Omega 40 (2012) 611–618

Contents lists available at SciVerse ScienceDirect

Omega

journal homepage: www.elsevier.com/locate/omega

DEA models for extended two-stage network structures

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ARTICLE INFO

Article history:
Received 6 August 2011
Accepted 27 November 2011
Processed by Associate Editor Huang
Available online 6 December 2011

Keywords:
Data envelopment analysis (DEA)
Two-stage
Game
Centralized
Efficiency

ABSTRACT

Liang et al. (2008) [1] developed DEA models based upon game approach to decompose efficiency for two-stage network structures where all outputs of the first stage are the only inputs to the second stage. This paper extends Liang et al. (2008) [1] by assuming that the inputs to the second stage include both the outputs from the first stage and additional inputs to the second stage. Two models are proposed to evaluate the performance of this type general two-stage network structures. One is a non-linear centralized model whose global optimal solutions can be estimated using a heuristic search procedure. The other is a non-cooperative model, in which one of the stages is regarded as the leader and the other is the follower. The newly developed models are applied to a case of regional R&D of China.

1. Introduction

Data envelopment analysis (DEA), developed by Charnes et al. [2], is a mathematical programming approach for analyzing the relative efficiency of peer decision making units (DMUs), which have multiple inputs and multiple outputs. Previous works have shown that DEA can be applied in various of settings, such as bank performance [3,4], production planning [5], bankruptcy assessment [6], R&D performance [7], agricultural economics [8], airport performance [9] and other applications [10]. In conventional DEA models,[1] DMUs are treated as black-boxes and the internal structure of DMUs is ignored. In recent years, a number of studies have looked at DMUs with network structures (see, e.g., Färe and Grosskopf [11], Tone and Tsutsui [12], Fukuyama and Weber [13], Castelli et al. [14], Kao [15], Kao and Hwang [16] and Liang et al. [1]). In a survey by Cook et al. [17], the authors point out several approaches in modeling DMUs with a two-stage network structure. Typically, models are developed based upon additive or geometric mean efficiency decompositions. While the network DEA approach of Färe and Grosskopf [11] can deal with different network structures, it cannot provide an efficiency decomposition or efficiency ratings for sub-DMUs that constitute the entire network DMUs. Using slacks-based models, Tone and Tsutsui [12] develop a network DEA model that evaluate both divisional and overall efficiencies of DMUs. Their paper assumes that (i) a network consists of several divisions, (ii) the divisional efficiency is a specific-division's index relative to its counterparts of other networks and (iii) the overall efficiency of a network is the weighted harmonic mean of its divisional scores with the weights set exogenously. By introducing dummy processes, Kao [15] transforms a general network structure system into series stages, which comprise of several parallel processes. Then, the author uses the approach developed by Kao and Hwang [16] to decompose series structure and the approach developed by Kao [18] to decompose parallel structure.

Cook et al. [19], on the other hand, develop models for DMUs with network structures based upon additive efficiency decomposition. Their approach can be viewed as a centralized model of Liang et al. [1]. The centralized model of Liang et al. [1] assumes the overall efficiency is a product or sum of divisional efficiencies. For example, consider the approach of Kao and Hwang [16] where a set of insurance companies are assumed to have a two-stage operations of premium acquisition and profit generation. The overall efficiency is then a product of premium acquisition efficiency and profit generation efficiency. Liang et al. [1] classify this type of modeling technique or efficiency decomposition as cooperative or centralized game approach, as the efficiency scores of all sub-DMUs or stages are simultaneously optimized.

Liang et al. [1] further introduce modeling two-stage network DMUs from the perspective of the non-cooperative game. The non-cooperative approach is characterized by the leader–follower, or Stackelberg game. For example, we assume that the first stage of premium acquisition is the leader, then the first

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[1] The conventional DEA models are the models developed by Charnes et al. [2] and their modifications (see Handbook on Data Envelopment Analysis, edited by Cooper et al. [10]).

0305-0483/$ - see front matter
doi:10.1016/j.omega.2011.11.007

stage performance is more important, and the efficiency of the second stage of profit generation is computed subject to the requirement that the efficiency of the first stage is to stay fixed. In a similar manner, we can also assume the second stage is the leader and the first stage is the follower.

Note that while the centralized model approach of Liang et al. [1] can be applied to DMUs with any network structures by assuming the overall efficiency is a weighted average of individual stage (or divisional) efficiencies, the leader–follower cannot be easily applied. Note also that the approach of Liang et al. [1] or Kao and Hwang [16] is developed under the assumption that the outputs from the first stage all become the only inputs to the second stage. The current paper extends Liang et al. [1] and Kao and Hwang [16] by assuming that the second stage has its own inputs in addition to outputs from the first stage.

For example, Liang et al. [20] study this type of two-stage network structure in analyzing the performance of a set of hypothetical supply chains. Other examples can be found in manufacturing with two sub-processes, one is production and the other is distribution. The inputs of first stage are manufacturing facilities, raw materials and components, laborers and operating fees of manufacturing department; the outputs of first stage are finished goods, which are also part of the inputs to the second stage. Another part of inputs to the second stage are advertisement fee, and employees of market department.

Due to the existence of additional inputs to the second stage, the approach of Liang et al. [1] or Kao and Hwang [16] will result a non-linear program that cannot be converted into linear programming problems if we assume that the overall efficiency is a geometric mean of two stages' efficiency. The current paper develops procedures to convert the resulting non-linear programs into parametric linear programs so that the global optimal solution can be found if one adopts the centralized and leader–follower approaches of Liang et al. [1]. Therefore, the current paper extends the approach of Liang et al. [1] to more general two-stage network structures.

The remainder of the paper is organized as follows. In the next section we extend the models of Liang et al. [1] to evaluate performance of the two-stage network structure with additional inputs to the second stage. Relations between the two approaches are established. The two approaches are then illustrated with an example about regional R&D process in China. We demonstrate how to estimate the global optimal solution from our converted non-linear program. Conclusions are given in the last section.

2. DEA models

Figs. 1 and 2 illustrate two types of two-stage network structures. Fig. 1 studied by Liang et al. [1] or Kao and Hwang [16] assumes that the outputs from the first stage all become the

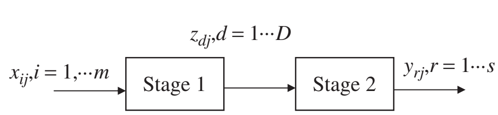

Fig. 1. Two-stage process of DMU_j.

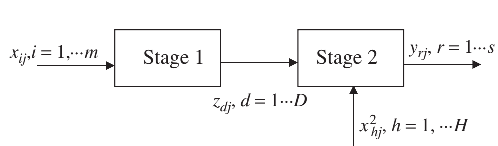

Fig. 2. Two-stage process with additional inputs to the second stage.

only inputs to the second stage. These measures in-between the two stages are called intermediate measures. Fig. 2 relaxes the above assumption by introducing inputs to the second stage in addition to the intermediate measures.

We assume that each DMU_j ($j=1, 2, \ldots, n$) has m inputs to the first stage, x_{ij}, ($i=1, 2, \ldots, m$) and D outputs (intermediate measures) from the first stage, z_{dj}, ($d=1, 2, \ldots, D$). These D outputs then become part of the inputs to the second stage. Another part of inputs are x^2_{hj} ($h=1, 2, \ldots, H$). The outputs from the second stage are y_{rj}, ($r=1, 2, \ldots, s$).

We next develop models based upon the approaches of Liang et al. [1] to analyze the performance of extended two-stage network structure as depicted in Fig. 2. Lastly, the study of relationships among efficiencies calculated through these models is presented.

2.1. Centralized model

There are many cases that each sub-DMU works together to reach the optimal performance of the overall DMU. For example, marketing and production departments would cooperate to maximize company's profit. Liang et al. [1] developed a centralized approach to analyze the performance of two-stage network structure described in Fig. 1. In their model, overall efficiency of the two-stage process is defined as the product of two stages' efficiencies. In a similar manner, based upon the ratio efficiency of the CCR model (Charnes et al. [2]), we can establish the following model for Fig. 2:

$$\theta^{cen} = \max \theta_1^o * \theta_2^o = \max \frac{\sum_{d=1}^{D} w_d z_{do}}{\sum_{i=1}^{m} v_i x_{io}} * \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{d=1}^{D} w_d z_{do} + \sum_{h=1}^{H} Q_h x^2_{ho}}$$

$$s.t. \quad \frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \quad \forall j \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj} + \sum_{h=1}^{H} Q_h x^2_{hj}} \le 1 \quad \forall j;$$

$$v_i, w_d, Q_h, u_r \ge 0, \forall i, d, h, r \tag{1}$$

where θ_1^o and θ_2^o represent the ratio efficiencies for stages 1 and 2, respectively. As in Liang et al. [1], it is assumed that a same set of weights (w_d) is applied to the intermediate measures (z_{dj}) for both stages. For example, the manufacturer and retailer jointly determine the price, order quantity, etc. to achieve maximum profit (Huang and Li [21]). Herein, as in Liang et al. [1], we also assume that the "worth" or value accorded to the intermediate variables is the same regardless of whether they are being viewed as inputs or outputs.

Due to the additional inputs to the second stage ($\sum_{h=1}^{H} Q_h x^2_{ho}$), model (1) cannot be converted into a linear program. We here introduce a heuristic method to solve this problem.

Consider the following model:

$$\theta_1^{o^{max}} = \max \frac{\sum_{d=1}^{D} w_d z_{do}}{\sum_{i=1}^{m} v_i x_{io}}$$

$$s.t. \quad \frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \quad \forall j \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj} + \sum_{h=1}^{H} Q_h x^2_{hj}} \le 1 \quad \forall j;$$

$$v_i, w_d, Q_h, u_r \ge 0, \forall i, d, h, r \tag{2}$$

In model (2), the two sets of constraints are the same to the ones in model (1), which ensure the efficiencies for the first and the second stage do not exceed one. Therefore, model (2) can be used to estimate the best possible efficiency for stage 1. Denote the optimal value to model (2) as $\theta_1^{o^{max}}$, then the efficiency for the first stage θ_1^o must satisfy $\theta_1^o \in [0, \theta_1^{o^{max}}]$.

Model (2) is a non-linear model, but can be converted into a linear program through the Charnes–Cooper transformation as follows:

$$\theta_1^{o^{max}} = \max \sum_{d=1}^{D} w_d z_{do}$$
$$s.t.\quad \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad \forall j$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{h=1}^{H} Q_h x_{hj}^2 - \sum_{d=1}^{D} w_d z_{dj} \le 0 \quad \forall j \quad \sum_{i=1}^{m} v_i x_{io} = 1;$$
$$v_i, w_d, Q_h, u_r \ge 0, \forall i,d,h,r \tag{3}$$

Therefore, the efficiency of the first stage θ_1^o can be treated as a variable $\theta_1^o \in [0, \theta_1^{o^{max}}]$ and the overall efficiency denoted as $\theta^{cen,1,*}$ can be considered as a function of θ_1^o as follows (or model (1) can be written as):

$$\theta^{cen,1,*} = \max \theta_1^o * \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{d=1}^{D} w_d z_{do} + \sum_{h=1}^{H} Q_h x_{ho}^2}$$
$$s.t.\quad \frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \quad \forall j \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj} + \sum_{h=1}^{H} Q_h x_{hj}^2} \le 1 \quad \forall j;$$
$$\frac{\sum_{d=1}^{D} w_d z_{do}}{\sum_{i=1}^{m} v_i x_{io}} = \theta_1^o \quad \theta_1^o \in \left[0, \theta_1^{o^{max}}\right]; \quad v_i, w_d, Q_h, u_r \ge 0, \forall i,d,h,r; \tag{4}$$

Model (4) now can be transformed via the Charnes–Cooper transformation as follows:

$$\theta^{cen,1,*} = \max \theta_1^o * \sum_{r=1}^{s} u_r y_{ro}$$
$$s.t.\quad \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad \forall j$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{h=1}^{H} Q_h x_{hj}^2 - \sum_{d=1}^{D} w_d z_{dj} \le 0 \quad \forall j$$
$$\sum_{h=1}^{H} Q_h x_{ho}^2 + \sum_{d=1}^{D} w_d z_{do} = 1 \quad \sum_{d=1}^{D} w_d z_{do} - \theta_1^o \sum_{i=1}^{m} v_i x_{io} = 0;$$
$$\theta_1^o \in [0, \theta_1^{o^{max}}] \quad v_i, w_d, Q_h, u_r \ge 0, \forall i,d,h,r; \tag{5}$$

Let $\theta_1^o = \theta_1^{o^{max}} - k\Delta\varepsilon$. Here $\Delta\varepsilon$ is a step size,[2] $k = 0,1,2 \ldots [k^{max}]+1$, where $[K^{max}]$ is the maximal integer, which is smaller than or equal to $\theta_1^{o^{max}}/\Delta\varepsilon$. Now, given each θ_1^o, model (5) can be solved as a linear program.

In solving model (5), we set the initial k value as the lower bound, $k=0$. Then we increase k at each step. We solve each linear program of model (5) corresponding to each k and denote the optimal value to model (5) as $\theta^{cen,1}(k)$. Therefore, the global optimal efficiency of the system under evaluation can be estimated as $\hat{\theta}^{cen,1,*} = \max_k \theta^{cen,1}(k)$.

Note, when the efficiency of the entire two-stage system under evaluation is $\hat{\theta}^{cen,1,*}$, the maximal efficiency for its first stage is $\hat{\theta}_1^{o+} = \theta_1^o(k^*)$, where $k^* = \min\{k \mid \hat{\theta}^{cen,1,*} = \theta^{cen,1}(k)\}$. Therefore, the minimal efficiency for its second stage is $\hat{\theta}_2^{o-} = ((\hat{\theta}^{cen,1,*})/(\hat{\theta}_1^{o+}))$.

Similarly, we can also treat the efficiency of stage 2 as a variable. The optimal efficiency of the second stage $\theta_2^{o^{max}}$ can be calculated using a model similar to model (2). Then, according to the above-mentioned algorithm, we can get the global efficiency $\hat{\theta}^{cen,2,*}$ and its corresponding maximal efficiency for the second stage $\hat{\theta}_2^{o+}$. Then the minimal efficiency for the first stage is $\hat{\theta}_1^{o-} = ((\hat{\theta}^{cen,2,*})/(\hat{\theta}_2^{o+}))$. (See Appendix A for the detailed development.)

[2] The smaller the $\Delta\varepsilon$ value we select, the more precise results we obtain.

Note that, no matter which stage's efficiency is assumed as a variable in deriving the efficiency for the entire two-stage system, the same optimal global optimal efficiency should be obtained, i.e., $\theta^{cen,1,*} = \theta^{cen,2,*}$. The efficiency decomposition is unique if $\theta_1^{o+} = \theta_1^{o-}$ and $\theta_2^{o+} = \theta_2^{o-}$.

2.2. Non-cooperative model

The models presented in previous section for analyzing the extended two-stage network structure with additional inputs to the second stage are under centralized decision-making environment. In this section, we extend the non-cooperative approach developed by Liang et al. [1] to analyze this extended two-stage network structure. We first treat stage 1 as the leader (this sub-process is assumed to be more important) and stage 2 as the follower. The efficiency of the first stage (the leader) for a specific DMU_o is calculated using the CCR model (Charnes et al. [2]) as follows:

$$e_1^{o*} = \max \sum_{d=1}^{D} w_d z_{do}$$
$$s.t.\quad \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad \forall j \quad \sum_{i=1}^{m} v_i x_{io} = 1;$$
$$v_i, w_d, Q_h, u_r \ge 0, \forall i,d,h,r \tag{6}$$

Let $v_i^*, i = 1 \cdots m$, $w_d^*, d = 1 \cdots D$ be a set of optimal weights associated with the efficiency of stage 1 e_1^{o*} in model (6). Since the two sub-processes are related with each other by intermediate measures, v_i^*, w_d^* need to be introduced to the next model for calculating the efficiency of stage 2. However, the weights v_i^*, w_d^* may not be unique. Doyle and Green [22] develop second goal models to solve a similar problem in DEA cross-efficiency. Following this idea, we develop a model that maximizes the efficiency of stage 2 as the objective function while fixing the efficiency of stage 1. The model is as follows:

$$e_2^{o*} = \max \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{d=1}^{D} w_d z_{do} + \sum_{h=1}^{H} Q_h x_{ho}^2} \quad s.t.\quad \frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \quad \forall j;$$
$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj} + \sum_{h=1}^{H} Q_h x_{hj}^2} \le 1 \quad \forall j \quad \frac{\sum_{d=1}^{D} w_d z_{do}}{\sum_{i=1}^{m} v_i x_{io}} = e_1^{o*};$$
$$v_i, w_d, Q_h, u_r \ge 0, \forall i,d,h,r \tag{7}$$

In model (7), the efficiency for the second stage of DMU_o is optimized based upon that the efficiency of the first stage e_1^{o*} remains unchanged. Model (7) can be transformed as

$$e_2^{o*} = \max \sum_{r=1}^{s} u_r y_{rj_0} \quad s.t.\quad \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad \forall j;$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{h=1}^{H} Q_h x_{hj}^2 - \sum_{d=1}^{D} w_d z_{dj} \le 0 \quad \forall j$$
$$\sum_{h=1}^{H} Q_h x_{ho}^2 + \sum_{d=1}^{D} w_d z_{do} = 1 \quad \sum_{d=1}^{D} w_d z_{do} - e_1^{o*} \sum_{i=1}^{m} v_i x_{io} = 0;$$
$$v_i, w_d, Q_h, u_r \ge 0, \forall i,d,h,r \tag{8}$$

Denote the optimal value to model (8) as e_2^{o*}, then the efficiency for the entire two-stage system or DMU_o is $e^{non,1} = e_1^{o*} * e_2^{o*}$.

In a similar manner, if we assume the second stage is the leader, the regular CCR DEA efficiency π_2^{o*} for that stage can be calculated via using the standard CCR model with inputs (z_{dj} and x_{hj}^2) and outputs (y_{rj}). Then, the efficiency score (π_1^{o*}) for the first stage (follower) can be obtained by solving a model with the restriction that the second stage score π_2^{o*} remains unchanged. (See Appendix B for detailed development). The overall efficiency of the entire system in this situation is $\pi^{non,2} = \pi_1^{o*} * \pi_2^{o*}$.

2.3. Relations between the two models

This section gives three theorems to illustrate the relations between the centralized model and the non-cooperative model.

Theorem 1. $e_1^{o*} \geq \pi_1^{o*}, e_2^{o*} \leq \pi_2^{o*}$, where $e_1^{o*}$and e_2^{o*} are the efficiencies for the first stage and the second stage, respectively, when stage 1 is assumed the leader. π_1^{o*} and π_2^{o*} are the efficiencies for the first stage and the second stage, respectively, when stage 2 is assumed the leader.

Proof. See the Appendix C. □

Theorem 2. (1) To each DMU, $\theta^{cen,1,*}=\theta^{cen,2,*}$ where $\theta^{cen,1,*}$ and $\theta^{cen,2,*}$ are the optimal efficiencies for the system based upon the centralized model when the efficiency of stage 1 and the efficiency of stage 2 are treated as variables, respectively; (2) $\theta^{cen} \geq e^{non,1,*}$, $\theta^{cen} \geq \pi^{non,2,*}$, where $\theta^{cen}=\theta^{cen,1,*}=\theta^{cen,2,*}$, and $e^{non,1,*}$ and $\pi^{non,2,*}$ are the optimal efficiencies for the system when stage 1 or stage 2 is assumed the leader, respectively.

Proof. See the Appendix C. □

Theorem 3. If there is only one intermediate measure, then the optimal efficiency for the system is unique based upon either the centralized model or non-cooperative model, such that $\theta^{cen}=\theta^{cen,1,*}=\theta^{cen,2,*}=e^{non,1,*}=\pi^{non,2,*}$. The efficiency decomposition is also unique such that $e_1^{o*}=\pi_1^{o*}=\theta_1^{CCR}, e_2^{o*}=\pi_2^{o*}=\theta_2^{CCR}$, where θ_1^{CCR} and θ_2^{CCR} are the efficiencies for the first and second stage as applying the standard CCR model.

Proof. See the Appendix C. □

3. An illustrative application

This section presents a real example about regional R&D process of 30 Provincial Level Regions in China. Fig. 3 shows a regional R&D process, which contains two sub-processes, one is technology development and the other is economic application.

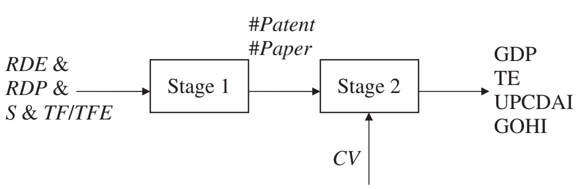

Fig. 3. A two-stage regional R&D system.

Table 2
Efficiency for Zhejiang Province (DMU26) corresponding to each k based upon the centralized model.

k	$\theta_1^o(k)=\theta_1^{o^{max}}-k*0.01$	$\theta^{cen,1,*}(k)$
0–10	0.9111–0.8111	0.4712–0.6231
11–18	0.8011–0.7311	06339–0.6682
19	0.7211	0.6688 (Global optimal efficiency)
20–30	0.7111–0.6111	0.6685–0.5926
31–40	0.6011–0.5111	0.5830–0.4971
41–50	0.5011–0.4111	0.4876–0.4017
51–60	0.4011–0.3111	0.3922–0.3063
61–70	0.3011–0.2111	0.2968–0.2109
71–80	0.2011–0.1111	0.2014–0.1155
81–90	0.1011–0.0111	0.1060–0.0201
91–92	0.0011–0.0000	0.0106–0.0000

Table 1
Inputs and outputs of R&D for 30 Provincial Level Regions in Mainland China.

Region type	DMU	Region	R&DP	R&DE	S&TF/TFE	Patent	Paper	CV	GDP	TE	UPCDAI	GOHI
Municipality												
	1	Beijing	10.34786	668.6351	5.445765	9157	65951	1236.245	12153.03	483.7932	26738.48	2757.14
	2	Chongqing	2.00665	79.45994	1.203814	834	13737	38.31581	6530.01	42.80071	15748.67	352.84
	3	Shanghai	6.46163	423.3774	7.201873	5997	32733	435.4108	15046.45	1417.96027	28837.78	5557.45
	4	Tianjin	2.8783	178.4661	3.023746	1889	12472	105.4611	7521.85	298.92719	21402.01	1901.07
Province												
	5	Anhui	3.01654	135.9535	1.702644	795	13699	35.61736	10062.82	88.86487	14085.74	460.31
	6	Fujian	2.27886	135.3819	1.975486	824	9075	23.25944	12236.53	533.1911	19576.83	1972.01
	7	Gansu	1.27445	37.26124	0.817095	227	7856	35.62869	3387.56	7.35512	11929.78	67.39
	8	Guangdong	12.97681	652.982	3.887568	11355	35773	170.985	39482.56	3589.54893	21574.72	17161.94
	9	Guizhou	0.77328	26.41343	1.040003	322	4946	1.780611	3912.68	13.56612	12862.53	293.64
	10	Hainan	0.17583	5.7806	1.249058	84	2726	0.555627	1654.21	13.08632	13750.85	54.75
	11	Hebei	3.8808	134.8446	1.125763	691	17970	17.21118	17235.48	156.88902	14718.25	629.17
	12	Heilongjiang	3.70197	109.1704	1.062859	1142	14553	48.855	8587	100.82127	12565.98	311.4
	13	Henan	4.79963	174.7599	1.222261	1129	21188	26.30461	19480.46	73.45376	14371.56	953.23
	14	Hubei	5.12124	213.449	1.211472	1478	25268	77.03287	12961.1	99.78796	14367.48	1039.52
	15	Hunan	3.49591	153.4995	1.339839	1752	21042	44.04324	13059.69	54.92034	15084.31	648.75
	16	Jiangsu	10.67826	701.9529	2.912858	5322	47441	108.2184	34457.3	1991.9919	20551.72	13015.35
	17	Jiangxi	1.83522	75.8936	0.857803	386	6811	9.78927	7655.18	73.68488	14021.54	755.65
	18	Jilin	2.60875	81.36019	1.28305	719	8987	19.75983	7278.75	31.24935	14006.27	537.66
	19	Liaoning	5.43947	232.3687	2.143081	1993	20801	119.7095	15212.49	334.14928	15761.38	1313.84
	20	Qinghai	0.30013	7.59379	0.982114	35	1240	8.496721	1081.27	2.51876	12691.85	19.22
	21	Shandong	8.33303	519.592	1.924254	2865	26941	71.9391	33896.65	794.90706	17811.04	4555.71
	22	Shanxi	2.52624	80.85633	1.127415	603	6757	16.20675	7358.31	28.37455	13996.55	196.47
	23	Shanxi	4.23465	189.5063	1.131443	1342	26403	69.80741	8169.8	39.88149	14128.76	717.04
	24	Sichuan	4.87863	214.459	0.79759	1596	22568	54.59769	14151.28	141.69447	13839.4	1766.76
	25	Yunnan	1.22051	37.23044	0.972869	476	7101	10.24687	6169.75	45.13252	14423.93	147.17
	26	Zhejiang	5.90844	398.8367	3.74258	4818	25638	56.45805	22990.35	1330.12954	24610.81	2672.09
Autonomous Region												
	27	Guangxi	1.56993	47.20277	1.114432	326	9982	1.766182	7759.16	83.7537	15451.48	273.67
	28	Inner Mongolia	1.27057	52.07259	0.937557	178	3214	14.76515	9740.25	23.15476	15849.19	236.61
	29	Ningxia	0.33954	10.44221	1.018058	52	1365	0.898229	1353.31	7.4293	14024.7	32.89
	30	Xinjiang	0.82683	21.80426	1.198296	120	5688	1.207767	4277.05	109.34563	12257.52	23.74

In the technology development process, the inputs are: R&D expenditure (R&DE), R&D personnel (R&DP) and the proportion of regional science and technology funds in regional total financial expenditure (S&TF/TFE), the outputs are: patents and papers. Among them, R&DE and R&DP are two core indexes in science and technology activities (see Zhong et al. [7]), S&TF/TFE is an important index in reflecting government's support. The outputs of the first stage are the number of patents and papers, which are also inputs to the second stage, namely, these are intermediate measures. The second stage also has an input of contract value (CV) in technology market. Economic application process transforms technology development into economic benefits. CV presents the function of intermediary services institution, which provides services in this process. The final outputs are complex economic indices, which embody the regional economic performance affected by R&D: GDP represents the macro-economy performance, total exports (TE) is important to depict international competency, urban per capita disposable annual income (UPCDAI) depicts people's living level and gross output of high-tech industry (GOHI) is directly to depict the condition of high-tech industry.

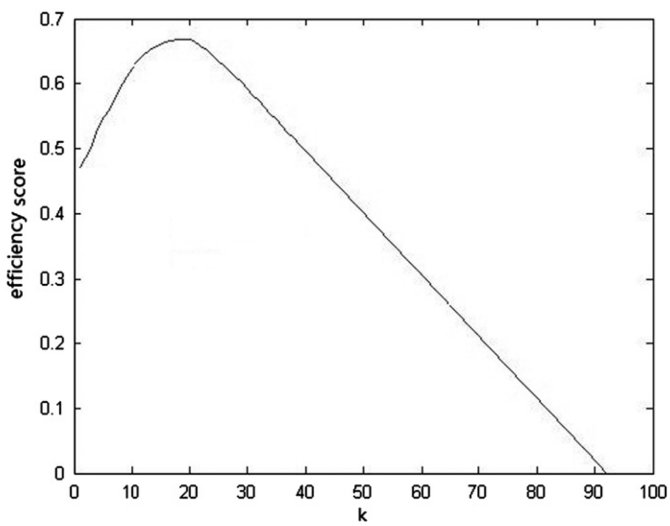

Fig. 4. Efficiency changes of Zhejiang Province (DMU 26) corresponding to each *k*.

Table 1 provides the data for the above R&D system for the 30 Provincial Level Regions in China. The data for Tibet Autonomous Region are incomplete and are not included in the current study. The data are derived from "China statistical yearbook, 2009" and "China science and technology statistical yearbook, 2009".

We now illustrate the proposed computation procedure in estimating the global optimal efficiency for each Provincial Level Region. Consider Zhejiang Province (DMU 26). The maximal score for its first stage is $\theta_1^{o^{max}} = 0.9111$ based upon model (3). Now, let $\theta_1^o = 0.9111 - k\Delta\varepsilon$, $k = 0,1,2...[k^{max}]+1$, and set the step size as$\Delta\varepsilon = 0.01$. Therefore $[k^{max}] = [\theta_1^{o^{max}}/\Delta\varepsilon] = [0.9111/0.01] = 91$, i.e., $k = 0, 1, 2..., 92$. Table 2 shows the results from model (5) for Zhejiang Province (DMU 26) corresponding to each *k* from 0 to 92.

Table 3
Results based upon the centralized model with $\Delta\varepsilon = 0.01$ and the non-cooperative model.

Region type	DMU	Region	Centralized model ($\Delta\varepsilon = 0.01$)						Non-cooperative model					
			Stage 1 as a variable			Stage 2 as a variable			Stage 1 as leader			Stage 2 as leader		
			$\hat{\theta}_1^{o+}$	$\hat{\theta}_2^{o-}$	$\hat{\theta}^{cen,1,*}$	$\hat{\theta}_1^{o-}$	$\hat{\theta}_2^{o+}$	$\hat{\theta}^{cen,2,*}$	e_1^{0*}	e_2^{0*}	$e^{non,1,*}$	π_1^{0*}	π_2^{0*}	$\pi^{non,2,*}$
Municipality														
	1	Beijing	1.0000	0.1598	0.1598	1.0000	0.1598	0.1598	1.0000	0.1598	0.1598	1.0000	0.1598	0.1598
	2	Chongqing	1.0000	0.2489	0.2489	1.0000	0.2489	0.2489	1.0000	0.2489	0.2489	1.0000	0.2489	0.2489
	3	Shanghai	0.8900	0.5394	0.4801	0.8842	0.5428	0.4799	1.0000	0.4433	0.4433	0.7942	0.5728	0.4549
	4	Tianjin	0.6774	0.5704	0.3864	0.6726	0.5704	0.3864	0.7426	0.4657	0.3459	0.6774	0.5704	0.3864
Province														
	5	Anhui	0.6697	0.3895	0.2609	0.6697	0.3895	0.2609	0.6697	0.3895	0.2609	0.6697	0.3895	0.2609
	6	Fujian	0.5668	1.0000	0.5668	0.5668	1.0000	0.5668	0.5668	1.0000	0.5668	0.5668	1.0000	0.5668
	7	Gansu	1.0000	0.2207	0.2207	0.9869	0.2221	0.2192	1.0000	0.2207	0.2207	0.3504	0.3121	0.1094
	8	Guangdong	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	9	Guizhou	0.9398	1.0000	0.9398	0.9398	1.0000	0.9398	0.9398	1.0000	0.9398	0.9398	1.0000	0.9398
	10	Hainan	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	11	Hebei	0.8885	0.8351	0.7420	0.8885	0.8351	0.7420	0.8885	0.8351	0.7420	0.8885	0.8351	0.7420
	12	Heilongjiang	0.9328	0.2648	0.2470	0.9328	0.2603	0.2428	0.9328	0.2648	0.2470	0.8536	0.2703	0.2308
	13	Henan	0.8504	0.7329	0.6233	0.8493	0.7373	0.6262	0.8504	0.7329	0.6233	0.8493	0.7373	0.6262
	14	Hubei	0.9060	0.2816	0.2551	0.9060	0.2760	0.2501	0.9060	0.2816	0.2551	0.4177	0.3360	0.1404
	15	Hunan	1.0000	0.3685	0.3685	1.0000	0.3680	0.3680	1.0000	0.3685	0.3685	0.9336	0.3780	0.3529
	16	Jiangsu	0.9225	1.0000	0.9225	0.9225	1.0000	0.9225	0.9225	1.0000	0.9225	0.9225	1.0000	0.9225
	17	Jiangxi	0.5647	0.9877	0.5577	0.5646	0.9900	0.5589	0.5647	0.9877	0.5577	0.4812	1.0000	0.4812
	18	Jilin	0.7158	0.4936	0.3533	0.7058	0.4984	0.3518	0.7158	0.4936	0.3533	0.6030	0.5184	0.3126
	19	Liaoning	0.6669	0.3669	0.2447	0.6713	0.3642	0.2445	0.6969	0.3488	0.2431	0.6335	0.3742	0.2371
	20	Qinghai	0.4573	1.0000	0.4573	0.4573	1.0000	0.4573	0.4573	1.0000	0.4573	0.4573	1.0000	0.4573
	21	Shandong	0.7101	0.8176	0.5806	0.7068	0.8198	0.5795	0.7101	0.8176	0.5806	0.5700	0.8498	0.4844
	22	Shanxi	0.5759	0.5109	0.2943	0.5664	0.5195	0.2942	0.5864	0.4996	0.2930	0.4774	0.5709	0.2725
	23	Shanxi	1.0000	0.1941	0.1941	1.0000	0.1909	0.1909	1.0000	0.1941	0.1941	0.4073	0.2509	0.1022
	24	Sichuan	1.0000	0.4566	0.4566	1.0000	0.4517	0.4517	1.0000	0.4566	0.4566	0.6851	0.4817	0.3300
	25	Yunnan	1.0000	0.5846	0.5846	0.9955	0.5859	0.5832	1.0000	0.5846	0.5846	0.8938	0.6159	0.5504
	26	Zhejiang	0.7311	0.9148	0.6688	0.7316	0.9141	0.6688	0.9111	0.5172	0.4712	0.6298	0.9541	0.6009
Autonomous Region														
	27	Guangxi	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	28	Inner Mongolia	0.3599	1.0000	0.3599	0.3599	1.0000	0.3599	0.3599	1.0000	0.3599	0.3599	1.0000	0.3599
	29	Ningxia	0.4300	1.0000	0.4300	0.4300	1.0000	0.4300	0.4300	1.0000	0.4300	0.4300	1.0000	0.4300
	30	Xinjiang	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

For example, when we set $k=10$ and $k=41$, the optimal efficiency for the Zhejiang Province (DMU 26) is 0.6231 and 0.3922, respectively.

Fig. 4 shows the change of the optimal value to model (5) as k increases from 1 to 92. It can be seen that its efficiency increases until $k=19$. When k exceeds 19, the optimal efficiency for Zhejiang province (DMU 26) starts to decrease. Thus, the global optimal efficiency for Zhejiang Province (DMU 26) is $\hat{\theta}^{cen,1,*}=0.6688$ when $k=19$.

Table 3 reports the results based upon the proposed approaches in this paper. The results based upon centralized models ($\Delta\varepsilon=0.01$) are shown in columns 4–9, where columns 4–6 are the efficiencies when the efficiency of stage 1 is assumed as a variable and columns 7–9 present the efficiencies when the efficiency of stage 2 is assumed as a variable. The results based upon non-cooperative models are shown in the last six columns, in which columns 10–12 show the results when stage 1 is assumed as leader, and the last three columns gives the efficiencies when stage 2 is assumed as leader.

First, the results in Table 3 verify our Theorem 1. For example, to each Provincial Level Region, its efficiency for the first stage in column 10 is always bigger than or equal to the one in column 13. Similarly, the efficiency for the second stage in column 11 is always less than or equal to the one in column 14. So the Theorem 1 can be verified such that $e_1^{o*} \geq \pi_1^{o*}, e_2^{o*} \leq \pi_2^{o*}$.

However, some results in Table 3 are not consistent with our Theorem 2. First, for some Provincial Level Regions, their efficiencies are not equal based upon the centralized model when stage 1's efficiency and stage 2's efficiency are treated as variables, respectively. For example, for Shanxi Province (DMU 22), $\hat{\theta}^{cen,1,*}=0.2943$, but $\hat{\theta}^{cen,2,*}=0.2942$. Therefore, the result does not support the first part of Theorem 2 such that $\theta^{cen,1,*}=\theta^{cen,2,*}$. The similar situation occurs to Shanghai (DMU 3), Gansu (DMU 7), Heilongjiang (DMU 12), Henan (DMU 13), Hubei (DMU 14), Hunan (DMU 15), Jiangxi (DMU 17), Jilin (DMU 18), Liaoning (DMU 19), Shandong (DMU 21), Shanxi (DMU 22), Sichuan (DMU 24) and Yunnan Province (DMU 25). The results for the other 16 Provincial Level Regions satisfy the first part of Theorem 2.

Furthermore, some results in Table 3 are not consistent with the second part of our Theorem 2. For example, for Jilin Province (DMU 18) $\hat{\theta}^{cen,2,*}=0.3518$, while $e^{non,1,*}=0.3533$. So the result is inconsistent with the second part of our Theorem 2 such that $\theta^{cen} \geq e^{non,1,*}, \theta^{cen} \geq \pi^{non,2,*}$. The similar situation occurs to Heilongjiang (DMU 12), Gansu (DMU 7), Henan (DMU 13), Hubei (DMU 14), Hunan (DMU 15), Shandong (DMU 21), Shanxi (DMU 22), Sichuan (DMU 24) and Yunnan Province (DMU 25). The results for the other 20 Provincial Level Regions satisfy the second part of Theorem 2.

The reason for the above inconsistency is due to the fact that the step size $\Delta\varepsilon$ we use is not small enough. If the $\Delta\varepsilon$ is adequately small, we can get the results in consistent with the theorems.

Table 4 reports the results based upon centralized model with $\Delta\varepsilon=0.0001$ and $\Delta\varepsilon=0.00001$, respectively. It shows there are only three Provincial Level Regions (Shandong (DMU 21), Hunan (DMU 15) and Shanxi (DMU 22)) whose two efficiencies are not equal, which is inconsistent with Theorem 2 when $\Delta\varepsilon=0.0001$. When

Table 4
Results based upon the centralized model when $\Delta\varepsilon=0.0001$ and 0.00001.

Region type	DMU	Region	$\Delta\varepsilon=0.0001$						$\Delta\varepsilon=0.00001$					
			Stage 1 as a variable			Stage 2 as a variable			Stage 1 as a variable			Stage 2 as a variable		
			$\hat{\theta}_1^{o+}$	$\hat{\theta}_2^{o-}$	$\hat{\theta}^{cen,1,*}$	$\hat{\theta}_1^{o-}$	$\hat{\theta}_2^{o+}$	$\hat{\theta}^{cen,2,*}$	$\hat{\theta}_1^{o+}$	$\hat{\theta}_2^{o-}$	$\hat{\theta}^{cen,1,*}$	$\hat{\theta}_1^{o-}$	$\hat{\theta}_2^{o+}$	$\hat{\theta}^{cen,2,*}$
Municipality														
	1	Beijing	1.0000	0.1598	0.1598	1.0000	0.1598	0.1598	1.0000	0.1598	0.1598	1.0000	0.1598	0.1598
	2	Chongqing	1.0000	0.2489	0.2489	1.0000	0.2489	0.2489	1.0000	0.2489	0.2489	1.0000	0.2489	0.2489
	3	Shanghai	0.8950	0.5365	0.4802	0.8949	0.5366	0.4802	0.8950	0.5365	0.4802	0.8950	0.5365	0.4802
	4	Tianjin	0.6774	0.5704	0.3864	0.6774	0.5704	0.3864	0.6774	0.5704	0.3864	0.6774	0.5704	0.3864
Province														
	5	Anhui	0.6697	0.3895	0.2609	0.6697	0.3895	0.2609	0.6697	0.3895	0.2609	0.6697	0.3895	0.2609
	6	Fujian	0.5668	1.0000	0.5668	0.5668	1.0000	0.5668	0.5668	1.0000	0.5668	0.5668	1.0000	0.5668
	7	Gansu	1.0000	0.2207	0.2207	1.0000	0.2207	0.2207	1.0000	0.2207	0.2207	1.0000	0.2207	0.2207
	8	Guangdong	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	9	Guizhou	0.9398	1.0000	0.9398	0.9398	1.0000	0.9398	0.9398	1.0000	0.9398	0.9398	1.0000	0.9398
	10	Hainan	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	11	Hebei	0.8885	0.8351	0.7420	0.8885	0.8351	0.7420	0.8885	0.8351	0.7420	0.8885	0.8351	0.7420
	12	Heilongjiang	0.9328	0.2648	0.2470	0.9326	0.2648	0.2470	0.9328	0.2648	0.2470	0.9328	0.2648	0.2470
	13	Henan	0.8493	0.7373	0.6262	0.8493	0.7373	0.6262	0.8493	0.7373	0.6262	0.8493	0.7373	0.6262
	14	Hubei	0.9060	0.2816	0.2551	0.9060	0.2815	0.2551	0.9060	0.2816	0.2551	0.9060	0.2816	0.2551
	15	Hunan	1.0000	0.3685	0.3685	1.0000	0.3684	0.3684	1.0000	0.3685	0.3685	1.0000	0.3685	0.3685
	16	Jiangsu	0.9225	1.0000	0.9225	0.9225	1.0000	0.9225	0.9225	1.0000	0.9225	0.9225	1.0000	0.9225
	17	Jiangxi	0.5644	0.9914	0.5595	0.5644	0.9913	0.5595	0.5644	0.9914	0.5595	0.5644	0.9914	0.5595
	18	Jilin	0.7152	0.4947	0.3538	0.7151	0.4947	0.3538	0.7152	0.4947	0.3538	0.7152	0.4947	0.3538
	19	Liaoning	0.6671	0.3668	0.2447	0.6671	0.3668	0.2447	0.6671	0.3668	0.2447	0.6671	0.3668	0.2447
	20	Qinghai	0.4573	1.0000	0.4573	0.4573	1.0000	0.4573	0.4573	1.0000	0.4573	0.4573	1.0000	0.4573
	21	Shandong	0.7101	0.8176	0.5806	0.7099	0.8177	0.5805	0.7101	0.8176	0.5806	0.7101	0.8176	0.5806
	22	Shanxi	0.5708	0.5156	0.2943	0.5708	0.5156	0.2943	0.5708	0.5156	0.2943	0.5708	0.5156	0.2943
	23	Shanxi	1.0000	0.1941	0.1941	1.0000	0.1940	0.1940	1.0000	0.1941	0.1941	1.0000	0.1941	0.1941
	24	Sichuan	1.0000	0.4566	0.4566	1.0000	0.4566	0.4566	1.0000	0.4566	0.4566	1.0000	0.4566	0.4566
	25	Yunnan	1.0000	0.5846	0.5846	1.0000	0.5847	0.5846	1.0000	0.5846	0.5846	1.0000	0.5846	0.5846
	26	Zhejiang	0.7293	0.9171	0.6688	0.7293	0.9171	0.6688	0.7293	0.9171	0.6688	0.7293	0.9171	0.6688
Autonomous Region														
	27	Guangxi	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	28	Inner Mongolia	0.3599	1.0000	0.3598	0.3599	1.0000	0.3599	0.3599	1.0000	0.3599	0.3599	1.0000	0.3599
	29	Ningxia	0.4300	1.0000	0.4300	0.4300	1.0000	0.4300	0.4300	1.0000	0.4300	0.4300	1.0000	0.4300
	30	Xinjiang	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$\Delta\varepsilon$=0.00001, the results for all the 30 Provincial Level Regions verify Theorem 2. This indicates that the choice of $\Delta\varepsilon$ is important and we should always use a very small $\Delta\varepsilon$ in order to reach the global optimal solution.

The results in the last six columns of Table 4 with $\Delta\varepsilon$=0.00001 shows that the efficiency decomposition is unique for all Provincial Level Regions. For example, for Zhejiang Province (DMU 26), $\hat{\theta}_1^{o+} = \hat{\theta}_1^{o-} = 0.7293$ and $\hat{\theta}_2^{o+} = \hat{\theta}_2^{o-} = 0.9171$.

Finally, note also that the two efficiencies based upon the centralized model and the non-cooperative model with stage 1 as the leader is the same for the majority of Provincial Level Regions. This may indicate that the first stage or the technology development stage is more important.

4. Conclusions

The current paper extends the approach of Liang et al. [1] to analyze the efficiency of two-stage network structures where the second stage has its own inputs in addition to the outputs from the first stage. In the current paper, a centralized model and a non-cooperative model are proposed to evaluate the efficiency of such a two-stage process and to further decompose the overall efficiency as a product of efficiency scores of the two individual stages as in Kao and Hwang [16].

Unlike the models in Liang et al. [1] or Kao and Hwang [16], the centralized model cannot be transformed to a linear program due to the existence of additional inputs to the second stage. The current paper proposes a heuristic method to estimate the global optimal efficiency. The proposed approaches are illustrated with a data set for measuring the R&D performance of 30 Provincial Level Regions in Mainland of China. As demonstrated in the application, the developed relations between the centralized and non-cooperative approaches can help test for whether a global optimal solution is found.

Although the current paper assumes that all the outputs from the first stage become inputs to the second stage, similar development can be made for cases when only portion of the outputs from the first stage become inputs to the second stage. That is, we can provide similar models for a more general two-stage network structure where each stage has its own inputs and outputs.

Finally, our models give solutions to the general two-stage network structure. It is desirable to improve these approaches to decompose efficiency for complex network structure in future research. And, the current models are under the assumption of CRS (constant return to scale), how to modify these models to decompose efficiency for general network structure by VRS (variable return to scale) model is also a direction for future research.

Acknowledgments

The authors are grateful for helpful comments and suggestions by two anonymous reviewers and an Associate Editor. This research is supported by National Natural Science Foundation of China under Grants (Nos. 71110107024, 70901070 and 70821001) and the Fundamental Research Funds for the Central Universities.

Appendix A. Centralized model when stage 2's efficiency is assumed to be a variable.

We obtain the best possible efficiency for stage 2 via the following model:

$$\theta_2^{o^{max}} = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{d=1}^{D} w_d z_{do} + \sum_{h=1}^{H} Q_h x_{ho}^2}$$
$$\text{s.t.}\quad \frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \quad \forall j$$
$$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj} + \sum_{h=1}^{H} Q_h x_{hj}^2} \le 1 \quad \forall j \quad v_i, w_d, Q_h, u_r \ge 0, \forall i,d,h,r; \tag{A.1}$$

Denote the optimal value to model (A.1) as $\theta_2^{o^{max}}$, then the efficiency for the second stage θ_2^o must satisfy $\theta_2^o \in [0, \theta_2^{o^{max}}]$.

Model (A.1) is a non-linear model, and can be converted into a linear program through the Charnes–Cooper transformation as follows:

$$\theta_2^{o^{max}} = \max \sum_{r=1}^{s} u_r y_{rj_0}$$
$$\text{s.t.}\quad \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad \forall j$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{h=1}^{H} Q_h x_{hj}^2 - \sum_{d=1}^{D} w_d z_{dj} \le 0 \quad \forall j$$
$$\sum_{h=1}^{H} Q_h x_{ho}^2 + \sum_{d=1}^{D} w_d z_{do} = 1 \quad v_i, w_d, Q_h, u_r \ge 0, \forall i,d,h,r; \tag{A.2}$$

The efficiency of the second stage θ_2^o can be treated as a variable $\theta_2^o \in [0, \theta_2^{o^{max}}]$ and the overall efficiency $\theta^{cen,2,*}$ can be considered as a function of θ_2^o as follows (or model (1) can be written as):

$$\theta^{cen,2,*} = \max \theta_2^o * \frac{\sum_{d=1}^{D} w_d z_{do}}{\sum_{i=1}^{m} v_i x_{io}}$$
$$\text{s.t.}\quad \frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \quad \forall j \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj} + \sum_{h=1}^{H} Q_h x_{hj}^2} \le 1 \quad \forall j;$$
$$\frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{d=1}^{D} w_d z_{do} + \sum_{h=1}^{H} Q_h x_{ho}^2} = \theta_2^o \quad \theta_2^o \in \left[0, \theta_2^{o^{max}}\right];$$
$$v_i, w_d, Q_h, u_r \ge 0, \forall i,d,h,r \tag{A.3}$$

Model (A.3) now can be transformed via the Charnes–Cooper transformation as follows:

$$\theta^{cen,2,*} = \max \theta_2^o * \sum_{d=1}^{D} w_d z_{do}$$
$$\text{s.t.}\quad \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad \forall j$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{h=1}^{H} Q_h x_{hj}^2 - \sum_{d=1}^{D} w_d z_{dj} \le 0 \quad \forall j \quad \sum_{i=1}^{m} v_i x_{io} = 1;$$
$$\sum_{r=1}^{s} u_r y_{ro} - \theta_2^o \quad \sum_{h=1}^{H} Q_h x_{ho}^2 + \sum_{d=1}^{D} w_d z_{do} \Bigg) = 0$$
$$\theta_2^o \in [0, \theta_2^{o^{max}}] \quad v_i, w_d, Q_h, u_r \ge 0, \forall i,d,h,r \tag{A.4}$$

Let $\theta_2^o = \theta_2^{o^{max}} - \rho\Delta\varepsilon$, $\rho = 0,1,2 \ldots [\rho^{max}]+1$, where $[\rho^{max}]$ is the maximal integer, which is smaller than or equal to $\theta_2^{o^{max}}/\Delta\varepsilon$. Given each θ_2^o, model (A.4) can be solved as a linear program.

By solving model (A.4), the global optimal efficiency of the system under evaluation can be estimated as $\theta^{cen,2,*} = \max_{\rho} \theta^{cen,2}(\rho)$. Then, when the efficiency of the entire two-stage system under evaluation is $\theta^{cen,2,*}$, the maximal efficiency for its second stage is $\theta_2^{o+} = \theta_2^o(\rho^*)$, where $\rho^* = \min\{\rho \mid \theta^{cen,2,*} = \theta^{cen,2}(\rho)\}$. Finally, the minimal efficiency for its first stage is $\theta_1^{o-} = ((\theta^{cen,2,*})/(\theta_2^{o+}))$.

Appendix B. Non-cooperative model when stage 2 is assumed to be the leader.

The efficiency of the second stage (the leader) for a specific DMU_o is calculated using the standard CCR model (Charnes et al. [2]) as follows:

$$\pi_2^{o*} = \max \sum_{r=1}^{s} u_r y_{rj_0}$$
$$s.t. \quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{h=1}^{H} Q_h x_{hj}^2 - \sum_{d=1}^{D} w_d z_{dj} \le 0 \quad \forall j$$
$$\sum_{h=1}^{H} Q_h x_{ho}^2 + \sum_{d=1}^{D} w_d z_{do} = 1 \quad w_d, Q_h, u_r \ge 0, \forall d,h,r; \tag{A.5}$$

The model for calculating the efficiency of follower (the first stage) is as follows:

$$\pi_1^{o*} = \max \frac{\sum_{d=1}^{D} w_d z_{do}}{\sum_{i=1}^{m} \nu_i x_{io}}$$
$$s.t. \quad \frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} \nu_i x_{ij}} \le 1 \quad \forall j \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj} + \sum_{h=1}^{H} Q_h x_{hj}^2} \le 1 \quad \forall j;$$
$$\frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{d=1}^{D} w_d z_{do} + \sum_{h=1}^{H} Q_h x_{ho}^2} = \pi_2^{o*} \quad \nu_i, w_d, Q_h, u_r \ge 0, \forall i,d,h,r \tag{A.6}$$

In model (A.6), the efficiency for the first stage of DMU_o is optimized based upon that the efficiency of the second stage π_2^{o*} remains unchanged. Model (A.6) can be transformed as

$$\pi_1^{o*} = \max \sum_{d=1}^{D} w_d z_{do}$$
$$s.t. \quad \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} \nu_i x_{ij} \le 0 \quad \forall j$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{h=1}^{H} Q_h x_{hj}^2 - \sum_{d=1}^{D} w_d z_{dj} \le 0 \quad \forall j$$
$$\sum_{i=1}^{m} \nu_i x_{io} = 1 \quad \sum_{r=1}^{s} u_r y_{ro} - \pi_2^{o*} \left(\sum_{h=1}^{H} Q_h x_{ho}^2 + \sum_{d=1}^{D} w_d z_{do} \right) = 0;$$
$$\nu_i, w_d, Q_h, u_r \ge 0, \forall i,d,h,r \tag{A.7}$$

In model (A.7), denote π_1^{o*} as the optimal efficiency for first stage. The overall efficiency of the entire system in this situation is $\pi^{non,2} = \pi_1^{o*} * \pi_2^{o*}$.

Appendix C. Proofs of theorems

Proof of Theorem 1.

Proof. Denote an optimal solution to model (8) as $(\nu_i^{non,1,*}, w_d^{non,1,*}, Q_h^{non,1,*}, u_r^{non,1,*}, \forall i,d,h,r)$ and the optimal efficiency for the stage 2 as e_2^{o*}.

Let $\varsigma = (w_d^{non,1,*}, Q_h^{non,1,*}, u_r^{non,1,*}, \forall d,h,r)$, then, ς is also a feasible solution to model (A.5). Note that, model (A.5) calculates the optimal efficiency for stage 2 when stage 2 is treated as leader. Therefore, its optimal efficiency is π_2^{o*}. Thus, the efficiency for stage 2 based upon ς is not bigger than π_2^{o*}. So we have $e_2^{o*} \le \pi_2^{o*}$.

Similarly, we can get the result $e_1^{o*} \ge \pi_1^{o*}$. □

Proof of Theorem 2.

Proof:.

(1) Either stage 1's efficiency or stage 2's efficiency is treated as a variable, the maximal efficiency for the system is unique. So $\theta^{cen,1,*} = \theta^{cen,2,*}$.

(2) Denote an optimal solution to model (7) as $(\nu_i^{non,1,*}, w_d^{non,1,*}, Q_h^{non,1,*}, u_r^{non,1,*}, \forall i,d,h,r)$, accordingly, the optimal efficiency for the system as $e^{non,1} = e_1^{o*} * e_2^{o*}$.

Let $\xi = (w_d^{non,1,*}, Q_h^{non,1,*}, u_r^{non,1,*}, \forall d,h,r)$, then ξ is also a feasible solution to model (1), therefore, the optimal efficiency based upon model (1) is bigger than or equal to the efficiency based upon the feasible solution ξ. So we get

$$\theta^{cen} \ge \frac{\sum_{d=1}^{D} w_d^{non,1,*} z_{do}}{\sum_{i=1}^{m} \nu_i^{non,1,*} x_{io}} * \frac{\sum_{r=1}^{s} u_r^{non,1,*} y_{ro}}{\sum_{d=1}^{D} w_d^{non,1,*} z_{do} + \sum_{h=1}^{H} Q_h^{non,1,*} x_{ho}^2} = e^{non,1,*}$$

Similarly, we can get $\theta^{cen} \ge \pi^{non,2,*}$. □

Proof of Theorem 3.

Proof: Liang et al. [1] has proven that $e_1^{o*} = \pi_1^{o*} = \theta_1^{CCR}$ and $e_2^{o*} = \pi_2^{o*} = \theta_2^{CCR}$. Based upon Theorem 2, we have $\theta^{cen} \ge e^{non,1,*}$. Thus $\theta^{cen} = \theta^{cen,1,*} \ge \theta_1^{CCR} * \theta_2^{CCR}$. On the other hand, the efficiency of the entire two-stage system based upon the centralized model is not bigger than the product of two efficiencies of the first and second stages using the standard CCR models. Therefore, $\theta_1^{CCR} * \theta_2^{CCR} \ge \theta^{cen}$. Thus, $\theta^{cen} = \theta_1^{CCR} * \theta_2^{CCR} = \theta^{cen,1,*} = e^{non,1,*}$.

Similarly, we can have $\theta^{cen} = \theta_1^{CCR} * \theta_2^{CCR} = \theta^{cen,2,*} = \pi^{non,2,*}$. □

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