Omega 58 (2016) 139–153

Contents lists available at ScienceDirect

Omega

journal homepage: www.elsevier.com/locate/omega

Pitfalls of decomposition weights in the additive multi-stage DEA model ☆

Sheng Ang [a], Chien-Ming Chen [b,*]

[a] School of Management, University of Science and Technology of China, 96 Jinzhai Road, Hefei, Anhui 230026, PR China
[b] Nanyang Business School, Nanyang Technological University, 50 Nanyang Avenue, Singapore 639798, Singapore

ARTICLE INFO

Article history:
Received 21 December 2014
Accepted 21 May 2015
Available online 5 June 2015

Keywords:
Data envelopment analysis
Additive efficiency decomposition
Multi-stage serial system

ABSTRACT

This paper examines limitations of the multi-stage DEA (data envelopment analysis) model in the literature. We focus on the DEA model with additive efficiency decomposition. We create taxonomy for the multi-stage DEA models and show when the decomposition weights can be non-increasing. When the decomposition weight for a stage is deemed reflective of the stage's relative importance, this property then implies that upstream stages (regardless the stage efficiency scores) in the model will obtain higher priority in efficiency decomposition. We also find that the non-increasing weights can affect the evaluation of overall and stage efficiency scores. We illustrate our findings through an empirical data set.

1. Introduction

Data envelopment analysis (DEA) is a popular method for evaluating the relative efficiency of decision-making units (DMUs) [2,15,4]. One of the unique features of the classical DEA method is that it does not make any assumption on the production system's internal structure. This property makes DEA a general and robust method. However, the classical DEA model leaves out information about the internal structure, which often reveals process improvement opportunities (e.g., [3]). In addition, few industrial production systems consist of only a single-stage process.

Recently, the classical DEA has been extended to multi-stage serial systems. One useful feature of the multi-stage DEA model is that it can estimate the technical efficiencies for a DMU as well as the stages inside it. The multi-stage model is gaining increasing prominence in both theoretical development and empirical applications of DEA [12,17,30,32,34,19]. However, the multi-stage model in its general form is a non-linear problem, in that the objective function is a weighted sum of stage efficiency indexes. Therefore, the usual assumption adopted by most researchers is to model the weights (i.e., *decomposition weights*), which implicitly reflect the relative importance of different stages in a DMU, as variables and assume that the weights have a specific structure, such that the original problem can be converted to an equivalent LP.

In this paper, we examine the unintended consequences of this assumption on decomposition weights and efficiency scores. We conclude that caution is required when interpreting and acting on estimation results under this assumption. Specifically, we find that the decomposition weights can be non-increasing from the first to the last stages in certain multi-stage DEA models, which implies that the model always assigns higher (or not lower) priority to the upstream stages in efficiency decomposition. This property can be problematic in practice, as the evaluator is likely to prefer not to set weights as such. As the weights and efficiency scores are both endogenous in the multi-stage model, we will show through an empirical dataset that the assumption on decomposition weights has influence not only on weights but also efficiency estimates as well. Regarding this property, we should note that Despotis et al. [13] discuss a similar issue for the two-stage DEA model without exogenous inputs and outputs (i.e., the simplest type of two-stage models). However, it still remains unclear whether this property also applies to a general two-stage or a multi-stage model, an issue which we examined in the current paper. Finally, we find that the analytical relationship between different stages' efficiency scores and weights for certain multi-stage DEA models: the stage-1 efficiency score decreases when its weight increases (and vice versa).

As an alternative to the models with endogenous weights, we investigate the additive DEA model in which weights are pre-determined by the evaluator based on her perception for different production stages. Since weights are set as constant in this model, the problem due

☆This manuscript was processed by Associate Editor Kao.

* Corresponding author. Tel.: +65 67906248.
E-mail addresses: shengang@mail.ustc.edu.cn (S. Ang), cmchen@ntu.edu.sg (C.-M. Chen).

http://dx.doi.org/10.1016/j.omega.2015.05.008
0305-0483/

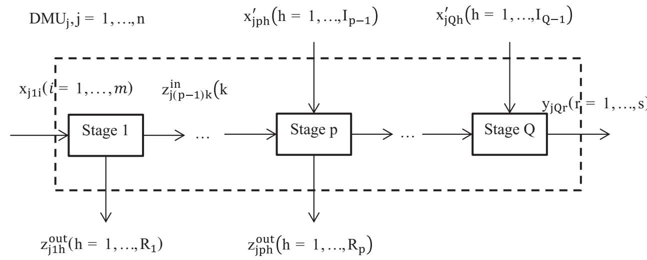

Fig. 1. A general serial multi-stage process.

to endogenous weights is circumvented. However, one limitation for this model lies in its nonlinearity. Li et al. [22] develop an algorithm applicable to a two-stage DEA model. We improve the existing algorithm for the constant weight model and outline how this improved algorithm can be applied to a general multi-stage DEA model. The proposed algorithm shortens the computation time by approximately 70% based on an application to evaluating twenty-four Taiwanese non-life insurance companies using a two-stage model. We expect that the difference in computation time between two algorithms will grow exponentially when the number of stages increases. Finally, we compare the constant weight model with the traditional two-stage model. We illustrate that the standard multi-stage model may generate spurious weights and efficiency scores when the importance of different production stages in the model can been articulated.

2. Additive efficiency decomposition in a multi-stage DEA model

Several recent studies extend the classical DEA model to a two-stage system and a multi-stage system [12,17]. An illustration of a multi-stage production process is shown in Fig. 1. Kao [17] provides a detailed survey of DEA studies on the serial multi-stage processes.

Three major distinctions can be drawn between the classical DEA model and the multi-stage DEA model. First, the classical DEA model only considers the initial inputs and final outputs of a system, whereas the multi-stage DEA model additionally considers the intermediate outputs that traverse through the entire process. Second, the classical DEA model aims to optimize the efficiency score of the DMU under evaluation, and the efficiency score is a ratio between weighted initial inputs and final outputs. In contrast, the objective function of the multi-stage DEA model (i.e., the overall efficiency) is an increasing function of the efficiency scores of all stages, and the stage efficiency scores are ratios of weighted inputs and outputs of different stages. Finally, the multistage DEA model calculates efficiency scores at the DMU and the stage levels, whereas the classical DEA can only calculate the DMU (overall) efficiency score. The multi-stage DEA model has been applied to measure the efficiencies of a wide variety of applications, such as cable TV service operation units [30], cement manufacturing production process [32], electric power companies [33,34], bank systems [28,19], Airlines [21] and vegetable oil industry [29].

The multi-stage DEA model that we study in the current paper employs ratio-form efficiency measures, and we begin by introducing the notation for it. Fig. 1 shows the Q-stage production process. For DMU$_j$, we denote the two output vectors of stage p by $\mathbf{z}^{out}_{jp} \in \mathfrak{R}^{R_p}_{+}$ and $\mathbf{z}^{in}_{jp} \in \mathfrak{R}^{S_p}_{+}$. The notation $\mathbf{z}^{out}_{jp}$ represents the exogenous outputs of stage p, and $\mathbf{z}^{in}_{jp}$ represents the outputs that serve as inputs to stage $p+1$. In addition, $\mathbf{x}'_{jp} \in \mathfrak{R}^{I_p}_{+}$ represents the exogenous input vector of stage p. We use an alternative formulation for the first stage (stage 1) and the last stage (stage Q). The inputs for stage 1 are denoted by $\mathbf{x}_{j1}$ and outputs of stage Q by $\mathbf{y}_{jQ}$. Thus it follows that $\mathbf{x}_{j1}' = \mathbf{x}_{j1}$, $\mathbf{z}^{in}_{j0} = 0$, $\mathbf{z}^{in}_{jQ} = 0$ and $\mathbf{z}^{out}_{jQ} = \mathbf{y}_{jQ}$.

We define three variants (M2, M3 and M4) of the general multi-stage model in Fig. 1:

- *Structure M1 (with external, intermediate inputs and outputs)*: if both $\mathbf{x}'_{jp} \neq 0$ and $\mathbf{z}^{out}_{jp} \neq 0$ for all $p = 1, …, Q$, where $\mathbf{x}'_{jp} \in \mathfrak{R}^{I_p}_{+}$ and $\mathbf{z}^{out}_{jp} \in \mathfrak{R}^{R_p}_{+}$.
- *Structure M2 (no external, intermediate inputs)*: if $\mathbf{x}'_{jp} = 0$ for $p = 2, …, Q$ and $\mathbf{z}^{out}_{jp} \neq 0$ for all $p = 1, …, Q$, where $\mathbf{x}'_{jp} \in \mathfrak{R}^{I_p}_{+}$ and $\mathbf{z}^{out}_{jp} \in \mathfrak{R}^{R_p}_{+}$.
- *Structure M3 (no external, intermediate outputs)*: if $\mathbf{x}'_{jp} \neq 0$ for all $p = 1, …, Q$ and $\mathbf{z}^{out}_{jp} = 0$ for $p = 1, …, Q-1$, where $\mathbf{x}'_{jp} \in \mathfrak{R}^{I_p}_{+}$ and $\mathbf{z}^{out}_{jp} \in \mathfrak{R}^{R_p}_{+}$.
- *Structure M4 (no external, intermediate inputs and outputs)*: if both $\mathbf{x}'_{jp} = 0$ for $p = 2, …, Q$ and $\mathbf{z}^{out}_{jp} = 0$ for $p = 1, …, Q-1$, where $\mathbf{x}'_{jp} \in \mathfrak{R}^{I_p}_{+}$ and $\mathbf{z}^{out}_{jp} \in \mathfrak{R}^{R_p}_{+}$.

Among these four structures, M1 (with external intermediate inputs and outputs) has the most general structure. M2, M3 and M4 are special cases of M1. As we did not find any specific properties for M1, our subsequent analysis and discussion will focus on M2, M3, and M4.

Following previous studies, we define the input-oriented efficiency for stage$p$ of DMU$_j$ as the ratio of weighted outputs and inputs[1]:

$$\theta_{jp} = \frac{\sum_{r=1}^{R_p} u_{pr} z^{out}_{jpr} + \sum_{k=1}^{S_p} \eta_{pk} z^{in}_{jpk}}{\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{in}_{jp-1k} + \sum_{i=1}^{I_p} v_{pi} x'_{jpi}}, \quad p = 2, …, Q-1. \tag{1}$$

Similarly, the efficiencies of the first and the last stage are respectively,

$$\theta_{j1} = \frac{\sum_{r=1}^{R_1} u_{1r} z^{out}_{j1r} + \sum_{k=1}^{S_1} \eta_{1k} z^{in}_{j1k}}{\sum_{i=1}^{m} v_i x_{ji}},$$

$$\theta_{jQ} = \frac{\sum_{r=1}^{s} u_{Qr} y_{jr}}{\sum_{k=1}^{S_{Q-1}} \eta_{Q-1k} z^{in}_{jQ-1k} + \sum_{i=1}^{I_Q} v_{Qi} x'_{jQi}}. \tag{2}$$

[1] The output-oriented formulation is shown in Appendix A.

In (1) and (2), u_{pr} represents the multiplier for z^{out}_{jpr}, the output from stage p. Similarly, v_{pi} represents the multiplier for the exogenous inputs x'_{jpi}, and η_{pk} represents the multiplier for the z^{in}_{jpk} (i.e., z^{in}_{jpk} is the outputs for stage p and inputs for stage $p+1$). These multipliers are the decision variables in the model. Eqs. (1) and (2) show that the numerator and denominator of the stage efficiency are weighted sums of the inputs and outputs of that stage, respectively. We refer to these sums as virtual inputs and virtual outputs.

2.1. Additive multi-stage models

Depending on how the stage efficiency scores enter the objective function, the efficiency decomposition in a multi-stage DEA model can be either additive or multiplicative. In an additive model, the objective function is a weighted average of the stage efficiency scores. In a multiplicative model, the objective is the product of the stage efficiency scores. This paper focuses only on the additive model.

We next present the formulation of the additive model under Structure M1 (with external, intermediate inputs and outputs). The additive multi-stage models for Structure M2, M3 and M4 are similar to that of M1. The overall efficiency of the multi-stage model (or DMU) is a weighted average of the efficiencies of individual stages [12]. The model maximizes the evaluated DMU's efficiency θ_o under the condition that the stage efficiencies of all DMUs θ_{jp} $(p=1,\ldots,Q)$ must not exceed unity.

$$\text{Max } \theta_o=\sum_{p=1}^{Q} w_p\theta_{op} \text{ s.t. } \theta_{jp}\le 1, j=1,\ldots,n; p=1,\ldots,Q, \tag{3}$$

where θ_{jp} are defined in (1) and (2).

We refer to w_p in (3) as the *decomposition weights*, as they decompose the overall efficiency into stage efficiencies. Cook et al. [12] define the weights w_p as the virtual inputs of stage p divided by total virtual inputs:

$$w_p=\begin{cases}\dfrac{\sum_{i=1}^{m} v_i x_{ji}}{\sum_{i=1}^{m} v_i x_{ji}+\sum_{p=2}^{Q}\left(\sum_{k=1}^{S_{p-1}}\eta_{p-1k}z^{in}_{jp-1k}+\sum_{i=1}^{I_p} v_{pi}x'_{jpi}\right)} \text{ when } p=1,\\[2ex] \dfrac{\sum_{k=1}^{S_{p-1}}\eta_{p-1k}z^{in}_{jp-1k}+\sum_{i=1}^{I_p} v_{pi}x'_{jpi}}{\sum_{i=1}^{m} v_i x_{ji}+\sum_{p=2}^{Q}\left(\sum_{k=1}^{S_{p-1}}\eta_{p-1k}z^{in}_{jp-1k}+\sum_{i=1}^{I_p} v_{pi}x'_{jpi}\right)}, \text{ for } p=2,\ldots,Q.\end{cases} \tag{4}$$

By (4), the θ_o in the objective function can be rewritten as

$$\theta_o=\frac{\sum_{p=1}^{Q-1}\left(\sum_{r=1}^{R_p} u_{pr}z^{out}_{opr}+\sum_{k=1}^{S_p}\eta_{pk}z^{in}_{opk}\right)+\sum_{r=1}^{s} u_r y_{or}}{\sum_{i=1}^{m} v_i x_{oi}+\sum_{p=2}^{Q}\left(\sum_{k=1}^{S_{p-1}}\eta_{p-1k}z^{in}_{op-1k}+\sum_{i=1}^{I_p} v_{pi}x'_{opi}\right)}. \tag{5}$$

In an output-oriented model, the weights are similar to the input-oriented counterparts, and can be defined as virtual outputs over the sum of virtual outputs across all stages. It is worth noting that by this construction, the decomposition weights w_p become endogenous in (3), in a sense that the weights and stage efficiencies are determined simultaneously within the model. Many subsequent studies on the additive model adopt this setting (e.g., [6,8,10,9,11]). The rationale is that the weights defined in this way can reflect the resource consumption of a stage compared with that of the entire DMU. This construction also provides great mathematical convenience, in that the original fractional linear problem can be transformed to an equivalent LP. Specifically, we can express the objective function of (3) as a ratio, and then use the Charnes-and-Cooper transformation [1] to convert the problem to the following LP:

$$\begin{aligned}
&\text{Max} \sum_{p=1}^{Q-1}\left(\sum_{r=1}^{R_p} u_{pr}z^{out}_{opr}+\sum_{k=1}^{S_p}\eta_{pk}z^{in}_{opk}\right)+\sum_{r=1}^{s}u_r y_{or}\\
&\text{s.t.} \sum_{i=1}^{m} v_i x_{oi}+\sum_{p=2}^{Q}\left(\sum_{k=1}^{S_{p-1}}\eta_{p-1k}z^{in}_{op-1k}+\sum_{i=1}^{I_p}v_{pi}x'_{opi}\right)=1,\\
&\sum_{r=1}^{R_1}u_{1r}z^{out}_{j1r}+\sum_{k=1}^{S_1}\eta_{1k}z^{in}_{j1k}\le\sum_{i=1}^{m}v_i x_{ji}, j=1,\ldots,n,\\
&\sum_{r=1}^{R_p}u_{pr}z^{out}_{jpr}+\sum_{k=1}^{S_p}\eta_{pk}z^{in}_{jpk}\le\sum_{k=1}^{S_{p-1}}\eta_{p-1k}z^{in}_{jp-1k}+\sum_{i=1}^{I_p}v_{pi}x'_{jpi}, p=2,\ldots,Q-1; j=1,\ldots,n,\\
&\sum_{r=1}^{s}u_r y_{jr}\le\sum_{k=1}^{S_{Q-1}}\eta_{Q-1k}z^{in}_{jQ-1k}+\sum_{i=1}^{I_Q}v_{Qi}x'_{jQi}, j=1,\ldots,n,\\
&u_{ph}, v_{pk}, \eta_{pk}, u_r, v_i\ge 0.
\end{aligned} \tag{6}$$

The output-oriented model of (6) can be developed analogously. See Appendix A for details.

2.2. Issue of the additive multi-stage model

Next, we will describe some properties associated with the weights (4). We find that the weights for different stages' efficiencies are non-increasing from stages 1 to Q. We start by looking at Structure M2, then M3, and finally M4.

In the input-oriented model for Structure M2 (no external, intermediate inputs), the efficiency score of each stage must not exceed one. Thus, it follows from (6) that

$$\sum_{r=1}^{R_1}u_{1r}z^{out}_{j1r}+\sum_{k=1}^{S_1}\eta_{1k}z^{in}_{j1k}\le\sum_{i=1}^{m}v_i x_{ji},$$

$$\sum_{r=1}^{R_p} u_{pr} z^{\text{out}}_{\text{jpr}} + \sum_{k=1}^{S_p} \eta_{pk} z^{\text{in}}_{\text{jpk}} \le \sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{\text{in}}_{\text{jp-1k}}, p=2,\ldots,Q-1,$$
$$\sum_{r=1}^{s} u_r y_{jr} \le \sum_{k=1}^{S_{Q-1}} \eta_{Q-1k} z^{\text{in}}_{\text{jQ-1k}}. \tag{7}$$

Therefore, the decomposition weights have the following relationship:

$$\begin{aligned} w_1 - w_2 &= \frac{\sum_{i=1}^m v_i x_{\text{ji}}}{\sum_{i=1}^m v_i x_{\text{ji}} + \sum_{p=2}^Q \left(\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{\text{in}}_{\text{jp-1k}}\right)} - \frac{\sum_{k=1}^{S_1} \eta_{1k} z^{\text{in}}_{\text{j1k}}}{\sum_{i=1}^m v_i x_{\text{ji}} + \sum_{p=2}^Q \left(\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{\text{in}}_{\text{jp-1k}}\right)} \\ &= \frac{\sum_{i=1}^m v_i x_{\text{ji}} - \sum_{k=1}^{S_1} \eta_{1k} z^{\text{in}}_{\text{j1k}}}{\sum_{i=1}^m v_i x_{\text{ji}} + \sum_{p=2}^Q \left(\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{\text{in}}_{\text{jp-1k}}\right)} \ge \frac{\sum_{r=1}^{R_1} u_{1r} z^{\text{out}}_{\text{j1r}}}{\sum_{i=1}^m v_i x_{\text{ji}} + \sum_{p=2}^Q \left(\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{\text{in}}_{\text{jp-1k}}\right)} \ge 0, \\ w_p - w_{p+1} &= \frac{\sum_{k=1}^{S_{p-1}} \eta_{p-1k} \mathbf{z}^{\text{in}}_{\text{jp-1k}}}{\sum_{i=1}^m v_i x_{\text{ji}} + \sum_{p=2}^Q \left(\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{\text{in}}_{\text{jp-1k}}\right)} - \frac{\sum_{k=1}^{S_p} \eta_{pk} z^{\text{in}}_{\text{jpk}}}{\sum_{i=1}^m v_i x_{\text{ji}} + \sum_{p=2}^Q \left(\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{\text{in}}_{\text{jp-1k}}\right)} \\ &= \frac{\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{\text{in}}_{\text{jp-1k}} - \sum_{k=1}^{S_p} \eta_{pk} z^{\text{in}}_{\text{jpk}}}{\sum_{i=1}^m v_i x_{\text{ji}} + \sum_{p=2}^Q \left(\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{\text{in}}_{\text{jp-1k}}\right)} \ge \frac{\sum_{r=1}^{R_p} u_{pr} z^{\text{out}}_{\text{jpr}}}{\sum_{i=1}^m v_i x_{\text{ji}} + \sum_{p=2}^Q \left(\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{\text{in}}_{\text{jp-1k}}\right)} \ge 0, p=2,\ldots,Q-1. \end{aligned} \tag{8}$$

Based on (8), we obtain following theorem.

Theorem 1. *In the input-oriented additive multi-stage model (Structure M2), it holds that* $w_1 \ge w_2 \ge \ldots \ge w_Q$.

For the input-oriented model for M2 (no external, intermediate inputs), Theorem 1 shows that the decomposition weights are non-increasing in the sequence of stages, regardless stage efficiency scores. This is against the intuition that in DEA, weights w_p should optimize the overall efficiency, given the stage efficiencies.

Given Theorem 1, one would reasonably expect that the output-oriented model for M3 (no external, intermediate outputs) may have symmetric property to Theorem 1. That is, the weights are expected to be non-decreasing in the sequence of stages. Interestingly, we could not establish this relationship for M3. Following our derivation for Theorem 1, we first note that (9) and (10) holds for the output-oriented model for M3.

$$\sum_{i=1}^{m} v_i x_{\text{ji}} \ge \sum_{k=1}^{S_1} \eta_{1k} z^{\text{in}}_{\text{j1k}}, j=1,..,n,$$
$$\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{\text{in}}_{\text{jp-1k}} + \sum_{i=1}^{I_p} v_{pi} x'_{\text{jpi}} \ge \sum_{k=1}^{S_p} \eta_{pk} z^{\text{in}}_{\text{jpk}}, p=2,\ldots,Q-1; j=1,..,n,$$
$$\sum_{k=1}^{S_{Q-1}} \eta_{Q-1k} z^{\text{in}}_{\text{jQ-1k}} + \sum_{i=1}^{I_Q} v_{Qi} x'_{\text{jQi}} \ge \sum_{r=1}^{s} u_r y_{jr}, j=1,..,n, \tag{9}$$

$$w_p - w_{p+1} = \frac{\sum_{k=1}^{S_p} \eta_{pk} z^{\text{in}}_{\text{jpk}} - \sum_{k=1}^{S_{p+1}} \eta_{p+1k} z^{\text{in}}_{\text{jp+1k}}}{\sum_{p=1}^{Q-1} \left(\sum_{k=1}^{S_p} \eta_{pk} z^{\text{in}}_{\text{jpk}}\right) + \sum_{r=1}^{s} u_r y_{jr}}, p=1,\ldots,Q-1,$$
$$w_{Q-1} - w_Q = \frac{\sum_{k=1}^{S_{Q-1}} \eta_{Q-1k} z^{\text{in}}_{\text{jQ-1k}} - \sum_{r=1}^{s} u_r y_{jr}}{\sum_{p=1}^{Q-1} \left(\sum_{k=1}^{S_p} \eta_{pk} z^{\text{in}}_{\text{jpk}}\right) + \sum_{r=1}^{s} u_r y_{jr}}. \tag{10}$$

However, we cannot establish a relationship between the weights for the two stages due to $\sum_{i=1}^{I_p} v_{pi} x'_{\text{jpi}} \ge 0$ in (9). If the property $w_1 \le w_2 \le \ldots \le w_Q$ exists, then $\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{\text{in}}_{\text{jp-1k}} + \sum_{i=1}^{I_p} v_{pi} x'_{\text{jpi}} \ge \sum_{k=1}^{S_p} \eta_{pk} z^{\text{in}}_{\text{jpk}}$ must be like $\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{\text{in}}_{\text{jp-1k}} + \sum_{i=1}^{I_p} v_{pi} x'_{\text{jpi}} \le \sum_{k=1}^{S_p} \eta_{pk} z^{\text{in}}_{\text{jpk}}$. But that cannot be true because of the restriction that the sum of virtual inputs must be not less than the sum of virtual outputs in both input- and output-oriented models by DEA.

We continue the same analysis to both input- and output-oriented models for other structures. Table 1 summarizes the results that we derive thus far.

Table 1
Implied weights relationship in additive multi-stage models.

Process structure		Additive multi-stage models	
		Input-oriented	Output-oriented
M1 (with external, intermediate inputs and outputs))	$\mathbf{x}'_{\text{jp}} \ne 0$ and $\mathbf{z}^{\text{out}}_{\text{jp}} \ne 0$	–	–
M2 (no external, intermediate inputs)	$\mathbf{x}'_{\text{jp}} = 0$ and $\mathbf{z}^{\text{out}}_{\text{jp}} \ne 0$	$w_1 \ge w_2 \ge \ldots \ge w_Q$	–
M3 (no external, intermediate outputs)	$\mathbf{x}'_{\text{jp}} \ne 0$ and $\mathbf{z}^{\text{out}}_{\text{jp}} = 0$	–	–
M4 (no external, intermediate inputs and outputs)	$\mathbf{x}'_{\text{jp}} = 0$ and $\mathbf{z}^{\text{out}}_{\text{jp}} = 0$	$w_1 \ge w_2 \ge \ldots \ge w_Q$	$w_1 \ge w_2 \ge \ldots \ge w_Q$

One might ask the following question: as we found $w_1 \geq w_2 \geq \ldots \geq w_Q$ for input-oriented model under M4, but why is not there a symmetric result that $w_1 \leq w_2 \leq \ldots \leq w_Q$ for output-oriented model? This can be explained as follows. Under M4 without exogenous inputs and outputs (i.e., when $\mathbf{x}'_{jp} = 0$ and $\mathbf{z}^{out}_{jp} = 0$), the relationship of the weights between stages p and $p+1$ depends on stages p's and $(p+1)$'s sum of virtual inputs (i.e., $\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{in}_{jp-1k}$, and $\sum_{k=1}^{S_p} \eta_{pk} z^{in}_{jpk}$) for input-oriented model, and stages p's and $(p+1)$'s sum of virtual outputs for output-oriented model (i.e., $\sum_{k=1}^{S_p} \eta_{pk} z^{in}_{jpk}$, and $\sum_{k=1}^{S_{p+1}} \eta_{p+1k} z^{in}_{jp+1k}$) according to the definition of decomposition weights in the additive multi-stage DEA model. We find the same relationship for weights in the input- and output-oriented model under M4 for two reasons. First, the intermediate products z^{in}_p served as both inputs (for stage $p+1$) and as outputs (for stage p). Second, the DEA constraints in both input-oriented model and output-oriented model yield the same inequality relationship between the sum of virtual inputs and the sum of virtual outputs for each stage.

However, the relationship may not exist in (i) both input and output-oriented models under M1, and M3, and (ii) the output-oriented model under M2. In conditions (i) with the input-oriented model and (iii) (i.e., when $\mathbf{x}'_{jp} \neq 0$), the relationship of the weights between stages p and $p+1$ depends on both stages p's and $(p+1)$'s sum of virtual inputs (i.e., $\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{in}_{jp-1k} + \sum_{i=1}^{I_p} v_{pi} x'_{jpi}$, and $\sum_{k=1}^{S_p} \eta_{pk} z^{in}_{jpk} + \sum_{i=1}^{I_{p+1}} v_{p+1i} x'_{jp+1i}$). However, stage p's efficiency is restricted to be less than or equal to one, and thus the sum of stage p's virtual inputs must be greater than or equal to the sum of its virtual outputs, i.e., $\sum_{r=1}^{R_p} u_{pr} z^{out}_{jpr} + \sum_{k=1}^{S_p} \eta_{pk} z^{in}_{jpk} \leq \sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{in}_{jp-1k} + \sum_{i=1}^{I_p} v_{pi} x'_{jpi}$ for M1. Note that $\sum_{k=1}^{S_p} \eta_{pk} z^{in}_{jpk}$ in the above inequality forms part of the outputs of stage p's as well as inputs for stage $p+1$'s. Similarly, for stage p under M3, it holds that $\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{in}_{jp-1k} + \sum_{i=1}^{I_p} v_{pi} x'_{jpi} \geq \sum_{k=1}^{S_p} \eta_{pk} z^{in}_{jpk}$, and $\sum_{k=1}^{S_p} \eta_{pk} z^{in}_{jpk}$ will be part of stage $p+1$'s inputs. However, the relationship of $\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{in}_{jp-1k} + \sum_{i=1}^{I_p} v_{pi} x'_{jpi}$ and $\sum_{k=1}^{S_p} \eta_{pk} z^{in}_{jpk} + \sum_{i=1}^{I_{p+1}} v_{p+1i} x'_{jp+1i}$ cannot be determined because the external input $\mathbf{x}'_{jp} \neq 0$. As a result, we could not identify the relationship between the decomposition weights of stages p and $p+1$. The same analysis applies to cases (i) with the output-oriented model and (ii) with the output-oriented models under M1 and M2 (i.e., when $\mathbf{z}^{out}_{jp} \neq 0$).

Finally, Theorem 2 shows additional properties about the decomposition weights and efficiencies of stage 1and stage Q under Structure M4 (no external, intermediate inputs and outputs). Interestingly, we find that the lower bound of the stage 1 decomposition weight decreases in both the number of stages (Q) and its stage efficiency. This means that the stage 1 decomposition weight w_1 can become lower when stage 1 becomes more efficient, and that w_1 can still be high (only bounded above by 1.0) even when Q becomes arbitrarily large. In summary, the theorem shows that w_1 in the standard multi-stage model is dominant compared with other w's. Part (ii) of Theorem 2 states a similar property for the output-oriented model.

Theorem 2. *(i) For Model (6) under Structure M4,* $w_1 \geq \frac{1}{1+(Q-1)\theta_1}$ *and* $w_Q \leq \frac{1}{Q\theta_Q}$. *(ii) For the output-oriented model (A.4) under Structure M4,* $w_Q \leq \frac{1}{(Q-1)\theta_Q+1}$ *and* $w_1 \geq \frac{1}{Q\theta_1}$.

Proof. In Structure M4 (no external, intermediate inputs and outputs),

(i). In the input-oriented additive multi-stage model, the stage efficiencies are $\theta_1 = \frac{\sum_{k=1}^{S_1} \eta_{1k} z^{in}_{j1k}}{\sum_{i=1}^{m} v_i x_{ji}}$, $\theta_p = \frac{\sum_{k=1}^{S_p} \eta_{pk} z^{in}_{jpk}}{\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{in}_{jp-1k}}$, $p = 2, \ldots, Q-1$, and $\theta_Q = \frac{\sum_{r=1}^{s} u_r y_{jr}}{\sum_{k=1}^{S_{Q-1}} \eta_{Q-1k} z^{in}_{jQ-1k}}$. The weights w_1 and w_Q are defined as $w_1 = \frac{\sum_{i=1}^{m} v_i x_{ji}}{\sum_{i=1}^{m} v_i x_{ji} + \sum_{p=2}^{Q}\left(\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{in}_{jp-1k}\right)}$ and $w_Q = \frac{\sum_{k=1}^{S_{Q-1}} \eta_{Q-1k} z^{in}_{Q-1k}}{\sum_{i=1}^{m} v_i x_{ji} + \sum_{p=2}^{Q}\left(\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{in}_{jp-1k}\right)}$. The restriction that stage efficiencies must be equal or less than unity shows $\sum_{i=1}^{m} v_i x_{ij} \geq \sum_{k=1}^{S_1} \eta_{1k} z^{in}_{j1k} \geq \ldots \geq \sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{in}_{jp-1k} \geq \sum_{k=1}^{S_p} \eta_{pk} z^{in}_{jpk} \geq \ldots \geq \sum_{k=1}^{S_{Q-1}} \eta_{Q-1k} z^{in}_{jQ-1k} \geq \sum_{r=1}^{s} u_r y_{jr}$. Then, $w_1 = \frac{\sum_{i=1}^{m} v_i x_{ji}}{\sum_{i=1}^{m} v_i x_{ji} + \sum_{p=2}^{Q}\left(\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{in}_{jp-1k}\right)} = \frac{1}{1+\sum_{p=2}^{Q}\left(\frac{\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{in}_{jp-1k}}{\sum_{i=1}^{m} v_i x_{ij}}\right)} \geq \frac{1}{1+\sum_{p=2}^{Q}\left(\frac{\sum_{k=1}^{S_1} \eta_{1k} z^{in}_{1k}}{\sum_{i=1}^{m} v_i x_{ij}}\right)} = \frac{1}{1+(Q-1)\theta_1}$, and $w_Q = \frac{\sum_{k=1}^{S_{Q-1}} \eta_{Q-1k} z^{in}_{Q-1k}}{\sum_{i=1}^{m} v_i x_{ji} + \sum_{p=2}^{Q}\left(\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z^{in}_{jp-1k}\right)} \leq \frac{\sum_{k=1}^{S_{Q-1}} \eta_{Q-1k} z^{in}_{Q-1k}}{Q\sum_{r=1}^{s} u_r y_{rj}} = \frac{1}{Q\theta_Q}$.

(ii). The proof is similar to (i) and is omitted ■

Table 2
A three-stage process and efficiencies.

DMU	Stage 1			DEA efficiency of the stage	Stage 2			DEA efficiency of the stage	Stage 3		DEA efficiency of the stage
	x	$\mathbf{z}^{out}_1$	$\mathbf{z}^{in}_1$		$\mathbf{z}^{in}_1$	$\mathbf{z}^{out}_2$	$\mathbf{z}^{in}_2$		$\mathbf{z}^{in}_2$	**y**	
A	5	10	10	1.0000	10	10	10	0.5000	10	5	0.5000
B	10	10	10	0.5000	10	20	20	1.0000	20	10	0.5000
C	10	10	10	0.5000	10	10	10	0.5000	10	10	1.0000

Table 3
Efficiencies and weights from the additive model.

DMU	θ	θ_1	θ_2	θ_3	w_1	w_2	w_3
A	0.9997	1.0000	0.5000	0.5000	0.9995	0.0004	0.0001
B	0.6666	0.5000	1.0000	0.5000	0.6666	0.3332	0.0002
C	0.5714	0.5000	0.5000	1.0000	0.5716	0.2857	0.1427

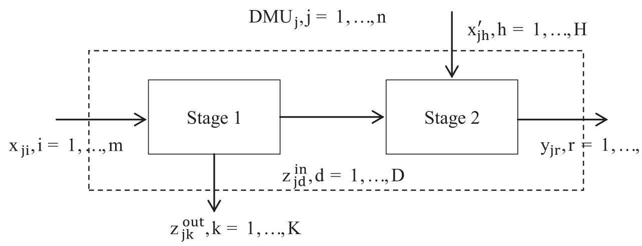

Fig. 2. A general two-stage process.

2.3. A three-stage example

We next demonstrate that the weights in the objective function as described above may bias the estimates of the stage and overall efficiencies. Tables 2 and 3 shows the data of three DMUs. Note that all three DMUs in the example have one efficient stage (i.e., efficiency score equal to 1 if we evaluate the stage with standard DEA) and two inefficient stages (i.e., efficiency score equal to 0.5 if we evaluate the stage with standard DEA). Specifically, stage 1 of DMU A, stage 2 of DMU B, and stage 3 of DMU C are efficient. Therefore, one would expect that these three DMUs should have the same overall efficiency in the multi-stage model, and that, across the three DMUs, the weights (i.e., w_1, w_2, and w_3) should follow a fixed pattern corresponding to the stage efficiencies.

However, the results are different from our intuitions. The weights are increasing ($w_1 > w_2 > w_3$) for all three DMUs, and DMU A's overall efficiency score is higher than DMU B's and DMU C's. This example illustrates that the decomposition weights can cause distortion of efficiency scores, and that the overall efficiency of some DMUs may not be optimal given their stage efficiencies.

3. Additive efficiency decomposition for two-stage processes

In this section, we turn to the two-stage model, which is a special case of the general serial Q-stage process (i.e., $Q = 2$). The purpose of discussing the two-stage model is two-fold. First, we obtain some properties about stage efficiencies and weights, which may not apply to the multi-stage model. Second, the applications of the two-stage model are more common in the literature. Therefore, a discussion about the properties of the two-stage model can be useful for future research. Fig. 2 depicts the two-stage process structure.

Similarly, we can categorize the two-stage models as follows:

- *Structure M1 (with external inputs and outputs)*: if both $\mathbf{x}'_j \neq 0$ and $\mathbf{z}^{out}_j \neq 0$, where $\mathbf{x}'_j \in \mathfrak{R}^H_+$ and $\mathbf{z}^{out}_j \in \mathfrak{R}^K_+$.
- *Structure M2 (no external inputs)*: if only $\mathbf{x}'_j = 0$, where $\mathbf{x}'_j \in \mathfrak{R}^H_+$ and $\mathbf{z}^{out}_j \in \mathfrak{R}^K_+$.
- *Structure M3 (no external outputs)*: if only $\mathbf{z}^{out}_j = 0$, where $\mathbf{x}'_j \in \mathfrak{R}^H_+$ and $\mathbf{z}^{out}_j \in \mathfrak{R}^K_+$.
- *Structure M4 (no external inputs and outputs)*: if both $\mathbf{x}'_j = 0$ and $\mathbf{z}^{out}_j = 0$, where $\mathbf{x}'_j \in \mathfrak{R}^H_+$ and $\mathbf{z}^{out}_j \in \mathfrak{R}^K_+$.

Despotis et al. [13] found that $w_2 \leq w_1$ in their input-oriented model for Structure M4, which is consistent with Theorem 1.

In addition to the property that $w_1 \geq w_2$ identified in Despotis et al. [13], we discover several interesting properties of the two-stage model, including the stage-1 efficiency decreases when w_1 increases which is contrary to practitioners' belief that higher decomposition weights should be associated with higher stage-wise efficiencies. To illustrate, first recall that the weights in the input-oriented two-stage model are $w_1 = \frac{\sum_{i=1}^m v_i x_{ji}}{\sum_{i=1}^m v_i x_{ji} + \sum_{d=1}^D \eta_d z^{in}_{jd}}$ and $w_2 = \frac{\sum_{d=1}^D \eta_d z^{in}_{jd}}{\sum_{i=1}^m v_i x_{ji} + \sum_{d=1}^D \eta_d z^{in}_{jd}}$. First, because $w_1 \geq w_2$ and $w_1 + w_2 = 1$, it follows that $w_1 \in [0.5,\ 1)$ and $w_2 \in (0,\ 0.5]$, indicating that stage 1 always receives at least as much weight as stage 2. Furthermore, for Structure M4, we can express the decomposition weights (w_1, w_2) as a function of the stage 1 efficiency:

$$w_1 = \frac{\sum_{i=1}^m v_i x_{ji}}{\sum_{i=1}^m v_i x_{ji} + \sum_{d=1}^D \eta_d z^{in}_{jd}} = \frac{1}{1 + \frac{\sum_{d=1}^D \eta_d z^{in}_{jd}}{\sum_{i=1}^m v_i x_{ji}}} = \frac{1}{1+\theta_{j1}},$$

$$w_2 = 1 - w_1 = \frac{\sum_{d=1}^D \eta_d z^{in}_{jd}}{\sum_{i=1}^m v_i x_{ji} + \sum_{d=1}^D \eta_d z^{in}_{jd}} = \frac{\theta_{j1}}{\theta_{j1}+1}. \tag{11}$$

Thus, w_1 will decrease and w_2 will increase as stage 1 gains higher efficiency. Further, $w_1 = w_2 = 0.5$ if and only if stage 1 is efficient.

Table 4
Classification of two-stage structures and related studies.

Structure	Two-stage additive models	Two-stage multiplicative models
M1 (with external inputs and outputs: $\mathbf{x}_j' \neq 0\ \mathbf{z}_j^{out} \neq 0, \mathbf{x}_j' \in \Re_+^H$ and $\mathbf{z}_j^{out} \in \Re_+^K$)	–	–
M2 (no external inputs: $\mathbf{x}_j' = 0, \mathbf{z}_j^{out} \neq 0, \mathbf{x}_j' \in \Re_+^H$ and $\mathbf{z}_j^{out} \in \Re_+^K$)	–	–
M3 (no external outputs: $\mathbf{x}_j' \neq 0, \mathbf{z}_j^{out} = 0, \mathbf{x}_j' \in \Re_+^H$ and $\mathbf{z}_j^{out} \in \Re_+^K$)	Liang et al. [25]	Chen et al. [7]; Li et al. [22]
M4 (no external inputs and outputs: $\mathbf{x}_j' = 0, \mathbf{z}_j^{out} = 0, \mathbf{x}_j' \in \Re_+^H$ and $\mathbf{z}_j^{out} \in \Re_+^K$)	Chen et al. [5]; Lu et al. [27]; Wang et al. [35]; Yang et al. [36]	Kao and Hwang [18]; Liang et al. [23]; Du et al. [14]

Similarly, we can express the weights (w_1, w_2) in the output-oriented model as

$$w_2 = \frac{\sum_{r=1}^{s} u_r y_{jr}}{\sum_{d=1}^{D} \eta_d z_{jd}^{in} + \sum_{r=1}^{s} u_r y_{jr}} = \frac{1}{\frac{\sum_{d=1}^{D} \eta_d z_{jd}^{in}}{\sum_{r=1}^{s} u_r y_{jr}} + 1} = \frac{1}{\theta_{j2}+1},$$
$$w_1 = 1 - w_2 = \frac{\theta_{j2}}{\theta_{j2}+1}, \tag{12}$$

which show that w_2 will decrease when stage 2's efficiency increases, and that w_1 will increase when stage 2's efficiency increases.

Table 4 lists the studies using the two-stage model. Note that several extensions of the basic two-stage model have also appeared in the literature. For example, Liang et al. [24] develop a two-stage model with a feedback loop from stage 2 to stage 1, and Chen et al. [6] propose a two-stage model with shared inputs for the two stages. Applications of the two-stage and network DEA models have been applied to measuring performance of bank branch with information technology [6], corporate governance on airline [27], Chinese commercial banking [35,26,37], and tourist hotels [16], among others. Cook et al. [12] extend the two-stage additive DEA model to multi-stage and network models. Cook et al. [12] and Kao [17] contain several real-world examples of the two-stage model.

4. Additive models with constant weights

The preceding sections illustrate the limitations of the multi-stage DEA model. To circumvent the problem due to variable decomposition weights, we turn to (13), which is an additive model with constant decomposition weights.

$$\text{Max } \theta_o = \sum_{p=1}^{Q} w_p \theta_{op}$$
$$\text{s.t.} \theta_{j1} = \frac{\sum_{r=1}^{R_1} u_{1r} z_{j1r}^{out} + \sum_{k=1}^{S_1} \eta_{1k} z_{j1k}^{in}}{\sum_{i=1}^{m} v_i x_{ji}} \leq 1, j = 1, \ldots, n,$$
$$\theta_{jp} = \frac{\sum_{r=1}^{R_p} u_{pr} z_{jpr}^{out} + \sum_{k=1}^{S_p} \eta_{pk} z_{jpk}^{in}}{\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z_{jp-1k}^{in} + \sum_{i=1}^{I_p} v_{pi} x_{jpi}'} \leq 1, p = 2, \ldots, Q-1; j = 1, \ldots, n,$$
$$\theta_{jQ} = \frac{\sum_{r=1}^{s} u_{Qr} y_{jr}}{\sum_{k=1}^{S_{Q-1}} \eta_{Q-1k} z_{jQ-1k}^{in} + \sum_{i=1}^{I_Q} v_{Qi} x_{jQi}'} \leq 1, j = 1, \ldots, n,$$
$$u_{ph}, v_{pk}, \eta_{pk}, u_r, v_i \geq 0, \tag{13}$$

where w_p are weights predetermined by the evaluator and $\sum_{p=1}^{Q} w_p = 1$.

Note that Model (13) is a fractional linear programming problem ([20], [31]). Unlike the standard DEA model, the problem cannot be expressed as a linear program by the Charnes-and-Cooper transformation. Therefore, we present a heuristic method to approximate its optimal solution. Our method extends Li et al.'s approach from their 2012 paper, which is developed for the two-stage multiplicative DEA model. Their approach aims to maximize the geometric mean of the two stages' efficiency scores (i.e., the DMU's efficiency expressed in the multiplicative form). The solution is obtained by a line search within the upper bound of the two stages' efficiency scores; we will introduce how the upper bound is determined later in this section. We improve Li et al.'s algorithm by finding a tighter lower bound, which can significantly reduce computation time. For example, the computation time is reduced by approximately 70% based on the dataset used in Section 5.[2]

Next we will introduce the method for the input-oriented two-stage DEA model with Structure M4 (no external inputs x_{hj}' and outputs z_{kj}^{out}), which we will use in the next section. The method can be extended and applied to solving a multi-stage model with different structures.

[2] For example, the computation time for the two algorithms is 515.05 seconds and 154.49 seconds, respectively, when the step length ε is 0.001.

For DMU$_o$, the input-oriented two-stage model with constant weights for M4 (no external inputs x'_{hj} for stage 2 and external outputs z^{out}_{kj} for stage 1) is

$$\begin{aligned}
&\text{Max } w_1\frac{\sum_{d=1}^{D}\eta_d z^{in}_{od}}{\sum_{i=1}^{m}v_i x_{oi}}+w_2\frac{\sum_{r=1}^{s}u_r y_{or}}{\sum_{d=1}^{D}\eta_d z^{in}_{od}}\\
&\text{s.t.}\frac{\sum_{d=1}^{D}\eta_d z^{in}_{jd}}{\sum_{i=1}^{m}v_i x_{ji}}\le 1,\ \forall j,\\
&\frac{\sum_{r=1}^{s}u_r y_{jr}}{\sum_{d=1}^{D}\eta_d z^{in}_{jd}}\le 1,\ \forall j,\\
&\eta_d, v_i, u_r\ge 0,\ \forall d, i, r,
\end{aligned}\tag{14}$$

where w_1 and w_2 are predetermined weights, and $w_1+w_2=1$.

We approximate the optimal solution of (14) as follows. In the first step, we calculate the bounds for the optimal stage efficiency scores. In the second step, we use the bounds from the first step to specify the value of one of the stage efficiency scores, and then we can solve the problem as an LP to obtain the other stage efficiency score.

Step 1. Calculating the bounds of the optimal value for θ_1 and θ_2 in Model (14).

To compute the ranges for θ_1 and θ_2, we calculate the maximal efficiency scores for stage 1 and stage 2 as follows:

$$\overline{\theta}_{o1}=\text{Max }\frac{\sum_{d=1}^{D}\eta_d z^{in}_{od}}{\sum_{i=1}^{m}v_i x_{oi}},\text{subject to the constraints of (14).}\tag{15}$$

$$\overline{\theta}_{o2}=\text{Max }\frac{\sum_{r=1}^{s}u_r y_{or}}{\sum_{d=1}^{D}\eta_d z^{in}_{od}},\text{subject to the constraints of (14).}\tag{16}$$

We denote the optimal objective values of Model (15) and (16) as $\overline{\theta}_{o1}$ and $\overline{\theta}_{o2}$, respectively. Next, Model (17) maximizes stage 1's efficiency given the stage 2's efficiency score $\overline{\theta}_{o2}$. Model (18) maximizes stage 2's efficiency given stage 1's efficiency score $\overline{\theta}_{o1}$.

$$\begin{aligned}
&\underline{\theta}_{o1}=\text{Max }\frac{\sum_{d=1}^{D}\eta_d z^{in}_{od}}{\sum_{i=1}^{m}v_i x_{oi}}\\
&\text{s.t. the same contraints as in (14),}\\
&\frac{\sum_{r=1}^{s}u_r y_{or}}{\sum_{d=1}^{D}\eta_d z^{in}_{od}}=\overline{\theta}_{o2}.
\end{aligned}\tag{17}$$

$$\begin{aligned}
&\underline{\theta}_{o2}=\text{Max }\frac{\sum_{r=1}^{s}u_r y_{or}}{\sum_{d=1}^{D}\eta_d z^{in}_{od}}\\
&\text{s.t. the same constraints as in (14),}\\
&\frac{\sum_{d=1}^{D}\eta_d z^{in}_{od}}{\sum_{i=1}^{m}v_i x_{oi}}=\overline{\theta}_{o1}.
\end{aligned}\tag{18}$$

The optimal objective values of Model (17) and (18) are denoted $\underline{\theta}_{o1}$ and $\underline{\theta}_{o2}$, respectively.

Theorem 3 shows that the optimal values of Models (15)–(18) constitute bounds for the optimal values for efficiency scores from Model (14).

Theorem 3. *Let $\overline{\theta}_{o1}$, $\overline{\theta}_{o2}$, $\underline{\theta}_{o1}$ and $\underline{\theta}_{o2}$ be the optimal objective values of Model (15–18). Then $\overline{\theta}_{o1}\le\theta_{o1}\le\overline{\theta}_{o1}$ and $\underline{\theta}_{o2}\le\theta_{o2}\le\overline{\theta}_{o2}$, where $\theta_{o1}$and θ_{o2} are the optimal values for stage 1 and stage 2 efficiencies by Model (14).*

Proof. Suppose θ_{o1} and θ_{o2} are the optimal efficiency scores of stage 1 and stage 2 by Model (14), and $0\le\theta_{o1}<\underline{\theta}_{o1}$. We can then have $w_1\theta_{o1}+w_2\theta_{o2}<w_1\underline{\theta}_{o1}+w_2\theta_{o2}\le w_1\underline{\theta}_{o1}+w_2\overline{\theta}_{o2}$. From Model (17), $\left(\underline{\theta}_{o1},\overline{\theta}_{o2}\right)$ can be a feasible solution to Model (14). This yields a contradiction that θ_{o1} is the optimum. Therefore, θ_{o1} must be larger than $\underline{\theta}_{o1}$. Adding that Model (15) shows the upper bound that $\theta_{o1}\le\overline{\theta}_{o1}$, we can derive that $\underline{\theta}_{o1}\le\theta_{o1}\le\overline{\theta}_{o1}$. A similar analysis is applied to proof of $\underline{\theta}_{o2}\le\theta_{o2}\le\overline{\theta}_{o2}$.

Step 2. Searching the optimal solutions of (14) based on the range obtained from Step 1.

Given the ranges obtained from Step 1, we can express Model (14) as a parametric programming problem with $\theta_{o1}\in\left[\underline{\theta}_{o1},\overline{\theta}_{o1}\right]$ as the parameter:

$$\begin{aligned}
&\text{Max}\quad w_1\theta_{o1}+w_2\frac{\sum_{r=1}^{s}u_r y_{or}}{\sum_{d=1}^{D}\eta_d z^{in}_{od}}\\
&\text{s.t.}\frac{\sum_{d=1}^{D}\eta_d z^{in}_{jd}}{\sum_{i=1}^{m}v_i x_{ji}}\le 1,\ \forall j,\\
&\frac{\sum_{r=1}^{s}u_r y_{jr}}{\sum_{d=1}^{D}\eta_d z^{in}_{jd}}\le 1,\ \forall j,\\
&\frac{\sum_{d=1}^{D}\eta_d z^{in}_{od}}{\sum_{i=1}^{m}v_i x_{oi}}=\theta_{o1},\\
&\theta_{o1}\in\underline{\theta}\left[\underline{\theta}_{o1},\overline{\theta}_{o1}\right],\\
&\eta_d, v_i, u_r\ge 0,\ \forall d, i, r.
\end{aligned}\tag{19}$$

With θ_{o1} assigned to a value, Model (19) maximizes the stage 2 efficiency score $\sum_{r=1}^{s} u_r y_{or} / \sum_{d=1}^{D} \eta_d z_{od}^{\text{in}}$. The optimal solution (η_d, v_i, u_r) of Model (19) can be approximated by solving Model (20):

$$\begin{aligned}
&\text{Max } \frac{\sum_{r=1}^{s} u_r y_{or}}{\sum_{d=1}^{D} \eta_d z_{od}^{\text{in}}}\\
&\text{s.t.} \frac{\sum_{d=1}^{D} \eta_d z_{jd}^{\text{in}}}{\sum_{i=1}^{m} v_i x_{ji}} \le 1, \forall j,\\
&\frac{\sum_{r=1}^{s} u_r y_{jr}}{\sum_{d=1}^{D} \eta_d z_{jd}^{\text{in}}} \le 1, \forall j,\\
&\frac{\sum_{d=1}^{D} \eta_d z_{od}^{\text{in}}}{\sum_{i=1}^{m} v_i x_{oi}} = \theta_{o1},\\
&\eta_d, v_i, u_r \ge 0, \forall d, i, r.
\end{aligned} \tag{20}$$

Model (20) is equivalent to the following LP after applying the Charnes-and-Cooper transformation:

$$\begin{aligned}
&\text{Max } \sum_{r=1}^{s} u_r y_{or}\\
&\text{s.t.} \sum_{d=1}^{D} \eta_d z_{od}^{\text{in}} = 1,\\
&\sum_{d=1}^{D} \eta_d z_{jd}^{\text{in}} - \sum_{i=1}^{m} v_i x_{ji} \le 0, \forall j,\\
&\sum_{r=1}^{s} u_r y_{jr} - \sum_{d=1}^{D} \eta_d z_{jd}^{\text{in}} \le 0, \forall j,\\
&\sum_{d=1}^{D} \eta_d z_{od}^{\text{in}} - \theta_{o1} \sum_{i=1}^{m} v_i x_{oi} = 0,\\
&\eta_d, v_i, u_r \ge 0, \forall d, i, r.
\end{aligned} \tag{21}$$

To obtain the optimal stage 2's efficiency θ_{o2}, we start by solving Model (21) with θ_{o1} equal to its upper bound, θ_{o1}. Then we stepwise reduce θ_{o1} by ε (ε=0.0001 for example), namely, $\theta_{o1}^k = \theta_{o1} - \varepsilon \times k$, $k = 1, 2, \ldots$ until the lower bound $\underline{\theta}_{o1}$ is reached. We denote the corresponding optimal objective value for Model (21) is θ_{o2}^k. The efficiency of the overall process can be estimated as $\theta_o = \max_k \left\{ w_1 \theta_{o1}^k + w_2 \theta_{o2}^k \right\}$, where the θ_{o1}^k and θ_{o2}^k associated with is the optimal stage 1 and stage 2 process's efficiency scores. We summarize this heuristic method as follows. We will discuss how to apply this algorithm to a general multi-stage model in the final section.

Heuristics to approximate the optimal solution of (14)

1. Initial $k = 0$, ε is a user-specify parameter.
2. Let $k = k + 1$, and $\theta_{o1}^k = \theta_{o1} - \varepsilon \times (k - 1)$.
3. Solve Model (21) with $\theta_{o1} = \theta_{o1}^k$ to obtain optimal objective value as θ_{o2}^k.
4. Compute $\theta_o(k) = w_1 \theta_{o1}^k + w_2 \theta_{o2}^k$.
5. If $\theta_{o1}^k \ge \underline{\theta}_{o1}$, go to step 2; otherwise, go to step 6.
6. Let $k^* = \arg\{\max\{\theta_o(k)\}\}$. The efficiency of the overall process can be approximated by $\theta_o = \theta_o(k^*)$, and the optimal stages efficiency scores are $\theta_{o1} = \theta_{o1}^{k^*}$ and $\theta_{o2} = \theta_{o2}^{k^*}$.

5. An illustration

In this section, we use the data of a two-stage model from Kao and Hwang [18] to demonstrate the issues that we discussed earlier in this paper. The data contain the inputs (Operation expenses and insurance expenses), intermediate outputs (direct written premiums and reinsurance premiums), and outputs (underwriting profit and investment profit) of 24 Taiwanese non-life insurance companies (see Appendix B). Table 5 shows the DMU's efficiencies and the optimal weights that one would obtain by using the two-stage model of Chen et al. [5].

Table 5 shows that stage 1's weight, w_1, is larger than stage 2's weight, w_2 for all DMUs except for DMUs #9, #12, #15, #19 and #24 ($w_1 = w_2 = 0.5$ for these five DMUs). Note that the weights are negatively associated with each stage's efficiencies. For example, both DMUs #3 and #22's stage 2 efficiency scores are 1.000, but DMU #3's stage 1's efficiency score (=0.690) is greater than DMU #22's (0.590). However, DMU #3's weight for stage 1 (0.592) is lower than that for DMU #22 (0.629). This is contrary to our expectation that a DMU (or the model) should assign a higher weight to the stage with a higher efficiency in order to maximize the DMU's overall efficiency score. This example illustrates the problem of interdependency between weights and efficiency scores, which arises because weights and stage efficiencies are both endogenous in the model. In other words, one cannot really tell how much of the efficiency score reflects technical efficiency status and how much is due to the influence from weights.

To illustrate the influence of weights on efficiency estimates, we compare the results from Chen et al.'s method with those from Model (14) with different predetermined weights. These predetermined weights reflect how the decision-maker values stage 1's efficiency over

Table 5
Additive efficiency scores and weights in Chen et al. [5].

DMU	θ_j	w_1	w_2	θ_{j1}	θ_{j2}
1	0.8491	0.5019	0.4981	0.9926	0.7045
2	0.8122	0.5004	0.4996	0.9985	0.6257
3	0.8166	0.5917	0.4083	0.6900	1.0000
4	0.5965	0.5799	0.4201	0.7243	0.4200
5	0.8727	0.5462	0.4538	0.8307	0.9233
6	0.6887	0.5100	0.4900	0.9606	0.4057
7	0.5804	0.5707	0.4293	0.7521	0.3522
8	0.5795	0.5795	0.4205	0.7256	0.3780
9	0.6116	0.5000	0.5000	1.0000	0.2233
10	0.7131	0.5372	0.4628	0.8615	0.5408
11	0.5088	0.5783	0.4217	0.7292	0.2066
12	0.8798	0.5000	0.5000	1.0000	0.7596
13	0.5565	0.5523	0.4477	0.8107	0.2431
14	0.5773	0.5798	0.4202	0.7246	0.3740
15	0.8069	0.5000	0.5000	1.0000	0.6138
16	0.6395	0.5303	0.4697	0.8856	0.3615
17	0.6126	0.5803	0.4197	0.7232	0.4597
18	0.5868	0.5576	0.4424	0.7935	0.3262
19	0.7056	0.5000	0.5000	1.0000	0.4112
20	0.7654	0.5173	0.4827	0.9332	0.5857
21	0.5412	0.5713	0.4287	0.7505	0.2623
22	0.7418	0.6291	0.3709	0.5895	1.0000
23	0.6854	0.5427	0.4573	0.8426	0.4989
24	0.5435	0.5000	0.5000	1.0000	0.0870

stage 2's efficiency. We use the same insurance companies' data, and experiment with different values of w_1 (from 0.1, 0.2,…, to 0.9) to solve Model (14). We refer to Model (14) with constant weights as the "constant-weight model." Note that in a two-stage model, the weights will sum up to one ($w_1 + w_2 = 1$); therefore when w_1 is assigned to a certain value, w_2 automatically receives a corresponding value.

Fig. 3 shows the differences between the efficiency scores of these two models. The horizontal axis represents the weight w_1 for stage 1. The vertical axis in Fig. 3(a) and (b) indicates the differences between the product of efficiency scores and weights (i.e., $w_1\theta_1$ in Fig. 3(a), $w_2\theta_2$ in Fig. 3(b), and $w_1\theta_1 + w_2\theta_2$ in Fig. 3(c)) for all 24 DMUs.[3] A positive value implies that the weighted efficiency score from the constant-weight model is higher than that obtained from Chen et al.'s model.

We have a number of interesting observations from the results. First, Fig. 3(a) shows that the differences regarding $w_1\theta_1$ increase from negative to positive values as w_1 increases, and that the differences approach zero when w_1 is within the region of [0.5, 0.6]. Observe that the optimal $w_1$values from Chen et al.'s [5] additive two-stage model shown in Table 5, fall in the range between 0.5000 and 0.6291, or approximately [0.50, 0.63]. This implies that stage 1's efficiency in Chen et al.'s model carries no less weight than stage 2's efficiency, and that a slightly higher preference is given to stage 1 for certain DMUs. Thus, when w_1 is set to lower than 0.500 in the constant-weight model, the value of $w_1\theta_1$ tends to be lower than the Chen et al. counterpart. The differences for $w_2\theta_2$ also exhibit a similar but opposite pattern described above (see Fig. 3(b)). Similarly, when w_1 is lower than 0.500, the differences for $w_2\theta_2$ tend to be positive. The above patterns demonstrate that Chen et al.'s model can generate stage-wise efficiency scores different from the optimal scores under a fixed value of w_1 and w_2. Therefore, Chen et al.'s model [5] would be inappropriate in a likely situation where the evaluator intends to attach a higher importance weight to stage 2 over stage 1.

Second, the two methods also give different overall efficiency scores. Fig. 3(c) shows the differences in the overall efficiency scores from two methods. It shows that the differences increase as w_1 moves further away from the range [0.50, 0.63], and that differences are positive for some DMUs and negative for others. The latter result is related to several DMU-level factors, including (1) whether stage 1 or stage 2 has better relative efficiency, and (2) the range of θ_1 and θ_2 for the constant-weight model. As an illustration, we pick a few representative DMUs (DMUs #9, #12, and #17) and plot their efficiency scores from the constant weights model in Fig. 4. The region between dashed lines is the range of θ_1, whereas the region between round-dotted lines is the feasible range of θ_2 The figure also shows the differences between the overall efficiency scores from two models, whose value corresponds to the right-hand-side vertical axis.

Fig. 4 shows the results for the three DMUs that we choose. The difference between the overall efficiency scores for DMU #9's increases in w_1. The constant-weight model generates a higher score when w_1 is higher than Chen et al.'s optimal w_1 value (marked as a "cross" in the horizontal axis). As noted, the main reason is that DMU #9's stage 1 has an edge over its stage 2's efficiency. Thus, its overall efficiency score would increase when w_1 takes a higher value. Note that through our computation we find that its stage 1 efficiency score (θ_1) can vary between [0.44, 1.0], while its stage 2 efficiency score (θ_2) can vary between [0.22, 0.29]. Increasing w_1 from 0.1 to 0.2 would result in a higher θ_1 at the cost of lower θ_2, and both scores have reached their respective bounds when $w_1 = 0.2$. Thus when $w_1 > 0.2$, the objective value, which is also the overall efficiency of DMU #9, will increase linearly at a fixed marginal rate of $\theta_1 - \theta_2$. In other words, the increase in overall efficiency when $w_1 > 0.2$ is due to the increase in w_1. The difference in overall efficiencies will increase from negative to zero when w_1 is set equal to the optimal w_1 value of the Chen et al.'s method, and will continue to increase linearly as w_1 increases. Note that the difference between the overall efficiency scores can be as high as 0.3.

[3] We choose to examine the product of efficiency and weight, $w_1\theta_1$ and $w_2\theta_2$, because both variables are endogenous in the Chen et al.'s method. Due to endogeneity, one cannot isolate the influence of weights on efficiency scores in the optimal solution.

Let us now turn to DMU #12. Observe that under the constant-weight model, the stage 1 and stage 2 efficiencies of DMU #12 have a unique solution. Thus, its overall efficiency will be a linear function of w_1. This stands in contrast to DMU #17: the bounds of DMU #17's stage 1 and stage 2 efficiencies have a substantial overlap (Fig. 4(c)), and therefore the difference between overall efficiency scores becomes nonlinear (in our case a concave function of w_1).

In summary, we show that the weights and efficiencies in the existing two-stage DEA model are endogenous, leading to difficulties in interpreting the results. We also show that the weight of stage 1 always carries at least as much weight as the stage 2. This implicit assumption is likely to lead to spurious efficiency scores, as was evidenced in the preceding analysis.

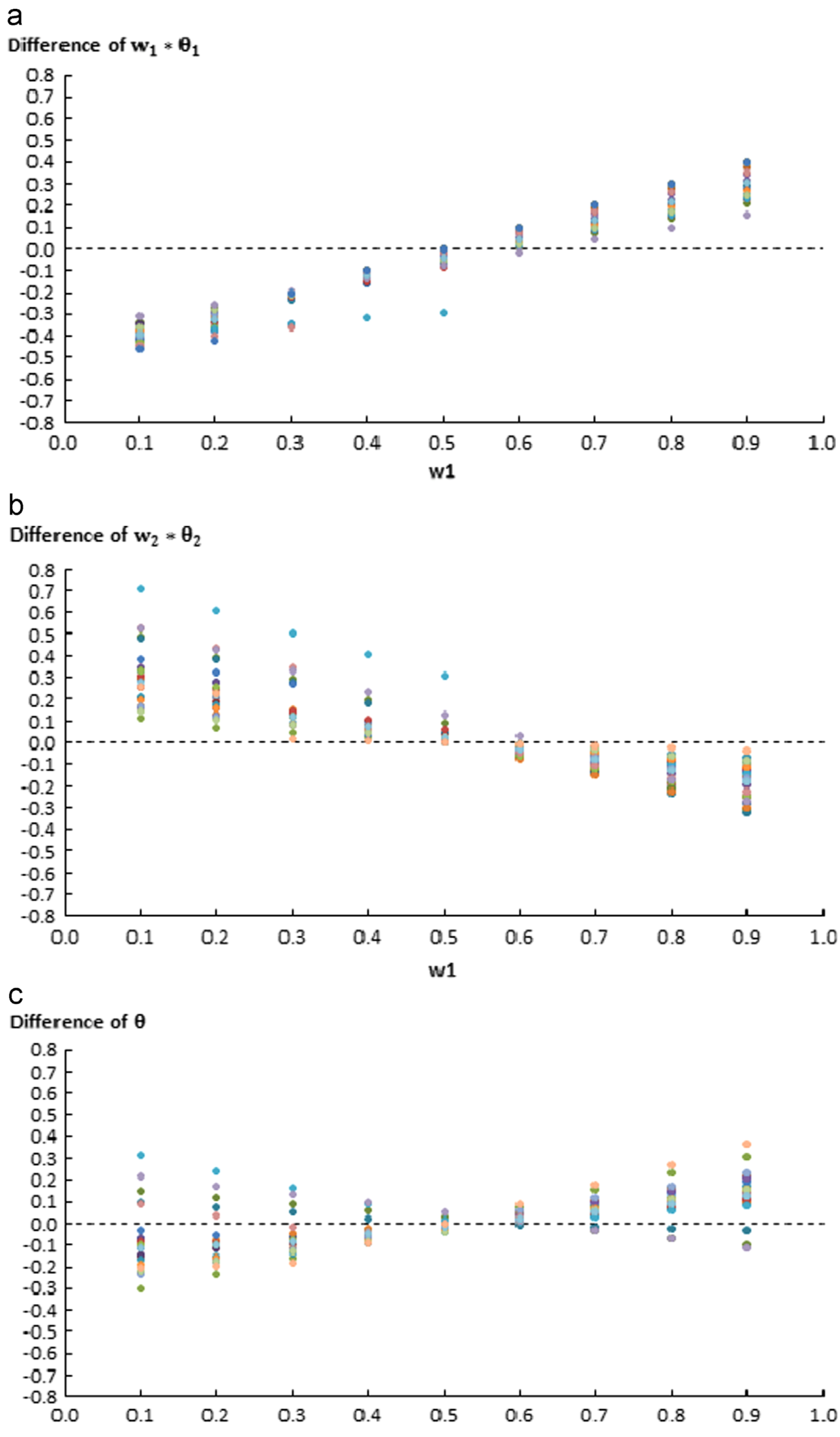

Fig. 3. Differences of the stage-wise and overall efficiency scores under different w_1. (a) Differences of the stage 1 $w_1{*}\theta_1$ in the optimal solutions from two models. (b) Differences of the stage 2 $w_2{*}\theta_2$ in the optimal solutions from two models. (c) Differences of the overall efficiencies from two models.

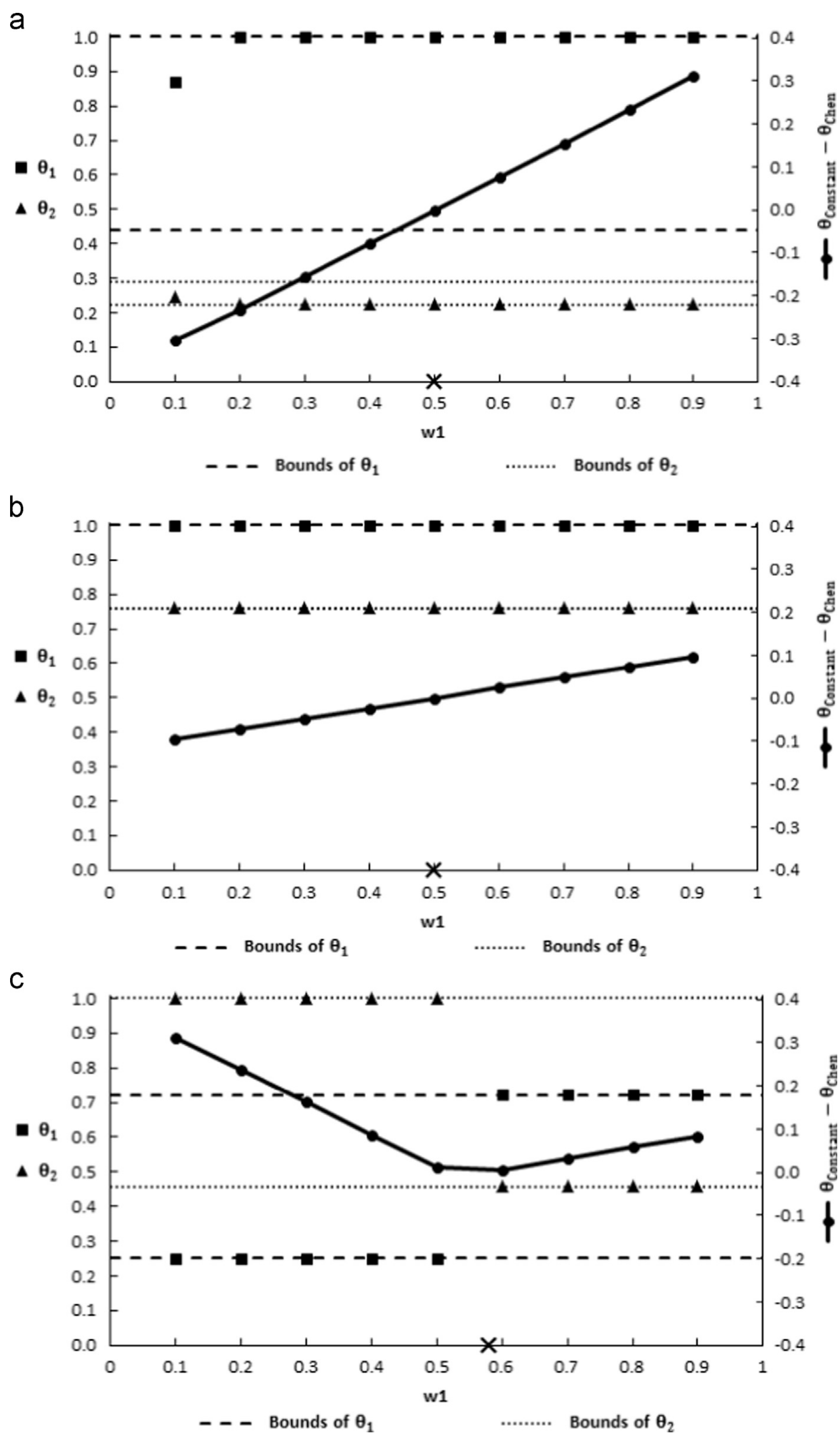

Fig. 4. Stage and overall efficiency scores (the cross marked on the horizontal axis is the optimal w_1 value obtained from Chen et al.'s model). (a) DMU #9's stage and overall efficiencies from two models. (b) DMU #12's stage and overall efficiencies from two models. (c) DMU #17's stage and overall efficiencies from two models.

6. Discussion and conclusion

This paper reveals several previously overlooked properties about the decomposition weights in the additive multi-stage DEA model. We summarize our findings as follows. First, we find that the decomposition weights for the input-oriented multi-stage models are non-increasing in the sequence of stages. This means that the earlier stages would obtain higher decomposition weights and therefore a greater influence on the overall efficiency. In the two-stage model, for example, this means that the first-stage decomposition weight will be at least as high as 0.5. It is not too difficult to find examples where managers may find this property at odds with the process context in question. For example, one may argue that stage 2 in a two-stage process should carry a higher weight because stage 2 is closer to the customer, whose perception on stage 2 would be more influential to future demand. The traditional two-stage (or multi-stage) model is not flexible enough to adapt to such environments. Another possible issue is that the endogenous weights may deviate substantially from the evaluator's preference and the best interest of the evaluated DMU (for example, if the evaluator prefers to set the weights as 0.1 and 0.9 for stages 1 and 2, respectively). Second, we demonstrate that the monotonic property of decomposition weights can interfere with the estimated stage and overall efficiency scores. The empirical application shows that the efficiency scores from a standard two-stage model are subject to the influence from decomposition weights. This also suggests the need to develop alternative models. We looked at one such alternative: the multi-stage DEA model with constant decomposition weights.

We improve the algorithm for solving the multi-stage DEA model with constant decomposition weights. In the case of two-stage models, the improved algorithm was able to reduce the computation time by approximately 70% than the competing algorithm. We believe the performance gap will increase exponentially in the number of stages, as the number of problems that needs be solved will also increase as such. This improved algorithm also easily lends itself to a multi-stage DEA model with a different internal structure. We outline the extension as follows. As noted earlier, we can obtain the upper bound of stage efficiency score, by solving one maximization problem for each stage efficiency score. The lower bound for the stage efficiency score for a particular stage can be obtained by solving a maximization problem similar to the one for the upper bound, but now the weighted sum of all the other stages' efficiency scores is set to its maximum value (which we obtain by first solving a separate problem). In other words, we approximate the lower bound of a particular stage's efficiency score by solving a problem similar to the two-stage method, in which we now fix the aggregate sum of all the other stages' efficiency scores at its maximum. Once the bounds are determined, we can proceed to compute the stage and overall efficiency scores by first discretizing the value range within the bounds and then solve problems based on combinations of the discretized values.

Further research into multi-stage DEA models seems warranted. First, we found that decomposition weights and efficiency scores are linked in the traditional model. Although we use a numerical example as evidence of this confounding effect, more research is needed to further validate this claim analytically. Second, the model with constant weight is one option to model the multi-stage problem, but we also acknowledged its limitation in computation, too. Future research can try to develop linear formulations that are not subjective to the same limitations identified by this paper.

Acknowledgments

The authors thank three anonymous referees for their constructive comments and suggestions, which have significantly improved this paper. The paper was written when S. Ang visited Nanyang Business School, Nanyang Technological University Singapore, under the financial support of the China Scholarship Council (20130634007). S. Ang also thanks the support from the National Natural Science Foundation of China (Grant nos. 71271195, 71301156 and 71322101).

Appendix A. The output-oriented multi-stage model under structure M1

The output-oriented efficiency for stage p process for $\mathrm{DMU_j}$ is defined as sum of virtual inputs divided by virtual outputs:

$$\theta_p = \frac{\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z_{jp-1k}^{in} + \sum_{i=1}^{I_p} v_{pi} x'_{jpi}}{\sum_{r=1}^{R_p} u_{pr} z_{jpr}^{out} + \sum_{k=1}^{S_p} \eta_{pk} z_{jpk}^{in}}, p = 2, \ldots, Q-1, \tag{A1}$$

and

$$\theta_1 = \frac{\sum_{i=1}^{m} v_i x_{ji}}{\sum_{r=1}^{R_1} u_{1r} z_{j1r}^{out} + \sum_{k=1}^{S_1} \eta_{1k} z_{j1k}^{in}},$$

$$\theta_Q = \frac{\sum_{k=1}^{S_{Q-1}} \eta_{Q-1k} z_{jQ-1k}^{in} + \sum_{i=1}^{I_Q} v_{Qi} x'_{jQi}}{\sum_{r=1}^{s} u_r y_{jr}}. \tag{A2}$$

The weights w_p is the sum of virtual outputs produced in stage p divided by total virtual outputs across all stages:

$$w_p = \begin{cases} \dfrac{\sum_{r=1}^{R_p} u_{pr} z_{jpr}^{out} + \sum_{k=1}^{S_p} \eta_{pk} z_{jpk}^{in}}{\sum_{p=1}^{Q-1}\left(\sum_{r=1}^{R_p} u_{pr} z_{jpr}^{out} + \sum_{k=1}^{S_p} \eta_{pk} z_{jpk}^{in}\right) + \sum_{r=1}^{s} u_r y_{jr}}, p = 1, \ldots, Q-1, \\ \dfrac{\sum_{r=1}^{s} u_r y_{jr}}{\sum_{p=1}^{Q-1}\left(\sum_{r=1}^{R_p} u_{pr} z_{jpr}^{out} + \sum_{k=1}^{S_p} \eta_{pk} z_{jpk}^{in}\right) + \sum_{r=1}^{s} u_r y_{jr}}, p = Q, \end{cases} \tag{A3}$$

where $\sum_{p=1}^{P} w_p = 1$. The overall efficiency measure of the system is still represented as a convex linear combination of the Q stage-level measures, $\theta = \sum_{p=1}^{P} w_p \theta_p$.

In the output-oriented model, the restrictions for the individual measures θ_p $(p = 1, \ldots, Q)$ are that they must exceed unity. Similarly, after the Charnes-and-Cooper transformation, the overall efficiency θ of $\mathrm{DMU_0}$ can be obtained by solving the following LP:

$$\text{Min} \sum_{i=1}^{m} v_i x_{0i} + \sum_{p=2}^{Q} \sum_{k=1}^{S_{Q-1}} \eta_{Q-1k} z_{0Q-1k}^{in} + \sum_{i=1}^{I_Q} v_{Qi} x'_{0Qi} \Bigg)$$

$$\text{s.t.} \sum_{p=1}^{Q-1}\left(\sum_{r=1}^{R_p} u_{pr} z_{0pr}^{out} + \sum_{k=1}^{S_p} \eta_{pk} z_{0pk}^{in}\right) + \sum_{r=1}^{s} u_r y_{0r} = 1,$$

$$\sum_{i=1}^{m} v_i x_{ji} \geq \sum_{r=1}^{R_1} u_{1r} z_{j1r}^{out} + \sum_{k=1}^{S_1} \eta_{1k} z_{j1k}^{in}, j = 1, .., n,$$

$$\sum_{k=1}^{S_{p-1}} \eta_{p-1k} z_{jp-1k}^{in} + \sum_{i=1}^{I_p} v_{pi} x'_{jpi} \geq \sum_{r=1}^{R_p} u_{pr} z_{jpr}^{out} + \sum_{k=1}^{S_p} \eta_{pk} z_{jpk}^{in}, p = 2, \ldots, Q-1; j = 1, .., n,$$

$$\sum_{k=1}^{S_{Q-1}} \eta_{Q-1k} z_{jQ-1k}^{in} + \sum_{i=1}^{I_Q} v_{Qi} x'_{jQi} \geq \sum_{r=1}^{s} u_r y_{jr}, j = 1, .., n,$$

$$u_{ph}, v_{pk}, \eta_{pk}, u_r, v_i \geq, 0. \tag{A4}$$

Appendix B. Data set of 24 Taiwanese non-life insurance companies

See Table B1

Table B1
Taiwanese non-life insurance companies 2001–2002 (in thousands of New Taiwan Dollars. Source: [18]).

DMU	Inputs		Intermediate outputs		Final outputs	
	Operation expenses	Insurance expenses	Direct written premiums	Reinsurance premiums	Underwriting profit	Investment profit
1	1,178,744	673,512	7,451,757	856,735	984,143	681,687
2	1,381,822	1,352,755	10,020,274	1,812,894	1,228,502	834,754
3	1,177,494	592,790	4,776,548	560,244	293,613	658,428
4	601,320	594,259	3,174,851	371,863	248,709	177,331
5	6,699,063	3,531,614	37,392,862	1,753,794	7,851,229	3,925,272
6	2,627,707	668,363	9,747,908	952,326	1,713,598	415,058
7	1,942,833	1,443,100	10,685,457	643,412	2,239,593	439,039
8	3,789,001	1,873,530	17,267,266	1,134,600	3,899,530	622,868
9	1,567,746	950,432	11,473,162	546,337	1,043,778	264,098
10	1,303,249	1,298,470	8,210,389	504,528	1,697,941	554,806
11	1,962,448	672,414	7,222,378	643,178	1,486,014	18,259
12	2,592,790	650,952	9,434,406	1,118,489	1,574,191	909,295
13	2,609,941	1,368,802	13,921,464	811,343	3,609,236	223,047
14	1,396,002	988,888	7,396,396	465,509	1,401,200	332,283
15	2,184,944	651,063	10,422,297	749,893	3,355,197	555,482
16	1,211,716	415,071	5,606,013	402,881	854,054	197,947
17	1,453,797	1,085,019	7,695,461	342,489	3,144,484	371,984
18	757,515	547,997	3,631,484	995,620	692,731	163,927
19	159,422	182,338	1,141,951	483,291	519,121	46,857
20	145,442	53,518	316,829	131,920	355,624	26,537
21	84,171	26,224	225,888	40,542	51,950	6491
22	15,993	10,502	52,063	14,574	82,141	4181
23	54,693	28,408	245,910	49,864	0.1	18,980
24	163,297	235,094	476,419	644,816	142,370	16,976

References

[1] Charnes A, Cooper WW. Programming with linear fractional functionals. Naval Research Logistics Quarterly 1962;9:181–5.
[2] Charnes A, Cooper WW, Rhodes E. Measuring the efficiency of decision making units. European Journal of Operational Research 1978;2(6):429–44.
[3] Chen C-M. A network-DEA model with new efficiency measures to incorporate the dynamic effect in production networks. European Journal of Operational Research 2009;194(3):687–99.
[4] Chen C-M, Delmas MA, Lieberman MB. Production frontier methodologies and efficiency as a performance measure in strategic management research. Strategic Management Journal 2015;36(1):19–36.
[5] Chen Y, Cook WD, Li N, Zhu J. Additive efficiency decomposition in two-stage DEA. European Journal of Operational Research 2009;196(3):1170–6.
[6] Chen Y, Du J, David Sherman H, Zhu J. DEA model with shared resources and efficiency decomposition. European Journal of Operational Research 2010;207(1):339–49.
[7] Chen Y, Liang L, Yang F. A DEA game model approach to supply chain efficiency. Annals of Operations Research 2006;145(1):5–13.
[8] Cook WD, Hababou M, Tuenter HJ. Multicomponent efficiency measurement and shared inputs in data envelopment analysis: an application to sales and service performance in bank branches. Journal of Productivity Analysis 2000;14(3):209–24.
[9] Cook WD, Harrison J, Imanirad R, Rouse P, Zhu J. Data envelopment analysis with nonhomogeneous DMUs. Operations Research 2013;61(3):666–76.
[10] Cook WD, Harrison J, Rouse P, Zhu J. Relative efficiency measurement: the problem of a missing output in a subset of decision making units. European Journal of Operational Research 2012;220(1):79–84.
[11] Cook WD, Zhu J. Multiple variable proportionality in data envelopment analysis. Operations Research 2011;59(4):1024–32.
[12] Cook WD, Zhu J, Bi G, Yang F. Network DEA: additive efficiency decomposition. European Journal of Operational Research 2010;207(2):1122–9.

[13] Despotis DK, Koronakos G, Sotiros D. Composition versus decomposition in two-stage network DEA: a reverse approach. Journal of Productivity Analysis 2014, http://dx.doi.org/10.1007/s11123-014-0415-x.
[14] Du J, Liang L, Chen Y, Cook WD, Zhu J. A bargaining game model for measuring performance of two-stage network structures. European Journal of Operational Research 2011;210(2):390–7.
[15] Emrouznejad A, Parker BR, Tavares G. Evaluation of research in efficiency and productivity: a survey and analysis of the first 30 years of scholarly literature in DEA. Socio-economic Planning Sciences 2008;42(3):151–7.
[16] Huang CW, Ho FN, Chiu YH. Measurement of tourist hotels' productive efficiency, occupancy, and catering service effectiveness using a modified two-stage DEA model in Taiwan. Omega 2014;48:49–59.
[17] Kao C. Network data envelopment analysis: a review. European Journal of Operational Research. 2014;239(1):1–16.
[18] Kao C, Hwang SN. Efficiency decomposition in two-stage data envelopment analysis: an application to non-life insurance companies in Taiwan. European Journal of Operational Research 2008;185(1):418–29.
[19] Kao C, Liu ST. Multi-period efficiency measurement in data envelopment analysis: the case of Taiwanese commercial banks. Omega 2014;47:90–8.
[20] Konno H, Yajima Y, Matsui T. Parametric simplex algorithms for solving a special class of nonconvex minimization problems. Journal of Global Optimization 1991;1(1):65–81.
[21] Lee CY, Johnson AL. Two-dimensional efficiency decomposition to measure the demand effect in productivity analysis. European Journal of Operational Research 2012;216(3):584–93.
[22] Li Y, Chen Y, Liang L, Xie J. DEA models for extended two-stage network structures. Omega 2012;40(5):611–8.
[23] Liang L, Cook WD, Zhu J. DEA models for two-stage processes: game approach and efficiency decomposition. Naval Research Logistics 2008;55(7):643–53.
[24] Liang L, Li ZQ, Cook WD, Zhu J. Data envelopment analysis efficiency in two-stage networks with feedback. IIE Transactions 2011;43(5):309–22.
[25] Liang L, Yang F, Cook WD, Zhu J. DEA models for supply chain efficiency evaluation. Annals of Operations Research 2006;145(1):35–49.
[26] Liu W, Zhou Z, Ma C, Liu D, Shen W. Two-stage DEA models with undesirable input-intermediate-outputs. Omega 2015;56:74–87.
[27] Lu WM, Wang WK, Hung SW, Lu ET. The effects of corporate governance on airline performance: production and marketing efficiency perspectives. Transportation Research Part E: Logistics and Transportation Review 2012;48(2):529–44.
[28] Matthews K. Risk management and managerial efficiency in Chinese banks: a network DEA framework. Omega 2013;41(2):207–15.
[29] Nouri J, Lotfi FH, Borgheipour H, Atabi F, Sadeghzadeh SM, Moghaddas Z. An analysis of the implementation of energy efficiency measures in the vegetable oil industry of Iran: a data envelopment analysis approach. Journal of Cleaner Production 2013;52:84–93.
[30] Park KS, Park K. Measurement of multiperiod aggregative efficiency. European Journal of Operational Research 2009;193(2):567–80.
[31] Shen PP, Wang CF. Global optimization for sum of linear ratios problem with coefficients. Applied Mathematics and Computation 2006;176(1):219–29.
[32] Tone K, Sahoo BK. Scale, indivisibilities and production function in data envelopment analysis. International Journal of Production Economics 2003;84(2):165–92.
[33] Tone K, Tsutsui M. Network DEA: a slacks-based measure approach. European Journal of Operational Research 2009;197(1):243–52.
[34] Tsutsui M, Goto M. A multi-division efficiency evaluation of US electric power companies using a weighted slacks-based measure. Socio-Economic Planning Sciences 2009;43(3):201–8.
[35] Wang K, Huang W, Wu J, Liu YN. Efficiency measures of the Chinese commercial banking system using an additive two-stage DEA. Omega 2014;44:5–20.
[36] Yang CH, Lin HY, Chen CP. Measuring the efficiency of NBA teams: additive efficiency decomposition in two-stage DEA. Annals of Operations Research 2014;217(1):565–89.
[37] Zha Y, Liang N, Wu M, Bian Y. Efficiency evaluation of Banks in China: a dynamic two-stage slacks-based measure approach. Omega 2015. http://dx.doi.org/10.1016/j.omega.2014.12.008.