Heart disease diagnosis based on mediative fuzzy logic

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1. Introduction

Uncertainty appears in different forms and affects decisions making. Frequently, information may be incomplete, imprecise, fragmentary, not fully reliable, vague, contradictory, or deficient in some other way. Much of this uncertainty can be handled using fuzzy logic type 1 and intensionist logic (IFS). The theory of fuzzy logic provides a mathematical theory to capture the uncertainties associated with human cognitive processes, such as thinking and reasoning. The conventional approaches to knowledge representation lack the means for representing the meaning of fuzzy concepts. As a consequence, the approaches based on first order logic do not provide an appropriate conceptual framework for dealing with the representation of common sense knowledge, since such knowledge is by its nature both lexically imprecise and non-categorical. The development of fuzzy logic was motivated in large measure by the need for a conceptual framework which can address the issue of lexical imprecision.

Intuitionistic fuzzy sets (IFSs), proposed as a generalization of the traditional fuzzy sets introduced by Zadeh [2], have the property to incorporate the uncertainty of the information. The IFSs offer a new possibility to represent imperfect knowledge and, therefore, to describe in a more adequate way many real problems. Such problems appear when we face with human opinions involving two or more answers of the type: “Yes”, “No”, “I do not know”, “I am not sure”, etc.

Many applications have highlighted the superiority of intuitionistic fuzzy logic compared to traditional fuzzy logic; in the following we mention some of them. In [8] a new approach for fuzzy inference in intuitionistic fuzzy system applied to monitoring a non-linear dynamic plant were described. In [9] an optimized method to reduce the points number to be used in order to identify a person using fuzzy fingerprints was described. The design of an intuitionistic fuzzy logic controller for heater fans on the basis of intuitionistic fuzzy systems were presented in [10]. In [11] an intuitionistic fuzzy logic controller to determine washing time for a washing machine was developed. Because the conventional traffic controller is unable to solve a set of problems (for instance, the congestion at the intersection) in [12] an intelligent transport system based of intuitionistic fuzzy logic is proposed. In [13] a method to construct type-1 intuitionistic fuzzy inference that is able to handle more uncertainty than type-1 fuzzy inference system was described. The results show that the intuitionistic fuzzy inference system performs better than other methods. Many works have been dedicated to improving the architectures and theory used in the construction of control systems. For instance, in [14] a new general procedure is proposed to construct the membership and non-membership functions of the fuzzy reliability using time-dependent intuitionistic fuzzy sets and in [15] an approach for graphically representing intuitionistic fuzzy sets for their use in Mamdani fuzzy inference systems were proposed. The importance of intuitionistic fuzzy sets is explained, also, in [16] was proved that any type-2 fuzzy set can be represented by intuitionistic fuzzy set.

Some questions appear. What happens if the knowledge base changes with time, and non-contradictory information becomes into doubtful or contradictory information, or any combination of these three situations? What inference system can be used? Mediative fuzzy logic, presented for first time in Montiel et al. [17], is a novel approach which is able to deal with kind of inconsistent information providing a common sense solution when contradiction exists; this is a mediate solution. Mediative fuzzy logic can be reduced to intuitionistic and traditional fuzzy logic, depending on how the membership functions (agreement or non-agreement) are established.

As is specified in [18], there is a lot of applications where information is inconsistent. In economics, for estimating the gross

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Abstract

Mediative fuzzy logic is an approach able to deal with inconsistent information providing a solution when contradiction exists. The aim of this paper is to design an expert system based on this type of fuzzy logic in order to diagnose a possible heart disease for a patient. Our proposed system is an extension of the standard Mamdani fuzzy logic controller and contains 44 rules of the type single input–single output. The system works with 11 variables as inputs and one variable as output.

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domestic product it is possible to use different variables and for estimating the exportation rates it is necessary to use a combination of different variables. In forecasting prediction, uncertainty always appears, because to obtain a reliable prediction it is necessary to have a number of decisions, each one based on a different group.

In medicine, information from experiments can be somewhat inconsistent because patients being might respond different to some experimental medication. The contradictory results from multiple clinical trials are attributed either to methodological deficiencies in the design of one of the trials or to small sample sizes [19].

After the introduction of mediative fuzzy logic, a number of works have demonstrated the benefits of its use [18,20–22]. In [21] mediative fuzzy logic were used with promising results for solving nonlinear optimization problems avoiding the problem of cycling, in [18] the results of mediative fuzzy logic were compared with those given by intuitionistic fuzzy logic and traditional fuzzy logic and the conclusion is: mediative fuzzy logic reflects contradictory knowledge at the system output and it can gives a softer transition when we have hesitation and contradiction fuzzy sets. In [22] mediative fuzzy logic is used to construct an intelligent method for controlling population size in evolutionary algorithms. Experimental results gave a significantly reduction in time and an improvement in precision.

In this paper we propose a reasoning system based on mediative fuzzy logic applied in a diagnosis medical problem. Further, this paper is organized as follows. In the second section we present the basic concepts concerning the mediative fuzzy logic; all these notions will be used in the next sections. In Section 3 we modify the standard Mamdani fuzzy logic controller in order to be used in mediative fuzzy logic. The Section 4 is devoted to application of the proposed system to heart disease diagnosis. The conclusions are discussed in Section 5.

2. Mediative fuzzy logic

A traditional fuzzy set in X is given as a set A = (x, μA(x)/x ∈ X) [2]. In [5] is defined the notion of intuitionistic fuzzy set as follows:

**Definition 1.** An intuitionistic fuzzy set A in X is defined as

\[ A = \{(x, \mu_A(x), \nu_A(x))/x \in X\} \]

where \( \mu_A, \nu_A : X \rightarrow [0, 1] \) satisfy the condition

\[ 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \quad \forall x \in X. \]  

(2)

The numbers \( \mu_A(x) \) and \( \nu_A(x) \) denote the degree of membership and non-membership of x to A, respectively. Obviously, a fuzzy set A corresponds to the following intuitionistic fuzzy set A = \{(x, \mu_A(x), 1 - \mu_A(x)), x \in X\}. For each intuitionistic fuzzy set A in X,

\[ \pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \]

(3)

is called the intuitionistic fuzzy index of x in A; it is a hesitation degree of x to A [5–7] and satisfies the inequality

\[ 0 \leq \pi_A(x) \leq 1 \quad \forall x \in X. \]

(4)

Therefore, if we want to describe an intuitionistic fuzzy set we must use any two functions from the triplet: membership function, non-membership function, intuitionistic fuzzy index.

Fuzzy inference in intuitionistic case implies working with the membership function \( \mu \) and the non-membership function \( \nu \). Hence, the output of an intuitionistic fuzzy system can be calculated as follows [23]:

\[ \text{IFS} = (1 - \pi)^{FS_\mu} + \pi^{FS_\nu} \]

(5)

where \( FS_\mu \) is the traditional output of a fuzzy system using the membership function \( \mu \) and \( FS_\nu \) is the output of the fuzzy system using the non-membership function \( \nu \). For \( \pi = 0 \) in the last relation, the IFS is reduced to the output of a traditional fuzzy system.

A contraction fuzzy set C in X is given by

\[ \zeta_C(x) = \min(\mu_C(x), \nu_C(x)). \]

(6)

In this case, the functions \( \mu_C \) and \( \nu_C \) are named agreement membership function and agreement non-membership function, respectively; as is specified in [18], these names are more adequate for contradictory fuzzy sets. In order to compute the system’s output, Montiel et al. [18] proposed the following expressions:

\[ \text{MFS} = \left(1 - \pi - \frac{\pi}{2}\right)^{FS_\mu} + \pi^{FS_\nu} \]

(7)

\[ \text{MFS} = \min\left(\left(1 - \pi\right)^{FS_\mu} + \pi^{FS_\nu}, 1 - \frac{\pi}{2}\right) \]

(8)

\[ \text{MFS} = \left((1 - \pi)^{FS_\mu} + \pi^{FS_\nu}\right)\left(1 - \frac{\pi}{2}\right) \]

(9)

In the case when the contradictory index \( \pi \) is equal to zero, the system’s output is reduced to an intuitionistic fuzzy output.

3. Knowledge representation

The knowledge base of a rule-based system may contain imprecisions which appear in the description of the rules given by the expert. In order to represent imprecise values, the majority of applications work with intuitionistic fuzzy numbers which was proposed by Burillo et al. [24], but various definitions were proposed in the literature.

**Definition 2 ([25]).** An intuitionistic fuzzy subset

\[ A = \{(x, \mu_A(x), \nu_A(x))/x \in \mathbb{R}\} \]

of the real line is called intuitionistic fuzzy number if:

(a) \( A \) is if-normal: there exists \( x_0 \in \mathbb{R} \) such that \( \mu_A(x_0) = 1 \) and \( \nu_A(x_0) = 0 \).

(b) \( A \) is if-convex: its membership function is fuzzy convex \( \mu_A(\lambda x_1 + (1 - \lambda) x_2) \leq \min(\mu_A(x_1), \mu_A(x_2)) \forall x_1, x_2 \in \mathbb{R}, \lambda \in [0, 1] \) and its non-membership function is fuzzy concave \( \nu_A(\lambda x_1 + (1 - \lambda) x_2) \geq \max(\nu_A(x_1), \nu_A(x_2)) \forall x_1, x_2 \in \mathbb{R}, \lambda \in [0, 1], \lambda \neq 0 \).

(c) \( \mu_A \) and \( \nu_A \) are continuous by parts.

For an easy implementation, majority applications work with trapezoidal intuitionistic fuzzy numbers.

**Definition 3.** An intuitionistic fuzzy number denoted as

\[ A = \{(x, \mu_A(x), \nu_A(x))/x \in \mathbb{R}\} \]

is given by

\[ \mu_A(x) = \begin{cases} 0 & \text{if } x < a_1 \\ \frac{x-a_1}{a_2-a_1} & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } a_2 \leq x \leq a_3 \\ \frac{x-a_3}{a_4-a_3} & \text{if } a_3 \leq x \leq a_4 \\ 0 & \text{for } a_4 < x \end{cases} \]

\[ \nu_A(x) = \begin{cases} 1 & \text{if } x < b_1 \\ \frac{x-b_1}{b_2-b_1} & \text{if } b_1 \leq x \leq b_2 \\ 0 & \text{if } b_2 \leq x \leq b_3 \\ \frac{x-b_3}{b_4-b_3} & \text{if } b_3 \leq x \leq b_4 \\ 1 & \text{if } b_4 < x \end{cases} \]

where \( a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4 \in \mathbb{R} \) and \( b_1 \leq a_1 \leq b_2 \leq a_2 \leq a_3 \leq b_3 \leq a_4 \leq b_4, A = (a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4). \)

The rule

if \( X \) is \( A \) then \( Y \) is \( B \)

is represented by its conditional possibility distribution \( \pi_{Y/X} \) defined as follows [26,27].
\[ \pi_{XY}(v, u) = \mu_A(u) \rightarrow \mu_B(v), \quad \forall \, u \in U, \quad \forall \, v \in V \]

where \( \rightarrow \) is an implication operator, \( \mu_A \) and \( \mu_B \) are the membership functions of the fuzzy sets \( A \) and \( B \), respectively, \( U \) is the domain of \( X \) and \( V \) is the domain of \( Y \).

**Definition 4.** A fuzzy implication is a function \( I: \{0, 1\}^2 \rightarrow \{0, 1\} \) satisfying the following conditions, for all values \( x, y, z \in \{0, 1\} \):

\begin{align*}
11: & \quad I(x, y) \leq I(x, z) \leq I(y, z) \\
12: & \quad I(y, z) \leq I(x, y) \leq I(y, z) \\
13: & \quad I(0, y) = 1 \quad (\text{falsity implies anything}) \\
14: & \quad I(x, 1) = 1 \quad (\text{anything implies tautology}) \\
15: & \quad I(1,0) = 0 \quad (\text{Booleanity})
\end{align*}

The following properties for fuzzy implication are required by different authors and they have their significance in various applications:

\begin{align*}
16: & \quad I(1, x) = x \quad (\text{tautology cannot justify anything}) \quad \forall \, x \in \{0, 1\} \\
17: & \quad I(x, y, z) = I(y, z, x) \quad (\text{exchange principle}) \quad \forall \, x, y, z \in \{0, 1\} \\
18: & \quad x \leq y \quad \text{if and only if} \quad I(x, y) = 1 \quad (\text{implication defines ordering}) \quad \forall \, x, y \in \{0, 1\} \\
19: & \quad I(x, 0) = N(x) \quad \forall \, x \in \{0, 1\} \quad (\text{is a strong negation}) \\
100: & \quad I(x, y) \leq y \quad \forall \, x, y \in \{0, 1\} \\
111: & \quad I(x, x) = 1 \quad (\text{identity principle}) \quad \forall \, x \in \{0, 1\} \\
112: & \quad I(x, y) = I(N(y), N(x)) \quad \forall \, x, y \in \{0, 1\} \quad \text{and a strong negation} \quad N \\
113: & \quad I(x) \quad \text{is a continuous function.}
\end{align*}

Czogala and Leski [28] analyzing a set of eight implications (Kleene-Dienes, Reichenbach, Lukasiewicz, Godel, Rescher-Gaines, Goguen, Zadeh, Fodor) concluded that the Lukasiewicz implication

\[ I(x, y) = \min(1 - x + y, 1) \]

satisfies all set of previous properties and, therefore, it is one of the most important implications.

### 4. Proposed mediative fuzzy system

In medicine we often find inconsistent information that can come from different human experts that do not fully agree all the time. In this case traditional and intuitionistic fuzzy logics are not effective while mediative fuzzy logic can be successfully used [29]. So is mentioned in first chapter, applications based on mediative fuzzy logic have shown its superiority to other fuzzy logics (traditional or intuitive). Starting from Mamdani fuzzy logic controller and from Fuzzy expert system from [30], in this section we describe a mediative fuzzy logic controller which can be used for heart disease diagnosis. For this we extend the system from [30] replacing fuzzy logic with mediative fuzzy logic, via intuitionistic fuzzy logic. In the experiment from [18] the best results were obtained using Eq. (7) in order to compute the system’s output; that is why we will use the same equation.

#### 4.1. Input and output variables

First step to designing an expert system is determination of input and output variables. We will work with linguistic variables. A linguistic variable is defined by a quin-tuple \( (x, T(x), U, G, M) \) where: \( x \) is the name of variable, \( T(x) \) is the set of terms (set of linguistic values of \( x \)), \( G \) is a syntactic rule for generating the name of the terms and \( M \) is a semantic rule for associating each term with its meaning.

We will use the same variables as in [30]; a value of a variable is a linguistic value or an integer number. A linguistic value is represented as an intuitionistic fuzzy number defined by its membership function and its non-membership function. For fuzzy numbers we will specify only the intervals with non-zero values.

![Fig. 1. The membership function of the input variable blood pressure.](image1)

- **Input variables are:**
  - **1. Chest pain.** This variable has numerical values: 1 = typical angina, 2 = atypical angina, 3 = non-angina pain and 4 = asymptomatic.
  - **2. Blood pressure.** This variable has 4 values: Low, Medium, High and Very High, defined as intuitionistic fuzzy sets. Low and Very high sets are represented by trapezoidal fuzzy numbers while Medium and High sets are triangular fuzzy numbers. The membership functions \( (\mu) \) and the non-membership functions \( (\nu) \) corresponding to these values are listed below (Figs. 1 and 2):

\[
\mu_{\text{Low}}(x) = \begin{cases} 
1 & \text{if } x \leq 111 \\
\frac{144 - x}{23} & \text{if } 111 \leq x \leq 134
\end{cases}
\]

\[
\nu_{\text{Low}}(x) = \begin{cases} 
\frac{x - 121}{21} & \text{if } 123 \leq x \leq 144 \\
1 & \text{if } 144 < x
\end{cases}
\]

\[
\mu_{\text{Medium}}(x) = \begin{cases} 
\frac{x - 122}{12} & \text{if } 127 \leq x \leq 139 \\
1 & \text{if } x = 139 \\
\frac{133 - x}{14} & \text{if } 139 \leq x \leq 153
\end{cases}
\]

\[
\nu_{\text{Medium}}(x) = \begin{cases} 
\frac{139 - x}{24} & \text{if } x < 115 \\
0 & \text{if } x = 115 \\
\frac{x - 130}{31} & \text{if } 139 \leq x \leq 170 \\
1 & \text{if } 170 < x
\end{cases}
\]

\[
\mu_{\text{High}}(x) = \begin{cases} 
\frac{x - 142}{15} & \text{if } 142 \leq x \leq 157 \\
1 & \text{if } x = 157 \\
\frac{172 - x}{15} & \text{if } 157 \leq x \leq 172
\end{cases}
\]

![Fig. 2. The non-membership function of the input variable blood pressure.](image2)
3. Cholesterol. For this input variable one uses the value of low density lipoprotein cholesterol. For to represent this value we use the intuitionistic fuzzy numbers given by the next functions (Figs. 3 and 4):

\[
\mu_{\text{Very Low}}(x) = \begin{cases} 
1 & \text{if } x < 125 \\
\frac{157 - x}{32} & \text{if } 125 \leq x \leq 157 \\
0 & \text{if } x = 157 \\
\frac{x - 157}{23} & \text{if } 157 < x \\
1 & \text{if } 190 < x 
\end{cases}
\]

\[
\nu_{\text{Very Low}}(x) = \begin{cases} 
\frac{x - 157}{17} & \text{if } 154 \leq x \leq 171 \\
1 & \text{if } 171 < x 
\end{cases}
\]

\[
\mu_{\text{Low}}(x) = \begin{cases} 
\frac{197 - x}{46} & \text{if } x \leq 151 \\
1 & \text{if } 151 < x \leq 197 
\end{cases}
\]

\[
\nu_{\text{Low}}(x) = \begin{cases} 
\frac{x - 174}{1} & \text{if } 174 < x \leq 210 \\
1 & \text{if } 210 < x 
\end{cases}
\]

\[
\mu_{\text{Medium}}(x) = \begin{cases} 
\frac{x - 188}{27} & \text{if } x \leq 215 \\
\frac{219 - x}{35} & \text{if } 215 < x \leq 250 \\
1 & \text{if } 250 < x 
\end{cases}
\]

\[
\nu_{\text{Medium}}(x) = \begin{cases} 
\frac{1}{1} & \text{if } x < 170 \\
\frac{215 - x}{45} & \text{if } 170 < x \leq 215 \\
0 & \text{if } 215 < x \leq 270 \\
1 & \text{if } 270 < x 
\end{cases}
\]

4. Blood sugar (diabetes). This input field has just one linguistic value: True, represented as (Figs. 5 and 6):

\[
\mu_{\text{True}}(x) = \begin{cases} 
\frac{x - 105}{15} & \text{if } 105 \leq x \leq 120 \\
1 & \text{if } 120 < x 
\end{cases}
\]

\[
\nu_{\text{True}}(x) = \begin{cases} 
\frac{x - 112}{32} & \text{if } x < 80 \\
1 & \text{if } 80 < x \leq 112 \\
0 & \text{if } 112 < x \leq 160 
\end{cases}
\]

5. Resting electrocardiography (ECG). This variable has 3 linguistic values: Normal, ST-T Abnormal, Hypertrophy. The intuitionistic fuzzy numbers corresponding to these values are given by the following functions (Figs. 7 and 8):

Fig. 3. The membership function of the input variable cholesterol.

Fig. 4. The non-membership function of the input variable cholesterol.

Fig. 5. The membership function of the input variable blood sugar.

Fig. 6. The non-membership function of the input variable blood sugar.
6. Maximum heart rate. The value of this input variable expresses the maximum rate of man in 24 hours and has the values: Low, Medium, High (Figs. 9 and 10):

\[
\mu_{\text{Low}}(x) = \begin{cases} 
1 & \text{if } 0 \leq x \leq 50 \\
\frac{50-x}{40} & \text{if } 50 < x \leq 90 \\
0 & \text{if } x > 90
\end{cases}
\]

\[
\nu_{\text{Low}}(x) = \begin{cases} 
0 & \text{if } 0 \leq x \leq 50 \\
\frac{50-x}{40} & \text{if } 50 < x \leq 90 \\
1 & \text{if } x > 90
\end{cases}
\]

\[
\mu_{\text{Medium}}(x) = \begin{cases} 
\frac{x-152}{124} & \text{if } x < 130 \\
\frac{152-x}{124} & \text{if } 130 < x \leq 154 \\
0 & \text{if } x > 154
\end{cases}
\]

\[
\nu_{\text{Medium}}(x) = \begin{cases} 
0 & \text{if } 0 \leq x \leq 130 \\
\frac{130-x}{124} & \text{if } 130 < x \leq 154 \\
1 & \text{if } x > 154
\end{cases}
\]

\[
\mu_{\text{High}}(x) = \begin{cases} 
\frac{x-152}{64} & \text{if } x < 152 \\
\frac{152-x}{64} & \text{if } 152 < x \leq 216 \\
0 & \text{if } x > 216
\end{cases}
\]

\[
\nu_{\text{High}}(x) = \begin{cases} 
0 & \text{if } 0 \leq x \leq 152 \\
\frac{152-x}{64} & \text{if } 152 < x \leq 216 \\
1 & \text{if } x > 216
\end{cases}
\]

7. Exercise. This input variable has 2 values: if doctor determines exercise test for patient, value is 1 and, otherwise, value is 0.

8. Old peak. This input variable means ST depression induced by exercise relative to rest and it has 3 values: Low, Risk, Terrible (Figs. 11 and 12):

\[
\mu_{\text{Low}}(x) = \begin{cases} 
1 & \text{if } 0 \leq x \leq 1 \\
2-x & \text{if } 1 < x \leq 2 \\
0 & \text{if } x > 2
\end{cases}
\]

\[
\nu_{\text{Low}}(x) = \begin{cases} 
0 & \text{if } 0 \leq x \leq 1 \\
\frac{x-1.5}{1.5} & \text{if } 1 < x \leq 3 \\
1 & \text{if } x > 3
\end{cases}
\]
9. **Thallium scan.** This input variable has 3 numerical values: 3, 6 and 7 named Normal, Fixed Defect and Reversible Defect, respectively.

10. **Sex.** This variable has 2 values: value 0 means that patient is male and value 1 means that patient is female.

11. **Age.** This input field has 4 linguistic values: Young, Mild, Old and Very Old, represented as intuitionistic fuzzy numbers given by the following functions (Figs. 13 and 14):

\[
\mu_{\text{Risk}}(x) = \begin{cases} 
\frac{x - 2.8}{4.2 - x} & \text{if } 2.8 \leq x \leq 4.2 \\
0 & \text{if } 2.8 < x < 1 \\
1 & \text{if } x = 2.8 \\
1 & \text{if } x = 2.8 \\
0 & \text{if } 1 \leq x \leq 2.8 \\
\frac{x - 2.8}{2.2} & \text{if } 2.8 \leq x \leq 5 \\
1 & \text{if } 5 < x 
\end{cases}
\]

\[
\nu_{\text{Risk}}(x) = \begin{cases} 
\frac{2.8 - x}{1.4} & \text{if } x < 1 \\
1 & \text{if } x = 2.8 \\
\frac{x - 2.8}{2.2} & \text{if } 2.8 \leq x \leq 5 \\
0 & \text{if } 1 \leq x \leq 2.8 \\
\frac{1}{1 - x} & \text{if } x = 2.8 \\
1 & \text{if } x = 2.8 \\
\frac{2.8 - x}{1.4} & \text{if } x < 1 \\
1 & \text{if } x = 2.8 \\
\end{cases}
\]

Output variable. The system has a single output variable that refers to presence of heart disease in the patient. It is an integer value from 0 to 4; these values are referred as Healthy, Sick1, Sick2, Sick3 and Sick4. By increasing of an integer value, heart disease risk increases for patient. The linguistic values for output variable are given by the following functions that define intuitionistic fuzzy numbers (Figs. 15 and 16):

\[
\mu_{\text{Young}}(x) = \begin{cases} 
\frac{x - 3.5}{10} & \text{if } 35 \leq x \leq 45 \\
1 & \text{if } 45 < x \\
\frac{x - 3.5}{10} & \text{if } 35 \leq x \leq 45 \\
1 & \text{if } 45 < x 
\end{cases}
\]

\[
\nu_{\text{Young}}(x) = \begin{cases} 
1 & \text{if } x = 29 \\
\frac{18 - x}{9} & \text{if } 29 \leq x \leq 38 \\
\frac{x - 35}{10} & \text{if } 35 \leq x \leq 45 \\
1 & \text{if } 45 < x 
\end{cases}
\]
The main part of a fuzzy inference system is rule base. Our system uses the same rule base as in [30], that contains 44 rules and 11 input variables. Every rule is of the type one input–one output. The rules are shown below.

\[ R_1. \text{ If Chest Pain is Typical Angina then Result is Healthy} \]
\[ R_2. \text{ If Chest Pain is Atypical Angina then Result is Sick1} \]
\[ R_3. \text{ If Chest Pain is Non-angina then Result is Sick2} \]
\[ R_4. \text{ If Chest Pain is Asymptomatic then Result is Sick3} \]
\[ R_5. \text{ If Chest Pain is Asymptomatic then Result is Sick4} \]
\[ R_6. \text{ If Sex is Female then Result is Sick1} \]
\[ R_7. \text{ If Sex is Male then Result is Sick2} \]
\[ R_8. \text{ If Blood Pressure is Low then Result is Healthy} \]
\[ R_9. \text{ If Blood Pressure is Medium then Result is Sick1} \]
\[ R_{10}. \text{ If Blood Pressure is High then Result is Sick2} \]
\[ R_{11}. \text{ If Blood Pressure is Very High then Result is Sick3} \]
\[ R_{12}. \text{ If Blood Pressure is Very High then Result is Sick4} \]
\[ R_{13}. \text{ If Cholesterol is Low then Result is Healthy} \]
\[ R_{14}. \text{ If Cholesterol is Medium then Result is Sick1} \]
\[ R_{15}. \text{ If Cholesterol is High then Result is Sick2} \]
\[ R_{16}. \text{ If Cholesterol is Very High then Result is Sick3} \]
\[ R_{17}. \text{ If Cholesterol is Very High then Result is Sick4} \]
\[ R_{18}. \text{ If Blood Sugar is True then Result is Sick2} \]
\[ R_{19}. \text{ If EKG is Normal then Result is Healthy} \]
\[ R_{20}. \text{ If EKG is Normal then Result is Sick1} \]
\[ R_{21}. \text{ If EKG is ST-T Abnormal then Result is Sick2} \]
\[ R_{22}. \text{ If EKG is Hypertrophy then Result is Sick3} \]
\[ R_{23}. \text{ If EKG is Hypertrophy then Result is Sick4} \]
\[ R_{24}. \text{ If Max Heart Rate is Low then Result is Healthy} \]
\[ R_{25}. \text{ If Max Heart Rate is Medium then Result is Sick1} \]
\[ R_{26}. \text{ If Max Heart Rate is Medium then Result is Sick2} \]
\[ R_{27}. \text{ If Max Heart Rate is High then Result is Sick3} \]
\[ R_{28}. \text{ If Max Heart Rate is High then Result is Sick4} \]
\[ R_{29}. \text{ If Exercise is True then Result is Sick2} \]
\[ R_{30}. \text{ If Old Peak is Low then Result is Healthy} \]
\[ R_{31}. \text{ If Old Peak is Low then Result is Sick1} \]
\[ R_{32}. \text{ If Old Peak is Terrible then Result is Sick2} \]
\[ R_{33}. \text{ If Old Peak is Terrible then Result is Sick3} \]
\[ R_{34}. \text{ If Old Peak is Risk then Result is Sick4} \]
\[ R_{35}. \text{ If Thallium is Normal then Result is Healthy} \]
\[ R_{36}. \text{ If Thallium is Normal then Result is Sick1} \]
\[ R_{37}. \text{ If Thallium is Fixed Defect then Result is Sick2} \]
\[ R_{38}. \text{ If Thallium is Reversible Defect then Result is Sick3} \]
\[ R_{39}. \text{ If Thallium is Reversible Defect then Result is Sick4} \]
\[ R_{40}. \text{ If Age is Young then Result is Healthy} \]
\[ R_{41}. \text{ If Age is Mild then Result is Sick1} \]
\[ R_{42}. \text{ If Age is Old then Result is Sick2} \]
\[ R_{43}. \text{ If Age is Old then Result is Sick3} \]
\[ R_{44}. \text{ If Age is Very Old then Result is Sick4} \]
4.3. Reasoning system

In order to infer imprecise conclusions from a set of imprecise premises, Zadeh gave a theory of approximate reasoning [27]; this theory is based on fuzzy logic inference processes and an important part of this reasoning is represented by Fuzzy Logic Control (FLC), that consists of four parts: Fuzzification Interface, Fuzzy Rule-Base, Fuzzy Inference Engine and Defuzzification Interface (Fig. 17).

Because the inputs and the outputs of a FLC system are fuzzy sets, we have to fuzzify the crisp inputs and to defuzzify the fuzzy outputs.

4.3.1. Fuzzification and firing level

A fuzzification operator transforms crisp data into intuitionistic fuzzy sets. For instance, \( x_0 \in U \) is fuzzified into \( x_0 \) according to the relations:

\[
\mu_{x_0}(x) = \begin{cases} 
1 & \text{if } x = x_0 \\
0 & \text{otherwise}
\end{cases}
\]

(10)

\[
\nu_{x_0}(x) = \begin{cases} 
0 & \text{if } x = x_0 \\
1 & \text{otherwise}
\end{cases}
\]

(11)

The \( \mu \)-firing level and the \( \nu \)-firing level of an intuitionistic fuzzy set \( A \) and a crisp value \( x_0 \) as input are \( \mu_A(x_0) \) and \( \nu_A(x_0) \), respectively.

4.3.2. Inferred conclusion

Let \( R \): if \( X \) is \( A \) then \( Y \) is \( C \) a rule from the rule base. Because we work with rules with one input, the firing level of a rule is the firing level of the fuzzy set from antecedent. Using the \( \mu \)-firing level \( l_\mu \) and the \( \nu \)-firing level \( l_\nu \) corresponding to rule \( R \), we compute the intuitionistic fuzzy index \( \pi = 1 - l_\mu - l_\nu \), the contradiction fuzzy index \( \zeta = \min(l_\mu, l_\nu) \) and the rule-firing level \( l = (1 - \pi - \frac{\zeta}{2})l_\mu + (\pi + \frac{\zeta}{2})l_\nu \). The rule is represented by Lukasiewicz implication and the conclusion is inferred using Mamdani’s model.

We get two conclusions:

- the \( \mu \)-conclusion \( C^\mu \), which is the traditional output of the rule computed using the membership function and the firing level \( l_\mu \) given by

\[
\mu_{C^\mu}(v) = l_\mu(l_\mu, \mu_C(v)), \forall v \in V.
\]

- the \( \nu \)-conclusion \( C^\nu \), which is the output of the rule computed using the non-membership function and the firing level \( l_\nu \) given by

\[
\mu_{C^\nu}(v) = l_\nu(l_\nu, \nu_C(v)), \forall v \in V.
\]

4.3.3. Defuzzification and output result

The conclusions \( C^\mu \) and \( C^\nu \) are defuzzified into \( y^\mu \) and \( y^\nu \) using the Middle of Maxima and Middle of Minima technique, respectively; this is defined as the main off all values of the universe of discourse having maximal (minimal, respectively) membership grades. The defuzzified output corresponding to conclusion \( C \) is computed as

\[
y = \left(1 - \pi - \frac{\zeta}{2}\right)y^\mu + \left(\pi + \frac{\zeta}{2}\right)y^\nu.
\]

The crisp output is computed by discrete Center of Gravity method:

If the number of fired rules is \( N \) then the final control action is

\[
y_0 = \frac{\sum_{j=1}^{N} y_j l_j}{\sum_{j=1}^{N} l_j}
\]

where \( l_j \) is the firing level and \( y_j \) is the crisp output of the \( j \)-th rule.

Fig. 17. Fuzzy logic controller.

Fig. 18. \( \mu \)-conclusion obtained with Lukasiewicz implication and firing level \( \alpha \).

Fig. 19. \( \nu \)-conclusion obtained with Lukasiewicz implication and firing level \( \alpha \).
5. System testing

We have tested the designed system with the same input values as in [30]:

- Chest Pain = 4 (Asymptomatic),
- Blood Pressure = 117 (Low),
- Cholesterol = 230 (High),
- Blood Sugar = 130 (True),
- ECG = 0 (Normal),
- Maximum Heart Rate = 160 (High),
- Exercise = 1 (True),
- Old Peak = 1.4 (Low),
- Thallium = 7 (Reversible),
- Sex = 1 (Male) and
- Age = 60 (Very Old).

The fired rules with their firing levels and crisp outputs are

Table 1
<table>
<thead>
<tr>
<th>Rule</th>
<th>Firing level</th>
<th>Crisp output</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_4$</td>
<td>1.0</td>
<td>3.0</td>
</tr>
<tr>
<td>$R_7$</td>
<td>1.0</td>
<td>2.0</td>
</tr>
<tr>
<td>$R_8$</td>
<td>0.19</td>
<td>0.4625</td>
</tr>
<tr>
<td>$R_{15}$</td>
<td>0.62</td>
<td>2.96</td>
</tr>
<tr>
<td>$R_{16}$</td>
<td>1.0</td>
<td>2.0</td>
</tr>
<tr>
<td>$R_{17}$</td>
<td>0.1</td>
<td>0.125</td>
</tr>
<tr>
<td>$R_{18}$</td>
<td>0.0156</td>
<td>2.38</td>
</tr>
<tr>
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</tr>
<tr>
<td>$R_{40}$</td>
<td>1.0</td>
<td>3.5</td>
</tr>
<tr>
<td>$R_{44}$</td>
<td>1.0</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Fig. 20. Graphical representation for cholesterol (x), blood sugar (y) and output system (z).

Fig. 21. Output surface generated by cholesterol and blood sugar.

Fig. 22. Graphical representation for blood pressure (x), blood sugar (y) and output system (z).

Fig. 23. Output surface generated by blood pressure and blood sugar.

Fig. 24. Graphical representation for blood pressure (x), cholesterol (y) and output system (z).
inference system. The possibility of complementing the knowledge with non-agreement doubtful information. As is mentioned in [22] the fact of having the working with intuitionistic fuzzy sets to represent the input and output presented in Figs. 20

6. Conclusion

Uncertainty appears in different forms and affects decision making. Nowadays, there are mathematical models to handle the uncertainty. But if we work with a knowledge base that changes with time, and with non-contradictory information that becomes doubtful or contradictory, or with any combination of these three situations then we need to use mediative fuzzy logic which is able to process inconsistent information.

As is mentioned in first chapter, applications based on mediative fuzzy logic (see papers [17–22]) have shown its superiority to other fuzzy logics (traditional or intuitive). For this reason, in this paper, we extend and improves the system from [30] based on fuzzy logic by working with intuitionistic fuzzy sets to represent the input and output variables and with mediative fuzzy logic for reasoning. Superiority of our system is given by the possibility to handle contradictory and doubtful information. As is mentioned in [22] the fact of having the possibility of complementing the knowledge with non-agreement functions give us the possibility of implementing a more realistic fuzzy inference system.

In future papers we intend to improve this system by

- tuning the membership functions and rules used in inference system; for instance, if-then rules can be obtained from training patterns
- using a procedure to generate inference rules so that each has a degree of certainty
- improving the reasoning system by using the degree of certainty for determining the inferred conclusion
- test with other equations to compute the mediate output; for instance (8) and (9).

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References


Fig. 25. Output surface generated by blood pressure and cholesterol.