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A gene–wavelet model for long lead time drought forecasting

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SUMMARY

Drought forecasting is an essential ingredient for drought risk and sustainable water resources management. Due to increasing water demand and looming climate change, precise drought forecasting models have recently been receiving much attention. Beginning with a brief discussion of different drought forecasting models, this study presents a new hybrid gene–wavelet model, namely wavelet–linear genetic programing (WLGP), for long lead-time drought forecasting. The idea of WLGP is to detect and optimize the number of significant spectral bands of predictors in order to forecast the original predictand (drought index) directly. Using the observed El Niño–Southern Oscillation indicator (NINO 3.4 index) and Palmer's modified drought index (PMDI) as predictors and future PMDI as predictand, we proposed the WLGP model to forecast drought conditions in the State of Texas with 3, 6, and 12-month lead times. We compared the efficiency of the model with those of a classic linear genetic programing model developed in this study, a neuro-wavelet (WANN), and a fuzzy-wavelet (WFL) drought forecasting models formerly presented in the relevant literature. Our results demonstrated that the classic linear genetic programing model is unable to learn the non-linearity of drought phenomenon in the lead times longer than 3 months; however, the WLGP can be effectively used to forecast drought conditions having 3, 6, and 12-month lead times. Genetic-based sensitivity analysis among the input spectral bands showed that NINO 3.4 index has strong potential effect in drought forecasting of the study area with 6–12-month lead times.

1. Introduction

Drought forecasting is an essential ingredient in watershed management. In recent years, its importance is being intensified owing to increasing water demand and looming climate change (Mishra and Singh, 2010). The success of drought preparedness and mitigation depends upon timely information on the drought onset and propagation in time and space (Özger et al., 2012). This information may be obtained through precise drought forecasting models, which is normally generated using drought indices.

Many drought forecasting models have been developed in recent years (e.g., Rao and Padmanabhan, 1984; Sen, 1990; Bogradi et al., 1994; Lohani and Loganathan, 1997; Mishra and Desai, 2005; Cancelliere et al., 2007; Modarres, 2007; Fernandez et al., 2009; Özger et al., 2012). Mishra and Singh (2011) have provided a comprehensive review on different drought forecasting approaches.

In recent years, artificial intelligence (AI) techniques such as artificial neural network (ANN), fuzzy logic (FL), and genetic programing (GP) have been pronounced as a branch of computer science to model wide range of hydro-meteorological processes (Pesti et al., 1996; Whigham and Crapper, 2001; Dolling and Varas, 2002; Morid et al., 2007; Kisi and Guven, 2010; Özger et al., 2012; Nourani et al., 2013a). Successful application of fuzzy rule-based modeling for short term regional drought forecasting using two forcing inputs, El Niño–Southern Oscillation (ENSO) and large scale atmospheric circulation patterns (CP), was described by Pongracz et al. (1999). Mishra and Desai (2006) used both recursive and direct multi-step ANNs for up to 6-month LT drought forecasting and found that the recursive multi-step model is the best suited for 1 month LT. When a LT longer than 4 months was considered, the direct multi-step model outperformed the recursive multi-step models. Morid et al. (2007) developed an ANN-based drought forecasting approach with the LTs of 1–12 months using Effective Drought Index (EDI), SPI, and different combinations of past rainfalls. The results indicated that forecasts using EDI were superior to those using SPI for all LTs. Barros and Bowden (2008) applied self-organizing maps and multivariate linear regression analysis to forecast SPI at Murray-Darling Basin in Australia up to 12 months in advance.

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Owing to the limited ability of the above-mentioned AI techniques to forecast non-stationary phenomena, hybrid AI models were developed and suggested to forecast drought and successful results have also been reported (Kim and Valdes, 2003; Mishra and Singh, 2010; Belayneh and Adamowski, 2012; Özger et al., 2012; Belayneh et al., 2014). Mishra et al. (2007), using the SPI series, developed a hybrid ANN-ARIMA model for drought forecasting in Kansabati River Basin in India. The hybrid model was found to be more accurate than individual stochastic and ANN models up to a 6-month LT. Bacanli et al. (2009) developed an adaptive neuro fuzzy inference system (ANFIS) for drought forecasting using SPI in central Anatolia, Turkey. The authors pointed out that the hybrid method performs better than the classic ANN model. Özger et al. (2012) developed a hybrid wavelet–FL (WFL) model for long lead time drought forecasting using Palmer modified drought index (PMDI) series across the State of Texas and compared the WFL results with those of an ANN and a coupled wavelet–ANN (WANN) models. They found that the WFL had a significant improvement over the ad hoc FL, ANN, and hybrid WANN models. Belayneh et al. (2014), using SPI time series, developed hybrid WANN and wavelet-support vector regression (WSVR) models to forecast long-term drought in the Awash River Basin of Ethiopia. They compared the effectiveness of these models with those of ARIMA, ANN, and ad hoc support vector regression models and stated that the WANN model is the best one for 6 and 12-months LT drought forecasting in their study area.

Despite providing plausible forecasting accuracy, all the aforementioned ANN-based models provide implicit formulations with huge matrix of synaptic weights and biases. Thus, necessity for further studies in order to develop not only precise but also explicit models is still receiving serious attention. In recent years, different variants/advancements of genetic programing (GP) approach has been pronounced as a robust explicit method to solve wide range of modeling problems in water resources engineering such as rainfall-runoff modeling (Dorado et al., 2003; Nourani et al., 2012), evapotranspiration (Kisi and Guven, 2010), unit hydrograph determination (Rabuñal et al., 2007), sediment transport (Aytek and Kisi, 2008), sea level forecasting (Ghorbani et al., 2010), streamflow prediction (Danandeh Mehr et al., 2013a) and others. A comprehensive review on application of hybrid wavelet–AI models in hydrology has been provided by Nourani et al. (2014). The authors also highlighted and discussed the importance of available hybrid models for drought forecasting. Moreover, our review indicated that there is no research in the relevant literature examining the performance of any hybrid GP technique in drought forecasting. It is also important to understand different modeling methods as well as their benefits and limitations (Mishra and Singh, 2011). These are the main reasons inspired us to develop an explicit model based on one of the advancements of GP namely linear genetic programing (LGP) GP to forecast drought in this study.

It is already proven that the drought process contains high non-stationary and long-term patterns (seasonality) and classic AI techniques such as ANN and FL may not be sufficient for long LT drought forecasting (Özger et al., 2012). Therefore, our study was commenced with a data pre-processing, i.e. de-noising our predictor time series using continuous wavelet transform technique, and accomplished by a LGP–based model. In this study, based upon lagged values of drought index across the State of Texas along with NINO 3.4 index, symbolizing the sea surface temperature anomalies, we developed a hybrid wavelet–linear genetic programing (WLGP) model (here after gene–wavelet model) for long LT drought forecasting. For this aim, we initially applied wavelet transform to decompose the predictor time series into its major sub-series and then we employed a LGP technique to make forecasts. The LGP component of the model can handle the nonlinearity elements, while the wavelet component can deal with periodicity of the hydro-climatic variables. Furthermore, the performance of the proposed gene–wavelet model was compared with those of hybrid WANN and WFL models previously reported by Özger et al. (2012).

Since the black-box models are often case-sensitive, in the present study, we do not attempt to claim or assert superiority of a particular model over the others. The main goal of this paper is, for the first time, to introduce a new explicit gene–wavelet model (WLGP) for drought forecasting.

2. Wavelet transform

Wavelet transform (WT) provides multi-resolution of a signal in time and frequency domains and has been employed for studying non-stationary time series, where it is difficult to detect the time of occurrence of a particular event if Fourier transform (FT) is used (Özger et al., 2012). In other words, while FT separates a signal into sine-waves of various frequencies, WT separates a signal into shifted and scaled version of the original (or mother) wavelet (Özger, 2010). WT allows the use of long-time intervals for low frequency signals and shorter intervals for high frequency signals and is able to expose some statistical features of time series like trend and jump that other signal analysis techniques such as FT might miss (Danandeh Mehr et al., 2013a). Since the ENSO indicators (such as NINO 3.4 index) and drought occurrence have long time intervals to develop, low frequency components gain importance in comparison with high frequency. High frequency components of the NINO 3.4 index and PMDI series are detected with lower scales that refer to a compressed wavelet (Özger et al., 2012).

2.1. Continuous wavelet transform (CWT)

In mathematics, an integral transform (Tf) is particular kind of mathematical linear operator, which has the following form:

$$Tf(u) = \int_{t1}^{t2} K(t,u)f(t)dt \tag{1}$$

where $f(t)$ is an square-integrable function such as a continuous time series and K is a two variable, t and u, function called kernel (Danandeh Mehr et al., 2013b).

According to Eq. (1), any integral transform is specified by a choice of the kernel function. If function K is chosen as wavelet function, then CWT is (Mallat, 1998):

$$T(a,b) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{+\infty} \Psi^*\left(\frac{t-b}{a}\right) f(t)dt \tag{2}$$

where $T(a, b)$ is the wavelet coefficients, $\Psi(t)$ is a mother wavelet function, in time and frequency domain, and $*$ denotes operation of complex conjugate.

The parameter a can be interpreted as a dilation ($a > 1$) or contraction ($a < 1$) coefficient of the $\Psi(t)$ corresponding to different scales of observation. The parameter b can be interpreted as a temporal translation (or shift) of the wavelet function, which allows the study of the signal $f(t)$ locally around the time b (Wu et al., 2009). The main property of wavelets is localized in both frequency (a) and time (b), whereas the Fourier transform is only localized in frequency (Danandeh Mehr et al., 2013b).

Appropriate selection of the type of mother wavelet to decompose input time series is one of the important tasks of modellers. It has been recommended that the suitable mother wavelet can be selected according to the shape pattern similarity between the mother wavelet and the investigated time series (Nourani et al., 2009b; Danandeh Mehr et al., 2013a; Onderka et al., 2013). A Brute-force search method has also been adopted as an alternative to find the best mother wavelet in practice (Nourani et al., 2009a, 2011, 2012, 2013a). Since successful application of Morlet wavelet has already been reported at drought forecasting studies (Özger et al., 2012), we considered this as our mother wavelet function in this study. Further information about Morlet wavelet functions can be found at Labat et al. (2000) and Labat (2005).

3. Linear genetic programing (LGP)

Genetic programming (GP) is an evolutionary computing technique that generates a structured representation of the system being studied using initial potential solutions and transformation operators (Koza, 1992). The nature of GP allows to gain additional information on how the system performs, i.e., it gives an insight into the relationship between input and output data (Nourani et al., 2013b). GP holds candidate solutions in a tree-based genome and the transformation operators (crossover and mutation) act on tree-based genomes (Koza, 1992). LGP is distinct from canonical GP systems in that the candidate solutions are programs and transformation operators act on a linear—not tree-based—genome (Banzhaf et al., 1998).

At the most brief level LGP is a steady-state, evolutionary algorithm using fitness-based tournament selection to continuously improve a population of machine-code functions (Francone, 2010). Generally, LGP solves any problem through the following six steps: (i) generation of an initial population (machine-code functions) using the user defined functions and terminals; (ii) Selection of two functions from the population randomly, Comparison of the outputs and designation of the function that is more fit as winner_1 and less fit as loser_1; (iii) Selection of two other functions from the population randomly and designation of the winner_2 and loser_2; (iv) Application of transformation operators to winner_1 and winner_2 to create two similar, but different evolved programs (i.e. offspring) as modified winners (v) replace The loser_1 and loser_2 in the population with modified winners and (vi) Repetition of steps (i)–(v) until the predefined run termination criterion. More information about the application of LGP in predictive modeling can be obtained from Poli et al. (2008).

In this study, different mathematical functions including basic arithmetic (+, −, ×, /), absolute value, square root, power, and trigonometric (Cosine, Sine) functions were utilized in modeling function sets. A set of random values between −1 and 1 in combination with the NINO 3.4 index and antecedent PMDI values are also defined as our terminal set. The mean square error (MSE) fitness function is used to rank the randomly generated initial programs and then new programs are evolved by using both crossover and mutation operators. As it is given in Table 1, in order to avoidance overfitting problem, the maximum size of the program and maximum number of generations was limited to 512 byte and 1000 generations, respectively. Further information about these parameters can be found at Francone (2010). We applied Discipulus®, the LGP soft-ware package developed by Francone (2010), to establish our LGP models.

Table 1
Parameter settings for the LGP system.

Parameter	Value
Initial populations (programs)	500
Mutation frequency	95%
Crossover frequency	50%
Initial program size	80 (Byte)
Maximum program size	512 (Byte)
Generation without improvement	300
Generation since start	1000

4. Gene–wavelet model

The proposed gene–wavelet model is a hybrid WLGP model, which means the pre-processed data via CWT are entered to the predictive LGP system in order to achieve powerful nonlinear approximation ability. In other words, the WLGP is a hybrid forecasting model that combines the power of CWT with LGP to improve the accuracy of ad hoc LGP. The schematic structure of the proposed WLGP model is illustrated in Fig. 1. The structure comprises two phases. In the first phase, pre-processing phase, the original predictor time series (i.e. NINO 3.4 and PMDI) are decomposed into sub-series of average wavelet spectra (A) through low-pass filter coefficients of a chosen mother wavelet. As mentioned previously, we implemented Morlet mother wavelet to decompose the original PMDI and NINO 3.4 series into their average wavelet spectra. Each of resulted average wavelet spectra may consists of several frequencies (bands). The prediction can be difficult and less accurate if the whole bands are taken into account without separation into significant bands and elimination of noises. Therefore, significant bands are determined in the next step of pre-processing phase. In this study, we used significant variances method in the wavelet spectra for the selection of significant frequency bands as suggested by Webster and Hoyos (2004). In the last step of the first phase, the corresponding time series of each significant band ($B_1, B_2, \ldots, B_i$) is determined by inverse wavelet filtering method which provides input variables for the LGP model.

In the second phase, simulation phase, at first, the LGP model is built such that the significant bands ($B_1, B_2, \ldots, B_i$) of the original time series are the input variables of the model. Then, training and validation processes are performed using the input variables and target variable to determine output time series. One important

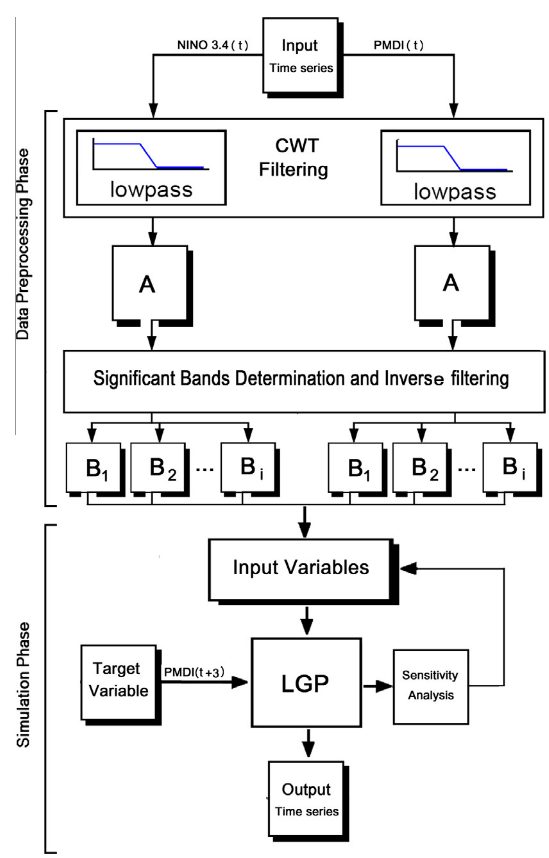

Fig. 1. Schematic structure of the proposed gene–wavelet drought forecasting model.

point in this phase that a modeller may encounter is the magnifying of prediction errors due to considerable rise in the number of input variables resulting from wavelet decomposition. In such cases, decreasing (or optimizing) of input significant band time series by selection of the most dominant ones was suggested (Nourani et al., 2012). The proposed LGP model can optimize the number of the original input signals (or sub-signals) via its heuristics-based evolutionary optimization feature which acts like an internal sensitivity analysis, whereas in WFL or WANN models an external sensitivity analysis is usually performed to optimize them (Partal and Kişi, 2007; Kişi, 2008). This is the reason why we prefer gene–wavelet models to other hybrid models suggested in the literature such as neuro-wavelet and/or neuro-fuzzy models.

5. Data and model precision criteria

As mentioned previously, the monthly time series of NINO 3.4 index and persistence in PMDI values (1951–2007, Fig. 2) were used as original drought predictor variables in this study. Future PMDI values were considered as target variable (predictand) of the model. PMDI is the modified version of PDSI which allows computation of PDSI operationally by taking the sum of the wet and dry terms after they have been weighted by their probability factors (Heim, 2002). The PMDI time series employed in current study is the average values across the State of Texas. The NINO 3.4 index, which is the mean sea surface temperature throughout the equatorial Pacific east of the dateline (5°North-5°South and 170–120°West), was used to represent the ENSO events. A strong relationship between the PDSI and ENSO events has well discussed by Piechota and Dracup (1996). The authors pointed out that dry condition at the region one of the Gulf of Mexico (GM1), which encompasses the entire State of Texas, Occur consistently during La Niña events. A long-term precipitation change in the State of Texas affected by ENSO is also informed by Özger et al. (2009).

The coefficient of determination (R^2) and the root mean square error (RMSE) measures, which widely used in the hydrological forecasting studies (e.g. Nourani et al., 2009b; Danandeh Mehr et al., 2013a), were applied to measure the efficiency, goodness of fit, of the proposed forecasting model in this study. Obviously, a high value for R^2 (up to one) and small values for RMSE indicate high efficiency of the model.

6. Results and discussion

According to the following expressions, we have considered 3, 6, and 12-month LTs for performing the proposed gene–wavelet drought forecasting model in this study. The given expressions have already been reported by Özger et al. (2012) as the best possible structures for long LT drought forecasting in the State of Texas.

- 3-Month LT: $PMDI_{t+3} = f(NINO3.4_t, PMDI_t, PMDI_{t-1})$.
- 6-Month LT: $PMDI_{t+6} = f(NINO3.4_t, PMDI_t, PMDI_{t-1}, PMDI_{t-2})$.
- 12-Month LT: $PMDI_{t+12} = f(NINO3.4_t, PMDI_t, PMDI_{t-1}, PMDI_{t-2}, PMDI_{t-3})$.

For each of these structures (i.e. different LT scenarios), firstly we applied the classic LGP modeling technique (Danandeh Mehr et al., 2013a) to develop our reference models. Then, the proposed WLGP model was performed for each scenario. As mentioned previously, a sensitivity analysis is carried out coincident with WLGP performance in order to optimize the inputs of the model. Eventually, the efficiency results of the best developed LGP and WLGP models at each scenario were compared with those of ANN, WANN, and WFL drought forecasting models which were already developed by Özger et al. (2012).

6.1. LGP results

Prior to applying the proposed WLGP model, an attempt has been done to assess the ability of ad hoc LGP to model the investigated phenomena using the original time series. For this aim, monthly observation data (the NINO3.4 and PMDI series during the period 1951–2006) was divided into two parts; namely, training (calibration) and testing (validation). The first 36 years of the entire data set (56 years or 672 months) was employed for the training period and the remaining part was used to test the validity of the model. Fig. 3 shows the observed and forecasted PMDI time series for 3-month LT. The figure illustrates that LGP reasonably forecasts the general behavior of the observed data. But it is not able to estimate extreme values satisfactorily. The obtained efficiency values are summarized in Table 2 along with comparison to those of FL and ANN models reported by Özger et al. (2012).

Table 2 indicates that the classic LGP likewise FL and ANN techniques are not able to produce sufficient accuracy for long LT drought forecasting except in 3-month scenario. It may be due to the presence of significant periodicity and seasonality in our drought index time series. The testing period performance results show that LGP in all scenarios yields slightly higher accuracy than those of FL and ANN. There is also a remarkable difference among the efficiency results of these models at testing period when they were applied for 12-month LT forecasting. Both FL and ANN models

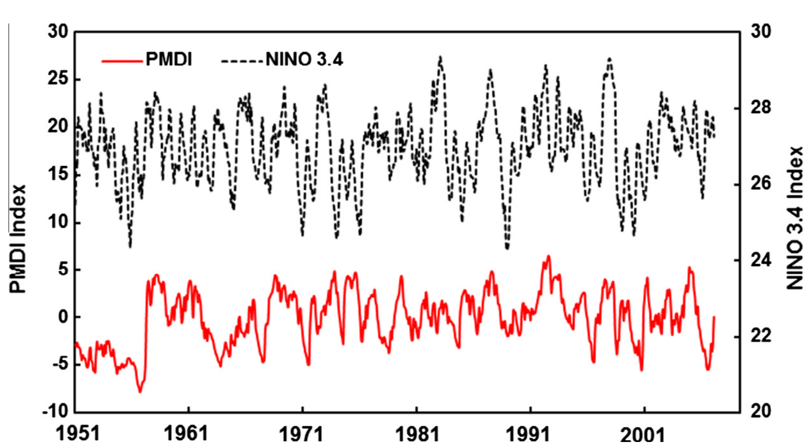

Fig. 2. PMDI and NINO 3.4 time series used in the study.

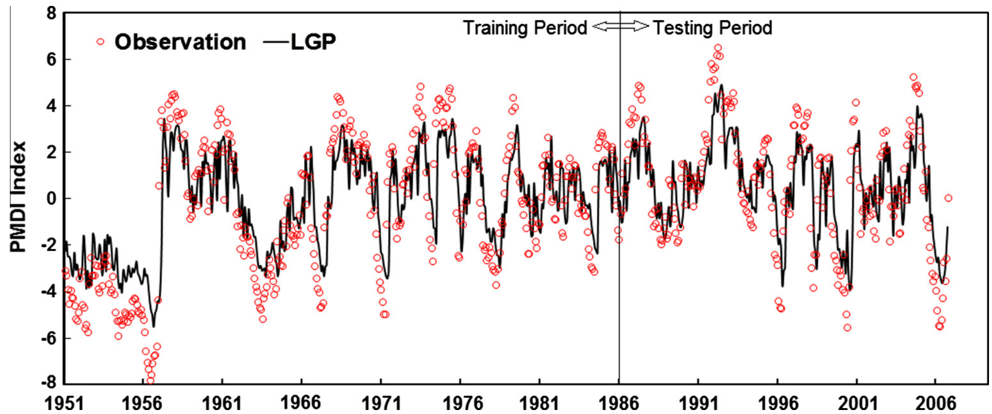

Fig. 3. Results of 3-month LT LGPdrought forecasting model in Texas state.

Table 2
Efficiency results of LGP, FL, and ANN drought forecasting models in Texas.

	LGP				FL				ANN			
	R^2		RMSE		R^2		RMSE		R^2		RMSE	
Forecasting scenario	Train	Test	Train	Test	Train	Test	Train	Test	Train	Test	Train	Test
3-month LT	0.642	0.516	1.75	1.78	0.666	0.501	1.69	1.84	0.673	0.474	1.67	1.89
6-month LT	0.402	0.144	2.25	2.39	0.434	0.106	2.21	2.43	0.476	0.097	2.13	2.44
12-month LT	0.109	0.034	2.72	2.55	0.348	−0.19	2.35	2.82	0.189	−0.12	2.62	2.73

resulted in negative R^2 values at testing period, however LGP still provides positive R^2 value. Considering the higher training R^2 values of FL and ANN models than LGP, it can be indicated that FL and ANN models suffer from overfitting (overtraining) problem in this scenario. It also implied the fact that the avoidance overfitting technique adopted for LGP in this study acted well as it was supposed.

6.2. WLGP results

Data pre-processing is the first phase of the proposed WLGP drought forecasting model. In this phase, each of the predictor time series is decomposed into their average wavelet spectra through the low-pass filter coefficients of continuous Morlet mother wavelet, and then, significant frequency bands are detected. For the

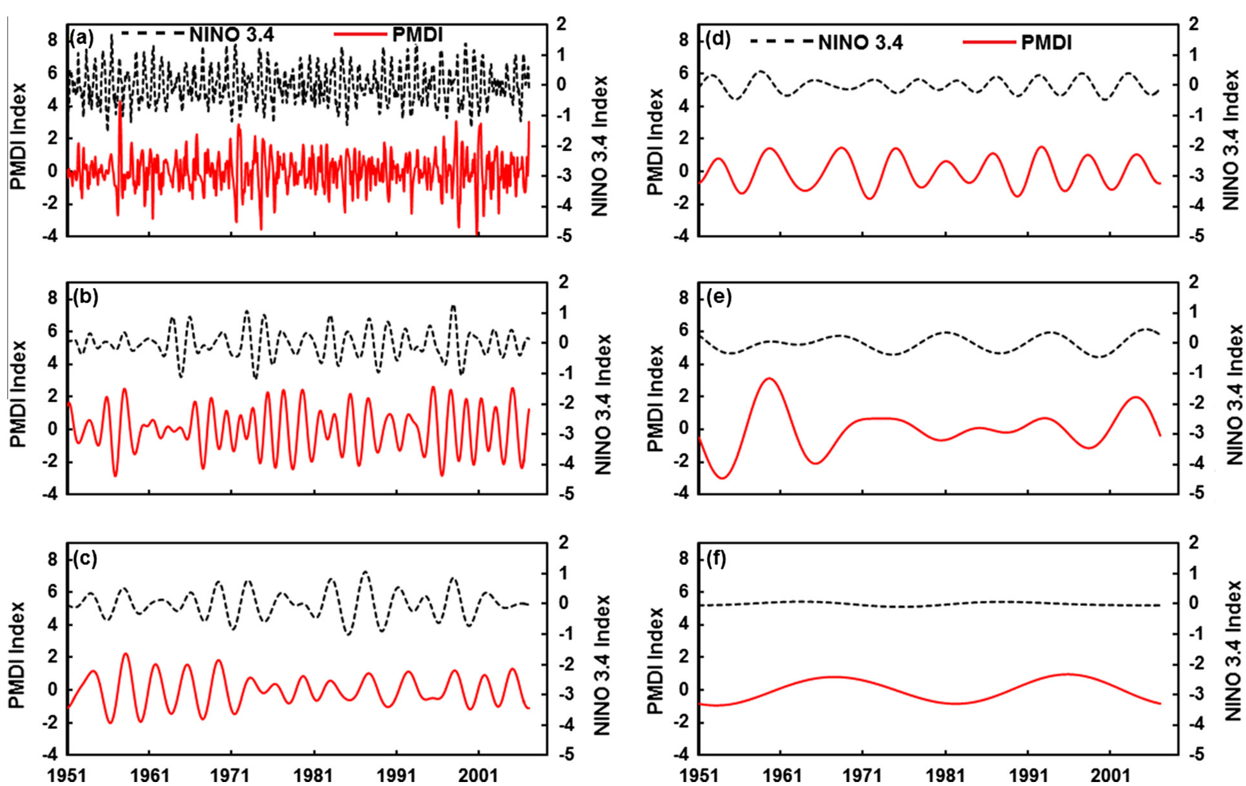

Fig. 4. Significant spectral bands of observed PMDI and NINO 3.4, (a) 7–16, (b) 17–33, (c) 34–56, (d) 57–93, (e) 94–222, and (f) >223 months bands.

employed data (see Fig. 1), six distinct frequency bands have already been detected and reported by Özger et al. (2012). The corresponding time series of wavelet bands obtained by the inverse wavelet filtering is given in Fig. 4, showing significant wavelet bands of each predictor at time t that are employed as input variables of WLGP at all 3, 6, and 12-month LT scenarios.

For different LT forecasting, it is required to establish different WLGP models by using the relevant inputs spectral bands. For instance, in 3-month LT scenario, the significant spectral bands of NINO 3.4_t, $PMDI_t$, and $PMDI_{t-1}$ are the input variables in the simulation phase of the proposed gene–wavelet model. In this phase, the significant spectral bands of $PMDI_{t-1}$ can be derived from significant spectral bands of $PMDI_t$ with one month lag. Fig. 5 illustrates WLGP forecasted PMDI values at different LTs in comparison with corresponding observations. The forecasting performance of the model for each scenario is also presented in Table 3 and compared with those of WANN and WFL models reported by Özger et al. (2012).

From the efficiency results given in Tables 2 and 3, it can be concluded that the WLGP model provided significant improvement for drought forecasting in comparison with the LGP model particularly at 6 and 12-month LT. At 12-month LT forecasting, The R^2 values about zero resulted in LGP modeling was improved up to 0.58 by WLGP. This remarkable improvement puts forward the advancement on explicit long LT drought forecasting models. This significant increment can be verified considering the elimination of some noisy data with the aid of wavelet transform. Since the ENSO-related drought occurrences developed in low frequencies, the removal of noisy and high frequency data captured by wavelet transform resulted in more consistent PMDI estimates. Table 3 also implies that WLGP is slightly less accurate than the WFL and WANN models. The reason behind this may be related to the fact that in the both WFL and WANN models, prior to training, the original predictand time series is decomposed into its average wavelet spectra bands as well as the original predictors. Each band of predictand was estimated from its corresponding predictor bands in these models. Then, spectral bands of the predictand were reconstructed to produce the original time series of future PMDI. But in WLGP, original time series of future PMDI entered to the model (i.e. target variable) and is estimated directly from the predictor bands.

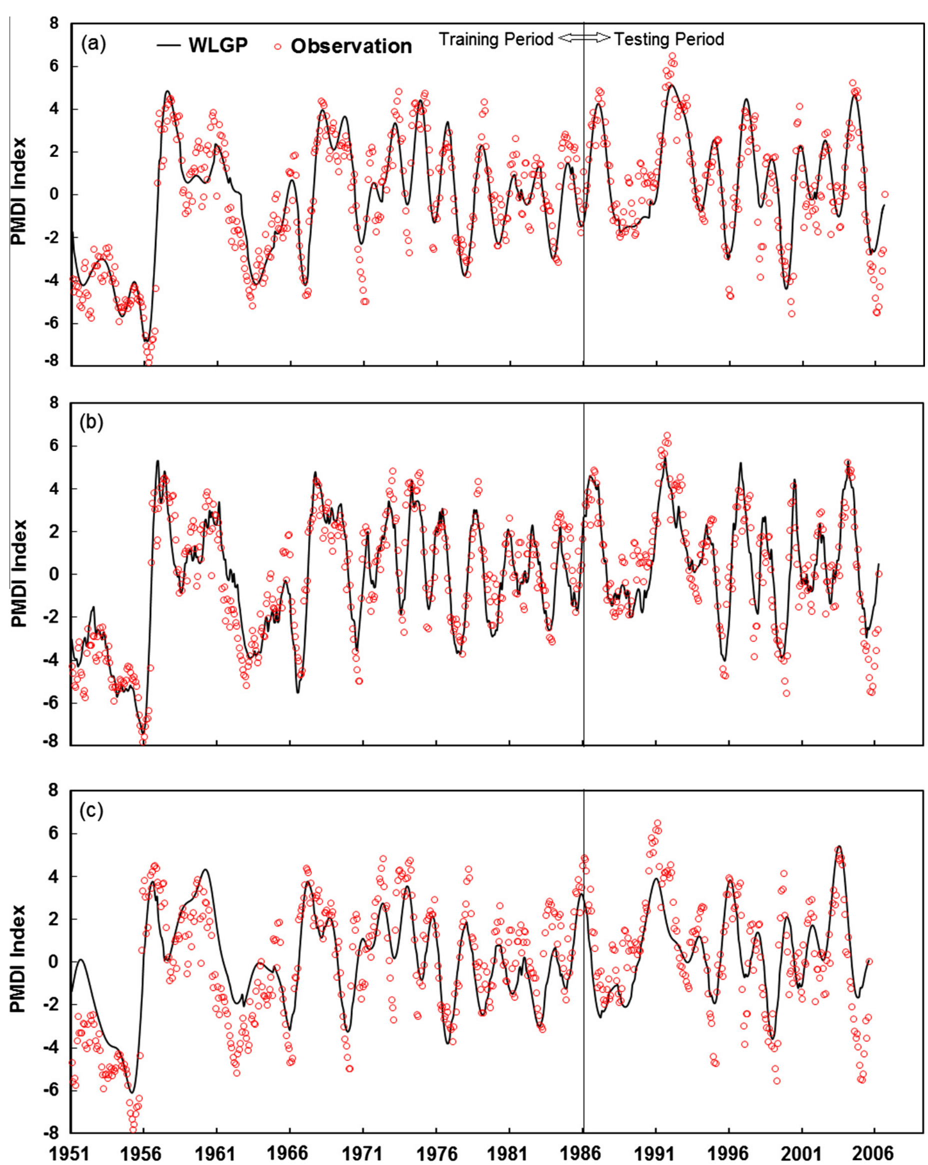

Fig. 5. Results of WLGPdrought forecasting model for the State of Texas, (a) 3 month LT, (b) 6 month LT, (c) 12 month LT.

Table 3
Efficiency results of WLGP, WFL and WANN drought forecasting models at Texas.

Prediction scenario	WLGP				WFL				WANN			
	R^2		RMSE		R^2		RMSE		R^2		RMSE	
	Train	Test	Train	Test	Train	Test	Train	Test	Train	Test	Train	Test
3 month LT	0.847	0.775	1.14	1.22	0.919	0.911	0.83	0.77	0.921	0.867	0.82	0.95
6 month LT	0.827	0.735	1.21	1.34	0.929	0.896	0.78	0.83	0.928	0.871	0.79	0.92
12 month LT	0.661	0.580	1.69	1.67	0.856	0.734	1.10	1.33	0.855	0.764	1.11	1.25

6.3. Sensitivity analysis results

It is inevitable that more input variables in evolutionary computing methods may lead to more complex formulations (Nourani et al., 2012). Owing to the wavelet filtering, considerable rise in the number of input variables might also magnify prediction errors (Danandeh Mehr et al., 2013a). Therefore, a genetic-based sensitivity analysis loop was embedded in the proposed gene–wavelet model, which can be applied during the simulation phase (see Fig. 1). By the aid of this loop, the most effective bands of input significant spectra are distinguished and re-entered to the LGP system as new input variable sets. The LGP system is performed once more for the same target variable and consequently population of new programs are generated that might lead the initial model to higher efficiency.

In order to identify the most effective bands, we considered the exceedance probability of each input band in the thirty best programs evolved by WLGP. Exceedance probabilities of input bands for the thirty best WLGP models in terms of frequency were listed in Table 4. The frequency values show what percentage of the best thirty programs from the model contained the referenced band. A similar sensitivity analysis among wavelet decomposed rainfall and runoff time series was also carried out by Nourani et al. (2012) counting the number of selections of each band at fifty GP runs.

According to Table 4, it can be concluded that at 3-month LT forecasting scenario, all input bands contributed in generation of the best thirty forecasting programs with more or less impact. The bands 2 through 4 of PMDI (t) have the most impact (100% frequency) in the value of PMDI ($t+3$). The first, second, and fifth bands of NINO 3.4 have the least impacts (less than 20% frequency) in this scenario.

At 6-month LT forecasting scenario, PMDI (t) and PMDI ($t-2$) have the most impact, respectively and NINO 3.4 bands are more effective than those of PMDI ($t-1$). The bands 2 through 4 of PMDI (t) are the most effective bands in prediction of PMDI ($t+6$) as well as PMDI ($t+3$). The bands 3 through 5 of NINO 3.4 (t) show significant increment in prediction of 6-month LT drought in comparison with corresponding bands at 3-month LT. Such a comparative significant increment is also observed at 12-month LT forecasting scenario among the bands 2 through 4 of NINO 3.4 (t) and corresponding bands in 3-month LT. It implies the high potential effect of the NINO 3.4 (t) index in 6-month through 12-month LT drought forecasting. Details on physical mechanism behind the fact that ENSO events are correlated with drought conditions in Gulf of Mexico region can be found in the literature (e.g., Kahya and Dracup, 1993, 1994; Dracup and Kahya, 1994; Piechota and Dracup, 1996; Rajagopalan et al., 2000). As given in the caption for Fig. 4, significant spectral bands of NINO 3.4 values at the bands 3 and 4 represent the 34–56 and 57–93-month spectrums, respectively. In other words, these bands roughly represent 3–8 year spectrums that are more or less equal to the prevalent frequency of ENSO events. It implies mid to long range forecasting potential of NINO 3.4 index that is consistent with results of the study conducted by Piechota and Dracup (1996). As it was mentioned previously, a strong relationship between dry condition and La Niña events in the State of Texas has been reported by Piechota and Dracup (1996). The authors pointed out that significant PDSI related La Niña signal appeared to be even negative during the period starting from November of event year continuing by the end of the following year. It is also important to emphasize that the magnitude of these negative PDSI anomalies in GM1 region were the largest among all regions in their study domain. This shows that the impacts of La Niña events on the PDSI pattern are long lasting in the Texas area implying a mid to long range forecasting potential.

Based upon frequency of input bands tabulated in Table 4, the bands possessing less than 50% frequency at the best thirty programs were eliminated from input variables of simulation phase of WLGP (see Fig. 1) and the rest of the bands spectra considered in re-performing of the WLGP as the effect of sensitivity analysis loop. For example, the NINO3.4 index does not have any band with more than 50% frequency at 3 and 6-month LT scenarios (see

Table 4
Frequency of each band in WLGP Model for PMDI prediction (1.0 = 100%).

Input parameter	Band 1	Band 2	Band 3	Band 4	Band 5	Band 6
3 month LT scenario						
NINO 3.4 (t)	0.07	0.17	0.2	0.23	0.13	0.47
PMDI ($t-1$)	0.13	1	0.47	0.2	0.57	0.53
PMDI (t)	0.1	1	1	1	0.83	0.33
6 month LT scenario						
NINO 3.4 (t)	0.2	0.17	0.3	0.4	0.43	0.4
PMDI ($t-2$)	0	1	0.77	0.23	0.5	0.3
PMDI ($t-1$)	0	0.13	0.23	0	0.33	0.17
PMDI (t)	0.43	1	1	1	0.93	0.03
12 month LT scenario						
NINO 3.4 (t)	0.03	0.5	0.27	0.57	0	0.2
PMDI ($t-3$)	0	0.83	1	0.63	0.5	0.4
PMDI ($t-2$)	0	0	0	0.03	0	0.33
PMDI ($t-1$)	0	0.17	0.27	0.17	0.43	0.2
PMDI (t)	0	1	0.5	0.83	0.7	0.07

Table 5
Efficiecny results of WLGP model without and with sensitivity analysis.

Prediction scenario	WLGP without sensitivity analysis				WLGP with sensitivity analysis			
	R^2		RMSE		R^2		RMSE	
	Train	Test	Train	Test	Train	Test	Train	Test
3 month LT	0.847	0.775	1.14	1.22	0.841	0.761	1.16	1.26
6 month LT	0.827	0.735	1.21	1.34	0.804	0.686	1.28	1.46
12 month LT	0.661	0.580	1.69	1.67	0.770	0.642	1.39	1.54

Table 4). Thus, none of the NINO 3.4 bands entered in re-performing the model for 3 and 6-month LT forecasting. However its second and fourth bands are considered in model re-performing 12-month LT.

The efficiency results of the WLGP model with the effect of sensitivity analysis for drought forecasting in the State of Texas with 3, 6, and 12-month LT were presented in Table 5 and were compared with those of WLGP without considering the sensitivity analysis loop. It is evident from the table that the use of sensitivity analysis generated more accurate forecasts of PMDI only in 12-month LT scenario. It indicates that: (i) The uncertainty feature of our data (noise) is formerly well diminished at 3 and 6-month LT forecasting due to the wavelet transform, (ii) increasing in the number of input sub-signals in 12-month LT scenario may magnify prediction errors and lead to unreliable outputs unless sensitivity analysis has been employed, and (iii) There is a strong potential in NINO 3.4 index to forecast drought with one year LT.

7. Conclusions

In this study, the LGP and wavelet transform concepts were combined to develop an explicit hybrid gene–wavelet model, WLGP for long LT drought forecasting using PMDI and NINO 3.4 values as predictors and forthcoming PMDI index as a predictand. The model is capable: (i) to obtain the average wavelet spectra, (ii) to detect the significant spectral bands (iii) to forecast future PMDI, and (iv) to optimize the number of significant spectral bands via its heuristics-based sensitivity analysis feature. The application of the WLGP across the State of Texas provided significant improvement in accuracy over the ad hoc LGP models particularly at 6 and 12-month LT forecasting. Sensitivity analysis among input variable bands indicated that the preceding values of PMDI have higher impact than NINO 3.4 for drought forecasting up to 6-month LT, whereas the latter has high potential to forecast drought for 6 through 12-month LT.

As a suggestion for future research, with the aid of the proposed model, other climatic indices impacts on drought condition can be investigated. The model also can be used to investigate the effectiveness of NINO 3.4 and PMDI indices in order to predict drought having LTs longer than a year. In this study, we used a fixed mother wavelet (Morlet wavelet functions) to decompose our input time series. Effect of different wavelet functions may be considered as a way to optimize the current model in future studies.

References

Aytek, A., Kisi, O., 2008. A genetic programming approach to suspended sediment modelling. J. Hydrol. 351 (3–4), 288–298.
Bacanli, U.G., Firat, M., Dikbas, E.F., 2009. Adaptive neuro-fuzzy inference system for drought forecasting. Stoch. Environ. Res. Risk Assess. 23, 1143–1154.
Banzhaf, W., Nordin, P., Keller, R. and Francone, F.D., 1998. Genetic programming-an introduction on the automatic evolution of computer programs and its application. dpunkt/Morgan Kaufmann: Heidelberg, San Francisco.
Barros, A.P., Bowden, G., 2008. Toward long-lead operational forecasts of drought: an experimental study in the Murray-Darling River Basin. J. Hydrol. 357 (3–4), 349–367.
Belayneh, A., Adamowski, J., 2012. Standard precipitation index drought forecasting using neural networks, wavelet neural networks, and support vector regression. Appl. Comput. Intell. Soft Comput. Article ID 794061, 13pages.
Belayneh, A., Adamowski, J., Khalil, B., Ozga-Zielinski, B., 2014. Long-term SPI drought forecasting in the Awash River Basin in Ethiopia using wavelet neural network and wavelet support vector regression models. J. Hydrol. 508, 418–429.
Bogradi, I., Matyasovsky, I., Bardossy, A., Duckstein, L., 1994. A hydroclimotological model of arial drought. J. Hydrol. 153, 245–264.
Cancelliere, A., Mauro, G.D., Bonaccorso, B., Rossi, G., 2007. Drought forecasting using the standardized precipitation index. Water Resour. Manage 21, 801–819.
Danandeh Mehr, A., Kahya, E., Olyaie, E., 2013a. Streamflow prediction using linear genetic programming in comparison with a neuro-wavelet technique. J. Hydrol. 505, 240–249.
Danandeh Mehr, A., Kahya, E., Bagheri, F., Deliktas, E. 2013b. Successive-station monthly streamflow prediction using neuro-wavelet technique. Earth Sci. Inform., doi: 10.1007/s12145-013-0141-3, in press.
Dolling, O.R., Varas, E.A., 2002. Artificial neural networks for streamflow prediction. J. Hydraul. Res. 40 (5), 547–554.
Dorado, J., Rabuñal, J.R., Pazos, A., Rivero, D., Santos, A., Puertas, J., 2003. Prediction and modeling of the rainfall-runoff transformation of a typical urban basin using ANN and GP. Appl. Artif. Intell. 17, 329–343.
Dracup, J.A., Kahya, E., 1994. The relationships between U.S. streamflows and La Nina events. Water Resour. Res. 30, 2133–2141.
Fernandez, C., Vega, J.A., Fonturbel, T., Jimenez, E., 2009. Streamflow drought time series forecasting: a case study in a small watershed in North West Spain. Stoch. Environ. Res. Risk Assess. 23, 1063–1070.
Francone, F.D., 2010. Discipulus™ with Notitia and solution analytics owner's manual. Register Machine Learning Technologies Inc., Littleton, CO, USA.
Ghorbani, M.A., Khatibi, R., Aytek, A., Makarynskyy, O., Shiri, J., 2010. Sea water level forecasting using genetic programming and artificial neural networks. Comput. Geosci. 36 (5), 620–627.
Heim Jr., Richard R., 2002. A review of twentieth-century drought indices used in the United States. Bull. Amer. Meteor. Soc. 83, 1149–1165.
Kahya, E., Dracup, J.A., 1993. US streamflows patterns in relation to the El Nino/Southern Oscillation. Water Resourc. Res. 29, 2491–2503.
Kahya, E., Dracup, J.A., 1994. The influence of type I El Nino and La Nina events on streamflows in the Pacific Southwest of the United States. J. Clim. 7 (6), 965–976.
Kim, T.W., Valdes, J.B., 2003. Nonlinear model for drought forecasting based on a conjunction of wavelet transforms and neural networks. J. Hydrol. Eng. 8 (6), 319–328.
Kişi, Ö., 2008. Stream flow forecasting using neuro-wavelet technique. Hydrol. Process. 22, 4142–4152.
Kisi, O., Guven, A., 2010. Evapotranspiration modeling using linear genetic programming technique. J. Irrig. Drain. Eng. 136 (10), 715–723.
Koza, J.R., 1992. Genetic programming: on the programming of computers by means of natural selection. MIT Press, Cambridge, MA, USA.
Labat, D., 2005. Recent advances in wavelet analyses: Part 1. A review of concepts. J. Hydrol. 314 (1–4), 275–288.
Labat, D., Ababou, R., Mangin, A., 2000. Rainfall–runoff relations for karstic springs. Part II: continuous wavelet and discrete orthogonal multiresolution analyses. J. Hydrol. 238, 149–178.
Lohani, V.K., Loganathan, G.V., 1997. An early warning system for drought management using the Palmer drought index. J. Am. Water Res. Assoc. 33 (6), 1375–1386.
Mallat, S., 1998. A Wavelet Tour of Signal Processing, 2nd Ed. Academic Press, San Diego, CA.
Mishra, A.K., Desai, V.R., 2005. Drought forecasting using stochastic models. J. Stoch. Environ. Res. Risk Assess. 19, 326–339.
Mishra, A.K., Desai, V.R., 2006. Drought forecasting using feed forward recursive neural network. Ecol. Model. 198, 127–138.
Mishra, A.K., Singh, V.P., 2010. A review of drought concepts. J. Hydrol. 391 (1–2), 202–216.
Mishra, A.K., Singh, V.P., 2011. Drought modelling – a review. J. Hydrol. 403, 157–175.
Mishra, A.K., Desai, V.R., Singh, V.P., 2007. Drought forecasting using a hybrid stochastic and neural network model. J. Hydrol. Eng. 12 (6), 626–638.
Modarres, R., 2007. Streamflow drought time series forecasting. Stoch. Environ. Res. Risk Assess. 15 (21), 223–233.
Morid, S., Smakhtin, V., Bagherzadeh, K., 2007. Drought forecasting using artificial neural networks and time series of drought indices. Int. J. Climatol. 27 (15), 2103–2111.
Nourani, V., Alami, M.T., Aminfar, M.H., 2009a. A combined neural-wavelet model for prediction of Ligvanchai watershed precipitation. Eng. Appl. Artif. Intell. 22 (3), 466–472.
Nourani, V., Komasi, M., Mano, A., 2009b. A multivariate ANN wavelet approach for rainfall–runoff modelling. Water Resour. Manage 23, 2877–2894.
Nourani, V., Kisi, O., Komasi, M., 2011. Two hybrid artificial intelligence approaches for modeling rainfall–runoff process. J. Hydrol. 402, 41–59.
Nourani, V., Komasi, M., Alami, M.T., 2012. Hybrid wavelet–genetic programming approach to optimize ANN modelling of rainfall-runoff process. J. Hydrol. Eng. 7 (6), 724–741.
Nourani, V., Hosseini Baghanam, A., Adamowski, J., Gebremichael, M., 2013a. Using self-organizing maps and wavelet transforms for space–time pre-processing of satellite precipitation and runoff data in neural network based rainfall–runoff modeling. J. Hydrol. 476, 228–243.
Nourani, V., Komasi, M., Alami, M.T., 2013b. Geomorphology-based genetic programming approach for rainfall-runoff modeling. J. Hydroinform. 15 (2), 427–445.
Nourani, V., Hosseini Baghanam, A., Adamowski, J., Kisi, O., 2014. Applications of hybrid Wavelet-Artificial Intelligence models in hydrology. A review. J. Hydrol. 514, 358–377.
Onderka, M., Banzhaf, S., Scheytt, T., Krein, A., 2013. Seepage velocities derived from thermal records using wavelet analysis. J. Hydrol. 479, 64–74.
Özger, M., 2010. Significant wave height forecasting using wavelet fuzzy logic approach. Ocean Eng. 37 (16), 1443–1451.
Özger, M., Mishra, A.K., Singh, V.P., 2009. Low frequency drought variability associated with climate indices. J. Hydrol. 364 (1–2), 152–162.
Özger, M., Mishra, A.K., Singh, V.P., 2012. Long lead time drought forecasting using a wavelet and fuzzy logic combination model. J. Hydrometeorol. 13, 284–297.

Partal, T., Kişi, Ö., 2007. Wavelet and neuro-fuzzy conjunction model for precipitation forecasting. J. Hydrol. 342 (1–2), 199–212.
Pesti, G., Shrestha, B., Duckstein, L., Bogardi, I., 1996. A fuzzy rule-based approach to drought assessment. Water Resour. Res. 32 (6), 1741–1747.
Piechota, T.C., Dracup, J.A., 1996. Drought and regional hydrologic variation in the United States: associations with the El Niño–Southern Oscillation. Water Resour. Res. 32 (5), 1359–1373.
Pongracz, R., Bogardi, B., Duckstein, L., 1999. Application of fuzzy rule-based modeling technique to regional drought. J. Hydrol. 224, 100–114.
Poli, R., Langdon, W.B. and McPhee, N.F., 2008. A Field Guide to Genetic Programming. Published via http://lulu.com and freely available at http://www.gp-field-guide.org.uk (With contributions by J.R. Koza)
Rabuñal, J.R., Puertas, J., Suárez, J., Rivero, D., 2007. Determination of the unit hydrograph of a typical urban basin using genetic programming and artificial neural networks. Hydrol. Process., 476–485.
Rajagopalan, B., Cook, E., Lall, U., Ray, B.K., 2000. Spatiotemporal variability of ENSO and SST teleconnections to summer drought over the United States during the twentieth century. J. Clim. 13, 4244–4255.
Rao, A.R., Padmanabhan, G., 1984. Analysis and modeling of palmer's drought index series. J. Hydrol. 68, 211–229.
Sen, Z., 1990. Critical drought analysis by second order Markov chain. J. Hydrol. 120, 183–202.
Webster, P.J., Hoyos, C.D., 2004. Prediction of monsoon rainfall and river discharge on 15–30-day time scales. Bull. Am. Meteorol. Soc. 85 (11), 1745–1765.
Whigham, P.A., Crapper, P.F., 2001. Modelling rainfall-runoff using genetic programming. Math. Comput. Model. 33, 707–721.
Wu, C.L., Chau, K.W., Li, Y.S., 2009. Methods to improve neural network performance in daily flows prediction. J. Hydrol. 372, 80–93.