Bar Layout and Weight Optimization of Special RC Shear Wall

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Abstract

Optimization problems can be defined and formulated with either discrete or continuous variables. This paper presents a continuous optimization method for the design of reinforced concrete shear walls, based on the concept of boundary element and with the reinforcement layout taken into consideration. Contrary to the discrete method, where algorithm must be provided with a set of previously prepared default designs, the continuous optimization algorithm generates and evaluates a wall design in each iteration. The objective function of the algorithm minimizes the cost of the wall, which depends on the reinforcement details (rebar diameter and layout) and the wall dimensions (the cost of concrete and formworking). This objective function consists of the boundary element dimensions and the reinforcement layout variables (cross-sectional area and spacing of rebars). Shear wall design requirements and restrictions are formulated as constraints in accordance with ACI318-11 provisions for special ductility. After obtaining optimal wall design for seismic loads, design details such as wall dimensions and reinforcement details are determined accordingly. The optimization is performed by the use of several metaheuristic algorithms, including PSO, FA, WOA, and CSA. The comparison of the results of continuous and discrete optimization methods show that the shear wall designs obtained by the continuous approach are less expensive and closer to the global optimum.


1. Introduction

With the rapid development and growing complexity of engineering optimization problems, the use of metaheuristic algorithms has become the method of choice in this regard. Metaheuristic algorithms are a family of computational method that estimate an initial solution and then improve it iteratively based on a set of rules to approach the global optimum. However, the final result of the algorithm is not guaranteed to be the optimal solution.

Many of the existing powerful algorithms are nature-inspired and population-based. One of the oldest of such algorithms is the Genetic Algorithm (GA) \cite{1}, which is inspired by biological mechanisms such as reproduction, mutation, and survival of the fittest. Other notable examples algorithms inspired by the collective and social behavior of animals include the Particle Swarm Optimization algorithm (PSO) \cite{2}, the Ant Colony Optimization algorithm (ACO) \cite{3}, and the Firefly Algorithm (FA) \cite{4}.

Researchers constantly develop new algorithms to accelerate convergence to the optimum and reduce the error of approximation for a specific set of problems. In other words, the purpose of any new algorithm is to improve solution accuracy, solving speed, or both. Notable among the algorithms introduced in recent years are the Harmony Search algorithm (HS) \cite{5}, Simulated Annealing algorithm (SA) \cite{6}, gradient evolution algorithm (GE) \cite{7} and Heat Transfer Search algorithm (HTS) \cite{8} developed in 2015, the Whale Optimization Algorithm (WOA) \cite{9}, Crow Search Algorithm (CSA) \cite{10} and Water Evaporation Optimization algorithm (WEO) \cite{11} developed in 2016, and the thermal exchange optimization \cite{12}, and Electro-Search algorithm \cite{13} developed in 2017.

In this study, a continuous optimization method is formulated for the RC shear wall design problem. The continuous nature of the optimization method provides better flexibility than discrete method, allowing the algorithm to further approach the Best Solution. Since the results produced by the continuous method may not be real-world applicable, the method is also combined with discrete approach in order to achieve more practical results. The shear wall problem is solved by PSO, FA, WOA, CSA algorithms and the results are compared.

This paper consists of six sections. In Section 2, optimization algorithms and their formulations are described. Section 3 explains the RC shear wall design problem, cost function, problem constraints, and the wall optimization method. Section 4 provides a numerical example of the shear wall problem, and Section 5 presents a parametric study performed on that wall. Finally, the results are presented in Section 6.
2. Swarm intelligence techniques

2.1. Particle Swarm Optimization

The Particle Swarm Optimization algorithm (PSO) is a nature-inspired algorithm developed by Kennedy and Eberhard [2]. This algorithm takes inspiration from the collective behavior of swarms of birds and fish. In this algorithm, each member (particle) of the population searches the space for food, which represents high-quality solutions.

Each particle has a velocity and a position, which is updated as follows:

\[ x_i^{t+1} = x_i^t + v_i^{t+1} \]  \hspace{1cm} (1)

In the above equation, \( x_i^t \) and \( x_i^{t+1} \) are the current and new positions of the particle, and \( v_i^{t+1} \) is the particle velocity. The velocity of particle \( i \) is given by:

\[ v_i^{t+1} = \omega v_i^t + c_1 r_1 (p_i - x_i^t) + c_2 r_2 (p_g - x_i^t) \]  \hspace{1cm} (2)

where \( v_i^t \) and \( v_i^{t+1} \) are the old and new velocities of particle \( i \), \( p_i \) is the best solution ever found by particle \( i \), \( p_g \) is the best solution ever found by any particle in the population, \( c \) is the random weight factor ranging between \([0, 2]\), \( r \) is a random number between \([0, 1]\) and \( \omega \) is the inertia weight, which ranges between \([0, 1.2]\).

2.2. Firewall Algorithm

Firewall Algorithm (FA) is a metaheuristic optimization algorithm inspired by how fireflies communicate with each other using the light emitted from their lower abdomen. FA was introduced by Yang (2008) based on the following assumptions [4]:

1. All fireflies are attracted to each other irrespective of their sex. The attraction intensity \( I \) is given by:

\[ I(r) = I_0 e^{-\gamma r^2} \]  \hspace{1cm} (3)

Where \( I_0 \) is the initial light intensity, \( \gamma \) is the constant light absorption coefficient, and \( r \) is the distance of fireflies from each other.

The attractiveness \( \beta(r) \) of a firefly, which is proportional to its light intensity as perceived by other fireflies, is given by:

\[ \beta(r) = \beta_0 e^{-\gamma^2 r^2} \]  \hspace{1cm} (4)

Where \( \beta_0 \) is the attractiveness at \( r = 0 \).

2. Attractiveness of a firefly is proportional to its brightness, and the perceived brightness and attractiveness are inversely proportional to distance. The distance between fireflies \( i \) and \( j \) located at \( x_i \) and \( x_j \) is given by:

\[ r_{ij} = |x_i - x_j| = \sqrt{\sum_{k=1}^{d} (x_{ik} - x_{jk})^2} \]  \hspace{1cm} (5)

Where \( x_{ik} \) is the \( k \)-th spatial coordinate of firefly \( i \), and \( d \) is the number of problem variables.

3. The brightness of a firefly is determined by the objective function defined for the problem. The equation of the motion of the dimmer firefly \( i \) toward the brighter firefly \( j \) is:

\[ x_i^{t+1} = x_i^t + \beta_0 e^{-\gamma r_{ij}^2} (x_j^t - x_i^t) + \alpha \epsilon_i \]  \hspace{1cm} (6)

In the above equation, \( \alpha \) is the motion randomization parameter, and \( \epsilon_i \) is a random vector with normal distribution in the interval \([0, 1]\).
2.3. Whale Optimization Algorithm

Introduced in 2016 by Mirjalili and Lewis, the Whale Optimization Algorithm (WOA) takes inspiration from the bubble-net feeding behavior of humpback whales [9]. These whales create bubbles as they swim in a spiral path around and below their preys to direct them to a hunting zone before attacking them. The formula of this algorithm is:

$$X_{w(t+1)} = D_e^{-b} \cdot \cos(2\pi t) + X_p$$  \hspace{1cm} (7)

where, $D$ is the distance between whale $W$ and prey $P$, $t$ is the algorithm iteration counter, constant $b$ is a constant that determines the shape of the logarithmic spiral, and $I$ is a random number between -1 and +1.

The spiral motion of the whale is updated according to the best-found solution by the following equation:

$$D = |C \cdot X_p(t) - X_w(t)|$$

$$X_w(t+1) = X_p(t) - A \cdot D$$  \hspace{1cm} (8)

In this equation, $C = 2r$ and $A = 2a \cdot r - a$ are the coefficient vectors that help the whale approach the hunting zone. The vector $r$ is generated randomly with a uniform distribution between [0, 1]. The parameter $a$ decreases from 2 to 0 over the course of iterations. Each whale swims around the prey according to the following equations:

$$X_w(t+1) = \begin{cases} 
X_p(t) - A \cdot D & \text{if } P_e < 0.5 \\
D_e^{-b} \cdot \cos(2\pi t) + X_p & \text{if } P_e < 0.5 
\end{cases}$$  \hspace{1cm} (9)

In these equations, $P_e$ is the probability of using either a spiral motion or an encircling motion with shrinking radius to update the position. To enhance the results, whales’ search for the prey is randomized. For this purpose, a random position is defined as follows:

$$D = |C \cdot X_{\text{rand}}(t) - X_w(t)|$$

$$X_w(t+1) = X_{\text{rand}}(t) - A \cdot D$$  \hspace{1cm} (10)

In this equation, $X_{\text{rand}}$ is a position vector randomly selected from among the existing population.

2.4. Crow Search Algorithm

Developed by Asgarzadeh in 2016, the Crow Search Algorithm (CSA) is another population-based algorithm from the family of swarm intelligence methods [10]. Research has shown that crows have a large brain compared to their body, and can be considered among the smartest birds. These birds have a strong visual memory and can easily detect strangers or remember far locations and where exactly they hid their food.

CSA is inspired by behavior of crows when they search for a place to store food. In other words, it mimics how a crow finds a safe spot for storing food so that it can dig up and eat the food when needs it.

Assume a murder of crows with a population size of $N$. Let the position of crow $i$ in iteration $t$ be $x_i^t$. Each crow remembers the position of its hidden food but constantly searches for the best position. This position in iteration $t$ is denoted by $m_i^t$. The equation of motion of crow $i$ when it does not know that it is being followed by crow $j$ is:

$$x_i^{t+1} = x_i^t + r_t \cdot f_i^t \cdot (m_i^t - x_i^t)$$  \hspace{1cm} (11)

But a crow that gives a $P_i^t$ probability that it is being followed by another crow changes its destination according to the following equation:

$$\text{if } r_i < P_i^t$$

$$x_i^{t+1} = \text{update to random position}$$  \hspace{1cm} (12)

Where $r_i$ and $r_j$ are random numbers in [0, 1], and $f_i^t$ denotes the flight length of crow $i$ in iteration $t$. 
3. RC shear wall problem

Reinforced concrete is a non-homogeneous construction material consisting of concrete and steel bars acting as reinforcement. Hence, one aim of optimization of RC shear walls is to minimize the cost of both of these elements simultaneously. However, the resulting economic design must also meet the design requirements. These requirements can be divided into two groups, one consisting of basic wall design formulations and another concerning the rules and limitations specified in building codes. In the present research, RC shear walls are designed based on special boundary elements in accordance with the requirements of ACI318-11 [14].

3.1. objective function

In general, the RC shear wall optimization problem is defined as follows:

\[
\min \quad \text{Cost} \quad \text{Shear Wall}
\]

subject to

\[
g_i \leq 0
\]

In this study, the objective function is the cost of the shear wall. In other words, the aim to minimize the cost of RC shear wall without violating the design constraints. The cost of the project consists of the costs of rebars, concrete, and formworking.

Thus, the objective function of the RC shear wall optimization problem can be formulated as follows:

\[
\text{Cost}^{\text{Shear Wall}} = \text{Cost}^{\text{steel}} + \text{Cost}^{\text{concrete}} + \text{Cost}^{\text{formwork}}
\]

where

\[
\text{Cost}^{\text{steel}} = C_s \left( A_{sf} \times H_w + N_v \times A_{sv} \times H_w + N_h \times A_{sh} \times l_w \right)
\]

\[
\text{Cost}^{\text{concrete}} = C_c \left( t_w \times l_w + 2t_f \times (t_f - t_w) \right) H_w
\]

\[
\text{Cost}^{\text{formwork}} = C_f \left( 4 (b_f + t_f - 0.5t_w) + 2 (l_w - 2bf) \right) H_w
\]

The unit price of steel is considered to be 0.9 $/kg. Assuming an equivalent reinforcing steel density of \( \gamma_s = 7850 \) kg/m\(^3\), the steel cost per unit volume will be \( C_s = 7065 \) $/m\(^3\). Also, the concrete cost per unit volume is \( C_c = 60 \) $/m\(^3\) and the formworking cost per unit area is \( C_f = 18 \) $/m\(^2\).

In the objective function, \( H_w \) is the total height of the wall; \( l_w \) is the total length of the wall; \( t_w \) is the thickness of the web of the wall; \( t_f \) the thickness of the flange of the wall; \( A_{sf} \) is the total cross-sectional area of vertical rebars in the flange of the wall; \( A_{sv} \) is the cross-sectional area of each pair of vertical rebars in the web of the wall, and \( A_{sh} \) is the cross-sectional area of each pair of horizontal rebars in the web of the wall (Fig. 1).

The parameters \( N_h \) and \( N_v \) are the numbers of horizontal and vertical rebars in the web of the wall, which are calculated by the following equations:

\[
N_h = 2 \times \left( \frac{h_f}{S_h} \right)
\]

\[
N_v = 2 \times \left( \frac{l_w - 2b_f}{S_v} \right)
\]
In the above equation, $S_h$ and $S_v$ denote the spacing between horizontal rebars and between vertical rebars, which, according to ACI318-11, are subject to following limitations:

$$S_h \leq \min \left(3 \frac{l_y}{S} , 450 \right) \quad (17)$$

$$S_v \leq \min \left(3 \frac{l_y}{S} , 450 \right) \quad (18)$$

To improve the construction condition and the performance of the shear wall, the wall is better to be thicker than 200 mm \[15\]. According to ACI318-11, the effective width of the flange of the wall under compression (boundary element) should be greater than 300 mm. Therefore:

$$t_w \geq 200 \text{ (mm) } \quad (19)$$

$$t_f \geq 300 \text{ (mm) } \quad (20)$$

According to ACI318-11, reinforcements in the web of the shear wall are subject to following limitations \[14\]:

- The allowable range of $\rho_f$ (the ratio of the cross-sectional area of longitudinal rebars in the boundary element to the cross-sectional area of concrete in the boundary element) is defined as follows:

$$0.01 \leq \rho_f \leq 0.06 \quad (21)$$

- Provided that the following condition is met, the ratio of horizontal and vertical shear reinforcement in the web of the shear wall ($\rho_h$ and $\rho_v$) should not be less than 0.0025.

$$V_n \geq 0.083 \sqrt{f_c' A_{cv}} \quad A_{cv} = t_w \times l_w \quad (22)$$

- Furthermore, shear reinforcement should be continuous and distributed across the shear plane. Thus, for the horizontal shear reinforcements:

$$0.0025 \leq \rho_h \leq 0.01 \quad (23)$$

- The ratio of vertical shear reinforcements is given by the following equation:

$$\rho_v = 0.0025 + 0.5 \left( 2.5 - \frac{H_v}{l_w} \right) \left( \rho \geq 0.0025 \right) \quad (24)$$

Furthermore, if $h_w / l_w \geq 2.5$, then the ratio of vertical reinforcements should be set equal to the minimum value, that is 0.0025. Therefore, these reinforcements are subject to following limitations:

$$0.0025 \leq \rho_v \leq 0.01 \quad (25)$$

### 3.2. Seismic design of the shear wall

In this study, the seismic design of the RC shear wall is optimized with the help of constraints defined in 9 groups. The effects of constraints on the objective function are formulated as an additive penalty function, in the sense that algorithm sums the penalty function with the objective function and minimizes the result (F function):

$$\min F = Cost_{Shear\ Wall} + f_{penalty}$$

$$f_{penalty} = \alpha V$$

$$V = \sum_{i=1}^{n} \left( \max \left[ 0, g_i \right] \right)$$

$$\alpha = \frac{4}{5}$$

$$n = 9$$
The penalty function consists of two parameters. First, the penalty factor which is a positive constant \( \omega = 5 \times 10^5 \), and second, the parameter \( V \), which represents the extent to which a constraint is violated.

When designing the boundary elements, each element can be considered as a short column where the axial force \( (P_a) \), which includes all gravity effects, and the bending moment \( (M_a) \) could be decomposed into a pure tensile force and a pure compressive force (Fig. 2). In doing so, the longitudinal reinforcements of the shear wall (boundary element) are designed based on the more critical effects of these forces [14, 16].

![Fig. 2. Decomposition of the shear wall forces into pure tensile and compressive forces](image)

Tensile axial force (Eq. 27) and tensile strength (Eq. 28) are:

\[
T = \frac{M_a}{l_u - b_f}, \quad (27)
\]

\[
T_a = \varphi_t A_{sf} F_y \quad (28)
\]

Where \( \varphi_t \) is the tensile strength degradation coefficient, which is considered to be 0.9. Thus, the first constraint is defined as:

\[
g_1 = \frac{T}{T_a} - 1 \leq 0 \quad (29)
\]

When the element is under compression:

\[
C = P_u + \frac{M_a}{l_u - b_f}, \quad (30)
\]

\[
C_a = \varphi_c \left\{ 0.85 f_c' \left( A_g - A_{sf} \right) + A_{sf} F_y \right\} \quad (31)
\]

Where \( \varphi_c \) is the compressive strength degradation coefficient, which is considered to be 0.65, and \( A_g = t_f \times b_f \) is the gross cross-sectional area of the boundary element. Thus, the second constraint is defined as follows:

\[
g_2 = \frac{C}{C_a} - 1 \leq 0 \quad (32)
\]

The nominal shear strength of the wall or the columns that contribute to the bearing of lateral forces should not exceed the following [14]:

\[
\]
Thus, the third constraint emerges as:

\[ g_3 = \frac{V_a}{V_{na}} - 1 \leq 0 \]  

(34)

Furthermore, the shear walls must have a special boundary element on any boundary where the maximum compressive stress at the extreme compression fiber is greater than 0.2\( f'_c \) (Fig. 3).

![Fig. 3. Placement of special stirrup in the boundary element [14].](image)

Therefore, the stress calculations for the extreme compression fiber are conducted based on the factored seismic-load-induced forces. This stress is obtained by the use of a linear elastic modulus and gross cross-section specifications:

\[ \sigma = \frac{P_u}{A_g} + \frac{M_u}{S_g} \]  

(35)

In this equation, \( A_g \) is the gross cross-sectional area and \( S_g \) is the gross section modulus area of the shear wall. Thus, the fourth constraint is defined in such a way that the shear wall requires the boundary element. Therefore:

\[ g_4 = \frac{0.2f'_c}{\sigma} - 1 \leq 0 \]  

(36)

In special RC shear walls, the nominal shear strength of the wall should be less than the following:

\[ V_{max} = A_{cy} \left( \alpha_c \sqrt{f_c} \rho_h F_{yr} \right) \]  

(37)

Where \( A_{cy} = l_c \times t_c \) is the gross cross-sectional area of the shear wall and \( \rho_h \) is the ratio of transverse reinforcements to the gross cross-sectional area of the shear wall. Therefore, the fifth constraint is defined as follows:

\[ g_5 = \frac{V_a}{V_{max}} - 1 \leq 0 \]  

(38)

The horizontal length of the boundary element \( (b_f) \) at the extreme compression fiber must be greater than the following [14]:

\[ b_{f_{max}} = \max \left( \frac{2}{2 \sqrt{c - 0.1l_w}} \right) \]  

(39)

Where \( c \) is the height of the compressive area of the concrete, which is approximated by the following equation (vertical reinforcements are ignored) [16]:

\[ V_{na} = 0.66 \sqrt{f_c t_w} \]  

(33)
\[
c = \frac{A_g \times F_g + P_u}{f' \times t_f}
\]

Thus, the sixth constraint is defined as follows:

\[
g_6 = \frac{b f_{\min}}{b_f} - 1 \leq 0
\]

Furthermore, since the boundary elements are symmetric, \( t_f \) must be at least as thick as the web of the wall [14]. Thus, the seventh constraint is:

\[
g_7 = \frac{t_w}{t_f} - 1 \leq 0
\]

To achieve proper seismic performance, the boundary element should constitute a confined area and thus requires special stirrup reinforcement. The special transverse reinforcement of the boundary element (\( A_{sb} \)) is defined such that the total cross-sectional area of rectangular stirrups on each side is not less than the following:

\[
A_{sb} = \max \left\{ \begin{array}{l}
0.09 b f' \frac{f_c}{F_{fy}} \\
0.3 b f' \left[ \left( A_g / A_c \right) - 1 \right]
\end{array} \right\}
\]

In this equation, \( b_f \) is the size of the core of the boundary element at each side, which equals the center-to-center distance of confining rebars, the parameter \( A_c \) is the cross-sectional area of the core of the boundary element, and the parameter \( A_g = b_f \times t_f \) is the total cross-sectional area of the element. The parameter \( s \) is the allowable spacing between stirrups, which is defined as follows:

\[
S \leq \min \left\{ \frac{h_{\min}}{4}, 6 \phi_f, S_0 = 100 + \frac{350 - h_c}{3} \right\}
\]

In the above equation, \( h_{\min} \) is the smallest dimension of the boundary element, \( \phi_f \) is the diameter of the longitudinal rebars and \( h_c \) is the greatest spacing between the stirrup hooks (\( x_i \)). The parameter \( S_0 \) is limited to a minimum 100 mm and a maximum of 150 mm (100 ≤ \( S_0 \) ≤ 150).

The minimum spacing between the vertical rebars of the boundary element (stirrup-confined compressive members) must be greater than:

1. \( 1.5 d_b \) (where \( d_b \) is the largest diameter of the vertical rebar).
2. 40 millimeter.

With the rebar diameter set to 32 mm and the net space considered, the minimum spacing is assumed to be 80 mm. On the other hand, the maximum spacing is limited to 200 mm (80 ≤ \( S_f \) ≤ 200). Therefore, to ensure the fulfillment of this condition, the eighth constraint is defined as follows:

\[
g_8 = \frac{S_f}{S_{fa}} - 1 \leq 0
\]

The algorithm calculates the area needed for the vertical reinforcements and provides an engineering design (\( N_f \phi D_f \@ S_f \)) based on the assumption of rebar diameter (\( D_f \)). Therefore, the number of rebars is addressed in the ninth constraint:
In this equation, $A_{sc}$ is the required area for vertical reinforcements and the parameters $(N_f)_x$ and $(N_f)_y$ are the algorithm outputs for the number of vertical rebars in the boundary element along $x$ and $y$ axes. After checking the possibility of placing this number of rebars in the boundary element with given dimensions, the algorithm provides an engineering design.

### 3.3. Optimization method

In general, optimization problems can be formulated in two ways: discrete and continuous. RC shear wall optimization problem can also be formulated in both ways, but so far researchers have preferred the discrete approach to this problem [17-19].

In the discrete design optimization, the user must introduce thousands of hypothetical designs to the algorithm. The algorithm then evaluates these designs one by one in search for the best solution, i.e. the design that yields the lowest costs without violating the constraints.

This approach has two major flaws. Firstly, preparing thousands of designs for a shear wall is a difficult and time-consuming task. Secondly, the designs to be introduced to the algorithm should be quite diverse and cover a wide range of specifications so that the algorithm can access at least one suitable design for any given force. Otherwise, each database will be applicable to only one specific load and will not be generalizable.

In this study, shear wall design optimization is performed by the continuous approach. Continuous optimization means that the algorithm is able to select any logical value for design parameters (length, cross-sectional area of reinforcement, etc.). Although the continuous nature of the method improves the algorithm’s ability to find a solution closer to the optimum (a better near-optimal solution), reaching a reasonable design requires very careful selection of design variables and constraints. This is because concrete structures, unlike steel structures, have a nonhomogeneous nature that makes their design more complex and difficult.

In other words, an RC shear wall can be designed with any dimension, and so there could be numerous variations in the number and diameter of rebars and even reinforcement layout in different parts of the section. In addition, continuous nature of the optimization method means that all these factors and requirements should be incorporated into the algorithm (contrary to the discrete optimization, where the designs must be prepared in advance by the user).

Supposing that the shear wall problem consists of only two variables, namely the cross-sectional area of reinforcements ($A_s$) and the wall thickness ($t_w$), the optimization can be implemented in four modes (Fig. 4):

1. **Continuous optimization (inapplicable to real-world construction):** The wall dimensions and the cross-sectional area of reinforcements can take any value (Plan-A)

2. **Continuous optimization - Discrete reinforcement cross-sectional area:** The wall dimensions can take any value, but the diameter of the rebars is given based on the required cross-sectional area.

3. **Continuous optimization - Discrete wall dimensions and reinforcement cross-sectional area:** The wall dimensions are given as real-world applicable discrete values and the diameter of the rebars is given based on the required cross-sectional area. In this study, dimensions of the shear walls are given in multiples of 5 mm (Plan-B) and 50 mm (Plan-C).

4. **Discrete optimization:** In this approach, wall designs are prepared in advance (Plan-D) [19].
4. Numerical Example

Consider an RC shear wall with a height of 42 m and a total length of 5.5 m (center to center distance of the boundary elements). The structure has 12 stories and each story is 3.5 meters high. This shear wall and the forces acting on each story are illustrated in Fig. 5 [19].

In this example, the yield stress of steel is 400 N/mm$^2$, the maximum 28-day compressive strength of concrete is 25 N/mm$^2$, and the diameter of the rebars is 32 mm.
4.1. Results of the shear wall design optimization

The shear wall design problem was optimized by four algorithms: PSO, FA, WOA, and CSA, each with 500 iterations. Figure (6) illustrates the wall cost calculated over different iterations of each algorithm.

Figure (7) shows the mean degree of constraint violation by each algorithm in each iteration. As can be seen, this value has decreased over iterations, ultimately converging to zero.

The results of wall optimization according to Plans A and B are presented in Tables (1). To ensure the accuracy of the results, the outputs were obtained from 300 independent runs of each algorithm. The comparisons are made based on minimum, mean and maximum values of these results.
Table. 1. The results of wall optimization according to Plans A and B

<table>
<thead>
<tr>
<th></th>
<th>Plan-A</th>
<th>Plan-B</th>
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<tbody>
<tr>
<td><strong>Algorithm</strong></td>
<td>PSO</td>
<td>FA</td>
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<td>$t_w$</td>
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<td>$t_f$</td>
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</table>

The figure (8) is plotted for Plan-B to illustrate the shear wall design performance of the algorithms in this mode of optimization. This circle diagram is graded from 1 to 300, which correspond to the number of runs of each algorithm. In this diagram, the optimal design costs in different runs relative to the Best Solution are connected together. Naturally, these relative values are less than or equal to one (they are equal to one when the algorithm solution equals the Best Solution). Therefore, any algorithm that covers a greater portion of the circle has approached the Best Solution more frequently and is, therefore, more desirable. Thus, it can be concluded that the best algorithms in the order of performance in this mode of optimization are CSA, WOA, PSO, and FA, respectively.
In the following, the shear wall designs obtained at a random iteration of CSA in Plan-B are examined as an example. The specifications of these walls and the cost details are presented in fig. 9.

Fig. 9. Shear walls generated at a random iteration of CSA in Plan-B

In Table 2, the results obtained by the continuous method (Plan-C) are compared with the results the discrete approach. For all algorithms the number of iteration are considered as 500, except in ref. [19], where it was 200 iterations.

As shown in Table (2), the wall cost in the three proposed plans (Plans A, B, and C) is about 5 % lower than the cost in the discrete method. One of the main causes of this reduced cost is the dimension of the boundary element, as the continuous method can choose any dimension that fits the problem without being confined to preset designs as it is in the discrete method. In the discrete method, the choice of best design based on cost and force criteria should be made from 7568 shear wall designs that are prepared in advance with full details and introduced to the algorithm before the optimization process. But in the continuous approach, the algorithm evaluates infinite states of reinforcement and dimensions to eventually obtain the most economical design. It can, therefore, be concluded that the continuous method is free of database restrictions, and can test the boundary elements with a wide variety of dimensions and reinforcement layouts; a task that is much more difficult in the discrete method. Because of these differences, the designs generated by the proposed method are more economical than those given by the discrete method. Further details about the presented plans are illustrated in Fig. (10).
Fig. 10. Details of the shear wall design: a) Plan-D [19], b) the present work (Plan-C), c) the present work (Plan-B)

Based on the results of Tables (2), the minimum wall costs obtained by the use of algorithms with the proposed methods are plotted in Fig. (11).

These results show that in most algorithms, the lowest cost has been obtained by the continuous method (Plan-A). This is because in this method, all possible solutions are selectable and the algorithm can further approach the Best Solution. However, the results of this mode of optimization are not applicable to real world. In contrast, the continuous method formulated with discrete dimensional variables (Plan-C) managed to satisfy the applicability requirement with a slight rise in cost (in this example, its solution is about $500 more expensive than the solution of Plan-A). In Plan-B, the construction accuracy has increased to 5 mm and the cost is very close to the cost of Plan-A.

According to the requirements discussed in Section (2.3), the stirrups in the boundary element are placed such that they confine every other longitudinal rebar, and their spacing does not exceed 350 mm. The details regarding this part of the results are presented in Fig. (12).
4.2. Evaluation of the optimum result

In this section, the cost contour of the wall designed with Plan-C is plotted to identify the optimum point. This contour covers the range of (450, 1400) for both length and width of the boundary element. The wall cost for the values in this range is calculated with other variables kept constant. In this contour, the coordinate axes represent the dimensions of the boundary element (in millimeters) and dashed lines represent the cost (Fig. 13). Naturally, the smaller is the size of the boundary element, the lower is the wall cost. The bold lines represent the constraints, and the intersection of these zones delimits the solution space (colored green in the figure).

The optimal point found by the algorithm and its data are displayed in the figure. It can be seen that the algorithm has found the best possible value for the dimension of the boundary element, as this solution matches the point where the cost is minimized without violating the constraints of the design.

5. Conclusion

The optimization of RC shear wall design should account for seismic conditions as well as economic criteria. In this study, RC shear wall design optimization was performed in accordance with seismic criteria suggested in ACI318-11.

The main advantage of the proposed method is the continuous nature of the design algorithm, which eliminates the need for a database of previously prepared designs (which itself requires extensive time and effort. Furthermore, the solutions given by the discrete approach are not necessarily optimal, because they are dictated by the prepared design database. The continuous method lacks this flaw, but is more difficult to code, since the algorithm must incorporate all primary elements of wall design (reinforcement layout, rebar spacing, etc.) into the optimization process.

A comparison between the discrete method and the proposed continuous method showed that, even after considering the real-world applicability conditions in the proposed method (Plan-C), its outputs were more economical (about 5%) than the results of the discrete method.
References


