We set the baseline parameter values in Table 1. We report the baseline results in Table 2 for the parameter values. Recall that the superscript * denotes symmetric information, while the superscript ** denotes asymmetric information. Under symmetric information, the low-cost firm prefers to sell out at $x_{L,1} = 1.03$, whereas the high-cost firm prefers to default at $x_{H,2} = 1.28$. Naturally, the low-cost firm's equity and debt values are higher than those of the high-cost firm. The low-cost firm's debt value $D_L^*(x)$ is equal to the face value $C/r = 16.667$, whereas the high-cost firm's debt value is discounted due to default risk.

Under asymmetric information, Case (III) holds. The low-cost firm lowers the sales trigger from $x_{L,1} = 1.03$ to $x_L^s = 0.851$ to signal asset quality to outsiders. Shareholders pay the signaling cost $E_L^*(x) - E_L^{**}(x) = 0.136$, whereas debt holders pay no cost due to asymmetric information. Notably, the informational cost is quite low, that is, $(E_L^*(x) - E_L^{**}(x))/E_L^*(x) = 0.00775$, although the impact on the sales trigger is large, that is, $(x_{L,1} - x_L^s)/x_{L,1} = 0.174$.

4.1.1. Effects of coupon C

Fig. 2 shows the equity and debt values, as well as the informational costs and bankruptcy triggers with varying levels of coupon C. Case (III) holds for $C \le 1.04$. In this region of the top right panel, we have $x_L^* = x_{L,1} = 1.03$ and $x_L^{**} = x_L^s$. Under asymmetric information, the low-cost firm sells out at the sales trigger $x_L^s(< x_{L,1} = 1.03)$ to signal the firm's type to outsiders. The high-cost firm chooses to sell out for $C \le 0.94$, while it chooses to default for $C > 0.94$. In the top right panel, the high-cost firm's change in bankruptcy choice generates a kink of x_L^{**} at $C = 0.94$ through ICC.

Case (IV) holds for $C \in (1.04, 1.06]$. In this region of the top right panel, we have $x_L^* = x_{L,1} = 1.03$ and $x_L^{**} = x_{L,2}$, and hence x_L^{**} jumps from x_L^s to $x_{L,2}$ at $C = 1.04$. The debt value $D_L^{**}(x)$ also jumps from C/r to $D_{L,2}(x)$ at $C = 1.04$. The low-cost firm changes its bankruptcy choice from selling out to liquidation bankruptcy with asymmetric information. From another viewpoint, the maximum risk-free debt level decreases from $C = 1.06$ to $C = 1.04$ due to asymmetric information. When we relate the maximum risk-free debt level to debt capacity, the result is consistent with the stylized fact that stronger asymmetric information (e.g., a higher ratio of intangible assets to tangible assets) decreases the debt capacity.

Case (I) holds for $C > 1.06$. In this region of the top right panel, we have $x_L^* = x_L^{**} = x_{L,2}$, and hence x_L^* jumps from $x_{L,1}$ to $x_{L,2}$ at $C = 1.06$. The debt value $D_L^*(x)$ also jumps from C/r to $D_{L,2}(x)$ at $C = 1.06$. In this case, the low-cost firm's first-best bankruptcy choice is default, and asymmetric information does not affect the bankruptcy procedure. For $C \in (1.06, 1.2]$ in the top right panel, $x_L^* = x_L^{**} = x_{L,2}$ is less than $x_{L,1} = 1.03$, which means liquidation bankruptcy. Note that we have a region of operating concern bankruptcy for much higher C.

We can see a novel result in the top right panel. Indeed, the sales trigger x_L^s increases in $C \in (0.94, 1.04]$ in Case (III). This result is not found in the existing literature. The standard literature (e.g., Mella-Barral and Perraudin (1997)) shows that as in (5), the sales timing is independent of C because the firm repays the face value of debt at the time of sales. With asymmetric information, however, C influences the sales timing through ICC. The mechanism is explained below. For $C \in (0.94, 1.04]$, the high-cost firm prefers to default in the first-best case. While a higher C increases the right-hand side of

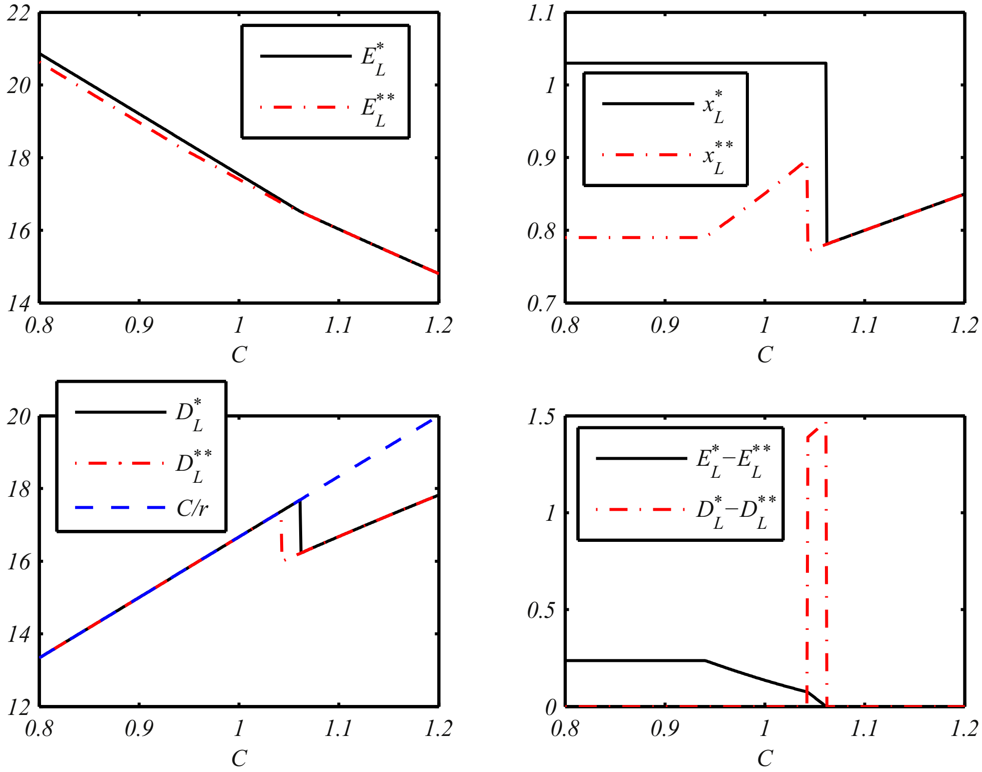

Fig. 2. Comparative statics with respect to C. The other parameter values are set in Table 1. Cases (III), (IV), and (I) hold for $C \le 1.04$, $C \in (1.04, 1.06]$, and $C > 1.06$, respectively.

(15), it does not change the left-hand side of (15). Then, a higher C increases x_L^s by (15). In other words, the low-cost firm can increase x_L^s because a higher C decreases the incentive for the high-cost firm to imitate the low-cost firm's sales timing.

Although Lambrecht and Myers (2008) also show that a higher C speeds up closure without default, the mechanism is quite different from ours. They focus on manager-shareholder conflicts rather than asymmetric information between insiders and outsiders. In their model, managers receive a fraction of the cash flows until liquidation, and due to managerial rents, managers have an incentive to delay liquidation. A higher C speeds up the closure because a higher C decreases managerial rents and the incentive to delay closure.

We also find some interesting results in the bottom right panel. Informational cost $E_L^*(x) - E_L^{**}(x)$ decreases in C in Cases (III) and (IV) because a higher C decreases the signaling cost. Recall that under symmetric information, the firm value in the sales case agrees with the maximum value, which is the unlevered firm value. On the other hand, under asymmetric information, the firm value with $C = 0.94$ is the highest because informational cost $E_L^*(x) - E_L^{**}(x)$ decreases in C in Cases (III). As we show in Corollary 1, in Case (IV), debt holders also pay an informational cost. Notably, we can see from the bottom right panel that $D_L^*(x) - D_L^{**}(x)$ is much higher than $E_L^*(x) - E_L^{**}(x)$ in Case (IV). This suggests that debt holders rather than shareholders suffer severe losses when asymmetric information triggers liquidation bankruptcy that does not occur under symmetric information.

4.1.2. Effects of scrap value θ

Fig. 3 shows the equity and debt values, as well as the informational costs and bankruptcy triggers with varying levels of scrap value θ. Case (I) holds for $\theta < 12.32$, where $x_L^* = x_L^{**} = x_{L,2} = 0.75$ holds. Case (IV) holds for $\theta \in (12.32, 12.52]$, where $x_L^* = x_{L,1}$ and $x_L^{**} = x_{L,2} = 0.75$ hold. Case (III) holds for $\theta > 12.52$, where $x_L^* = x_{L,1}$ and $x_L^{**} = x_L^s$ hold. As in Fig. 2, we have a kink at $\theta = 13.66$ because the high-cost firm chooses to default for $\theta \le 13.66$ and to sell out for $\theta > 13.66$. For $\theta \in [12.52, 15]$, $x_{L,2}$ is less than $x_{L,1}$, which indicates liquidation bankruptcy.

We find a novel result in the top right panel. The sales trigger x_L^s decreases in $\theta \in (12, 52, 13.66]$, meaning that a higher scrap value θ delays the sales timing. This result is novel and is not reported in the existing literature. Indeed, this is opposite to the standard result (cf. Mella-Barral and Perraudin (1997) and Lambrecht and Myers (2008)) that a higher scrap value θ accelerates the sales timing. We can explain the mechanism though ICC as follows. For $\theta \in (12, 52, 13.66]$, the high-cost firm prefers to default in the first-best case; hence, its first-best equity value is independent of θ. While the right-hand side of (15) is constant, the left-hand side of (15), that is, the imitation value, increases in θ. Then, the sales trigger x_L^s decreases in θ. In other words, the low-cost firm lowers x_L^s because a higher θ increases the incentive for the high-cost firm

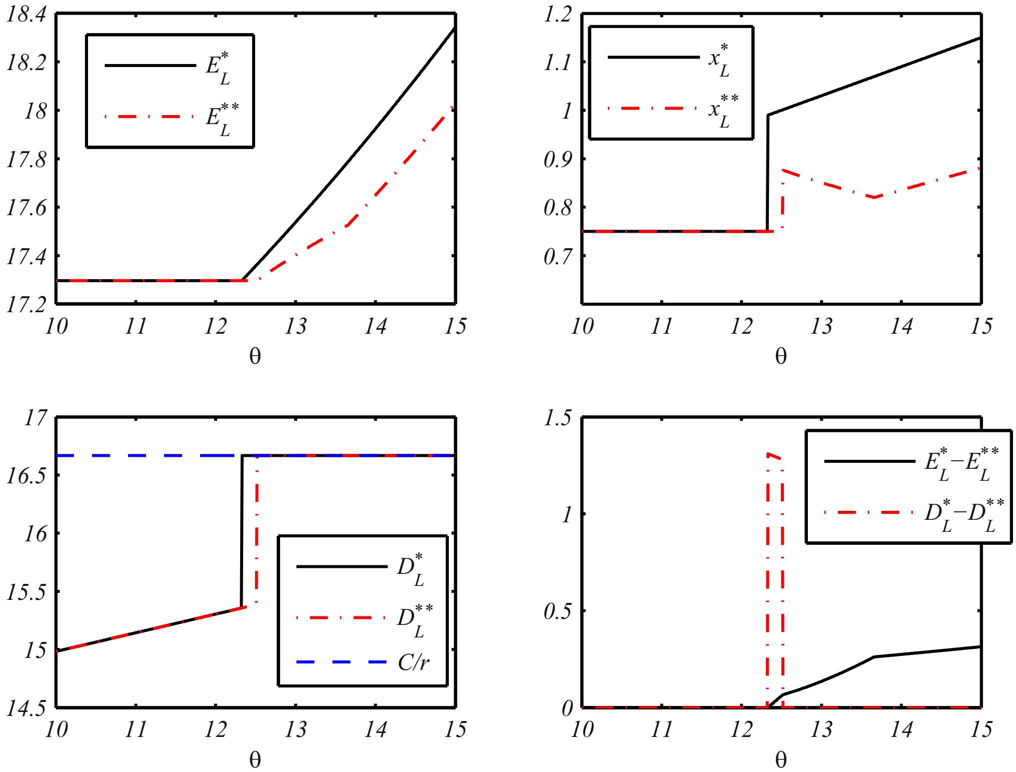

Fig. 3. Comparative statics with respect to θ. The other parameter values are set in Table 1. Cases (I), (IV), and (III) hold for $\theta < 12.32$, $\theta \in (12.32, 12.52]$, and $\theta > 12.52$, respectively.

to imitate the low-cost firm's sales timing. Note that for $\theta > 13.66$, both the left- and right-hand sides of (15) increase in θ, and x_L^s straightforwardly increases in θ.

The bottom right panel indicates several results. Informational cost $E_L^*(x) - E_L^{**}(x)$ increases in θ in Cases (III) and (IV). Only in Case (IV), debt holders also pay an informational cost. As in the bottom right panel of Fig. 2, $D_L^*(x) - D_L^{**}(x)$ is much higher than $E_L^*(x) - E_L^{**}(x)$ in Case (IV). The impacts of parameters a and b are similar to those of θ, and we omit their depiction.

4.1.3. Effects of cash flow volatility σ

Fig. 4 shows the equity and debt values, as well as the informational costs and bankruptcy triggers with varying levels of volatility σ. Case (III) holds for $\sigma \leq 0.223$, where $x_L^* = x_{L,1}$ and $x_L^{**} = x_L^s$ hold. For $\sigma \in [0.1, 0.223]$, $x_{L,2}$ is below $x_{L,1}$, which indicates liquidation bankruptcy. As in Figs. 2 and 3, we have a kink at $\sigma = 0.175$ because the high-cost firm chooses to sell out for $\sigma \leq 0.175$ and to default for $\sigma > 0.175$. Case (IV) holds for $\sigma \in (0.223, 0.236]$, where $x_L^* = x_{L,1}$ and $x_L^{**} = x_{L,2}$ hold. Case (I) holds for $\sigma > 0.236$, where $x_L^* = x_L^{**} = x_{L,2}$ holds.

In the top panels, regardless of whether the information is symmetric or asymmetric, we can see that a higher σ increases the option value of bankruptcy and decreases the bankruptcy trigger. This aligns with the standard volatility effects (e.g., Dixit and Pindyck (1994)) that a higher σ increases the option value of waiting and delays the exercise of the option. In the left panels, we find that a higher σ causes asset substitution from debt holders to shareholders. This result is consistent with the standard result (e.g., Jensen and Meckling (1976) and Shibata and Nishihara (2010)).

In the bottom right panel, we can see more interesting results. Indeed, $E_L^*(x) - E_L^{**}(x)$ increases in $\sigma \in [0.1, 0.175]$ and decreases in $\sigma \in [0.175, 0.223]$ in Case (III). We can explain the non-monotonic result by the option convexity. As we explain in the last of Section 3.1, due to the option convexity, a higher σ increases the option value of default more than the option value of selling out. For $\sigma \in [0.175, 0.223]$, the high-cost firm chooses to default under symmetric information, and hence, a higher σ increases the first-best value more than the imitation value. Thus, a higher σ mitigates ICC for $\sigma \in [0.175, 0.223]$. On the other hand, for $\sigma \in [0.1, 0.175]$, the high-cost firm chooses to sell out under symmetric information. Due to the size effect, a higher σ increases the imitation value more than the first-best value. Thus, a higher σ tightens ICC for $\sigma \in [0.1, 0.175]$. We can also see that as in the bottom right panels of Figs. 2 and 3, $D_L^*(x) - D_L^{**}(x)$ is much higher than $E_L^*(x) - E_L^{**}(x)$ in Case (IV), that is, debt holders suffer much greater losses due to the distortion in the bankruptcy choice than shareholders do.

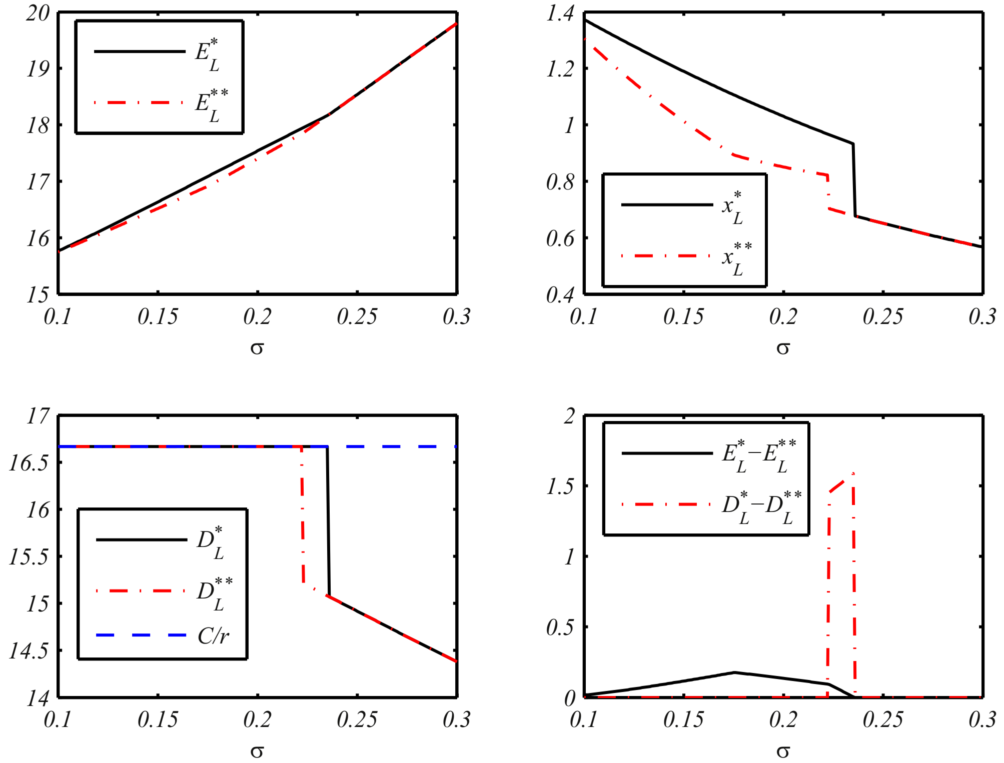

Fig. 4. Comparative statics with respect to σ. The other parameter values are set in Table 1. Cases (III), (IV), and (I) hold for $\sigma \le 0.223$, $\sigma \in (0.223, 0.236]$, and $\sigma > 0.236$, respectively.

4.1.4. Effects of bankruptcy cost α

As seen in the bottom right panels of Figs. 2–4, we find that in Case (IV), debt holders pay rather high informational costs compared to equity holders. We wonder if bankruptcy cost $\alpha = 0.3$ causes this result because the post-default asset value falls to $(1-\alpha)$ times the pre-default value. In this subsection, we investigate the effects of α on the debt value and informational costs. Fig. 5 shows the debt values and informational costs with varying levels of bankruptcy cost α. To focus on Case (IV), we set $\theta = 12.5$. We set the other parameter values besides α and θ in Table 1. We omit equity values and bankruptcy triggers, which are independent of α. In the left panel, $D^*_L(x)$ agrees with the risk-less debt value C/r, whereas $D^{**}_L(x)$ is discounted due to default risk.

In the right panel, we find that $D^*_L(x) - D^{**}_L(x) > E^*_L(x) - E^{**}_L(x)$ holds, even for $\alpha = 0$, and that $D^*_L(x) - D^{**}_L(x)$ increases linearly with α. For a realistic $\alpha \in [0.1, 0.5]$, $D^*_L(x) - D^{**}_L(x)$ is much higher than $E^*_L(x) - E^{**}_L(x)$. In conclusion, we argue that debt holders pay much higher informational costs than shareholders when under asymmetric information, shareholders change the bankruptcy choice from selling out to default. Although we assume that shareholders cannot directly transmit asset quality to outsiders, we now suppose that they can inform outsiders of asset quality with a transmission cost. In such a case, shareholders transmit asset quality and sell the firm to outsiders if and only if the transmission cost is lower than $E^*_L(x) - E^{**}_L(x)$. However, shareholders do not care about the debt holders' informational cost $D^*_L(x) - D^{**}_L(x)$. Then, shareholders can greatly damage debt holders by choosing default in their self-interest.

In this study, we do not consider renegotiation between equity and debt holders. In reality, however, debt holders may negotiate with shareholders to sell the firm at trigger $x_{L,2}$ without formal bankruptcy to save the bankruptcy costs associated with ownership changes. In such a case, debt holders may be able to decrease the informational cost to $D^*_L(x) - D^{**}_L(x)$ with $\alpha = 0$, but they are likely to pay a renegotiation cost.

4.2. Testable implications and related empirical findings

Our analysis of the asymmetric information model yields several new predictions that have not been found in previous papers on dynamic bankruptcy decisions (e.g., Leland et al. (1994), Mella-Barral and Perraudin (1997), and Lambrecht and Myers (2008)). We summarize them here.

1. Firms sell out later to signal asset quality to outsiders.
2. Firms with more debt can incur lower signaling costs and sell out earlier.
3. Firms with higher asset values can incur higher signaling costs and sell out later.

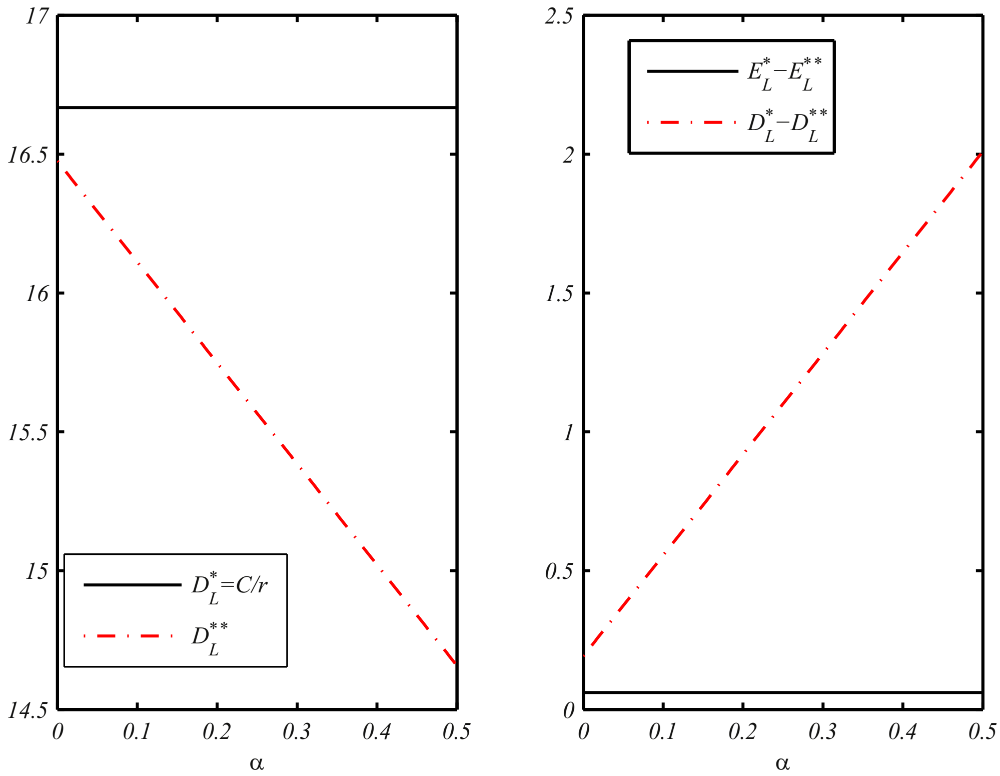

Fig. 5. Comparative statics with respect to α in Case (IV) of Proposition 3. To focus on Case (IV), we set $\theta = 12.5$. The other parameter values are set in Table 1.

4. Informational cost due to asymmetric information can change the bankruptcy choice from selling out to liquidation bankruptcy.
5. In liquidation bankruptcy driven by asymmetric information, debt holders suffer severe losses, while shareholders suffer limited losses.

These results enlighten the roles of asymmetric information between insiders and outsiders in the dynamic bankruptcy procedure. Before providing empirical implications, we begin with explaining proxies for asymmetric information. First, as we discussed in Section 2.1, target firms with lower accounting quality[17] have higher asymmetric information. Indeed, firms with low accounting quality tend to increase the short-term firm values through earnings management such as accrual and real activities manipulation (e.g., Healy and Wahlen (1999) and Leuz et al. (2003)). Second, asymmetric information greatly depends on the relationship between the selling and acquiring firms. For instance, asymmetric information is low when the seller and the acquirer have business relations in the same industry. If the former owner buys back the distressed firm, there is little asymmetric information. On the other hand, asymmetric information increases when the acquirer is an industry outsider and has no prior relation to the seller.

Other factors of target firms are relevant to asymmetric information. Due to few financial reporting requirements, unlisted firms allow insiders to have more private information about asset quality. They can potentially conceal losses and increase earnings. Because outsiders find it more difficult to evaluate intangible assets than tangible assets, firms with higher levels of intangible assets tend to have higher asymmetric information between insiders and outsiders. Our results show that these components greatly distort the bankruptcy choice and timing as well as the equity and debt values.

Now, we explain empirical findings related to our results. Above all, the most notable result is prediction 1 because any static model cannot lead to the result about liquidation timing. To our knowledge, in virtue of our dynamic bankruptcy model with asymmetric information, we unveil the effect of asymmetric information on the bankruptcy timing. Mason (2005), who examines the liquidation procedure of failed banks, shows the empirical evidence of prediction 1. He shows that banks with high levels of volatile assets, such as commercial, industrial, and real estate loans, tend to delay liquidation time. He also shows that such firms tend to gain higher liquidation value by delaying liquidation time. He argues this can be explained by the real options theory that a higher volatility delays liquidation and increases the liquidation threshold. Prediction 1 leads to a more plausible interpretation of the empirical finding. In our view, banks delay the sales

[17] Accounting quality is usually measured by accruals quality proposed by Dechow and Dichev (2002).

of assets with higher asymmetric information to signal asset quality, and then they obtain higher liquidation value. Actually, Mason (2005) states that assets, such as commercial, industrial, and real estate loans, are more difficult to evaluate and entail higher asymmetric information compared to stable assets, such as cash and government securities. Our result can explain his findings in terms of signaling through the sales timing.

We can also see the empirical evidence of prediction 1 in the literature on M&A. Marquardt and Zur (2015) show that target firm accounting quality is positively associated with the speed of the sales process. Although they argue that acquiring firms tend to take longer time for due diligence of target firms with lower accounting quality, our paper complements their explanation by adding the following view: As was explained above, lower accounting quality increases asymmetric information between insiders and outsiders. Prediction 1 suggests that target firms with lower accounting quality information can delay the sales timing to obtain higher sales values. We can argue that not only the strict due diligence process but also the delayed sales timing itself guarantees asset quality of targets. In addition, prediction 1 is in line with the empirical evidence of Adelino et al. (2017) who, although they examine privately-securitized mortgages instead of distressed firms, show a positive relation between time-to-sale and mortgage performance, which means that informed sellers tend to signal quality and obtain higher prices by delaying sales.

Although predictions 2 and 3 have not been tested in the literature to date, they point out the possibility that the straight-forward results do not always hold true in the presence of asymmetric information. For example, normally, a distressed firm with a higher asset value can sell out earlier, but this may not be the case if a distressed firm has a high level of intangible assets. Indeed, prediction 3 argues that such a firm delays the asset sales timing to signal asset quality as the asset value is higher.

As to prediction 4, we can see the empirical evidence in the following papers. For instance, Marquardt and Zur (2015) show that higher target firm accounting quality increases the likelihood that the proposed deal is ultimately completed. McNichols and Stubben (2015) show that higher target firm accounting quality facilitates better bidding decisions by acquiring firms, while Cain et al. (2017) document that earnings management risk in target-initiated auction sales decreases shareholder wealth of both targets and bidders. These empirical findings in the accounting literature are consistent with prediction 4. Actually, lower target firm accounting quality increases the signaling cost, and firms give up sales if the signaling cost is sufficiently high. Hotchkiss and Mooradian (1998), Stromberg (2000), and Thorburn (2000) show that distressed firms are likely to be acquired by former owners and firms in the same industry, rather than industry outsiders. As was discussed above, these acquirers in the same industry have more information about asset quality of target firms. Then, their findings are in line with prediction 4, which argues that lower asymmetric information encourages the sales.

Prediction 5 is related to empirical findings on the literature of fire sales of distressed firms. The following mechanism of fire sales is proposed by Shleifer and Vishny (1992). When a firm goes in liquidation bankruptcy, potential bidders in the same industry can also be financially distressed. In such a situation, efficient bidders (say, industry insiders) cannot pay for the distressed firm's assets, and inefficient bidders (say, industry outsiders) acquire assets. Because of this inefficiency, the distressed firm, compared to a normal firm, is forced to sell assets at depressed prices. Although our model does not consider industry-level distress, we can explain the reason why sales to industry outsiders are inefficient in terms of asymmetric information.

In the context of fire sales, Pulvino (1998) finds the fire sale discount of air crafts is deepened to a non-airline during industry distress, while Acharya et al. (2007) also show that creditor recoveries are depressed during industry distress. Stromberg (2000) shows that asset sales to industry outsiders decrease sales prices. Thorburn (2000) demonstrates that debt holders recover more when former owners buy back firms. These findings are consistent with prediction 5, which states that higher asymmetric information is related to inefficient liquidation bankruptcy, where the firm sells assets at the lower price, and debt holders suffer severe losses. It should be noted that our model generates not only prediction 5 but also prediction 1, which, as was discussed above, highlights the possibility that a distressed firm can mitigate the fire sales discount by delaying the sales timing.

5. Conclusion

In this study, we examined corporate bankruptcy decisions in a contingent claim model. We reveal how asymmetric information about asset quality between a firm's insiders and outsiders affects the bankruptcy choice between selling out and default, their timing, and the debt and equity values. This paper contributes to the literature on dynamic bankruptcy models, the asset sales of distressed firms, the effects of target accounting quality on M&A process, and dynamic signaling games by showing the following novel results.

Most notably, the low-cost firm can delay the sales timing to signal asset quality to outsiders. This result suggests that distressed firms can potentially avoid fire sales by delaying asset sales. In contrast to the standard results, more debt and lower asset value can accelerate the low-cost firm's asset sales timing because they reduce the high-cost firm's incentive to imitate the low-cost firm. When the signaling cost in asset sales is higher than the direct cost, that is, the asset value minus the face value of debt, the low-cost firm changes the bankruptcy choice from selling out to liquidation bankruptcy, which greatly lowers the firm value. In this case, debt holders suffer severe losses, although shareholders suffer limited losses. The existing literature does not report such results, and they can account for many empirical findings of how asymmetric information (e.g., low accounting quality, industry outsider's acquisition, and intangible assets) affects the length of the sales procedure, sales prices, shareholder wealth, and creditor recovery rates.

Appendix A. Proof of Proposition 1

By the first order condition in (2), we have (5) and (6). Similarly, by the first order condition in (3), we have (7) and (8). The inequality (6) ≥ (8) is equivalent to

$$\left(\frac{1}{x_{i,1}}\right)^{\gamma}\left(\frac{(a-1)x_{i,1}}{r-\mu}+\frac{(1-b)w_i}{r}+\theta\right)\geq\left(\frac{1}{x_{i,2}}\right)^{\gamma}\left(-\frac{x_{i,2}}{r-\mu}+\frac{w_i+C}{r}\right)$$
$$\Leftrightarrow\left(\frac{1-a}{(1-b)w_i+r\theta}\right)^{\gamma}\left(\frac{(1-b)w_i}{r}+\theta\right)\geq\left(\frac{1}{w_i+C}\right)^{\gamma}\frac{w_i+C}{r}$$
$$\Leftrightarrow (4).$$

Suppose that (4) does not hold. By the discussion above, the firm chooses to default at the default trigger $x_{i,2}$ defined by (7). Now, we can derive the debt value at the default trigger $x_{i,2}$ as follows:

$$\begin{aligned}
&\sup_T \mathbb{E}_{x_{i,2}}[\int_0^T \mathrm{e}^{-rt}(1-\alpha)(X(t)-w_i)\mathrm{d}t+\mathrm{e}^{-rT}(1-\alpha)\left(\frac{aX(T)}{r-\mu}-\frac{bw_i}{r}+\theta\right)]\\
&=(1-\alpha)\left(\frac{x_{i,2}}{r-\mu}-\frac{w_i}{r}+\sup_T\mathbb{E}[\mathrm{e}^{-rT}\left(\frac{(a-1)X(T)}{r-\mu}+\frac{(1-b)w_i}{r}+\theta\right)]\right)\\
&=(1-\alpha)\left(\frac{x_{i,2}}{r-\mu}-\frac{w_i}{r}+\sup_{\tilde{x}(\leq x_{i,2})}\left(\frac{x_{i,2}}{\tilde{x}}\right)^{\gamma}\left(\frac{(a-1)\tilde{x}}{r-\mu}+\frac{(1-b)w_i}{r}+\theta\right)\right)\\
&=\begin{cases}(1-\alpha)\left(\frac{ax_{i,2}}{r-\mu}-\frac{bw_i}{r}+\theta\right) & (x_{i,2}\leq x_{i,1})\\ (1-\alpha)\left(\frac{x_{i,2}}{r-\mu}-\frac{w_i}{r}+\left(\frac{x_{i,2}}{x_{i,1}}\right)^{\gamma}\left(\frac{(a-1)x_{i,1}}{r-\mu}+\frac{(1-b)w_i}{r}+\theta\right)\right) & (x_{i,2}>x_{i,1}),\end{cases}
\end{aligned}\tag{20}$$

where in (20), $x_{i,1}$ and $x_{i,2}$ are defined by (5) and (7), respectively. Eq. (20) means that the former debt holders sell the total assets instantly after taking over the firm if and only if $x_{i,2} \le x_{i,1}$ holds. By (5) and (7), we have

$$\frac{x_{i,2}}{x_{i,1}}=(1-a)\frac{w_i+C}{(1-b)w_i+r\theta}.\tag{21}$$

By (21) and the upper equation in (20), we can derive the debt values (10) if (9) holds. By (21) and the lower equation in (20), we can derive the debt values (11) if (9) does not hold. The proof is complete.

Appendix B. The proof of $D_{i,2}(x) < C/r$ under symmetric information

Because $\alpha > 0$, we have

$$E_{i,1}(x)+D_{i,1}(x)\geq E_{i,2}(x)+D_{i,2}(x).\tag{22}$$

By substituting $D_{i,1}(x) = C/r$ into (22), we have

$$D_{i,2}(x)\leq E_{i,1}(x)-E_{i,2}(x)+C/r.\tag{23}$$

Suppose that (4) does not hold, that is, the firm chooses to default. In this case, we have $E_{i,1}(x) < E_{i,2}(x)$, and hence, by (23), we have $D_{i,2}(x) < C/r$.

Appendix C. Pooling equilibria

In this appendix, we derive pooling equilibria where both types of firms sell out at the same time, and then, we show that all pooling equilibria are removed by the Intuitive Criterion in Cho and Kreps (1987). In pooling equilibria, outsiders cannot distinguish between high- and low-cost types, and hence the asset value is equal to the expectation of (1) under the prior probability, namely,

$$\frac{aX(t)}{r-\mu}-\frac{b\bar{w}}{r}+\theta,\tag{24}$$

where $\bar{w} = qw_L + (1-q)w_H$. Then, for each type i, as in (2), the equity value of the firm that sells out at the sales trigger x^p becomes

$$E_i^p(x)=\frac{x}{r-\mu}-\frac{w_i+C}{r}+\left(\frac{x}{x^p}\right)^{\gamma}\left(\frac{(a-1)x^p}{r-\mu}+\frac{w_i-b\bar{w}}{r}+\theta\right)\quad(i=L,H),\tag{25}$$

where superscripts x^p denotes a pooling equilibrium. The debt value is $D_i^p(x) = C/r$ $(i = L, H)$ (risk-less debt). We denote the optimal pooling sales trigger for type i by

$$x_i^p = \arg\max_{x^p} \left(\frac{x}{x^p}\right)^{\gamma} \left(\frac{(a-1)x^p}{r-\mu} + \frac{w_i - b\bar{w}}{r} + \theta\right)$$
$$= \frac{\gamma(r-\mu)}{(\gamma-1)r} \frac{w_i - b\bar{w} + r\theta}{1-a} \quad (i = L, H). \tag{26}$$

Then, outsiders rationally believe that $x_L^p \le x^p \le x_H^p$. These are pooling equilibria as long as $E_L^p(x) \ge \max\{E_L^s(x), E_{L,2}(x)\}$ and $E_H^p(x) \ge \max\{E_{H,1}(x), E_{H,2}(x)\}$ hold.[18]

Next, we show that pooling equilibria are eliminated by the Intuitive Criterion in Cho and Kreps (1987). Fix any pooling equilibrium with $x^p \in [x_L^p, x_H^p]$. We can find a sales trigger $\tilde{x} \in (0, x_L^p)$ satisfying

$$E_L^p(x) < \frac{x}{r-\mu} - \frac{w_L + C}{r} + \left(\frac{x}{\tilde{x}}\right)^{\gamma} \left(\frac{(a-1)\tilde{x}}{r-\mu} + \frac{(1-b)w_L}{r} + \theta\right) \tag{27}$$

and

$$E_H^p(x) > \frac{x}{r-\mu} - \frac{w_H + C}{r} + \left(\frac{x}{\tilde{x}}\right)^{\gamma} \left(\frac{(a-1)\tilde{x}}{r-\mu} + \frac{w_H - bw_L}{r} + \theta\right) \tag{28}$$

as follows.

Denote the right-hand sides of (27) and (28) by $f_L(\tilde{x})$ and $f_H(\tilde{x})$, respectively. $f_L(\tilde{x})$ increases in $\tilde{x}$ for $\tilde{x} \le x_{L,1}$ and decreases in $\tilde{x}$ for $\tilde{x} \ge x_{L,1}$, where $x_{L,1}$ is defined by (5) with $i = L$. By (25), (26), and (5), we can easily show that $E_L^p(x) < f_L(x_L^p) < f_L(x_{L,1})$ and that $x_L^p < x_{L,1}$. Then, we have $\tilde{x}' \in (0, x_L^p)$ satisfying $E_L^p(x) = f_L(\tilde{x}')$. For this $\tilde{x}'$, we have

$$E_H^p(x) - f_H(\tilde{x}') = \left(\frac{x}{x^p}\right)^{\gamma} \left(\frac{(a-1)x^p}{r-\mu} + \frac{w_H - b\bar{w}}{r} + \theta\right) - \left(\frac{x}{\tilde{x}'}\right)^{\gamma} \left(\frac{(a-1)\tilde{x}'}{r-\mu} + \frac{w_H - bw_L}{r} + \theta\right)$$
$$= \underbrace{E_L^p(x) - f_L(\tilde{x}')}_{=0} + \left(\frac{x}{x^p}\right)^{\gamma} \frac{w_H - w_L}{r} - \left(\frac{x}{\tilde{x}'}\right)^{\gamma} \frac{w_H - w_L}{r}$$
$$= \left(\left(\frac{x}{x^p}\right)^{\gamma} - \left(\frac{x}{\tilde{x}'}\right)^{\gamma}\right) \frac{w_H - w_L}{r}$$
$$> 0,$$

where the last inequality follows from $\tilde{x}' < x_L^p \le x^p$ and $\gamma < 0$. By the continuity of $f_H(\tilde{x})$, we find a positive (sufficiently small) ϵ satisfying $E_H^p(x) > f_H(\tilde{x}' + \epsilon)$. We also have $E_L^p(x) = f_L(\tilde{x}') < f_L(\tilde{x}' + \epsilon)$ because $f_L(\tilde{x})$ increases in $\tilde{x}$ for $\tilde{x} \le x_{L,1}$. By taking $\tilde{x} = \tilde{x}' + \epsilon$, we have $\tilde{x} \in (0, x_L^p)$ satisfying both (27) and (28).

Now, fix any $\tilde{x} \in (0, x_L^p)$ satisfying (27) and (28). Because of (28) the high-cost type is strictly worse off selling out at the sales trigger $\tilde{x}$ than in the pooling equilibrium, while because of (27) the low-cost type can be better off selling out at the sales trigger $\tilde{x}$ than in the pooling equilibrium. Then, under the Intuitive Criterion in Cho and Kreps (1987), outsiders rationally believe that the firm is the low-cost type with probability one for the out-of-equilibrium sales trigger $\tilde{x}$. This belief causes the low-cost type to defect from the pooling equilibrium because the low-cost type gain $f_L(\tilde{x}) > E_L^p(x)$ by choosing the sales trigger $\tilde{x}$. Thus, any pooling equilibrium does not survive the Intuitive Criterion. The above discussion is essentially the same as that of Section V in Cho and Kreps (1987), where they prove that the Riley's separating outcome is the unique outcome that survives the Intuitive Criterion in the Spence signaling model with two types. Although we focus only on pure strategies, the same discussion holds for mixed strategies. Only the separating equilibrium survives the Intuitive Criterion if we include mixed strategies. For details of the Intuitive Criterion, refer to Cho and Kreps (1987).

Appendix D. The single crossing condition

Define the payoff function in the sales case by

$$u(\tilde{x}, \tilde{w}, w) = \frac{x}{r-\mu} - \frac{w + C}{r} + \left(\frac{x}{\tilde{x}}\right)^{\gamma} \left(\frac{(a-1)\tilde{x}}{r-\mu} + \frac{w - b\tilde{w}}{r} + \theta\right),$$

where arguments $\tilde{x}, \tilde{w}$, and w stand for the sales trigger, outsiders' belief about the firm's type, and the firm's real type, respectively. We have

$$\frac{\partial u}{\partial \tilde{x}}(\tilde{x}, \tilde{w}, w) = \left(\frac{x}{\tilde{x}}\right)^{\gamma} \left(\frac{(1-\gamma)(a-1)\tilde{x}}{r-\mu} - \frac{\gamma}{\tilde{x}} \left(\frac{w - b\tilde{w}}{r} + \theta\right)\right),$$
$$\frac{\partial u}{\partial \tilde{w}}(\tilde{x}, \tilde{w}, w) = -\left(\frac{x}{\tilde{x}}\right)^{\gamma} \frac{b}{r}.$$

[18] Bustamante (2012) studies the least-cost equilibrium for a good type. The equilibrium corresponds to the maximum of $E_L^s(x)$ and $E_L^p(x)$ with $x^p = x_L^p$.

Then, we can check the single crossing condition as follows:

$$\frac{\partial}{\partial w}\left(\frac{\frac{\partial u}{\partial \tilde{x}}(\tilde{x},\tilde{w},w)}{\frac{\partial u}{\partial \tilde{w}}(\tilde{x},\tilde{w},w)}\right)=\frac{\partial}{\partial w}\left(-\frac{r}{b}\left(\frac{(1-\gamma)(a-1)\tilde{x}}{r-\mu}-\frac{\gamma}{\tilde{x}}\left(\frac{w-b\tilde{w}}{r}+\theta\right)\right)\right)$$
$$=\frac{\gamma}{b\tilde{x}}<0.$$

Appendix E. Proof of Proposition 3

Case (II): $x_{L,1}$ is the solution to the unconstrained problem (2). When $x_{L,1}$ satisfies (14), $x_{L,1}$ is the solution to problem (13) subject to (14).
Case (III): In the binding case, the optimal solution to problem (13) subject to (14) is equal to (14). By equating (14), we obtain two candidates $x_L^s \in (0, x_{L,1})$ and $\tilde{x}_L^s (> x_{L,1})$ for the optimal solution, where

$$\left(\frac{1}{x_L^s}\right)^{\gamma}\left(\frac{(a-1)x_L^s}{r-\mu}+\frac{w_H-bw_L}{r}+\theta\right) \tag{29}$$

$$=\left(\frac{1}{\tilde{x}_L^s}\right)^{\gamma}\left(\frac{(a-1)\tilde{x}_L^s}{r-\mu}+\frac{w_H-bw_L}{r}+\theta\right) \tag{30}$$
$$=\max\left\{\left(\frac{1}{x_{H,1}}\right)^{\gamma}\left(\frac{(a-1)x_{H,1}}{r-\mu}+\frac{(1-b)w_H}{r}+\theta\right),\left(\frac{1}{x_{H,2}}\right)^{\gamma}\left(-\frac{x_{H,2}}{r-\mu}+\frac{w_H+C}{r}\right)\right\}$$

holds. Using (29) = (30), we have

$$\left(\frac{1}{x_L^s}\right)^{\gamma}\left(\frac{(a-1)x_L^s}{r-\mu}+\frac{(1-b)w_L}{r}+\theta\right) \tag{31}$$

$$=\left(\frac{1}{x_L^s}\right)^{\gamma}\left(\frac{(a-1)x_L^s}{r-\mu}+\frac{w_H-bw_L}{r}+\theta\right)-\left(\frac{1}{x_L^s}\right)^{\gamma}\frac{w_H-w_L}{r}$$
$$=\left(\frac{1}{\tilde{x}_L^s}\right)^{\gamma}\left(\frac{(a-1)\tilde{x}_L^s}{r-\mu}+\frac{w_H-bw_L}{r}+\theta\right)-\left(\frac{1}{x_L^s}\right)^{\gamma}\frac{w_H-w_L}{r}$$
$$>\left(\frac{1}{\tilde{x}_L^s}\right)^{\gamma}\left(\frac{(a-1)\tilde{x}_L^s}{r-\mu}+\frac{w_H-bw_L}{r}+\theta\right)-\left(\frac{1}{\tilde{x}_L^s}\right)^{\gamma}\frac{w_H-w_L}{r} \tag{32}$$

$$=\left(\frac{1}{\tilde{x}_L^s}\right)^{\gamma}\left(\frac{(a-1)\tilde{x}_L^s}{r-\mu}+\frac{(1-b)w_L}{r}+\theta\right), \tag{33}$$

where we use $x_L^s < \tilde{x}_L^s$ and $\gamma < 0$ in (32). Because (31) > (33), the objective value for x_L^s in the constrained problem (13) is higher than the objective value for $\tilde{x}_L^s$. Then, x_L^s is the optimal solution.

By (16), we have

$$E_L^s(x)=\frac{x}{r-\mu}-\frac{w_L+C}{r}+\left(\frac{x}{x_L^s}\right)^{\gamma}\left(\frac{(a-1)x_L^s}{r-\mu}+\frac{(1-b)w_L}{r}+\theta\right)$$
$$\geq\frac{x}{r-\mu}-\frac{w_L+C}{r}+\left(\frac{x}{x_{L,2}}\right)^{\gamma}\left(-\frac{x_{L,2}}{r-\mu}+\frac{w_L+C}{r}\right)$$
$$=E_{L,2}(x).$$

Then, shareholders prefer to sell out at the trigger x_L^s. In the sales case, the debt is riskless.
Case (IV): When (16) does not hold, we have $E_L^s(x) < E_{L,2}(x)$. Then, shareholders prefer to default at the trigger $x_{L,2}$. Because (12) does not hold in Case (IV), we have

$$\frac{w_L+C}{(1-b)w_L+r\theta}\leq\left(\frac{1}{1-a}\right)^{\frac{-\gamma}{1-\gamma}}\leq\frac{1}{1-a}, \tag{34}$$

which leads to $x_{L,2} \leq x_{L,1}$ (liquidation bankruptcy). Then, we have the equity and debt values, $E_{L,2}(x)$ and $D_{L,2}(x)$ defined by (8) and (10), respectively. The proof is complete.

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