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Production routing problems with reverse logistics and remanufacturing

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ABSTRACT

This paper introduces a mixed integer programming model for production routing problems with reverse logistics and remanufacturing, which are closed-loop production routing problems addressed for the first time. A solution method of branch-and-cut guided search algorithm is developed. Computational results from instances adapted from benchmarks of production routing problems show that, the algorithm is more effective when pickup requests are relative high. The problem is also easier to solve when production or transportation costs are lower. The optimal decisions are insensitive to the location of remanufacturing depot whether it is geographically centered or centered with gravity.

1. Introduction

Concerns about the environmental impact of transportation and logistic activities have greatly increased in recent years. Since integrated operations can help achieve goals of lesser harm to the environment, while remaining operational effective (Qiu et al., 2017), supply chain optimization problems have attracted much research efforts. Among these problems, the production routing problem (PRP) that jointly optimizes decisions of production, inventory, distribution and routing has recently received a considerable attention (Adulyasak et al., 2015a). This integrated optimization problem is of practical relevance to success in business competitions, especially in modern logistical practices of vendor managed inventory (VMI) and just-in-time (JIT).

Besides integrating operations forward, closed-loop supply chain optimization showed a further reduction in environmental impact (Savaskan et al., 2004). Return flow processes in a closed-loop supply chain usually consists of (1) product collection from consumers; (2) reverse logistics to take collected products back; (3) screening, assorting and disposal to specify the most economically attractive reuse alternatives; (4) remanufacturing; and (5) remarketing to produce and utilize new markets (Iassinovskaia et al., 2017). A closed-loop PRP naturally integrates reverse logistics and remanufacturing. This problem is important because in addition to economic benefits, environmental benefits due to extension of the product useful life, reduced energy and material consumption, pollution prevention, and other sustainability benefits can be expected.

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After the importance of considering production, inventory and routing decisions simultaneously was stressed by Chandra (1993), the PRP was extended in various ways to consider, e.g., multiple plants and heterogeneous fleets of vehicles (Lei et al., 2006), incapacitated production (Archetti et al., 2011), multiple homogeneous capacitated vehicles (Adulyasak et al., 2014a), demand uncertainty (Adulyasak et al., 2015b), multi-item back-order (Brahimi and Aouam, 2015), perishable products(Vahdani et al., 2017), and multiscale production(Zhang et al., 2017) in the past decade. The environmental impact of the PRP has seldom been addressed, with only a few notable exceptions, such as the PRP with carbon emissions (Qiu et al., 2017), and multi-objective production and pollution routing problem with time window (Kumar et al., 2015). However, reverse logistics and remanufacturing are completely ignored to the best of our knowledge.

Battarra et al. (2014) addressed pickup-and-delivery problems for goods transportation, and reviewed various available algorithms. The vehicle routing problem with simultaneous pickups and delivery (VRPSPD), also known as the most studied and most general variant of the one-to-many-to-one (1-M-1) problems, has become increasingly popular. The electric appliances industry, beverage industry, and returnable/reusable transport items (RTI), or returnable/reusable logistical packaging have witnessed the application of the VRPSPD. Exact algorithms such as branch-and-cut method (Subramanian et al., 2013), branch-price-and-cut method (Cherkesly et al., 2015; Qu and Bard, 2015), are relatively new. Inventory routing problems with simultaneous pickups and deliveries (IRPSPD) have been explored only recently (Soysal, 2016; van Anholt et al., 2016; Iassinovskaia et al., 2017). Extending these problems and methods to the PRP is a natural step forward.

Besides, since the importance of considering remanufacturing in closed-loop supply chain was stressed by Savaskan et al. (2004), the research on remanufacturing had mainly focused on inventory system with remanufacturing (DeCroix, 2006; Tao and Zhou, 2014), economic aspect of remanufacturing (Geyer et al., 2007; Chen and Chang, 2012), and marketing issues (Atasu et al., 2008; Agrawal et al., 2015). Applications of remanufacturing cover personal computers and peripherals, B2B information technology equipment, tires, and construction equipment. Concerns were also on practical issues such as supply chain-based barriers (Zhu et al., 2014) and third-party remanufacturing mode selection (Zou et al., 2016). However, little has been revealed when remanufacturing is not only involved with inventory decisions but also with routing decisions.

Our aim is to bridge these gaps by designing a model and algorithm for a closed-loop production routing problem with remanufacturing, simultaneous pickups and deliveries (PRPRPD). The PRP involves combinatorial optimization of both delivery and routing decisions. Exact algorithms, such as branch-and-price (Bard and Nananukul, 2009a, 2010; Qiu et al., 2017) and branch-and-cut (Archetti et al., 2011; Adulyasak et al., 2014a), can solve small and medium sized problems. Heuristics are often applied in other research, e.g., approximation algorithm (Chandra and Fisher, 1994), decoupled heuristic (Fumero and Vercellis, 1999), greedy randomized adaptive search procedure (Boudia et al., 2007), memetic algorithm (Boudia and Prins, 2009), tabu search (Bard and Nananukul, 2009b; Armentano et al., 2011), adaptive large neighborhood search (Adulyasak et al., 2014b), iterative mixed integer programming (Absi et al., 2015), particle-swarm optimization (Kumar et al., 2015), mathematical programming heuristic (Russell, 2017), and multiphase heuristic (Solyalıand Süral, 2017). A hybrid algorithm combining branch-and-cut and heuristic search is promising for the PRPRPD.

The contributions of this paper can be summarized as follows. First, we introduce a real-world variant of the PRP with reverse logistics and remanufacturing. Reverse logistics characterized with simultaneous pickups and deliveries is now mixed with capacitated vehicle routing problems (CVRP), which is unique from the PRP literature. Second, we formulate the PRPRPD as a mixed integer linear programming (MILP) problem with discussions on feasibility and optimality. Third, we introduce valid inequalities to tighten the MILP formulation and design a novel branch-and-cut guided search algorithm as the solution method. Finally, we conduct extensive computational experiments to assess the performance of the proposed algorithm and develop managerial insights through sensitivity analysis. We find that the algorithm is more effective when pickup requests are relative high. The problem is also easier to solve when production or transportation costs are lower. The optimal decisions are insensitive to the location of remanufacturing depot whether it is geographically centered or centered with gravity. When remanufacturing rate is sufficiently high, the manufacturing activities can be totally replaced by remanufacturing ones, resulting a large drop in total costs, given there are enough pickups inventory and new pickups to cover the delivery requests. Our PRPRPD model, algorithm and computational results can serve as a stepping stone for further research of the PRP with return flow (Adulyasak et al., 2015a).

The rest of the paper is organized as follows. Section 2 describes the PRPRPD and introduces a mathematical formulation. Section 3 elaborates a solution method of branch-and-cut guided search algorithm. Extensive computational results are provided in Section 4. We conclude in Section 5 with discussions on future research directions.

2. Problem description and mathematical formulation

2.1. Problem description

The PRP with remanufacturing, pickups and deliveries (PRPRPD) is defined on a complete directed graph $\mathscr{G} = (\mathscr{V}, \mathscr{A})$, where the node set $\mathscr{V} = (\mathscr{N} \cup \mathscr{M} \cup \mathscr{R})$ consists of a set $\mathscr{N}$ of customers, a set $\mathscr{M}$ of manufacturing depots, and a set $\mathscr{R}$ of remanufacturing depots, and the arc set is $\mathscr{A} = \{(i,j): i,j \in \mathscr{V}, i \neq j\}$. A finite fleet or set $\mathscr{K}$ of heterogeneous vehicles at manufacturing depots or remanufacturing depots, respectively, are available to serve the customers over a finite set $\mathscr{T}$ of planning periods. In every period, each customer has pickup and delivery requests, new products can be manufactured with unit production costs c^{m}, or remanufactured from pickups with unit remanufacturing costs c^{r} at the depot.

We assume that remanufacturing a new product by using a returned one is less costly than manufacturing a new one, i.e., $c^{\mathrm{r}} < c^{\mathrm{m}}$. This assumption is in line with Savaskan et al. (2004). Furthermore, we assume that fixed setup cost for remanufacturing is also smaller than that of manufacturing, i.e., $c^{\mathrm{fr}} < c^{\mathrm{fm}}$. These assumptions ensure that remanufacturing can reduce production cost. Therefore, ceteris paribus, a higher product pickup rate is strictly preferred if vehicle capacity allows.

We also assume that returned units cannot be fully remanufactured because some returned units are irreparable by technical constraints. This assumption implies that the remanufacturing rate from returned units ρ satisfies $0 \leqslant \rho \leqslant 1$.

Products can be shipped to customers while pickups can be collected simultaneously in each period. Given initial inventory levels of deliveries and pickups at depots and customers, the problem is to determine the manufacturing and remanufacturing amount at each depot, the pickup and delivery amount at each customer, and the set of routes in each period with minimum total costs of production, inventory and routing.

2.2. Feasibility and optimality concerns

To ensure the feasibility of formulations and solutions, the following conditions must be satisfied:

- (Vehicle Availability) A finite fleet of heterogeneous vehicles are available at manufacturing depots or remanufacturing depots, respectively. Moreover, the load on each vehicle should not exceed its capacity. This restriction is less severe for vehicles at manufacturing depots because the problem is the well-known capacitated vehicle routing problem. For vehicles at remanufacturing depots, the problem is now vehicle routing problems with simultaneous pickups and deliveries. The load on vehicles after they leaving from remanufacturing depots fluctuates throughout the route before they return to remanufacturing depots. Thus, the load should be monitored after the vehicle leaving from each customer. Fortunately, when we find an infeasible route, we can possibly obtain a feasible route by reverse the route if the total amount of pickups and deliveries of all customers is less than the sum of vehicle capacity and the minimum arc load of the infeasible route.
- (Manufacturing Capacity) The quantity of new products manufactured should not exceed the capacity of manufacturing depot.
- (Remanufacturing Capacity) The quantity of new products remanufactured from pickups should not exceed the capacity of remanufacturing depot.
- (Inventory Capacity) The inventory quantity of delivery products and pickup products at customers and depots should not exceed inventory capacity, respectively.

To ensure the optimality of solutions, the following conditions must be satisfied:

Proposition 1. *If a feasible solution satisfying all the feasibility conditions, it is an optimal solution only if the delivery inventories at both manufacturing depot and remanufacturing depot in the last period are zero.*

Proof. Suppose we obtain an optimal solution that the delivery inventories at either manufacturing depot or remanufacturing depot in the last period are not zero, we can either reduce manufacturing quantity or remanufacturing quantity so that the delivery inventories at either manufacturing depot or remanufacturing depot in the last period reduce to zero. Thus, the variable manufacturing or remanufacturing cost can be reduced, and the cost of carrying delivery inventories will also decrease, while all other feasibility conditions still hold. This contradicts with the optimality assumption. □

Proposition 2. *When remanufacturing rate is sufficiently high, if a feasible solution satisfying all the feasibility conditions, it is an optimal solution only if the manufacturing activities is totally replaced by remanufacturing ones, given there are enough pickups inventory and new pickups to cover the delivery requests.*

Proof. Suppose we obtain an optimal solution that manufacturing activities is not totally replaced by remanufacturing ones, given there are enough pickups inventory and new pickups to cover the delivery requests, we can always reduce the manufacturing quantity to zero and increase remanufacturing quantity. Thus, the total cost will decrease because of the assumptions that remanufacturing activities are less expensive than manufacturing activities. This contradicts with the optimality assumption. □

2.3. A MILP formulation for the PRPRPD

To formulate the problem, following notations for indices, parameters and decision variables are used.

Indices:

- $\mathscr{N}$: Set of pickup and delivery nodes, $\mathscr{N} = \{1,2,...,|\mathscr{N}|\}$.
- $\mathscr{M}$: Set of manufacturing depots, $\mathscr{M} = \{1,2,...,|\mathscr{M}|\}$.
- $\mathscr{R}$: Set of remanufacturing depots, $\mathscr{R} = \{1,2,...,|\mathscr{R}|\}$.
- $\mathscr{V}$: Set of all locations, $\mathscr{V} = (\mathscr{N} \cup \mathscr{M} \cup \mathscr{R})$.
- $\mathscr{A}$: Set of arcs, $\mathscr{A} = \{(i,j): i,j \in \mathscr{V}, i \neq j\}$.
- $\mathscr{G}$: A complete directed graph, $\mathscr{G} = (\mathscr{V},\mathscr{A})$.
- $\mathscr{K}_{\mathrm{m}}$: Set of vehicles that serve manufacturing depots, $\mathscr{K}_{\mathrm{m}} = \{1,2,...,|\mathscr{K}_{\mathrm{m}}|\}$.
- $\mathscr{K}_{\mathrm{r}}$: Set of vehicles that serve remanufacturing depots, $\mathscr{K}_{\mathrm{r}} = \{1,2,...,|\mathscr{K}_{\mathrm{r}}|\}$.
- $\mathscr{K}$: Set of vehicles, $\mathscr{K} = (\mathscr{K}_{\mathrm{m}} \cup \mathscr{K}_{\mathrm{r}})$.
- $\mathscr{T}$: Set of planning periods, $\mathscr{T} = \{1,2,...,|\mathscr{T}|\}$.
- $\mathscr{N}_{kt}$: Set of pickup and delivery nodes that can be served by vehicle k in period t, $\mathscr{N}_{kt} = \{1,2,...,|\mathscr{N}_{kt}|\}$, and $|\mathscr{N}_{kt}| \leqslant |\mathscr{N}|$.
- $\mathscr{M}_{kt}$: Set of manufacturing depots that can be served by vehicle k in period t, $\mathscr{M}_{kt} = \{1,2,...,|\mathscr{M}_{kt}|\}$, and $|\mathscr{M}_{kt}| \leqslant |\mathscr{M}|$.
- $\mathscr{R}_{kt}$: Set of remanufacturing depots that can be served by vehicle k in period t, $\mathscr{R}_{kt} = \{1,2,...,|\mathscr{R}_{kt}|\}$, and $|\mathscr{R}_{kt}| \leqslant |\mathscr{R}|$.
- $\mathscr{V}_{kt}$: Set of all locations that can be visited by vehicle k in period t, $\mathscr{V}_{kt} = (\mathscr{N}_{kt} \cup \mathscr{M}_{kt} \cup \mathscr{R}_{kt})$.
- $\mathscr{A}_{kt}$: Set of arcs for vehicle k in period t, $\mathscr{A}_{kt} = \{(i,j): i,j \in \mathscr{V}_{kt}, i \neq j\}$.

Parameters:

- Q_k: Capacity of vehicle k ($k \in \mathscr{K}$).
- C_i^{m}: manufacturing capacity at manufacturing depot i;
- C_i^{r}: remanufacturing capacity at remanufacturing depot i;
- c^{fm}: fixed manufacturing setup costs;
- c^{m}: unit manufacturing costs;
- c^{fr}: fixed remanufacturing setup costs;
- c^{r}: unit remanufacturing costs;
- c_{ij}: transportation cost over arc (i,j);
- δ_{it}: delivery requests of customer i in period t;
- π_{it}: pickup requests of customer i in period t;
- h_i^{d}: unit inventory holding cost of deliveries at customer i or depot i;
- h_i^{p}: unit inventory holding cost of pickups at customer i or at remanufacturing depot i;
- L_i^{d}: storage capacity for deliveries at customer i or depot i;
- L_i^{p}: storage capacity for pickups at customer i or depot i;
- I_{i0}^{d}: initial delivery inventory at customer i or depot i;
- I_{i0}^{p}: initial pickup inventory at customer i or depot i;
- B_{1it}: $=\min\{C_i^{\mathrm{m}}, L_i^{\mathrm{d}}, \sum_{\tau=t}^{|T|} \sum_{i \in \mathscr{N}} \delta_{i\tau}\}$;
- B_{2it}: $=\min\{C_i^{\mathrm{r}}, L_i^{\mathrm{p}}, \sum_{\tau=t}^{|T|} \sum_{i \in \mathscr{N}} \delta_{i\tau}\}$;
- M_{1itk}: $=\min\{Q_k, L_i^{\mathrm{d}}, \sum_{\tau=t}^{|T|} \delta_{i\tau}\}$;
- M_{2itk}: $=\min\{Q_k, L_i^{\mathrm{p}}, I_{i0}^{\mathrm{p}} + \sum_{\tau=1}^{t} \pi_{i\tau}\}$;

Variables:

- m_{it}: manufacturing quantity at manufacturing depot i in period t;
- r_{it}: remanufacturing quantity at remanufacturing depot i in period t;
- I_{it}^{d}: delivery inventory at customer i or depot i at the end of period t;
- I_{it}^{p}: pickup inventory at customer i or depot i at the end of period t;
- d_{ikt}: delivery amount to customer i by vehicle k in period t;
- p_{ikt}: pickup amount at customer i by vehicle k in period t;
- u_{ijkt}: pickup amount over arc (i,j) by vehicle k in period t if arc (i,j) is traversed by vehicle k in period t, 0 otherwise;
- v_{ijkt}: delivery amount over arc (i,j) by vehicle k in period t if arc (i,j) is traversed by vehicle k in period t, 0 otherwise;
- x_{ijkt}: binary variable, equal to 1 if arc (i,j) is traversed by vehicle k in period t, 0 otherwise;
- y_{it}: binary variable, equal to 1 if manufacturing is set up for production at manufacturing depot i in period t, 0 otherwise;
- z_{it}: binary variable, equal to 1 if remanufacturing is set up for production at remanufacturing depot i in period t, 0 otherwise;

Given the notations above, the arc-flow-based formulation of the PRPRPD is given as follows:

$$\text{minimize} \quad \sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{M}}(c^{\mathrm{m}}m_{it}+c^{\mathrm{fm}}y_{it})+\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{R}}(c^{\mathrm{r}}r_{it}+c^{\mathrm{fr}}z_{it}) \tag{1a}$$

$$+\sum_{t\in\mathcal{T}}\sum_{i\in\mathcal{V}}(h_i^{\mathrm{d}}I_{it}^{\mathrm{d}}+h_i^{\mathrm{p}}I_{it}^{\mathrm{p}}) \tag{1b}$$

$$+\sum_{t\in\mathcal{T}}\sum_{k\in\mathcal{K}}\sum_{(i,j)\in\mathcal{A}}c_{ij}x_{ijkt} \tag{1c}$$

subject to

$$I_{i,t-1}^{\mathrm{d}}+m_{it}-\sum_{k\in\mathcal{K}_{\mathrm{m}}}\sum_{j\in\mathcal{N}}d_{jkt}=I_{it}^{\mathrm{d}}, \qquad \forall\, i\in\mathcal{M},t\in\mathcal{T}, \tag{2}$$

$$I_{i,t-1}^{\mathrm{d}}+r_{it}-\sum_{k\in\mathcal{K}_{\mathrm{r}}}\sum_{j\in\mathcal{N}}d_{jkt}=I_{it}^{\mathrm{d}}, \qquad \forall\, i\in\mathcal{R},t\in\mathcal{T}, \tag{3}$$

$$I_{i,t-1}^{\mathrm{p}}-r_{it}/\rho+\sum_{k\in\mathcal{K}_{\mathrm{r}}}\sum_{j\in\mathcal{N}}p_{jkt}=I_{it}^{\mathrm{p}}, \qquad \forall\, i\in\mathcal{R},t\in\mathcal{T}, \tag{4}$$

$$I_{i,t-1}^{\mathrm{d}}+\sum_{k\in\mathcal{K}}d_{ikt}-\delta_{it}=I_{it}^{\mathrm{d}}, \qquad \forall\, i\in\mathcal{N},t\in\mathcal{T}, \tag{5}$$

$$I_{i,t-1}^{\mathrm{p}}-\sum_{k\in\mathcal{K}}p_{ikt}+\pi_{it}=I_{it}^{\mathrm{p}}, \qquad \forall\, i\in\mathcal{N},t\in\mathcal{T}, \tag{6}$$

$$r_{it}\leqslant\rho I_{i,t-1}^{\mathrm{p}}, \qquad \forall\, i\in\mathcal{R},t\in\mathcal{T}, \tag{7}$$

$$m_{it}\leqslant B_{1it}y_{it}, \qquad \forall\, i\in\mathcal{M},t\in\mathcal{T}, \tag{8}$$

$$r_{it}\leqslant B_{2it}z_{it}, \qquad \forall\, i\in\mathcal{R},t\in\mathcal{T}, \tag{9}$$

$$I_{it}^{\mathrm{d}}\leqslant L_i^{\mathrm{d}}, \qquad \forall\, i\in\mathcal{V},t\in\mathcal{T}, \tag{10}$$

$$I_{it}^{\mathrm{p}}\leqslant L_i^{\mathrm{p}}, \qquad \forall\, i\in\mathcal{V},t\in\mathcal{T}, \tag{11}$$

$$\sum_{j\in\mathcal{V}}x_{ijkt}\leqslant 1, \qquad \forall\, i\in\mathcal{V}_{kt},k\in\mathcal{K},t\in\mathcal{T}, \tag{12}$$

$$\sum_{j\in\mathcal{V}}x_{ijkt}-\sum_{j\in\mathcal{V}}x_{jikt}=0, \qquad \forall\, i\in\mathcal{V}_{kt},k\in\mathcal{K},t\in\mathcal{T}, \tag{13}$$

$$\sum_{j\in\mathcal{V}}v_{jikt}-\sum_{j\in\mathcal{V}}v_{ijkt}=d_{ikt}, \qquad \forall\, i\in\mathcal{N}_{kt},k\in\mathcal{K},t\in\mathcal{T}, \tag{14}$$

$$\sum_{j\in\mathcal{V}}u_{ijkt}-\sum_{j\in\mathcal{V}}u_{jikt}=p_{ikt}, \qquad \forall\, i\in\mathcal{N}_{kt},k\in\mathcal{K},t\in\mathcal{T}, \tag{15}$$

$$v_{ijkt}+u_{ijkt}\leqslant Q_kx_{ijkt}, \qquad \forall\, (i,j)\in\mathcal{A}_{kt},k\in\mathcal{K},t\in\mathcal{T}, \tag{16}$$

$$d_{ikt}\leqslant M_{1itk}\sum_{j\in\mathcal{V}}x_{ijkt}, \qquad \forall\, i\in\mathcal{N}_{kt},k\in\mathcal{K},t\in\mathcal{T}, \tag{17}$$

$$p_{ikt}\leqslant M_{2itk}\sum_{j\in\mathcal{V}}x_{ijkt}, \qquad \forall\, i\in\mathcal{N}_{kt},k\in\mathcal{K}_{\mathrm{r}},t\in\mathcal{T}, \tag{18}$$

$$m_{it}\geqslant 0,y_{it}\in\{0,1\}, \qquad \forall\, t\in\mathcal{T}, \tag{19}$$

$$r_{it}\geqslant 0,z_{it}\in\{0,1\}, \qquad \forall\, t\in\mathcal{T}, \tag{20}$$

$$I_{it}^{\mathrm{d}},I_{it}^{\mathrm{p}}\geqslant 0, \qquad \forall\, i\in\mathcal{V},t\in\mathcal{T}, \tag{21}$$

$$d_{ikt},p_{ikt}\geqslant 0, \qquad \forall\, i\in\mathcal{N}_{kt},k\in\mathcal{K},t\in\mathcal{T}, \tag{22}$$

$$u_{ijkt}, v_{ijkt} \geqslant 0\,, \qquad \forall\ (i,j) \in \mathscr{A}_{kt}, k \in \mathscr{K}, t \in \mathscr{T}, \tag{23}$$

$$x_{ijkt} \in \{0,1\}\,, \qquad \forall\ (i,j) \in \mathscr{A}_{kt}, k \in \mathscr{K}, t \in \mathscr{T}. \tag{24}$$

The objective function (1a)–(1c) minimizes the total operational costs, where (1a), (1b), and (1c) measure production, inventory, and routing costs, respectively. Constraints (2) and (3) enforce delivery inventory flow balance at manufacturing and remanufacturing depots, respectively. Pickup inventory flow balance at remanufacturing depots is ensured by constraints (4). Constraints (5) and (6) present delivery and pickup inventory flow balance at the customers, respectively. Constraints (7) state that remanufacturin its objective function. The constraints g quantity is limited by pickup inventory in the last period and remanufacturing rate. Constraints (8) guarantee that the setup binary variable is one if manufacturing takes place in each period. Constraints (8) also limit manufacturing quantity to the minimum between manufacturing capacity and total delivery requests in the remaining periods. Similarly, constraints (9) restrict remanufacturing quantity and setup binary variable for remanufacturing. Constraints (10) and (11) stipulate that the product and pickup inventory do not exceed their corresponding capacity, respectively. Constraints (12) serve as degree constraints. Constraints (13) represent vehicle flow balance. Constraints (14) and (15) are the flow conservation constraints for deliveries and pickups, respectively. Constraints (16) bound the product flow transporting over each arc to be at most equal to the vehicle capacity. Constraints (17) specify that each customer is visited if the delivery quantity at the customer is nonzero. Constraints (18) restrict that each customer is visited if the pickup quantity at the customer is nonzero. Finally, constraints (19)–(24) introduce the model's decision variables.

Note that the distinctive features of the proposed model relative to the literature in related areas are as follows. First, remanufacturing and reverse logistics are integrated together naturally within the context of production routing, whereas most literature in related areas usually deal with one aspect only. Second, remanufacturing decisions are combined within the context of production routing, so that remanufacturing decisions are not only competed with manufacturing decisions which most literature in related areas usually deal with. In this model, remanufacturing decisions are also influenced by the extra routing costs conducting pickup activities. Third, locations of remanufacturing depots are distinguished from those of manufacturing depots which further complicates the routing part of the production routing problem. The vehicle routing subproblem now is a mix of capacitated vehicle routing problems (CVRP) and VRP with simultaneous pickups and deliveries (VRPSPD), while most literature in related areas dealt with CVRP or VRP with time windows only. Finally, heterogeneous vehicles with varying capacities locating at manufacturing and remanufacturing depots are considered, while most literature in related areas dealt with either homogeneous vehicles, or vehicles locating at manufacturing depots only.

3. Solution method

In this section, after we strengthen the linear relaxation of the formulation (2)–(24) with valid inequalities, we propose a branch-and-cut guided search algorithm.

3.1. Valid inequalities

Denote by $I^{\mathrm{d}}_{i0,s}$: $=\max\{0, I^{\mathrm{d}}_{i0} - \sum_{\tau=1}^{s} \delta_{i\tau}\}$ the delivery product quantity remaining from the initial delivery product inventory at customer $i \in \mathscr{N}$ at the end of period s, and let $I^{\mathrm{d}}_{i0,0} := I^{\mathrm{d}}_{i0}$. The residual delivery requests at customer i in period s can also be defined as $\hat{\delta}_{is}$: $=\max\{0, \delta_{is} - I^{\mathrm{d}}_{i0,s-1}\}$. With these notations, we can obtain the following propositions.

Proposition 3. *The linear relaxation of the formulation (2)–(24) can be strengthened with the following valid inequalities:*

$$\sum_{\tau=1}^{t} \sum_{j\in\mathscr{V}} \sum_{k\in\mathscr{K}} x_{ijk\tau} \geqslant \left\lceil \frac{\sum_{\tau=1}^{t} \hat{\delta}_{i\tau}}{\min\{\max_{k\in\mathscr{K}} Q_k, L_i^{\mathrm{d}}\}} \right\rceil, \qquad \forall\ i \in \mathscr{N}, t \in \mathscr{T}, \tag{25}$$

$$\sum_{\tau=1}^{t} \sum_{j\in\mathscr{V}} \left(\sum_{i\in\mathscr{M}} \sum_{k\in\mathscr{K}_{\mathrm{m}}} x_{ijk\tau} + \sum_{i\in\mathscr{R}} \sum_{k\in\mathscr{K}_{\mathrm{r}}} x_{ijk\tau} \right) \geqslant \left\lceil \frac{\sum_{i\in\mathscr{N}} \sum_{\tau=1}^{t} \hat{\delta}_{i\tau}}{\max_{k\in\mathscr{K}} Q_k} \right\rceil, \qquad \forall\ t \in \mathscr{T}, \tag{26}$$

$$\sum_{\tau=1}^{t} \sum_{j\in\mathscr{V}} \sum_{k\in\mathscr{K}_{\mathrm{r}}} x_{ijk\tau} \geqslant \left\lceil \frac{I^{\mathrm{p}}_{i0} + \sum_{\tau=1}^{t} \pi_{i\tau} - L_i^{\mathrm{p}}}{\min\{\max_{k\in\mathscr{K}} Q_k, L_i^{\mathrm{p}}\}} \right\rceil, \qquad \forall\ i \in \mathscr{N}, t \in \mathscr{T}. \tag{27}$$

Proof. First, consider lower bounds on the number of visits to each customer in each period by inspection of the customer's accumulated residual deliveries. The number of visits are bounded below by how many times vehicle with maximum capacity need to accommodate the accumulated residual deliveries, only if the inventory capacity allows. Thus, we obtain inequalities (25). Similarly, consider lower bounds on the number of visits to each customer in each period by inspection of the customer's accumulated pickups, we obtain inequalities (27). Finally, consider lower bounds on the number of vehicles used in each period, we calculate total residual delivery requests of all customers instead of one customer, then we obtain inequalities (26). □

Proposition 4. *The linear relaxation of the routing constraints (12)–(18) can be strengthened with the following valid inequalities:*

$$\sum_{i\in\mathcal{N}} d_{ikt} \leqslant Q_k \sum_{j\in\mathcal{V}}\sum_{i\in\mathcal{M}} x_{ijkt}, \quad \forall\, k\in\mathcal{K}_{\mathrm{m}}, t\in\mathcal{T}, \tag{28}$$

$$\sum_{i\in\mathcal{N}} d_{ikt} \leqslant Q_k \sum_{j\in\mathcal{V}}\sum_{i\in\mathcal{R}} x_{ijkt}, \quad \forall\, k\in\mathcal{K}_{\mathrm{r}}, t\in\mathcal{T}, \tag{29}$$

$$\sum_{i\in\mathcal{N}} \pi_{ikt} \leqslant Q_k \sum_{j\in\mathcal{V}}\sum_{i\in\mathcal{R}} x_{ijkt}, \quad \forall\, k\in\mathcal{K}_{\mathrm{r}}, t\in\mathcal{T}, \tag{30}$$

$$\sum_{j\in\mathcal{V}} x_{ijkt} \leqslant \sum_{j\in\mathcal{V}}\sum_{i\in\mathcal{M}} x_{ijkt}, \quad \forall\, i\in\mathcal{N}, k\in\mathcal{K}_{\mathrm{m}}, t\in\mathcal{T}, \tag{31}$$

$$\sum_{j\in\mathcal{V}} x_{ijkt} \leqslant \sum_{j\in\mathcal{V}}\sum_{i\in\mathcal{R}} x_{ijkt}, \quad \forall\, i\in\mathcal{N}, k\in\mathcal{K}_{\mathrm{r}}, t\in\mathcal{T}, \tag{32}$$

Proof. Consider the total delivery quantities that a vehicle at manufacturing depot and remanufacturing depot can carry on without violation of vehicle capacity, respectively, we obtain inequalities (28) and (29). Similarly, for total pickup quantities, we obtain inequalities (30). Finally, each customer can be visited by a vehicle only if the corresponding depot is also traversed by the same vehicle, we thus obtain inequalities (31) and (32). □

Proposition 5. *The linear relaxation of the formulation (2)–(24) can be strengthened with the following valid inequalities:*

$$\left(\sum_{l=0}^{s} \delta_{i,t-l}\right)\left(1-\sum_{k\in\mathcal{K}}\sum_{l=0}^{s}\sum_{j\in\mathcal{V}} x_{ijk,t-l}\right) \leqslant I^{\mathrm{d}}_{i,t-s-1}, \quad \forall\, i\in\mathcal{N}, t\in\mathcal{T}, 0\leqslant s\leqslant t-1, \tag{33}$$

$$L_i^{\mathrm{p}}-\left(\sum_{l=0}^{s} \pi_{i,t-l}\right)\left(1-\sum_{k\in\mathcal{K}_{\mathrm{r}}}\sum_{l=0}^{s}\sum_{j\in\mathcal{V}} x_{ijk,t-l}\right) \geqslant I^{\mathrm{p}}_{i,t-s-1}, \quad \forall\, i\in\mathcal{N}, t\in\mathcal{T}, 0\leqslant s\leqslant t-1, \tag{34}$$

Proof. Consider the relationship between delivery inventory and total accumulated delivery requests, inventory should be enough to cover requests in each time instant. Thus, we obtain inequalities (33). Similarly, consider accumulated pickup requests and the corresponding pickup inventory, we obtain inequalities (34). □

3.2. Branch-and-cut guided search algorithm

To solve the model, we propose a branch-and-cut guided search algorithm.

3.2.1. Branch-and-cut guided search

The idea motivating Branch-and-cut guided search (BCGS) is that the adjustment of manufacturing, remanufacturing, pickup and delivery schedules is guided by the branch-and-cut process. The proposed search algorithm is thus called branch-and-cut guided search algorithm. The proposed branch-and-cut guided search is carried on a branch-and-cut tree managed by the CPLEX engine. Thus, we may be able to solve the problem quickly and obtain the optimality gap if we cannot solve the problem in a limited time. The optimality gap information on the other hand is usually impossible to obtain with other meta-heuristics.

In addition to the search for feasible solutions we outline in the next section, we can strengthen the linear relaxation of the formulation (2)–(24) with the valid inequalities introduced in Section 3.1. Moreover, we further strengthen the linear relaxation of the formulation (2)–(24) with the following subtour elimination constraints:

$$\sum_{(i,j)\in\mathcal{A}(S)} x_{ijt} \leqslant \sum_{i\in S}\sum_{j\in\mathcal{V}} x_{ijt} - \sum_{j\in\mathcal{V}} x_{ejt}, \quad \forall\, S\subseteq N, |S|\geqslant 2, e\in S, t\in\mathcal{T}, \tag{35}$$

where $A(S) = \{(i,j): i,j \in S, i \neq j\}$. These cuts are separated by a connected component heuristic adapted from Lysgaard et al. (2004).

In this study, the highest branching priority is assigned to variables y_t. After the branching on variables y_t is done, higher branching priorities are assigned to variables x_{0it} and x_{i0t}. Finally, the branching is carried on the rest of variables x_{ijt}, where $i,j \neq 0$. The exploration of enumeration tree is achieved through a best-bound search strategy.

Algorithm 1. Branch-and-cut guided search algorithm

```
1  Initialize the upper bound U* and the incumbent solution.
2  Initialize the node pool N with the root node.
3  Generate and insert the proposed delivery and pickup valid inequalities
     into the program at the root node of the search tree.
4  repeat
5      Selection: Select the next node in N to evaluate and remove it from
         N.
6      Lower bound: Solve the LP relaxation at the current node, let U^l be
         the obtained lower bound of the current node:
7      if the current solution is feasible then
8          if U^l > U* then
9              go to the termination check.
           else
10             U* ← U^l.
11             Update the incumbent solution.
12             Prune nodes with lower bound U > U*.
           end
       else
13         Search heuristically based on current linear relaxation solutions.
14         if the solution obtained is feasible then
15             go to line 8.
           end
       end
16     Cut generation:
17     if the current solution of the LP relaxation violates any cuts, then
18         determine the violated subtour elimination cuts by connected
             component heuristics adapted from Lysgaard et al. (2004).
19         Add violated cuts.
       end
20     Branching: If U^l > U*, go to the termination check.
   until N = ∅ or time limit is met (termination check)
21 Stop with the optimal solution and the corresponding cost U*.
```

This algorithm is outlined in Algorithm 1. Line 13 in Algorithm 1 needs a further explanation. When the solution of the LP relaxation at the current node does not satisfy integer restrictions, we use this solution as a guide to search for a solution that satisfies all integer restrictions. More specifically, we round the pickup and delivery quantities in the LP solution to form a new pickup and delivery schedule. Then, we solve VRPSPD subproblems for each period and solve the production-inventory problem the same way as we obtain the initial solution. The procedure to obtain the initial solution is outlined in the next section.

3.2.2. Initial solution

At the root node of the branching tree, an upper bound U^* and an incumbent solution is obtained through the following heuristic.

First, if there are plenty of remanufacturing products available, delivery quantity from the remanufacturing depot to each customer is set to the customer's residual delivery requests in each period. If remanufacturing products are not enough to cover all customers' residual delivery requests, then remanufacturing products are distributed to each customer proportionally. The total remaining delivery requests are then satisfied by products from the manufacturing depot. Delivery quantity from the manufacturing depot to each customer is set to the customer's remaining delivery requests.

Second, pickup quantity of each customer is also set to the customer's pickup requests in each period, except that the pickup quantity in the first period also covers the initial pickup inventory. In each period, with these pickup and delivery quantities as inputs to CVRP or VRPSPD subproblems, we use the well-known Clark and Wright heuristic to obtain an initial feasible solution quickly. The VRPSPD solution is then improved by a guided variable neighborhood descent which has also been among the fastest improvement heuristics for the VRP (Kytöjoki et al., 2007). We modify the original guided variable neighborhood descent (GVND) mainly by replacing "intra-route" operators with the fast and efficient Lin-Kernighan heuristic (Lin and Kernighan, 1973). The modified procedure of the guided variable neighborhood descent is provided in Algorithm 2.

Algorithm 2. Guided variable neighborhood descent

Function GVND(s)
1 **foreach** $m \in \mathcal{M}$ *or* $m \in \mathcal{R}$ **do**
2   **foreach** $t \in \mathcal{T}$ **do**
3   Retrieve R_{mt}, the set of routes in period t for depot m, from the feasible solution s
4   **repeat**
5   **repeat**
6   Select the next route pair $\{R_1, R_2\}, R_1 \in R_t, R_2 \in R_{mt}$.
7   Set the threshold $\tau = \lambda \times \max\{c_{ij} | (i,j) \in \mathcal{A}(\{R_1, R_2\})\}$, where $A(\{R_1, R_2\})$ is the set of arcs in the route pair. Set $P_{ij} = 0$ for all arcs in $\{R_1, R_2\}$.
8   Apply *exchange*, *relocate*, *2-opt**, and *cross-exchange* to route pair $\{R_1, R_2\}$ until no more improvements can be found. If an improvement has been found, apply the LKH implementation of the Lin-Kernighan heuristic to both R_1 and R_2 separately.
9   Select an long arc to be penalized according to Kytöjoki et al. (2007) and increase its P_{ij} by 1. If $\sum P_{ij} \times \lambda > \tau$, goto line 7; otherwise, goto line 6 evaluating moves according to Kytöjoki et al. (2007). The selection of λ and τ values can be found in Kytöjoki et al. (2007)
            until *all route pairs have been considered*
        until *no more improvements can be found for any pair of routes*
    end
end
return s

In the modified GVND, well-known "inter-route" operators *exchange*, *relocate*, *2-opt** and *cross-exchange* that modify several routes simultaneously are implemented with the VRPH library (Groër et al., 2010). For each route pair, we first apply the inter-route variable neighborhood descent (VND) step and if an improvement is found, the LKH implementation (Helsgaun, 2012) of the Lin-Kernighan heuristic is applied to each route. Other details of the guided local search strategy can be found in Kytöjoki et al. (2007).

After we obtain VRPSPD solutions for each period, we solve a production-inventory problem for the depot with a MIP solver. This problem uses (1a) and (1b) as its objective function. The constraints 2,3,4,5,6,7,8 and (17) and (18) serve as restrictions except that the pickup and delivery quantities are set to known values as above mentioned. The production and inventory quantities, combined with the pickup and delivery quantities, and VRPSPD solutions for each period form the initial solution for the PRPRPD.

4. Computational results

This section summarizes computational experiments conducted to assess the performance of our algorithm and investigate how variations in key parameters related to remanufacturing affect solutions.

The algorithm was coded in C + + with IBM ILOG CPLEX version 12 release 7 as the LP solver. The experiments were run on a 64-bit Windows 7 PC with Intel Core i7-6700 3.40 GHz CPU and 16 GB RAM.

In Tables 1 and 2, we report average CPU times in seconds in column **CPU (s)**. Column **Gap (%)** shows the difference between the best upper bound and the final lower bound as a percentage of the best upper bound. In Tables 3 and 4, we present comparison of costs under different combinations of remanufacturing parameters.

4.1. Data and experiment settings

We generate instances by adapting the data set of Adulyasak et al. (2014a), adding the parameters related to remanufacturing and pickup requests of customers. The data set of Adulyasak et al. (2014a) was created from a subset of the Archetti et al. (2011) dataset.

Table 1
Performance of the algorithm on the first class of instances: base settings.

			Low Pickups		High Pickups	
n	$\|T\|$	$\|K\|$	CPU (s)	Gap (%)	CPU (s)	Gap (%)
10	3	2	21	0.0	17	0.0
15	3	2	129	0.0	41	0.0
20	3	2	580	0.0	208	0.0
25	3	2	600	1.8	387	0.0
30	3	4	600	3.2	500	0.0
35	3	4	600	4.8	600	3.6
40	3	4	600	5.0	600	4.9
45	3	4	600	5.0	600	4.8
50	3	4	600	4.9	600	3.8
10	6	2	490	0.0	119	0.0
15	6	2	600	1.0	600	0.8
20	6	2	600	1.9	600	1.6
25	6	2	600	2.8	600	2.1
30	6	4	600	3.9	600	3.8
35	6	4	600	4.7	600	4.2
40	6	4	600	5.9	600	4.7
45	6	4	600	7.0	600	6.5
50	6	4	600	7.2	600	6.9
100	20	25	600	19.0	600	18.3
50	6	4	28,800	3.2	28,800	2.0
100	20	25	28,800	9.2	28,800	8.8

Note. $\rho = 0.9, \gamma = 0.3, \alpha = 0.5$.

The Archetti et al. (2011) dataset for the PRP consists four classes of instances, which considers many different aspects, e.g., inventory costs at customers, initial inventory at customers, and varying transportation and production costs. Each instance type has five instances with different node coordinates. The first class contains standard instances. The second and the third classes are identical to the first but with higher unit production costs and higher transportation costs, respectively. The fourth class consists of instances from the first and second classes but with no customer inventory cost.

Based on Archetti et al. (2011) dataset, Adulyasak et al. (2014a) reduced initial inventory level for instances with 3 periods, and set up production capacity and depot inventory capacity accordingly. Based on Adulyasak et al. (2014a) dataset, we introduce varying locations of remanufacturing depot, different vehicle capacities, and low and high pickup requests of customers to obtain a robust data set. Instance size varies and results in 210 instances for Table 1 and 120 instances for Table 2, respectively. Initial pickup inventory level at remanufacturing depot is set to a proportion of the total residual delivery requests, i.e., $I_{00}^{p} = \alpha \sum_{i \in \mathscr{N}} \sum_{\tau=1}^{|\mathscr{T}|} \hat{\delta}_{i\tau}$. Finally, we assume $c^{r} = \gamma c^{m}$, where unit remanufacturing cost rate γ satisfies $0 \leqslant \gamma \leqslant 1$.

In addition to Archetti et al. (2011) dataset, the benchmark instances introduced by Boudia et al. (2007) for the PRP have constraints on production capacity, plant inventory capacity, and maximum number of vehicles. Besides these constraints, customer demand is dynamic and inventory holding costs at the customers are neglected. The demand in period 1 is satisfied by the initial inventory at the plant. These instances are considered large to very-large size ones with 50/100/200 customers and 20 time periods.

Table 2
Performance of the algorithm on instances with 10, 15, and 20 customers: four classes of instances

		Low Pickups		High Pickups	
Class	n	CPU (s)	Gap (%)	CPU (s)	Gap (%)
I	10	21	0.00	17	0.00
II	10	24	0.00	16	0.00
III	10	86	0.00	21	0.00
IV	10	12	0.00	14	0.00
I	15	109	0.00	41	0.00
II	15	118	0.00	109	0.00
III	15	600	0.07	172	0.00
IV	15	600	0.02	500	0.00
I	20	580	0.00	208	0.00
II	20	590	0.00	327	0.00
III	20	600	0.23	600	0.08
IV	20	600	0.06	600	0.04

Note. $\rho = 0.9, \gamma = 0.3, \alpha = 0.5, |\mathscr{T}| = 3, |\mathscr{K}| = 2$.
Class I: base settings; Class II: high production unit costs.
Class III: large transportation costs; Class IV: no retailer inventory costs.

Table 3
Comparison of costs under different combinations of remanufacturing parameters: moderate remanufacturing rate.

ρ	γ	geoCenter	TC	RIC	RC	MIC	RMIC	MPC	RPC
0.5	0.1	1	16,667	2947	5065	0	691.5	5625	1189.5
0.5	0.1	2	16676.5	2922	5101	0	690	5625	1189.5
0.5	0.1	3	16,598	3020	4956	0	679.5	5625	1189.5
0.5	0.3	1	19075.5	3084	5088	0	687	5625	3508.5
0.5	0.3	2	19051.5	3015	5070	0	697.5	5670	3495
0.5	0.3	3	18,951	2971	5018	0	679.5	5625	3508.5
0.5	0.5	1	21,824	3020	5571	0	685.5	5625	5827.5
0.5	0.5	2	21331.5	3168	4947	0	678	5625	5827.5
0.5	0.5	3	21,248	3020	4968	0	679.5	5625	5827.5
0.5	0.7	1	23947.5	2839	5465	0	756	5625	8146.5
0.5	0.7	2	23,937	3167	5131	0	673.5	5895	7957.5
0.5	0.7	3	24,254	2725	5803	0	793.5	5745	8062.5
0.5	0.9	1	26081.5	3472	4809	0	670.5	5910	10,209
0.5	0.9	2	26570.5	3416	5138	0	670.5	6630	9561
0.5	0.9	3	25901.5	3221	4778	0	672	5895	10222.5
0.7	0.1	1	14827.6	3115.6	4673	0	669.9	3852	1366.8
0.7	0.1	2	14811.5	3097.4	4677	0	677.4	3843	1367.7
0.7	0.1	3	14869.5	3097.4	4735	0	677.4	3843	1367.7
0.7	0.3	1	17591.2	3096.6	4782	0	669.9	3852	4040.4
0.7	0.3	2	17460.2	3115.6	4632	0	669.9	3852	4040.4
0.7	0.3	3	17506.1	3185.4	4649	0	672.6	3843	4043.1
0.7	0.5	1	20,361	3084	4830	0	673.5	3843	6718.5
0.7	0.5	2	20184.4	3123.2	4648	0	669.9	3894	6693
0.7	0.5	3	20777.5	3140	5258	0	684	3843	6718.5
0.7	0.7	1	22846.4	3115.6	4671	0	669.9	3852	9387.6
0.7	0.7	2	23217.6	3531.2	4591	0	673.5	3843	9393.9
0.7	0.7	3	22818.6	3123.2	4617	0	669.9	3894	9358.2
0.7	0.9	1	25654.2	3111	4769	0	669.9	4020	11,910
0.7	0.9	2	25377.8	3369.6	4241	0	669.9	4362	11602.2
0.7	0.9	3	25421.2	3366.4	4308	0	669.9	4278	11677.8

Note. $n = 20, |T| = 3, |K| = 2, \alpha = 0.5$.
TC: total cost; RC: routing cost.
RIC: retailer inventory cost; MIC: manufacturing inventory cost.
RMIC: remanufacturing inventory cost.
MPC: manufacturing production cost; RPC: remanufacturing production cost.

Table 4
Comparison of costs under different combinations of remanufacturing parameters: high remanufacturing rate.

ρ	γ	geoCenter	TC	RIC	RC	MIC	RMIC	MPC	RPC
0.9	0.1	1	10976.7	3424	4201	0	653.7	0	1452
0.9	0.1	2	11032.1	3276	4429	0	680.1	0	1452
0.9	0.1	3	10915.1	3270	4300	0	692.1	0	1452
0.9	0.3	1	13802.1	3270	4343	0	692.1	0	4296
0.9	0.3	2	13750.1	3270	4291	0	692.1	0	4296
0.9	0.3	3	13781.7	3424	4162	0	653.7	0	4296
0.9	0.5	1	16664.7	3424	4201	0	653.7	0	7140
0.9	0.5	2	16625.7	3424	4162	0	653.7	0	7140
0.9	0.5	3	16603.9	3254	4309	0	675.9	0	7140
0.9	0.7	1	19490.9	3254	4352	0	675.9	0	9984
0.9	0.7	2	19469.7	3424	4162	0	653.7	0	9984
0.9	0.7	3	19469.7	3424	4162	0	653.7	0	9984
0.9	0.9	1	22384.1	3270	4393	0	692.1	0	12,828
0.9	0.9	2	22282.1	3270	4291	0	692.1	0	12,828
0.9	0.9	3	22282.1	3270	4291	0	692.1	0	12,828

Note. $n = 20, |T| = 3, |K| = 2, \alpha = 0.5$.
TC: total cost; RC: routing cost.
RIC: retailer inventory cost; MIC: manufacturing inventory cost.
RMIC: remanufacturing inventory cost.
MPC: manufacturing production cost; RPC: remanufacturing production cost.

We only test the instance of 100 customers to show the ability of the proposed algorithm on this difficult benchmark. The details of the test sets used in our computational experiments are provided in the online supplement.

4.2. Results and discussion

In Table 1, we report the performance of the algorithm. The CPU time limit was set to 10 min, except for two medium and large-size instance types. Given the notation $ac/bp/cv$, where a,b, and c are the number of customers, periods, and vehicles, respectively, these tests showed that the proposed algorithm can obtain the optimal solutions for instances up to $20c/3p/2v$ and $10c/6p/2v$ for the base settings when pickup requests are relatively low. However, the proposed algorithm can obtain the optimal solutions for instances up to $30c/3p/4v$ for the base settings when pickup requests are relatively high.

For instances up to $50c/3p/4v$, the algorithm can obtain near optimal solutions within a gap of at most 5% in 10 min. For instances up to $50c/6p/4v$, the solutions obtained by the algorithm are also within gaps less than 8% in 10 min. For instances of $50c/6p/4v$ and $100c/20p/25v$, we also run the algorithm up to 8 h, and the algorithm can obtain near optimal solutions with a gap of 3.2% and 9.2% for the base settings when pickup requests are relatively low, respectively. On the other hand, the algorithm can obtain near optimal solutions with a gap of 2.0% and 8.8% for the base settings when pickup requests are relatively high, respectively.

Current exact algorithms for multi-vehicle PRP can only solve instances of up to 35c/3p/3v in 2 h (Adulyasak et al., 2014a). Since vehicle routing subproblems in the PRPRPD involve pickups, the routing subproblems are notably more difficult than that in the original PRP. The overall performance of our algorithm for the PRPRPD is thus quite acceptable.

Moreover, we compare the performance of the algorithm for different classes of instances in Table 2. It can be seen that class I are easiest to solve, whereas instances in class III and IV are more difficult to solve. This implies that the problem is easier to solve when production or transportation costs are lower. As shown from Table 1 and 2, instances are also easier to solve when pickup requests are relatively high.

Finally, in Table 3 and 4, we provide a comparison of costs under different combinations of remanufacturing parameters. The column "geoCenter" indicates the location of the remanufacturing depot. The remanufacturing depot is located in geographic center of all customers when the index is 1. The remanufacturing depot is located in gravity center of all customers with the delivery or pickup requests as weights when the index is 2 or 3, respectively. It is interesting to note that, the optimal decisions are not sensitive to the location of the remanufacturing depot whether it is geographically centered or centered with gravity. To help reading other key information, we illustrate selected costs and parameters in Fig. 1.

As shown in Fig. 1, given remanufacturing rate, when remanufacturing cost rate increases, both remanufacturing cost and production cost increase. However, given remanufacturing cost rate, when remanufacturing rate increases, remanufacturing cost increases, but production cost decreases because more and more manufacturing activities are replaced by remanufacturing ones. It is

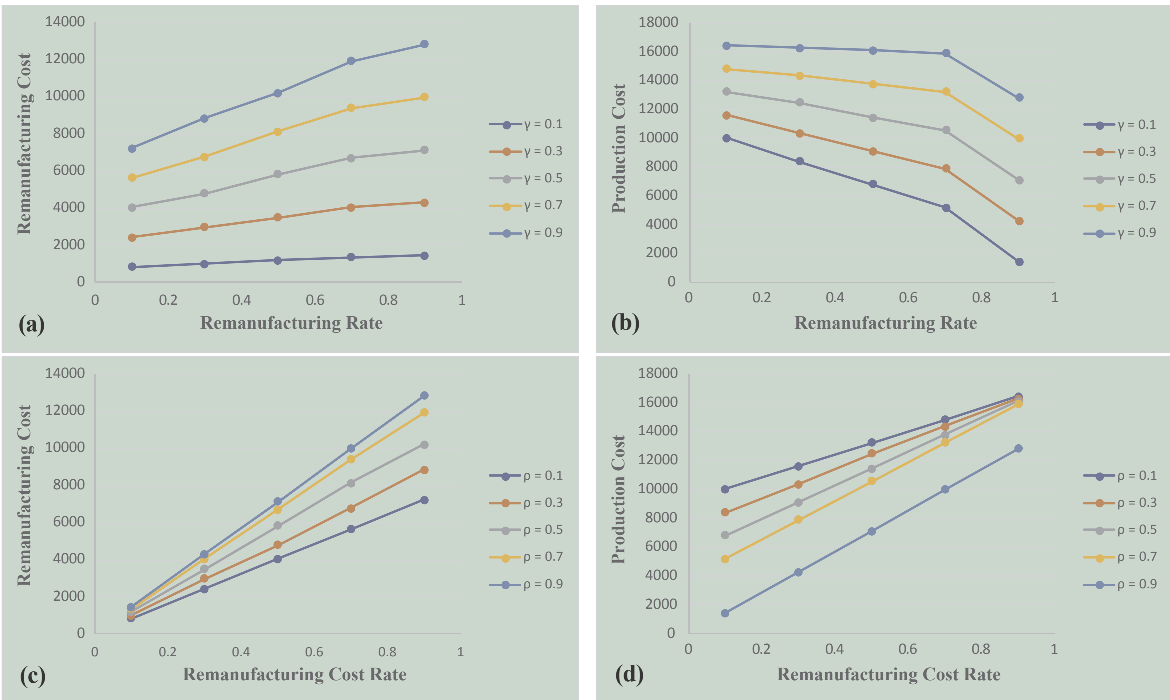

Fig. 1. Sensitivity Analysis.

also interesting to note that when remanufacturing rate is sufficiently high, e.g., $\rho = 0.9$, the manufacturing activities can be totally replaced by remanufacturing ones, resulting a large drop in total costs, given there are enough pickups inventory and new pickups to cover the delivery requests.

5. Conclusions

We have introduced, modeled and analyzed the PRPRPD, a generalization of the VRPSPD and IRPSPD. The contributions of this paper are (i) to describe a modeling approach enriching the production routing problems with remanufacturing, simultaneous pickups and deliveries; (ii) to offer a mixed integer linear programming formulation for the PRPRPD; (iii) to provide a branch-and-cut guided search algorithm; and (iv) to present extensive computational analyses that capture the effects of key remanufacturing parameters on the balance between manufacturing and remanufacturing, from which managerial insight can be drawn.

Several extensions are possible for the PRPRPD. One worth mentioning here is the possibility of incorporating carbon emissions in vehicle routing. Another extension would be to consider the simultaneous pickups and deliveries in multi-level production and routing problems with time windows. Finally, a branch-price-and-cut algorithm could be developed when dealing with a limited number of customers per vehicle.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.tre.2018.01.009.

References

Absi, N., Archetti, C., Dauzere-Peres, S., Feillet, D., 2015. A two-phase iterative heuristic approach for the production routing problem. Transport. Sci. 49 (4), 784–795.
Adulyasak, Y., Cordeau, J.-F., Jans, R., 2014a. Formulations and branch-and-cut algorithms for multivehicle production and inventory routing problems. INFORMS J. Comput. 26 (1), 103–120.
Adulyasak, Y., Cordeau, J.-F., Jans, R., 2014b. Optimization-based adaptive large neighborhood search for the production routing problem. Transport. Sci. 48 (1), 20–45.
Adulyasak, Y., Cordeau, J.-F., Jans, R., 2015a. The production routing problem: a review of formulations and solution algorithms. Comput. Oper. Res. 55, 141–152.
Adulyasak, Y., Cordeau, J.-F., Jans, R., 2015b. Benders decomposition for production routing under demand uncertainty. Oper. Res. 63 (4), 851–867.
Agrawal, V.V., Atasu, A., van Ittersum, K., 2015. Remanufacturing, third-party competition, and consumers' perceived value of new products. Manage. Sci. 61 (1), 60–72.
Archetti, C., Bertazzi, L., Paletta, G., Speranza, M.G., 2011. Analysis of the maximum level policy in a production-distribution system. Comput. Oper. Res. 38 (12), 1731–1746.
Armentano, V., Shiguemoto, A., Lokketangen, A., 2011. Tabu search with path relinking for an integrated production-distribution problem. Comput. Oper. Res. 38 (8), 1199–1209.
Atasu, A., Sarvary, M., Wassenhove, L.N.V., 2008. Remanufacturing as a marketing strategy. Manage. Sci. 54 (10), 1731–1746.
Bard, J.F., Nananukul, N., 2009a. Heuristics for a multiperiod inventory routing problem with production decisions. Comput. Ind. Eng. 57 (3), 713–723.
Bard, J.F., Nananukul, N., 2009b. The integrated production-inventory-distribution-routing problem. J. Scheduling 12 (3), 257–280.
Bard, J.F., Nananukul, N., 2010. A branch-and-price algorithm for an integrated production and inventory routing problem. Comput. Oper. Res. 37 (12), 2202–2217.
Battarra, M., Cordeau, J.-F., Iori, M., 2014. Pickup-and-delivery problems for goods transportation. In: Toth, P., Vigo, D. (Eds.), Vehicle Routing: Problems, Methods, and Applications, MOS/SIAM Series on Optimization, vol. 18, SIAM, pp. 161–192.
Boudia, M., Louly, M., Prins, C., 2007. A reactive GRASP and path relinking for a combined production-distribution problem. Comput. Oper. Res. 34 (11), 3402–3419.
Boudia, M., Prins, C., 2009. A memetic algorithm with dynamic population management for an integrated production-distribution problem. Eur. J. Oper. Res. 195 (3), 703–715.
Brahimi, N., Aouam, T., 2015. Multi-item production routing problem with backordering: a MILP approach. Int. J. Prod. Res. 54 (4), 1076–1093.
Chandra, P., 1993. A dynamic distribution model with warehouse and customer replenishment requirements. J. Oper. Res. Soc. 44 (7), 681–692.
Chandra, P., Fisher, M.L., 1994. Coordination of production and distribution planning. Eur. J. Oper. Res. 72 (3), 503–517.
Chen, J.-M., Chang, C.-I., 2012. The co-opetitive strategy of a closed-loop supply chain with remanufacturing. Transport. Res. Part E: Logist. Transp. 48 (2), 387–400.
Cherkesly, M., Desaulniers, G., Laporte, G., 2015. Branch-price-and-cut algorithms for the pickup and delivery problem with time windows and last-in-first-out loading. Transport. Sci. 49 (4), 752–766.
DeCroix, G.A., 2006. Optimal policy for a multiechelon inventory system with remanufacturing. Oper. Res. 54 (3), 532–543.
Fumero, F., Vercellis, C., 1999. Synchronized development of production, inventory, and distribution schedules. Transport. Sci. 33 (3), 330–340.
Geyer, R., Wassenhove, L.N.V., Atasu, A., 2007. The economics of remanufacturing under limited component durability and finite product life cycles. Manage. Sci. 53 (1), 88–100.
Groër, C., Golden, B., Wasil, E., 2010. A library of local search heuristics for the vehicle routing problem. Math. Program. Comput. 2 (2), 79–101.
Helsgaun, K., 2012. LKH. < http://www.akira.ruc.dk/~keld/research/LKH > .
Iassinovskaia, G., Limbourg, S., Riane, F., 2017. The inventory-routing problem of returnable transport items with time windows and simultaneous pickup and delivery in closed-loop supply chains. Int. J. Prod. Econ. 183, 570–582.

Kumar, R.S., Kondapaneni, K., Dixit, V., Goswami, A., Thakur, L., Tiwari, M., 2015. Multi-objective modeling of production and pollution routing problem with time window: a self-learning particle swarm optimization approach. Comput. Ind. Eng. 99, 29–40.
Kytöjoki, J., Nuortio, T., Bräysy, O., Gendreau, M., 2007. An efficient variable neighborhood search heuristic for very large scale vehicle routing problems. Comput. Oper. Res. 34 (9), 2743–2757.
Lei, L., Liu, S., Ruszczynski, A., Park, S., 2006. On the integrated production, inventory, and distribution routing problem. IIE Trans. 38 (11), 955–970.
Lin, S., Kernighan, B.W., 1973. An effective heuristic algorithm for the traveling-salesman problem. Oper. Res. 21, 498–516.
Lysgaard, J., Letchford, A.N., Eglese, R.W., 2004. A new branch-and-cut algorithm for the capacitated vehicle routing problem. Math. Program. 100 (2), 423–445.
Qiu, Y., Qiao, J., Pardalos, P., 2017. A branch-and-price algorithm for production routing problems with carbon cap-and-trade. Omega 68, 49–61.
Qu, Y., Bard, J.F., 2015. A branch-and-price-and-cut algorithm for heterogeneous pickup and delivery problems with configurable vehicle capacity. Transport. Sci. 49 (2), 254–270.
Russell, R.A., 2017. Mathematical programming heuristics for the production routing problem. Int. J. Prod. Econ. 193, 40–49.
Savaskan, R.C., Bhattacharya, S., Wassenhove, L.N.V., 2004. Closed-loop supply chain models with product remanufacturing. Manage. Sci. 50 (2), 239–252.
Solyalı, O., Süral, H., 2017. A multi-phase heuristic for the production routing problem. Comput. Oper. Res. 87, 114–124.
Soysal, M., 2016. Closed-loop inventory routing problem for returnable transport items. Transport. Res. Part D: Transp. Environ. 48, 31–45.
Subramanian, A., Uchoa, E., Pessoa, A.A., Ochi, L.S., 2013. Branch-cut-and-price for the vehicle routing problem with simultaneous pickup and delivery. Optim. Lett. 7 (7), 1569–1581.
Tao, Z., Zhou, S.X., 2014. Approximation balancing policies for inventory systems with remanufacturing. Math. Oper. Res. 39 (4), 1179–1197.
Vahdani, B., Niaki, S., Aslanzade, S., 2017. Production-inventory-routing coordination with capacity and time window constraints for perishable products: heuristic and meta-heuristic algorithms. J. Clean. Prod. 161, 598–618.
van Anholt, R.G., Coelho, L.C., Laporte, G., Vis, I.F.A., 2016. An inventory-routing problem with pickups and deliveries arising in the replenishment of automated teller machines. Transport. Sci. 50 (3), 1077–1091.
Zhang, Q., Sundaramoorthy, A., Grossmann, I.E., Pinto, J.M., 2017. Multiscale production routing in multicommodity supply chains with complex production facilities. Comput. Oper. Res. 79, 207–222.
Zhu, Q., Sarkis, J., Lai, K., 2014. Supply chain-based barriers for truck-engine remanufacturing in china. Transport. Res. Part E: Logist. Transp. 68, 103–117.
Zou, Z.-B., Wang, J.-J., Deng, G.-S., Chen, H., 2016. Third-party remanufacturing mode selection: outsourcing or authorization? Transport. Res. Part E: Logist. Transp. 87, 1–19.