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Optimization of train energy saving based on golden ratio genetic algorithm

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Abstract——**In order to reduce the energy consumption of train operation, an optimization method based on genetic algorithm of golden section is proposed. Firstly, the Multi-particle train model is established. Secondly, the optimal operation strategy of subway trains is analyzed according to different ramps. Then, a golden section genetic algorithm (GR-GA) is proposed to solve the problem that genetic algorithm is easy to fall into local optimum. A golden section genetic algorithm (GR-GA) is proposed to search for the optimal transfer position of train and the best adaptive point of searching crossover and mutation operator with golden ratio is introduced, which improves the local optimization ability and convergence performance. Taking Yizhuang line as a simulation case, the results show that the proposed algorithm has a better optimization effect.**

Keywords—*Subway train; Energy-saving optimization; Genetic algorithm; Local optimum; Golden ratio*

I. Introduction

With the rapid development of urban rail transit, huge energy consumption has attracted widespread attention of scholars. The energy consumption accounts for nearly half of the total energy consumption of the subway system. Therefore, the reduction in energy consumption of the subway makes a lot of sense. Domestic and foreign scholars have done a lot of research on the optimization of urban rail transit trains and achieved many results. Generally, research methods are divided into two categories [1]: analytical algorithms and numerical algorithms.

The analytical algorithm is based on the factors of trains and lines and solves the problem of train energy saving in the form of mathematical expressions. Howlett [2] used maximum principle to search for the key operating points and then got the optimal solution of the problem. Khmelnitsky [3] used the maximum value principle to optimum the control of train with different road conditions; Liu [4] used the maximum value principle to optimize the train running curve combining the optimal control strategy with the train motion model; In order to minimize energy consumption, Liang[5] made the analysis of the optimal control under different road conditions, taking slopes and speed limits into consideration. In general, it is more likely to get the best results using analytical method, but it requires complex derivation, which appears to be the difficulty of solving the problem.

On the other hand, the numerical method is also based on the train and road condition, but it is a method to complete the solution within a specified time in an iterative way. Compared with the analytical method, the numerical method is easier to deal with complex objective functions, so it has attracted the attention of many scholars. Wong [6] applied genetic algorithm to determine the number of coasting point; Jin [7] applied the neural network combined with genetic algorithm to optimize the speed curve of the train. However, the use of neural network for optimization may cause the problem of long training time and slow calculation speed; In 2016, Mohammad [8] established a multi-mass train motion model of the train and compared the optimization performance of the ant colony algorithm and the genetic algorithm. And it turned out to be better when using genetic algorithm to solve the problem of energy-saving of train; Li [9] proposed the principle of handle-level variation of the dichotomy of slopes, and used genetic algorithm to solve the relationship between train handle level and train position; Wang [10] used an adaptive genetic algorithm based on a phase-adapted adaptive strategy to search for the operating point.. However, this method has a good effect at the later stage of evolution, but is not favorable to evolution in the early stage of evolution which is more likely to fall into a local optimum during evolution.

For these purposes, this paper aims at finding the optimal solution of energy-saving problem. The main organization of this paper is as follows: the motion model of the train is established and control strategies are analyzed in section Ⅱ. The golden ratio genetic algorithm is introduced to solve the problem of optimal controlling of the train in section Ⅲ. The method is verified under actual subway conditions in section Ⅳ and conclusions are shown in section Ⅴ.

II. Optimal Train Control

A. *Train movement model*

When studying the train motion model, the length of the train needs to be taken into account. This paper regards the train as a continuous rigid chain, which can greatly reduce the error caused by the calculation of the train length. The traction and acceleration of the train can be given as [1]:

$$\begin{cases} \mathbf{F} = \mathbf{F}_T - \mathbf{F}_R - \mathbf{F}_B \\ \mathbf{a} = \dfrac{\mathbf{F}}{M(1+\beta)} \end{cases} \tag{1}$$

In the equation, $\mathbf{F}$ represents the resultant force acting on the train, $\mathbf{F}_T$ represents the traction of the train, $\mathbf{F}_R$ represents the resistance of the train, and $\mathbf{F}_B$ represents the train braking force. $\mathbf{a}$ represents the acceleration of the train, M represents the total mass of the train, and β represents the coefficient of train's rotating mass. And train acceleration model is established as equation (2):

$$\mathbf{a}(t)=\mathbf{a}_T(t)-\mathbf{a}_R(t)-\mathbf{a}_B(t) \qquad (2)$$

In the equation, $\mathbf{a(t)}$ represents the acceleration of the vehicle at the time of t, $\mathbf{a_T(t)}$ represents the train traction acceleration at the time of t, $\mathbf{a_R(t)}$ represents the train resistance acceleration at the time of t, and $\mathbf{a_B(t)}$ represents the train braking acceleration at the time of t.

The resistance to the train includes basic resistance and additional resistance. The basic resistance acceleration is usually defined as formula (3):

$$\mathbf{a}_{\mathbf{w}_0}(t)=a+b\mathbf{v}(t)+c\mathbf{v}(t)^2 \qquad (3)$$

The additional resistance is mainly due to the resistance of the curve, ramp resistance, and the air resistance in the tunnel. Considering the multi-mass movement model of the train, the acceleration of the curve can be expressed as (4):

$$\mathbf{a}_{\mathbf{w}_r}=[\frac{600}{R_1}\times l_1+\frac{600}{R_2}\times l_2+\cdots+\frac{600}{R_n}\times l_n]\cdot\frac{\mathbf{g}}{10^3 L} \qquad (4)$$

where L means the length of the train, $R_1,R_2,\cdots,R_n$ represents the slope of the line, $l_1,l_2,\cdots,l_n$ means the length of the slope and $l_1+l_2+\cdots+l_n=L$.

The acceleration of air resistance in the tunnel is shown in equation (5):

$$\mathbf{a}_{\mathbf{w}_s}=\frac{1.3L_s\mathbf{g}}{10^7} \qquad (5)$$

Where L_s is the length of the tunnel.

B. Train optimization indicators

This paper regards energy consumption as an important optimization index, and regards running time and arrival accuracy as constraints.

1) Criterion of energy consumption

The energy consumption of a train is affected by many factors. During the operation of a train, traction work accounts for a large proportion of the energy. Therefore, the expression of energy consumption is shown in Equation (6):

$$\mathbf{J}_E=\sum_{k=0}^{n}\mathbf{F}_T(t)\cdot\Delta s \qquad (6)$$

$\mathbf{F}_T(t)$ is the traction force of the train at the time of t, Δs is the train running distance, n is the number of divided sections of the road.

2) Constraints

The train will be subject to many constraints, including safety constraints, time constraints, parking accuracy constraints, etc., shown in equation (7):

$$\begin{cases} x(0)=0, x(T)=x_T \\ v(0)=0, v(x_T)=0 \\ 0\le v\le v_{\max} \\ |S_{run}-S|<\Delta S \\ J_T=|T-T_{tarket}| \end{cases} \qquad (7)$$

where v represents the speed of the train, $v_{\max}$ is the maximum speed limit, $x(0)$ and $x(T)$ represents the starting and ending position of the train, T means the actual running time of the train, T_{tarket} means the tarket time for the train, S_{run} means the actual running distance of the train, and the parking error is usually within 50cm.

C. Train Operation Optimization Strategy

The actual road conditions are very complicated. Therefore, it is necessary to analyze control strategies according to different road conditions. The method of discretization is usually used to divide the road section which lays a solid fundamental on the study of train controlling strategy in each section of the line. As shown in Figure 1, the road is divided into n segments to ensure the speed limit and slope of each section are the same.

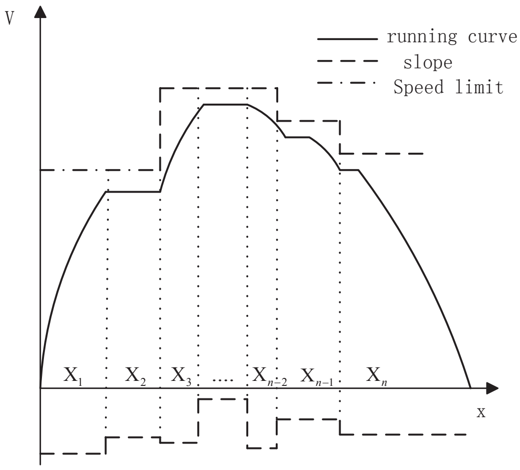

Fig.1. Line division

The ramps are generally divided into three categories: straight roads, gentle slopes, and steep slopes [13]. Each ramp corresponds to a different controlling strategy.

1) Straight roads: When the train runs on a straight road, Liu Wei et al.[14] verified that it is more energy-efficient to run at constant speed than tracking and coasting. Therefore, it is more reasonable to use the former to control the train when running on the straight road.

2) Gentle slope: The way of controlling the train on gentle slope is similar to the way of controlling on the straight roads due to the slope is so small that can be ignored.

3) Steep slope: When the train is operating in traction or brake mode, the train cannot maintain its speed, and the slope may be considered as a steep slope. Here we take the example of a long downhill slope for analysis.

a) Steep downhill: Before the train runs to point p_1 shown in Fig 2, it needs to switch to the coasting mode in advance. If the switching position is too close to p_2, the train may exceed the speed limit when going downhill, triggering unnecessary braking and increasing energy consumption. When the train arrives at q, the operating mode of the train is switched from coasting mode to constant speed mode.

b) Steep uphill: Before the train runs to point p_1 shown in Fig 3, it needs to switch to the motoring mode in advance. If the switching position is close to the change point p_2, the train may not be able to successfully go through the slope. When the train arrives at q, the train is switched from motoring mode to constant speed mode.

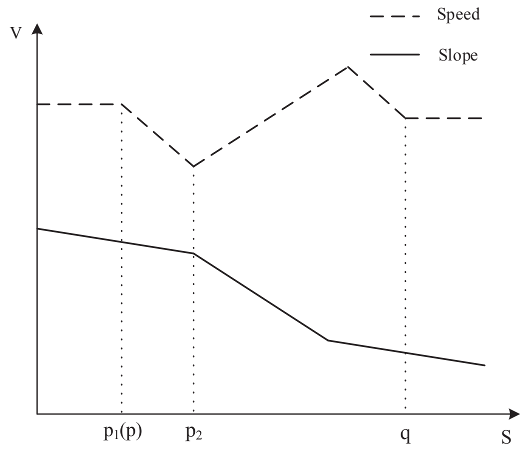

Fig.2. Steep downhill

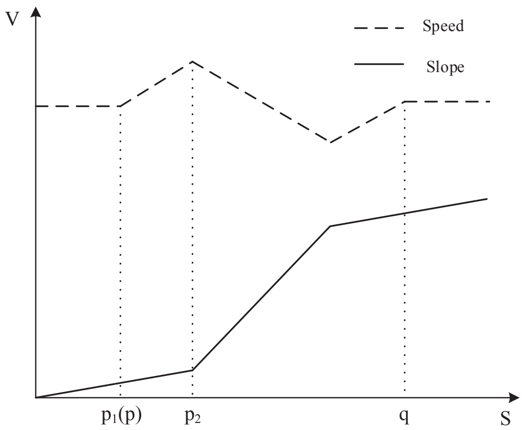

Fig.3. Steep uphill

III. Solving Train Handling Model Based on Golden Section Genetic Algorithm

A. Genetic Algorithm (GA)

Genetic Algorithm (abbreviated as GA) is a random search algorithm inspired by the evolutionary principles of the biological world. Probabilistic optimization methods are used to automatically obtain and guide the optimization of the search space and adaptively adjust the search direction without any rules.

There are three basic operators in the genetic algorithm: selection operator, crossover operator and mutation operator. The choice of these three operators determines the performance of the genetic algorithm. Among them, the crossover operator provides a coarse-grained, large-step search strategy for genetic algorithms. Although this large-step search is beneficial to global search, it has an adverse effect on the local search performance of GA. The basic function of the mutation operation is to make up for the loss of some important genes and to ensure the diversity of the population during evolution. However, the mainstream variation methods have poor performance in the middle and late stages of evolution. Therefore, this paper introduces the golden section to choose the crossover and mutation probability, so as to achieve the optimization effect combining local and global.

B. Genetic Algorithm Based on Golden Section

The Golden Ratio is a mathematical proportional relationship that embodies the internal harmony and balance of things and is particularly evident in the biosphere [15]. The golden section is a local optimization method. Based on the principle of "reducing the inferiority and saving excellence", symmetric and equal contraction methods are used to gradually narrow down the search scope. It is a local optimization method, based on the principle of "reducing bad and keeping good", and adopting symmetrical and equal contraction methods to gradually narrow the search scope. The solution space is reduced to 0.32 or 0.618 times each time, so that the optimal position between the two points can be found quickly. Combining the golden section idea with the genetic algorithm, using its advantage of strong local optimization to find better individuals between the two local optimal individuals, this point is selected in the initial population individual and the best adaptive point of the crossover and mutation operator. The determination is well reflected.

C. golden section genetic algorithm optimization steps

The optimization process based on the Golden Ratio Genetic Algorithm (GR-GA) is shown in Figure 4. The main steps are as follows:

Step 1: Initializing the population:

The size of the initial population directly affects the final result and execution efficiency of the genetic algorithm. Therefore, it needs to be selected in combination with the slope change point and speed limit requirements. In this paper, the initial population PA and PB are randomly generated based on the actual road data. PA is the main population based on GA global optimization, and PB is the subpopulation based on GR local optimization. The initialization parameters, including the primary and secondary population sizes are M, N, evolutionary generation, crossover probability, mutation probability, and fitness value. With binary coding, since the maximum speed is 80km/h, 7-bit coding is selected. Setting traction level from 1 to 4, brake level from 1 to 4 and coasting level 0. Therefore, 5 bits are selected, and when the highest bit is 1, it indicates operation mode is traction or coasting. When is 0, it indicates the operation mode is brake. And the remaining 4 position value is control level. For example: if the speed is 30km/h, and the representation of the 3-stage traction is shown in Table Ⅰ.

TABLE I. Coding Of Operating Mode

	Speed(km/h)	Operating Mode	Control Level
Original Value	30	Motoring	3
Binary Code	11110	1	011

Step 2: Fitness function calculation

The constraint condition is added to the objective function as a penalty function. The objective function is shown in formula (8):

$$f = J_E + \alpha(|T - T_{tarket}|) + \beta(|S - S_{tarket}|) \quad (8)$$

In the equation, J_E represents the energy consumption of the train, $|T - T_{tarket}|$ represents the arrival time deviation constraint, $|S - S_{tarket}|$ represents the arrival position deviation constraint, α and β represents the penalty factor, and f represents the final fitness evaluation value.

Step 3: Termination condition

If the fitness value meets the minimum requirement or the number of iterations reaches the maximum, the calculation is terminated; otherwise, it is transferred to the next step.

Step 4: Individual acceptance criteria

The Metropolis criterion was used to accept the new individuals in the population PB after the local optimization, and the worst individual of the population PA was replaced in turn with the best preserved individuals.

Step 5: Local Optimization

Select the best 2N individuals from each generation of GA-optimized individuals to replace the worst individuals in PB, and select N outstanding individuals from the PB as the initial individuals, and use the golden section operator to perform local optimization. Update PB.

Step 6: Genetic Operation

Genetic manipulation includes three basic operators: selection, crossover, and mutation.

1). Select operator

Calculate the fitness value of all individuals in the population and select individuals with high fitness from the current population to reproduce

one generation, because highly-adapted individuals will be more likely to inherit to the next generation, then copy the selected individuals from the previous generation as parents to the next generation with a certain probability.

2) Crossover and mutation operators

A golden section rate is introduced to search for the best adaptive point with the probability of 0.382 or 0.618 to improve the search efficiency and speed up the operation. Compared with the traditional adaptive mechanism, it avoids the problem of affecting the convergence performance of the evolutionary process by adjusting the value in the evolution phase, and is not easy to fall into the local optimum. Based on the fitness rate of crossover rate and mutation rate, as shown in formula (9)(10):

$$\begin{cases} P_c = \{\exp(-0.382)\dfrac{f' - f_{avg}}{f_{\max} - f_{avg}}\} & f' > f_{avg} \\ P_{c\max} & f' \le f_{avg} \end{cases} \quad (9)$$

$$\begin{cases} P_m = \{\exp(-0.618)\dfrac{f - f_{avg}}{f_{\max} - f_{avg}}\} & f \ge f_{avg} \\ P_{m\max} & f < f_{avg} \end{cases} \quad (10)$$

where f_{avg} denotes the average fitness value, $f_{\max}$ denotes the maximum fitness value, f' denotes the larger fitness value between the two individuals, and f denotes the fitness value of the individual to be changed, $P_{c\max}$ denotes the maximum crossover probability, $P_{m\max}$ denotes the maximum mutation probability.

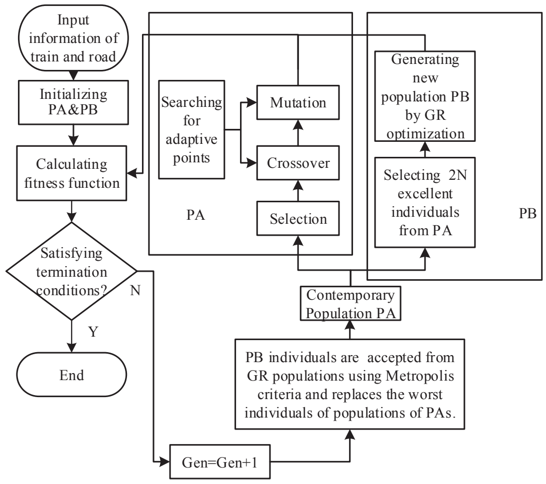

Fig.4. Optimization flow chart

IV. SIMULATION RESEARCH

In this section, we will test above method with Beijing's Yizhuang line. The distance between two stations is 2682m. The train's specified running time is 185s. The arrival time error is within 5s. The distance error is within 50cm. The train parameters are shown in Table Ⅱ. The speed limit and slope conditions are shown in Table Ⅲ and Figure 4.

TABLE II. TRAIN PARAMETER

Parameters name	Parameter characteristics
Train weight(N)	199
Max speed (km/h)	80
Maximum acceleration(m/s^-2)	1.45
Basic resistance(N)	3.4818+0.1449v+0.0852v2
Traction(N)	20-0.4779V
Brake(N)	0.372((17v+100)/(60V+100))

TABLE III. SPEED LIMIT PARAMETER

Distance(m)	Speed limits(km/h)
0-271.3	47.16
271.3-1275.6	61.96
1275.6-2339.9	79.99
2339.9-2628.2	57.17

Considering the influence of actual slope and speed limit, the golden ratio genetic algorithm is used to optimize the operating point of the train in the vicinity of the slope change point. As shown in Figure 5, the red line represents speed limit, the blue line represents running speed curve of train and green line represents slopes. It can be seen that the train fully utilized the coasting mode during the operation, reduced unnecessary braking, saved energy and satisfied the prescribed running time.

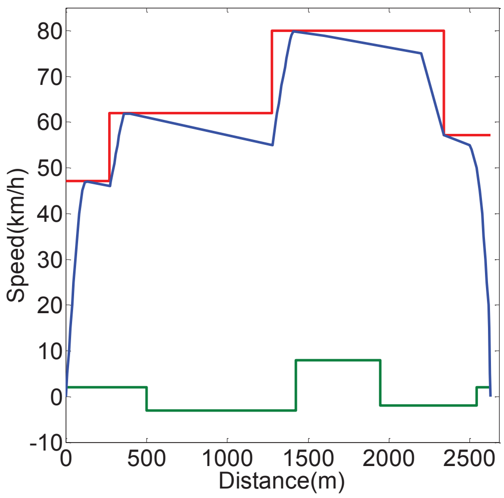

Fig.5. Train distance-speed curve optimized by GR-GA

Table Ⅳ shows the position of the operating condition change point. The train only carried out 8 conversions of operating conditions between the two stations, the optimization effect was better, and the energy consumption was reduced by about 14.2%.

TABLE IV. OPTIMAL CONTROL PROCESS OF TRAIN

NO	Switching position(m)	Tarket time(s)	Running time (s)	Energy consumption (kw·h)
1	120	190	183	16.9
2	273.3			
3	360.2			
4	1280.5			
5	1410.8			
6	2200.1			
7	2340.3			
8	2500			

This paper compares the golden section genetic algorithm (GR-GA) with the standard genetic algorithm (GA), and selects the same population size of 100, the maximum evolutionary generation of 80, GA crossover probability, mutation probability of 0.6, 0.01. As shown in Figure 6, GR-GA converges faster than GA and has a better fitness value. Table Ⅴ shows the optimized performance of the two algorithms.

TABLE V. PERFORMANCES OF GA AND GR-GA

Algorithm	Iterations	Energy(kw·h)	Fitness
GA	61	19.7	6112
GR-GA	40	16.9	5572

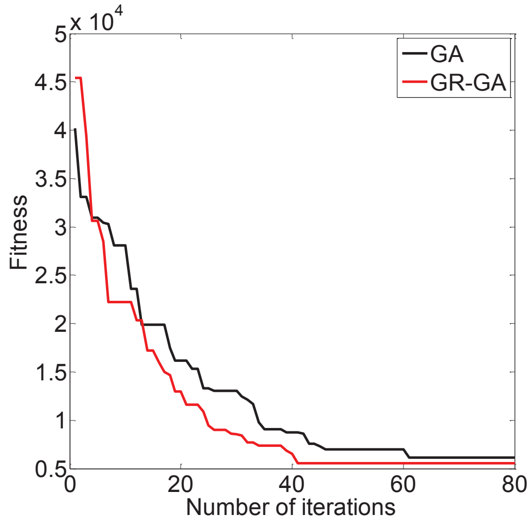

Fig.6. Comparison of convergence curve using GR-GA and GA

V. CONCLUSION

In this paper, energy consumption of the train is regarded as the objective. Considering speed limit and the slope, the optimal controlling strategy of the train on different slopes is analyzed. Based on this, a golden section genetic algorithm is proposed to find out the position of the optimal operating point of the train, so as to obtain the optimal train controlling sequence. The effectiveness of the algorithm is verified by Matlab simulation. The results show that the algorithm converges faster than the traditional genetic algorithm and the reduction of the energy consumption is significant. To study the optimization of multi-objective of train is our next research direction.

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