Stock and bond return relations and stock market uncertainty: Evidence from wavelet analysis

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PII: S1059-0560(17)30520-8
DOI: 10.1016/j.iref.2017.07.013
Reference: REVECO 1458


Received Date: 1059-0560 1059-0560
Revised Date: 1059-0560 1059-0560
Accepted Date: 1059-0560 1059-0560


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Stock and Bond Return Relations and Stock Market Uncertainty: Evidence from Wavelet Analysis

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Abstract
This paper adopts continuous wavelet analysis to investigate the time variation features of stock-bond return relations across different frequencies from 1988 to 2014. We also examine whether the time variation features of stock-bond return relations can be linked to two dimensions: fundamental economic factors and stock market uncertainty. The empirical results show that the short-term and long-term dependencies between stocks and bonds did vary over time. In addition, the relations between stock and bond returns have positive sign sensitivity to the short rate and the slope of term structure, while their sensitivity to stock market volatility is negative. Moreover, the impact of crises on the long-term stock-bond relation is significantly negative and the impact on short-term relation is significantly positive. Hence, the fundamental economic factors which drive the stock-bond relations do not vary across time frequencies; however, the impacts of crises do vary across the time frequencies. The findings have economic implications to help investors determine their portfolio allocations. Furthermore, policy makers monitor the financial markets and adjust the macroeconomic policies by observing changes in these state variables.

Keywords: Stock-bond return relations, Stock market uncertainty, Wavelet analysis

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1. Introduction

Stocks and bonds are the two most important asset classes traded on financial markets, which play a crucial role in asset allocation and portfolio management. It is now the stylized fact that the stock-bond relations have changed dramatically over the last two decades, shifting from sizably positive to predominantly negative in the late 1990s (Baele, Bekaert, and Inghelbrecht, 2010; Bansal, Connolly, and Stivers, 2014, Chiang, Li, and Yang, 2015; Lee, Marsh, Maxim, and Pfleiderer, 2006). Efforts also have been made to explore various economic forces driving the time-variation in stock-bond relations, including macroeconomic state variables (Baele, et al., 2010; Yang, Zhou, and Wang) and financial market uncertainty (Connolly, Stivers, and Sun, 2005; Chiang, Li, and Yang, 2015). Although the aforementioned studies have provided insight on the time-variations in stock-bond relations, few have ever been on investigating the dynamics of their relations across frequencies. This paper expands extant literature to explore their dynamics simultaneously across various frequencies over time and further examine whether the relations can be linked to fundamental economic factors and stock market uncertainty.

Understanding the dynamics of stock-bond relations across various frequencies over time and their determinants is important for deciding the asset allocation and making macroeconomic policy. For example, investors with various investing horizons concern the characteristics of asset returns in the corresponding investing frames respectively. Thus, exploring the time-frequency relations of stock and bond returns is crucial to asset allocation as the market participants pursue different investing horizons. Moreover, given stocks and government bonds account for a dominant share in all traded financial assets, the main economic forces driving their
relations also become of interest for regulatory and monetary authorities. The determinants of their relations provide useful information for policy maker on understanding the status of financial markets and the expectations of investors.

In order to capture the time-varying structures of stock-bond relations, most existing studies have investigated by the rolling-window correlation (Gulko, 2002; Ilmanen, 2003; Andersson et al., 2008) or worked with the family of GARCH models (DeGoeij and Marquering, 2004; Connolly et al., 2005; Chiang et al., 2015). In addition, copula method has been applied to characterize extreme dependence of stock and bond returns recently (Chui and Yang, 2012; Jammazi, Tiwari, Ferrer, and Moya, 2015; Sun, Rachev, Stoyanov, and Fabozzi, 2008; Wu and Liang, 2011). The benefit of copula method is that it can provide the whole dependence between asset returns beyond only linear association and result holds regardless of the distribution of returns. However, as mentioned above, investors with various investing horizons concern the corresponding dependences of investing frames respectively. Aforementioned approaches failed to capture the time-varying dependences across different frequencies.

To overcome the shortcoming, this study adopts wavelet analysis to capture the dynamics of stock-bond relations across frequencies over time simultaneously. Wavelet analysis decomposes data into different time scale components to capture different cyclical features of the data. Here, the local relation of between stock and bond returns is measured by wavelet coherence in time-frequency domain. By design, wavelet analysis provides more rich features in describing the stock-bond return relations simultaneously across different frequencies and over time, and helps to deepen the understanding of the determinants of these time-frequency relations. Since investors may have different investment horizons due to their different patterns of consumption. Different investors have different portfolio management needs.
Therefore, this study helps different type of investors in identifying the portfolio diversification opportunities as facing different investment horizons or holding periods of assets.

Naturally, it is of interest to analyze the economic forces driving the stock-bond return relations and quantify how much of the relation dynamics can be attributed to fundamentals. Generally, the stock-bond return correlation can be explained by their common exposure to macroeconomic factors. For example, the stock and bond prices can be represented as the expected future cash flows discounted the discount rate. Therefore, interest rate shocks are likely to move stock and bond prices in the same direction. Since stock and bond markets are both exposed to similar macroeconomic conditions, a positive stock-bond return relation is expected in the long term. However, inflation may generate different exposures between stock and bond returns because bond has fixed nominal cash flows. Ilmanen (2003) and Andersson et al. (2008) identify inflation as a key determinant of the stock-bond correlation. Li (2002) further documents that uncertainty about expected inflation plays an important role in determining the major trends in stock-bond return relation. Moreover, Yang et al. (2009) investigate the correlations over the last 150 years and also document significant differences across the business cycle. However, as discussed above, the stock-bond return relation has shifted from sizably positive to predominantly negative over the last two decades. There are a number of empirical papers from risk factors’ perspective that bear some explanations for the negative correlations, mostly relate the negative correlations to a fight-to-safety phenomenon. For instance, Gulko (2002) finds that the stock-bond correlation becomes significantly negative during the drawdown period of the stock market. Connolly et al. (2005) also argue that stock market uncertainty is a major determinant of the changing stock-bond correlation. Investors rebalance portfolios from stocks to bonds in times of increased stock market
uncertainty. Chiang et al. (2015) further specify that the financial market uncertainty attributes to the bond market uncertainty as well as the stock market uncertainty.

In sum, previous empirical studies have been attempt to predict differentiated impacts of economic fundamentals based on either macroeconomic determinants or risk determinants. In this study, we emphasize on examining whether the impacts of such determinants behave same at the different frequency. This study adopts continuous wavelet analysis to look at all time-frequency stock-bond relations simultaneously and show how fundamental factors affected these relations differently. We make two important contributions to the extant literature in the present study. First, our study contributes to the wide literature on the time variation in stock-bond relations. Many previous studies (e.g., Gulko, 2002; Ilmanen, 2003; DeGoeij and Marquering, 2004; Connolly et al., 2005; Andersson et al., 2008; Chiang et al., 2015) adopt rolling window or GARCH family model to capture time-varying correlations features. These approaches always contain a mix of short-run and long-run data smooths the trend and fail to extract the intrinsic direction of their correlations across different time frequencies. Instead, this paper is the first which applies continuous wavelet analysis to capture all time-frequency stock-bond relations simultaneously. Thanks to continuous wavelet analysis mapping the stock-bond relations in a two-dimensional figure that allows us to easily identify and interpret patterns or hidden information of time-frequency stock-bond return relations.

Next, extant literature exploring the economic determinants of the stock-bond relations is mainly based upon low frequency data (e.g., Li, 2002; Connolly et al. 2005; d’Addona and Kind, 2006 ; Andersson et al., 2008; Yang et al., 2009; Baele et al., 2010). However, the findings derived from above fails to reveals the stock-bond return variations in reacting to short-run shocks to financial markets. Therefore, this study emphasizes on the short-run state variables in detecting the impact of
macro-finance determinants on the stock-bond relations. Moreover, unlike existing literature by using different time frequencies for the same sample period (Chiang et al., 2014), this study directly uses the wavelet coherence results in time-frequency domain to investigate the differentiated impacts of fundamental drivers on stock-bond relations across different frequencies. This study can simultaneously reveal the stock-bond relations variations in reacting to short-run shocks across different frequencies. Our evidence finds that the impacts of financial market uncertainty on stock-bond relations at different frequency are mixed. This highlights the value of continuous wavelet analysis on investigating the determinants of stock-bond relations by observing the low and high frequency coherence simultaneously.

The remainder of the paper is organized as follows. Section 2 explains methodological issues and introduces our wavelet analysis procedure. Section 3 describes the data and presents the stock-bond return relation over different time periods and frequencies. Section 4 examines whether the relations can be linked to economic fundamental factors and stock market uncertainty. Section 5 concludes this paper and provides the interpretation for our main findings.

2. Methodology

This study puts emphasis on exploring the time-frequency relationship of stock-bond returns. Previous studies primarily use rolling-window method or work with the family of GARCH models to capture the time-varying relation structures in the time domain. However, these methods whose realizations recorded at a predetermined frequency are unable to explore the relationships across frequencies. On the other hand, Fourier analysis deals with the frequency dependencies, but fails to estimate the spectrum as a function of time. Unlike above methods, wavelet analysis
decomposes the data into time-frequency domain to capture the different cyclical features of the data. Using wavelet analysis, we are able to examine their dynamics simultaneously across various frequencies over time. Here, the time-frequency dependencies between stock and bond returns are measured by wavelet coherency analysis.

Wavelet analysis has been successfully used for many studies in social science during the last decade. Researchers are usually familiar with the use of the discrete wavelet transform (DWT) and the maximal overlap discrete wavelet transform (MODWT). DWT and MODWT have been used in the empirical studies of financial literature (e.g. Ramsey, 2002; Kim and In, 2003 and 2005; In, Kim, and Gençay, 2011; Sun, Rezania, Racchev, and Fabozzi, 2011; Meinl and Sun, 2012; Sun, and Meinl, 2012; Sun, Chen, and Yu, 2015; Chen, Sun, and Yu, 2015; Barunik, Kočenda and Vácha, 2015). More recently, the tools associated with the continuous wavelet transform (CWT) are becoming more widely used to analyze the interaction of financial and economic time series (e.g. Aguiar-Conraria, Azevedo, and Soares, 2008; Aguiar-Conraria and Soares, 2011a, 2011b and 2011c; Rua and Nunes, 2009; Rua, 2010; Tonn, Li and Marthy, 2010; Graham, Kiviaho, and Nikkinen, 2013; Lin, Chen, and Yang, 2015). One major benefit of continuous wavelet analysis has over discrete wavelet analysis, as Aguiar-Conraria et al., (2008) and Rua and Nunes (2009) pointed out, is that we need not define the number of wavelets (time-scales) in continuous wavelet analysis which generates itself according to the length of data. This allows us to easily identify and interpret patterns or hidden information. Therefore, continuous wavelet analysis is applied here to evaluate the time and frequency-varying features of stock-bond relations. The following gives briefly introduction of continuous wavelet method and wavelet coherency analysis.

Typically, the continuous wavelet transform decomposes a time series in terms
of some elementary functions, which are derived from a time-localized mother wavelet by translation and dilation. These wavelets result from a mother wavelet $\psi(t)$ that is a function of time parameter $t$ and can be defined as follows,

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t - \tau}{s}\right)$$

(1)

where mother wavelet $\psi(t)$ is a continuous function of the time position $\tau$ (translation parameter) and the scale $s$ (dilation parameter), which is related with the frequency. Here, $\tau$ acts to translate the function across the data and $s$ acts to vary the time scale of the mother wavelet function $\psi$. In addition, $1/\sqrt{s}$ is a normalization factor to ensure energy normalization that wavelet transform is comparable across time-frequency scale. Consequently, in the time domain we have defined a sequence of functions that are doubly indexed, once by location in the time domain, and once by the scale. To be a mother wavelet, $\psi(t)$ must satisfy the following three conditions: it must have zero mean, i.e. $\int \psi(t) dt = 0$; its square integrates to unity, i.e. $\int \psi^2(t) dt = 1$; and it must meet the so-called admissibility condition.

The most commonly used continuous mother wavelet is the Morlet wavelet. A Morlet wavelet is composed of a complex exponential multiplied by a Gaussian envelope and is defined as follows,

$$\psi(t) = \pi^{-1/4} e^{i\omega t} e^{-t^2/2}$$

(2)

here $\omega$ is the parameter which controls the number of oscillations within the Gaussian envelope. The Gaussian envelope gives less weight to pixels further from the center and makes this wavelet localized in time. In practice, $\omega$ is set between 5 and 6 for providing a good balance between time and frequency localization, as suggested by
Aguiar-Conraria and Soares (2011c). Moreover, a Morlet wavelet consists of a complex structure, given by the term $e^{i\omega t}$, within a Gaussian envelope that is captured by the term $e^{-t^2/2}$. Therefore, we can compute the phase of wavelet transform of each series from complex part, which gives us information about the position in the cycle of a time-series.

Given a time-series $x(t)$, its continuous wavelet transform with respect to the mother wavelet $\psi(t)$ can be represented as follows,

$$W_x(\tau, s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} x(t)\psi^*\left(\frac{t-\tau}{s}\right)dt$$

(3)

where $\tau$ is the time position and $s$ is the scale that is related with the frequency and the (*) denotes complex conjugation. For a discrete time series, $x(t), t=1,\ldots, T$, we have

$$W_x(\tau, s) = \frac{1}{\sqrt{s}} \sum_{t=1}^{T} x(t)\psi^*\left(\frac{t-\tau}{s}\right)$$

(4)

Here, a Morlet wavelet is chosen as the mother wavelet. Since the mother wavelet $\psi(t)$ is complex-valued, the corresponding wavelet transform $W_x(\tau, s)$ is also complex-valued. With a complex-valued wavelet, the amplitude and phase information of a time series can be separated into real parts and imaginary parts, respectively.

Given two time series $x(t)$ and $y(t)$, with wavelet transforms $W_x(\tau, s)$ and $W_y(\tau, s)$, where $\tau$ is the time position, $s$ is the scale. To examine the relationship between $x(t)$ and $y(t)$ in the time-scale plane, we consider the cross wavelet spectrum $W_{xy}(\tau, s)$, which is defined as follows,

$$W_{xy}(\tau, s) = W_x(\tau, s)W_y^*(\tau, s)$$

(5)

where $\tau$ is the time position and $s$ is the scale that is related with the frequency and
the (*) denotes complex conjugation. As discussed above, since the mother wavelet is complex, the cross wavelet spectrum is also valued of complexity and can be decomposed into real and imaginary parts. We can then separate the information about amplitude and phase of the two series into their real and imaginary parts, respectively. Based on the setting, we can obtain both time-dependent amplitude and phase across frequency. Here, the cross wavelet power can be defined as the absolute value of cross wavelet spectrum, $|W_{XY}(\tau, s)|$. The magnitude of the cross wavelet spectrum can be interpreted as the absolute value of the local covariance between two time-series at each time and frequency.

In the wavelet analysis, the localized correlation coefficient in time frequency space can be captured by the wavelet coherency. Following Rua (2010), we define the wavelet coherency as the real part of cross-wavelet spectrum, which is normalized by the smoothed wavelet power spectra as follows,

$$R_{X,Y}(\tau, s) = \frac{\mathcal{R}(W_{XY}(\tau, s))}{\sqrt{\text{WPS}_X(\tau, s)\text{WPS}_Y(\tau, s)}}$$  \hspace{1cm} (6)

where $\tau$ is the time position and $s$ is the scale. In addition, $\mathcal{R}(\cdot)$ represents the real part of cross wavelet spectrum, $W_{XY}(\tau, s)$. $\text{WPS}_X(\cdot)$ and $\text{WPS}_Y(\cdot)$ are the wavelet power spectrum of time series $x(t)$ and $y(t)$, respectively. For example, the wavelet power spectrum of time series $x(t)$ can be defined as follows,

$$\text{WPS}_X(\tau, s) = |W_X(\tau, s)|^2$$  \hspace{1cm} (7)

where $\tau$ is the time position and $s$ is the scale. Similar to the terminology used in Fourier transform, the wavelet power spectrum (WPS) can be defined as the square of the wavelet transform’s amplitude. In contrast to the Fourier power spectrum, the wavelet power spectrum $\text{WPS}_X(\tau, s)$ can be interpreted as the local variance of time series $x(t)$ across frequencies over time. Therefore, we can observe risk characteristics of stock and bond in time domain as well as in frequency domain.
Here, similar to the usual correlation coefficient between two time series, 
\( R_{X,Y}(\tau, s) \) has a value between -1 and 1. In this specification, 
\( R_{X,Y}(\tau, s) \) captures the wavelet correlation features of specific investment horizons. In particular, by inspecting the contour plot of the above measure, one can identify the regions in the time-frequency domain where the existence of correlation and captures both time and frequency varying features of the dependence. Therefore, the suggested wavelet-based measure provides richer insights on the relations between stock and bond returns.

3. Empirical Results

3.1 Data and Preliminary Statistics

We compute the daily stock and bond returns using the value-weighted NYSE/AMEX/Nasdaq return from CRSP and implied return calculated by the daily movements in the ten-year Treasury constant maturity series from Federal Reserve. Moreover, we use three-month Treasury Bill rate for the short-term interest rate. The slope of term structure, as in Lee et al. (2006) and Bansal, Connolly and Stivers (2014), is defined as the difference of ten-year and six-month Treasury constant maturity series yields, namely yields spread. Here, yields spread is used as a proxy for business conditions, while the short rate is for the discount rate. In addition, the TED spread is used as a proxy for credit market uncertainty and defined as the difference between the short-term three-month LIBOR Euro-dollar rate and the short-term three-month Treasury Bill rate. To capture stock market uncertainty, we use the implied volatility index, as constructed by the Chicago Board Options Exchange (CBOE). The implied volatility index is constructed from S&P 500 options. The data covers the sample period from January 5, 1998 through December 31, 2014. Since the CBOE's VIX is first reported in 1986, so we exclude the stock market crash of Oct 19,

[Please insert Table 1 here]

Table 1 reports the mean, volatility, realized correlation, and other summary statistics for stock and bond returns over the full sample and two sub-periods, 1988-1999, and 2000-2014. We divide our sample into two sub-samples (1988-1999, 2000-2014), since the stylized fact that the stock-bond relations have shifted from sizably positive to predominantly negative in the late 1990s. For the entire period from January 1988 to December 2014, as shown in Panel A of Table 1, the annualized daily mean return is 0.043% for stocks and 0.024% for bonds. The return risk as measured by the standard deviation of return is 1.095% for stock and 0.464% for bond. The stock market has higher return accompanied by higher volatility over the entire period. The realized correlation of stock-bond return is -0.181 over the entire period, implying that there are opportunities for diversification and for the hedging of risks.

The results of two sub-samples further reveal two main features. First, the stock market has higher return accompanied by higher volatility for each sub-period. Note that the volatilities of stocks are about 1.96, and 2.54 times higher than those of bonds over the former and latter sub-periods respectively. Second, the stock-bond return correlations change over time, from the positive correlation (0.280) in the former period to the negative correlation (-0.370) in the latter period. The preliminary statistics in Table 1 are consistent with the finding of “flight-to-safety” phenomenon by Connolly et al. (2005). The shift of correlation implies that a rise in stock market uncertainty induces investors to shift their funds from stocks to bonds, thereby depressing stock price, bidding up bond price and hence decoupling the stock-bond correlation.

3.2 Realized Correlation between Stock and Bond Returns
Figure 1 shows the realized correlations for stock and bond returns from January 1988 to December 2014. Realized correlations are estimated on a non-overlapping quarterly and annual basis by using daily returns. This figure illustrates a pattern similar to that in Table 1, where the stock-bond return correlations have changed dramatically over the last two decades. The correlations shift from sizably positive to predominantly negative since the late 1990s. This is consistent with the findings of Baele et al. (2010) and Bansal et al. (2014).

[Please insert Figure 1 here]

3.3 Wavelet Coherency between Stock and Bond Returns

Since the realized correlations estimated in session 3.2 contain both short-run and long-run information, a mix of short-run and long-run data smooths the trend and fails to extract the intrinsic direction of their correlations across different time frequencies. This paper further uses the wavelet coherency to extract the intrinsic short-run and long-run correlations between the stock and bond returns.

Figure 2 demonstrates the wavelet coherency of stock and bond returns, where a contour plot with three dimensions is depicted, the time dimension, the frequency dimension and the color code. In Figure 2, the cone of influence, which indicates the region affected by edge effects, is depicted by the bold black line. The color code for wavelet coherency ranges from the blue (negative coherency) to the red (positive coherency). The grey outer and black inner contours are respectively correspond to derived 10% and 5% significance level which is estimated from 1,000 Monte Carlo simulations based on an ARMA(1,1) null. Time and frequency are presented on the horizontal and the vertical axes, respectively. Frequency is converted into the annual scale.

[Please insert Figure 2 here]
As shown in Figure 2, a significant positive coherency (red code) in the higher frequency band (less than 0.5 year frequency) as well as lower frequency band (1.5 to 5 year frequency) are observed before 1997. These indicate that there existed positive short-run and long-run dependencies between stock and bond returns before 1997. However, from the late 1990s, there is a strong negative coherency (blue code) at the 0.5-2 year frequency band, positive coherency is observed only around 2000 and 2008 at the 0.25 year frequency. These findings confirm that the short-run and long-run dependencies between stocks and bonds did vary over time. Overall, Figure 2 confirms that the short-run and long-run dependencies between stocks and bonds did vary over time. In particular, the long-run wavelet coherency of stock-bond returns has changed dramatically over the last two decades. The wavelet coherency has shifted from sizably positive to predominantly negative in the late 1990s, which is consistent with the realized correlation estimated in session 3.2. Also, it is in line with Baele et al. (2010) and Bansal et al. (2014). The long-run negative relationship ensures bonds as the instruments of portfolio diversification as long as the investors pursue a long-run investment horizon.

However, a significant positive coherency is observed in the high frequency area, especially in the periods of Asian crisis (1997), the dot-com bubble (2000-2001) and the subprime crisis (2008-2009). A positive coherency indicates a positive relationship between the returns of stocks and bonds in the short run. The short-run relational shifts imply that bonds were no longer a short-run hedging asset during the periods of crises.

4. Determinants of Stock-Bond Relations

4.1 Macroeconomic Factors

The stock-bond return correlation is intrinsically explained by their exposures to
the macroeconomic factors. They are valued by a series of expected future cash flows discounted by the associated discount rates. Since the real interest rate determines the discount rates, the shocks of interest rates push the prices of stock and bond to move toward the same direction. On the other side, Harvey (1988; 1991) found the slope of term structure forecasts both consumption growth and economic growth. Investors believe that the shape of the yield curve reflects the market's future expectation of interest rates and monetary policies. Moreover, Viceira (2012) documented that both the short rate and slope of term structure were crucial in determining the covariance of stock and bond returns. Therefore, the short rate and the slope of term structure are chosen to investigate whether the time variation of stock-bond return correlations is determined by the macroeconomic factors.

Most studies exploring the economic determinants of the stock-bond correlation were using low frequency data, which renders limited information in capturing the variations of correlation in response to the short-run shocks to financial markets. Therefore, this study emphasizes the short-run state variables in characterizing the time-varying stock-bond return relations.

Since the wavelet coherency of stock-bond returns is, by definition, restricted to the range \([1, -1]\), thus in order to make the dependent variable unrestricted, Fisher transformation of the relation is applied to transform the range of wavelet coherency (Li, 2002; Andersson et al., 2008; Chiang et al.; 2015). Consequently, the first model uses a linear regression to regress the stock-bond return relation on the macroeconomic factors as follows,

\[
\ln\left(\frac{1 + R_{XY}(t,s)}{1 - R_{XY}(t,s)}\right) = \alpha_s + \beta_s Macro_t + \varepsilon_{t,s} \tag{8}
\]

where \(R_{XY}(t,s)\) is the real part of complex wavelet coherency of stock \((X)\) and bond \((Y)\) returns with scale \(s\) at time \(t\). In addition, \(\beta_s\) is a vector of estimated parameters at
scale \( s \), and \( \text{Macro}_t \) is a vector of macroeconomic factors at time \( t \). Here, two macroeconomic factors, the short rate and the slope of term structure reflecting the state of economy are chosen. Similar to Viceira (2012), we take the slope of term structure to proxy for business conditions, while the short rate is used as proxy for the discount rate. We measure the slope of term structure by taking the spread between a long-term yield and a short-term yield, namely yields spread.

[Please insert Table 3 here]

Table 3 reports the impact of macroeconomic determinants on daily, weekly, quarterly and annual wavelet coherence between stock and bond returns from January 5, 1988 to December 31, 2014. As shown in Table 3, the coefficients on short rate are significantly positive over the daily, weekly, quarterly and annual frequencies, indicating that the discount rate has a positive impact on the short-term as well as long-term stock-bond return relations. In addition, we also use yields spread as a proxy for business conditions and find that the long-term and short-term stock-bond relations are positively and significantly related to business conditions. These findings are in line with Aslanidis and Christiansen (2012, 2014), confirming that a positive stock-bond relation exists during the periods of favorable economic conditions.

4.2 Stock Market Uncertainty

The price and return dynamics of stocks and bonds are affected by the market uncertainty (David, 1997; Veronesi, 1999; Ozoguz, 2009). The portfolio theory suggests that as the stock market uncertainty occurs, investors fly to the safety heaven by shifting their funds from stock markets to other safe markets. If the bonds are chosen as the safe assets, the prices of stocks and bonds deviate. This study uses the implied volatility of equity index options as the proxy for stock market uncertainty.
and investigates how stock market uncertain impacts on the stock-bond relations. As most literature did (e.g. Connolly et al., 2005; d’Addona and Kind, 2006; Andersson et al., 2008; Chiang et al., 2015), we expect a negative relationship between the stock-bond return relations and the stock market uncertainty.

Furthermore, we choose TED spread as a proxy for credit market uncertainty and examine its impact on stock-bond return relations. The TED spread is defined as the difference between the short-term 3-month LIBOR Euro-dollar rate and the short-term three-month Treasury Bill rate. TED spread is widely used as a signal of global fluctuations in credit risk (Taylor and Williams, 2009; Chiang et al., 2015). As the TED spread widens, investors believe that credit risk is growing, this leads to the collapse of both stock and bond prices. Thus, we expect that there is a positive relationship between TED spread and stock-bond return relations. In addition, the empirical studies from Chordia, Sarkar, and Subrahmanyam (2005) indicate that increased uncertainty induced the dynamic cross-market hedging through frequent portfolio reallocations during the periods of financial crises. Following Chiang, Jeon, and Li (2007), the crisis dummies are also included to capture the impacts of global market turmoil on the stock-bond return relations during the periods of financial crises.

This study examines whether time-varying stock-bond relations can be linked to financial market uncertainty as follows,

\[
\ln \left( \frac{1 + R_{X,Y}(t,s)}{1 - R_{X,Y}(t,s)} \right) = \alpha_s + \beta_s Macro_t + \lambda_{sVIX} VIX_t + \lambda_{sTED} TED_t + \lambda_{sDUM} DUM_{t crises} + \varepsilon_{t,s}
\]

where \( R_{X,Y}(t,s) \) is the real part of complex wavelet coherency of stock \( (X) \) and bond
(Y) returns with scale s at time t. In addition, $\beta_s$, $\lambda_s^{VIX}$, $\lambda_s^{TED}$ and $\lambda_s^{DUM}$ are the vectors of estimated parameters at scale s. $\text{Macro}_t$, $VIX_t$, $TED_t$ and $DUM_t^{crisis}$ are the proxies for macroeconomic factors, implied volatility index, TED spread and the crisis dummies at time t, respectively. Here, we use the implied volatility index, TED spread and the crisis dummies as the proxies for financial market uncertainty. The crisis dummies, DUM97, DUM00 and DUM08, are used to control the impacts on the stock-bond relations from the Asian crisis (1997), the dot-com bubble (2000-2001) and the subprime crisis (2008-2009), respectively.

[Please insert Table 4 here]

Table 4 reports the impacts of stock market volatility, TED spread and crisis dummies on daily, weekly, quarterly and annual wavelet coherency between stocks and bonds. Evidence from Table 4 suggests that the impact of implied volatility index on the stock-bond relation is statistically significant with a negative sign over the daily, weekly, quarterly and annual frequencies. This evidence is consistent with the findings by Connolly et al. (2005) and Baele et al. (2010). The negative impact of volatility index on stock-bond relation could contribute to the “flight-to-safety” effect. Also, the “decoupling” effect between stock and bond returns is not a temporary phenomenon. Instead, for the past two decades, the shifting relationship between stock and bond returns implies that as the volatility of stock market increases, the prices of the stocks and bonds deviate and result in the negative relations between stock and bond returns.

Table 4 also presents the impacts of these crises and TED spread on long-term and short-term stock-bond relations. We find the impacts of crisis dummy of 2000 on stock-bond relation at different frequencies are all significantly negative. However,
there are mixed signs for crisis dummies of 2000 and 2008. We find that stock-bond relations are significantly negative related to the crisis dummies of 2000 and 2008 at daily, weekly and quarterly frequencies; whereas the crisis impacts of 2000 and 2008 on their relations are negative at annual frequency. In addition, as shown in Table 4, there are also mixed signs for the TED at different frequency. We find significantly positive relationship between TED and stock-bond relations at quarterly and annual frequencies; whereas the impacts of TED on stock-bond relations are negative at daily and weekly frequencies. The short-term negative impact of TED confirms that the effect of credit risk on liquidity in the short term is severe, and puts downward pressure on stock prices. The short-term negative impact reveals the hidden information beyond the low frequency data. It highlights the application of continuous wavelet analysis by observing the both low and high frequency data simultaneously.

For a robust check, we further investigate the model based on two sub-periods: January 5, 1988-December 31, 1999 and January 4, 2000-December 31, 2014. The sub-sample selections are based on the stylized fact that the stock-bond relations have shifted from sizably positive to predominantly negative in the late 1990s. Tables 4 reports the regression estimates of stock-bond relations regressed on macroeconomic factors and financial uncertainty determinants for each period. Consistent with previous findings, as shown in Table 4, the stock-bond relations are positively related to short rate and yield spread at all frequencies. These confirm that a positive stock-bond relation exists during the periods of favorable economic conditions. Moreover, the estimated coefficients of VIX variable consistently show the expected sign with negative value at all frequencies. That is, a rise of stock market uncertainty tends to move stock-bond correlations in opposite directions. Thus, the flight-to-safety behavior during times of high stock market uncertainty can play an important role in explaining the predominantly negative stock-bond relations after the late 1990s.
Overall, these results are consistent with main findings of full sample.

5. Conclusions

The relation between stock and bond returns has been received considerable attention in literature. The stylized fact is that the stock-bond relations have changed dramatically over the last two decades, shifting from sizably positive to predominantly negative in the late 1990s. This study adopts continuous wavelet analysis to investigate the time variation features of stock-bond return relations across different frequencies and examines whether the time-varying stock-bond relations can be linked to two dimensions: macroeconomic factors, and financial market uncertainty. Analyzing the daily U.S. stock and bond returns for the period 1988-2014, we derive several important empirical conclusions.

First, the results of wavelet coherence show that the short-run and long-run dependences between stocks and bonds vary across frequencies over time. The long-term stock-bond returns relation has shifted from sizably positive to predominantly negative in the late 1990s. However, a significant positive coherency is found in the high frequency area, especially in the periods of crisis. Second, we examine whether the time-varying stock-bond return relations is related to two fundamental economic factors: the short rate and slope of term structure. The empirical findings support that the discount rate and the slope of term structure both have positive impacts on the short-term as well as long-term stock-bond relations. These confirm that a positive stock-bond relation exists during the periods of favorable economic conditions. In addition, we investigate the impacts of financial market uncertainty on the time-varying stock and bond return relations. The evidence indicates that the implied volatility index has negative impact on the time-varying stock-bond relations at the daily, weekly, quarterly, and annual frequency. This is
consistent with the findings by Connolly et al. (2005) and Baele et al. (2010), implying a “flight-to-safety” effect. However, the impacts of crisis dummies and TED at different frequency are mixed. In sum, these finding highlight the value of continuous wavelet analysis on investigating the determinants of stock-bond relations by observing the both low and high frequency data simultaneously.

The findings of this paper provide rich economic implications to help investors determine their portfolio allocations. For example, investors with various investing horizons usually concern the characteristics of asset returns in the corresponding investing frames respectively. Thus, measuring the time-frequency relations of stock and bond returns is crucial to asset allocation as the market participants pursue different investing horizons. Asset allocation directly relies on the relations of underlying assets, where negative relations across assets provide opportunities for diversification. This study bears on these issues by promoting a better understanding of how diversification benefits vary with state of economy and financial market uncertainty. The evidence suggests somewhat limited diversification potential of bonds to stock investors especially in the times of crisis, which is presumably more likely to occur given shorter investment horizon. In addition, we find a significantly positive short-run relationship between the stock-bond relations and crisis dummies of 1997 and 2008. The short-term positive impact reveals the hidden information beyond the low frequency data. From the short-term perspective of portfolio management, bonds are not chosen as the hedging tools as facing the global crises. Investors rebalance their portfolios by selling the stocks as well as the bonds and park their funds in the safe heaven, such as the gold, which drives the prices of stocks and bonds downward and increases in the stock-bond correlation during the periods of financial crises. Thus, the portfolio management theory which favors the hedging features of bonds during the crisis periods is feasible in the long term of the annual base only.
References


Table 1 Summary statistics of stock and bond returns
This table provides preliminary statistics of the value-weighted NYSE/AMEX/Nasdaq return from CRSP and implied return calculated by the daily movements in the 10-year Treasury constant maturity series from Federal Reserve. We provide the results over the entire sample period of 1988-2014 in Panel A and two sub-sample periods of 1988-1999, and 2000-2014 in Panels B, and C, respectively. Statistical significance at the 1, 5, and 10% levels are denoted by ***, ** and *, respectively.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Median</th>
<th>Std. dev.</th>
<th>Min.</th>
<th>Max.</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Realized ( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Entire sample period from January 5, 1988 to December 31, 2014</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stock</td>
<td>0.043</td>
<td>0.087</td>
<td>1.095</td>
<td>-8.976</td>
<td>9.527</td>
<td>-0.358</td>
<td>10.572</td>
<td>-0.181***</td>
</tr>
<tr>
<td>Bond</td>
<td>0.024</td>
<td>0.018</td>
<td>0.464</td>
<td>-2.438</td>
<td>4.470</td>
<td>-0.008</td>
<td>5.887</td>
<td></td>
</tr>
<tr>
<td><strong>Panel B : Sub-sample 1 from January 5, 1988 to December 31, 1999</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Stock</td>
<td>0.069</td>
<td>0.090</td>
<td>0.801</td>
<td>-6.526</td>
<td>4.836</td>
<td>-0.485</td>
<td>8.306</td>
<td>0.280***</td>
</tr>
<tr>
<td>Bond</td>
<td>0.028</td>
<td>0.021</td>
<td>0.408</td>
<td>-2.438</td>
<td>1.901</td>
<td>-0.137</td>
<td>5.044</td>
<td></td>
</tr>
<tr>
<td><strong>Panel C : Sub-sample 1 from January 4, 2000 to December 31, 2014</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stock</td>
<td>0.023</td>
<td>0.080</td>
<td>1.282</td>
<td>-8.976</td>
<td>9.527</td>
<td>-0.277</td>
<td>9.099</td>
<td>-0.370***</td>
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<tr>
<td>Bond</td>
<td>0.021</td>
<td>0.013</td>
<td>0.505</td>
<td>-1.966</td>
<td>4.470</td>
<td>0.053</td>
<td>5.868</td>
<td></td>
</tr>
</tbody>
</table>
Table 2 Estimates of stock-bond relations regressed on macroeconomic fundamental determinants

This table reports the impact of macroeconomic determinants on daily, weekly, quarterly and annual wavelet coherence between stocks and bonds from January 5, 1988, to December 31, 2014. Fisher transformation of the relation is applied to transform the range of wavelet coherency. We specify that the time variation in the stock-bond relations can be linked to the measures of macroeconomic factors as follows.

\[
\ln \left( \frac{1 + R_{XY}(t,s)}{1 - R_{XY}(t,s)} \right) = \alpha_s + \beta_s \text{Macro}_t + \varepsilon_{t,s}
\]

where \( R_{XY}(t,s) \) is the real part of complex wavelet coherency of stock \((X)\) and bond \((Y)\) returns with scale \(s\) at time \(t\). \( \text{Macro}_t \) is a vector of macroeconomic factors at time \(t\). Two macroeconomic factors, short rate and the slope of term structure are considered, here. The slope of term structure is defined as the difference of ten-year and six-month Treasury constant maturity series yields. The numbers in parentheses are values of \(t\)-statistics. Statistical significance at the 1, 5, and 10% levels are denoted by \(***\), \(**\), and \(*\), respectively.

<table>
<thead>
<tr>
<th></th>
<th>Daily</th>
<th>Weekly</th>
<th>Quarterly</th>
<th>Annual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short rate</td>
<td>0.542 (^{***})</td>
<td>0.716 (^{***})</td>
<td>0.633 (^{***})</td>
<td>0.374 (^{***})</td>
</tr>
<tr>
<td></td>
<td>(35.02)</td>
<td>(50.05)</td>
<td>(42.22)</td>
<td>(22.94)</td>
</tr>
<tr>
<td>Yields spread</td>
<td>0.235 (^{***})</td>
<td>0.336 (^{***})</td>
<td>0.320 (^{***})</td>
<td>0.161 (^{***})</td>
</tr>
<tr>
<td></td>
<td>(15.20)</td>
<td>(23.47)</td>
<td>(21.37)</td>
<td>(9.87)</td>
</tr>
<tr>
<td>(N)</td>
<td>6620</td>
<td>6620</td>
<td>6620</td>
<td>6620</td>
</tr>
<tr>
<td>(\text{adj. } R^2)</td>
<td>0.172</td>
<td>0.293</td>
<td>0.223</td>
<td>0.082</td>
</tr>
</tbody>
</table>
Table 3 Estimates of stock-bond relations regressed on macroeconomic factors and financial uncertainty determinants

This table reports the impacts of macroeconomic factors and financial uncertainty determinants on daily, weekly, quarterly and annual wavelet coherence between stocks and bonds from January 5, 1988, to December 31, 2014. Fisher transformation of the relation is applied to transform the range of wavelet coherency. We specify the time variation in the stock-bond relations can be linked to macroeconomic factors and financial market uncertainty as follows.

\[
\ln \left( \frac{1 + R_{XY}(t,s)}{1 - R_{XY}(t,s)} \right) = \alpha_s + \beta_y Macro_t + \lambda_s Vol_t + \epsilon_{t,s}
\]

where \( R_{XY}(t,s) \) is the real part of complex wavelet coherency of stock (X) and bond (Y) returns with scale s at time t. Macro and Vol are vectors of two macroeconomic factors and three financial market uncertainty proxies, respectively. Two macroeconomic factors, short rate and slope of term structure are considered. Three financial market uncertainty proxies, the implied volatility index, TED spread, and crisis dummy variables reflecting the state of financial market uncertainty are chosen. The crisis dummy variables DUM\text{crisis97}, DUM\text{crisis00} and DUM\text{crisis08} are used to control for impacts of the Asian crisis (1997), the dot-com bubble (2000-2001) and the subprime crisis (2008-2009), respectively. The numbers in parentheses are values of t-statistics. The notations ***, **, and * denote statistical significance at the 1, 5, and 10% levels, respectively.

<table>
<thead>
<tr>
<th></th>
<th>Daily</th>
<th>Weekly</th>
<th>Quarterly</th>
<th>Annual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short Rate</td>
<td>0.547***</td>
<td>0.696***</td>
<td>0.661***</td>
<td>0.280***</td>
</tr>
<tr>
<td></td>
<td>(31.19)</td>
<td>(43.99)</td>
<td>(39.87)</td>
<td>(18.08)</td>
</tr>
<tr>
<td>Yields Spread</td>
<td>0.212***</td>
<td>0.329***</td>
<td>0.364***</td>
<td>0.130***</td>
</tr>
<tr>
<td></td>
<td>(13.42)</td>
<td>(23.04)</td>
<td>(24.32)</td>
<td>(9.27)</td>
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<tr>
<td>Implied Volatility Index</td>
<td>-0.142***</td>
<td>-0.231***</td>
<td>-0.054***</td>
<td>-0.526***</td>
</tr>
<tr>
<td></td>
<td>(-10.62)</td>
<td>(-19.15)</td>
<td>(-4.23)</td>
<td>(-44.47)</td>
</tr>
<tr>
<td>TED Spread</td>
<td>-0.049***</td>
<td>-0.006</td>
<td>0.060***</td>
<td>0.058***</td>
</tr>
<tr>
<td></td>
<td>(-2.81)</td>
<td>(-0.38)</td>
<td>(3.68)</td>
<td>(3.82)</td>
</tr>
<tr>
<td>DUM\text{crisis97}</td>
<td>0.046***</td>
<td>0.100***</td>
<td>0.059***</td>
<td>0.015</td>
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<tr>
<td></td>
<td>(4.10)</td>
<td>(10.03)</td>
<td>(5.64)</td>
<td>(1.49)</td>
</tr>
<tr>
<td>DUM\text{crisis00}</td>
<td>-0.054***</td>
<td>-0.025**</td>
<td>0.051***</td>
<td>-0.071***</td>
</tr>
<tr>
<td></td>
<td>(-4.63)</td>
<td>(-2.40)</td>
<td>(4.62)</td>
<td>(-6.88)</td>
</tr>
<tr>
<td>DUM\text{crisis08}</td>
<td>0.026*</td>
<td>0.025*</td>
<td>0.235***</td>
<td>-0.085***</td>
</tr>
<tr>
<td></td>
<td>(1.80)</td>
<td>(1.93)</td>
<td>(17.55)</td>
<td>(-6.80)</td>
</tr>
<tr>
<td>N</td>
<td>6620</td>
<td>6620</td>
<td>6620</td>
<td>6620</td>
</tr>
<tr>
<td>adj. $R^2$</td>
<td>0.203</td>
<td>0.351</td>
<td>0.287</td>
<td>0.378</td>
</tr>
</tbody>
</table>
Table 4 Estimates of stock-bond relations regressed on macroeconomic factors and financial uncertainty determinants based on two sub-periods: January 5, 1988-December 31, 1999 and January 4, 2000 - December 31, 2014.

This table reports the impacts of macroeconomic factors and financial uncertainty determinants on daily, weekly, quarterly and annual wavelet coherence between stocks and bonds based on two sub-periods: January 5, 1988-December 31, 1999 and January 4, 2000 - December 31, 2014. Fisher transformation of the relation is applied to transform the range of wavelet coherency. We specify the time variation in the stock-bond relations can be linked to macroeconomic factors and financial market uncertainty as follows.

$$\ln \left( \frac{1 + R_{XY}(t, s)}{1 - R_{XY}(t, s)} \right) = \alpha_s + \beta_s \text{Macro}_t + \lambda_s^{VIX_t} \text{VIX}_t + \lambda_s^{TED_t} \text{TED}_t + \lambda_s^{DUM_t} \text{DUM}_t^{\text{crisis}_t} + \epsilon_{t,s}$$

where $R_{XY}(t, s)$ is the real part of complex wavelet coherency of stock ($X$) and bond ($Y$) returns with scale $s$ at time $t$. $\text{Macro}_t$, $\text{VIX}_t$, $\text{TED}_t$ and $\text{DUM}_t^{\text{crisis}_t}$ are the proxies for macroeconomic factors, implied volatility index, TED spread and the crisis dummies at time $t$, respectively. Two macroeconomic factors, the short rate and the slope of term structure are considered. Here, we use the implied volatility index, TED spread and the crisis dummies as the proxies for financial market uncertainty. The crisis dummy variables $\text{DUM}_{\text{crisis}97}$, $\text{DUM}_{\text{crisis}00}$ and $\text{DUM}_{\text{crisis}08}$ are used to control for impacts of the Asian crisis (1997), the dot-com bubble (2000-2001) and the subprime crisis (2008-2009), respectively. The numbers in parentheses are values of $t$-statistics. The notations ***, **, and * denote statistical significance at the 1, 5, and 10% levels, respectively.

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td></td>
<td>Daily</td>
<td>Weekly</td>
</tr>
<tr>
<td>Short Rate</td>
<td>0.164***</td>
<td>0.403***</td>
</tr>
<tr>
<td></td>
<td>(5.45)</td>
<td>(14.24)</td>
</tr>
<tr>
<td>Yields Spread</td>
<td>0.145***</td>
<td>0.275***</td>
</tr>
<tr>
<td></td>
<td>(5.71)</td>
<td>(11.49)</td>
</tr>
<tr>
<td>Implied Volatility Index</td>
<td>-0.024</td>
<td>-0.218***</td>
</tr>
<tr>
<td></td>
<td>(-1.05)</td>
<td>(-10.18)</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>Variable</th>
<th>TED Spread</th>
<th>DUM\textsubscript{crisis97}</th>
<th>DUM\textsubscript{crisis00}</th>
<th>DUM\textsubscript{crisis08}</th>
<th>N</th>
<th>adj. $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.110***</td>
<td>-0.017</td>
<td>0.093***</td>
<td>0.119***</td>
<td>-0.044*</td>
<td>-0.008</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.026</td>
<td>0.161***</td>
<td>0.046***</td>
<td>-0.012</td>
<td></td>
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</tr>
</tbody>
</table>

| N         | 2945  | 2945  | 2945  | 2945  | 3675  | 3675  | 3675  | 3675  |
| adj. $R^2$ | 0.021 | 0.139 | 0.187 | 0.313 | 0.068 | 0.144 | 0.180 | 0.328 |
Figure 1 Realized correlation between stock and bond returns

This figure depicts the realized correlation of stock and bond returns from January 5, 1988 to December 31, 2014, with a total of 6,745 observations. The realized correlations are estimated on a non-overlapping quarterly and an annual basis, using daily returns.
Figure 2 Wavelet coherency between stock and bond returns
Wavelet coherency - the cone of influence, which indicates the region affected by edge effects, is depicted by the bold black line. The color code for wavelet coherency ranges from the blue (negative coherency) to the red (positive coherency). The grey outer and black inner contours are respectively correspond to derived 10% and 5% significance level which is estimated from 1,000 Monte Carlo simulations based on an ARMA(1,1) null. Time and frequency are presented on the horizontal and the vertical axes, respectively. Frequency is converted into the annual scale.