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Research Article

A Scientific Decision Framework for Supplier Selection under Interval Valued Intuitionistic Fuzzy Environment

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This paper proposes a new scientific decision framework (SDF) under interval valued intuitionistic fuzzy (IVIF) environment for supplier selection (SS). The framework consists of two phases, where, in the first phase, criteria weights are estimated in a sensible manner using newly proposed IVIF based statistical variance (SV) method and, in the second phase, the suitable supplier is selected using ELECTRE (ELimination and Choice Expressing REality) ranking method under IVIF environment. This method involves three categories of outranking, namely, strong, moderate, and weak. Previous studies on ELECTRE ranking reveal that scholars have only used two categories of outranking, namely, strong and weak, in the formulation of IVIF based ELECTRE, which eventually aggravates fuzziness and vagueness in decision making process due to the potential loss of information. Motivated by this challenge, third outranking category, called moderate, is proposed, which considerably reduces the loss of information by improving checks to the concordance and discordance matrices. Thus, in this paper, IVIF-ELECTRE (IVIFE) method is presented and popular TOPSIS method is integrated with IVIFE for obtaining a linear ranking. Finally, the practicality of the proposed framework is demonstrated using SS example and the strength of proposed SDF is realized by comparing the framework with other similar methods.

1. Introduction

Uncertainty and vagueness are an integral part of SS process [1, 2]. In SS, the decision maker (DM) or the expert committee rates each supplier based on a set of criteria. This rating can be either linguistic, numeric, or both depending on the choice of the DM. Such rating styles introduce an implicit vagueness in the process and are hence difficult to arrive at a concrete consensus. To better deal with such issues, Atanassov [3] came up with the idea of intuitionistic fuzzy set (IFS), in which a pair consisting of both membership and nonmembership value for every instance was used to better represent fuzziness in preferences. Later, Atanassov and Gargov [4] combined interval numbers and IFS to form IVIF sets. They claim that IVIF had better scope for representing fuzziness and vagueness. From then on, researchers widely explored IVIF by proposing several new theories and concepts. Let us now discuss some of them here. A brief discussion on classical operators of IVIF is also given in [5]. Bustince and Burillo [6] introduced the idea of correlation between IVIF set and developed decomposition theorems for the same. Xu and Chen [7] proposed ordered weighted and hybrid aggregation operators for IVIF sets and applied them for group decision making (GDM). Few researchers have also used IVIF-TOPSIS for solving GDM problems [8, 9]. Wang and Liu [10] proposed new Einstein aggregation operators based on sum, exponent, and product theories and demonstrated the same for selecting better propulsion systems. Following this, some researchers have also formulated new IVIF based entropy measures for solving pattern recognition and GDM problems [11–13]. Also, new distance measures under IVIF context have attracted authors that provide effective GDM process [14, 15]. Mukherjee and Das [16, 17] developed a theoretic framework of IVIF soft sets and effectively solved investor selection problem. The fusing of IVIF judgment matrix is an interesting research topic which has attracted researchers to contribute with some novel mechanisms [18, 19]. Motivated by the power of possibility theory, Wan and Dong [20] proposed a new method for comparison of IVIF values. Dadgostar and Afsari [21] further came up with a new idea for accurate edge detection in image steganography using IVIF concepts. Rashmanlou et al. [22] formulated the concept of IVIF graphs and gave a detailed description on IVIF graphs and their properties. Dymova and Sevastjanov [23] extended Dempster-Shafer theory over IVIF for solving GDM problems effectively. Another interesting area of concern for researchers in IVIF lies in consistency checks and preference completion [24, 25]. Researchers have also extended popular AHP [26] and MULTIMOORA [27] ranking schemes for IVIF domain and applied these methods for better GDM process.

Based on the discussion made above, we observe that IVIF based decision making methods are computationally attractive and effective for handling fuzziness and vagueness. In general, MCDM (multicriteria decision making) methods are classified into two groups, namely, utility based and outranking based methods. In utility based approach, alternatives are selected based on the utility function. On the other hand, in outranking based approach, alternatives are selected based on the preference function. Liao and Xu [28] claim that utility based methods are weak and pointed out few limitations of the methods. They are as follows: “(i) aggregation of different conflicting and commensurable criteria values for each alternative into a single entity is logically not correct and (ii) correlation between the criteria affects aggregation.” To circumvent these issues, researchers came up with the concept of outranking based methods. The two popular methods of this category are PROMETHEE and ELECTRE. Both follow outranking relations and are affected by rank reversals. Details on PROMETHEE and its family can be found in [29]. Since the focus of this paper is pertaining to ELECTRE method, we confine our discussion with ELECTRE and its family.

Roy [30] fine drafted the initial idea of Benayoun to formulate ELECTRE method which is one of the ranking schemes used for solving MCDM problems. The method follows outranking relationship concepts. The family of ELECTRE consists of popular schemes like ELECTRE I, ELECTRE IS, ELECTRE II, ELECTRE III, ELECTRE IV, and ELECTRE TRI. Among these, ELECTRE I and ELECTRE TRI IS are used for selection and others are used for ranking. ELECTRE I gave the basic idea for concordance and discordance, while ELECTRE IS proposed the concept of indifference and preference thresholds. ELECTRE II introduced strong and weak concordance and discordance estimates but was poor in handling uncertain and fuzzy data. To compensate the issue, ELECTRE III was proposed which combined the ideas of ELECTRE II and ELECTRE IS along with fuzzification of indices. ELECTRE IV, on the other hand, used the ideas of ELECTRE II and ELECTRE III along with elimination of weights for indices estimation. ELECTRE IV used number of criteria in each outranking category instead of membership values for ranking. ELECTRE TRI is used when the number of alternatives is high. Detailed discussions on ELECTRE family can be found in [31]. Many scholars inspired by the power of ELECTRE applied the method to different fuzzy variants. Some of the interesting fuzzy variant based ELECTRE methods are discussed here. Wu and Chen [32] proposed a new ELECTRE scheme under IFS domain which is an extension to the classical ELECTRE, where three categories of outranking relations were introduced and fuzziness was better handled. Though the handling of fuzziness was taken care of by the three outranking categories, the problem that still remained unsolved was the proper representation of fuzziness and vagueness. To throw light on this issue, Wu [33] proposed an IVIF based ELECTRE which uses score and accuracy metrics along with membership and hesitation index. Though the concept of IVIF was used to effectively tackle the problem of data representation, still, these parameters (metrics and indices) complicate the ranking scheme by further aggravating the vagueness in judgments by converting interval numbers to the single valued entity. This also causes loss of information which eventually affects the ranking process. In the view of proposing a simplified framework for decision making under uncertainty, Xu and Shen [34] extended ELECTRE I method under IVIF environment for supplier selection which uses simple score and accuracy measure for the construction of concordance and discordance matrices. But, even IVIF-ELECTRE from [32] suffers the loss of potential information as the interval values were converted into single valued entities for analysis. Further, Chen [35] derived a new mechanism for ranking using IVIF based ELECTRE method. He adopted inclusion comparison approach along with distance measure and score function. These parameters also increase the vagueness in judgments as they work with single valued terms rather than interval numbers. Also, the estimation of these parameters complicates the decision making process. Hashemi et al. [36] also proposed IVIF based ELECTRE III method for investor ranking. They also used score method for constructing concordance and discordance matrices which again causes loss of information due to single term processing rather than interval numbers. Further, they considered only single category for outranking relation which again aggravates fuzziness and vagueness in the decision making process. Chen [37] again extended the idea of concordance and discordance over many IVIF based likelihood preference functions and proposed a new ranking involving permutation and scoring mechanisms. Though this method handled loss of information to a certain extent, the complexity of implementation increased.

To address such issues and to give a simple and straightforward approach for decision making, in this paper, we make efforts to extend the Wu and Chen [28] approach of IFS based ELECTRE. Also, to the best of our knowledge, there is no direct extension to [28] and there is no procedure for estimating lower and upper bounds of concordance and discordance under three-way outranking categories (strong, moderate, and weak). We also affirm our claim based on the work done by Govindan and Jepsen [51] in deeply investigating ELECTRE and its extensions. Hence, we set our proposal to address such research lacuna. To serve the context, in this paper, we propose a novel IVIFE scheme which estimates both lower and upper concordance and discordance separately for all three outranking categories and also provides a systematic procedure for ranking which is a simple and direct extension of [28]. The estimation of

Table 1: Survey on supplier selection under IVIF environment from 2013 to 2016.

Ref #	Year	Aggregation	Fuzzy methods	Application(s)	Discussion
[2]	2016	Yes	IVIF based ranking	Supplier selection	Linguistic terms were converted to IVIF terms and aggregation was performed to fuse different opinions into a single matrix. TOPSIS method was used for criteria weight estimation. Finally, ranking is done using linear programming model and SS example is used to demonstrate the practicality.
[38]	2016	No	IVIF based ranking	Supplier selection	A new framework was proposed to select suitable business intelligent system supplier under IVIF domain. Criteria weights were estimated using ordered pairwise comparison method and weight of experts was determined using entropy method. A new algorithm was used for ranking suppliers and its practicality was tested using SS example.
[39]	2016	No	IVIF based ranking	Green supplier selection	IVIF terms were used for evaluation. Delphi method was used for choosing criteria for SS. Ranking was done using Choquet integral and fuzzy measures.
[40]	2016	Yes	IVIF based ranking	Supplier selection	IVIF terms were used for rating suppliers. AHP method was used for estimation of criteria weights and TOPSIS method was extended to IVIF domain for ranking suppliers. Preference values were aggregated using weighted arithmetic operator under interval domain.
[41]	2015	Yes	IVIF based ranking	Supplier selection	The DMs gave their preferences in IVIF fashion. Criteria and DMs weights were calculated using cross entropy measures and ranking of suppliers was done using optimization mechanism.
[42]	2015	No	IVIF based ranking	Green supplier selection	Heterogeneous set of terms were used for evaluation. IVIF based LINMAP approach was used for ranking of suppliers. Criteria weights were also determined using fuzzy programming model.
[34]	2014	Yes	IVIF based ranking	Supplier selection	IVIF information was used for rating suppliers. ELECTRE 1 was extended to IVIF domain for ranking suppliers. Score and accuracy were used for estimating concordance and discordance matrix. Criteria weights were calculated using entropy method.
[43]	2013	No	IVIF based ranking	Supplier selection	Heterogeneous values were used for evaluation of suppliers. LINMAP method was extended to IVIF domain and new distance measure was adopted to rank suppliers.
[44]	2013	Yes	Dynamic fuzzy set (DFS) based ranking	Supplier selection	The preferences were made in DFS fashion. The proposal discusses few weight models and ranking was done using a newly proposed DFS function. DFS was compared with IVIF over SS example and the power of DFS was realized.

Table 1: Continued.

Ref #	Year	Aggregation	Fuzzy methods	Application(s)	Discussion
[45]	2013	Yes	IVIF based ranking	Supplier selection	A new decision framework was proposed under IVIF domain for SS. The input to the framework was assessment in the form of IFVs and ranking was done using aggregated index.
[46]	2013	No	IVIF based ranking	Supplier selection	Heterogeneous information was used to rate suppliers. The LINMAP method was extended over IVIF and distance measure was used to rank suppliers. Criteria weights were calculated using fuzzy optimization model.
[47]	2013	No	IVIF based ranking	Supplier selection	The DMs rate the suppliers using IVIF values. The criteria weights were determined using newly proposed entropy measures. Ranking was also done using score and accuracy methods.
[48]	2013	No	IVIF based ranking	Supplier selection	The DMs adopt IVIF information to rate suppliers. The criteria were weighed using cotangent function based entropy measure. Ranking of suppliers was done using score and accuracy function and suitable supplier was selected for the job.
[49]	2013	Yes	IVIF based ranking	Supplier selection	IVIF information was used by the DM to gauge suppliers. A new aggregation method was proposed under IVIF environment for fusing preferences and final ranking was done using this method. Power of the proposed model was validated using sensitivity analysis and results infer better practicality and robustness of the model.
[50]	2013	No	IVIF based ranking	Supplier selection	IVIF values were used for rating. Criteria weights were estimated using AHP method and VIKOR was extended to IVIF for ranking suppliers.

lower and upper concordance and discordance makes the scheme more effective against vagueness by mitigating loss of information and acts as a better choice for MCDM problems involving uncertainty. To demonstrate the practicality of the proposed IVIF-ELECTRE method, SS problem is used and the proposed SDF is compared with other methods to realize its strengths.

The rest of the paper is organized as literature review in Section 2, followed by prerequisites in Section 3 where basic definitions of IFS, IVIF, and interval numbers are discussed. Section 4 makes a discussion on proposed SDF where the IVIF based SV method is proposed for criteria weight estimation and IVIFE method is proposed for ranking of suppliers. Section 5 demonstrates an illustrative example of SS for expressing the applicability of the proposed framework. The comparison of proposed IVIFE method with other IVIF based ranking methods is conducted in Section 6 and conclusion is given in Section 8.

2. Literature Review

In this section, literature review is conducted using two-stage approach, where both application and method are concentrated. The application considered here is SS and the method is IVIF based MCDM methods. So far, in the field of survey and analysis, this two-stage approach is considered to be effective and straightforward. Thus, our study also uses this approach for the review process. We apply filters to the year of publication from 2013 to 2017, owing to the extension of work done by [53, 54]. The two survey papers cover a total lifespan of 2000 to 2012 and hence, we set our filters in this manner. Based on the keywords and the year of publication, we obtained 19 potential papers. The discussion of these papers is tabulated in Table 1.

From Table 1, we infer the following points:

(1) Supplier selection is an attractive area for research in the field of SCM which has gained welcoming interest from the researchers under the MCDM perspective.

(2) The use of IVIF based MCDM methods for SS problem has just started and exploration in this field of study is becoming essential. Moreover, researchers have started realizing the strength of IVIF over IFS and hence, the use of IVIF based MCDM concepts to SS problem becomes an interesting and challenging task.

(3) Many scholars have extended the LINMAP approach under IVIF environment for solving SS problem and the extension of ELECTRE method to SS problem has just started.

(4) Estimation of criteria weights is another attractive area for research, where many scholars have focused their attention. These scholars have dominantly extended entropy measures and optimization models for calculating criteria weights, but Liu et al. [55] claimed that these methods yield unreasonable weight values and hence, to alleviate the issue, they proposed SV method.

Based on these inferences, the following research lacunas can be identified:

(1) From inference (1), we are motivated towards the supplier selection problem and hence make efforts to propose a new framework called SDF, for solving SS problem.

(2) Inference (2) motivated us to adopt IVIF environment for SS problem. Table 1 clarifies the power of IVIF in decision making and hence we set our proposal under IVIF domain.

(3) Inference (3) motivated us to propose a new extension to ELECTRE method under IVIF environment for solving SS problem. Table 1 clarified the fact that ELECTRE method is less explored for SS and hence we set out proposal towards this direction.

(4) The claim from inference (4) motivated us to propose a new extension to SV method under IVIF domain for producing sensible and rational weight values for the criteria.

3. Prerequisites

Definition 1 (see [3]). Consider a fixed set $M = (m_1, m_2, \ldots, m_p)$ such that $N \subset M$ is also fixed. Now IFS $\overline{N}$ in M is an object which is of the form given by

$$\overline{N} = \{(m_k, \mu_{\overline{N}}(m_k), \upsilon_{\overline{N}}(m_k)) \mid m_k \in M\}, \tag{1}$$

where $\mu_{\overline{N}}(m_k)$ and $\upsilon_{\overline{N}}(m_k)$ represent the membership and nonmembership values, respectively. Here $0 \le \mu \le 1$, $0 \le \upsilon \le 1$, and $\mu + \upsilon \le 1$.

Remark 2. For the ease of understanding Atanassov [5] called $\alpha = (\mu_\alpha, \upsilon_\alpha)$ which is an intuitionistic fuzzy value (IFV). Szmidt and Kacprzyk [56] introduced an additional parameter to IFV called the hesitation or indeterminacy and it is derived from

$$\pi_\alpha = 1 - (\mu_\alpha + \upsilon_\alpha), \tag{2}$$

where π_α is the indeterminacy value with $0 \le \pi \le 1$.

Definition 3 (see [4]). Consider a fixed set M such that $A \subset M$ is also fixed. Now IVIF set $\overline{A}$ in M is an object which is given by

$$\overline{A} = \{(m_k, \mu_{\overline{A}}(m_k), \upsilon_{\overline{A}}(m_k)) \mid m_k \in M\}, \tag{3}$$

where $\mu_{\overline{A}} = [\mu^l_{\overline{A}}, \mu^u_{\overline{A}}] \in [0,1]$ is the membership degree, $\upsilon_{\overline{A}} = [\upsilon^l_{\overline{A}}, \upsilon^u_{\overline{A}}] \in [0,1]$ is the nonmembership degree, and hesitation $\pi_{\overline{A}} = [\pi^l_{\overline{A}}, \pi^u_{\overline{A}}] \in [0,1]$ with $\pi^l_{\overline{A}} = 1 - (\mu^u_{\overline{A}} + \upsilon^u_{\overline{A}})$ and $\pi^u_{\overline{A}} = 1 - (\mu^l_{\overline{A}} + \upsilon^l_{\overline{A}})$.

Definition 4 (see [57]). The IFS obeys certain operational laws given in (4). Let one consider two IFVs, $P = (\mu_p, \upsilon_p)$ and $Q = (\mu_q, \upsilon_q)$, which follow

$$\begin{aligned} P \oplus Q &= (\mu_P + \mu_Q - \mu_P\mu_Q, \upsilon_P\upsilon_Q), \\ P \otimes Q &= (\mu_P\mu_Q, \upsilon_P + \upsilon_Q - \upsilon_P\upsilon_Q), \\ \bigoplus_{k=1}^{l} P &= \left(1 - \prod_{k=1}^{l}(1-\mu_k), \prod_{k=1}^{l}\upsilon_k\right), \\ \bigotimes_{k=1}^{l} P &= \left(\prod_{k=1}^{l}\mu_k, \prod_{k=1}^{l}(1-\upsilon_k)\right), \end{aligned} \tag{4}$$

where the resultant of all these operations also yields an IFS which obeys all properties mentioned in Definition 1.

Definition 5 (see [58]). The interval numbers also obey some operational laws which are given by (5). Let one consider $V = [v^l, v^u]$ and $W = [w^l, w^u]$ be two interval numbers in the range $[-\infty, +\infty]$; then one obtains the following:

$$\begin{aligned} V + W &= (v^l + w^l, v^u + w^u), \\ V - W &= (v^l - w^u, v^u - w^l), \\ V \times W &= (v^l \times w^l, v^u \times w^u), \\ \frac{V}{W} &= \left(\frac{v^l}{w^u}, \frac{v^u}{w^l}\right), \end{aligned} \tag{5}$$

where the operations yield an interval number which is also in the range $[-\infty, +\infty]$.

4. Proposed Scientific Decision Framework

4.1. Workflow of SDF. In this section, we demonstrate the working procedure of proposed SDF. The flow chart idea is adopted for depicting the working procedure. Figure 1 illustrates the workflow of SDF and since the flow chart is self-contained and straightforward, we confine our discussion for brevity.

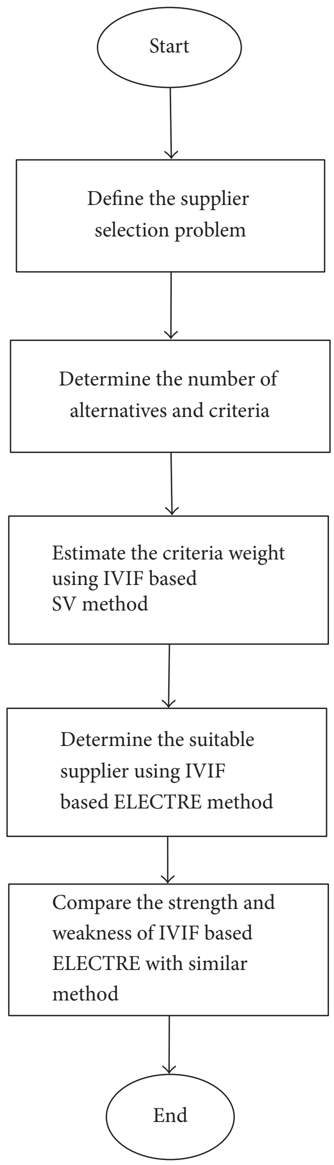

Figure 1: Workflow of proposed SDF.

4.2. Proposed IVIF Based SV Method. The estimation of weights or relative importance for each criterion is an attractive area for research. Many scholars have worked towards this field of study and proposed new methods for weight estimation. Broadly, the idea of criteria weight estimation can be classified into two groups, namely, manual weight estimation and methodical weight estimation. In manual weight estimation, DMs give weights to the criteria directly without following any procedure for estimation. On the other hand, methodical type of weight estimation is a systematic procedure used for estimation of criteria weights. Popular among them are entropy based [15, 59, 60], AHP based [61], and optimization model based weight estimation [62]. Liu et al. made an argument that though these methods estimate weight values, all these methods yield unrealistic and unreasonable weights for the criteria. Hence, to circumvent the issue, they came up with an idea of statistical variance (SV) method for weight estimation. They claim that (1) SV method yields much sensible and reasonable weight values, (2) unlike other methods that concentrate on the extremes, SV method concentrates on all points and then decides the distribution, and (3) SV method is simple and straightforward. These strengths of SV method motivated us to extend the SV method to IVIF environment and to propose a new method for criteria weight estimation under IVIF domain.

The steps involved in IVIF based SV method is discussed below.

Step 1. Construct a criteria weight evaluation matrix of order $(m \times n)$, where m is the total trials made by the committee for evaluation and n is the number of criteria taken for consideration.

Step 2. Calculate the mean of each instance and convert IVIF values to IFVs. Calculate the accuracy for each instance and convert IFV to a single value term.

Step 3. Calculate the mean of each criterion (values taken from Step 2) and estimate the variance value for each criterion using (6). This forms a vector of order $(1 \times n)$, where n is the number of criteria.

$$\text{var} = \sum_{k=1}^{m} \left(r_{ab}^{k} - \overline{r_{ab}} \right)^2, \tag{6}$$

where m is the total trials and $\overline{r_{ab}}$ is the mean value of a particular criterion.

Step 4. Normalize these variance values using (7) to obtain the relative importance (weights) for each criterion.

$$\omega_i = \frac{\text{var}_i}{\sum_{i=1}^{n} \text{var}_i}, \tag{7}$$

where ω_i is the weight vector for each of the criterion and $\sum \text{var}^* = 1$.

4.3. Proposed IVIF Based ELECTRE Method. The IVIFE method is a novel approach for solving MCDM problems, which integrates IFS and interval numbers into ELECTRE ranking scheme. We also combine classical TOPSIS approach with this method to obtain final ranking. The procedure for IVIFE is given below.

Step 1. Construct a judgment matrix as in (8). Let $\mu = [\mu^l, \mu^u]$, $\upsilon = [\upsilon^l, \upsilon^u]$, and $\pi = [\pi^l, \pi^u]$.

$$J_{ab} = \left(\mu_{ab}^{l}, \mu_{ab}^{u}, \upsilon_{ab}^{l}, \upsilon_{ab}^{u}, \pi_{ab}^{l}, \pi_{ab}^{u} \right), \tag{8}$$

where $(\mu_{ab}^{l}, \mu_{ab}^{u})$, $(\upsilon_{ab}^{l}, \upsilon_{ab}^{u})$, and $(\pi_{ab}^{l}, \pi_{ab}^{u})$ are the lower and upper limits of the membership values, nonmembership values, and indeterminacy, respectively. Let r and c be the number of alternatives and criteria; then the judgment matrix J is defined as a matrix of order $r \times c$ with $\mu_{ab}^{l} \le \mu_{ab}^{u}$, $\upsilon_{ab}^{l} \le \upsilon_{ab}^{u}$, $\pi_{ab}^{l} \le \pi_{ab}^{u}$, $\mu_{ab}^{l} + \upsilon_{ab}^{l} \le 1$, $\mu_{ab}^{u} + \upsilon_{ab}^{u} \le 1$, $\pi_{ab}^{l} = 1 - (\mu_{ab}^{u} + \upsilon_{ab}^{u})$, and $\pi_{ab}^{u} = 1 - (\mu_{ab}^{l} + \upsilon_{ab}^{l})$.

Table 2: Condition for concordance and discordance.

Constraints		Concordance	Discordance
Strong	Lower	$\left(c \mid \mu_{ec}^{l} \geq \mu_{fc}^{l}, \nu_{ec}^{l} < \nu_{fc}^{l}, \pi_{ec}^{l} < \pi_{fc}^{l}\right)$	$\left(c \mid \mu_{ec}^{l} < \mu_{fc}^{l}, \nu_{ec}^{l} \geq \nu_{fc}^{l}, \pi_{ec}^{l} \geq \pi_{fc}^{l}\right)$
	Upper	$\left(c \mid \mu_{ec}^{u} \geq \mu_{fc}^{u}, \nu_{ec}^{u} < \nu_{fc}^{u}, \pi_{ec}^{u} < \pi_{fc}^{u}\right)$	$\left(c \mid \mu_{ec}^{u} < \mu_{fc}^{u}, \nu_{ec}^{u} \geq \nu_{fc}^{u}, \pi_{ec}^{u} \geq \pi_{fc}^{u}\right)$
Moderate	Lower	$\left(c \mid \mu_{ec}^{l} \geq \mu_{fc}^{l}, \nu_{ec}^{l} < \nu_{fc}^{l}, \pi_{ec}^{l} \geq \pi_{fc}^{l}\right)$	$\left(c \mid \mu_{ec}^{l} < \mu_{fc}^{l}, \nu_{ec}^{l} \geq \nu_{fc}^{l}, \pi_{ec}^{l} < \pi_{fc}^{l}\right)$
	Upper	$\left(c \mid \mu_{ec}^{u} \geq \mu_{fc}^{u}, \nu_{ec}^{u} < \nu_{fc}^{u}, \pi_{ec}^{u} \geq \pi_{fc}^{u}\right)$	$\left(c \mid \mu_{ec}^{u} < \mu_{fc}^{u}, \nu_{ec}^{u} \geq \nu_{fc}^{u}, \pi_{ec}^{u} < \pi_{fc}^{u}\right)$
Weak	Lower	$\left(c \mid \mu_{ec}^{l} \geq \mu_{fc}^{l}, \nu_{ec}^{l} \geq \nu_{fc}^{l}\right)$	$\left(c \mid \mu_{ec}^{l} < \mu_{fc}^{l}, \nu_{ec}^{l} < \nu_{fc}^{l}\right)$
	Upper	$\left(c \mid \mu_{ec}^{u} \geq \mu_{fc}^{u}, \nu_{ec}^{u} \geq \nu_{fc}^{u}\right)$	$\left(c \mid \mu_{ec}^{u} < \mu_{fc}^{u}, \nu_{ec}^{u} < \nu_{fc}^{u}\right)$

For each of the c criteria in the judgment matrix, there is a weight value associated which represents the importance of each criterion. It is denoted by $\omega = (\omega_1, \omega_2, \ldots, \omega_c)$ with $0 \leq \omega \leq 1$ and $\sum_{i=1}^{c} \omega_i = 1$.

Step 2. Form the concordance and discordance matrix (CM and DM) using the conditions given in Table 2. These matrices are squared in nature with an order $(r \times r)$ defined for each of the r alternatives. The entries in the matrix are the column set that satisfies the conditions specified in Table 2.

The three categories of concordance and discordance, namely, strong (s), moderate (m), and weak (w) are assigned weights by the DM(s). These are represented by

$$\omega_{\text{con}} = \left(\omega_s^{\text{con}}, \omega_m^{\text{con}}, \omega_w^{\text{con}}\right), \tag{9}$$

$$\omega_{\text{dis}} = \left(\omega_s^{\text{dis}}, \omega_m^{\text{dis}}, \omega_w^{\text{dis}}\right). \tag{10}$$

Step 3. Calculate the concordance payoff matrix (CPM). This is a square matrix of order $(r \times r)$ defined for each of the r alternatives. Henceforth, the CPM value between any two alternatives is defined as the product of its corresponding concordance weights and criteria sum. Mathematically, it is given by

$$\text{CPM}_{(x,y)}^{1} = \omega_s^{\text{con}} \sum_{c \in \text{con}_s^l} \omega_c + \omega_m^{\text{con}} \sum_{c \in \text{con}_m^l} \omega_c + \omega_w^{\text{con}} \sum_{c \in \text{con}_w^l} \omega_c, \tag{11}$$

$$\text{CPM}_{(x,y)}^{2} = \omega_s^{\text{con}} \sum_{c \in \text{con}_s^u} \omega_c + \omega_m^{\text{con}} \sum_{c \in \text{con}_m^u} \omega_c + \omega_w^{\text{con}} \sum_{c \in \text{con}_w^u} \omega_c, \tag{12}$$

where x and y are any two alternatives; Step 1 and (11) and (12) give the weights of criteria, concordance, and discordance, respectively.

Equation (11) operates with the lower bounds of the criteria while (12) operates with the upper bounds of the criteria. Here, we take up only those criteria weights which have satisfied the desired outranking condition in each of the outranking categories.

Step 4. Calculate the discordance payoff matrix (DPM). This is also a square matrix of order $(r \times r)$ defined for each of the r alternatives. This is defined as the ratio of matching distance to total distance. Mathematically, it is expressed as in

$$\text{DPM}_{(x,y)}^{1} = \frac{\bigvee\left(\omega_{\text{dis}}^{*} \times d\left(\lambda_{xc^*}^{l}, \lambda_{yc^*}^{l}\right)\right)}{\bigvee\left(d\left(\lambda_{xc}^{l}, \lambda_{yc}^{l}\right)\right)}, \tag{13}$$

$$\text{DPM}_{(x,y)}^{2} = \frac{\bigvee\left(\omega_{\text{dis}}^{*} \times d\left(\lambda_{xc^*}^{u}, \lambda_{yc^*}^{u}\right)\right)}{\bigvee\left(d\left(\lambda_{xc}^{u}, \lambda_{yc}^{u}\right)\right)}, \tag{14}$$

where x and y are any two alternatives, ω_{dis}^{*} is the corresponding weight values from different categories of discordance, namely, strong, moderate, and weak, $\bigvee$ is the maximum operator, c^* is the criteria present in the corresponding discordance matrix, $d(\lambda_1, \lambda_2)$ is the distance measure, and c is the number of criteria.

Equations (13) and (14) also operate with lower and upper bounds, respectively. In (13) and (14), the denominator yields a single value which is considered to be the maximum value obtained from the distance estimate for each of the criterion. On the other hand, the numerator of (13) is also a single value which is considered to be the maximum value obtained from the distance estimate of only those criteria which satisfies the outranking condition in each of the categories. A clear understanding of (11)–(14) can be obtained in Section 4.

The distance measure is calculated using

$$d\left(\lambda_{xc}, \lambda_{yc}\right) = \sqrt{0.5\left\{\left(\mu_{xc} - \mu_{yc}\right)^2 + \left(\upsilon_{xc} - \upsilon_{yc}\right)^2 + \left(\pi_{xc} - \pi_{yc}\right)^2\right\}}. \tag{15}$$

Step 5. Calculate the dominance concordance (DC) and dominance discordance (DD) matrix using

$$\text{DC} = \left(\frac{\text{CPM}_{ij}^{1} + \text{CPM}_{ij}^{2}}{2}\right), \quad \text{DD} = \left(\frac{\text{DPM}_{ij}^{1} + \text{DPM}_{ij}^{2}}{2}\right), \tag{16}$$

where i and j are row and column index of the matrix.

Step 6. Identify the largest value from DC matrix and name it as DC^*. Subtract every element of DC by DC^*. Similarly subtract every element of DD by DD^* (largest element in

DD matrix). These matrices are named T and S, respectively. Equation (17) gives the T and S matrix.

$$\begin{aligned} T &= \left(\mathrm{DC}^{*} - \mathrm{DC}\right), \\ S &= \left(\mathrm{DD}^{*} - \mathrm{DD}\right). \end{aligned} \tag{17}$$

We note that T and S are also square matrices of order $(r \times r)$ defined for each of the r alternatives. Also T and S are matrices with single valued terms.

Step 7. Now, form the rank matrix ψ to identify the preference order using (18). From the preference order optimal compromise solution is chosen.

$$\psi_{ij} = \frac{S_{ij}}{\left(S_{ij} + T_{ij}\right)}. \tag{18}$$

From (18), we note that ψ is also a square matrix of order $(r \times r)$ defined for each of the r alternatives.

Step 8. Determine the optimal ranking order using TOPSIS method. The rank estimate φ is given by (19). The greater the value of φ is, the better that alternative is preferred.

$$\varphi_i = \left(\frac{1}{r-1}\left(\sum_{j=1}^{r}\psi_{ij}\right)\right) \quad \text{for } i = 1, 2, \ldots, r. \tag{19}$$

5. An Illustrative Example

This section demonstrates the working of SDF with the popular SS example in the context of auto parts manufacturing agents in an automobile factory. Here, we consider 8 suppliers for the initial analysis and based on the prescreening process, 3 suppliers were removed and 5 potential suppliers were chosen for further investigation. These potential suppliers were judged based on 6 criteria. The criteria used for evaluation are cost, product delivery time, service satisfaction, quality of end product, risk, and supplier profile. These are functional criteria inspired from [63] for the evaluation of suitable supplier. The rating is done in the form of IVIF based information. Now, based on Step 1, construct J matrix as below.

$$\begin{pmatrix} (0.34, 0.45, 0.42, 0.5) & (0.42, 0.51, 0.24, 0.33) & (0.32, 0.44, 0.43, 0.52) & (0.44, 0.52, 0.25, 0.36) & (0.7, 0.8, 0.04, 0.1) & (0.67, 0.72, 0.21, 0.26) \\ (0.41, 0.47, 0.34, 0.44) & (0.44, 0.5, 0.14, 0.25) & (0.44, 0.5, 0.13, 0.24) & (0.34, 0.42, 0.44, 0.5) & (0.65.0.7, 0.06, 0.2) & (0.7, 0.74, 0.22, 0.23) \\ (0.52, 0.6, 0.21, 0.3) & (0.35.0.41, 0.22, 0.32) & (0.37, 0.42, 0.4, 0.5) & (0.42, 0.5, 0.31, 0.4) & (0.57, 0.62, 0.11, 0.3) & (0.77, 0.82, 0.05, 0.12) \\ (0.54, 0.62, 0.12, 0.2) & (0.62, 0.67, 0.25, 0.3) & (0.27, 0.34.0.45, 0.52) & (0.37, 0.46, 0.27, 0.33) & (0.55, 0.63, 0.24, 0.3) & (0.72, 0.76, 0.08, 0.15) \\ (0.48, 0.53, 0.23, 0.36) & (0.57, 0.63, 0.25, 0.3) & (0.2, 0.4, 0.15, 0.33) & (0.31, 0.4, 0.46, 0.5) & (0.72, 0.81, 0.06, 0.14) & (0.78, 0.83, 0.12, 0.16) \end{pmatrix}. \tag{20}$$

The order of above matrix is (5×6). The instance is of the form $(\mu^l, \mu^u, \upsilon^l, \upsilon^u)$ with $\mu^l \le \mu^u$, $\upsilon^l \le \upsilon^u$, $\mu^l + \upsilon^l \le 1$, $\mu^u + \upsilon^u \le 1$. Equation (2) is used to calculate the indeterminacy limits.

Step 2 deals with the construction of CM and DM using Table 2. We form 3 matrices for CM and 3 for DM based on s, m, and w category, which are given below.

$$\mathrm{CM}_s^l = \begin{pmatrix} \text{—} & 5 & \text{—} & 4,3 & \text{—} \\ \text{—} & \text{—} & 2 & \text{—} & 4,3 \\ \text{—} & \text{—} & \text{—} & 6,3 & 1 \\ \text{—} & \text{—} & \text{—} & \text{—} & \text{—} \\ 6 & 6 & 5 & 5 & \text{—} \end{pmatrix},$$

$$\mathrm{CM}_m^l = \begin{pmatrix} \text{—} & 4 & 5,4 & 5 & 4 \\ 3,2,1 & \text{—} & 5,3 & 5,3 & \text{—} \\ 6,3,1 & 6,4,1 & \text{—} & 5 & 4 \\ 6,1 & 6,4,1 & 1 & \text{—} & 4,1 \\ 1 & 1 & \text{—} & 5 & \text{—} \end{pmatrix},$$

$$\mathrm{CM}_w^l = \begin{pmatrix} \text{—} & \text{—} & 2 & \text{—} & 3 \\ 6 & \text{—} & \text{—} & \text{—} & \text{—} \\ \text{—} & \text{—} & \text{—} & 4 & 3 \\ 2 & 2 & 2 & \text{—} & 3,2 \\ 5,2 & 5,2 & 6,2 & 6 & \text{—} \end{pmatrix},$$

$$\mathrm{DM}_s^l = \begin{pmatrix} \text{—} & \text{—} & \text{—} & \text{—} & 6 \\ 5 & \text{—} & \text{—} & \text{—} & 6,5 \\ \text{—} & 2 & \text{—} & \text{—} & 5 \\ 4,3 & \text{—} & 6,3 & \text{—} & 5 \\ \text{—} & 4,3 & 4,1 & 2 & \text{—} \end{pmatrix},$$

$$\mathrm{DM}_m^l = \begin{pmatrix} \text{—} & 3,2,1 & 6,3,1 & 6,1 & 1 \\ 4 & \text{—} & 6,4,1 & 6,4,1 & 1 \\ 5,4 & 5,3 & \text{—} & 1 & \text{—} \\ 5 & 5,3 & 5 & \text{—} & \text{—} \\ 4 & \text{—} & \text{—} & 4,1 & \text{—} \end{pmatrix},$$

$$\mathrm{DM}_w^l = \begin{pmatrix} \text{—} & 6 & \text{—} & 2 & 5,2 \\ \text{—} & \text{—} & \text{—} & 2 & 2 \\ 2 & \text{—} & \text{—} & 2 & 6,2 \\ \text{—} & \text{—} & 4 & \text{—} & 6 \\ 3 & \text{—} & 3 & 3 & \text{—} \end{pmatrix},$$

$$\mathrm{CM}_s^u = \begin{pmatrix} \text{—} & 5 & 5 & \text{—} & \text{—} \\ 6 & \text{—} & 5,2 & \text{—} & 3 \\ \text{—} & \text{—} & \text{—} & 6,3 & 1 \\ 2 & \text{—} & 2 & \text{—} & \text{—} \\ 6,2 & 5 & 5,2 & \text{—} & \text{—} \end{pmatrix},$$

$$\mathrm{CM}_m^u = \begin{pmatrix} \text{—} & 4 & 4 & 5 & 4 \\ 3,1 & \text{—} & 3 & 5,3 & \text{—} \\ 6,1 & 6,4,1 & \text{—} & \text{—} & 4 \\ 6,1 & 6,4,1 & 1 & \text{—} & 4,1 \\ 1 & 6,1 & \text{—} & 5,3 & \text{—} \end{pmatrix},$$

$$\mathrm{CM}_w^u = \begin{pmatrix} \text{—} & 2 & 3,2 & 4,3 & 3 \\ \text{—} & \text{—} & \text{—} & \text{—} & 4 \\ \text{—} & \text{—} & \text{—} & 4 & 3 \\ \text{—} & 2 & 5 & \text{—} & 2 \\ 5 & 2 & 6 & 6 & \text{—} \end{pmatrix},$$

$$\mathrm{DM}_s^u = \begin{pmatrix} \text{—} & 6 & \text{—} & 2 & 6,2 \\ 5 & \text{—} & \text{—} & \text{—} & 5 \\ 5 & 5,2 & \text{—} & 5,2 & 5,2 \\ 3 & \text{—} & 6,3 & \text{—} & \text{—} \\ \text{—} & 3 & 1 & 2 & \text{—} \end{pmatrix},$$

$$\mathrm{DM}_m^u = \begin{pmatrix} \text{—} & 3,1 & 6,1 & 6,1 & 1 \\ 4 & \text{—} & 6,4,1 & 6,4,1 & 6,1 \\ 4 & 3 & \text{—} & 1 & \text{—} \\ 5 & 5,3 & \text{—} & \text{—} & 5,3 \\ 4 & 4 & 4 & 4,1 & \text{—} \end{pmatrix},$$

$$\mathrm{DM}_w^u = \begin{pmatrix} \text{—} & \text{—} & \text{—} & \text{—} & 5 \\ 2 & \text{—} & \text{—} & 2 & 2 \\ 3,2 & \text{—} & \text{—} & \text{—} & 6 \\ 4 & \text{—} & 4 & \text{—} & 6 \\ 3 & \text{—} & 3 & \text{—} & \text{—} \end{pmatrix}. \tag{21}$$

Just as an example, we will consider single entry (1×2) of CM_s^l. This gives a value 5, which means that criterion 5 matches the constraint in Table 1 which gives $(c \mid \mu_{ec}^l \geq \mu_{fc}^l, \upsilon_{ec}^l < \upsilon_{fc}^l, \pi_{ec}^l < \pi_{fc}^l)$. Let e be 1 and f be 2 and c be the criteria that match the constraint. Look at the judgment matrix given above. Clearly from the first two rows, column 5 is the only column matching the constraint and so criterion 5 is selected. Similarly all other matrices are formed.

Table 2 depicts the IVIF based rating for different criteria which is in turn converted into IFVs (depicted in Table 3) by finding the mean of lower and upper bounds. Table 4 depicts the accuracy value $(\mu + \nu)$ of each of the IFVs. This table now yields a single value for rating each criterion. The variance value for each criterion is given by $\mathrm{var}_1 = 0.00334$, $\mathrm{var}_2 = 0.00655$, $\mathrm{var}_3 = 0.00640$, $\mathrm{var}_4 = 0.00065$, $\mathrm{var}_5 = 0.00175$, and $\mathrm{var}_6 = 0.00570$. From (7), we can obtain the criteria weights as $\omega_c = (0.1, 0.3, 0.2, 0.1, 0.2, 0.1)$.

Steps 3 and 4 are used to calculate CPM and DPM using (7)–(10). These are estimated for lower and upper limits separately with an order of (5×5). The concordance weights and discordance weights are given by $(\omega_s^{\mathrm{con}}, \omega_m^{\mathrm{con}}, \omega_w^{\mathrm{con}}) = (0.732, 0.613, 0.333)$ and $(\omega_s^{\mathrm{dis}}, \omega_m^{\mathrm{dis}}, \omega_w^{\mathrm{dis}}) = (0.700, 0.542, 0.432)$:

$$\mathrm{CPM}^1_{(x,y)} = \begin{pmatrix} \text{—} & 0.2077 & 0.2838 & 0.3422 & 0.1279 \\ 0.4011 & \text{—} & 0.4686 & 0.2452 & 0.2196 \\ 0.2452 & 0.1839 & \text{—} & 0.3755 & 0.2011 \\ 0.2225 & 0.2838 & 0.1612 & \text{—} & 0.2891 \\ 0.3010 & 0.3010 & 0.2796 & 0.1797 & \text{—} \end{pmatrix},$$

$$\mathrm{CPM}^2_{(x,y)} = \begin{pmatrix} \text{—} & 0.3076 & 0.3742 & 0.2225 & 0.1279 \\ 0.2571 & \text{—} & 0.4886 & 0.2452 & 0.1797 \\ 0.1226 & 0.1839 & \text{—} & 0.2529 & 0.2011 \\ 0.3422 & 0.2838 & 0.3475 & \text{—} & 0.2225 \\ 0.4207 & 0.3689 & 0.3993 & 0.2785 & \text{—} \end{pmatrix}, \tag{22}$$

$$\mathrm{DPM}^1_{(x,y)} = \begin{pmatrix} \text{—} & 0.5420 & 0.5420 & 0.5420 & 0.3458 \\ 0.2821 & \text{—} & 0.3041 & 0.3847 & 0.4320 \\ 0.2873 & 0.5420 & \text{—} & 0.4320 & 0.3697 \\ 0.3561 & 0.5420 & 0.2406 & \text{—} & 0.5200 \\ 0.4320 & 0.6809 & 0.4320 & 0.4320 & \text{—} \end{pmatrix},$$

$$\mathrm{DPM}^2_{(x,y)} = \begin{pmatrix} \text{—} & 0.5420 & 0.4950 & 0.5420 & 0.3184 \\ 0.2992 & \text{—} & 0.3084 & 0.4526 & 0.4320 \\ 0.7000 & 0.5420 & \text{—} & 0.7000 & 0.5252 \\ 0.4034 & 0.5420 & 0.1670 & \text{—} & 0.5420 \\ 0.4320 & 0.6505 & 0.4320 & 0.2867 & \text{—} \end{pmatrix}.$$

Table 3: IVIF values for relative importance of criteria.

Criteria weights	C_1	C_2	C_3	C_4	C_5	C_6
T_1	([0.1, 0.3] [0.2, 0.2])	([0.11, 0.33] [0.32, 0.36])	([0.2, 0.4] [0.2, 0.3])	([0.2, 0.3] [0.1, 0.3])	([0.24, 0.3] [0.1, 0.14])	([0.2, 0.4] [0.14, 0.3])
T_2	([0.2, 0.4] [0.1, 0.1])	([0.2, 0.28] [0.16, 0.32])	([0.2, 0.26] [0.18, 0.28])	([0.1, 0.4] [0.12, 0.34])	([0.1, 0.3] [0.2, 0.26])	([0.1, 0.3] [0.2, 0.24])
T_3	([0.2, 0.2] [0.2, 0.4])	([0.2, 0.26] [0.15, 0.29])	([0.2, 0.4] [0.1, 0.26])	([0.22, 0.42] [0.12, 0.2])	([0.2, 0.3] [0.1, 0.2])	([0.25, 0.35] [0.12, 0.2])

Table 4: IFVs for relative importance of criteria.

Criteria weights	C_1	C_2	C_3	C_4	C_5	C_6
T_1	(0.2, 0.2)	(0.22, 0.34)	(0.30, 0.25)	(0.25, 0.20)	(0.25, 0.12)	(0.3, 0.22)
T_2	(0.3, 0.1)	(0.24, 0.24)	(0.23, 0.23)	(0.25, 0.23)	(0.20, 0.23)	(0.2, 0.22)
T_3	(0.2, 0.3)	(0.23, 0.22)	(0.30, 0.18)	(0.32, 0.16)	(0.25, 0.15)	(0.30, 0.16)

Table 5: Single value for relative importance of criteria.

Criteria weights	C_1	C_2	C_3	C_4	C_5	C_6
T_1	0.4	0.56	0.55	0.45	0.37	0.52
T_2	0.4	0.48	0.46	0.48	0.43	0.42
T_3	0.5	0.45	0.48	0.48	0.40	0.46

As an example, let us consider the entry (1×2) for $\mathrm{CPM}^1_{(x,y)}$ and $\mathrm{DPM}^2_{(x,y)}$. Applying (7) we get

$$\begin{aligned}\mathrm{CPM}^1_{(1,2)} &= 0.732\,(0.2) + 0.613\,(0.1) + 0.333\,(0) \\ &= 0.2077.\end{aligned} \tag{23}$$

Applying (10) we get

$$\begin{aligned}&\mathrm{DPM}^2_{(1,2)} \\ &\quad = \frac{\max(0.029, 0.138, 0.014)}{\max(0.053, 0.085, 0.255, 0.125, 0.100, 0.026)} \\ &\quad = 0.5420.\end{aligned} \tag{24}$$

Steps 5 and 6 are used to estimate DC, DD, T, and S values. These matrices are of order (5×5). Apply (12)–(15) to obtain these matrices. For brevity we show T and S directly.

$$\begin{aligned}T &= \begin{pmatrix} — & 0.2191 & 0.1477 & 0.1944 & 0.3488 \\ 0.1476 & — & — & 0.2315 & 0.2771 \\ 0.2928 & 0.2928 & — & 0.1625 & 0.2756 \\ 0.1944 & 0.1929 & 0.2224 & — & 0.2209 \\ 0.1159 & 0.1418 & 0.1372 & 0.2476 & — \end{pmatrix}, \\ S &= \begin{pmatrix} — & 0.1237 & 0.1472 & 0.1237 & 0.3336 \\ 0.3751 & — & 0.3595 & 0.2471 & 0.2337 \\ 0.1721 & 0.1237 & — & 0.0997 & 0.2182 \\ 0.2860 & 0.1237 & 0.4619 & — & 0.1347 \\ 0.2337 & — & 0.2337 & 0.3064 & — \end{pmatrix}.\end{aligned} \tag{25}$$

We obtain the preference order using Steps 7 and 8. Equations (18) and (19) are incorporated from TOPSIS scheme which gives the final linear ranking from which an optimal alternative is elected as compromise solution.

$$\psi = \begin{pmatrix} — & 0.3610 & 0.4992 & 0.3890 & 0.4889 \\ 0.7176 & — & 1 & 0.5163 & 0.4576 \\ 0.3701 & 0.2970 & — & 0.3803 & 0.4419 \\ 0.5954 & 0.3908 & 0.6751 & — & 0.3788 \\ 0.6686 & — & 0.6300 & 0.5531 & — \end{pmatrix}, \tag{26}$$

$\varphi_1 = 0.4345$, $\varphi_2 = 0.6729$, $\varphi_3 = 0.3724$, $\varphi_4 = 0.5100$, $\varphi_5 = 0.4629$. Thus, the preference order is given by $\varphi_2 \succ \varphi_4 \succ \varphi_5 \succ \varphi_1 \succ \varphi_3$ and φ_2 is the compromise solution.

6. Comparison of Proposed IVIFE with Other IVIF Based MCDM Methods

6.1. Theoretical Analysis of IVIF Based Methods. In this section, we compare our proposed IVIFE method with other state-of-the-art methods in the same IVIF environment for maintaining homogeneity. We consider VIKOR, PROMETHEE, and ELECTRE ranking methods under IVIF environment as potential candidates for comparison with IVIFE. Table 5 shows the preference order obtained by using different ranking schemes.

From Table 2, we observe that φ_2 can be a potential alternative for the task (based on the ranking order obtained from proposed IVIFE and IVIFP [33] methods). On the other hand, IVIFV [52] and IVIFE [53] methods produce different ranking orders which claim φ_1 and φ_5 to be better choice for the task, respectively. Now, there arises an implicit confusion in the DMs' mind about the selection which can only be resolved using intuition. Thus, in the view of helping DMs, Table 3 shows the characteristics of different IVIF based ranking methods. Based on such investigation, DMs can easily choose a particular ranking method (or the ranking order obtained from that method) for the decision making process. To further help DMs, Spearman correlation [64] is adopted and the consistency of the proposed IVIFE method is realized.

From Table 6, we infer certain advantages of IVIFE method over other methods and they are as follows:

(1) Supplier φ_2 is a potential candidate for the task based on the majority wins concept (both proposed IVIFE and IVIFP methods select this alternative).

(2) Unlike the method described in [53], the proposed IVIFE method handles fuzziness and vagueness better by setting up additional constraint checks for concordance and discordance estimation. This mechanism improves the study on dominance of each criterion between any two suppliers taken at a time for consideration. The proposed method not only concentrates on strong and weak dominance of suppliers over

Table 6: Comparison of proposed IVIFE with other IVIF methods.

Method(s)	Alternatives					Preference order
	φ_1	φ_2	φ_3	φ_4	φ_5	
IVIFE (proposed)	4	1	5	2	3	$\varphi_2 \succ \varphi_4 \succ \varphi_5 \succ \varphi_1 \succ \varphi_3$
IVIFV [52]	1	4	2	5	3	$\varphi_1 \succ \varphi_3 \succ \varphi_5 \succ \varphi_2 \succ \varphi_4$
IVIFP [28]	4	1	5	2	3	$\varphi_2 \succ \varphi_1 \succ \varphi_5 \succ \varphi_4 \succ \varphi_3$
IVIFE [34]	5	2	4	3	1	$\varphi_5 \succ \varphi_2 \succ \varphi_4 \succ \varphi_3 \succ \varphi_1$

Note. In [52], the first three criteria are taken as cost and last three are taken as benefit. In [28], v function is used, IVIF values are changed to IFV by taking the mean, q value is taken as 0, and p value is taken as $(1/n)$, where n is the number of criteria. In [34], the linear order is achieved using TOPSIS method.

each criterion but also pays significant attention to those situations where suppliers behave as moderately dominant over the criteria.

(3) Unlike method [53], the proposed IVIFE method handles information loss effectively by preserving the interval concept throughout the decision making process. The conversion of interval numbers into single valued entity causes certain loss of information, which affects the process of decision making by aggravating imprecision and vagueness in the study.

(4) In case of proposed IVIFE method, the final ranking values that are used for constructing the preference order set are broader in nature. This helps DMs to clearly form rank value set and to easily make decisions under uncertainty by providing well distinguishable values for clear evaluation of suitable supplier for the process. Such rank value set can also help DMs to make backup suppliers ready either for other processes or for the same process. On the other hand, the method [34] yields a narrow preference value and hence, it becomes difficult for the DMs to arrive at some concrete evidence for selecting a suitable supplier for the job. The values shown below can help in realizing such claim. In order to understand the percentage of contribution, we normalize the rank value set of the method (when values exceed 1).

IVIFE is $\varphi_1 = 0.4345$, $\varphi_2 = 0.6729$, $\varphi_3 = 0.3724$, $\varphi_4 = 0.5100$, $\varphi_5 = 0.4629$.

IVIFE [53] is $\varphi_1 = 0.055$, $\varphi_2 = 0.2728$, $\varphi_3 = 0.082$, $\varphi_4 = 0.1854$, $\varphi_5 = 0.3949$.

Based on the values shown above, we can affirm that proposed IVIFE method yields a reasonable and rational preference order than [34]. The estimation of percentage for each rank value clarifies the fact that proposed IVIFE method yields much sensible and rational rank value set rather than its counterpart method [34]. This helps DMs to clearly form backups to serve other tasks under consideration.

(5) The proposed IVIFE method is moderately consistent (inference is gained from Spearman rank correlation; see Figure 2) with other methods and provides a much sensible preference order by preserving interval values throughout the evaluation process and this also closely resembles the DMs' behavior in decision making by imposing additional constraint checks which handles imprecision and vagueness better.

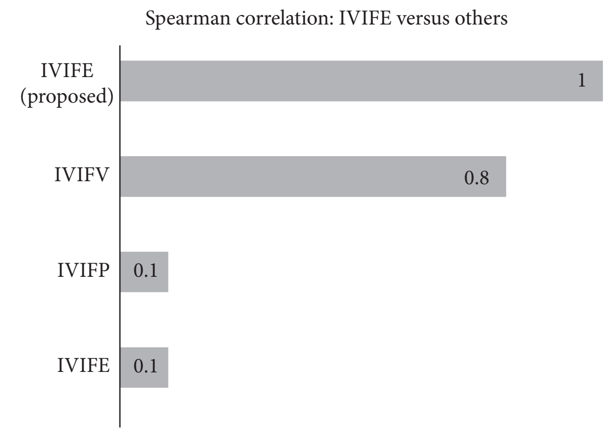

Figure 2: Spearman correlation for different ranking methods.

(6) As we observe, these advantages are realized from the theoretical lens, but one inference (see point (4)) of broad rank value set adds a logical meaning to the claim and this ensures that preserving of interval value and IFS property throughout the evaluation handles imprecision and vagueness better, rather than conversion to single valued entity for the ease of processing.

Though the proposed IVIFE method enjoys such attractive advantages, it does suffer from some disadvantages as well. Some disadvantages of IVIFE method are as follows:

(1) The extra constraint check causes an additional overhead in manipulation and thereby increases the complexity of estimation.

(2) Also, linear ranking by TOPSIS method is done with single valued entity which causes some loss of information. These disadvantages can be addressed in the future work.

6.2. Numerical Analysis of IVIF Based Methods. In the previous section, we investigated the strength of IVIFE method from the theoretic perspective (see Table 7 for details). Now, in this section, we make efforts to further analyze the strength of proposed IVIFE method from the numerical perspective.

Table 7: Characteristics of different IVIF based ranking methods.

Characteristics	IVIFE	IVIFV [52]	IVIFP [28]	IVIFE [34]
Type of input	Interval values	Interval values	Interval values are initially used and these are converted to IFS using mean operator	Interval values
Total preorder	Yes	Yes	Yes	Yes
Criteria weights	Given by DM	Given by DM	Given by DM	Given by DM
Concept adopted	Pareto dominance	Compromise solution	Preference function	Pareto dominance
Algorithm feature	Concordance and discordance measure	Ideal solution	Preference and indifference measure	Concordance and discordance measure
Metric adopted	Three categories of outranking relation	L_p metric	Partial and total outranking	Two categories of outranking relation
Ranking procedure	Concordance and discordance matrices for both lower and upper bounds are constructed under all three categories. Finally, TOPSIS is used for linear ranking	Group utility, individual regret, and merit function are determined for both lower and upper bounds and finally linear ranking is obtained using ranking measures	Score measure is used along with V function to form preference relations. Finally, linear ranking is obtained using partial and total ranking order	Score measure is used for the construction of concordance and discordance matrices. Single valued matrices are processed to obtain linear ranking using TOPSIS method
Ranking category	Outranking	Utility	Outranking	Outranking
Fuzziness handled	Better because the estimation is done in three levels and interval values are retained till the final stage of ranking	Normal	Normal	Weak because the estimation is done only in two levels and interval values are initially converted into single valued terms using score measure
Alternative chosen	φ_2	φ_1	φ_2	φ_5

We consider method [32] as a close counterpart for comparison. The investigation considers 4 potential parameters, namely, adequacy to alternatives changes, adequacy to criteria changes, number of suppliers and criteria, and agility to judgment making which are inspired from [65]. All these 4 parameters are used over IVIFE and [53] to understand the numerical difference between the two methods.

(1) *Adequacy to Alternatives Changes.* This parameter is used to test the stability of the method from the alternative perspective. The rank reversal issue is also discussed for both methods from the perspective of repetition of alternatives. In the example demonstrated above, we consider a decision matrix of order (5×6). Here 5 suppliers are evaluated over 6 criteria. We now form 5 test cases by repeating each supplier instance. The inference of the analysis is made below:

(i) These 5 test cases are given as input to IVIFE method [53] and ranking order is investigated. The normal ranking order given by [53] is $\varphi_5 \succ \varphi_2 \succ \varphi_4 \succ \varphi_3 \succ \varphi_1$. The repetition of supplier 1 causes change in suitable supplier order and thus the stability of the method is affected. When supplier 1 is repeated, the order changes to $\varphi_1 \succ \varphi_5 \succ \varphi_2 \succ \varphi_4 \succ \varphi_3$.

(ii) On the other hand, when these 5 test cases are given as input to proposed IVIFE method, the ranking order remains unaffected and the stability of the method is also ensured.

(iii) The crux of this investigation is that though adequate changes are made to the alternatives, unlike method [53], the proposed IVIFE method is stable and remains unaffected by rank reversal issue.

(2) *Adequacy to Criteria Changes.* This parameter is also used to test the stability of the methods from the perspective of changes to criteria. In the example above, we considered 5 suppliers and 6 criteria for the decision making process. Now, we repeat these 6 criteria and form 6 different test cases for the analysis. The inference of the analysis is made below:

(i) These 6 test cases are given as input to the method discussed in [53] and we observe that there is no change in the ranking order. The normal order is $\varphi_5 \succ \varphi_2 \succ \varphi_4 \succ \varphi_3 \succ \varphi_1$ and this order is maintained throughout the analysis which infers that method [53] is stable.

(ii) On the other hand, when these 6 test cases are given as input to the proposed IVIFE method, we observe that the rank order changes for criteria 3, 4, and 5. The normal rank order is $\varphi_2 \succ \varphi_4 \succ \varphi_5 \succ \varphi_1 \succ \varphi_3$ and it changes to $\varphi_2 \succ \varphi_4 \succ \varphi_1 \succ \varphi_5 \succ \varphi_3$. Though the ranking order changes, the suitable supplier chosen for the process remains unchanged.

(iii) The crux of this investigation is that proposed IVIFE method suffers from rank reversal issue when adequate changes are made to the criteria. This can be considered as a weakness of the proposed method and can be addressed in the future.

(3) *Number of Suppliers and Criteria*. This parameter is used to validate the scalability of the methods. Since proposed IVIFE method and method [53] follow concordance and discordance estimation, the size of the matrix increases as the number of suppliers increases. Similarly, the increase in number of criteria also increases computation. Thus, to balance the trade-off between alternatives and criteria, we adopt the concept of [66], which states that any human's cognitive thought process can handle at most 9 items at a given instance and hence, we maintain this value in the decision making process.

(4) *Agility to Judgment*. This parameter is used to estimate the total judgment needed by the DM to arrive at a proper consensus. The procedure to estimate the judgment for both methods is as follows:

(i) The method discussed in [53] follows concordance and discordance concepts and hence their agility to judgment is given by $(i(i-1))$, where i is the order of the matrix. In the example above, since there are concordance and discordance matrices for both strong and weak zone, we have $(4m(m-1))$ judgments, where m is the total number of alternatives $((2m(m-1)) + (2m(m-1)))$.

(ii) On the other hand, proposed IVIFE method uses three zones for evaluation of concordance and discordance matrix and hence the judgment needed for consensus is given by $(6m(m-1))$.

(iii) Thus, for the example demonstrated above, method discussed in [53] needs $(4 \times 5(5-1)) = 80$ judgments and proposed IVIFE method needs $(6 \times 5(5-1)) = 120$ judgments.

(iv) Clearly from the discussion made above, proposed IVIFE method is slow in arriving at a particular consensus compared to method [34]. Though the agility of the proposed IVIFE method is low, the method prevents information loss and yields much sensible rank value set which helps DMs to make clarified and rational decision.

7. Another Example for Validating the Strength of Proposed IVIFE Method

In this section, we clearly bring out the power of proposed IVIFE method by conducting an experiment using simulation process. The experiment clarifies the need for an additional outranking category, that is, moderate. We generate 300 decision matrices with IVIF information under 3 categories, namely, (a) (3×4) with 3 alternatives and 4 criteria; (b) (4×5) with 4 alternatives and 5 criteria; and (c) (6×5) with 6 alternatives and 5 criteria. We generate 100 matrices in each category with constraints defined in Definition 3. The order of the matrix is chosen arbitrarily and scalability is ensured by varying the order $(m \times n)$. The criteria weights are considered to be unbiased for the ease of evaluation. The procedure for conducting the experiment is given below.

Step 1. Form 300 decision matrices with IVIF information under 3 categories mentioned above. For brevity, we consider all these 100 matrices to be complete and they all obey the condition given in Definition 3.

Step 2. These decision matrices are given as input to the proposed ranking method and different ranking order is observed for each matrix. We use unbiased weights for each criterion and weights of concordance and discordance in each outranking category are taken from Section 5.

Step 3. In order to maintain homogeneity and closeness in comparison, we give these decision matrices as input to IVIFE [53]. We also normalize the rank value set in the desired places, where the values exceed 1. This normalization clarifies the percentage of contribution of each alternative in the process.

Step 4. From the ranking order obtained for each matrix by using different ranking methods as in Steps 2 and 3, we calculate the standard deviation. The deviation clearly shows that the proposed IVIFE method is much sensible (broad rank value set) and the alternatives are easily distinguishable.

Step 5. To better realize the need for an additional outranking category moderate in the proposal, we plot the deviation vectors of each method as shown in Figure 3. The deviation calculated for each method is a vector of order (1×300). We obtain 4 such vectors and Figure 3 clearly shows that the proposed IVIFE method produces much sensible and broad ranking order to certain extent that helps DMs to make better decisions with proper backup to address critical and uncertain scenarios.

Based on the analysis conducted in Section 6 and the inference which we gain from Figure 3, we observe that

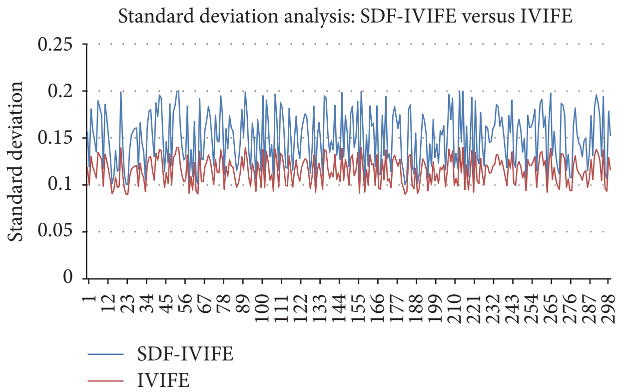

Figure 3: Analysis of standard deviation of rank values for different ranking methods.

proposed IVIFE method and IVIFE method presented in [53] are close counterparts which perform similar to certain extent. But the introduction of moderate category in the proposal adds high value to the proposed IVIFE method by producing broad and sensible rank value set which offers DMs some confidence for better backup management.

8. Conclusion

In this study, we have demonstrated a new computational framework for cloud vendor selection by proposing IVIF, which is an extension to ELECTRE under IVIF environment with three categories of outranking relation for the construction of concordance and discordance matrices. This new scheme also combines TOPSIS ranking method for achieving linear preference order. The proposed IVIFE method represents vagueness better by adopting IVIF concept. The method tackles the issue of information loss better by preserving the interval concept during concordance and discordance estimation. The proposed IVIFE method also handles information loss and vagueness better by constructing concordance and discordance matrices for all three outranking categories under both lower and upper bounds separately. The introduction of the third category (moderate) mitigates the problem of vagueness and information loss within the data by bringing out additional constraint checks for concordance and discordance matrices. This proposal also helps DMs to make rational decisions at uncertain and critical times. An illustrative numerical example is also demonstrated to verify the practicality of the proposed IVIFE scheme.

Some key contributions of the proposed SDF are given below:

(1) The proposed SDF is the first framework for supplier selection under IVIF environment which uses a combination of SV method for criteria weight estimation and ELECTRE method for ranking suppliers.

(2) The SDF complements the work done in [34] by proposing three zones (strong, moderate, and weak) for evaluation of concordance and discordance matrix rather than using the traditional two-zone model. Also, the SDF uses outranking relations for evaluation of concordance and discordance rather than using score and accuracy concepts (used in [34]). This formulation helps DMs to handle information loss better and provides sensible rank value set for providing better backup to manage task.

(3) The power of SDF is realized from both theoretic and numeric perspective and to the best of our knowledge, this is the first time such an analysis is carried out over a decision framework under IVIF environment.

(4) From the analysis, we infer that both proposed IVIFE and method [34] suffer from rank reversal issue, but proposed IVIFE method performs better in terms of the following ways: (1) handling imprecision and vagueness, (2) preventing information loss by incorporating additional constraint checks, (3) producing broad and sensible rank value set, and (4) being moderately consistent with VIKOR method under IVIF environment.

Along with the future works planned above, we also have ideas for developing some new fuzzy concepts which might be integrated along with ELECTRE for solving MCDM problems. We also have plans to extend the existing fuzzy concepts over some new outranking methods. Also, the proposed framework, SDF, can be extended to other applications like medical for nurse selection, equipment selection, and so forth, management for manager selection, personnel selection, and so forth, and manufacturing for material selection and so on.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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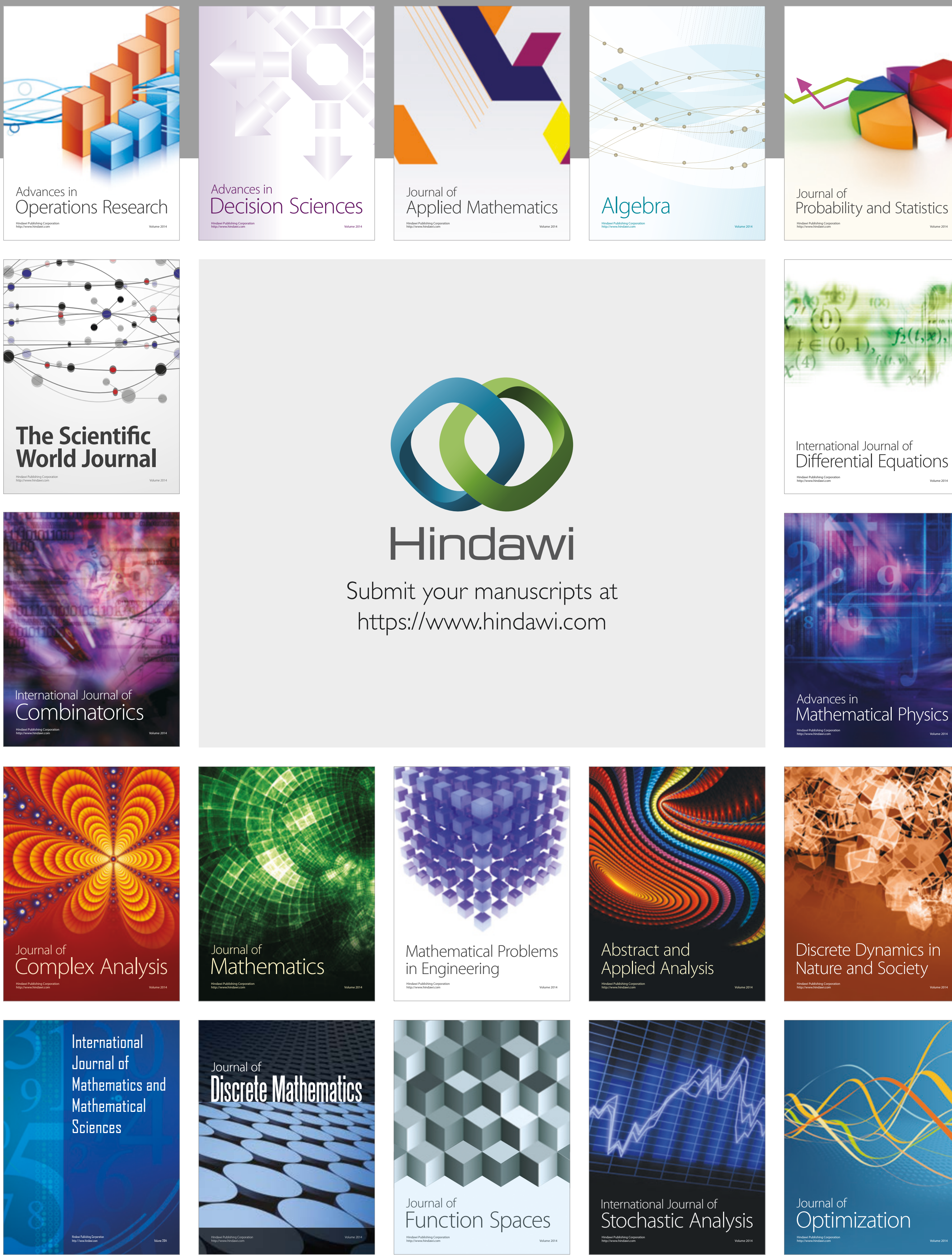
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