



Porosity estimation of unsaturated soil using Brutsaert equation



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ARTICLE INFO

Keywords:

Brutsaert model
Compressional wave velocity
Dynamic cone penetration test
Energy dissipation
Porosity

ABSTRACT

Although the porosity is a crucial parameter for understanding the soil behavior under static and dynamic loading, estimating the porosity of unsaturated soil is difficult owing to the various input parameters required. The objective of this study is to suggest a technical method to obtain the level of porosity based on the elastic wave velocity in unsaturated soil. The Brutsaert model, which utilizes the theory of wave propagation, is applied to modify the proposed method in terms of the porosity of unsaturated soil. The soil compressional wave velocity is gathered through a seismic refraction survey, and the porosity distribution at different depths is estimated. A dynamic cone penetration test is applied to verify the converted porosity based on the compressional wave. The two porosities estimated through the wave propagation and penetration show similar trends. Furthermore, a special validation is performed to determine whether the energy dissipation can be ignored in the compressional wave propagations under this experiment condition. The results of this study indicate that the suggested technique is useful for obtaining the porosity in unsaturated soil.

1. Introduction

The elastic wave velocity has been used to investigate subsurface structures and obtain the design parameters when applying the theory of wave propagation in a medium. Among the various design parameters, porosity is essential for assessing the stability of soil under static and dynamic loadings, including compression, consolidation, earthquakes and liquefaction [39,9]. Porosity can be obtained through a laboratory testing of an extracted sample. However, obtaining reliable porosity data is difficult because the extraction procedure is limited to special soil types, and extracting and moving the soil causes disturbances that hinder the reliability [38]. To overcome the above mentioned disadvantages, the seismic wave velocity has been used to obtain the porosity as an in-situ method. Wood [36] expressed porosity in terms of compressional wave velocity using compressibility, a reciprocal of the elastic modulus, as an intermediary. Gassmann, whose work was translated by Berryman [8], proposed a relationship between porosity and elastic wave velocity for a low frequency range in an isotropic porous medium. Applying [10] theory of linear poroelasticity, Foti et al. [16] suggested a method for estimating the porosity under the assumptions that pore water is an undrained condition and the number of soil particles is infinite. Lee and Yoon [20] recently summarized techniques for evaluating the porosity using a variety of assumptions, and assessed the sensitivities of every parameter to compare the

resolutions of these techniques.

A study was also conducted to investigate the characteristics of porosity in unsaturated soils using elastic waves. Lu and Sabatier [25] monitored the elastic wave velocity according to changes in the water potential, moisture content, and soil temperature during a two-year period. Shin et al. [30] used the elastic wave velocity to estimate the effects of soil structure, moisture content, and strength on plant growth in unsaturated soil. Although these previous researches include no direct description of the change in porosity, the alternation of porosity can be indirectly estimated through the measured total potential, effective stress, and moisture content. Gao et al. [17] showed the possibility of estimating the porosity in unsaturated soil using matric suction deduced through the measured elastic wave velocity, and Whalley et al. [35] demonstrated the elastic wave velocity as a function of porosity using Bishop's equation. Changes in elastic wave velocity in shallow surface soil have recently been addressed to determine seasonal and weather effects [24], and the results of this study show that the elevation of the elastic wave velocity is caused by increased rigidity and decreased porosity of the surface soil. Previous studies have shown that the porosity has a significant influence on the elastic wave velocity under unsaturated soil conditions; however, there has been a lack of methodological content to directly obtain the porosity. Therefore, in the present study, we propose a practical method for estimating the porosity of unsaturated soil using the seismic velocity.

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An unsaturated medium consists of one soil particle and two immiscible fluids, including water and air, and thus the medium is a complicated multiphase system with three distinct phases [14,2,29]. Brutsaert [13] first suggested a comprehensive model of wave propagation in unsaturated soil, which can be applied under partial water saturation [23]. This model was extended by several different researchers [18,32]. The reason many researchers have selected the Brutsaert model is its treatment of wave propagation in unsaturated soil. The model has been applied to establish the relationship between the elastic wave velocity and water contents of soil [15,31,7]. However, the Brutsaert model has a porosity parameter and can be used to estimate the porosity using the elastic wave velocity for unsaturated conditions. Thus, the Brutsaert model was also selected for the present study.

This study suggests a method for evaluating the porosity based on the Brutsaert model. First, the Brutsaert model is introduced, discussed and then rearranged in terms of the porosity. The sensitivity of every parameter is determined to enhance the measurement reliability. Profiles of compressional waves at several depths are determined and the porosity distribution is plotted. The porosity estimated from the elastic wave velocity is compared to the porosity deduced using a the dynamic cone penetration test, and the reliability of this estimate is described. Finally, the low frequency range under the experiment conditions is verified to satisfactorily fulfill of the assumptions of the Brutsaert model.

2. Background theory

The theory of wave propagation in an unsaturated medium was first proposed by Brutsaert [13] for a three-phase porous medium including solid, liquid, and gas. A random aggregation of spherical soil grains is assumed to idealize the contact force between particles. The liquid phase is assumed to form a liquid-gas interface with the atmosphere and the Kirchhoff-Helmholtz theory is applied to define the dissipation coefficients during the wave propagation.

$$V_p = \left[\frac{0.306aP_{effective}^{\frac{1}{3}}Z}{\rho_{mass}nb^{\frac{2}{3}}} \right]^{\frac{1}{2}} \quad (1)$$

where V_p is the compressional wave velocity. The empirical parameters a and b depend on the granular properties of the material. In addition, $P_{effective}$, ρ_{mass} , and n are the effective pressure under the unsaturated conditions, the mass density of the material and the porosity, respectively. The interstitial effects between the total density and effective pressure, Z , are expressed mathematically in Eq. (2).

$$Z = \frac{\left\{ 1 + \frac{30.75[(1-S)B_{gas} + SB_{liquid}]^{\frac{3}{2}}b}{P_{effective}^{\frac{1}{2}}} \right\}^{\frac{5}{3}}}{\left\{ 1 + \frac{46.12[(1-S)B_{gas} + SB_{liquid}]^{\frac{3}{2}}b}{P_{effective}^{\frac{1}{2}}} \right\}} \quad (2)$$

where B_{gas} and B_{liquid} are the bulk moduli of the gas and liquid, respectively and parameter S indicates the saturation of the medium. The remaining parameters are the same as those used in Eq. (1). According to a study by Shin et al. [31], the value of Z is almost unity under unsaturated conditions, and Z is known to have less of an effect in Eq. (1) at a shallow depth [11,15]. Furthermore, Adamo et al. [1] also suggested that Z is equal to 1 because this value is insensitive to changes in saturation, porosity, and depth of wave propagation for a range of values. Thus, Eq. (1) can be rearranged into Eq. (3) with an added amplification factor (Φ).

$$V_p = \Phi \left[\frac{0.306P_{effective}^{\frac{1}{3}}}{\rho_{mass}n} \right]^{\frac{1}{2}} \quad (3)$$

where the amplification factor (Φ) consists of empirical constants a and b ($\Phi = a^{1/2}/b^{1/3}$), and Z is assumed to be 1. Eq. (3) shows that the porosity of unsaturated soil can be obtained using the compressional wave velocity, as shown in Eq. (4).

$$n = \frac{0.306\Phi^2P_{effective}^{0.33}}{V_p^2\rho_{mass}} \quad (4)$$

2.1. Sensitivity of each parameter in the Brutsaert model

The sensitivity of the Brutsaert model to changes in each parameter is estimated using the error norm technique, which has been widely applied to verify the model [19,20,37]. The sensitivity can be estimated using the ratios of the calculated porosity based on the true values to the predicted porosity based on various ranges of input values. The true values of the compressional wave velocity, mass density of the material, effective pressure and amplification factor are 137 m/s, 1530 kg/m³, 28,000 Pa, and 614, respectively, according to results from Adamo et al. [1]. The minimum and maximum values of each parameter are determined by decreasing and increasing the true value of 100%, respectively. Finally, the sensitivity is calculated using Eq. (5).

$$Sensitivity = \frac{Porosity_{true\ value} - Porosity_{various\ ranges\ input\ values}}{Porosity_{true\ value}} \quad (5)$$

where $porosity_{true\ value}$ and $porosity_{various\ ranges\ input\ values}$ denote the porosity based on the true value and various ranges of input values, respectively.

The results are shown in Fig. 1, where the x-axis represents the ratio of the true value to the changed values. As a result, a value of 1 means that the true value is the same as the changed value, and values of zero and 2 indicate that the changed values are decreased and increased 100% compared to the true value, respectively. The compressional wave velocity and mass density were shown to be more sensitive to the decreased input values and less sensitive to the increased input values.

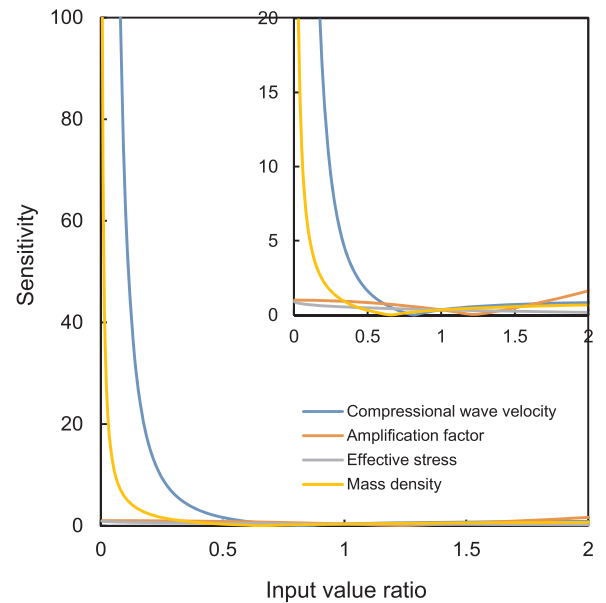


Fig. 1. Sensitivity of the Brutsaert model. The ratio of input values is the ratio between the true value and changed values, which were decreased and increased in comparison to the true value. The order of the relative effects of the parameters in the Brutsaert model: compressional wave velocity \approx mass density $>$ effective stress \approx amplification factor.

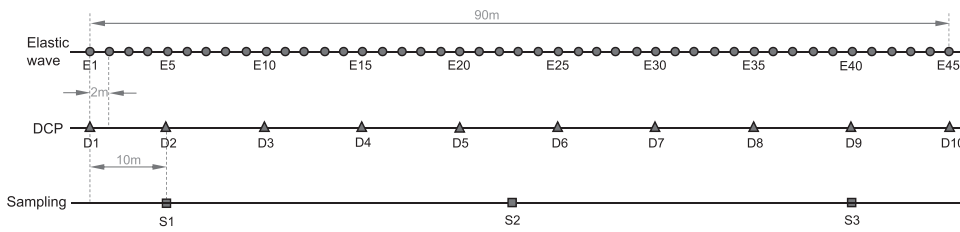


Fig. 2. Profiles of field tests (elastic wave velocity and DCP) and soil sampling.

On the other hand, the sensitivities of the amplification factor and effective stress were relatively low irrespective of the true value. Eq. (4) has the greatest effect on the compressional wave velocity and mass density, and thus more accurate input values are needed to obtain the porosity under unsaturated conditions using a high-resolution elastic wave.

3. Field test

3.1. Site description

The landslide selected as the testing site provided unsaturated conditions for this study. The site is on Geohwa Mountain in South Korea, a location where a debris-flow occurred approximately 5 years ago and that consists of many different streams. Among these streams, one main stream, which is 90 m in length, was selected for the field test because of the deposited volume of the materials.

The materials in the landslide were extracted using a sampler. The positions of the samples, labeled S1, S2, and S3, were separated at horizontal intervals of approximately 25 m, shown in Fig. 2. The samples obtained were used to estimate the material properties through experiments including sieve test [3], minimum and maximum void ratio [4], and specific gravity [6]. Fig. 3 shows the results of the sieve test, where the average coefficients of the curvature (Cc) and uniformity (Cu) are approximately 1.5 and 13, respectively. The extracted materials are classified as well graded sands (SW) based on the unified soil classification system (USCS) using the estimated coefficients. The ranges of the maximum and minimum void ratios are 1.04 and 0.62, respectively, and the average specific gravity is 2.63. Detailed material properties are provided in Table 1.

3.2. Elastic wave survey

The elastic wave survey captures the wave profiles propagated into the medium through an induced artificial impact on the surface. The wave profiles obtained can be converted into the elastic wave velocity, which is used to characterize the medium because the elastic wave is propagated into individual grains of the medium. Therefore, in the

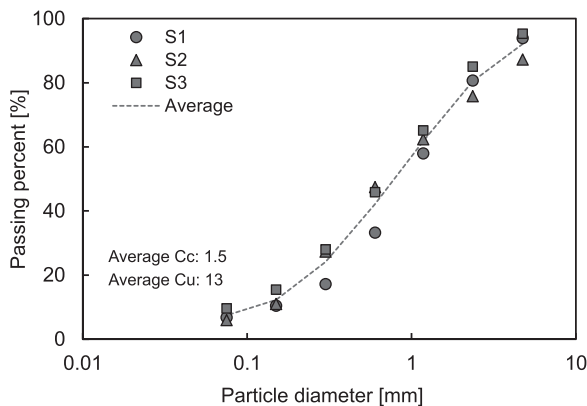


Fig. 3. Sieve test results. Note that Cc and Cu are the coefficients of curvature and uniformity, respectively.

Table 1
Properties of extracted materials.

	D ₁₀ (mm)	D ₃₀ (mm)	D ₆₀ (mm)	C _c	C _u	USCS	e _{max}	e _{min}	G _s
S1	0.08	0.5	1.2	2.6	15	SW	1.07	0.60	2.62
S2	0.08	0.3	1.1	1.0	13	SW	1.00	0.68	2.64
S3	0.08	0.3	1.0	1.1	12	SW	1.05	0.54	2.63
Average	0.08	0.36	1.1	1.5	13	-	1.04	0.62	2.63

*D10, D30, and D60 denote the diameters at passing percentages of 10%, 30%, and 60%, respectively. Cc and Cu are coefficients of curvature and uniformity. e and G_s are the void ratio and specific gravity.

study, the elastic refraction method was applied to obtain the compressional wave velocity.

The total length used to conduct the elastic wave survey was 90 m, and geophones were installed at 2 m intervals to increase the resolution at shallow depths. Fig. 2 outlines the elastic wave survey. A sledgehammer was selected to generate artificial vibrations, and subsequent impactions were conducted in three different locations near E1, E20, and E45 to obtain high quality signals, as shown in Fig. 2.

3.3. Dynamic cone penetration test

A dynamic cone penetration (DCP) test measures the penetrated depth using a penetrometer, and this study utilized a dropped hammer (78.8 N) with a constant height of 575 mm [5]. The penetrated depth per blow is called the dynamic cone penetration index (DPI), which is useful for understanding the strength of a material. Material properties including the porosity, elastic modulus, dry unit weight and friction angle were characterized using a DCP test [12,26,27]. Thus, in the study, the DCP test was selected for determining the reference value for a comparison with the porosity estimated using the elastic wave velocity.

The DCP tests were carried out in the same stream where the elastic wave survey was conducted. As shown in Fig. 2, the DPI was gathered at 10 m intervals, and thus the experiments were carried out ten times.

4. Results

4.1. Elastic wave velocity

The waveforms were gathered using the seismic refraction method, and the travel time-distance curve was determined using the SeisImager-Poltrefa program (OYO Co.). Fig. 4 shows the distribution of the analyzed elastic wave velocity as a function of elevation based on the geographic information system. The test site consists of four layers based on the ranges of the compressional wave velocity: 0–0.7 km/s (colluvial soil), 0.8–1.2 km/s (weathering soil), 1.3–1.9 km/s (weathering rock) and over 2.0 km/s (soft rock). The thicknesses of the soil and weathering rock are approximately 1 and 3 m, respectively and exhibit similar trends, whereas the soft rock has much greater thicknesses at distances of around approximately 40 and 60 m.

4.2. Dynamic cone penetration index (DPI)

The measured penetration depth per blow was converted into the

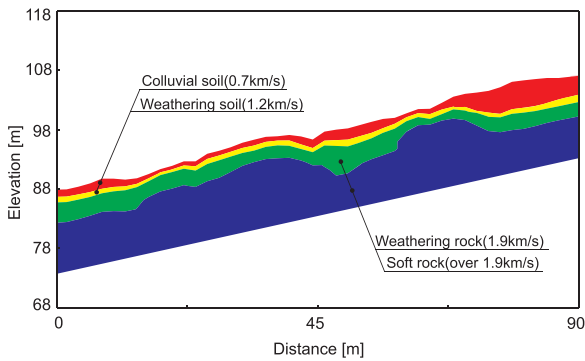


Fig. 4. Measured elastic wave velocity profiles.

dynamic cone penetration index (DPI), and the values obtained at ten sites are plotted in Fig. 5. The average initial DPI is approximately 227 mm/blow, and positions D1, D7 and D9 are where the initial DPI is below the average value, indicating that the surface may have loosened. A high stiffness of the surface soil may be predicted at positions D4 and D10 owing to the high initial DPI (approximately greater than 300 mm/blow). The final penetrated depths of the DCP at positions D1, D2, D3, D4, and D10 are almost 1 m, whereas positions D5, D6, D7, and D8 demonstrate relatively small penetrated depths (approximately less than 1 m) owing to deposits of stiff soil. However, position D10 shows a high penetration depth (approximately 2 m) where loose soil was deposited.

4.3. Porosity

The elastic wave velocity was chosen at positions E1, E5, E10, E15, E20, E25, E30, E35, E40, and E45, which are the same positions used for the DCP tests. The velocity as a function of depth is plotted in Fig. 6, which shows that the elastic wave velocity increases as the depth increases owing to the growth of the effective stress and the contact force

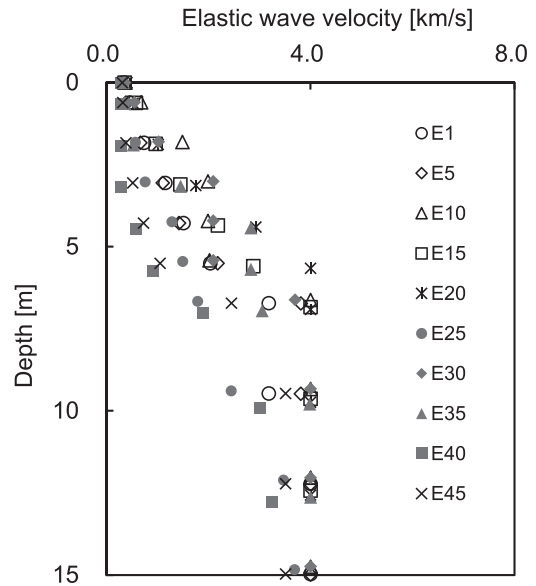


Fig. 6. Distribution of measured elastic wave velocity for different depths.

between particles. The velocity finally converges to 4 km/s, indicating bedrock. The measured compressional wave velocity is converted into the porosity for these depths using Eq. (4). The compressional wave velocity is shown in Fig. 6 (measured values). The values of the mass density, effective pressure, and amplification factor in Eq. (4) are 1530 kg/m³ (measured value), 28,000 Pa [1] and 614 [1], respectively. Fig. 7 shows the porosity distribution, based on the compressional wave velocity, has a significant range of 0.85 to 0.004. Fig. 7 shows almost the opposite trend as in Fig. 6 because the compressional wave velocity is highly affected by Eq. (4), as shown in Fig. 1. Note that Fig. 7 shows that it is possible to obtain the porosity in unsaturated soil using the compressional wave velocity.

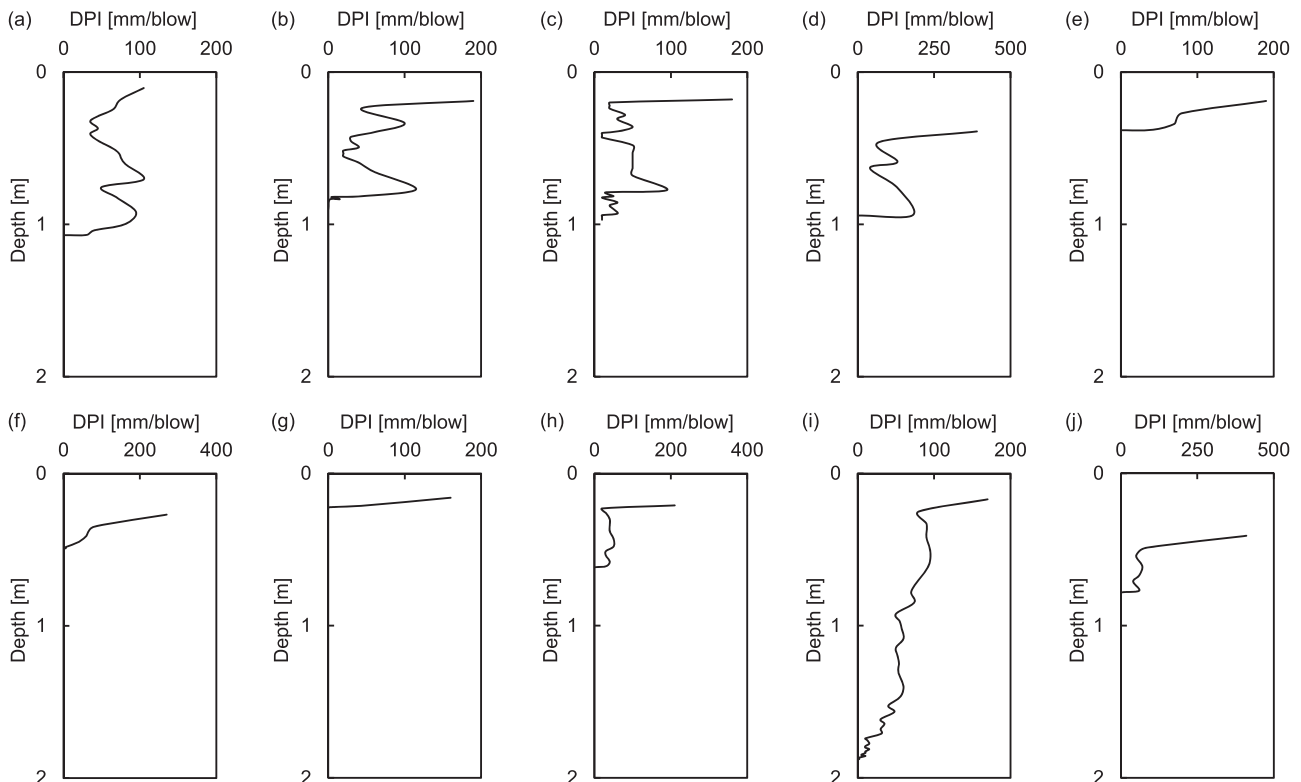


Fig. 5. Measured DPI by a dynamic cone penetrometer at positions of: (a) D1; (b) D2; (c) D3; (d) D4; (e) D5; (f) D6; (g) D7; (h) D8; (i) D9; (j) D10. The locations are shown in Fig. 2.

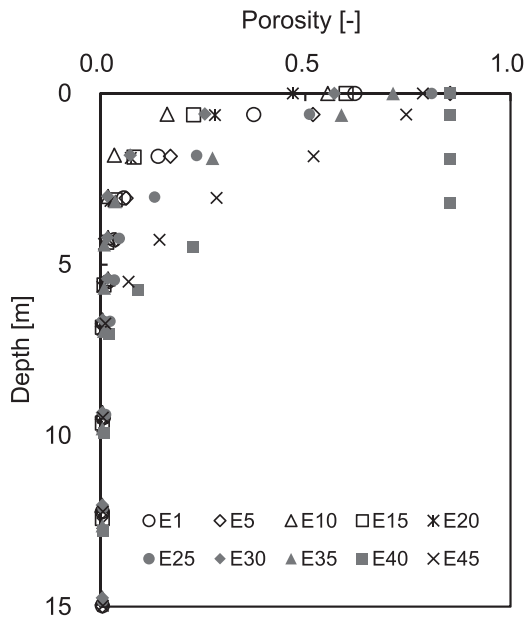


Fig. 7. Converted porosity profiles for different depths.

Lee et al. [21] suggested an equation to obtain the porosity using the DPI, and thus the deduced porosity based on velocity is compared with the porosity estimated through the DPI for verification. In the case of the porosity determined from Eq. (4), the mass density is a secondary factor affecting the porosity. However, the mass density at the soil surface is used to determine the porosity because the extraction of deeper samples is limited. Thus, 20 porosities determined from the surface soil were selected for comparison with the DPI. The results are compared in Fig. 8, which shows that the two different porosities have a linear relationship with a coefficient of determination of 0.4, as indicated in Eq. (6) below.

$$n_{DPI} = 0.302n_{velocity} + 0.3 \quad (R^2 = 0.4) \quad (6)$$

where n_{DPI} and $n_{velocity}$ denote the porosities based on the DPI and

elastic wave velocity, respectively. Fig. 8 shows that there are two different dispersed relationships which ranges A and B based on a porosity of under and over 0.6, respectively. The coefficients of determination are 0.7 and 0.2 for ranges A and B, respectively, and thus, the relationship of range A is higher than that of range B. The results shows that Eq. (4) is appropriate for medium to dense soil under a porosity of approximately less than 0.6.

5. Discussion

The results indicate that the Brutsaert model is suitable for estimating the porosity in unsaturated soil, and thus, a detailed verification of the measured compressional wave velocity is performed: First, the reliability of the measured compressional wave velocity is reviewed through a comparison between the elastic moduli estimated using the compressional wave velocity and the DPI. Second, the validation of Brutsaert model is examined under these experiment conditions with consideration of the energy dissipation.

5.1. Elastic modulus

The relationship between the seismic wave velocity and the DPI is estimated to verify the measured values using the exponential function between the Young's modulus (E) and DPI, as suggested by Mohammadi et al. [26].

$$E = 55.033 \cdot DPI^{-0.5459} \quad (7)$$

A seismic wave propagates in an elastic medium without causing any disturbances or altering the on-going processes [28]; thus, the seismic wave velocity is related to the elastic modulus including the Young's modulus (E), constraint modulus (M), shear modulus (G) and bulk modulus (B) [33]. The compressional wave velocity (V_p), which is obtained through a seismic survey, can be converted into a constraint modulus (M) with the mass density (ρ) because particle motion is parallel to the direction of the wave propagation.

$$M = V_p^2 \cdot \rho \quad (8)$$

The constraint modulus consists of shear and bulk moduli as

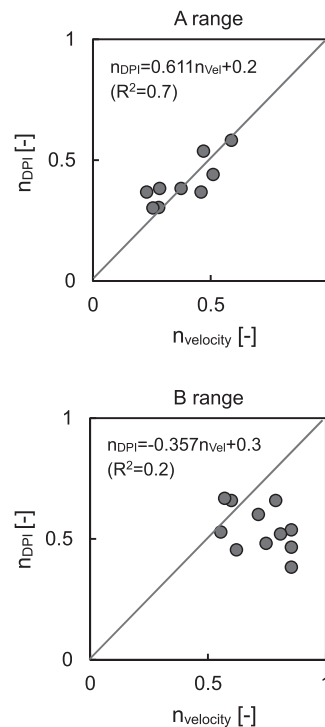
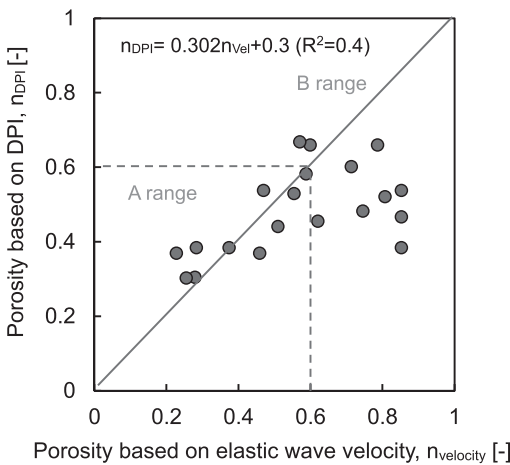


Fig. 8. Comparison of porosities based on elastic wave velocity and DPI.

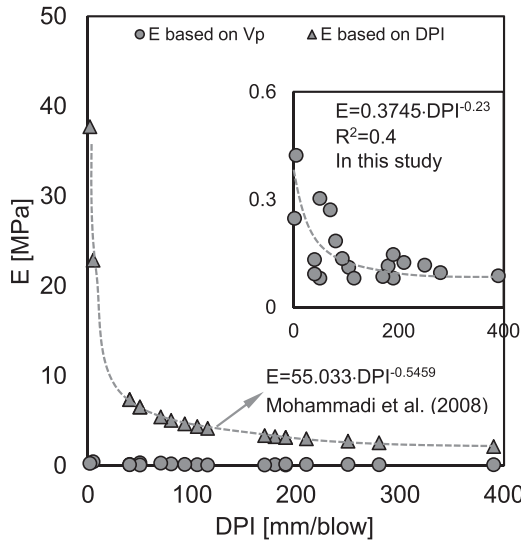


Fig. 9. Relationship between Young's modulus (E) and DPI. The subplot shows a change of E in the range of 0 MPa–0.6 MPa.

follows.

$$V_p^2 \cdot \rho = B + \frac{4}{3} \cdot G \tag{9}$$

Eq. (9) can be rearranged in terms of the Young's modulus, as shown in Eq. (11) using Eq. (10).

$$B = M - \frac{4}{3} \cdot G \tag{10}$$

$$E = \frac{9BG}{3B + G} = \frac{9MG - 12G^2}{3M - 4G + G} = \frac{\rho(9V_p^2 V_s^2 - 12V_s^4)}{\rho(3V_p^2 - 4V_s^2 + V_s^2)} \tag{11}$$

where V_s is the shear wave velocity.

Finally, the compressional wave velocity is deduced in terms of the Young's modulus because the compressional wave velocity is generally 0.6 times faster than the shear wave velocity [22].

$$E \approx 0.9V_p^2 \tag{12}$$

The relationship between the Young's modulus and the DPI, as measured in this study, is plotted in Fig. 9. Although the converted Young's modulus shows a smaller range than in previous studies [26] owing to the different characteristics at this site, the relationship can still be described using an exponential function as in Eq. (13).

$$E = 0.3736 \cdot \text{DPI}^{-0.284} \tag{13}$$

This relationship demonstrates that the measured compressional wave velocity and the DPI have a reliable relationship for this particular site. Furthermore, additional support of the appropriateness of this relationship is suggested through the low range of values (0–0.6 MPa) of the Young's modulus based on the low compressional wave velocity.

5.2. Validation of the Brutsaert model

Wave propagation leads to relative motion between phases, and thus the Brutsaert [13] model is recommended at sufficiently low frequency ranges when neglecting the dissipation of energy. The inertia-viscosity factor (β) is used to estimate whether dissipation has occurred [1]. If β is less than 1, the condition of low frequency is satisfied.

$$\beta = \sqrt{\frac{5.13f_s k_i}{nS}} < 1 \tag{14}$$

where f_s , k_i , n and S are the frequency of the output signal, the hydraulic conductivity under unsaturated conditions, the porosity, and the saturation, respectively. The value of k_i can be obtained using the

hydraulic conductivity under saturated conditions (k_{is}) based on Van Genuchten's model [34].

$$k_i = k_{is} S^{\frac{1}{2}} [1 - (1 - S^{\frac{1}{m}})^m]^2 \tag{15}$$

where S is the saturation and m is a dimensionless parameter. The value of m is deduced from the Van Genuchten curve shape (VG_S).

$$m = 1 - \frac{1}{VG_S} \tag{16}$$

The range of output frequencies of the measured signals is approximately 30–50 Hz, and the input frequency in Eq. (14) has an average value of 40 Hz. The hydraulic conductivity under standard conditions is estimated as 0.01 mm/s based on the Hazen equation, and the average saturation is calculated as 10% using measured values, which range from 3% to 27%, based on the time domain reflectometry. The value of VG_S was assumed to be 0.6 in the study by Van Genuchten [34] and the dimensionless parameter (m) was calculated to be 0.62. Finally, the viscosity factor deduced using the input values is 0.02, which is less than the reference value of unity, and thus the experimental conditions satisfy the assumption of a low frequency range without a dissipation of energy. Note that the measured value in this study is fully satisfied when applying the Brutsaert model.

5.3. Advantage and limitation of suggested methodology

The compressional wave velocity shows highly affecting factors in Brutsaert model and the Fig. 1 show the sensitivity increases in an inverse relationship with the growth of compressional wave velocity. The compressional wave velocity depends on effective stress of materials and thus, the velocity increase when the soil particles are highly combined. Note that the compressional wave velocities are approximately 60–230 m/s for soils and 2200–7000 m/s for rocks [28]. Given that the reference compressional wave velocity is 137 m/s in this study, the sensitivity is small in soil and rocks because the velocities of the materials are generally larger than the reference velocity, however when the velocity is smaller than reference velocity, the correct value should be considered. The small compressional wave velocity means that the effective stress of the soil is weak, which is called as loose soils. And thus, the limitation is the same as the results of the porosity comparison based on DPI and elastic wave velocity (Fig. 8), showing that the suggested model is more reliable in medium and dense soil.

6. Conclusion

In this study, the porosity in unsaturated soils was obtained using the compressional wave velocity, and the converted porosity was compared to the DPI using a dynamic cone penetrometer test to verify reliability. The detailed results of this study are as follows.

- The Brutsaert model was applied to deduce the porosity, and the sensitivity of every parameter in this model was estimated. The compressional wave velocity is a highly sensitive parameter in this model, indicating that the selected model is suitable for evaluating the porosity based on the elastic wave velocity.
- Detailed porosity profiles as a function of depth were determined, and porosities based on the compressional wave velocity and the DPI were compared. Two different ranges of the dispersed relationship were shown.
- To verify the measured compressional wave velocity, a relationship between the compressional wave velocity and the Young's modulus was proposed. The energy dissipation was also identified to confirm the low frequency range of the measured seismic wave.

Acknowledgement

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIP) (NRF-2017R1A2B4008157).

References

- [1] Adamo F, Andria G, Attivissimo F, Giaquinto N. An acoustic method for soil moisture measurement. *IEEE Trans Instrum Meas* 2004;53(4):891–8.
- [2] Arora A, Tomar SK. The effect of inertial coupling on seismic reflection amplitudes. *Geophys Prospect* 2008;56(5):643–54.
- [3] ASTM. C136. Standard test method for sieve analysis of fine and coarse aggregates; 1984.
- [4] ASTM. D4254. Standard test methods for minimum index density and unit weight of soils and calculation of relative density; 2016.
- [5] ASTM. D6951. Standard test method for use of the dynamic cone penetrometer in shallow pavement applications; 2015.
- [6] ASTM. D854. Standard test methods for specific gravity of soil solids by water pycnometer; 2014.
- [7] Attivissimo F, Cannazza G, Cataldo A, De Benedetto E, Fabbiano L. Enhancement and metrological characterization of an accurate and low-cost method based on seismic wave propagation for soil moisture evaluation. *IEEE Trans Instrum Meas* 2010;59(5):1216–23.
- [8] Berryman JG. Origin of Gassmann's equations. *Geophysics* 1999;64(5):1627–9.
- [9] Bian H, Shahrour I. Numerical model for unsaturated sandy soils under cyclic loading: application to liquefaction. *Soil Dyn Earthq Eng* 2009;29(2):237–44.
- [10] Biot MA. Theory of propagation of elastic waves in a fluid-saturated porous solid. I. Low-frequency range. *J Acoust Soc Am* 1956;28(2):168–78.
- [11] Blum A, Flammer I, Friedli T, Germann P. Acoustic tomography applied to water flow in unsaturated soils. *Vadose Zone J* 2004;3(1):288–99.
- [12] Brough M, Stirling A, Ghataora G, Madelin K. Evaluation of railway trackbed and formation: a case study. *NDT E Int* 2003;36(3):145–56.
- [13] Brutsaert W. The propagation of elastic waves in unconsolidated unsaturated granular mediums. *J Geophys Res* 1964;69(2):243–57.
- [14] Conte E, Cosentini RM, Troncone A. Shear and dilatational wave velocities for unsaturated soils. *Soil Dyn Earthq Eng* 2009;29(6):946–52.
- [15] Flammer I, Blum A, Leiser A, Germann P. Acoustic assessment of flow patterns in unsaturated soil. *J Appl Geophys* 2001;46(2):115–28.
- [16] Foti S, Lai CG, Lancellotta R. Porosity of fluid-saturated porous media from measured seismic wave velocities. *Géotechnique* 2002;52:359–73.
- [17] Gao W, Watts CW, Ren T, Shin HC, Taherzadeh S, Attenborough K, Jenkins M, Whalley WR. Estimating penetrometer resistance and matric potential from the velocities of shear and compression waves. *Soil Sci Soc Am J* 2013;77(3):721–8.
- [18] Garg SK, Nayfeh AH. Compressional wave propagation in liquid and/or gas saturated elastic porous media. *J Appl Phys* 1986;60(9):3045–55.
- [19] Lee JS, Fernandez AL, Santamarina JC. S-wave velocity tomography: Small-scale laboratory application; 2005.
- [20] Lee JS, Yoon HK. Porosity estimation based on seismic wave velocity at shallow depths. *J Appl Geophys* 2014;105:185–90.
- [21] Lee C, Kim KS, Woo W, Lee W. Soil Stiffness Gauge (SSG) and Dynamic Cone Penetrometer (DCP) tests for estimating engineering properties of weathered sandy soils in Korea. *Eng Geol* 2014;169:91–9.
- [22] Lillie RJ. Whole earth geophysics: an introductory textbook for geologists and geophysicists (No. 550.8 LIL); 1999.
- [23] Lo WC, Sposito G. Acoustic waves in unsaturated soils. *Water Resour Res* 2013;49(9):5674–84.
- [24] Lu Z. Feasibility of using a seismic surface wave method to study seasonal and weather effects on shallow surface soils. *J Environ Eng Geophys* 2014;19(2):71–85.
- [25] Lu Z, Sabatier JM. Effects of soil water potential and moisture content on sound speed. *Soil Sci Soc Am J* 2009;73(5):1614–25.
- [26] Mohammadi SD, Nikoudel MR, Rahimi H, Khamsehchiyan M. Application of the Dynamic Cone Penetrometer (DCP) for determination of the engineering parameters of sandy soils. *Eng Geol* 2008;101(3):195–203.
- [27] Salgado R, Yoon S. Dynamic cone penetration test (DCPT) for subgrade assessment. *Jt Transp Res Program* 2003:73.
- [28] Santamarina JC, Klein A, Fam MA. Soils and waves: particulate materials behavior, characterization and process monitoring. *J Soils Sediment* 2001;1(2). [130–130].
- [29] Sawada S, Tsukamoto Y, Ishihara K. Residual deformation characteristics of partially saturated sandy soils subjected to seismic excitation. *Soil Dyn Earthq Eng* 2006;26(2):175–82.
- [30] Shin HC, Taherzadeh S, Attenborough K, Whalley WR, Watts CW. Non-invasive characterization of pore-related and elastic properties of soils in linear Biot–Stoll theory using acoustic-to-seismic coupling. *Eur J Soil Sci* 2013;64(3):308–23.
- [31] Shin HC, Whalley WR, Attenborough K, Taherzadeh S. On the theory of Brutsaert about elastic wave speeds in unsaturated soils. *Soil Tillage Res* 2016;156:155–65.
- [32] Tuncay K, Yavuz Corapcioglu M. Body waves in poroelastic media saturated by two immiscible fluids. *J Geophys Res: Solid Earth* 1996;101(B11):25149–59.
- [33] Uhlemann S, Hagedorn S, Dashwood B, Maurer H, Gunn D, Dijkstra T, Chambers J. Landslide characterization using P- and S-wave seismic refraction tomography—the importance of elastic moduli. *J Appl Geophys* 2016;134:64–76.
- [34] Van Genuchten MT. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil Sci Soc Am J* 1980;44(5):892–8.
- [35] Whalley WR, Jenkins M, Attenborough K. The velocity of shear waves in unsaturated soil. *Soil Tillage Res* 2012;125:30–7.
- [36] Wood AB. A textbook of sound. London: G. Bell and Sons Ltd.; 1949.
- [37] Yoon HK, Lee JS. Field velocity resistivity probe for estimating stiffness and void ratio. *Soil Dyn Earthq Eng* 2010;30(12):1540–9.
- [38] Yoon HK, Jung SH, Lee JS. Characterisation of subsurface spatial variability using a cone resistivity penetrometer. *Soil Dyn Earthq Eng* 2011;31(7):1064–71.
- [39] Zhang M, Wang X, Yang G, Xie L. Solution of dynamic Green's function for unsaturated soil under internal excitation. *Soil Dyn Earthq Eng* 2014;64:63–84.