Accepted Manuscript

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PII: S0304-4068(18)30034-X
DOI: https://doi.org/10.1016/j.jmateco.2018.03.004
Reference: MATECO 2227

To appear in: *Journal of Mathematical Economics*

Received date : 20 March 2016
Revised date : 17 October 2017
Accepted date : 12 March 2018

Please cite this article as: Vachadze G., Credit market imperfection, minimum investment requirement, and endogenous income inequality. *Journal of Mathematical Economics* (2018), https://doi.org/10.1016/j.jmateco.2018.03.004

Credit Market Imperfection, Minimum Investment Requirement, and Endogenous Income Inequality

George Vachadze*

March 24, 2018

Abstract

The main goal of this paper is to describe an endogenous feedback mechanism through which imperfection in the credit market may amplify income inequality. When entrepreneurs are subject to a minimum investment requirement and entrepreneurs' future revenue is not fully pledgeable for debt repayment, then the highest interest rate entrepreneurs can credibly offer to depositors depends not only on the marginal product of capital but also on entrepreneurs' net wealth. This dependence creates an entrepreneurial rent which has both direct and indirect impacts on income inequality. On the one hand, entrepreneurial rent magnifies income inequality because it changes the balance between marginal product on capital (collected by entrepreneurs) and the interest rate (collected by depositors) and alters young agents saving decision. Entrepreneurial rent indirectly affects the labor income inequality because it distorts young agents' labor supply decision and thus indirectly affects labor income earned by borrowers and lenders. Under some configuration of parameter values, the model predicts a Kuznets curve, i.e., an inverted-U relationship between per capita income and income inequality.

Keywords: Credit market imperfection, income inequality, Kuznets curve

JEL Classification: D63; E44; J22; O16

*I am indebted to two anonymous referees and the associate editor for their constructive and valuable comments. I also benefited from comments and suggestions made by Linda Coull, Thomas Hauner, Richard Nugent, and Bryan Weber. This work was supported by Provost's Research Scholarship Award and City University of New York PSC-CUNY Research Award TRADA-46-127. Department of Economics, College of Staten Island and Graduate Center, City University of New York, Staten Island, NY 10314. E-mail: george.vachadze@csi.cuny.edu

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1 Introduction

Many papers have argued that income inequality is a result of historically shaped inequalities of opportunities transmitted across generations through education, social position, place of birth, etc.[1] Frictions in the credit market generate entry barriers, offer fewer opportunities for the poor, and cause the borrowing rate and thus the access to credit to depend on wealth and social status. Poor individuals under-invest in both physical and human capital because of limited ability to borrow. As a result, long run living standards depend on initial inequality which may persist or even be magnified over time. What happens when inequality is not transmitted across generations? Can credit market imperfections still cause amplification of income inequality? How does the income inequality depend on the severity of credit market imperfection and per capita income?

The main goal of this paper is to argue that imperfections in the credit market can be responsible for income inequality even when intergenerational transmission mechanism of income inequality is not present. When entrepreneurs are subject to a minimum investment requirement and entrepreneurs' future revenue is not fully pledgeable for debt repayment, then the highest interest rate entrepreneurs can credibly offer to depositors depends not only on entrepreneurs' future revenue but also on entrepreneurs' net wealth. This dependence creates an entrepreneurial rent which has direct and indirect impacts on income inequality. On the one hand, entrepreneurial rent magnifies income inequality because it directly affects the capital income earned by old agents. On the other hand, entrepreneurial rent magnifies income inequality because it distorts young agents' labor supply decision and thus indirectly affects labor incomes earned by borrowers and lenders. The logic of endogenous inequality described in this paper does not suggest that the transmission of inequality across generations is unimportant. On the contrary, it suggests that income inequality may exist in a society in which inequality is not transmitted across generations and inequality transmitted across generations may be amplified to create larger observed income inequality.

I start with the basic model in order to describe the mechanism through which imperfections in the credit market may affect the income inequality. In the basic model, I assume that young agents of every generation are endowed with equal amounts of labor and are equally productive workers. In such setup, I demonstrate the presence of direct and indirect effects of credit market imperfection on income inequality. The theoretical model presented in this paper is similar to one presented in Matsuyama

[1] See for example, Banerjee and Newman (1993), Galor and Zeira (1993), Piketty (1997), Matsuyama (2000), and Mookherjee and Ray (2002, 2003), among others.

(2004, 2007). The only modifications which I made in such setup are the introduction of an endogenous labor supply and endogenous saving decisions of young agents. Without such modifications, both direct and indirect effects of entrepreneurial rent on income inequality are switched off because all young agents supply the same amount of labor and save the same fraction of their labor income.

In order to expose the direct and indirect impacts of entrepreneurial rent on income inequality, I assume that young agents decide on how much labor to supply and how much to save while young. Since entrepreneurs face a minimum investment requirement and the maximum amount of credit entrepreneurs can obtain is proportional to the value of their pledgeable income, it follows that in equilibrium, some agents would work harder, save more, become entrepreneurs, and earn entrepreneur rent, while others work and save less and become depositors. At the end of the paper, I generalize the basic model by allowing young agents to be ex-ante heterogeneous by possessing different amounts of labor endowment and/or being unequally productive workers. This way I demonstrate that ex-ante heterogeneity assumption does not eliminate the channel through which imperfections in the credit market affects the income inequality.

The endogenous feedback mechanism described in this paper does not require any exogenous source of variation because of the circular causation between labor income and individual incentive to become an entrepreneur and earn higher capital income. That is, not only does entrepreneurship imply a higher second-period capital income but also a higher labor income and higher saving are needed to overcome the borrowing constraint and become an entrepreneur. Such circular causality implies the existence of an asymmetric equilibrium in which the pool of young agents is divided into two sub-groups, lenders and borrowers earning different levels of capital and labor incomes. In other words, this paper explains the variation of income purely as an outcome of the internal mechanisms of the system, in a self-organized manner. A contribution of this paper follows the literature in which endogenous heterogeneity emerges in a strategic setting. For example, Neal and Rosen (2000), and Acemoglu (2001) argue that both high and low wage jobs are created endogenously among ex-ante heterogeneous firms due to a presence of search frictions which may break the link between wage and the marginal product of labor and introduce rent-sharing between firms and workers. Amir and Wooders (1999), Reynolds and Wilson (2000) and Besanko and Doraszelski (2004) demonstrate how exogenous idiosyncratic technology shocks and demand uncertainty can cause ex-ante homogeneous firms to invest differently in production capacity, research, and development. Matsuyama (2002) provides a general discussion on how endogenous diversity across homogeneous agents may emerge as an equilibrium outcome.

Endogenous inequality of capital and labor incomes is not the only prediction of the model. Under some configurations of parameter values, the model predicts an inverted-U relationship between per capita income and income inequality consistent with the Kuznets curve. The inverted-U relationship between per capita income and income inequality occurs as a result of a mechanism distinct from those already discussed in the literature. For example, Kuznets (1955), Baumol (1967), Kuznets (1973), Baumol et al. (1985), Laitner (2000), Kongsamut et al. (2001), and Gollin (2002) propose structural transformation models in which the change in the sectoral composition of the economy is the main force behind the widening and shrinking of the labor income inequality. The emergence of a new sector offering higher labor income causes a gradual shift of the labor force into the new sector. This shift causes an initial rise followed by a gradual decline in the labor income inequality.

The rest of the paper is organized as follows. In section 2, I present a basic model through which I explain how imperfection in the credit market may magnify income inequality. In section 3, I examine agents' optimal behavior. In Section 4, I define competitive equilibrium and establish its existence and uniqueness. In section 5, I study the impact of credit market imperfection on the equilibrium income inequality. In section 6, I discuss the robustness of the basic model under alternative specifications. In section 7, I summarize the main results and draw some conclusions. All proofs are located in the Appendix.

2 Model Setup

I consider a discrete time economy populated by an infinite sequence of two-period lived, overlapping generations. There are two generations alive in each period. Individuals in the first period of life are referred to as *young agents*, while individuals in the second period of life are referred to as *old agents*. A single final commodity is produced by a large number of identical firms using capital and labor as inputs. The technology of the final goods producing firm is described by a constant returns to scale production function. Aggregate output produced at time t is $Y_t = L_t y_t$, where L_t is the aggregate amount of labor supplied by young agents and $y_t = f(k_t)$ is the output per labor. $f : \mathbb{R}_+ \to \mathbb{R}_+$ is the production function in intensive form, K_t is the aggregate capital supplied by old agents and $k_t = K_t/L_t$ is the capital-labor ratio.[2] Factor markets are competitive and rewards on physical capital and labor are determined by the marginal product rule, i.e., $\rho_t = f'(k_t)$ is the rate of return on one unit of capital and $w_t = W(k_t) = f(k_t) - k_t f'(k_t)$ is the wage rate. I assume that $f : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable on $\mathbb{R}_{++}$ and strictly increasing and strictly concave on $\mathbb{R}_+$. The final commodity produced at time t may be either consumed or invested in the capital which becomes available in period $t + 1$. Capital depreciates fully within a period.

In period $t = 0$ there is an *initially* old generation of measure one. Members of this generation live for one period and are endowed with K_0 units of capital. In

[2] "Labor" can be interpreted broadly to include any endowment held by young agents, while "capital" can be interpreted broadly to include human capital or any other reproducible good used in production.

each period, $t = 0, 1, ...$, a young generation of measure one is born so that in period $t = 1, 2, ...$ the economy is populated by agents of two successive generations - an old generation born in period $t-1$ and a young generation born in period t. I begin by assuming ex-ante *homogeneity* among young agents.

Young agents possess one unit of labor endowment and they are equally productive workers. In the first period of life, young agents decide how much labor to supply to the competitive labor market and how much to save. Let $\ell_t \in [0, 1]$ and $s_t \in [0, 1]$ denote the first period labor supply and first period savings rate respectively. Then $\ell_t w_t$ represent first period labor income, while $s_t \ell_t w_t$ and $c_{1t} = (1 - s_t)\ell_t w_t$ represent first period saving and first period consumption respectively. Young agents have two options to transfer the current saving into second period consumption. First, they may become depositors by lending first period saving to the competitive credit market at gross interest rate r_{t+1} and consume $c_{2t+1} = s_t \ell_t w_t r_{t+1}$ during the second period (Option 1). Second, they may become entrepreneurs by borrowing $i_t - s_t \ell_t w_t$ and investing $i_t \geq m$ units of the final commodity in period t in order to produce i_t units of capital good in period $t+1$ (Option 2). As in Banerjee and Newman (1993), Galor and Zeira (1993), Piketty (1997), and Matsuyama (2000, 2004, 2007), parameter $m > 0$ represents a minimum investment requirement for setting up a new firm and producing capital. Beyond the fixed investment cost, justifications for a minimum investment requirement include a fixed cost for entrepreneurs to acquire know-how in the organization, management, marketing and other areas in setting-up a production facility, successfully launching a new product, managing labor, producing, and profitably reaching a viable market.

Entrepreneurs rent produced capital to the final commodity producing firm at the rate of $\rho_{t+1} = f'(k_{t+1})$ and generate the revenue $i_t \rho_{t+1}$. After repaying the debt, $(i_t - s_t \ell_t w_t) r_{t+1}$ carried over from the previous period, entrepreneur's profit is

$$\pi_{t+1} = i_t \rho_{t+1} - (i_t - s_t \ell_t w_t) r_{t+1} = (\theta_{t+1} + s_t \ell_t w_t) r_{t+1} \quad \text{where} \quad \theta_{t+1} = \frac{\rho_{t+1} - r_{t+1}}{r_{t+1}} \tag{1}$$

denotes the entrepreneurial rent at time $t+1$. Entrepreneurs consume the entirely profit during the second period, $c_{2t+1} = \pi_{t+1}$. If $\theta_{t+1} = 0$ then entrepreneurs and depositors achieve the same levels of second period consumption while if $\theta_{t+1} > 0$ then entrepreneurs consume more during the second period than depositors do.

Young agents face a minimum investment requirement because they must invest at least $m > 0$ units of the final commodity at time t in order to become entrepreneurs and produce capital. If $i_t < m$ than young agents cannot set up a new firm, cannot produce any capital, and thus cannot consume anything during the second period. At the same time, young agents face the borrowing constraint because of limited pledgeability of entrepreneur's future revenue for debt repayment,

$$(i_t - s_t \ell_t w_t) r_{t+1} \leq (1 - \lambda) i_t \rho_{t+1} \quad \Leftrightarrow \quad i_t \leq \frac{s_t \ell_t w_t}{\lambda - (1 - \lambda)\theta_{t+1}}. \tag{2}$$

First inequality in (2) can be referred as a *borrowing constraint* because the maximum loan size young agent can take while becoming an entrepreneur, $(i_t - s_t\ell_t w_t)r_{t+1}$, is limited by a $1-\lambda \in (0,1)$ fraction of entrepreneur's future revenue, $i_t\rho_{t+1}$. Justifications for limited pledgeability of entrepreneurs' future revenue include asymmetric information between borrowers and lenders and limitations of legal institutions. For example, creditors might have difficulty in collecting a loan because of entrepreneurs' ability to hide a fraction of their revenue from financiers or because of imperfect law enforcement. If an entrepreneur can hide a $\lambda \in (0,1)$ fraction of firm's revenue from a creditor, than the maximum amount of credit depositors extend to entrepreneurs is limited by $(1-\lambda)i_t\rho_{t+1}$, because a loan above this value will not be repaid. Alternatively, if the default cost is proportional to entrepreneur's revenue then depositors can avoid a strategic default only by limiting borrower's debt obligation by $(1-\lambda)i_t\rho_{t+1}$. In other words, parameter λ measures the imperfection in the credit market.[3]

Young agents' have a logarithmic utility function

$$(\ell_t, c_{1t}, c_{2t+1}) \mapsto (1-\gamma)\log(1-\ell_t) + (1-\beta)\gamma \log c_{1t} + \beta\gamma \log c_{2t+1},$$

where parameter $\beta \in (0,1]$ measures the relative importance of second period consumption (in terms of first period consumption), while parameter $\gamma \in (0,1]$ measures the relative importance of first and second period consumptions of final commodities (in terms of first period leisure). I intentionally exclude the case with $\beta\gamma = 0$, because in such case young agents do not value second period consumption, they do not save, and thus capital does not accumulate at all.

3 Agents' Optimal Behavior

Young agents take values of wage – w_t, interest rate – r_{t+1}, and rate of return on capital – ρ_{t+1}, as given. Young agents decide how much leisure to consume during the first period, what fraction of labor income to save, whether to become a depositor or an entrepreneur during the second period and how much to invest as an entrepreneur.

[3]Microeconomic literature offers several alternative justifications (apart from the two mentioned above) for a partial pledgeability of project revenue. There is a costly-state-verification approach of Townsend (1979), Bernanke and Gertler (1989), Boyd and Smith (1997), Bhattacharya and Chakraborty (2005), and others, a moral hazard approach of Holmström and Tirole (1997) and others, and an adverse selection approach of Hart and Moore (1994), Kiyotaki and Moore (1997), and others. In this paper, I will not argue which of the above stories offer a more plausible explanation for credit market imperfection. Instead, I will rely on the reduced form approach of Matsuyama (2004, 2007) and Holmström and Tirole (2011), to parameterize the severity of credit market imperfection and analyze its implication on income and income inequality.

3.1 Depositors' Optimal Behavior

For a given pair, (ℓ_t, s_t), depositor's first and second period consumptions are $c_{1t} = (1-s_t)\ell_t w_t$ and $c_{2t+1} = s_t \ell_t w_t r_{t+1}$ respectively. Resulting lifetime utility of a depositor is $\log V^d + \gamma \log w_t + \beta\gamma \log r_{t+1}$, where

$$V^d = \max_{\substack{0\le s\le 1\\ 0\le \ell\le 1}} \left\{(1-\ell)^{1-\gamma}\ell^{\gamma}(1-s)^{(1-\beta)\gamma}s^{\beta\gamma}\right\}. \tag{3}$$

As a result, depositor's optimal labor supply and optimal savings rate are $\ell^d = \gamma$ and $s^d = \beta$ respectively, while $V^d = (1-\gamma)^{1-\gamma}\gamma^{\gamma}(1-\beta)^{(1-\beta)\gamma}\beta^{\beta\gamma}$. Labor income earned by young depositors of generation t is $y^d_{1t} = \gamma w_t$ while the capital income earned by old depositors of generation $t-1$ is $y^d_{2t-1} = \beta\gamma w_{t-1} r_t$.

3.2 Entrepreneurs' Optimal Behavior

For a given pair (ℓ_t, s_t), entrepreneur's first and second period consumptions are $c_{1t} = (1-s_t)\ell_t w_t$ and

$$c_{2t+1} = i_t\rho_{t+1} - (i_t - s_t\ell_t w_t)r_{t+1} = \left(\frac{i_t\theta_{t+1}}{\ell_t w_t} + s_t\right)\ell_t w_t r_{t+1} \tag{4}$$

respectively. As seen in (4), entrepreneur's second period consumption depends on entrepreneurial rent, which is defined in (1). Resulting lifetime utility of an entrepreneur is $\log V^e(w_t, \theta_{t+1}) + \gamma \log w_t + \beta\gamma \log r_{t+1}$, where

$$V^e(w,\theta) = \max_{\substack{0\le s\le 1\\ 0\le \ell\le 1}} \left\{(1-\ell)^{1-\gamma}\ell^{\gamma}(1-s)^{(1-\beta)\gamma}\left(\frac{i\theta}{\ell w}+s\right)^{\beta\gamma} \,\middle|\, m \le i \le \frac{s\ell w}{\lambda-(1-\lambda)\theta}\right\}.$$

Proposition 1 *For a given pair (w_t, θ_{t+1}) entrepreneur's optimal labor supply, saving, and investment decisions are*

$$\ell^e_t = \min\left\{\max\left\{\gamma, \frac{(1-\beta)\gamma}{1-\beta\gamma} + \frac{1-\gamma}{1-\beta\gamma}\frac{(\lambda-(1-\lambda)\theta_{t+1})m}{w_t}\right\}, 1\right\}, \tag{5}$$

$$s^e_t = \min\left\{\max\left\{\beta, \frac{1}{\ell^e_t}\frac{(\lambda-(1-\lambda)\theta_{t+1})m}{w_t}\right\}, 1\right\}, \quad \textit{and} \quad i^e_t = \frac{s^e_t\ell^e_t w_t}{\lambda-(1-\lambda)\theta_{t+1}}. \tag{6}$$

Resulting $V^e(w_t, \theta_{t+1})$ is

$$V^e(w,\theta) = \begin{cases} 0 & \textit{if} \quad \dfrac{(\lambda-(1-\lambda)\theta)m}{w} \ge 1 \\ V^d\left(\dfrac{\lambda(1+\theta)}{\lambda-(1-\lambda)\theta}\right)^{\beta\gamma} & \textit{if} \quad \dfrac{(\lambda-(1-\lambda)\theta)m}{w} \le \beta\gamma, \end{cases} \tag{7}$$

and

$$V^e(w,\theta) = \left(\frac{1-\gamma}{1-\beta\gamma}\right)^{1-\gamma} \left(\frac{(1-\beta)\gamma}{1-\beta\gamma}\right)^{(1-\beta)\gamma} \left(1 - \frac{(\lambda-(1-\lambda)\theta)m}{w}\right)^{1-\beta\gamma} \left(\frac{\lambda\,(1+\theta)\,m}{w}\right)^{\beta\gamma} \quad (8)$$

if $\frac{(\lambda-(1-\lambda)\theta)m}{w} \in (\beta\gamma, 1)$.

Proof of Proposition 1 can be found in the Appendix. Proposition 1 implies that when wage is at the intermediate level then entrepreneurs are willing to supply more labor and save a higher portion of their labor income than depositors do, $\ell_t^e > \ell^d = \gamma$ and $s_t^e > s^d = \beta$. This is so because entrepreneurs cannot overcome the borrowing constraint without earning high labor income and without saving a higher fraction of their labor income. When the wage is sufficiently large then entrepreneurs can overcome the minimum investment requirement and the borrowing constraint by supplying the same amount of labor and by saving the same fraction of their labor income as depositors do.

Labor income earned by young entrepreneurs of generation t is $y_{1t}^e = \ell_t^e w_t$ while capital income earned by old depositor of generation $t-1$ is $y_{2t-1}^e = i_{t-1}^e \rho_t - (i_{t-1}^e - s_{t-1}^e \ell_{t-1}^e w_{t-1}) r_t$. The difference between labor incomes earned by young entrepreneurs and young depositors is $y_{1t}^e - y_{1t}^d = (\ell_{t-1}^e - \gamma) w_{t-1}$ while the difference between capital incomes earned by old entrepreneurs and old depositors is $y_{2t-1}^e - y_{2t-1}^d = i_{t-1}^e(\rho_t - r_t) + (s_{t-1}^e \ell_{t-1}^e - \beta\gamma) w_{t-1} r_t$. It is worth noting at this point that when $\theta_t > 0$ then the entrepreneurial rent has direct and indirect effects on capital and labor income inequalities. One the one hand, for a given pair (w_{t-1}, r_t) entrepreneurial rent directly affects capital income earned by old agents because $\rho_t - r_t = \theta_t r_t$ and $s_{t-1}^e - \beta$ both increase with θ_t. On the other hand, entrepreneurial rent indirectly affects labor incomes earned by young agents because $\ell_{t-1}^e - \gamma$ increases with θ_t.[4]

3.3 Optimal Occupation Choice

In sections 3.1 and 3.2, I discussed optimal behavior of depositors and entrepreneurs. While making optimal occupational choice young agents of each generation compare values of V^d and $V^e(w_t, \theta_{t+1})$, which are given by (3), (7), and (8) respectively. If $V^e(w_t, \theta_{t+1}) < V^d$ then young agents supply γ units of labor, save β fraction of their labor income, and become depositors during second period (Option 1). If $V^e(w_t, \theta_{t+1}) > V^d$ then young agents supply ℓ_t^e units of labor, save s_t^e fraction of their labor income, borrow $i_t^e - s_t^e \ell_t^e w_t$ units of final commodity during the first period, and become entrepreneurs during the second period (Option

[4]Entrepreneurial rent is present in Matsuyama (2004, 2007) as well, however its direct and indirect impacts on income inequality is absent because young agents only decide whether to become a depositor or an entrepreneur and they do not decide about how much labor to supply and how much to save.

2). When $V^e(w_t, \theta_{t+1}) = V^d$ then young agents are indifferent among these two options. If $\frac{(\lambda-(1-\lambda)\theta_{t+1})m}{w_t} \geq 1$ then it follows from (7) that young agents strictly prefer Option 1 over Option 2 because $V^e(w_t, \theta_{t+1}) = 0 < V^d$. If $\frac{(\lambda-(1-\lambda)\theta_{t+1})m}{w_t} \leq \beta\gamma$ then it follows from (7) that $V^e(w_t, \theta_{t+1}) \gtreqless V^d \Leftrightarrow \theta_{t+1} \gtreqless 0$. What happens when $\frac{(\lambda-(1-\lambda)\theta_{t+1})m}{w_t} \in (\beta\gamma, 1)$?

Proposition 2 *For a given wage, $w_t \in \left(0, \frac{\lambda m}{\beta\gamma}\right)$, there exists a unique $\theta_{t+1} = \Theta(w_t) \in (0, \frac{\lambda}{1-\lambda})$ which solves $V^e(w_t, \theta_{t+1}) = V^d$. $w \mapsto \Theta(w)$ is a continuous and strictly decreasing function satisfying boundary conditions $\lim_{w\downarrow 0} \Theta(w) = \frac{\lambda}{1-\lambda}$ and $\lim_{w\uparrow \frac{\lambda m}{\beta\gamma}} \Theta(w) = 0$.*

Proof of Proposition 2 can be found in the Appendix. Proposition 2 implies that when wage is sufficiently small, $w_t \in (0, \frac{\lambda m}{\beta\gamma})$, then young agents are indifferent between Options 1 and 2 only in the presence of a positive entrepreneurial rent. Without entrepreneur rent, the agent's second period consumption is the same while the first period labor supply and the first period saving of entrepreneurs exceed the first period labor supply and the first period saving of depositors. As a result, depositors are better off than entrepreneurs because depositors consume more leisure and consumption commodity during the first period. Positive entrepreneurial rent pushes up the second period consumption of entrepreneurs and compensates them for an extra work and low first period consumption. It is clear that if $\theta_{t+1} \gtrless \Theta(w_t)$ then $V^e(w_t, \theta_{t+1}) \gtrless V^d$ because high entrepreneurial rent implies high second period consumption for capital producers. If wage is sufficiently large, $w_t \geq \frac{\lambda m}{\beta\gamma}$, then the indifference between Options 1 and 2 is achieved only with zero entrepreneurial rent.

4 Equilibrium

Definition 1 *For a given $K_0 > 0$ a competitive equilibrium is a sequence of quantities, $\{(L_t, K_t, Y_t)\}_{t=0}^{\infty}$, and a sequence of prices $\{(w_t, \rho_t, r_t)\}_{t=0}^{\infty}$, such that (i) young agents behave optimally, (ii) final commodity producing firm pays rental rates on capital and labor inputs according to a marginal product rule, and (iii) capital and labor markets clear simultaneously in each period $t = 0, 1, ...$*

In order to establish the existence and uniqueness of a competitive equilibrium, I rely on the following strategy. Firstly, for a given $K_{t+1} \geq 0$, I derive capital market clearing quantities of wage, employment, and the entrepreneurial rent. Secondly, for a given $K_{t+1} \geq 0$, I derive capital and labor market clearing quantity of K_t. Thirdly, for a given $K_t \geq 0$, I demonstrate the existence and uniqueness of K_{t+1} which is consistent with agents optimal behavior and clears capital and labor markets simultaneously.

4.1 Equilibrium in the Capital Market

For a given $K_{t+1} \in (0, m)$, what are the values of (w_t, θ_{t+1}) and (ℓ_t^e, s_t^e) which are consistent with agents' optimal behavior and at the same time clear capital market? To answer this question, I consider two cases separately. Case I corresponds to the binding credit constraint, while Case II corresponds to the slack credit constraint.

Case I (binding credit constraint): Suppose $\frac{(\lambda-(1-\lambda)\theta_{t+1})m}{w_t} \geq 1$. Then it follows from Proposition 1 that $s_t^e = 1$, $\ell_t^e = 1$, $i_t^e = \frac{w_t}{\lambda-(1-\lambda)\theta_{t+1}} \leq m$, and thus $V^e(w_t, \theta_{t+1}) = 0 < V^d$. Since every young agent strictly prefers to become a depositor rather than an entrepreneur, it follows that $K_{t+1} = 0$. This is in contradiction with $K_{t+1} \in (0, m)$ and thus $\frac{(\lambda-(1-\lambda)\theta_{t+1})m}{w_t} \geq 1$ cannot happen in equilibrium. Suppose $\frac{(\lambda-(1-\lambda)\theta_{t+1})m}{w_t} \in (\beta\gamma, 1)$. Then it follows from Proposition 1 that $i_t^e = m$, $s_t^e \ell_t^e w_t = (\lambda - (1-\lambda)\theta_{t+1})m$, and thus $i_t^e - s_t^e \ell_t^e w_t = (1-\lambda)(1+\theta_{t+1})m$. Each entrepreneur transforms m units of final commodity in period t into m units of capital in period $t+1$. In order for the aggregate capital to be K_{t+1} in period $t+1$, the mass of entrepreneurs in period t must be $\frac{K_{t+1}}{m}$. Since the size (i.e., the mass) of young agents is normalized to unity, it follows that the mass of depositors in period t is $1 - \frac{K_{t+1}}{m}$. This implies that the pair (w_t, θ_{t+1}) being consistent with agents' optimal behavior and at the same time clearing the capital market must satisfy the system of equations,

$$V^e(w_t, \theta_{t+1}) = V^d \quad \text{and} \quad \beta\gamma w_t \left(1 - \frac{K_{t+1}}{m}\right) = (1-\lambda)(1+\theta_{t+1})m\frac{K_{t+1}}{m}. \tag{9}$$

On the one hand, if $V^e(w_t, \theta_{t+1}) > V^d$ then the supply of credit is zero while demand of credit is positive because young agents strictly prefer to become entrepreneurs rather than depositors. On the other hand, if $V^e(w_t, \theta_{t+1}) < V^d$ then the demand of credit is zero while supply of credit is positive because young agents strictly prefer to become depositors rather than entrepreneurs. Thus the condition $V^e(w_t, \theta_{t+1}) = V^d$ ensures co-existence of entrepreneurs (or borrowers) and depositors (or lenders). Depositors save $\beta\gamma w_t$ units of final commodity, while entrepreneurs save $s_t^e \ell_t^e w_t = (\lambda - (1-\lambda)\theta_{t+1})m$, invest $i_t^e = m$, and thus borrow $i_t^e - s_t^e \ell_t^e w_t = (1-\lambda)(1+\theta_{t+1})m$ units of final commodity. Aggregate supply of credit is equal to the aggregate demand of credit holds when second equation of (9) is satisfied. After expressing θ_{t+1} from second equation of (9), I obtain that the entrepreneurial rent consistent with equilibrium in the capital market is

$$\theta_{t+1} = \frac{\beta\gamma w_t}{K_{t+1}} \frac{m - K_{t+1}}{(1-\lambda)m} - 1. \tag{10}$$

This with (3), (8), and the first equation of (9) implies that the wage consistent with equilibrium in the capital market is

$$\frac{m}{w_t} = \frac{\beta\gamma m}{K_{t+1}} + (1-\beta\gamma)\left(1 - \left(\frac{(1-\lambda)K_{t+1}}{\lambda(m - K_{t+1})}\right)^{\frac{\beta\gamma}{1-\beta\gamma}}\right). \tag{11}$$

Case II (slack credit constraint): Suppose $\frac{(\lambda-(1-\lambda)\theta_{t+1})m}{w_t} \leq \beta\gamma$, then it follows from Proposition 1 that $\ell_t^e = \gamma$, $s_t^e = \beta$, $i_t^e = \frac{\beta\gamma w_t}{\lambda-(1-\lambda)\theta_{t+1}} \geq m$, and thus $V^e(w_t, \theta_{t+1}) = V^d \left(\frac{\lambda(1+\theta_{t+1})}{\lambda-(1-\lambda)\theta_{t+1}}\right)^{\beta\gamma}$. As a result, the pair (w_t, θ_{t+1}) being consistent with agents' optimal behavior and at the same time clearing the capital market is

$$\theta_{t+1} = 0 \quad \text{and} \quad w_t = \frac{K_{t+1}}{\beta\gamma}. \tag{12}$$

On the one hand, if $\theta_{t+1} > 0$ then the supply of credit is zero while the demand of credit is positive because everyone strictly prefers to become an entrepreneur. On the other hand, if $\theta_{t+1} < 0$ then the demand of credit is zero while supply of credit is positive because everyone strictly prefers to become a depositor. Competition among entrepreneurs and depositors will cause the disappearance of equilibrium rent, $\theta_{t+1} = 0$. In such case young agents are indifferent among Options 1 and 2 and everyone supplies γ units of labor and saves β fraction of their labor income. Since entrepreneurs save $s_t^e \ell_t^e w_t = \beta\gamma w_t$, invest $i_t^e = m$, and thus borrow $m - \beta\gamma w_t$ units of final commodity, it follows that the capital market clears when

$$\beta\gamma w_t \left(1 - \frac{K_{t+1}}{m}\right) = (m - \beta\gamma w_t)\frac{K_{t+1}}{m} \quad \Leftrightarrow \quad w_t = \frac{K_{t+1}}{\beta\gamma}.$$

After combining Cases I and II (expressions (10), (11), and (12) in particular), I obtain that capital market clearing pair is $(w_t, \theta_{t+1}) = (\mathcal{W}(K_{t+1}), \mathcal{R}(K_{t+1}))$, where

$$\mathcal{W}(K) = \begin{cases} \frac{K}{\beta\gamma + \frac{(1-\beta\gamma)K}{m}\left(1-\left(\frac{(1-\lambda)K}{\lambda(m-K)}\right)^{\frac{\beta\gamma}{1-\beta\gamma}}\right)} & \text{if} \quad K \in (0, \lambda m) \\ \frac{K}{\beta\gamma} & \text{if} \quad K \in [\lambda m, m), \end{cases} \tag{13}$$

and

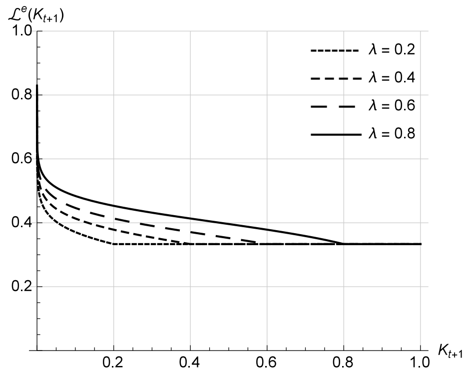
$$\mathcal{R}(K) = \begin{cases} \frac{\beta\gamma\mathcal{W}(K)}{K}\frac{m-K}{(1-\lambda)m} - 1 & \text{if} \quad K \in (0, \lambda m) \\ 0 & \text{if} \quad K \in [\lambda m, m). \end{cases} \tag{14}$$

It follows from (5), (6), (13), and (14), that entrepreneurs' optimal labor supply and optimal savings rate consistent with agents' optimal behavior and equilibrium in the capital market are $\ell_t^e = \mathcal{L}^e(K_{t+1})$ and $s_t^e = \mathcal{S}^e(K_{t+1})$ respectively, where

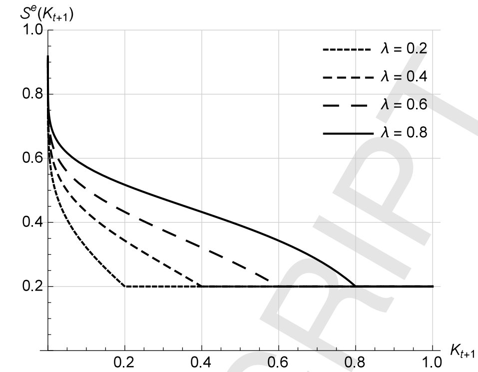
$$\mathcal{L}^e(K) = \begin{cases} 1 - (1-\gamma)\left(\frac{(1-\lambda)K}{\lambda(m-K)}\right)^{\frac{\beta\gamma}{1-\beta\gamma}} & \text{if} \quad K \in (0, \lambda m) \\ \gamma & \text{if} \quad K \in [\lambda m, m) \end{cases} \tag{15}$$

and

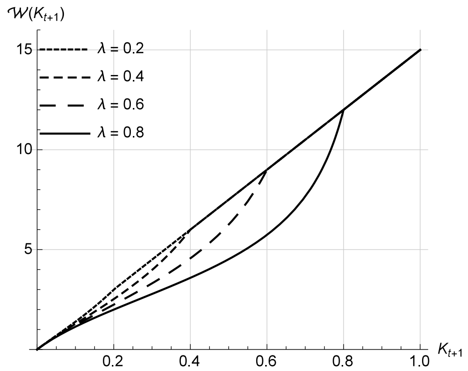
$$\mathcal{S}^e(K) = \frac{1-\beta\gamma}{1-\gamma} - \frac{(1-\beta)\gamma}{1-\gamma}\frac{1}{\mathcal{L}^e(K)}.$$



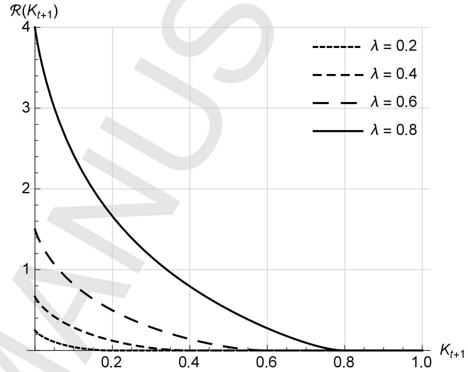
(a) Entrepreneur's labor supply for different values of parameter λ.



(b) Entrepreneur's savings rate for different values of parameter λ.



(c) Equilibrium wage for different values of parameter λ.



(d) Equilibrium entrepreneurial rent for different values of parameter λ.

Figure 1: The above figures are constructed when $\beta = 1/5$, $\gamma = 1/3$, and $m = 1$. All figures presented in this paper are constructed by using a computational program Mathematica, version 9.0. The code is available upon request.

Figure 1 visualizes the configurations of $\ell_t^e = \mathcal{L}^e(K_{t+1})$, $s_t^e = \mathcal{S}^e(K_{t+1})$, $w_t = \mathcal{W}(K_{t+1})$ and $\theta_{t+1} = \mathcal{R}(K_{t+1})$ for different values of parameter λ. As the figure indicates, for a sufficiently small value of next period capital stock, $K_{t+1} \in (0, \lambda m)$, entrepreneurs supply more labor and save a higher portion of their labor income than the depositors do. As a result, entrepreneurs can offer a higher interest rate to creditors without violating the borrowing constraint. By sacrificing first period leisure, and first period consumption, entrepreneurs earn the entrepreneurial rent, which in equilibrium compensates them from a disutility of extra work and low first period consumption. In equilibrium, ex-ante homogeneous agents make ex-post heterogeneous choices of labor supply, saving, occupation, labor, and capital incomes. That is why ex-ante homogeneous agents become ex-post heterogeneous despite the fact that they achieve the same levels of lifetime utility.

When $K_{t+1} \in (0, \lambda m)$, then the equilibrium allocation in which every agent supplies γ units of labor and saves β fraction of their labor income is not consistent with agents' optimal behavior. To verify this, let us assume the existence of a symmetric equilibrium in which all young agents supply the same amount of labor and save the same fraction of their labor income. This, on the one hand implies that $\theta_{t+1} = 0$, and on the other hand implies that $K_{t+1} = \beta\gamma w_t \in (0, \lambda m)$. However, these two equations are inconsistent with the borrowing constraint because,

$$m - \beta\gamma w_t > (1-\lambda)m \quad \Leftrightarrow \quad (m - \beta\gamma w_t)r_{t+1} > (1-\lambda)m\rho_{t+1}.$$

Instead of a symmetric equilibrium there exists a unique and asymmetric equilibrium in which $\ell_t^e \in (\gamma, 1)$ and $s_t^e \in (\beta, 1)$ for $K_{t+1} \in (0, \lambda m)$. By supplying more labor and saving higher fraction of their labor income, young agents are able to offer a higher interest rate to depositors without violating the borrowing constraint. Competition between young agents will drive up both entrepreneurs' labor supply, saving rate, and equilibrium interest rate, so that

$$m - s_t^e \ell_t^e w_t = m - \mathcal{S}^e(K_{t+1})\mathcal{L}^e(K_{t+1})\mathcal{W}(K_{t+1}) = (1-\lambda)(1+\mathcal{R}(K_{t+1}))m = (1-\lambda)m\frac{\rho_{t+1}}{r_{t+1}}.$$

4.2 Equilibrium in the Labor Market

What is the pair of current and next period capital stocks, (K_t, K_{t+1}), which is consistent with agents' optimal behavior and clears capital and labor markets simultaneously? For a given $K_{t+1} \in (0, m)$, the size of young agents who become entrepreneurs at time t is $\frac{K_{t+1}}{m}$, while size of young agents who become depositors at time t is $1 - \frac{K_{t+1}}{m}$. Each entrepreneur supplies $\mathcal{L}^e(K_{t+1})$ units of labor while each depositor supplies γ units of labor. As a result, the aggregate labor supply at time t is

$$L_t = \mathcal{L}(K_{t+1}) = \mathcal{L}^e(K_{t+1})\frac{K_{t+1}}{m} + \gamma\left(1 - \frac{K_{t+1}}{m}\right).$$

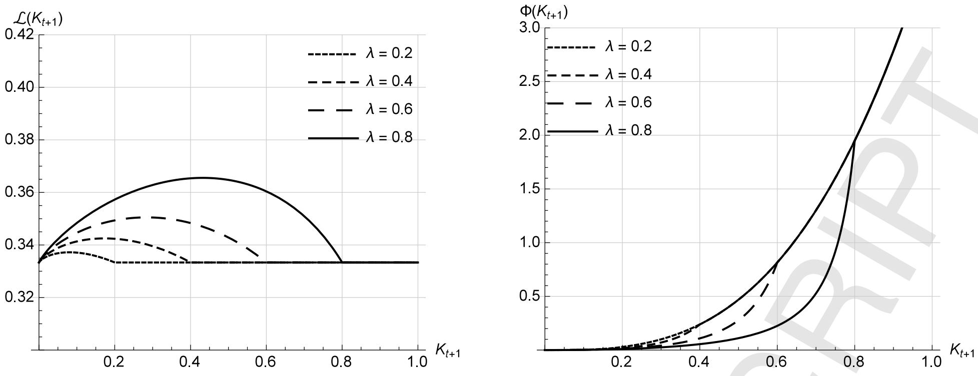
This with (15) implies that, for a given $K_{t+1} \in (0, m)$,

$$\mathcal{L}(K) = \begin{cases} \gamma + (1-\gamma)\dfrac{K}{m}\left(1 - \left(\dfrac{(1-\lambda)K}{\lambda(m-K)}\right)^{\frac{\beta\gamma}{1-\beta\gamma}}\right) & \text{if} \quad K \in (0, \lambda m) \\ \gamma & \text{if} \quad K \in [\lambda m, m). \end{cases} \tag{16}$$

Based on (13), one can easily verify that $\mathcal{L}$ can be also be expressed as

$$\mathcal{L}(K) = \frac{(1-\beta)\gamma}{1-\beta\gamma} + \frac{1-\gamma}{1-\beta\gamma}\frac{K}{\mathcal{W}(K)}. \tag{17}$$

Proposition 3 *For a given $\beta \in (0, 1]$, $\gamma \in (0, 1]$, $\lambda \in (0, 1)$, and $m > 0$,*



(a) Equilibrium aggregate labor supply for different values of parameter λ.

(b) Configuration of Φ for different values of parameter λ. This figure is constructed when $f(k) = Ak^{\alpha}$ with $A = 10$, and $\alpha = 0.33$.

Figure 2: Both figures are constructed when $\beta = 1/5$, $\gamma = 1/3$, and $m = 1$.

(1) *$K \mapsto \mathcal{W}(K)$ is a continuous and strictly increasing function satisfying boundary conditions, $\lim_{K\downarrow 0} \mathcal{W}(K) = 0$, $\lim_{K\uparrow \lambda m} \mathcal{W}(K) = \frac{\lambda m}{\beta\gamma}$, and $\mathcal{W}(K) \equiv \frac{K}{\beta\gamma}$ for $K \geq \lambda m$.*

(2) *$K \mapsto \mathcal{R}(K)$ is a continuous and strictly decreasing function satisfying boundary conditions, $\lim_{K\downarrow 0} \mathcal{R}(K) = \frac{\lambda}{1-\lambda}$, $\lim_{K\uparrow \lambda m} \mathcal{R}(K) = 0$, and $\mathcal{R}(K) \equiv 0$ for $K \geq \lambda m$.*

(3) *$K \mapsto \mathcal{L}(K)$ is a continuous and inverted "U" shaped curve satisfying boundary conditions, $\lim_{K\downarrow 0} \mathcal{L}(K) = \gamma$, $\lim_{K\uparrow \lambda m} \mathcal{L}(K) = \gamma$, and $\mathcal{L}(K) \equiv \gamma$ for $K \geq \lambda m$.*

Proof of Proposition 3 can be found in the Appendix. Figure 2(a) visualizes the configuration of $\mathcal{L}(K)$ for different values of the parameter λ. As the figure indicates equilibrium labor supply initially increases and then decreases with aggregate capital stock. Aggregate labor supply is non-monotonic due to the presence of positive income and negative substitution effects. As K_{t+1} increases, the size of young agents who become entrepreneurs increases. Aggregate employment increases because entrepreneurs supply more labor (positive effect). At the same time increase of K_{t+1} implies the increase of wage. When wage is high then young entrepreneurs become able to substitute the current leisure for future consumption because they can overcome the minimum investment requirement and the borrowing constraint by reducing labor supply and savings rate. As a result, entrepreneurs' labor supply decreases (negative effect). Among these two effects the positive effect is dominant for small values of K_{t+1} while the negative effect becomes dominant for large values of K_{t+1}. The aggregate labor supply, consistent with agents optimal behavior and equilibrium in the capital market, displays an inverted "U" shape. Properties of $\mathcal{W}$ along with (17) implies that $\mathcal{L}$ is a continuous function satisfying the boundary conditions, $\lim_{K\downarrow 0} \mathcal{L}(K) = \gamma$, $\lim_{K\uparrow \lambda m} \mathcal{L}(K) = \gamma$, and $\mathcal{L}(K) \equiv \gamma$ for $K \in [\lambda m, m)$.

For a given K_t, the wage and aggregate employment pair, (w_t, L_t), clearing the labor market satisfies

$$L_t W^{-1}(w_t) = K_t. \tag{18}$$

Since the wage and aggregate employment pair consistent with agents' optimal behavior and equilibrium in the capital market is $(w_t, L_t) = (\mathcal{W}(K_{t+1}), \mathcal{L}(K_{t+1}))$, it follows from (18) that the equilibrium pair of current and next period capital stocks, (K_t, K_{t+1}), must satisfy,

$$\Phi(K_{t+1}) = K_t \quad \text{where} \quad \Phi(K_{t+1}) = \mathcal{L}(K_{t+1})W^{-1}[\mathcal{W}(K_{t+1})]. \tag{19}$$

Before establishing the existence and uniqueness of equilibrium, at this point it is worthwhile to discuss the benchmark case in which either there is no imperfection in the credit market, $\lambda = 0$, or there is no minimum investment requirement, $m = 0$, or both. If $\lambda m = 0$ then it follows from (13) and (16) that the equilibrium wage and aggregate labor supply are $\mathcal{W}(K_{t+1}) = \frac{K_{t+1}}{\beta\gamma}$ and $\mathcal{L}(K_{t+1}) = \gamma$ respectively. This with (19) implies that the equilibrium pair (K_t, K_{t+1}) satisfies,

$$\gamma W^{-1}\left(\frac{K_{t+1}}{\beta\gamma}\right) = K_t \quad \Leftrightarrow \quad K_{t+1} = \phi_0(K_t) \quad \text{where} \quad \phi_0(K_t) = \beta\gamma W\left(\frac{K_t}{\gamma}\right). \tag{20}$$

In other words, when $\lambda m = 0$ then the existence and uniqueness of equilibrium capital stock is always guaranteed by the monotonicity of W. [5]

If $\lambda m > 0$ then monotonicity of Φ can no longer be guaranteed by the monotonicity of W because $\mathcal{L}$ is a non-monotonic function. In order to prove the existence and uniqueness of equilibrium $K_{t+1} \in (0, m)$ solving $K_t = \Phi(K_{t+1})$ for any $K_t \in (0, m)$, I impose the following restrictions of the production function. First, I assume that the elasticity of substitution between capital and labor inputs is at least unity. Second, I assume that the aggregate investment made by young agents is smaller than m when young agents supply γ units of labor and save β fraction of their labor income.

Assumption 1 *f is such that*

(a) $\dfrac{f'(k)W(k)}{f(k)W'(k)} = -\dfrac{f'(k)}{kf''(k)}\left(1 - \dfrac{kf'(k)}{f(k)}\right) \geq 1$ *for any* $k \geq 0$.

(b) $\beta\gamma W\left(\dfrac{m}{\gamma}\right) < m.$

It is easy to verify that Assumption 1 is satisfied when the production function is Cobb-Douglas, $f(k) = Ak^\alpha$, with capital share parameter $\alpha \in (0, 1)$ and the total factor productivity parameter $A \in (0, \frac{1}{(1-\alpha)\beta}(\frac{m}{\gamma})^{1-\alpha})$.

[5]Monotonicity of W follows from concavity of f because $W'(k) = -kf''(k) > 0$.

Proposition 4 ***(Existence and Uniqueness of Equilibrium):*** *If Assumption 1 is satisfied then for a given* $K_t \in (0, m)$ *there exists a unique* $K_{t+1} = \phi(K_t) \in (\phi_0(K_t), m)$ *(where* ϕ_0 *is defined in (20)) which solves* $K_t = \Phi(K_{t+1})$*.* ϕ *is a continuous and a strictly increasing function satisfying boundary properties,* $0 \leq \phi(0) < \phi(m) < m$ *and* $\phi(K_t) \equiv \phi_0(K_t)$ *for* $K_t \in [\lambda m, m)$*.*

Proof of this proposition is given in the Appendix. The dynamical system describing the evolution of capital stock is $(\phi, (0, m))$. At this point it is worthwhile to discuss the role of Assumption 1. Assumption 1.(a) guarantees the monotonicity of Φ. As a result, there exists at most one K_{t+1} solving $K_t = \Phi(K_{t+1})$. Assumption 1.(a) is a sufficient but not a necessary condition for uniqueness of equilibrium. For example, when $\gamma = 1$ then Φ is still monotonic even when Assumption 1.(a) is violated. However, for any $\gamma \in (0, 1)$, one may not rule out non-monotonicity of Φ (and thus multiplicity of equilibrium) after relaxing the Assumption 1.(a). Assumption 1.(b) is needed in order to guarantee the uniqueness of the equilibrium capital stock K_{t+1} belonging to the interval $(0, m)$. If Assumption 1.(b) is relaxed then the equilibrium capital stock might become $K_{t+1} > m$ for some values of $K_t \in (0, m)$. In other words, Assumption 1.(b) implies that $K_{t+1} \in (0, m) \Leftrightarrow \mathcal{W}(K_{t+1}) \in (0, \frac{m}{\beta\gamma})$ and thus guarantees young agents need to borrow in order to become an entrepreneur.

4.3 Dynamics

If Assumption 1 is satisfied then the dynamics of capital stock is described by the following time one map

$$K_{t+1} = \phi(K_t) \in (0, m) \quad \text{for any} \quad K_t \in (0, m).$$

Let ϕ^t denotes the t^{th} iterate of ϕ, where $t = 1, 2, 3, ...$ Then for a given K_0, the equilibrium capital stock is given by

$$\mathcal{O}(K_0) = \left\{K_0, \phi^1(K_0), \phi^2(K_0), ...\right\}. \tag{21}$$

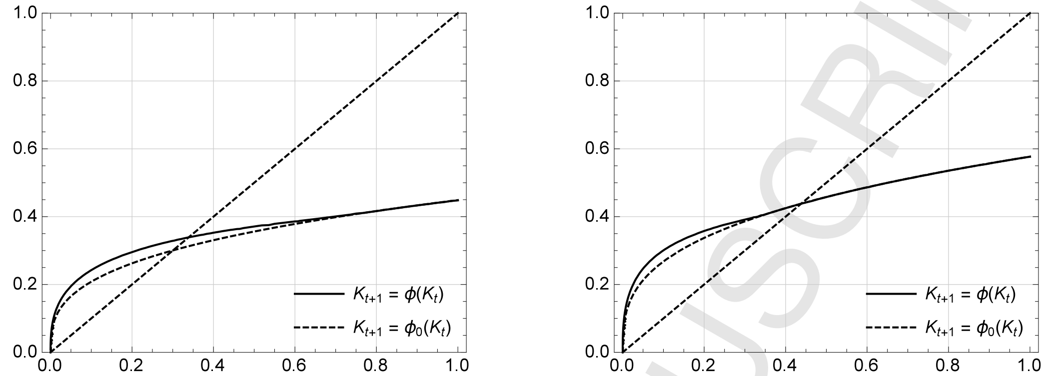
Since ϕ is a strictly increasing and bounded function, it follows that the above sequence converges to K_∞. Monotonicity of ϕ also implies that if $K_0 < K_\infty$ ($K_0 > K_\infty$) then equilibrium capital stock increases (decreases) over time. Equilibrium values of aggregate labor supply, capital labor ratio, and aggregate output are

$$L_t = \mathcal{L}[\phi^{t+1}(K_0)], \quad K_t = \phi^t(K_0), \quad k_t = \frac{K_t}{L_t}, \quad \text{and} \quad Y_t = L_t f(k_t),$$

while equilibrium values of the rental rate of capital, wage, and interest rate are

$$w_t = W(k_t), \quad \theta_{t+1} = \Theta(w_t), \quad \rho_{t+1} = f'(k_{t+1}), \quad \text{and} \quad r_{t+1} = \frac{\rho_{t+1}}{1 + \theta_{t+1}}.$$

If Assumption 1.(b) is satisfied then $K_t \in (0, m)$ is the forward invariant set of capital stock because $K_{t+1} = \phi(K_t) \in (0, m)$ for any $K_t \in (0, m)$. This with monotonicity of ϕ implies that the economy converges to a steady state in which the capital stock is $K_\infty \in (0, m)$ and thus entrepreneurial need to borrow in order to satisfy the minimum investment requirement.



(a) Case with $\beta\gamma W(\frac{\lambda m}{\gamma}) < \lambda m \Leftrightarrow A < \frac{1}{(1-\alpha)\beta}\left(\frac{\lambda m}{\gamma}\right)^{1-\alpha}$. Under such configuration, $K_{t+1} \in (0, \lambda m)$ for any $K_t \in (0, \lambda m)$. This figure is constructed when $A = 7$ implying that $K_\infty = 0.34 \in (0, \lambda m)$.

(b) Case with $\beta\gamma W(\frac{\lambda m}{\gamma}) \geq \lambda m \Leftrightarrow A \geq \frac{1}{(1-\alpha)\beta}\left(\frac{\lambda m}{\gamma}\right)^{1-\alpha}$. Under such configuration, $K_{t+1} \in [\lambda m, m)$ for some $K_t \in (0, \lambda m)$. This figure is constructed when $A = 9$ implying that $K_\infty = 0.44 \in [\lambda m, m)$.

Figure 3: Two possible configurations of the time one map of capital accumulation. Both figures are constructed when $f(k) = Ak^\alpha$, $\beta = 1/5$, $\gamma = 1/3$, $m = 1$, $\alpha = 1/3$, and $\lambda = 0.4$ implying that $W(\frac{\lambda m}{\gamma}) = \frac{\lambda m}{\beta\gamma} \Leftrightarrow A = \frac{1}{(1-\alpha)\beta}\left(\frac{\lambda m}{\gamma}\right)^{1-\alpha} = 8.47$.

Figure 3 displays configurations of $K_{t+1} = \phi(K_t)$ (solid line) and $K_{t+1} = \phi_0(K_t)$ (dotted line) when $\beta\gamma W(\frac{\lambda m}{\gamma}) < \lambda m$ and $\beta\gamma W(\frac{\lambda m}{\gamma}) > \lambda m$. If $\beta\gamma W(\frac{\lambda m}{\gamma}) < \lambda m$, then imperfections in the credit market has long run implication on capital accumulation because the imperfection in the credit market affects agents' decision making in a steady state. If $\beta\gamma W(\frac{\lambda m}{\gamma}) > \lambda m$ then imperfections in the credit market has only a short run implication on capital accumulation because imperfections in the credit market does not affect agents' decision making in a steady state.

5 Equilibrium Income Inequality

How imperfection in the credit market affects income inequality? In order to answer this question I start by discussing the benchmark case. If $\lambda m = 0$ then there is no entrepreneurial rent, $\theta_t = 0$, all young agents supply $\ell^e_{t-1} = \ell^d_{t-1} = \gamma$ units of labor, and save the same fraction of their labor income, $s^e_{t-1} = s^d_{t-1} = \beta$. As a result, young agents earn the same labor income, $y^d_{1t} = y^e_{1t} = \gamma w_t$, while old agents earn the same capital income, $y^d_{2t-1} = y^e_{2t-1} = \beta\gamma w_{t-1}\rho_t = K_t\rho_t$ (because equilibrium capital stock satisfies $K_t = \beta\gamma w_{t-1}$). As a result, there is neither labor income nor capital income inequality.

Lemma 1 *Suppose population consists of I groups. Let the relative size of group $i = 1, 2, ..., I$ is p_i and everyone in that group earns the income y_i. Then the Gini index of income inequality in the entire population is*

$$G = \frac{\sum_{i=1}^{I}\sum_{j=1}^{I} p_i p_j |y_i - y_j|}{2\sum_{i=1}^{I} p_i y_i}.$$

Proof of Lemma 1: can be found in Kendal and Stuart (1963).

What is income inequality in benchmark case and how does it depend on per capita income? If $\lambda m = 0$ then the aggregate labor supply is $L_t = \gamma$ and thus the capital to labor ratio is $k_t = \frac{K_t}{\gamma}$. Since 50% of all agents (i.e., all young agents) earn the labor income $\gamma w_t = \gamma W(k_t)$ which is $\frac{\gamma W(k_t)}{\gamma f(k_t)} = \frac{W(k_t)}{f(k_t)}$ fraction of aggregate income, while another 50% of all agents (i.e., all old agents) earn the capital income $K_t\rho_t = \gamma k_t f'(k_t)$ which is $\frac{\gamma k_t f'(k_t)}{\gamma f(k_t)} = \frac{k_t f'(k_t)}{f(k_t)}$ fraction of total income, it follows from Lemma 1 that the Gini index of income inequality in benchmark case is

$$\frac{1}{2}\left|\frac{k_t f'(k_t)}{f(k_t)} - \frac{W(k_t)}{f(k_t)}\right| = \left|0.5 - \frac{k_t f'(k_t)}{f(k_t)}\right| \quad \text{because} \quad \frac{W(k_t)}{f(k_t)} = 1 - \frac{k_t f'(k_t)}{f(k_t)}.$$

If the production function is Cobb-Douglas with $f(k) = Ak^\alpha$ then the capital share in production is $\frac{kf'(k)}{f(k)} = \alpha$ and thus the Gini index of income inequality is a constant $|0.5 - \alpha|$ and independent from per capita income. In the rest of this section I discuss the case with $\lambda m > 0$ and argue that imperfection in the credit market magnifies income inequality.

5.1 Labor Income Inequality

At time t, the pool of young agents is divided into two subgroups, depositors of size $1 - \frac{K_{t+1}}{m}$ earning labor income $y^d_{1t} = \gamma w_t$ and entrepreneurs of size $\frac{K_{t+1}}{m}$ earning labor income $y^e_{1t} = \ell^e_t w_t$. Aggregate labor income earned by young agents is

$$\left(1 - \frac{K_{t+1}}{m}\right)\gamma\mathcal{W}(K_{t+1}) + \frac{K_{t+1}}{m}\mathcal{L}^e(K_{t+1})\mathcal{W}(K_{t+1}) = \mathcal{L}(K_{t+1})\mathcal{W}(K_{t+1}).$$

This along with Lemma 1 implies that the Gini index of labor income inequality at time t is $g_{Lt} = \mathcal{G}_L(K_{t+1})$, where

$$\mathcal{G}_L(K) = \left(1 - \frac{K}{m}\right)\frac{K}{m}\frac{\mathcal{L}^e(K) - \gamma}{\mathcal{L}(K)} = \left(1 - \frac{K}{m}\right)\left(1 - \frac{\gamma}{\mathcal{L}(K)}\right), \tag{22}$$

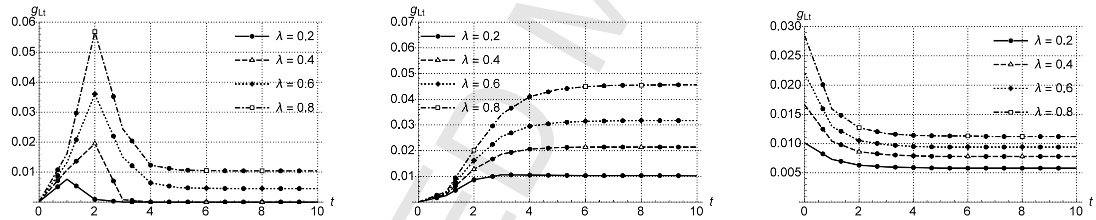
because $(\mathcal{L}^e(K) - \gamma)K = (\mathcal{L}(K) - \gamma)m$ follows from (15) and (16). This with (21) implies that the equilibrium labor income inequality is $\{\mathcal{G}_L[\phi^{t+1}(K_0)]\}_{t=0}^{\infty}$.

Proposition 5 *Let fix the values of $\beta \in (0, 1]$, $\gamma \in (0, 1]$, and $m > 0$.*

(a) *For a given $\lambda \in (0, 1)$, $\lim_{K\downarrow 0} \mathcal{G}_L(K) = \lim_{K\uparrow\lambda m} \mathcal{G}_L(K) = 0$ and there exists a unique $K_L^c \in (0, \lambda m)$ which solves $\mathcal{G}_L'(K) = 0$.*

(b) *For a given $K_t \in (0, \lambda m)$, $\lambda \mapsto \mathcal{G}_L(K_t)$ is a strictly increasing function.*

Proof of this proposition can be found in the Appendix. As Proposition 5 indicates, for a given $\lambda \in (0, 1)$, $K \mapsto \mathcal{G}_L(K)$ is an inverted "U" shaped curve. The maximum inequality achieved depends on parameter γ. If $\gamma = 1$ then the channel through which entrepreneurial rent indirectly affects the labor income inequality is entirely shut down because when $\gamma = 1$ then young agents supply one unit of labor endowment inelastically and thus the labor income inequality is zero. The indirect effect of entrepreneurial rent on labor income inequality strengthens as γ decreases. This is so because for a given K_{t+1} the difference $1 - \frac{\gamma}{\mathcal{L}(K_{t+1})}$ decreases with parameter γ. The inverted "U" shape of $K \mapsto \mathcal{G}_L(K)$ implies the existence of three distinct equilibrium dynamics for labor income inequality. All three cases are visualized on Figure 4



(a) When $K_0 < K_L^c < K_\infty$ then the labor income inequality initially increases and then decreases. This figure is constructed when $A = 10$ and $K_0 = 10^{-12}$.

(b) When $K_0 < K_\infty < K_L^c$ then the labor income inequality increases over time. This figure is constructed when $A = 3$, $K_0 = 10^{-12}$.

(c) When $K_L^c < K_0 < K_\infty$ then the labor income inequality increases over time. This figure is constructed when $A = 1$ and $K_0 = 0.50$.

Figure 4: Three possible equilibrium dynamics of labor income inequality for different values of parameter λ. All three figures are constructed when $\alpha = 0.33$, $\beta = 1/5$, $\gamma = 1/3$, and $m = 1$.

If $K_0 < K_L^c < K_\infty$ then the equilibrium labor income inequality has an inverted "U" shape. Why does labor income inequality first increase and then decrease with per capita income? There are two opposite effects of per capita income on labor income inequality. On the one hand, an increase in per capita income causes the labor income earned by depositors and entrepreneurs to converge (negative effect). On the other hand, an increase in per capita income causes the size of entrepreneurs to increase (positive effect). When per the capital income is relatively low (high), then the positive (negative) effect is the dominant one implying the labor income inequality first to increase and then decrease with per capita income. The hypothesis of offsetting income and substitution based effects proposed in this paper provides a possible explanation of the location of the "turning point" income which reflects a trade-off between the rising ability for agents to become entrepreneurs and a declining incentive for agents to substitute the first period leisure for future consumption.

The mechanism for generating inverted-U Kuznets curve discussed in this paper differs from the mechanism proposed in the existing literature. In particular, the existing models discuss the so-called demand-pull channel through which inverted-U Kuznets curve can be generated by an inter-sectoral transition from agriculture to industry. Kuznets (1955) argued that industrial development is accompanied by an urban-rural income gap and rising labor income inequality. Inequality would rise as the differential between city and countryside came to dominate the development landscape. Eventually, however, the scarcity of workers in the rural sector becomes such that the wages for rural workers start to increase as well, which has an effect of decreasing inequality. Theoretical models displaying the impact of sectoral composition of labor income inequality include Baumol (1967), Baumol et al. (1985), Laitner (2000), Kongsamut et al. (2001), Gollin et al. (2002), Ngai and Pissarides (2007), and Blum (2008), among others. Consistent with the above mentioned papers, this paper argues that the labor income inequality widens in early phases of development and might narrow in later stages. The proposed model identifies the imperfection in the credit market as the major factor behind inverted-U Kuznets curve.

If $K_0 < K_\infty < K_L^c$ then equilibrium labor income inequality increases over time because the rise of per capita income boost agents ability to become entrepreneurs. The migration from depositors to entrepreneurs boosts further the labor income inequality because entrepreneur's labor supply does not change too much. If $K_L^c < K_0 < K_\infty$ then the rise of income still boosts agents ability to become entrepreneurs, but at the same time more wealthy agents will find it more optimal to substitute future consumption for current leisure. The decrease of entrepreneur's labor supply causes the reduction of equilibrium labor income inequality.

5.2 Capital Income Inequality

At time t, the pool of old agents is divided into two subgroups, depositors of measure $1-\frac{K_t}{m}$ earning capital income $y^d_{2t-1} = \beta\gamma w_{t-1} r_t$ and entrepreneurs of measure $\frac{K_t}{m}$ earning capital income $y^e_{2t-1} = m\rho_t - (m - s^e_{t-1}\ell^e_{t-1}w_{t-1})r_t$. If $K_t \in (0, \lambda m)$ then $s^e_{t-1}\ell^e_{t-1}w_{t-1} = (\lambda - (1-\lambda)\theta_t)m$, $1+\theta_t = \frac{\beta\gamma w_{t-1}}{K_t}\frac{m-K_t}{(1-\lambda)m}$, and $r_t = \frac{\rho_t}{1+\theta_t}$. As a result, capital incomes earned by old depositors and old entrepreneurs are,

$$y^d_{2t-1} = \frac{(1-\lambda)m}{m-K_t} K_t \rho_t \quad \text{and} \quad y^e_{2t-1} = \frac{\lambda m}{K_t} K_t \rho_t,$$

respectively. If $K_t \in [\lambda m, m)$ then $s^e_{t-1}\ell^e_{t-1}w_{t-1} = \beta\gamma w_{t-1}$, $\theta_t = 0$, and $r_t = \rho_t$. As a result, capital incomes earned by old depositors and old entrepreneurs are the same, $y^d_{2t-1} = y^e_{2t-1} = \beta\gamma w_{t-1}\rho_t = K_t\rho_t$. By combining cases $K_t \in (0, \lambda m)$ and $K_t \in [\lambda m, m)$, I obtain that capital incomes earned by old depositors and old entrepreneurs are

$$y^d_{2t-1} = \min\left\{\frac{(1-\lambda)m}{m-K_t}, 1\right\} K_t\rho_t \quad \text{and} \quad y^e_{2t-1} = \max\left\{\frac{\lambda m}{K_t}, 1\right\} K_t\rho_t,$$

respectively. This along with Lemma 1 implies that the Gini index of capital income inequality, at time t, is $g_{Kt} = \mathcal{G}_K(K_t)$, where

$$\begin{aligned} \mathcal{G}_K(K) &= \left(1 - \frac{K}{m}\right)\frac{K}{m}\left(\max\left\{\frac{\lambda m}{K}, 1\right\} - \min\left\{\frac{(1-\lambda)m}{m-K}, 1\right\}\right) = \\ &= \begin{cases} \lambda - \dfrac{K}{m} & \text{if} \quad K \in (0, \lambda m) \\ 0 & \text{if} \quad K \in [\lambda m, m). \end{cases} \end{aligned} \tag{23}$$

This with (21) implies that the equilibrium capital income inequality is $\{\mathcal{G}_K[\phi^t(K_0)]\}_{t=0}^{\infty}$. Some important properties of capital income inequality can be highlighted here. Since $\mathcal{G}_K$ is a non-increasing function, it follows that equilibrium capital income inequality decreases along the capital accumulation path. This is in contrast with labor income inequality which may display three distinct equilibrium dynamics.

5.3 Income Inequality

At time t the pool of young agents is divided into two subgroups, young depositors of size $p^d_{1t} = \frac{1}{2}(1 - \frac{K_{t+1}}{m})$ earning labor income $y^d_{1t} = \gamma w_t$ and young entrepreneurs of size $p^e_{1t} = \frac{1}{2}\frac{K_{t+1}}{m}$ earning labor income $y^e_{1t} = \ell^e_t w_t$. At the same time the pool of old agents is divided into two subgroups as well, old depositors of size $p^d_{2t-1} = \frac{1}{2}(1 - \frac{K_t}{m})$ earning capital income $y^d_{2t-1} = \min\left\{\frac{(1-\lambda)m}{m-K_t}, 1\right\} K_t\rho_t$ and old entrepreneurs of size $p^e_{2t-1} = \frac{1}{2}\frac{K_t}{m}$ earning capital income $y^e_{2t-1} = \max\left\{\frac{\lambda m}{K_t}, 1\right\} K_t\rho_t$. This implies that the per capita income is

$$(p^d_{1t}y^d_{1t} + p^e_{1t}y^e_{1t}) + (p^d_{2t-1}y^d_{2t-1} + p^e_{2t-1}y^e_{2t-1}) = \frac{L_t w_t + K_t\rho_t}{2} = \frac{Y_t}{2}.$$

If the production function is Cobb-Douglas then $L_t w_t = (1-\alpha)Y_t$ and $K_t\rho_t = \alpha Y_t$. This with Lemma 1 implies that the Gini index of income inequality is

$$g_{KLt} = \mathcal{G}_{KL}(K_{t+1}) = \sum_{i=1}^{6} \mathcal{G}^i_{KL}(K_{t+1}),$$

where

$$\mathcal{G}^1_{KL}(K) = \tfrac{1-\alpha}{2}\left(1-\tfrac{K}{m}\right)\tfrac{K}{m}\tfrac{\mathcal{L}^e(K)-\gamma}{\mathcal{L}(K)} = \tfrac{1-\alpha}{2}\mathcal{G}_L(K)$$

$$\mathcal{G}^2_{KL}(K) = \tfrac{1-\alpha}{2}\left(1-\tfrac{K}{m}\right)\left(1-\tfrac{\Phi(K)}{m}\right)\left|\tfrac{\gamma}{\mathcal{L}(K)} - \min\left\{\tfrac{(1-\lambda)m}{m-\Phi(K)},1\right\}\tfrac{\alpha}{1-\alpha}\right|$$

$$\mathcal{G}^3_{KL}(K) = \tfrac{1-\alpha}{2}\left(1-\tfrac{K}{m}\right)\tfrac{\Phi(K)}{m}\left|\tfrac{\gamma}{\mathcal{L}(K)} - \max\left\{\tfrac{\lambda m}{\Phi(K)},1\right\}\tfrac{\alpha}{1-\alpha}\right|$$

$$\mathcal{G}^4_{KL}(K) = \tfrac{1-\alpha}{2}\tfrac{K}{m}\left(1-\tfrac{\Phi(K)}{m}\right)\left|\tfrac{\mathcal{L}^e(K)}{\mathcal{L}(K)} - \min\left\{\tfrac{(1-\lambda)m}{m-\Phi(K)},1\right\}\tfrac{\alpha}{1-\alpha}\right|$$

$$\mathcal{G}^5_{KL}(K) = \tfrac{1-\alpha}{2}\tfrac{K}{m}\tfrac{\Phi(K)}{m}\left|\tfrac{\mathcal{L}^e(K)}{\mathcal{L}(K)} - \max\left\{\tfrac{\lambda m}{\Phi(K)},1\right\}\tfrac{\alpha}{1-\alpha}\right|$$

$$\mathcal{G}^6_{KL}(K) = \tfrac{\alpha}{2}\left(1-\tfrac{\Phi(K)}{m}\right)\tfrac{\Phi(K)}{m}\left|\max\left\{\tfrac{\lambda m}{\Phi(K)},1\right\} - \min\left\{\tfrac{(1-\lambda)m}{m-\Phi(K)},1\right\}\right| = \tfrac{\alpha}{2}\mathcal{G}_K[\Phi(K)].$$

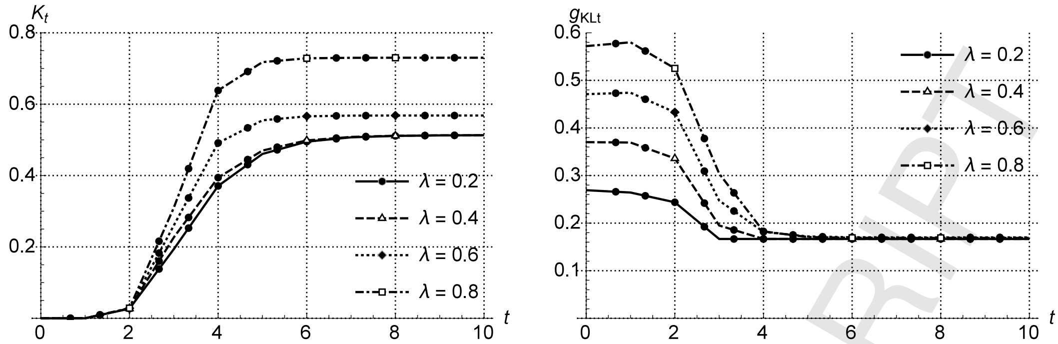
If $K_{t+1} \in (0, \lambda m)$ and $K_t = \Phi(K_{t+1}) \in (0, \lambda m)$ then the total population at time t consists of four subgroups and six subgroup pairs. That's why the Gini index of income inequality can be decomposed into six terms, each one representing income inequality within a subgroup. If $K_{t+1} \in [\lambda m, m)$ and $K_t = \Phi(K_{t+1}) \in (0, \lambda m)$ then the total population at time t consists of three subgroups and three subgroup pairs because $\mathcal{L}(K_{t+1}) = \mathcal{L}^e(K_{t+1}) = \gamma$ and thus young agents earn the same labor income. If $K_{t+1} \in (0, \lambda m)$ and $K_t = \Phi(K_{t+1}) \in [\lambda m, m)$, then the total population at time t again consists of three subgroups and three subgroup pairs because $\max\left\{\frac{\lambda m}{\Phi(K_{t+1})}, 1\right\} = \min\left\{\frac{(1-\lambda)m}{m-\Phi(K_{t+1})}, 1\right\} = 1$ and thus old agents earning the same capital income. If $K_{t+1} \in [\lambda m, m)$ and $K_t = \Phi(K_{t+1}) \in [\lambda m, m)$, then the total population at time t consists of two subgroups and one subgroup pair because young agents earn the same labor income while old agents earn the same capital income. As a result, $\mathcal{G}_{KL}(K_{t+1}) = |\alpha - 0.5|$.

Proposition 6 *If either $K_{t+1} \in (0, \lambda m)$ or $K_t = \Phi(K_{t+1}) \in (0, \lambda m)$ then $\mathcal{G}_{KL}(K_{t+1}) > |\alpha - 0.5|$. If $K_{t+1} \in [\lambda m, m)$ and $K_t = \Phi(K_{t+1}) \in [\lambda m, m)$ then $\mathcal{G}_{KL}(K_{t+1}) = |\alpha - 0.5|$.*

Proposition 6 indicates that the imperfection in the credit market magnifies income inequality. Figure 5(a) plots equilibrium capital stock for different values of parameter λ. As expected, per capita income increases along the accumulation path. Figure 5(b) plots equilibrium income inequality for different values of parameter λ. As the figure indicates, imperfection in the credit market magnifies the equilibrium income inequality.

5.4 Steady State Income Inequality

In this section I analyze how steady state income inequality depends on parameter λ. I consider the case when the production function is given by, $f(k) = Ak^{\alpha}$. It



(a) Equilibrium capital stock for different values of parameter λ.

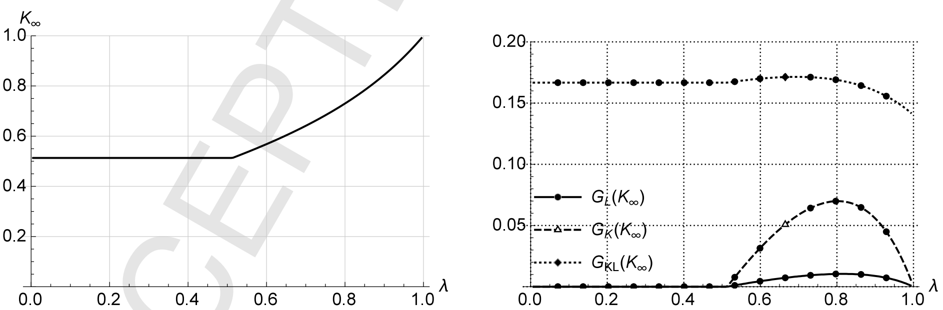
(b) Equilibrium income inequality for different values of parameter λ.

Figure 5: Dynamics of equilibrium capital stock and income inequality. Both figures are constructed when $A = 10$, $\alpha = 1/3$, $\beta = 1/5$, $\gamma = 1/3$, $m = 1$, and $K_0 = 10^{-12}$.

follows from (19) that the steady state capital, K_∞, satisfies

$$K_\infty = \mathcal{L}(K_\infty)W^{-1}[\mathcal{W}(K_\infty)]. \tag{24}$$

If imperfection in the credit market is sufficiently weak, $\lambda m \leq \gamma[A(1-\alpha)\beta]^{\frac{1}{1-\alpha}}$, then the expression for steady state capital stock can be obtained analytically, $K_\infty = \gamma[A(1-\alpha)\beta]^{\frac{1}{1-\alpha}}$, because $\mathcal{L}(K_\infty) = \gamma$ and $\mathcal{W}(K_\infty) = \frac{K_\infty}{\beta\gamma}$. This is not an interesting case however, because there is no capital and labor income inequalities in such steady state. In contrast, if imperfection in the credit market is sufficiently strong, $\lambda m > \gamma[A(1-\alpha)\beta]^{\frac{1}{1-\alpha}}$, then solution of (24) can be obtained only numerically. Figure 6(a) plots the numerical solution of (24) for $\lambda \in (0, 1)$. Figure 6(b) plots how $\mathcal{G}_L(K_\infty)$, $\mathcal{G}_K(K_\infty)$, and $\mathcal{G}_{KL}(K_\infty)$ depends on parameter λ. As the figure indicates labor income, capital income, and income inequalities are magnified by imperfection in the credit market for sufficiently large values of parameter λ.



(a) Steady state capital stock.

(b) Steady state income inequality.

Figure 6: Both figures are constructed when $A = 10$, $\alpha = 1/3$, $\beta = 1/5$, $\gamma = 1/3$, and $m = 1$.

6 Alternative Specifications

So far, I made several simplifying assumptions in order to minimize the dimension of the parameter space and avoid unnecessary complications while analyzing the model. Of course, some of the results obtained in this paper depend on these simplifying assumptions, but the main results are robust to alternative specifications as well. The main features of the model would remain as long as (i) agents face minimum investment requirement for producing capital, (ii) credit market is imperfect, and (iii) elasticity of substitution between capital and labor inputs is at least unity. As long as these features of the model are maintained, alternative specifications of the model would not invalidate the key results, although they might considerably complicate the analysis. The rest of the paper gives a brief sketch of how the analysis needs to be modified under alternative specifications.

6.1 CES Production Function

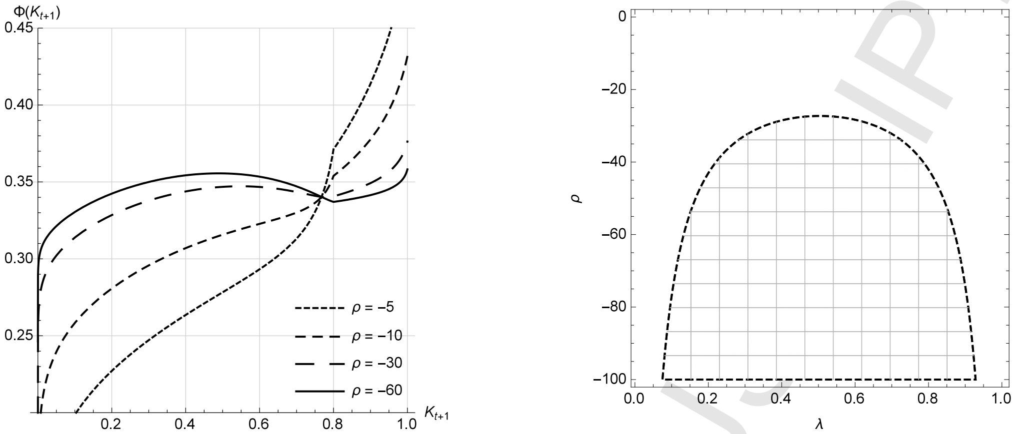
It is worthwhile at this stage to discuss the role of Assumptions 1.(a). In particular, I would like to demonstrate based on a numerical example that if Assumption 1.(a) is violated then there may exist multiple equilibria. Suppose the final commodity is produced according to a constant elasticity of substitution (CES) production function,

$$f(k) = \begin{cases} A(1-\alpha+\alpha k^{\rho})^{\frac{1}{\rho}} & \text{if} \quad \rho \in (-\infty,0)\cup(0,1) \\ Ak^{\alpha} & \text{if} \quad \rho = 0, \end{cases} \tag{25}$$

where $A > 0$ represents the total factor productivity of a firm, $\alpha \in (0,1)$ measures the importance of capital stock in production, and

$$\frac{f'(k)W(k)}{f(k)W'(k)} = -\frac{f'(k)}{kf''(k)}\left(1-\frac{kf'(k)}{f(k)}\right) = \frac{1-\alpha+\alpha k_t^{\rho}}{(1-\rho)(1-\alpha)}\frac{1-\alpha}{1-\alpha+\alpha k_t^{\rho}} = \frac{1}{1-\rho}$$

measures the elasticity of substitution between capital and labor inputs. This implies that Assumption 1.(a) is violated when $\rho \in (-\infty, 0)$ and thus capital and labor inputs complement to each other. Figure 2(b) visualizes the configuration of Φ for different values of the parameter $\rho \in (-\infty, 0)$. As the figure demonstrates Φ is monotonic for sufficiently large values of parameter ρ but becomes non-monotonic for sufficiently small values of parameter ρ. Figure 2(b) is constructed when $\beta = 0.20$, $\gamma = 0.33$, $m = 1$, $\lambda = 0.80$, $A = 15$, $\alpha = 0.33$, and the elasticity of substitution between capital and labor inputs are 0.17, 0.09, 0.03, and 0.02 respectively. Monotonicity property of Φ is lost only after $\rho < -30$ implying the elasticity of input substitution to be $\frac{1}{1-\rho} < 0.03$. This critical value is so small that it cannot be reconciled with the empirically observed values of input substitution parameter.[6] When Φ is non-monotonic then $K_t = \Phi(K_{t+1})$ may admit multiple solutions of $K_{t+1} \in (0, m)$ for some values of $K_t \in (0, m)$. I Assume 1.(a) in order to rule out such possibility.



(a) Possibility of multiple equilibria for different values of parameter ρ. This figure is constructed when $\lambda = 0.80$.

(b) (λ, ρ) parameter region with non-monotonic Φ implying the possibility of multiple of equilibria.

Figure 7: Possibility of multiple equilibria. Both figures are constructed when $A = 15$, $\alpha = 1/3$, $\beta = 1/5$, $\gamma = 1/3$, and $m = 1$.

Proposition 7 *If the final commodity is produced by a CES production function given by (25) then*

$$\frac{1-\beta\gamma}{\lambda\beta(1-\gamma)(1-\rho)} + (1-\alpha)\left(\frac{A(1-\alpha)\beta\gamma}{\lambda m}\right)^{\frac{\rho}{1-\rho}} \geq 1. \tag{26}$$

is a necessary and sufficient condition for monotonicity of Φ.

Proof of Proposition 7 can be found in the Appendix. One can easily verify that when $\rho \in [0, 1)$ then (26) is automatically satisfied because the first term in (26) is more than unity while the second term is positive. When $\rho \in (-\infty, 0)$ however, (26) may not be satisfied for all values of parameter ρ. Figure 7(b) displays the configuration of parameter region (λ, ρ) for which Φ is non-monotonic function and thus there exists possibility of multiple equilibria for intermediate values of K_t. As the figure indicates, non-monotonicity of Φ requires an extremely small value of parameter ρ. In particular, when $A = 15$, $\alpha = 1/3$, $\beta = 1/5$, $\gamma = 1/3$, and $m = 1$ then the possibility of multiple equilibria exists for $\rho < -27$ which corresponds to the elasticity of substitution between capital and labor inputs to satisfy $\frac{1}{1-\rho} < 0.036$.

[6] Berndt (1976) and Hamermesh (1996) surveyed a number of studies and reported the elasticity of factor substitution in the United States in the ranging from 0.32 to 1.16. Chirinko (2008) reported considerable cross country variation of elasticity parameter in a range of 0.4 to 0.6.

6.2 Power Utility Function

In the basic model of section 2, I assumed that young agents have a logarithmic utility. In this section, I demonstrate that imperfection in the credit market magnifies income inequality under alternative utility specification as well. In this section I modify the basic model in two respects. Firstly, I simplify the model by assuming that $\gamma = 1$, so that I abstract from agents labor supply decisions, and secondly, I assume that young agents value first and second period consumption according to the following power utility function,

$$(c_{1t}, c_{2t+1}) \mapsto (1-\beta)\frac{c_{1t}^{1-\sigma}-1}{1-\sigma} + \beta\frac{c_{2t+1}^{1-\sigma}-1}{1-\sigma},$$

where parameter $\sigma \in (0,1) \cup (1,\infty)$ represents the curvature of the utility function. Under such specification, I assume that young agents do not obtain a disutility from the first period labor supply. This simplification implies that every young agent supplies one unit of labor endowment inelastically and the labor income inequality is always zero. In other words, I eliminate the indirect effect of entrepreneurial rent on labor income inequality and only consider the direct effect of entrepreneurial rent on capital income inequality. In other words, when $\sigma = 1$ then I recover the basic model with $\gamma = 1$.

Depositor's first and second period consumptions are $c_{1t} = (1-s_t)w_t$ and $c_{2t+1} = s_t w_t r_{t+1}$. As a result, depositor's optimal savings rate is $s_t^d = s^d(r_{t+1})$ while lifetime utility is $(w_t^{1-\sigma}V^d(r_{t+1})-1)/(1-\sigma)$, where

$$s^d(r) = \frac{(\beta r^{1-\sigma})^{\frac{1}{\sigma}}}{(1-\beta)^{\frac{1}{\sigma}} + (\beta r^{1-\sigma})^{\frac{1}{\sigma}}} \quad \text{and} \quad V^d(r) = \left((1-\beta)^{\frac{1}{\sigma}} + (\beta r^{1-\sigma})^{\frac{1}{\sigma}}\right)^{\sigma}. \tag{27}$$

Entrepreneur's first and second period consumptions are $c_{1t} = (1-s_t)w_t$ and $c_{2t+1} = \left(\frac{i_t\theta_{t+1}}{w_t} + s_t\right) w_t r_{t+1}$. As a result, entrepreneur's optimal savings rate and investment are

$$s_t^e = \min\left\{\max\left\{s^d(r_{t+1}), \frac{(\lambda-(1-\lambda)\theta_{t+1})m}{w_t}\right\}, 1\right\} \quad \text{and} \quad i_t^e = \frac{s_t^e w_t}{\lambda-(1-\lambda)\theta_{t+1}}.$$

Resulting lifetime utility of an entrepreneur is $(w_t^{1-\sigma}V^e(w_t,\theta_{t+1},r_{t+1})-1)/(1-\sigma)$, where

$$V^e(w,\theta,r) = \begin{cases} 0 & \text{if} \quad \dfrac{(\lambda-(1-\lambda)\theta)m}{w} \geq 1 \\ V^d(r) + \beta(s(r)r)^{1-\sigma}\left(\left(\dfrac{\lambda(1+\theta)}{\lambda-(1-\lambda)\theta}\right)^{1-\sigma} - 1\right) & \text{if} \quad \dfrac{(\lambda-(1-\lambda)\theta)m}{w} \leq s^d(r), \end{cases}$$

and

$$V^e(w,\theta,r) = (1-\beta)\left(1 - \frac{(\lambda-(1-\lambda)\theta)m}{w}\right)^{1-\sigma} + \beta\left(\frac{\lambda m(1+\theta)r}{w}\right)^{1-\sigma} \tag{28}$$

if $\frac{(\lambda-(1-\lambda)\theta)m}{w} \in (s^d(r), 1)$.

Case I (binding credit constraint): If $K_{t+1} < \lambda m$ then aggregate supply of credit is equal to the aggregate demand for credit implies that

$$s^d(r_{t+1})w_t\left(1-\tfrac{K_{t+1}}{m}\right) = (1-\lambda)(1+\theta_{t+1})m\tfrac{K_{t+1}}{m} \quad\Leftrightarrow\quad 1+\theta_{t+1} = \frac{s^d(r_{t+1})w_t}{K_{t+1}}\frac{m-K_{t+1}}{(1-\lambda)m}.$$

This with $\rho_{t+1} = (1+\theta_{t+1})r_{t+1}$, $\rho_{t+1} = f'(K_{t+1})$, and labor market clearing conditions, $L_t = 1$ and $w_t = W(K_t)$, implies that for a given pair (K_t, K_{t+1}), equilibrium interest rate, r_{t+1}, solves

$$r_{t+1}s^d(r_{t+1}) = \frac{(1-\lambda)m}{m-K_{t+1}}\frac{K_{t+1}f'(K_{t+1})}{W(K_t)}. \tag{29}$$

Let $r_{t+1} = r(K_t, K_{t+1})$ denotes the solution of (29). It is worthwhile to mention at this point than if consumers' have logarithmic utility (i.e., when $\sigma = 1$), then $s^d(r_{t+1}) \equiv \beta$ and thus $r(K_t, K_{t+1})$ can be expressed in closed form. In contrast, when $\sigma \neq 1$ then $r(K_t, K_{t+1})$ can only be obtained numerically. Since $r \mapsto rs^d(r)$ is strictly increasing and satisfies the boundary conditions $\lim_{r\downarrow 0} rs^d(r) = 0$ and $\lim_{r\uparrow\infty} rs^d(r) = \infty$, it follows that there exists a unique $r_{t+1} = r(K_t, K_{t+1})$ solving (29) for any $(K_t, K_{t+1}) \in [0, m] \times [0, m]$. In addition, monotonicity properties of $K \mapsto W(K)$ and $K \mapsto Kf'(K)$ imply that $K_t \mapsto r(K_t, K_{t+1})$ is strictly decreasing while $K_{t+1} \mapsto r(K_t, K_{t+1})$ is strictly increasing.

It follows from (27) and (28) that for a given K_t, equilibrium K_{t+1} must solve

$$(1-\beta)\left(1-\left(1-\frac{(1-\lambda)f'(K_{t+1})}{r(K_t, K_{t+1})}\right)\frac{m}{W(K_t)}\right)^{1-\sigma} + \beta\left(\frac{\lambda m f'(K_{t+1})}{W(K_t)}\right)^{1-\sigma} = V^d[r(K_t, K_{t+1})]. \tag{30}$$

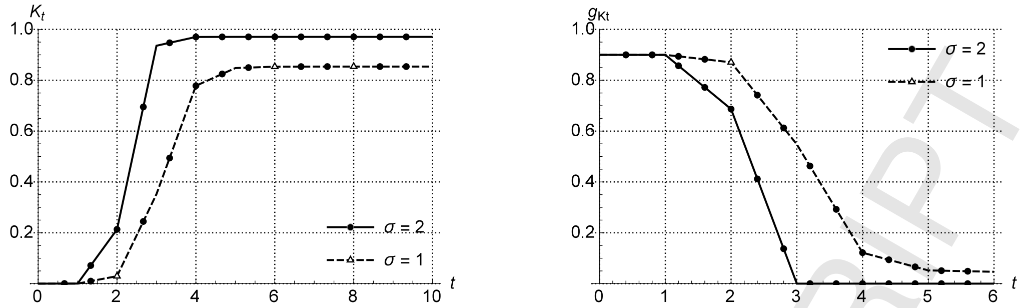
Monotonicity properties of r and f' imply that if $\sigma \in (0, 1)$ ($\sigma \in (1, \infty)$) then the left hand side of (30) is strictly decreasing (increasing) with respect to K_{t+1}. At the same time if $\sigma \in (0, 1)$ ($\sigma \in (1, \infty)$) then the right hand side of (30) is strictly increasing (decreasing) with respect to K_{t+1}. This implies the existence a unique K_{t+1} solving (30).

Case II (slack credit constraint): If $K_{t+1} \geq \lambda m$ then $\theta_{t+1} = 0$ and $K_{t+1} = s^d(r_{t+1})w_t$ where $r_{t+1} = f'(K_{t+1})$. This with labor market clearing conditions, $L_t = 1$ and $w_t = W(K_t)$, imply that for a given K_t, equilibrium K_{t+1} solves

$$\frac{K_{t+1}}{s^d[f'(K_{t+1})]} = W(K_t). \tag{31}$$

One can easily verify that the left hand side of (31) is always strictly increasing. As a result, I can conclude that (31) always admits a unique solution.

It is worthwhile at this stage to mention that the existence and uniqueness of equilibrium is guaranteed even when Assumption 1.(a) is violated. This is so because



(a) Dynamics of capital stock for different values of parameter σ.

(b) Dynamics of capital income inequality for different values of parameter σ.

Figure 8: Both figures are constructed when $A = 5$, $\alpha = 1/3$, $m = 1$, $\lambda = 0.90$, *and* $\beta = 1/5$.

when $\gamma = 1$ then the aggregate employment is fixed and thus the labor market clearing wage is $w_t = W(K_t)$ for any $K_t \in (0, m)$. Figure 8(a) visualized the time evolution of equilibrium capital stock for different values of parameter σ.

Capital Income Inequality: At time t, the pool of old agents is divided into two subgroups, depositors of measure $1 - \frac{K_t}{m}$ earning capital income $y^d_{2t-1} = w_{t-1}s^d(r_t)r_t$ and entrepreneurs of measure $\frac{K_t}{m}$ earning capital income $y^e_{2t-1} = m\rho_t - (m - s^e_{t-1}w_{t-1})r_t$. If $K_t \in (0, \lambda m)$ then $s^e_{t-1}w_{t-1} = (\lambda - (1-\lambda)\theta_t)m$, $1 + \theta_t = \frac{s^d(r_t)w_{t-1}}{K_t}\frac{m-K_t}{(1-\lambda)m}$, and $r_t = \frac{\rho_t}{1+\theta_t}$. As a result, capital incomes earned by old depositors and old entrepreneurs are

$$y^d_{2t-1} = \frac{(1-\lambda)m}{m-K_t}K_t\rho_t \quad \text{and} \quad y^e_{2t-1} = \frac{\lambda m}{K_t}K_t\rho_t,$$

respectively. If $K_t \in [\lambda m, m)$ then $s^e_{t-1}w_{t-1} = s^d(r_t)w_{t-1}$, $\theta_t = 0$, and $r_t = \rho_t$. As a result, capital incomes earned by old depositors and old entrepreneurs are the same, $y^d_{2t-1} = y^e_{2t-1} = K_t\rho_t$. By combining cases $K_t \in (0, \lambda m)$ and $K_t \in [\lambda m, m)$, I obtain that capital incomes earned by old depositors and old entrepreneurs are

$$y^d_{2t-1} = \min\left\{\frac{(1-\lambda)m}{m-K_t}, 1\right\}K_t\rho_t \quad \text{and} \quad y^e_{2t-1} = \max\left\{\frac{\lambda m}{K_t}, 1\right\}K_t\rho_t,$$

respectively. This along with Lemma 1 implies that Gini index of capital income inequality, at time t, is $g_{Kt} = \mathcal{G}_K(K_t)$, where $\mathcal{G}_K(K)$ is given by (23). This implies that the equilibrium income inequality is a robust feature of the model and does not disappear by relaxing the logarithmic utility assumption. Figure 8(b) visualizes the time evolution of equilibrium capital income inequality for different values of parameter σ.

6.3 Ex-ante Heterogeneity Among Consumers

In the basic model of section 2, I assumed that young agents are ex-ante homogeneous by possessing equal amounts of labor endowment and being equally productive workers. Limited pledgeability along with minimum investment requirement makes entrepreneurs earn entrepreneurial rent. As a result, there is an endogenous income inequality which depends on per capita income. Income inequality completely disappears if either future profit becomes fully pledgeable (i.e., $\lambda = 0$), if there is no minimum investment requirement (i.e., $m = 0$), or if per capita income is sufficiently large, $K_{t+1} > \lambda m$. This observation may lead someone to conjecture that the relationship between income and income inequality would also disappear if there is enough ex-ante heterogeneity within a generation so that competition between rich enough agents will always drive entrepreneurial rent to zero and prevent endogenous inequality. In this section, I demonstrate that such a conjecture is false. In order to do so, I modify the basic model in two respects. Firstly, I simplify the model by assuming that $\beta = 1$ and $\gamma = 1$, so that I abstract from agents labor supply and saving decisions, and second, I generalize the model by assuming that young agent $j \in [0, 1]$ possesses $n(j) : [0, 1] \to [0, \infty)$ units of labor endowment (alternatively, one may think that agent j possesses one unit of labor endowment and $n(j)$ represents j's labor productivity). Let $N(j') = \int_0^{j'} n(j)dj$ denote the cumulative labor endowment of agents $j \in [0, j']$. Without loss of generality, I assume that (a) n is a strictly increasing function, and (b) aggregate labor endowment is normalized to unity, $N(1) = 1$.

Agents' Optimal Behavior: By setting $\beta = 1$ and $\gamma = 1$, I assume that young agents supply the entire labor endowment inelastically and save the entire labor income. It follows from (3) that the lifetime utility of a depositor j is $\log V^d(j) + \log w_t + \log r_{t+1}$, where

$$V^d(j) = n(j). \tag{32}$$

At the same time, it follows from Proposition 1 that the lifetime utility of an entrepreneur j is $\log V^e(j, w_t, \theta_{t+1}) + \log w_t + \log r_{t+1}$ where

$$V^e(j, w_t, \theta_{t+1}) = \begin{cases} 0 & \text{if} \quad n(j) \leq \frac{(\lambda-(1-\lambda)\theta_{t+1})m}{w_t} \\ n(j)\dfrac{\lambda(1+\theta_{t+1})}{\lambda-(1-\lambda)\theta_{t+1}} & \text{if} \quad n(j) \geq \frac{(\lambda-(1-\lambda)\theta_{t+1})m}{w_t}. \end{cases} \tag{33}$$

It follows from (32) and (33) that it is optimal for agent j (a) to save $n(j)w_t$ units of final commodity and become a depositor if $n(j) < \frac{(\lambda-(1-\lambda)\theta_{t+1})m}{w_t}$, and (b) to save $n(j)w_t$ units of final commodity and become an entrepreneur by borrowing $i_t^e(j) - n(j)w_t = \frac{(1-\lambda)(1+\theta_{t+1})}{\lambda-(1-\lambda)\theta_{t+1}} n(j)w_t$ and investing $i_t^e(j) = \frac{n(j)w_t}{\lambda-(1-\lambda)\theta_{t+1}} \geq m$ units of final commodity if $n(j) \geq \frac{(\lambda-(1-\lambda)\theta_{t+1})m}{w_t}$.

Equilibrium in the Capital and Labor Markets: As above, I consider two cases separately. Case I corresponds to the binding credit constraint, while Case II corresponds to the slack credit constraint.

Case I (binding credit constraint): Suppose $w_t \in \left(0, \frac{\lambda m}{n(N^{-1}[1-\lambda])}\right)$. Equilibrium in the capital market is established when the aggregate supply of credit is equal to the aggregate demand of credit. This happens when the pair (w_t, θ_{t+1}) satisfies the equation

$$w_t \int_0^{n^{-1}\left(\frac{(\lambda-(1-\lambda)\theta_{t+1})m}{w_t}\right)} n(j)dj = \frac{(1-\lambda)(1+\theta_{t+1})}{\lambda-(1-\lambda)\theta_{t+1}} w_t \int_{n^{-1}\left(\frac{(\lambda-(1-\lambda)\theta_{t+1})m}{w_t}\right)}^1 n(j)dj. \quad (34)$$

The left hand side of (34) is the aggregate supply of credit and the right hand side of (34) is the aggregate demand for credit. After eliminating w_t from both sides of (34) and using the fact that $\int_x^1 n(j)dj = 1 - \int_0^x n(j)dj$, I obtain that the equilibrium pair (w_t, θ_{t+1}) satisfies

$$N\left[n^{-1}\left(\frac{(\lambda-(1-\lambda)\theta_{t+1})m}{w_t}\right)\right] = (1-\lambda)(1+\theta_{t+1}) \quad \Leftrightarrow \quad w_t = \frac{(\lambda-(1-\lambda)\theta_{t+1})m}{n(N^{-1}[(1-\lambda)(1+\theta_{t+1})])}. \quad (35)$$

Proposition 8 *For a given $w_t \in \left(0, \frac{\lambda m}{n(N^{-1}[1-\lambda])}\right)$ there exists a unique $\theta_{t+1} = \Theta(w_t) \in (0, \frac{\lambda}{1-\lambda})$ which solves (34). $w \mapsto \Theta(w)$ is a continuous and strictly decreasing function satisfying boundary conditions, $\lim_{w\downarrow 0}\Theta(w) = \frac{\lambda}{1-\lambda}$ and $\lim_{w\uparrow \frac{\lambda m}{n(N^{-1}[1-\lambda])}} \Theta(w) = 0$.*

Case II (slack credit constraint): Suppose $w_t \geq \frac{\lambda m}{n(N^{-1}[1-\lambda])}$. Then it follows from (35) that equilibrium rent is zero, $\theta_{t+1} = 0$, because a large set of young agents, $j \in [N^{-1}(1-\lambda), 1]$, become able to borrow and produce capital. Competition among young agents would drive the entrepreneurial rent to zero.

Under the assumption of ex-ante heterogeneity, young agents no longer achieve the same levels of lifetime utility as it happened in the basic model. After combining (34) and (35), I obtain that for a given w_t

$$J(w_t) = N^{-1}[(1-\lambda)(1+\Theta(w_t))] \quad (36)$$

defines a marginal agent who becomes an entrepreneur.

The aggregate supply of labor in every period is $L_t = N(1) = 1$ because $\gamma = 1$ and thus young agents supply their labor endowment inelastically. As a result, labor market clearing wage is $w_t = W(K_t)$. Since agent $j \in [J(w_t), 1]$ becomes an entrepreneur by investing $i_t^e(j) = \frac{n(j)w_t}{\lambda-(1-\lambda)\Theta(w_t)}$ units of final commodity, it follows that the next period capital stock is equal to aggregate investment

$$K_{t+1} = \frac{w_t}{\lambda-(1-\lambda)\Theta(w_t)} \int_{J(w_t)}^1 n(j)dj = w_t, \quad (37)$$

because

$$\int_{J(w_t)}^{1} n(j)dj = 1 - N[J(w_t)] = 1 - (1-\lambda)(1+\Theta(w)) = \lambda - (1-\lambda)\Theta(w). \quad (38)$$

It follows from (38) that the evolution of capital stock is given by $K_{t+1} = W(K_t)$. It is worthwhile at this stage to mention that the existence and uniqueness of equilibrium is guaranteed even when Assumption 1.(a) is violated. This is so because when $\gamma = 1$ then the aggregate employment is fixed and thus the labor market clearing wage is $w_t = W(K_t)$ for any $K_t \in (0, m)$. Assumption 1.(b) (which can be written as $W(m) < m$) guarantees that $K_{t+1} \in (0, m)$ for any $K_t \in (0, m)$.

Labor Income Inequality: At time t agent j earns the labor income $n(j)w_t$. As a result, the aggregate labor income earned by j fraction of relatively poor agents is $\frac{N(j)w_t}{N(1)w_t} = N(j)$, i.e., $N(j)$ represents the equilibrium Lorenz curve. Resulting equilibrium Gini index of labor income inequality is $\mathcal{G}_L = 2\int_0^1 (j - N(j))dj = 1 - 2\mathcal{N}(1) \in (0,1)$, where $\mathcal{N}(j') \stackrel{def}{=} \int_0^{j'} N(j)dj$. As expected, imperfection in the credit market has no impact on labor income inequality (the indirect effect of entrepreneurial rent) because the channel through which entrepreneurial rent affects labor income inequality is switched off. Of course the Gini index of labor income inequality is positive but it is so because of ex-ante heterogeneity in agents labor endowment.

Capital Income Inequality: At time t, agent $j \in [0, J(K_t))$ becomes a depositor and earns capital income $n(j)w_{t-1}r_t = \frac{n(j)}{1+\theta_t}K_t\rho_t$ because $\rho_t = (1+\theta_t)r_t$ and $K_t = w_{t-1}$. In contrast, agent $j \in [J(K_t), 1]$ becomes an entrepreneur and earns capital income

$$\frac{n(j)w_{t-1}}{\lambda - (1-\lambda)\theta_t}(\rho_t - r_t) + n(j)w_{t-1}r_t = \frac{\lambda n(j)}{\lambda - (1-\lambda)\theta_t}K_t\rho_t. \quad (39)$$

If $K_t \geq \frac{\lambda m}{n[N^{-1}(1-\lambda)]}$ then $\theta_t = \Theta(K_t) = 0$ and thus the equilibrium Lorenz curve is $N(j)$ because capital income earned by agent $j \in [0,1]$ is $n(j)K_t\rho_t$. The resulting Gini index of capital income inequality is $\mathcal{G}_K = 2\int_0^1 (j - N(j))dj = 1 - 2\mathcal{N}(1) \in (0,1)$.

Proposition 9 *If $K_t < \frac{\lambda m}{n[N^{-1}(1-\lambda)]}$ then the capital income inequality is*

$$\mathcal{G}_K(K_t) = \mathcal{G}_K + 2(1-\lambda)\Theta(K_t)\left(\frac{\mathcal{N}[J(K_t)]}{N[J(K_t)]} + \frac{1 - J(K_t)}{1 - N[J(K_t)]} - \frac{\mathcal{N}[1] - \mathcal{N}[J(K_t)]}{1 - N[J(K_t)]}\right). \quad (40)$$

$K \mapsto \mathcal{G}_K(K)$ is a continuous and strictly decreasing function satisfying boundary conditions $\lim_{K\downarrow 0}\mathcal{G}_K(K) = \mathcal{G}_K + \lambda(1-\mathcal{G}_K)$ and $\lim_{K\uparrow\frac{\lambda m}{n[N^{-1}(1-\lambda)]}}\mathcal{G}_K(K) = \mathcal{G}_K$.

Proof of this proposition can be found in the Appendix. As Proposition 9 indicates, credit market imperfection magnifies capital income inequality when $K_t < \frac{\lambda m}{n[N^{-1}(1-\lambda)]}$.

The endogenous capital income inequality, $\mathcal{G}_K(K_t) - \mathcal{G}_K$ weakens if either K_t or $\mathcal{G}_K \in (0,1)$ increases and strengthens if $\lambda \in (0,1)$ increases.

Numerical Example: In order to make the model more transparent, I consider a numerical example in which I assume that labor endowment/labor productivity of agent $j \in [0,1]$ is given by $n(j) = 1 - \epsilon + 2\epsilon j$. This assumption basically implies that young agents' labor endowment is uniformly distributed on the interval $[1-\epsilon, 1+\epsilon]$ with average labor endowment being $\int_0^1 n(j)dj = 1$. Parameter $\epsilon \in [0,1]$ measures the ex-ante heterogeneity of young agents endowment. Large value of parameter ϵ indicates higher variability of agents labor endowment, while $\epsilon = 0$ recovers the basic model because $n(j) = 1$ for any $j \in [0,1]$.

$N(j) = \int_0^j n(x)dx = (1-\epsilon)j + \epsilon j^2$ along with (34) and (35) implies that if $w_t < \frac{\lambda m}{2\sqrt{1-\lambda}}$ then the equilibrium entrepreneurial rent $\theta_{t+1} = \Theta(w_t)$ solves

$$w_t = \frac{(\lambda - (1-\lambda)\theta_{t+1})m}{\sqrt{(1-\epsilon)^2 + 4\epsilon(1-\lambda)(1+\theta_{t+1})}}.$$

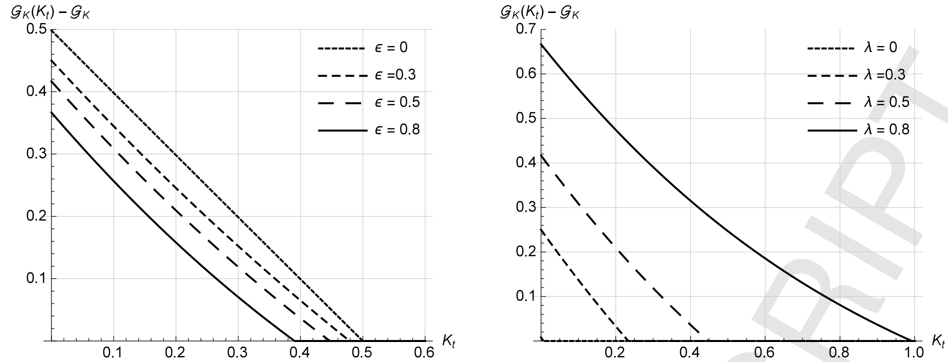
The resulting equilibrium entrepreneurial rent is

$$\Theta(w_t) = \begin{cases} \frac{1}{1-\lambda}\left(\lambda - \frac{w_t}{m}\left(\sqrt{(1+\epsilon)^2 + \left(\frac{2\epsilon w_t}{m}\right)^2} - \frac{2\epsilon w_t}{m}\right)\right) & \text{if} \quad w_t < \frac{\lambda m}{\sqrt{(1+\epsilon)^2 - 4\lambda\epsilon}} \\ 0 & \text{if} \quad w_t \geq \frac{\lambda m}{\sqrt{(1+\epsilon)^2 - 4\lambda\epsilon}}. \end{cases}$$

This with (36) implies that the marginal agent who becomes an entrepreneur is

$$J(w_t) = -\frac{1-\epsilon}{2\epsilon} + \sqrt{\left(\frac{1-\epsilon}{2\epsilon}\right)^2 + \frac{(1-\lambda)(1+\Theta(w_t))}{\epsilon}}.$$

$N(j) = (1-\epsilon)j + \epsilon j^2$ and $\mathcal{N}(j) = \frac{(1-\epsilon)j^2}{2} + \frac{\epsilon j^3}{3}$ implies that $\mathcal{N}(1) = \frac{3-\epsilon}{6}$ and thus $\mathcal{G}_K = \frac{\epsilon}{3}$. This with (40) implies that the endogenous capital income inequality can be determined analytically. Figure 9(a) displays the configuration of $K \mapsto \mathcal{G}_K(K) - \mathcal{G}_K$ for different values of parameter ϵ and for a fixed value of parameter λ. As the figure indicates, endogenous capital inequality weakens either as capita stock, K, increases or as young agents ex-ante heterogeneity, ϵ, increases. Figure 9(b) displays the configuration of $K \mapsto \mathcal{G}_K(K) - \mathcal{G}_K$ for different values of parameter λ and for a fixed value of parameter ϵ. As the figure indicates, endogenous capital income inequality weakens through an increase of capital stock, K, and strengthens through an increase of parameter λ measuring the imperfection in the credit market. Endogenous capital income inequality entirely disappears when the credit market becomes perfect, $\lambda = 0$.



(a) Endogenous capital income inequality for different values of parameter ϵ.

(b) Endogenous capital income inequality for different values of parameter λ.

Figure 9: Both figures are constructed when $\beta = 1$, $\gamma = 1$, $m = 1$, $\lambda = 0.5$, and $\epsilon = 0.5$.

7 Summary and Conclusions

The two main goals of this paper are: (1) to present a mechanism through which credit market imperfection may magnify income inequality, and (2) to propose an alternative explanation of Kuznets' inverted-U Hypothesis. I have shown that credit market imperfection along with minimum investment requirement creates an entrepreneurial rent which has both direct and indirect effects on income inequality. One major advantage of the model presented in this paper is its analytical tractability. However, some cautionary remarks should be pointed out about the predictions of the model. I do not argue that the credit market imperfection alone is responsible for increased income inequality, or that other sources of policy change, structural change, globalization, education policy, etc., are unimportant sources behind increased income inequality. Instead, I argue that credit market imperfection may also magnify income inequality.

At this point, I would like to point out some limitations of the model presented in this paper. First, the model has only one type of capital good and one type of final good. Second, the model does not allow for growth either in technology or labor force. Third, the economy is closed and thus does not interact with other economies. Due to these limitations, I can think of many ways in which the model can be extended. First, there is a shortage of theoretical and empirical research studying the impact of financial sector policies, such as bank regulations and securities law, on persistent inequality, and second, there is no conceptual framework developed in the literature which considers the joint and endogenous evolution of finance, inequality, and economic growth. The present paper represents a step towards research in this direction.

8 Appendix

Proof of Proposition 1: Entrepreneur's optimization problem is (subscripts are eliminated for notational convenience):

$$V^e(w,\theta) \overset{def}{=} \max_{\substack{0\leq s\leq 1\\ 0\leq \ell\leq 1}} \left\{ (1-\ell)^{1-\gamma}\ell^{\gamma}(1-s)^{(1-\beta)\gamma}\left(\frac{\theta i}{\ell w}+s\right)^{\beta\gamma} \middle| \, m \leq i \leq \frac{s\ell w}{\lambda-(1-\lambda)\theta} \right\}. \tag{41}$$

If $\theta > 0$ then entrepreneur's objective function is strictly increasing with respect to i. As a result, $i = \frac{\ell s w}{\lambda-(1-\lambda)\theta}$ and thus the entrepreneur's optimization problem becomes

$$\max_{\substack{0\leq s\leq 1\\ 0\leq \ell\leq 1}} \left\{ (1-\ell)^{1-\gamma}\ell^{\gamma}(1-s)^{(1-\beta)\gamma}s^{\beta\gamma} \middle| \, m \leq i \leq \frac{s\ell w}{\lambda-(1-\lambda)\theta} \right\}. \tag{42}$$

Solution of (42) is $s^e = \min\left\{\max\left\{\beta, \frac{(\lambda-(1-\lambda)\theta)m}{\ell w}\right\}, 1\right\}$. As a result, there are three cases to be considered.

1) If $w_t \leq (\lambda-(1-\lambda)\theta)m$ and $\theta > 0$ then $s^e = 1$, $\ell^e = 1$, $i^e = \frac{w}{\lambda-(1-\lambda)\theta} \leq m$, and thus it follows from (41) that $V^e(w,\theta) = 0$.

2) If $w_t \in \left((\lambda-(1-\lambda)\theta)m, \frac{(\lambda-(1-\lambda)\theta)m}{\beta\gamma}\right)$ and $\theta > 0$ then $s^e = \frac{(\lambda-(1-\lambda)\theta)m}{\ell w}$. Resulting $\ell^e = \frac{(1-\beta)\gamma}{1-\beta\gamma} + \frac{1-\gamma}{1-\beta\gamma}\frac{(\lambda-(1-\lambda)\theta)m}{w}$, $i^e = \frac{s_t^e \ell_t^e w}{\lambda-(1-\lambda)\theta} = m$, and thus it follows from (41) that

$$V^e(w,\theta) = \left(\frac{1-\gamma}{1-\beta\gamma}\right)^{1-\gamma}\left(\frac{(1-\beta)\gamma}{1-\beta\gamma}\right)^{(1-\beta)\gamma}\left(1-\frac{(\lambda-(1-\lambda)\theta)m}{w}\right)^{1-\beta\gamma}\left(\frac{\lambda(1+\theta)m}{w}\right)^{\beta\gamma}.$$

3) If $w_t \geq \frac{(\lambda-(1-\lambda)\theta)m}{\beta\gamma}$ and $\theta > 0$ then $s^e = \beta$, $\ell^e = \gamma$, $i^e = \frac{\beta\gamma w}{\lambda-(1-\lambda)\theta}$, and thus it follows from (41) that

$$V^e(w,\theta) = V^d\left(\frac{\lambda(1+\theta)}{\lambda-(1-\lambda)\theta}\right)^{\beta\gamma}.$$

If $\theta = 0$ then entrepreneur's optimization problem becomes

$$\max_{\substack{0\leq s\leq 1\\ 0\leq \ell\leq 1}} \left\{ (1-\ell)^{1-\gamma}\ell^{\gamma}(1-s)^{(1-\beta)\gamma}s^{\beta\gamma} \middle| \, m \leq i \leq \frac{s\ell w}{\lambda} \right\}. \tag{43}$$

Solution of (43) is $s^e = \min\left\{\max\left\{\beta, \frac{\lambda m}{\ell w}\right\}, 1\right\}$, $\ell^e = \frac{(1-\beta)\gamma}{1-\beta\gamma} + \frac{1-\gamma}{1-\beta\gamma}\frac{\lambda m}{w_t}$, and $i^e \in [m, \frac{s^e \ell_t^e w}{\lambda}]$.

QED.

Proof of Proposition 2: Suppose $w < \frac{\lambda m}{\beta\gamma}$. Then it follows from (7) that $V^e(w,\theta) = V^d$ is equivalent to solving

$$\Delta(w,\theta) = (\beta\gamma)^{\beta\gamma}(1-\beta\gamma)^{1-\beta\gamma} \quad \text{where} \quad \Delta(w,\theta) = \left(1 - \frac{(\lambda-(1-\lambda)\theta)m}{w}\right)^{1-\beta\gamma}\left(\frac{\lambda(1+\theta)m}{w}\right)^{\beta\gamma}. \tag{44}$$

Since $\theta \mapsto \Delta(w,\theta)$ is a continuous, strictly increasing function for $\frac{1}{1-\lambda}\left(\lambda - \frac{w}{m}\right) < \theta < \frac{\lambda}{1-\lambda}$, and satisfies boundary conditions

$$\lim_{\theta\downarrow\frac{1}{1-\lambda}\left(\lambda-\frac{w}{m}\right)} \Delta(w,\theta) = 0 \quad \text{and} \quad \lim_{\theta\uparrow\frac{\lambda}{1-\lambda}} \Delta(w,\theta) = \left(\frac{\lambda m}{(1-\lambda)w}\right)^{\beta\gamma} > (\beta\gamma)^{\beta\gamma}(1-\beta\gamma)^{1-\beta\gamma},$$

because $w < \frac{\lambda m}{\beta\gamma}$, $\lambda \in (0,1)$, and $\beta\gamma \in (0,1]$. Existence and uniqueness of $\theta = \Theta(w) \in (0, \frac{\lambda}{1-\lambda})$ solving (44) follows from an implicit function theorem. Monotonicity and boundary behavior of Θ follows from monotonicity and boundary behavior of $\Delta(.,\theta)$.

QED.

Proof of Proposition 3: Monotonicity and boundary behavior of $\mathcal{W}$ follows from (11) and (12). In order to demonstrate monotonicity property of $\mathcal{R}$, I rely on (13), (14) and observe that when $K < \lambda m$ then

$$\frac{\beta\gamma}{1-\lambda}\frac{1}{1+\mathcal{R}(K)} = \frac{\beta\gamma m + (1-\beta\gamma)K}{m-K} - (1-\beta\gamma)\left(\frac{1-\lambda}{\lambda}\right)^{\frac{\beta\gamma}{1-\beta\gamma}}\left(\frac{K}{m-K}\right)^{\frac{1}{1-\beta\gamma}}. \tag{45}$$

Since the derivative of the right hand side of (45) is

$$\frac{m}{(m-K)^2}\left(1 - \left(\frac{(1-\lambda)K}{\lambda(m-K)}\right)^{\frac{\beta\gamma}{1-\beta\gamma}}\right) > 0$$

for $K \in (0, \lambda m)$, it follows from (45) that $\mathcal{R}$ is a strictly decreasing function. Boundary properties of $\mathcal{R}$ follows from (14) and from the boundary properties of $\mathcal{W}$.

QED.

Proof of Proposition 4: (a) Monotonicity and concavity of f implies that W is a strictly increasing function and the capital share in production $\frac{kf'(k)}{f(k)}$ belongs to the interval $(0,1)$. This with Assumption 1.(a) implies that $\frac{kW'(k)}{W(k)} \leq \frac{kf'(k)}{f(k)} < 1$ and thus $k \mapsto \frac{k}{W(k)}$ is a strictly increasing function because $\frac{kW'(k)}{W(k)} \in (0,1)$ and $\left(\frac{k}{W(k)}\right)' = \frac{1}{W(k)}\left(1 - \frac{kW'(k)}{W(k)}\right) > 0$. This with monotonicity of W implies that $w \mapsto \frac{W^{-1}(w)}{w}$ is also a strictly increasing function. Since

$$\Phi(K) = \frac{(1-\beta)\gamma}{1-\beta\gamma}W^{-1}[\mathcal{W}(K)] + \frac{1-\gamma}{1-\beta\gamma}K\frac{W^{-1}[\mathcal{W}(K)]}{\mathcal{W}(K)},$$

it follows from monotonicity of $K \mapsto \mathcal{W}(K)$ and monotonicity of $w \mapsto \frac{W^{-1}(w)}{w}$ that $K \mapsto \Phi(K)$ is also a strictly increasing function.

(b) Since $\mathcal{L}(m) = \gamma$ and $\mathcal{W}(\lambda m) = \frac{m}{\beta\gamma}$, it follows from (19) and from Assumption 1.(b) that

$$\Phi(m) = \gamma W^{-1}\left(\frac{m}{\beta\gamma}\right) < \gamma\frac{m}{\gamma} = m.$$

Monotonicity of Φ with its boundary behavior implies the existence and uniqueness of $K_{t+1} = \phi(K_t) \in (0, m)$ which solves $K_t = \Phi(K_{t+1}) \in (0, m)$ for any $K_t \in (0, m)$.

QED.

Proof of Proposition 5: (a) Boundary behavior of $\mathcal{G}_L$ follows from the fact that $\lim_{K\downarrow 0} \mathcal{L}^e(K) = \gamma$ and $\lim_{K\uparrow\lambda m} \mathcal{L}^e(K) = \gamma$. It follows from (22) that solving $\mathcal{G}_L'(K) = 0$ is equivalent to solving

$$-[\mathcal{L}^e]'(K)\frac{\gamma m}{\mathcal{L}^e(K) - \gamma} = \frac{\gamma(m-K)^2 - \mathcal{L}^e(K)K^2}{K(m-K)}. \tag{46}$$

Since

$$-[\mathcal{L}^e]'(K) = \frac{\beta\gamma}{1-\beta\gamma}\frac{m(1-\mathcal{L}^e(K))}{K(m-K)}, \tag{47}$$

it follows from (46) that solving $\mathcal{G}_L'(K) = 0$ is equivalent to solving $\Delta(K) = \frac{\beta(\gamma m)^2}{1-\beta\gamma}$ where

$$\Delta(K) \stackrel{def}{=} \frac{\mathcal{L}^e(K) - \gamma}{1 - \mathcal{L}^e(K)}(m-K)^2\left(\gamma - \left(\frac{K}{m-K}\right)^2 \mathcal{L}^e(K)\right). \tag{48}$$

Monotonicity of $\mathcal{L}^e(K)$ along with $\gamma \in (0,1)$ implies that $K \mapsto \frac{\mathcal{L}^e(K)-\gamma}{1-\mathcal{L}^e(K)}$ is a strictly decreasing function. At the same time it follows from (47) that $K \mapsto \left(\frac{K}{m-K}\right)^2 \mathcal{L}^e(K)$ is a strictly increasing function. This implies that $K \mapsto \Delta(K)$ is a strictly decreasing function. This with boundary behavior $\lim_{K\downarrow 0} \Delta(K) = \infty$ and $\lim_{K\uparrow\lambda m} \Delta(K) = 0$, implies the existence and uniqueness of K_L^c solving $\mathcal{G}_L'(K) = 0$.

Since $\mathcal{L}^e$ increases with λ and $K \in (0, \lambda m)$ it follows from (22) that $\mathcal{G}_L$ increases with λ as well. Since $\lim_{\lambda\downarrow 0} \mathcal{L}^e(K) = \gamma$ and $\lim_{\lambda\uparrow 1} \mathcal{L}^e(K) = \gamma$ it follows from (48) that

$$\lim_{\lambda\downarrow 0} K_L^c = 0 \quad \text{and} \quad \lim_{\lambda\uparrow 1} K_L^c = \tfrac{\sqrt{\gamma}}{1+\sqrt{\gamma}}m. \tag{49}$$

(b) It follows from (22) that $\mathcal{G}_L$ increases with λ. This along with implicit function theorem implies that $\lambda \mapsto \mathcal{G}_L^c(\lambda)$ is a strictly increasing function as well. After substituting (49) into (22), I obtain that

$$\lim_{\lambda\downarrow 0} \mathcal{G}_L^c(\lambda) = 0 \quad \text{and} \quad \lim_{\lambda\uparrow 1} \mathcal{G}_L^c(\lambda) = \frac{1-\sqrt{\gamma}}{1+\sqrt{\gamma}}.$$

QED.

Proof of Proposition 6: Since the fraction of total income received by old agents is α and the fraction of total income received by young agents is $1-\alpha$, it follows from $|x-y|+|y-z| \geq |x-z|$ that

$$\mathcal{G}_{KL}(K) > \left|\frac{1-\alpha}{2} - \frac{\alpha}{2}\right| = |\alpha - 0.5|$$

QED.

Proof of Proposition 7: If $K \in [\lambda m, m]$ then $\mathcal{L}(K) = \gamma$ and $\mathcal{W}(K) = \frac{K}{\beta\gamma}$. This with monotonicity of W^1 and with (19) implies that Φ is a strictly increasing function on the interval $[\lambda m, m]$. Based on numerical procedure, I demonstrated that when f is the CES production function and $K \in [0, \lambda m)$ then Φ can be either a monotonically increasing function or may have an inverted "U" shape.

After taking a natural logarithm of both sides of (19) and then differentiating it I obtain that

$$\frac{K\Phi'(K)}{\Phi(K)} = \frac{K\mathcal{L}'(K)}{\mathcal{L}(K)} + \frac{\mathcal{L}(K)}{\Phi(K)} \frac{W[\Phi(K)/\mathcal{L}(K)]}{W'[\Phi(K)/\mathcal{L}(K)]} \frac{K\mathcal{W}'(K)}{\mathcal{W}(K)} \tag{50}$$

It follows from (13) and (16) that

$$\lim_{K\uparrow\lambda m} \frac{K\mathcal{W}'(K)}{\mathcal{W}(K)} = \frac{1}{1-\lambda} \quad \text{and} \quad \lim_{K\uparrow\lambda m} \frac{K\mathcal{L}'(K)}{\mathcal{L}(K)} = -\frac{\beta(1-\gamma)}{1-\beta\gamma}\frac{\lambda}{1-\lambda}.$$

This with (50) implies that $\lim_{K\uparrow\lambda m} \frac{K\Phi'(K)}{\Phi(K)} \geq 0$ if and only if

$$\frac{W(k)}{kW'(k)} \geq \frac{\lambda\beta(1-\gamma)}{1-\beta\gamma} \quad \text{where} \quad k = \lim_{K\uparrow\lambda m} \frac{\Phi(K)}{\mathcal{L}(K)} = W^{-1}\left(\frac{\lambda m}{\beta\gamma}\right).$$

Since

$$\frac{W(k)}{kW'(k)} = \frac{1}{1-\rho}\frac{1-\alpha+\alpha k^\rho}{\alpha k^\rho} \quad \text{and} \quad W(k) = A(1-\alpha)(1-\alpha+\alpha k^\rho)^{\frac{1-\rho}{\rho}}$$

it follows that $\lim_{K\uparrow\lambda m} \frac{K\Phi'(K)}{\Phi(K)} \geq 0$ if and only if

$$\frac{1-\beta\gamma}{\lambda\beta(1-\gamma)(1-\rho)} + (1-\alpha)\left(\frac{A(1-\alpha)\beta\gamma}{\lambda m}\right)^{\frac{\rho}{1-\rho}} \geq 1.$$

QED.

Proof of Proposition 8: Let

$$\Delta(\theta) \stackrel{def}{=} \frac{(\lambda-(1-\lambda)\theta)m}{n\left(N^{-1}[(1-\lambda)(1+\theta)]\right)}.$$

Monotonicity and boundary properties of n and N imply the monotonicity of Δ along with the following boundary behavior

$$\lim_{\theta\downarrow 0}\Delta(\theta) = \frac{\lambda m}{n[N^{-1}(1-\lambda)]} \quad \text{and} \quad \lim_{\theta\uparrow\frac{\lambda}{1-\lambda}}\Delta(\theta) = 0.$$

As a result I conclude that $w = \Delta(\theta)$ admits a unique solution $\theta = \Theta(w) \in (0, \frac{\lambda}{1-\lambda})$ for $w < \frac{\lambda m}{n[N^{-1}(1-\lambda)]}$. This with implicit function theorem implies that Θ is a strictly decreasing function satisfying boundary properties $\lim_{w\downarrow 0}\Theta(w) = \frac{\lambda}{1-\lambda}$ and $\lim_{w\uparrow\frac{\lambda m}{n[N^{-1}(1-\lambda)]}}\Theta(w) = 0$.

QED.

Proof of Proposition 9: I drop time index for notational convenience. If $K < \frac{\lambda m}{n[N^{-1}(1-\lambda)]}$ then $\Theta(K_t) > 0$. This along with $N[J(K)] = (1-\lambda)(1+\Theta(K))$ implies the following equilibrium Lorenz curve,

$$\mathcal{LC}(j,K) = \begin{cases} (1-\lambda)\dfrac{N(j)}{N[J(K)]} & \text{if} \quad j \in [0, J(K)) \\ 1-\lambda\dfrac{1-N(j)}{1-N[J(K)]} & \text{if} \quad j \in [J(K), 1]. \end{cases}$$

Resulting Gini index of capital income inequality is

$$\mathcal{G}_K(K) = 2\int_0^1 (j - \mathcal{LC}(j,K))dj = \mathcal{G}_K + 2\int_0^1 (N(j) - \mathcal{LC}(j,K))dj. \tag{51}$$

Since

$$2\int_0^{J(K)} (N(j) - \mathcal{LC}(j,K))dj = 2(1-\lambda)\Theta(K)\frac{\mathcal{N}[J(K)]}{N[J(K)]} \tag{52}$$

and

$$2\int_{J(K)}^1 (N(j) - \mathcal{LC}(j,K))dj = 2(1-\lambda)\Theta(K)\frac{1-\mathcal{N}[1]+\mathcal{N}[J(K)]-J(K)}{1-N[J(K)]} \tag{53}$$

the claim of the proposition follows from (51), (52), and (53).

QED.

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