# Investigating the interaction between rough surfaces by using the Fisher–Shannon method: Implications on interaction between tectonic plates

Lucia Rebeca Moreno-Torres, Armando Gomez-Vieyra, Michele Lovallo,
Alejandro Ramírez-Rojas, Luciano Telesca

PII: S0378-4371(18)30453-9
DOI: https://doi.org/10.1016/j.physa.2018.04.023
Reference: PHYSA 19455

To appear in: *Physica A*

Received date : 16 January 2018
Revised date : 21 February 2018

Please cite this article as: L.R. Moreno-Torres, A. Gomez-Vieyra, M. Lovallo, A. Ramírez-Rojas, L. Telesca, Investigating the interaction between rough surfaces by using the Fisher–Shannon method: Implications on interaction between tectonic plates, *Physica A* (2018), https://doi.org/10.1016/j.physa.2018.04.023

## Highlights

1. The interaction between surfaces with asperities is studied
2. Voltage series are recorded by stick-slip experiments
3. The Shannon entropy of the voltage series decreases with the runs
4. The FIM increases with the runs
5. The experimental model mimics aging effect of a tectonic fault

# Investigating the interaction between rough surfaces by using the Fisher-Shannon method: implications on interaction between tectonic plates

Lucia Rebeca Moreno-Torres[1], Armando Gomez-Vieyra[2], Michele Lovallo[3], Alejandro Ramírez-Rojas[2], Luciano Telesca[4]

[1]Escuela Superior de Fisica y Matematicas, Instituto Politecnico Nacional, Unidad Profesional Zacatenco, Mexico City, Mexico
[4]Departamento de Ciencias Básicas, Universidad Autónoma Metropolitana Azcapotzalco, Av. San Pablo 180, 02200 Mexico City, Mexico
[3]ARPAB, Potenza, Italy
[2]CNR-IMAA, Potenza Italy, It, Potenza, Italy

## Abstract

We investigated the properties of the interaction between surfaces with asperities by the analysis of voltage time series recorded by an experimental stick-slip model. We found that: 1) the standard deviation of the voltage time series decreases with the number of consecutive runs, reflecting the decrease of the displacements' variation due to the decrease of friction between the two interacting surfaces; 2) the Shannon entropy (the Fisher Information Measure) of the voltage time series decreases (increases) with the number of consecutive runs, suggesting a decrease of the randomness of the system; 3) this experimental model mimics the aging effect that characterizes the interaction between tectonic faults.

**Keywords**: stick-slip; friction; earthquakes; aging; Shannon-Fisher method, analogic modelling

## Introduction

When two surfaces with asperities interact with each other in a relative movement, a stick-slip event occurs when the potential energy that is accumulated during the process becomes larger than the static friction, and the energy released produces a displacement [1]. This is generally the underlying mechanism of earthquake generation in seismic areas. The interaction between huge surfaces, like tectonic plates, is a complex phenomenon that needs theoretical and experimental approaches to be deeply understood. The well-known spring-block model simulates the interaction between tectonic plates by means of masses connected by linear springs [2]. Vargas et al. (2008) [3] set up an experimental stick–slip model, simulating the relative movement of two rough surfaces by the interaction of two blocks covered by sandpapers with a certain roughness degree. In their experimental model the interaction between rough surfaces (sandpapers), moving in opposition to each other, produces stick-slip events (synthetic seismicity), which mimic the real seismicity. Analyses of synthetic seismicity have been performed by calculating the Gutenberg–Richter law [3], the correlation properties based on the detrended fluctuation analysis [4], the order parameter that is defined in natural time domain (NTD) [5-9], the connectivity degree by applying the visibility graph method [10].

In our paper, we present an experimental setting rather similar to that described in [3]. It consists of 2 Kg aluminum block (A) sliding over 1.5 m long surface with asperities (B) (Fig. 1). Both the surface (B) and the bottom side of the block (A) are covered by a sandpaper of roughness degree 36 (36 grains per cm$^2$). A low friction suspension system formed by two glass plates is employed; the superior glass plate has a thickness of 6 mm and is placed on a set of small steel spheres, with diameter of 4 mm that can roll on the inferior glass plate of 0.012 m thickness. The suspension system is placed over a metallic frame to maintain it in a leveled position. A charge cell (C) acts as a bumper against the metallic frame, allowing the recording of the force exerted by the inferior plate over the cell (C) when the elastic rope (D) is kept in tension. The rope is a fishing string with a diameter of $5\times10^{-4}$ m and a charge limit of 400 N. The fishing string connects the aluminum block with the motor (E) through a pulley. To pull the block we use a DC motor with a speed control (F). During the experimental runs the string is pulled with a constant speed of 0.133 m/min and the charge cell (C) converts the force into voltage that is measured by a digital voltmeter with sampling frequency of 10 Hz.

In this study we focus on the properties of the voltage time series that would convey information about the dynamical interaction between the two surfaces and their friction properties. In particular, we focus on the aging effects that would arise when the same block slides several times consecutively on the same surface, simulating the aging phenomenon of the interaction between tectonic plates [11].

We apply the Fisher-Shannon method to the time series of the voltage time series in order to investigate the properties of order and organization of the surface interaction, and to find possible patterns linked with the aging phenomenon.

## The method

The Fisher-Shannon (FS) method is very effective in investigating the complex behavior of nonstationary signals. It combines two quantities: Fisher Information Measure (FIM) and Shannon entropy ($H_X$). The FIM is a quantification of the level of organization or order in a time series, while $H_X$ is a measurement of the uncertainty or disorder. The FIM was proposed for the first time by Fisher [12] within the information theory, and Frieden [13] employed it for describing evolving physical processes. Martin et al. [14-15] used the FIM to study the dynamical behavior of electroencephalographic signals to identify relevant dynamical changes linked with pathologic states. Lovallo and Telesca [16], Telesca and Lovallo [17] and Telesca [18] employed The FIM to gain insight into the complex dynamics of several geophysical and environmental processes. Further studies were performed on the use of the FIM to detect precursory signs of critical phenomena [19-20]. The Shannon entropy is a measure of uncertainty of the prediction of the outcome of a probabilistic event [21], and it is null for deterministic events.

Let's indicate with $f(x)$ the density of $x$; thus, its FIM $I$ is given by

$$I = \int_{-\infty}^{+\infty} \left( \frac{\partial}{\partial x} f(x) \right)^2 \frac{dx}{f(x)}, \quad (1)$$

while its Shannon entropy is defined as [21]:

$$H_X = -\int_{-\infty}^{+\infty} f(x) \log f(x) dx . \quad (2)$$

An equivalent form of the Shannon entropy is the Shannon entropy power $N_X$, given by

$$N_X = \frac{1}{2\pi e} e^{2H_X} . \quad (3)$$

To calculate the FIM and the Shannon entropy, we firstly need to estimate the density $f(x)$ (pdf). It was recently demonstrated [22] that the kernel-based approach [23-24] to estimate the density $f(x)$ is more efficient than the discrete-based approach [25-26]. By using the kernel density estimator technique, we approximate f(x) as

$$\hat{f}_M(x) = \frac{1}{Mb}\sum_{i=1}^{M} K\left(\frac{x - x_i}{b}\right), \quad (4)$$

with $b$ the bandwidth, $M$ the length of the series and $K(u)$ the kernel function, which is a continuous non-negative and symmetric function satisfying the following two constrains

$$K(u) \geq 0 \text{ and } \int_{-\infty}^{+\infty} K(u) du = 1. \quad (5)$$

The pdf $f(x)$ was, then, estimated by using the algorithm proposed in [27] joined with that proposed by Raykar and Duraiswami [28], which employs a Gaussian kernel with zero mean and unit variance, with fast and more efficient computability characteristics:

$$\hat{f}_M(x) = \frac{1}{M\sqrt{2\pi b^2}} \sum_{i=1}^{M} e^{-\frac{(x-x_i)^2}{2b^2}} \ . \ (6)$$

## Results and discussion

In our experiment we covered by a sandpaper of roughness degree 36 both the upper surface of the track and the bottom surface of the moving block. Keeping the same sandpaper on both interacting surfaces, we performed 16 consecutive runs. Fig. 2 shows, as an example, three of the performed runs.

Fig. 3 shows the standard deviation of the amplitude of the voltage time series measured in each run; we can clearly see that the standard deviation decreases with the number of consecutive runs. In order to characterize each time series in terms of order and organization we applied the Fisher-Shannon method to each time series. Fig. 4 shows the variation of the FIM and the Shannon entropy with the number of consecutive runs. We observe a certain pattern in both the informational measures: in particular, the Shannon entropy has a decreasing trend with the number of consecutive runs, while the FIM is characterized by an increasing trend. This indicates that the interaction between the two sliding surfaces becomes more ordered and organized with the increase of the number of runs performed consecutively.

On the basis of the obtained results we can point out to the following findings:

1) Although in a different scientific context, like the micro-positioning, in [29] found that between the average step size of a slider and the applied voltage there is a direct relationship, and that the coefficient of friction determines the step size, as the coefficient of friction increases the average step size increases. Proportionality between output voltage and total displacement of in confined liquid in the stick-slip regime was also observed in [30]. In our experiment, the decreasing trend of the standard deviation of the voltage time series with the number of runs could suggest that also the standard deviation of the fluctuations of the displacements of the moving block decreases with the number of consecutive runs. The displacement of the block is controlled both by the roughness degree (which is the number of sandpaper's asperities per unit area) and by the height of the same asperities. The higher the asperities, the larger the friction between the two surfaces, the larger the displacement's variation [29]. Since in each consecutive run, the average height of the asperities (which are the grains of the sandpaper) of the surfaces of both the sliding block and the track becomes lower and lower, also the friction between the two interacting surfaces (that become even smoother after each run) is reduced, and this hinders the largest variations of the relative displacement.
2) Nosonovky [31] investigated the relationship between entropy and friction. Considering a surface profile as a random process, and calculating its Shannon entropy, it can be found that a surface

profile with a small value of Shannon entropy is more ordered than another surface profile whose value of Shannon entropy is higher. Therefore, a smooth surface is characterized by a zero value of the Shannon entropy. Thus, the Shannon entropy can be considered as a measure of the roughness of a surface [31]. In our experiment, we found that the Shannon entropy (the FIM) of the voltage time series decreases (increases) with the number of consecutive runs, suggesting a decrease (increase) of the disorder or randomness. At each successive run, the interacting surfaces become smoother and smoother, thus reducing the randomness of the corresponding profiles. This effect reflects on the measured voltage time series that, in turn, are a measurement of the variation of the displacements of the moving block.

3) Kanamori [32] introduced the concept of “seismic coupling” to quantify the interaction between tectonic plates at subduction zones. Ruff and Kanamori [11] found a significant correlation between the earthquake size and two other variables, which are the age of the subducting lithosphere and the convergence rate. Strongest events occur mainly in zones with young lithosphere and fast convergence rate. The size of an earthquake depends on the fault length and also on the asperity distribution on the fault plane, and in our experiment it depends on the amplitude of fluctuations of the displacements of the moving block, that could reflected in the standard deviation of the voltage time series. The Kanamori’s model of the “seismic coupling” seems well

reproduced in our experimental model, since we found that after consecutive runs, the amplitude of the fluctuations of the displacements decreases, implying a decrease of the “earthquake size” with the age of the interacting surfaces. The smoother the asperities, the smaller the “earthquake size”, the older the interacting surfaces.

## Conclusions

This study presents the analysis of voltage time series recorded during consecutive runs of a stick-slip experiment consisting in a rough block sliding on a rough surface (both characterized by identical roughness degree) pulled by a DC motor with a constant velocity. The aim of the experiment was to characterize the properties of the interaction between rough surfaces; such rough surfaces can mimic quite well a tectonic fault, and the number of consecutive runs can model the age of the tectonic fault, so that the last run models a status of the fault which is older than that modeled by the first run of the experiment. We employed the Fisher-Shannon method to investigate the order/organization properties of the voltage time series and our findings indicate a tendency toward a state of higher order and organization of the stick-slip system as the roughness of the interacting surfaces decreases.

## Acknowledgements

The study was performed in the framework of the bilateral project 2015-2017 by SRE (Mexico) and MAECI (Italy). ARR thanks Carlos Vargas, head of AFPI and Basic Sciences Dept. of UAM for the financial support.

## References

[1]Sholtz, C. H., 2002. The Mechanics of Earthquakes and Faulting, 2nd Ed., Cambridge University Press

[2]Burridge, R., Knopoff, L., 1967. Model and theoretical seismicity, Bull. Seismol. Soc. Am., 57, 341–371.

[3]Vargas, C. A., Basurto, E., Guzman-Vargas, L., Angulo-Brown, F., 2008. Sliding size distribution in a simple spring-block system with asperities, Physica A, 387, 3137–3144.

[4]Vargas, C. A., Basurto, E., Guzman-Vargas, L., Angulo-Brown, F., 2012. Correlations and scaling in a simple sliding spring-block model, Rev. Mex. Fís., 58, 116–120.

[5]Varotsos, P. A., Sarlis, N. V., Skordas, E. S., 2011. Natural Time Analysis: The New View of Time, Springer, Praxis Pub., UK

[6]Sarlis, N. V., Skordas, E. S., Lazaridou, M. S., Varotsos, P. A., 2008. Investigation of seismicity after the initiation of a Seismic electric signal activity until the main shock, Proc. Japan Acad. B, 84, 331–343.

[7]Sarlis, N. V., Skordas, E. S., Varotsos, P. A., 2010. Order parameter fluctuations of seismicity in natural time before and after mainshocks, Europhys. Lett., 91, 59001.

[8]Ramírez-Rojas, A., Flores-Márquez, E. L., 2013. Order parameter analysis of seismicity of the Mexican pacific coast, Physica A, 392, 2507–2512.

[9]Flores-Márquez, E. L., Vargas, C. A., Telesca, L., Ramírez-Rojas, A., 2014. Analysis of the distribution of the order parameter of synthetic seismicity generated by a simple spring–block system with asperities, Physica A, 393, 508–512.

[10]Telesca, L., Lovallo, M., Ramirez-Rojas, A., Flores-Marquez, L., 2014. Relationship between the Frequency Magnitude Distribution and the Visibility Graph in the Synthetic Seismicity Generated by a Simple Stick-Slip System with Asperities, PlosOne, 9, e106233

[11]Ruff, L., Kanamori, H., 1983. Seismic coupling and uncoupling at subduction zones, Tectonopjhysics, 99, 99-117

[12]Fisher, R. A., 1925. Theory of statistical estimation. Proceedings of the Cambridge Philosophical Society 22, 700-725.

[13]Frieden, B. R., 1990. Fisher information, disorder, and the equilibrium distributions of physics. Physical Review A 41, 4265-4276.

[14]Martin, M. T., Pennini, F., Plastino, A., 1999. Fisher's information and the analysis of complex signals. Physics Letters A 256, 173–180.

[15]Martin, M. T., Perez, J., Plastino, A., 2001. Fisher information and nonlinear dynamics. Physica A 291, 523–532

[16]Lovallo, M., Telesca, L., 2011. Complexity measures and information planes of X-ray astrophysical sources, Journal of Statistical Mechanics, P03029.

[17]Telesca, L., Lovallo, M., 2011. Analysis of the time dynamics in wind records by means of multifractal detrended fluctuation analysis and the Fisher–Shannon information plane, Journal of Statistical Mechanics, P07001.

[18]Telesca, L., Lovallo, M., Hsu, H.-L., Chen, C.-C., 2011. Analysis of dynamics in magnetotelluric data by using the Fisher-Shannon method. Physica A 390, 1350-1355.

[19]Telesca, L., Lovallo, M., Carniel, R., 2010. Time-dependent Fisher Information Measure of volcanic tremor before 5 April 2003 paroxysm at Stromboli volcano, Italy. Journal of Volcanology and Geothermal Research 195, 78-82.

[20]Telesca, L., Lovallo, M., Ramirez-Rojas, A., Angulo-Brown, F., 2009. A nonlinear strategy to reveal seismic precursory signatures in earthquake-related self-potential signals. Physica A 388, 2036-2040.

[21]Shannon, C.E., 1948. A Mathematical Theory of Communication. Bell System Technical Journal 27, 379-423, 623-656.

[22]Telesca, L., Lovallo, M., 2017. On the performance of Fisher Information Measure and Shannon entropy estimators, Physica A, 484 569–576

[23]Devroye, L. A Course on Density Estimation, Birkhauser, Boston, 1987.

[24]Janicki A, Weron A. Simulation and Chaotic Behavior of Stable Stochastic Processes. Marcel Dekker, New York, 1994.

[25]Chelani, A. B., 2014. Irregularity analysis of CO, NO2 and O3 concentrations at traffic, commercial and low activity sites in Delhi, Stoch. Environ. Res. Risk Assess., 28, 921-925

[26]Fath, B. D., Cabezas, H., Pawlowski, C. W., 2003. Regime changes in ecological systems: an information theory approach, J. Theoret. Biol., 222, 517-530

[27]Troudi, M., Alimi, A. M., Saoudi, S., 2008. Analytical Plug-In Method for kernel density estimator applied to genetic neutrality study, EURASIP Journal on Advances in Signal Processing, Vol. 2008, Article ID 739082, 8 pages

[28]Raykar VC, Duraiswami R. Fast optimal bandwidth selection for kernel density estimation. In Proceedings of the sixth SIAM International Conference on Data Mining, Bethesda, April 2006, 524-528

[29]Anantheshwara, K., Murali, N. S., Bobji, M. S., 2008. Effect of friction on the performance of inertial spider, Sadhana, 33, 221–226

[30] Demirel, A. L., Granick, S., 1996. What is fluctuating in a confined liquid in the stick-slip regime?, In Physics of sliding friction, Persson, B. N. J., and Tosatti, E. (Eds.), Springer, 93-102

[31]Nosonovsky, M., 2010. Entropy in Tribology: in the Search for Applications, Entropy, 12, 1345-1390

[32]Kanamori, H., 1973. Great earthquakes at island arcs and lithosphere. Tectonophysics, 12, 187-198

**Figure**

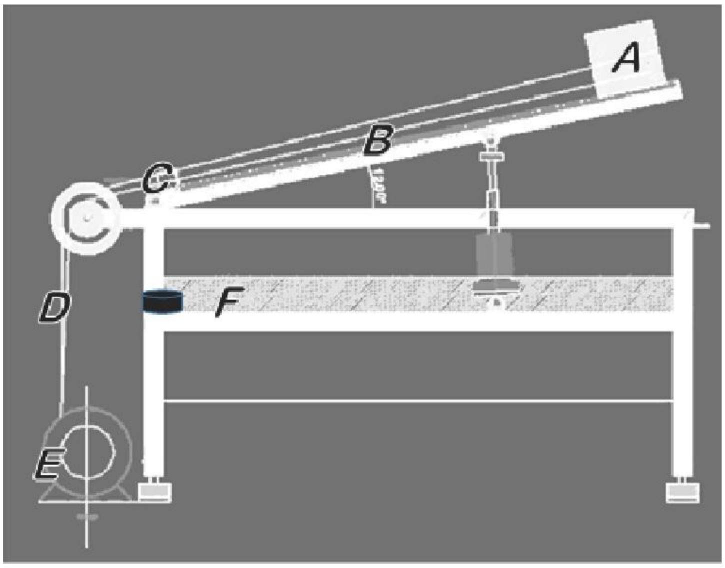


Fig. 1. Stick-slip experimental setup. The 2 Kg aluminum block (A) slides over 1.5 m long surface with asperities (B). The charge cell (C) allows the recording of the force exerted by the inferior plate when the elastic rope (D) is kept in tension. The string connects the aluminum block with the motor (E) through a pulley. The DC motor has a speed control (F).

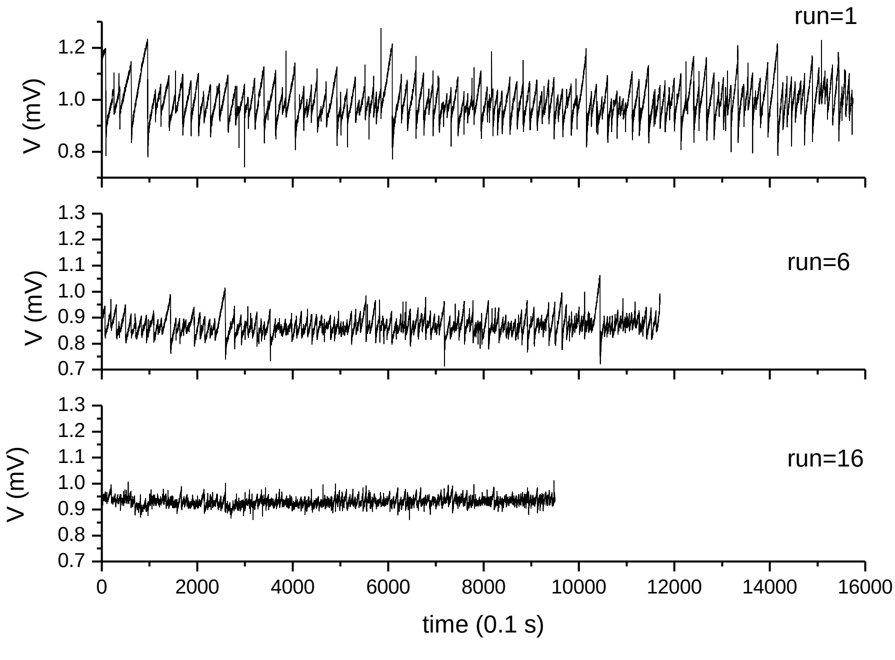


Fig. 2. Voltage time series recorded during the first run (a), the sixth run (b) and the last run (c).

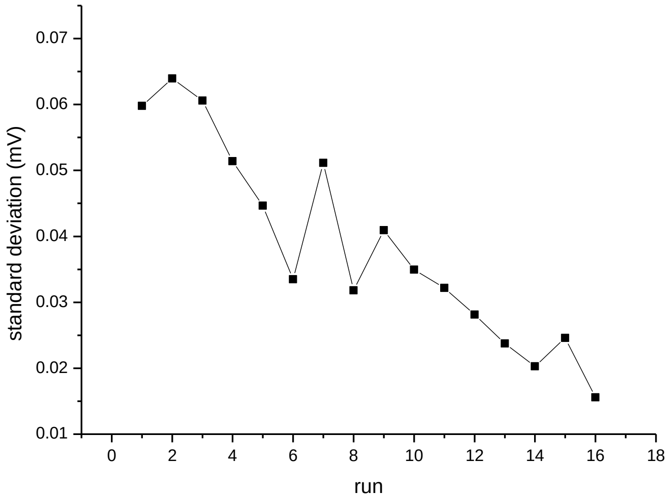
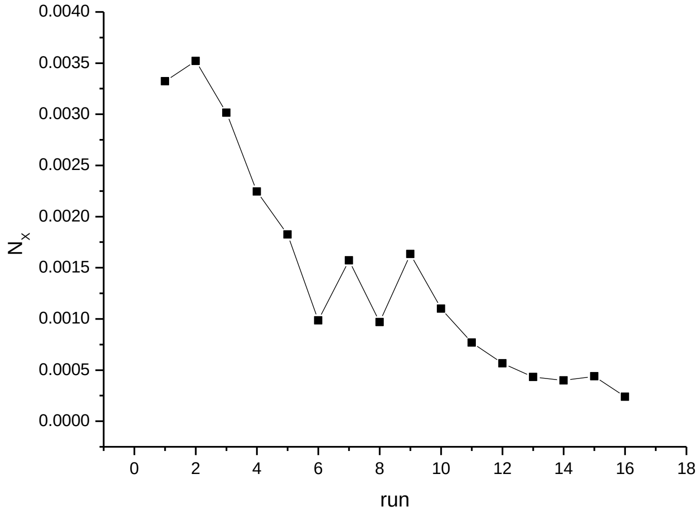
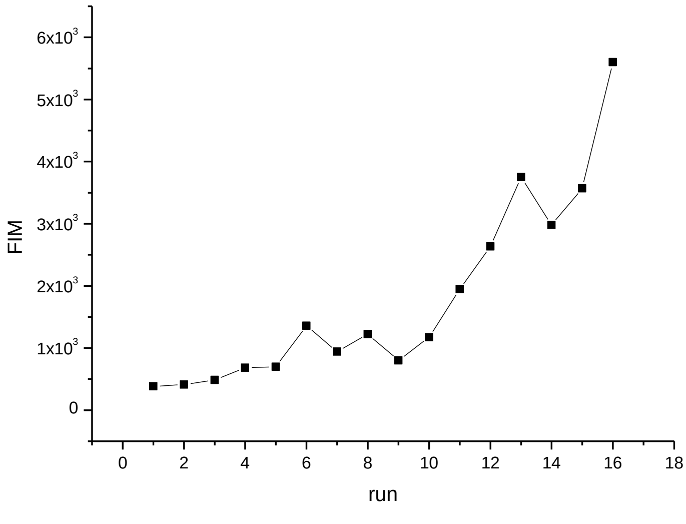


Fig. 3. Standard deviation of the voltage time series versus run number



a)



b)

Fig. 4. Shannon entropy (a) and FIM (b) versus run number