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A study of DEA models without explicit inputs ☆

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ABSTRACT

In performance evaluations, data without explicit inputs (such as index data, pure output data) are widely used. To directly use such data, this paper presents a study on building DEA models without explicit inputs, so-called DEA-WEI models, which are applicable to the evaluation applications where inputs are not directly considered. We provide an axiom foundation of these kinds of models, and further discuss how to incorporate value judgments of decision makers into these DEA-WEI models. Several such models are derived. Finally, applications of the DEA-WEI models are presented.

1. Introduction

In performance evaluations, index indicators are widely used in assessment of business (e.g. [1]), human development (e.g. [2,3]), health service (e.g. [4,5]), competitiveness or wealth of countries (e.g. [6], World competitiveness yearbook 2006 by International Institute for Management Development, IMD) and others. Let x_i and y_r be the input and output of a decision making unit (DMU), then the index data have the form $e_{ir}=y_r/x_i$, such as GDP per capita (if manpower is considered to be an input), and citations per article or profit margin, etc. Furthermore, in the evaluation of efficacy or effectiveness such as countries' power and students' performance, only outputs are explicitly used. Thus it is sometimes difficult if not impossible to recover the explicit input–output relationship among the data as required in the evaluation applications of the standard DEA models.

In practical applications, some aggregation techniques are often employed in order to produce a single score of performance from index data. The most widely used technique is to calculate the weighted sum of indexes: $(\sum w_{ir}e_{ir})$ to arrive at an aggregate measure of performance of a DMU (referred to as Ratio Approach or Comprehensive Analysis). However, how to properly select the weights (w_{ir}) is a main source of difficulty in the application of this technique. Popular methods to determine the weights include peer review through Delphi or analytic hierarchy process (AHP), statistics methods such as regression analysis and principal component analysis (PCA), and entropy method, see [7–12]. The same weights are used for all the DMUs in the above methods. However this is often the source of controversies for the final evaluation results.

Data envelopment analysis (DEA) is a non-parametric method to identify the best-practice frontier rather than the central-tendency, and then only the DMUs on the frontier are classified efficient. In this method, DMUs can freely select their weights to maximize their performance scores. Since the first DEA paper was published in EJOR in 1978 [13], it has become an attractive tool of performance evaluation in both non-profit and for-profit sectors. The standard DEA models have been formulated via input and output data of DMUs. However, as mentioned above, data sets are sometimes given without inputs, or the original input–output data cannot be easily recovered. For example, in an evaluation of research institutes in Chinese Academy of Sciences (CAS), the index data used are publications per staff, research funding per staff, citations per publication and others. It is clearly difficult to recover the original inputs and outputs directly from these indexes as the publications are used both as numerator and denominator here, although in this particular case the original inputs and outputs are in fact available from the CAS database. In some cases, CAS just used the outputs to evaluate the research institutes without explicating considering the inputs at all. Furthermore, in practical applications, often only a part of the indexes is available or meaningful. In DEA literature DEA models for the index data, such as [5,14,3,15].

The aim of this paper is to present more systematic theoretical background for these models in the previous studies.

The paper is organized as follows: Section 2 presents a mathematical derivation of some basic DEA models without explicit inputs. Section 3 discusses value judgement in these DEA models. Section 4 presents an empirical study of these DEA models, and the conclusion is given in Section 5.

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2. Basic DEA models without explicit inputs

Motivated by the above different applications, let us examine the DEA models without explicit inputs (hereinafter called DEA-WEI models) by taking an axiom approach in order to cover as wide applications as possible (see [13,16,17] for the details of axiom approach for the classic DEA models). It is clear such DEA models can be used to assess efficiency, as well as efficacy, where inputs are not taken into account as seen in assessing examination performances of students, or overall economic power of countries. To this end, we first define:

Definition 1. (**Attainable set**): A attainable set AS is a nonempty subset in R_+^n. It is said to be **Free-disposal**, if an element $Y \in AS$, $Z \le Y$, then $Z \in AS$. It is said to be **Convex,** if Z, $Y \in AS$, then $\lambda Z+(1-\lambda)Y \in AS$, for any $0 \le \lambda \le 1$.

Let $\{Y_j | j=1,\ldots,n\}$ be a group of data in R_+^n. Then the smallest closed convex and free-disposal attainable set that contains the observations can be further expressed as follows:

$$AS = \left\{ Y \,|\, Y \le \sum_{j=1}^{n} \lambda_j Y_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \ge 0. \right\}$$

Then a DEA model for the observation under evaluation (Y_0) is to identify the virtual elements in AS to have the largest residual performance (with a given measure) over Y_0, see [18]. Using the radial measure and the classic arguments of economics, we obtain the DEA-WEI model: $\max\{\theta | \theta Y_0 \in AS\}$, that is:

$$\begin{aligned} \theta^* = \max \quad & \theta \\ \text{subject to} \quad & \sum_{j=1}^{n} \lambda_j Y_j \ge \theta Y_0, \\ & \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \ge 0, \\ & j=1,\ldots,n. \end{aligned} \tag{1}$$

Notice here we do not explicitly consider the input variables in the attainable set. Thus we will only consider bounded attainable sets since otherwise will render infeasibility of the programme—unbounded solutions. In the following proposition, we list some possible ways of building an attainable set:

Proposition 1. *Let $P=\{(X,Y)\}$ be a bounded production possibility set (PPS), which is a free-disposal and closed convex technology set. Then its projection of all outputs*:

$$ASI = \{Y : \text{There is an } X \text{ such that} (X,Y) \in P\}$$

defines a bounded closed convex and free-disposal attainable set.

Let $\{(X_j,Y_j) | j=1,\ldots,n\}$ be a group of input and output data. Then

$$ASII = \left\{ F \le \sum_{j=1}^{n} \lambda_j \frac{Y_j}{X_j}, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \ge 0 \right\}$$

defines a bounded closed convex and free-disposal attainable set, where the divided data

$$\left\{ \frac{Y}{X} = \left(\frac{y_1}{x_1}, \frac{y_2}{x_1}, \ldots, \frac{y_s}{x_1}, \frac{y_1}{x_2}, \ldots, \frac{y_s}{x_2}, \ldots, \frac{y_1}{x_m}, \ldots, \frac{y_s}{x_m} \right) \right\}$$

are $s \times m$ dimension vectors, with $X=(x_1,x_2,\ldots,x_m)$ and $Y=(y_1,y_2,\ldots,y_s)$.

Proof. We first show conclusion one:

P is bounded, so is its subset ASI. If Z, $Y \in ASI$, then there exist X, W such that (W,Z), $(X,Y) \in P$. Since P is convex, so that $\lambda(W,Z)+(1-\lambda)(X,Y) \in P$, for any $0 \le \lambda \le 1$.Thus $\lambda Z+(1-\lambda)Y \in ASI$ according to its definition. Finally let $Y \in ASI$ and $Z \le Y$. It follows from the definition that there is an X:$(X,Y) \in P$. Since P is free-disposal, thus $(X,Z) \in P$ and therefore $Z \in ASI$. □

For $ASII$, clearly it is bounded and free-disposal. Suppose that $F^1, F^2 \in ASII$. Then

$$F^1 \le \sum_{j=1}^{n} \lambda_j^1 \frac{Y_j}{X_j}, \quad \sum_{j=1}^{n} \lambda_j^1 = 1, \lambda_j^1 \ge 0,$$

$$F^2 \le \sum_{j=1}^{n} \lambda_j^2 \frac{Y_j}{X_j}, \quad \sum_{j=1}^{n} \lambda_j^2 = 1, \lambda_j^2 \ge 0.$$

Thus for any $0 \le \lambda \le 1$

$$\lambda F^1 + (1-\lambda) F^2 \le \sum_{j=1}^{n} (\lambda \lambda_j^1 + (1-\lambda)\lambda_j^2) \frac{Y_j}{X_j},$$

$$\sum_{j=1}^{n} (\lambda \lambda_j^1 + (1-\lambda)\lambda_j^2) = 1.$$

Therefore $\lambda F^1 + (1-\lambda) F^2 \in ASII$.

It is clear that using ASI leads to DEA-WEI models that are used to assess efficacy or effectiveness, while $ASII$ is for the applications of index data, although they both have the same form. It can be argued that in $ASII$ constant return to scale (CRS) is assumed.

Model (1) was used by Lovell and Pastor [19], where the authors regarded this model as the output-oriented BCC model with the inputs being assumed equal to the unit. Further improvement was given by Hollingsworth and Smith [20]. Halkos and Salamouris [1] proposed the assessment measurement of the Greek commercial banks by using Model (1) with six financial ratios (index data), such as profit/loss per employee, return to total assets, net interest margin and others.

On the other hand we show it is natural and possible to apply the DEA principle to handle indexes directly. For $j=1,\ldots,n$, let $(x_1^j, x_2^j, \ldots, x_m^j)$ be inputs and $(y_1^j, y_2^j, \ldots, y_s^j)$be outputs of DMU$_j$. DEA models for index data are to directly use some of the indexes: $e_{ir}^j = y_r^j / x_i^j$ (as perhaps not all these indexes are relevant) to evaluate performance of the DMUs. Here we assume that all the inputs and outputs are desirable so that one wishes to maximize the weighted sum. Consequently, one would like to estimate the performance score by solving the following DEA model:

$$\begin{aligned} \max \quad & h = \sum w_{ir} e_{ir}^0 \\ \text{subject to} \quad & \sum w_{ir} e_{ir}^j \le 1, \quad j=1,\ldots,n, \\ & w_{ir} \ge 0, \quad i=1,\ldots,m, \quad r=1,\ldots,s. \end{aligned} \tag{2}$$

Thus the weights w_{ir} are decided to give the maximum score for the DMU$_0$. This model was firstly discussed by Fernandez-Castro and Smith [21] and has been used in [14]. Model (2) also looks like a DEA model without inputs; see [2,3].

We then investigate the relationship of the two models. The dual of Model (2) reads:

$$\begin{aligned} \min \quad & \sum_{j=1}^{n} \lambda_j \\ \text{subject to} \quad & \sum_{j=1}^{n} \lambda_j Y_j \ge Y_0, \\ & \lambda_j \ge 0, \quad j=1,\ldots,n \end{aligned} \tag{3}$$

If we let $t=\sum_{j=1}^{n}\lambda_j, \lambda_j'=\lambda_j/t$, and $\theta=1/t$, and substitute λ_j for λ_j', then Model (3) can be reformulated as

$$\begin{array}{ll} \max & \theta \\ \text{subject to} & \sum_{j=1}^{n}\lambda_j y_{rj} \geq \theta y_{r0}, \\ & \sum_{j=1}^{n}\lambda_j = 1, \quad \lambda_j \geq 0, \\ & j=1,\ldots,n, \quad r=1,\ldots,s. \end{array}$$

which is just Model (1). Thus the optimal value of Model (1) is the reciprocal of that of Model (2) with $h^*=1/\theta^*$. Therefore the DEA-WEI models with multiplier form do not have a direct dual relationship with their envelopment form as the standard DEA models.

Furthermore we will discuss relationship between the above models and the standard DEA models.

Assume now that DMUs have the unit input, and index outputs y_{rj}. Then standard ratio CCR DEA model reads:

$$\begin{array}{ll} \max & h_0 = \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i} \\ \text{subject to} & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i} \leq 1, \quad j=1,\ldots,n, \\ & v_i, u_r \geq 0, \quad i=1,\ldots,m, \quad r=1,\ldots,s. \end{array} \tag{4}$$

Taking the new weights, one has

$$\begin{array}{ll} \max & h_0 = \sum_{r=1}^{s}\mu_r y_{r0} \\ \text{subject to} & \sum_{t=1}^{s}\mu_r y_{rj} \leq 1, \quad j=1,\ldots,n \\ & v_i, \mu_r \geq 0, \quad i=1,\ldots,m, \quad r=1,\ldots,s. \end{array}$$

This is the exactly same as Model (2).

Let us now examine the multiplier output oriented BCC model:

$$\begin{array}{ll} \min & h = \sum_{i=1}^{m} v_i x_{i0} + u_0 \\ \text{subject to} & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - u_0 \leq 0, \quad j=1,\ldots,n, \\ & \sum_{r=1}^{s} u_r y_{r0} = 1, \\ & u_r, v_i \geq 0, \quad u_0 \text{free} \end{array}$$

Now letting the input be the unit and using a new non-negative variable $\sum_{i=1}^{m} v_i + u_0 = \lambda$, then the model reads:

$$\begin{array}{ll} \min & \lambda \\ \text{subject to} & \sum_{r=1}^{s} u_r y_{rj} \leq \lambda, \quad j=1,\ldots,n, \\ & \sum_{r=1}^{s} u_r y_{r0} = 1, \\ & u_r, \lambda \geq 0. \end{array}$$

Then taking the new weighs $w_r = u_r/\lambda$, one has

$$\begin{array}{ll} \max & \sum_{r=1}^{s} w_r y_{r0} \\ \text{subject to} & \sum_{r=1}^{s} w_r y_{rj} \leq 1, \quad j=1,\ldots,n, \\ & w_r \geq 0. \end{array}$$

This is again exactly model (2). Similarly one can show that either the output oriented dual CCR or BCC model will reduce to Model (1) based on the fact that the constraint $\sum_{j=1}^{n}\lambda_j=1$ in Model (1) can be equivalently replaced by $\sum_{j=1}^{n}\lambda_j \leq 1$.

In this case the additive DEA-WEI model will have the following form:

$$\begin{array}{ll} \max & \sum_{r=1}^{s} s_r^+ \\ \text{subject to} & \sum_{j=1}^{n}\lambda_j y_{rj} - s_r^+ = y_{r0}, \\ & \sum_{j=1}^{n}\lambda_j = 1, \\ & s_r^+ \geq 0, \quad r=1,\ldots,s, \\ & \lambda_j \geq 0, \quad j=1,\ldots,n. \end{array} \tag{5}$$

Noting that there are no input slacks in above model, one can use the following SBM formula to replace the objective function in (5) to have the SBM DEA-WEI model:

$$\begin{array}{ll} \min & e = \frac{1}{1+(1/s)\sum_{r=1}^{s}(s_r^+/y_{r0})} \\ \text{subject to} & \sum_{j=1}^{n}\lambda_j y_{rj} - s_r^+ = y_{r0}, \\ & \sum_{j=1}^{n}\lambda_j = 1, \\ & s_r^+ \geq 0, \quad r=1,\ldots,s, \\ & \lambda_j \geq 0, \quad j=1,\ldots,n. \end{array} \tag{6}$$

Model (5) is the additive DEA-WEI model that was used by Cai and Wu [22] to evaluate the financial position of eleven IT companies, where four synthetic financial indexes were used. The authors used this model by setting the input as the unit, but without further explanation on why this model can be used to handle their case. One of the advantages of this model is that it can handle negative data directly. To handle negative data using Model (1), one may need to use suitable transformation (shifting) or the directional distance, see [23] for more details. Similarly we can discuss the DEA-WEI model using the Russell measurement (see Model (14) in Section 3).

Let us now not assume that the inputs are unit but assume that the input variable is single. Then dual output oriented CCR model reads:

$$\begin{array}{ll} \max & \theta \\ \text{subject to} & \sum_{j=1}^{n} x_{1j}\lambda_j' = x_{10}, \\ & \sum_{j=1}^{n} z_{rj}\lambda_j' \geq \theta z_{r0}, \\ & \lambda_j' \geq 0, \quad r=1,\ldots,s, \quad j=1,\ldots,n. \end{array}$$

Note that the constraint for input now must be binding so it is replaced by the equation. Let $y_{rj}=z_{rj}/x_{1j}$, $y_{r0}=z_{r0}/x_{10}$ and $\lambda_j = x_{1j}\lambda_j'/x_{10}$, then we can see that it is exactly Model (1). However it does not seem to be possible to carry out similar converting for a BCC dual model. Thus this shows that the concept of "Return to Scale" is generally tricky in the DEA models for index data, as they include the case where inputs are not taken into account. In fact it is not appropriate to rescale the outputs in such DEA models.

In summary, by applying classical DEA models to data with unit inputs, we can derive their corresponding DEA-WEI models. In this case, both CCR and BCC models reduce to the same forms of DEA-WEI models. If PPS of *ASII* type is used, then CRS is implicitly assumed. In the case of single input, one can directly convert the CCR model into a DEA-WEI model. However Models (1) and (2) do cover more applications (e.g., the cases of outputs data only).

Finally we briefly discuss the issue of correlated data. Although the index data may come from divisions of original data, and thus may increase correlations among the divided data, it follows from the equivalence between the CCR and Model (1) with the single input that the possible effects on the final DEA efficiency results should be similar to the standard CCR case as discussed in [24], and thus omission of variables purely on grounds of correlation should be avoided.

3. Value judgement in DEA models without explicit inputs

As shown above the use of ratios is sometimes unavoidable in real applications. This introduces some important considerations that require careful treatment, as first pointed out by Fernandez-Castro and Smith [21]. One particular point discussed is that the concept of "Return to Scale" is generally tricky in the DEA-WEI models. Another important issue is value judgment, which will be addressed below.

In many DEA applications, the value judgments of decision makers (DM) need to be incorporated. Mahlberg and Obersteiner [2] and Despotis [3,15] presented an application in human development analysis, where three indexes indicators—longevity, educational attainment and standard of living, were used as three outputs to reflect major dimensions of human development. In these models each indicator is regarded as equally important and not substitutable, where DMU_0 can be efficient if any one of the three outputs is extremely high. However, the two assumptions may not be appropriate in many situations, due to value judgements of decision makers (DMs) as illustrated in [25,26,18]. We will discuss these issues via the "Envelopment form" of DEA-WEI models in this section, which can incorporate value judgement more directly by changing the preferences as seen in [27–29].

Let 3-dimensional vector Y_i denote the examination marks of maths, physics and chemistry of the student-i in a mathematics department. Assume the department now wishes to evaluate students' performance. Traditionally such an assessment will be carried out without considering inputs, and thus DEA-WEI models can be employed. If all the subject marks are regarded as equally important and not substitutable, the basic DEA models discussed in Section 2 will be adequate. However, this may not always be the case. For example, the department may think "the marks of maths to be more important". One possible approach is to use weight restrictions in Model (2). Here we only examine the most useful type. Very often this value judgment is reflected by adding the constraints $(\mu_1/\mu_2) \geq \alpha, (\mu_2/\mu_3) \geq \beta$ in Model (2), where $\alpha, \beta \geq 1$ are constants—Assurance Region of type I (ARI), see [25]. In order to understand the implicit meanings contained in this model, we first derive its dual as follows:

$$
\begin{aligned}
&\max && \theta \\
&\text{subject to} && \sum_{j=1}^{n} \lambda_j y_{1j} \geq \theta y_{10} + \gamma_1, \\
& && \sum_{j=1}^{n} \lambda_j y_{2j} \geq \theta y_{20} - \alpha\gamma_1 + \gamma_2, \\
& && \sum_{j=1}^{n} \lambda_j y_{3j} \geq \theta y_{30} - \beta\gamma_2, \\
& && \sum_{j=1}^{n} \lambda_j = 1, \quad \gamma_1, \gamma_2, \lambda_j \geq 0, \quad j = 1, \ldots, n.
\end{aligned}
$$

Then we can transform the above model to the following equivalent one:

$$
\begin{aligned}
&\max && \theta \\
&\text{subject to} && \sum_{j=1}^{n} \lambda_j y_{1j} \geq \theta y_{10}, \\
& && \sum_{j=1}^{n} \lambda_j (\alpha y_{1j} + y_{2j}) \geq \theta(\alpha y_{10} + y_{20}), \\
& && \sum_{j=1}^{n} \lambda_j (\beta\alpha y_{1j} + \beta y_{2j} + y_{3j}) \geq \theta(\beta\alpha y_{10} + \beta y_{20} + y_{30}), \\
& && \sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geq 0, \quad j = 1, \ldots, n.
\end{aligned}
\tag{7}
$$

Then it is clear that three weighted sums of the marks are used for comparisons in this DEA model, and the precise meaning of "Maths mark is more important" is clearer. For more general ARI, we can follow the framework of the Cone-Ratio model proposed by Charnes et al. [30], to formulate the counterpart cone-ratio DEA-WEI model, which confines weights in a cone U:

$$
\begin{aligned}
&\max && u^T y_0 \\
&\text{subject to} && u^T y_j \leq 1, \quad j = 1, \ldots, n, \\
& && u \in U \subset R_s^+.
\end{aligned}
\tag{8}
$$

When $U = R_s^+$, it is back to the DEA Model (2). In practical applications, value judgments are incorporated into the model by defining a suitable polyhedral cone U.

We say that a cone $U \subset R_s^+$ is a polyhedral cone, if it has the so-called half space form: $U = \{u \mid Du \geq 0\}$, where D is a $l \times s$ matrix, and u is a $s \times 1$ vector. Generally, the constraints of ARI can be regarded to incorporate value judgement in this way, see [25,26] for details. One can easily identify the matrix D in the above example.

However it is not always easy to find the dual of Model (8). In the case where the cone is generated by a finite set of vectors, that is it can be rewritten: $U = \{u \mid u = B^T\gamma, \gamma \geq 0\}$, where B^T is a $s \times l$ matrix, and γ is a $l \times 1$ vector, then by using a data transformation we can rewrite Model (8) as Model (9), and then find its dual Model (10) below:

Cone-ratio multiplier form Model	Cone-ratio envelopment form Model
$\begin{aligned} &\max && h = \gamma^{\mathrm{T}}(BY_0) \\ &\text{subject to} && \gamma^T(BY) \leq 1, \\ & && \gamma \geq 0. \end{aligned} \quad (9)$	$\begin{aligned} &\max && \theta \\ &\text{subject to} && (BY)\lambda \geq \theta(BY_0), \\ & && \lambda^T e = 1, \quad \lambda \geq 0. \end{aligned} \quad (10)$

where the optimal value of Model (9) is the reciprocal of that of Model (10) with $h^* = 1/\theta^*$.

In our example of students' performance evaluation, B is $\begin{pmatrix} 1 & 0 & 0 \\ \alpha & 1 & 0 \\ \alpha\beta & \beta & 1 \end{pmatrix}$. Unfortunately, it is neither easy nor always possible to find matrix B to rewrite U in the form of $U = \{u \mid u = B^T\gamma, \gamma \geq 0\}$. Charnes et al. [31] and Brockett et al. [40] discussed relationships between the matrixes B and D under various conditions. For example, suppose D is $s \times s$ and has an inverse, then we can take $B^T = D^{-1}$.

Below we will find it is possible to express value judgements via preferences. A preference is a precise relationship to clarify the meanings for the vague expressions like "better, worse". Clearly one should have some understandings on these meanings before an evaluation is carried out. The most classic example of preferences is the numerical order (preference) for the real numbers like "$5>3$" and "$4<6$". Such an order can be generalized to a column or a table of real numbers—like the Pareto preference widely used in economics, which compares each component of the vectors. In the above example, the department may think student-*i* is better than student-*j* if his or her maths mark and total mark are higher, that is, the preference reads: student-*i* is better than student-*j* if

$$\begin{array}{ccc} y_{1i} & \geq & y_{1j} \\ y_{1i}+y_{2i}+y_{3i} & \geq & y_{1j}+y_{2j}+y_{3j} \end{array} \tag{11}$$

If the department thinks all the subject marks are equally important and substitutable, then the totals are to be used to compare so that student-*i* is better than student-*j* if

$$y_{1i}+y_{2i}+y_{3i} \quad \geq \quad y_{1j}+y_{2j}+y_{3j} \tag{12}$$

The above inequalities can be conveniently presented with matrixes. For (11) we can let $B=\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$, so performance of student-*i* is better than that of student-*j* in the sense of the matrix preference if and only if $BY_i \geq BY_j$ in Pareto preference. For the average preference (12), $B=(1\ \ 1\ \ 1)$. Finally if we let $B=I=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ or $B=\begin{pmatrix} 1 & 0 & 0 \\ \alpha & 1 & 0 \\ \alpha\beta & \beta & 1 \end{pmatrix}$ then the matrix preference is just the Pareto preference or the preference used in Model (7) respectively. More precisely in Model (7) implicitly it assumes the preference adopted as follows: For given inputs level, DMU_i is better than DMU_j if and only if

$$\begin{array}{ccc} y_{1i} & \geq & y_{1j} \\ \alpha y_{1i}+y_{2i} & \geq & \alpha y_{1j}+y_{2j} \\ \beta\alpha y_{1i}+\beta y_{2i}+y_{3i} & \geq & \beta\alpha y_{1j}+\beta y_{2j}+y_{3j}, \end{array}$$

There are in fact many more useful preferences, like Lexicographic preference, see [18,28] for more details.

Therefore, value judgements may be expressed via preferences. We then see another approach to incorporate value judgment: to use the measures and preferences in the dual DEA models like Model (1) to incorporate value judgments, see [27] for more details. For example, replacing the Pareto Preference by the matrix preference in Model (1), we then have the following DEA model:

$$\begin{array}{ll} \max\limits_{\theta,\lambda} & \theta \\ \text{subject to} & \sum\limits_{j=1}^{n} BY_j\lambda_j \geq \theta\, BY_0, \\ & \sum\limits_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geq 0. \end{array} \tag{13}$$

where B is the matrix in the definition of the matrix preference to specify compensations between these outputs. Let us note that Model (13) looks identical to Model (10). However one has to know U to apply Model (10), while Model (13) can be used directly if one can express the value judgement in the matrix preference as we did above. This model will be used in Section 4.

Furthermore, in many applications, it may not be rational to assume radial contraction or expansion, see [32]. Then one can adopt the Russell measurement, and have the following model:

$$\begin{array}{ll} \max & \frac{1}{s}\sum\limits_{r=1}^{s} \theta_r \\ \text{subject to} & \sum\limits_{j=1}^{n} \lambda_j y_{rj} = \theta_r y_{r0} \\ & \sum\limits_{j=1}^{n} \lambda_j = 1 \\ & \theta_r \geq 1, \quad r=1,\ldots,s \\ & \lambda_j \geq 0, \quad j=1,\ldots,n. \end{array} \tag{14}$$

If the constraint $\theta_r \geq 1$ is replaced by $\theta_r \geq 0$ in this model, then the average preference is actually used for the outputs. Similarly, one can use DEA-WEI models of SBM type to deal with such applications. Let us note that SBM DEA models can be transformed into Russell DEA models using some variable transformation, see [33] for details.

Let us mention that the models above can be applied to effectiveness evaluations where only outputs need to be evaluated.

4. Empirical studies

In this section we present some empirical studies on DEA-WEI models. We first compare the standard DEA and DEA-WEI models – this basically means we apply the standard DEA models directly by using the original input and output data. Then we standardize the original data and apply the DEA-WEI models. After that, we apply DEA-WEI models in a case study on performance evaluation of research institutes at Chinese Academy of Sciences (CAS), where value judgements have to be incorporated in the assessment.

4.1. Comparisons of the standard DEA and DEA-WEI models

Greenberg and Nunamaker [4] pointed out that incorporation of conventional performance measures and ratios should promote practical acceptance of the DEA approach because this method could overcome some disadvantages of ratio analysis. Greenberg and Nunamaker [4] and Thanassoulis et al. [5] presented practical applications to compare DEA with ratio approach, where the standard DEA models were still used by setting some indexes as inputs and some were outputs. In [34], some inherent relationship between DEA frontier DMUs and output–input ratios was discussed so that one sometimes can find DEA frontiers directly from ratios analysis. In [35], relationship between DEA efficiency estimates and financial ratios was discussed.

Zhu [36] presented comparisons between DEA and PCA in aggregating multiple inputs and multiple outputs in the evaluation of DMUs. In Zhu's [36] paper, super-efficiency scores were first computed via a modified input-oriented CCR model, where the DMU under evaluation was excluded from the virtual sums. The purpose of this exclusion is to discriminate those efficient DMUs. Then indexes were defined by using (three) outputs divided by each of two inputs separately, and the principle components for the defined indexes were then determined. Finally, a single score is obtained by summing the weighted principal components, where the weights could be decided by their eigenvalues. The results indicated that the ranking by DEA was consistent with the PCA ranking, although there should be no surprise had these two methods produced very different results since they are based on very different principles anyway. Here we extend the comparison using the standard DEA models (CCR, BCC, and SBM), Zhu's model, the radial DEA-WEI model (Model 1), and the SBM DEA-WEI model (Model 6) to evaluate these 18 cities again. Both input and output

Table 1
Comparison results.

	Cities	CCR score	BCC score	Zhu's score	Model (1)	SBM score	Model (6)
2	QingHuangDao	1.0000	1	12.6785	1.0000	1	1
10	WenZhou	1.0000	1	2.2848	1.0000	1	1
6	WeiHai	1.0000	1	1.6724	1.0000	1	1
12	ZhangJiang	0.7866	0.9202	0.7866	0.7866	0.4133	0.24049
13	BeiHai	0.7514	1	0.7513	0.7514	0.3888	0.25531
9	NingBo	0.6577	1	0.6577	0.6869	0.4810	0.37556
5	YanTai	0.6311	1	0.6311	0.6485	0.4735	0.35018
4	Qingdao	0.5022	0.9421	0.5022	0.5022	0.3058	0.16268
8	LianYunGang	0.4959	0.5148	0.4959	0.4959	0.3366	0.23701
16	ShanTou	0.4704	1	0.4704	0.4704	0.1392	0.03012
1	Dalian	0.4691	1	0.4691	0.4691	0.2797	0.1429
7	ShangHai	0.3580	1	0.3581	0.3580	0.1823	0.07041
17	XiaMen	0.3059	0.3095	0.3060	0.3059	0.1172	0.01424
11	GuangZhou	0.3010	1	0.3010	0.3010	0.1693	0.07997
3	TianJin	0.2779	0.8656	0.2779	0.2779	0.0918	0.03498
18	HaiNan	0.1953	0.1967	0.1953	0.1953	0.0861	0.03834
15	ZhuHai	0.1867	0.2040	0.1867	0.1867	0.0525	0.00808
14	ShenZHen	0.1366	0.2625	0.1382	0.1366	0.0454	0.01159

orientated models were computed for the standard DEA models and little difference was found. Thus we just present the results with output-orientated models. For the standard DEA models, we just use the original input and output data directly. To apply the DEA-WEI models, we divided the outputs by each input to form the index data. The efficiency scores are shown in Table 1.

In Section 2 we have seen that for the case of single input the standard CCR model is equivalent to a DEA-WEI model, while this is not true for the BCC model. The above experiment showed a similar picture although this time the CCR model has two inputs. In Table 1, the efficiency scores of DEA-WEI Model (1) are almost the same as the results of CCR model. For inefficient DMUs, the efficiency scores could be a bit different, but the rankings are the same. Furthermore if we exclude the DMU under evaluation from virtual sums in the DEA-WEI model (thus have DEA-WEI super-efficiency scores), we then obtained the exactly same ranking as Zhu's. However the scores of BCC model are quite different from those produced by Model (1).

We also test the standard SBM model, and Model (6) using the SBM score formula e. This time although the scores of the two models are quite different, the ranks produced by the two scores are very similar: the spearman correlation coefficient between the two ranks is actually 0.977. Model (14) is found to produce similar results to Model (6).

4.2. A case study

In this section, we carry out a pilot study on applying DEA methodology to assess performance of 15 basic research institutes in Chinese Academy of Sciences (CAS), and also compare CCR, BCC, and two DEA-WEI models.

Since the late 1990s, CAS has carried out a set of performance assessments for its research institutes. The assessments used to consist of quantitative part and qualitative part (mainly peer reviews). Since 2002, the CAS evaluation system has focused on not only effectiveness but also efficiency. The quantitative indicators have become more or less stable. However, one of the main criticisms to this evaluation system is the fairness of the weights selection, which has been questioned since the assessments started. Another criticism is that the indexes, which have been used to measure efficiency of research investment, such as publications per staff, publications per expenditures, graduates enrolment per staff, etc., seemingly change every year. In this pilot study,

Table 2
Inputs and outputs of basic research institutes in CAS.

DMU	Staff	Res. Expen.	SCI Pub.	High Pub.	Grad. Enroll.	Exter. Fund.
Unit 1	380	59,880	201	28	386	35,368
Unit 2	418	79,910	480	196	354	69,763
Unit 3	68	13,150	78	72	57	5747
Unit 4	1105	92,710	153	45	642	49,074
Unit 5	248	18,920	68	18	165	13,801
Unit 6	828	134,240	167	64	229	73,748
Unit 7	481	52,460	38	13	136	32,797
Unit 8	493	40,840	94	6	115	12,743
Unit 9	198	23,110	43	16	79	15,964
Unit 10	243	32,580	42	11	48	20,731
Unit 11	553	62,100	156	34	105	67,927
Unit 12	347	49,510	64	8	190	31,616
Unit 13	445	78,280	440	162	529	62,448
Unit 14	260	27,530	113	23	137	33,952
Unit 15	304	59,450	94	19	263	70,015

Value of Res. Expen. and Exter. Fund. are in RMB thousand.

we try to explore the possibility of using DEA-WEI models as a possible ranking tool in the efficiency evaluation of basic research institutes of CAS in order to address the issue of weights selection. Here we select the most important inputs and outputs, and carry out some comparisons between the original results and the results by using the DEA-WEI models.

Often the number of full-time research staff (Staff) and total research expenditures (Res. Expen.) are major research inputs, while research outputs are quite variable depending on different stakeholders' view. The following represents a view from the level of the Bureau of Basic-research in CAS, where publications include the international papers indexed by Science Citation Index (SCI Pub.), high quality papers published in top research journals (High Pub.), graduate students' enrolment (Grad. Enroll.), and the external research funding obtained (Exter. Fund) are the major indicators to judge performance. These are also the key indicators to assess the basic institutes in Comprehensive Evaluation System (CES) of CAS in 2002. Although only index data are needed for the DEA-WEI models, we wish to carry out some comparisons between the standard DEA and DEA-WEI models. Thus the original input–output data are provided in Table 2, which come from CES of CAS in 2002.

As shown in Table 2, Unit 4 had the largest size and the highest graduate students' enrolments. Unit 6 obtained the maximum external research funding, while its research expenditure were the highest as well. Unit 2 would be rank on the top according to its SCI publications and high quality publications.

In this study, we employ the output-oriented CCR, BCC and DEA-WEI models to do comparison analysis. To apply the DEA-WEI models we translate these indicators into indexes by using outputs/inputs separately, and we have 8 indexes defined as $Y_j=(y_{1j},y_{2j},\ldots,y_{8j})^T$, where:

y_{1j} = SCI Pub./staff
y_{2j} = SCI pub./Res. Expen.
y_{3j} = High Pub./Staff
y_{4j} = High Pub./Res. Expen.
y_{5j} = Exter. Fund./Staff
y_{6j} = Grad. Enroll./Staff
y_{7j} = Exter. Fund./Res. Expen.
y_{8j} = Grad. Enroll./Res. Expen.

Since the percentage of external funding used for research is unknown, and the expenditure for graduate education are normally very small in China, here we exclude y_{7j} and y_{8j}, and these indexes are also excluded in the CES 2002.

Table 3 presents the standardized indexes. The formula to standardize these indexes is $y_{rj}=(y_{rj}/\mathrm{Max}_j y_{rj})\times 100$. The purpose to standardize indexes is to remove measurement differences in these weighted sums.

We firstly apply the CCR and BCC model to evaluate performance of these institutes. The results of CCR and BCC are quite close except Unit 4, which is efficient in BCC but drops to 82.37% in CCR. Therefore, we only present the BCC results in the second column of Table 4 for further comparison. Next by using the radial measurement and Pareto preference, and the Russell measurement and Pareto preference [37,38], we apply Model (1) and Model (14) for the index data. The results are provided in Table 4.

In these models, all the outputs and the six indexes are regarded as non-substitutable and equally important. However, this assumption may not be suitable for the current situations in CAS. For instance, according to the questionnaire analysis in [39], there still exist two groups of basic research institutes. One group focuses on publishing papers in high quality journals to pursue higher research impact, while the other group is still on the quantity expansion stage by increasing SCI publications. Therefore, we need to include both the indicators and regard y_{1j} and y_{3j} directly substitutable in order to give fairer consideration for both groups. Meanwhile, the external research funding per staff y_{5j} is regarded to be very important and non-substitutable by the DMs because it is often used to reflect the competitiveness of the institutes. Furthermore, the sum of these indexes should indicate the overall performance. Hence, we combine the six indexes into three new equally important and non-substitutable indexes, and have the following model:

$$\begin{aligned} &\max_{\theta,\lambda} && \theta \\ &\text{Subject to} && BY\lambda \geq \theta\, BY_0, \\ & && \sum_{j=1}^{n}\lambda_j = 1, \quad \lambda_j \geq 0, \\ & && B=\begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \end{aligned} \tag{15}$$

The efficiency scores by Model (15) are shown in the fifth column of Table 4 and followed by the ranking orders.

In Table 4, the 15 DMUs are ranked via the scores of Model (15) and divided into three groups (A, B, C) for clearer comparisons.

Table 3
Standardized indexes.

DMU	y_{1j}	y_{2j}	y_{3j}	y_{4j}	y_{5j}	y_{6j}
Unit 1	46.06	55.88	6.96	8.54	40.41	85.45
Unit 2	100.00	100.00	44.28	44.80	72.46	71.24
Unit 3	99.89	98.75	100.00	100.00	36.70	70.51
Unit 4	12.06	27.47	3.85	8.86	19.28	48.87
Unit 5	23.88	59.83	6.85	17.38	24.16	55.97
Unit 6	17.56	20.71	7.30	8.71	38.67	23.27
Unit 7	6.88	12.06	2.55	4.53	29.61	23.78
Unit 8	16.60	38.32	1.15	2.68	11.22	19.62
Unit 9	18.91	30.98	7.63	12.64	35.01	33.56
Unit 10	15.05	21.46	4.28	6.17	37.04	16.62
Unit 11	24.57	41.82	5.81	10.00	53.33	15.97
Unit 12	16.06	21.52	2.18	2.95	39.56	46.06
Unit 13	86.10	93.57	34.38	37.80	60.93	100.00
Unit 14	37.85	68.33	8.35	15.26	56.70	44.33
Unit 15	26.93	26.32	5.90	5.84	100.00	72.78

Table 4
Efficiency scores based on DEA models.

DMU	BCC	Model (1)		Model (14)		Model (15)	
	Scores	Scores	Ranks	Scores	Ranks	Scores	Ranks
Unit 2	100	100	A	100	A	100	A
Unit 3	100	100	A	100	A	100	A
Unit 15	100	100	A	100	A	100	A
Unit 13	100	100	A	100	A	92.5	A
Unit 14	100	74.87	B	30.82	B	66.85	B
Unit 11	65.12	62.78	B	18.71	B	56.27	B
Unit 1	93.15	85.45	B	25.46	B	56.1	B
Unit 12	65.12	51.68	B	6.7	C	43.18	B
Unit 6	54.6	42.98	C	15.42	B	41.45	B
Unit 5	100	62.40	B	20.72	B	40.88	B
Unit 9	54.57	43.48	B	18.39	B	40.87	B
Unit 10	98.28	41.03	C	10.74	C	38.68	C
Unit 7	42.23	31.85	C	6.8	C	30.56	C
Unit 4	100	48.87	B	11.43	B	27.51	C
Unit 8	66.61	38.32	C	4.29	C	19.37	C
Average scores	82.64	65.58		37.97		56.95	

Group A represents the top 25% research institutes, while Group C indicates the bottom 25%. Hence, DMUs of the top four research institutes are classified as A, the worst four are in Group C, and the rest are in Group B. As Table 4 shows, there are 7 efficient DMUs using the standard BCC model with the average score 82.64%. The results from the CCR model are almost the same. This implies that too many indicators have diminished the DEA discrimination, though this could be dealt with by weighs restrictions.

By using the DEA-WEI model with the Radial measurement (Model 1) and Russell measurement (Model 14), the efficiency scores of BCC model have been sharply reduced. When Model (15) is employed, only Units 2, 3 and 15 are efficient. Model (15) allows that indexes y_{1j} and y_{3j} can compensate each other directly, and thus focuses on the average level according to the DM's value judgments. In addition, index y_{5j} (external research funding per staff) is emphasized as it is now substitutable. Because value judgements are incorporated into the model and possible substitutions are considered among the indexes, the discrimination power of this DEA-WEI model is much enhanced.

Table 5 provides the original evaluation scores of CES 2002 of the 15 basic research institutes. The second column presents the aggregated final scores E, which combine the scores in qualitative and quantitative evaluations, and produce Rank E. The fourth column presents the scores E_1, which reflect the research institutes' quantitative output evaluation without consideration of inputs, and produce Rank 1. The sixth column presents the scores E_2 for

Table 5
Original evaluation scores of CES 2002.

DMU	Aggregated final scores (E)	Rank E	Research outputs (E_1)	Rank 1	Research sustainability (E_2)	Rank 2	Innovative culture (E_3)
Unit 3	66.53	A	51.19	B	79.09	A	93
Unit 2	83.63	A	85.91	A	78.44	A	93
Unit 13	82.75	A	91.77	A	68.4	A	95
Unit 15	60.79	B	48.81	B	68.21	A	91
Unit 1	57.21	B	42.41	B	66.49	B	94
Unit 6	67.44	A	67.32	A	61.94	B	90
Unit 12	50.08	C	37.48	C	56.6	B	87
Unit 11	51.34	C	39.13	C	56.44	B	92
Unit 10	51.63	B	40.84	B	56.26	B	87
Unit 9	47.71	C	33.86	C	54.19	B	91
Unit 14	58.3	B	56	B	53.75	B	88
Unit 7	58.77	B	58.12	B	51.78	C	90
Unit 5	47.56	C	35.78	C	50.94	C	93
Unit 4	55.04	B	53.82	B	48.09	C	89
Unit 8	52.89	B	50.89	B	45.87	C	91

$E = 0.5E_1 + 0.4E_2 + 0.1E_3$

their research sustainability evaluation, which actually are more or less the CES efficiency evaluation, and produce Rank 2. In Table 5, the 15 basic research institutes are ranked according to the research efficiency Rank 2 for easy comparisons with Table 4. Table 5 and 2 have clearly shown several institutes have higher total scores due to their big sizes. For instance, Unit 6 was ranked A mainly due to its higher research outputs score. Units 4–5, 7–8 were all ranked C in efficiency but three of them were ranked overall B again due to their output scores.

Next we compare the DEA results with the CES research sustainability evaluation results (Rank 2), where the weighted sum of the six indexes in Table 3 is used as the score. Firstly we compare the group ranking based on Model (14) (Ranks of Model 14) in Table 4 and Rank 2 based on the research institutes' research sustainability in Table 5. The top 4 DMUs have the same ranking results. For the worst performed DMUs, Unit 4 and Unit 5 are ranked B in Ranks of Model (14) but C in Rank 2, while Units 10 and 12 are also ranked differently. Thus there are substantial differences (four out of eight—50%) in the classification of Group C via the two approaches. Such results are understandable as the two approaches are based on very different principles,

Then we compare the Ranks of Model (15) in Table 4 (Rank of Model (1) is very similar) with Rank 2 in Table 5. Model (15) has taken some value judgements of the DMs into account. The research institutes in Group A of Model (15) are still the same as those of Rank 2. The classifications of Group C under the two approaches are much closer—now only 25% difference. For example, Unit 12 is in Group C by using Model (14), but has relatively strong performance in obtaining external research funding per staff (7th out of 15). Thus it is ranked B by using Model (15). In terms of the group ranking, the two ranks above are more consistent. Therefore, we believe this DEA model is applicable to the efficiency evaluation of CAS. The ranks of the research institutes in the same group are quite different. Nevertheless, similar ranks are not expected since two every different approaches are employed. In summary, we think it is feasible to apply DEA-WEI in future CAS performance evaluation, and it is necessary to apply the DEA-WEI models incorporating value judgements of the DMs.

5. Conclusion

Motivated by the different applications of the DEA models without explicit inputs, in this paper we take an axiom approach to examine these DEA models in order to cover as wide applications as possible. This approach leads to a uniform presentation of the DEA-WEI models, some of which are derived explicitly in this paper. One advantage is that it is much easier to use these models when only index indicators are available. Furthermore these DEA models are applicable to efficacy evaluations where inputs are not directly taken into account. The multiplier form DEA-WEI models look similar to ratio approach, but allow flexibility of weights selection for the assessed DMUs.

Furthermore, we discuss how to incorporate the DMs' value judgment in these DEA models using weight restrictions and preferences, and further present a practical application of research evaluation in CAS. We find that the DEA models without explicit inputs have some unique advantages and should be applied to more real-life applications. The empirical results show that it is feasible to apply DEA-WEI in future CAS evaluation, and it is necessary to incorporate value judgements of the DMs into DEA-WEI models. Thus these DEA-WEI models provide a possible approach to deal with the main controversies of the existing CAS research evaluations.

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