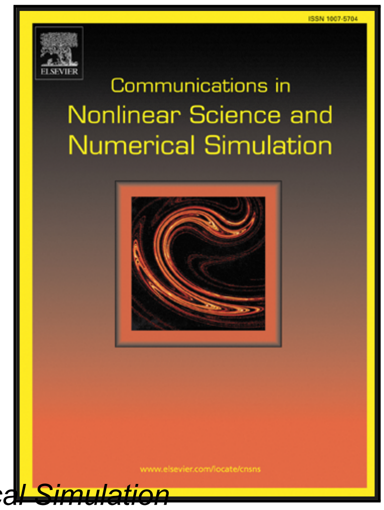
# Accepted Manuscript




Dendritic growth model of multilevel marketing

James Christopher S. Pang, Christopher P. Monterola

PII: S1007-5704(16)30229-5
DOI: 10.1016/j.cnsns.2016.06.030
Reference: CNSNS 3917

To appear in: *Communications in Nonlinear Science and Numerical Simulation*

Received date: 4 November 2015
Revised date: 13 June 2016
Accepted date: 19 June 2016

Please cite this article as: James Christopher S. Pang, Christopher P. Monterola, Dendritic growth model of multilevel marketing, *Communications in Nonlinear Science and Numerical Simulation* (2016), doi: 10.1016/j.cnsns.2016.06.030

**Highlghts**

- The complexity and inherent dynamics of multilevel marketing (MLM) is introduced
- A greedy branching model is used to capture the observed tree-like structures in MLM
- In particular, biologically inspired dendritic network growth is used to model MLM
- Model accuracy is demonstrated using known statistics of previously studied MLM
- Paradigm reported captures MLM dynamics better than previously reported models

# Dendritic growth model of multilevel marketing

James Christopher S. Pang

*School of Physics, University of Sydney, New South Wales 2006, Australia*

Christopher P. Monterola*

*Institute of High Performance Computing, Agency for Science, Technology and Research (A*STAR), 1 Fusionopolis Way, #16-16 Connexis 138632, Singapore*

*Complexity Institute, Nanyang Technological University, 50 Nanyang Avenue 639798, Singapore*

**Abstract**

Biologically inspired dendritic network growth is utilized to model the evolving connections of a multilevel marketing (MLM) enterprise. Starting from agents at random spatial locations, a network is formed by minimizing a distance cost function controlled by a parameter, termed the balancing factor $bf$, that weighs the wiring and the path length costs of connection. The paradigm is compared to an actual MLM membership data and is shown to be successful in statistically capturing the membership distribution, better than the previously reported agent based preferential attachment or analytic branching process models. Moreover, it recovers the known empirical statistics of previously studied MLM, specifically: ($i$) a membership distribution characterized by the existence of peak levels indicating limited growth, and ($ii$) an income distribution obeying the 80-20 Pareto principle. Extensive types of income distributions from uniform to Pareto to a "winner-take-all" kind are also modeled by varying $bf$. Finally, the robustness of our dendritic growth paradigm to random agent removals is explored and its implications to MLM income distributions are discussed.

*Keywords:* dendritic networks, multilevel marketing, balancing factor, Pareto principle

*Corresponding author.

*Email addresses:* `james.pang@sydney.edu.au` (James Christopher S. Pang), `monterolac@ihpc.a-star.edu.sg` (Christopher P. Monterola)

## 1. Introduction

Biologically inspired models continue to drive the development of wide range of tools for understanding physical and social systems. Artificial neural networks and dendritic cell algorithms, to name a few, have been successfully utilized to aid in the development of tools and/or understanding of robotics [1], finance [2], artificial immunology [3], public opinion [4], differential equations [5, 6], and physics education [7]. Here, we demonstrate that dendritic growth model in a confined substrate can be used to capture the empirical features of multilevel marketing (MLM) schemes.

Multilevel marketing is a form of direct selling where distribution and selling of products is administered through a direct contact between a buyer and an independent distributor [8]. In contrast with traditional media sources like television, radio, or newspaper, MLM makes use of a more direct *word of mouth* marketing to sell products and exploits the network connections of its members through recruitment processes. In recent years, the number of MLM companies is projected to continuously grow in the worldwide scene [9]. In the Philippines alone, the World Federation of Direct Selling Association (WFDSA) reported that the country has more than 100 MLM companies involving almost 4 million members as of 2014 with a combined direct selling retail sales of 1.2 billion US Dollars. Globally, WFDSA reported that the total sales of MLM companies for 2014 is about 180 billion US dollars, more than twice the sales of video gaming industry (∼$76 billion) and about a dozen times of the music industry (∼$15 billion).

The MLM scheme naturally entices its stakeholders because it promises a personal financial stake for it consumers/inverstors, in terms of profit sharing and/or commission-based income [10]. However, MLM companies are closely watched by government agencies because of the similarity of their operating mechanism to illegal pyramid schemes [8]. The risk of MLM companies to degenerate into unethical and fraudulent pyramid schemes provides a compelling reason on why we need to better understand the mechanisms behind the growth of their networks. An accurate model of the connectivity of agents within the network is one of the most crucial step to advance this understanding. Previously, observed features such as parity, membership evolution, and Pareto distributed income are attributed to the fact that the networks underlying the simulations have scale free connectivity patterns and not random nor ordered [11].

In this article, we show that the morphological features of MLM networks can be replicated by synthetic biologically inspired dendritic networks. We take advantage of this structural similarity and show by example that the characteristics manifested by MLM networks might not be due to the network itself but is possibly driven by the underlying dynamics during the member recruitment process. We also highlight the rich kind of networks and income distributions that can emerge from a very simple dendritic growth paradigm, driven by a single parameter called the balancing factor ($bf$) that accounts for the costs of recruitment. Finally, we report the sensitivity of our dendritic networks to random node removals and comprehensively probe the large span of parameters that can define the networks' properties.

## 2. Methods

### *2.1. Multilevel marketing business model*

Earnings in an MLM enterprise are based on the combination of product sales and recruitment process. Each member sells a product and is allowed to recruit as many new members as he/she can who will work under his/her supervision. The members are motivated to expand their personal network of recruits since successful recruiters are entitled to a commission from the product sales of their direct and/or indirect recruits. This simple chain of recruiter-recruit dynamics extends the membership and network of an MLM enterprise.

Here, we present two common types of MLM network configurations; the unilevel [see Fig. 1(a)] and binary [see Fig. 1(b)] structures. Members in the same row belong to the same level $m$, with $m = 1$ assigned to the founder of the MLM enterprise. Person $B$ is referred to as the downline of person $A$ if he/she is in a level below $A$ and belongs to the income portfolio of $A$. Theoretically, each member has an opportunity to recruit unlimited number of new recruits but the architecture of the MLM enterprise imposes restrictions to the maximum number of immediate downlines. For a unilevel MLM, a member in level $m$ can position all of his/her recruits as immediate downlines in a single level $m + 1$. On the other hand, for a binary MLM, a member is only allowed to have two maximum immediate downlines. However, he/she can strategically position his/her recruits as downlines of his/her downlines or as downlines of downlines of his/her downlines, and so on. He/she does

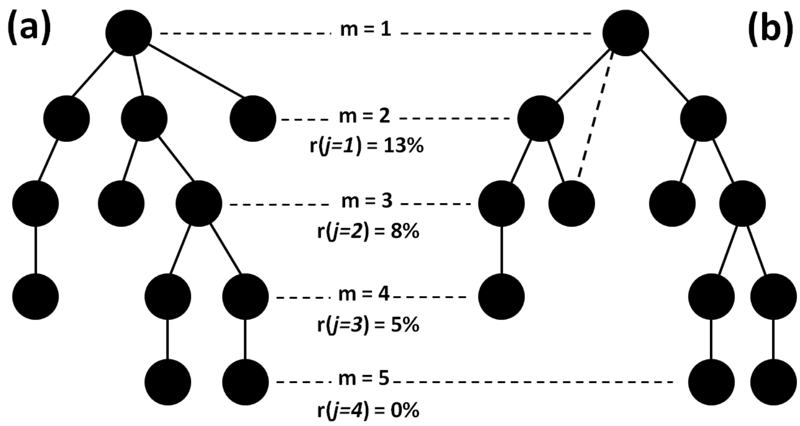



Figure 1: Common MLM structures. (a) Unilevel and (b) binary structure where the topmost member at level $m = 1$ is the founder of the enterprise. Members at a particular level are connected to members at another level via solid lines. For the binary structure, dashed line represents a direct recruit of a member but is strategically repositioned under his/her downlines to maintain lateral symmetry. Shown also are the representative commission rates received by level $m$ from the income of level $j$ under his/her portfolio.

this strategic repositioning to maintain lateral symmetry for the MLM network and maximize his/her income gain. Each member is motivated to symmetrize his/her downlines distribution since he/she is given a pairing income whenever he/she completes a pair of recruits as downlines for levels below him/her.

A member's compensation plan can then be separated into three types: $(i)$ commission, $(ii)$ referral, and $(iii)$ pairing income (adopted from [11]).

Consider the number of members in a level $m$ to be $f(m)$. A commission income, $P_{\text{commission}}(m)$, is awarded to a member proportional to the sales made by his/her downlines below a certain depth of his portfolio $j$, and can be mathematically expressed as:

$$P_{\text{commission}}(m) = \sum_{j=1}^{D} r(j) f(m+j) C_{\text{sales}}, \tag{1}$$

where $D$ is the maximum depth a member in level $m$ is entitled to percentage of sales of the members in that level, $r(j)$ is the commission rate, and $C_{\text{sales}}$ is the amount of sales made to those outside the MLM organization (non-members). Throughout the paper, without loss of generality, we use a standard company practice of having a maximum depth of $D = 3$ and commission rates of $r(1) = 13\%$, $r(2) = 8\%$, and $r(3) = 5\%$. This will let level $m$ earn a commission income from the sales of levels $m+1$, $m+2$, and $m+3$ with commission rates of 13%, 8%, and 5%, respectively.

A referral income $P_{\text{referral}}(m)$ is the income earned by a member proportional to the number of his/her downlines. This is the collective incentive of each level for recruiting new members. It is mathematically expressed as

$$P_{\text{referral}}(m) = \xi(m) C_{\text{referral}}, \tag{2}$$

where $\xi(m)$ is the number of recruits made by all members in level $m$ and $C_{\text{referral}}$ is the referral bonus.

The pairing income $P_{\text{pairing}}(m)$, only exists in a binary structure, is awarded to a member each time he/she is able to fill his/her two immediate downlines with a pair of recruits. This pairing income will drive and influence members to preserve the symmetry of the binary network. It is modeled as

$$P_{\text{pairing}}(m) = \frac{1}{2} \sum_{j=1}^{D} f(m+j) C_{\text{pairing}}, \tag{3}$$

where $f(m+j)$ is the number of members in level $m+j$ and $C_{\text{pairing}}$ is the pairing bonus.

For simplification purposes, the product price, referral bonus, and pairing bonus are set at $C_{\text{sales}} = 1$, $C_{\text{referral}} = 0.1$, and $C_{\text{pairing}} = 0.1$, respectively. Note that we deliberately carried out the relation $C_{\text{sales}} \gg C_{\text{referral}}$ to not let the recruitment process be the dominant contributor to a member's income following rules of a legitimate MLM company. Finally, the total income per level $P_{\text{total}}(m)$ is just the sum of Eqs (1) and (2) for the unilevel structure; while it is the sum of Eqs (1) to (3) for the binary structure. This in turn provides an average income of $P_{\text{total}}(m)/f(m)$ for each member of level $m$.

It is important to note that the set of income quantities we chose to model is limited and is only a subset of the different income types commonly utilized by real-world MLM companies. The three types shown in Eqs (1) to (3) are specifically chosen to gain an insight on how the network structure can directly impact the income earned by an individual. In addition, we have to clear that our basic assumption in this model is that the agents are homogenous and perfect - have the ability to sell the same volume of sales.

Thus, we disregarded the commission earned by own personal sales as this will only result to a constant additive term to every member's total income and will not significantly change the entire income dynamics. Finally, we emphasize that the resulting total income $P_{\text{income}}$ only represents the total positive flow of income and disregards cost factors associated with MLM participation (e.g., payments, joining fees, trainings [12]) .

*2.2. Dendritic network generation, comparison, and conversion*

This article aims to use a biophysical perspective to model the evolving connections underlying the recruiter-recruit network of an MLM enterprise. We do this by utilizing a dendritic network adopted from [13], that follows a branching process governed by a greedy algorithm. We generate the network by initializing $N$ nodes at random locations inside a circular substrate of radius 5 units including a root node at the center. At the next simulation step, nodes surrounding the root node within a threshold distance of 0.5 units are connected to it and the result is considered to be the initial network. For succeeding steps, an unconnected node is chosen one at a time. It is then connected to a node, already belonging to the network, nearest to it satisfying the same threshold distance while minimizing a cost function $\Psi$. The cost function is composed of two distance factors: ($i$) a *wiring cost* $d$ represented by the Euclidean distance of the unconnected node to a node already in the network, and ($ii$) a *path length cost* $pl$ described by the length of the path along the network from the root node to the unconnected node. These two cost factors are weighed by a balancing factor $bf$ as shown by the equation

$$\Psi = d + bf \cdot pl, \tag{4}$$

where $bf \in [0, 1]$. A $bf$ close to 0 will produce a network with nodes having a strong preference to smaller $d$. On the other hand, a high $bf$ close to 1 will have nodes favoring a smaller $pl$. We speculate the relevance of these parameters to an MLM context in the succeeding discussions.

Figure 2 illustrates the different network structures that one can recover using a circular substrate architecture, by just spanning $bf$ values from 0 to 1. The structures range from dendritic-like network, where each node follows a branching process and that only nodes close to each other tend to have a connection, to that of a star-like network where each node is directly connected to the root node. Furthermore, addition of nodes using the same radius denses the substrate without changing the structural form of the network produced.

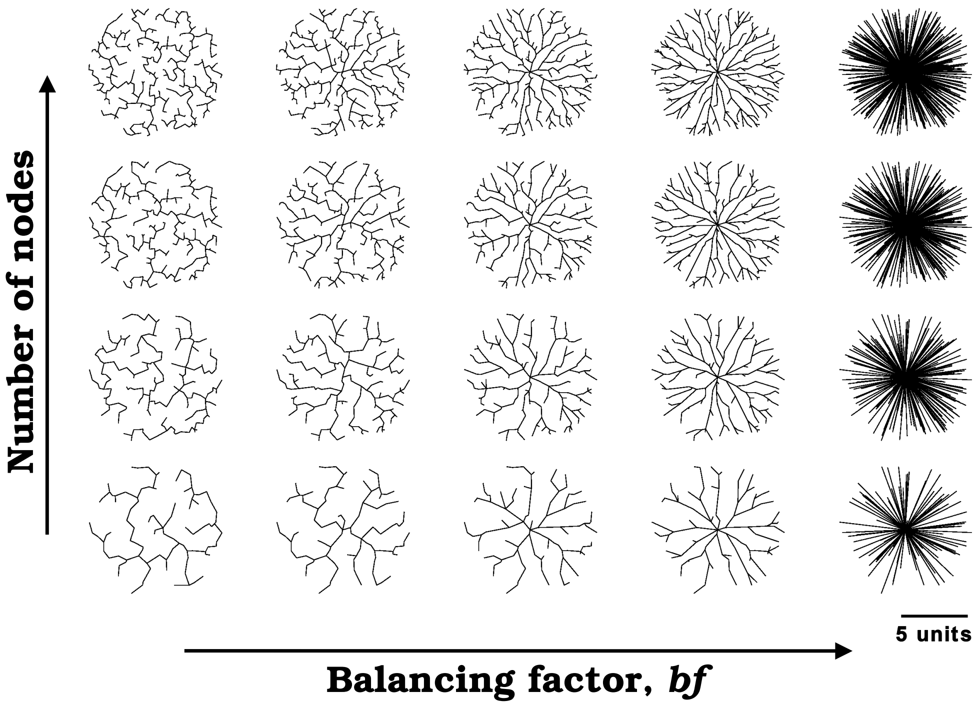



Figure 2: Example of networks created by randomly distributing nodes in a circular substrate of radius 5 units. Plotted as a function of the number of initial nodes/members $N$ = 100, 200, 300, and 400 (bottom to top) and balancing factor $bf$ = 0.0, 0.25, 0.5, 0.75, and 1.0 (left to right). The formed networks range from having nearest neighbor connections only, to a stellate structure where each node is directly connected to the root node.

To verify that the structure of the formed dendritic networks are comparable to a typical MLM network, we utilize their associated $M \times M$ adjacency matrices $A$ and compare their morphological features. Matrix $A$ is a connectivity matrix with elements $a_{ij} = 1$ if two nodes are connected and $a_{ij} = 0$ otherwise, for $i, j \in \{1, 2, \ldots, M\}$. We do the comparison by first representing $A$ in terms of its eigendecomposition $A = \Gamma\Lambda\Gamma^{-1}$ where $\Lambda = \text{diag}\,(\lambda_1, \lambda_2, \ldots, \lambda_M)$ is the diagonal matrix of eigenvalues $\lambda_k$ and $\Gamma = (\gamma_1, \gamma_2, \ldots, \gamma_M)$ is the matrix of eigenvectors $\gamma_k$. We then quantify the morphological signature based on the adjacency spectrum $\sigma$ defined as the set of ordered eigenvalues $\sigma = \{\lambda_1, \lambda_2, \ldots, \lambda_M\}$ with ordering $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_M$.

We show in Fig. 3 the spectrum of dendritic networks, formed using $bf = \{0.0, 0.5, 0.8\}$, and an actual MLM network both having 2122 nodes. We see that independent of the balancing factor $bf$ used, the spectrum of the dendritic network follows the trend (no perfect matching) of an actual MLM validating the consistency of our claim that we can indeed compare the two network types. Note that the data for the dendritic networks are averages of 20 different realizations while we only have one dataset for the actual MLM. We conjecture that as we increase the number of datasets for the MLM network, the curve will eventually follow a smooth trend and our stated claim may become more apparent.

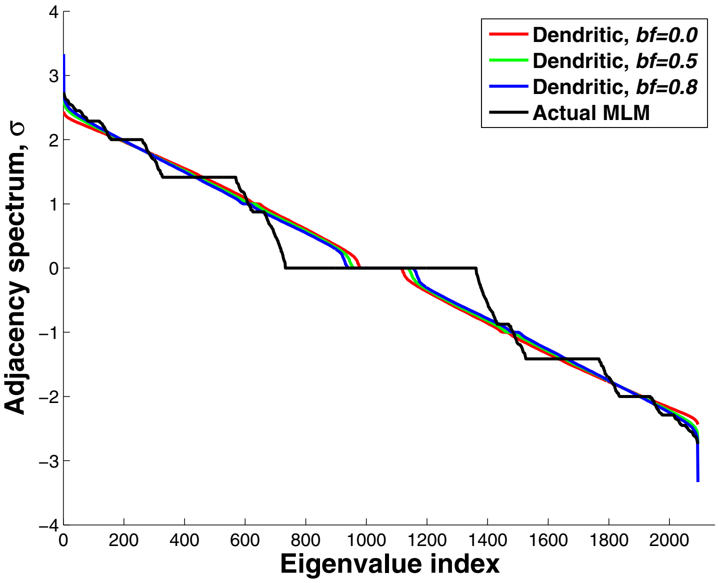



Figure 3: Spectrum of dendritic and actual MLM networks. The dendritic networks were created using $bf = 0.0$, 0.5, and 0.8.

From the results of Fig. 3, we speculate that in terms of an MLM framework, the wiring cost $d$ can be thought as the vulnerability of potential recruits to be recruited. Values of $d$ closer to 1 mean that a new recruit has a higher probability to be recruited by nodes within a shorter distance. We stress that the distance can be the nodes' actual geographical separation and/or social affinity. The path length cost $pl$, on the other hand, can be regarded as the diffusion of influence from top members of the network. Values of $pl$ closer to 1 mean that potential recruits want to be hierarchically close to the top node as they want to mimic their success and maximize their profit. Finally, the balancing factor $bf$ can be thought of as the inherent ability of a potential recruit to choose between which cost, either $d$ or $pl$, will have a dominant influence in his/her decision to join the MLM network. For this case, our network only considers homogenous agents, thus having a uniform $bf$. Another possible interpretation for $bf$ is the ability of the MLM company's management (here represented as the root node) to control the growth of the network. The management can choose lower $bf$ values if they want the network to diffuse locally, or higher $bf$ values if they want to have direct control over majority of the potential recruits.

In terms of the network topology, we can imagine the dendritic network to be of unilevel structure without the need for post-processing since a node cannot be simultaneously connected to two nodes existing in the network prior to its connection time. On the other hand, a binary structure is just a reorientation of the formed unilevel structure wherein a member does strategic repositioning to maintain lateral symmetry for the MLM network. Examples of the conversion of a created dendritic network using $bf$ = 0.0, 0.35, 0.75, and 1.0 to a unilevel and binary MLM structure are shown in Fig. 4.

## 3. Results and Discussion

### *3.1. MLM limited growth distribution features*

The profitability of an MLM enterprise for a new recruit can be gauged by looking at the number of individuals that can be a potential recruit once he/she joins at level $m$, since most of the income earned in our model is proportional to the members below his/her portfolio. Figure 5 shows example average member distributions, as a function of level $m$, of a unilevel MLM network for different network sizes $N$. In general, for any value of $N$, membership increases almost linearly until a peak level, after which growth starts to decline nonlinearly until $f(m)$ approaches 0. Interestingly, the peak level shifts to a higher value as network size $N$ increases. This is due to the fact that more members can be accommodated at lower levels because the substrate becomes dense and there is a higher probability for nodes to be located closer to the root node (at level $m = 1$ and spatially at the center of the substrate). The same trend is observed for a binary network structure (see Fig. 6).

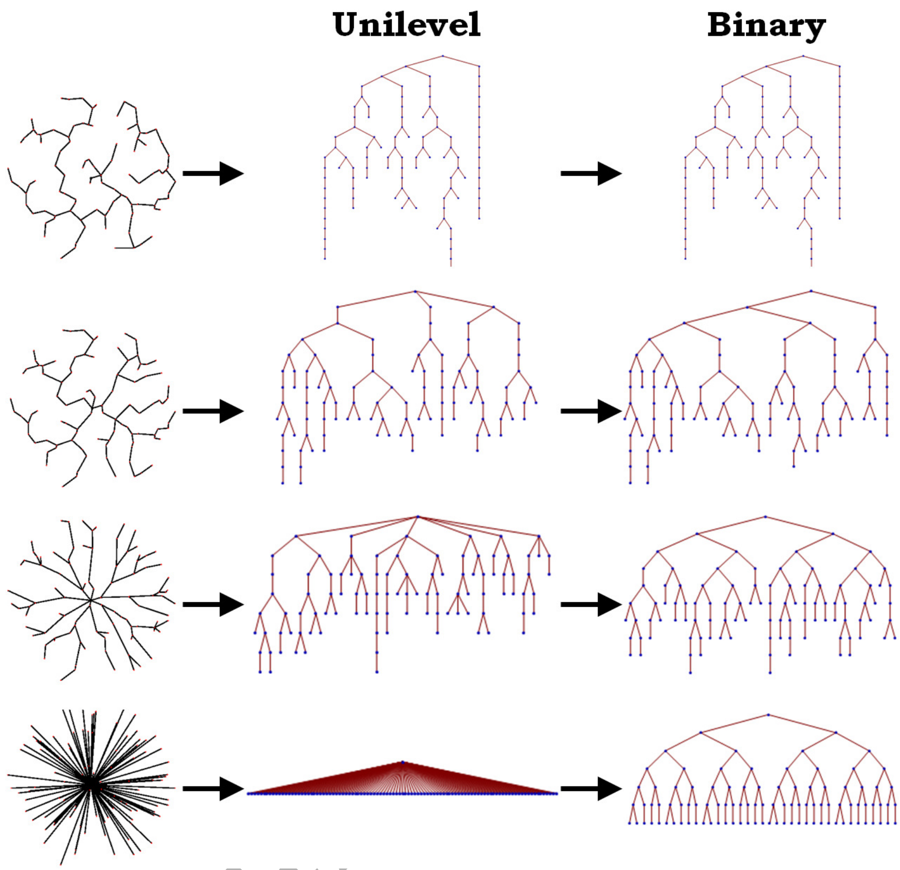



Figure 4: Conversion of dendritic networks to unilevel and binary MLM structures. Rows starting from the top show the dendritic networks formed using $bf$ = 0.0, 0.35, 0.75, and 1.0, respectively, and their corresponding representations/conversions as unilevel and binary MLM structures.

To establish the accuracy of our results, we transform the formed dendritic networks to a binary structured MLM (as discussed in Sec. 2.2) and compare it with an actual data from a real-world binary structured MLM company (*Legacy Philippines*) having $N = 2122$ members. As a proof of the model's reliability, we also compare it with the preferential attachment and branching process models developed by [11]. In the preferential attachment model, each member who joined at a time $t_i$ can recruit a new member at a later time $t$ with probability $\exp[-\beta(t - t_i)]$, where $\beta$ is a free parameter serving as a

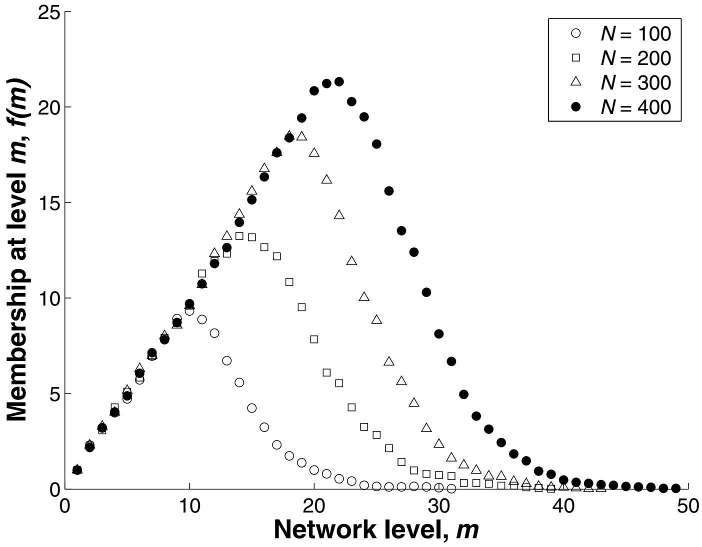



Figure 5: Average distribution of members among levels. The network sizes used are $N = 100$, 200, 300, and 400; while the balancing factor is fixed at $bf = 0.05$. For each network size $N$, there exists a peak level where the number of members start to decrease thereafter.

decay constant. On the other hand, the analytic branching process model established that the number of members at level $m$, $f(m)$, approximately follows the dynamical equation

$$\frac{\mathrm{d}f(m)}{\mathrm{d}N} = \frac{\lambda}{N}\frac{2f(m-1) - f(m)}{N+1}, \tag{5}$$

where $\lambda/N$ is the fraction of members responsible for a new recruit situated at level $m$. In this study, we adopt the parameters $\beta = 0.0085$ and $\lambda/N = 0.0068$ which [11] have shown to be the values that can best demonstrate the recruitment mechanism of real-world MLMs.

Figure 6 visually demonstrates that our dendritic growth model accurately captures the membership distribution of a real-world MLM and is better than existing MLM models especially in the attainment of the peak level. The peak level is very important because it suggests that after such level, the company will no longer flourish and will start its decline. Since income earned by members partially rely on percentage of sales of members below their level within a maximum depth, scarcity of members in levels after the peak level means less income for members in those levels.

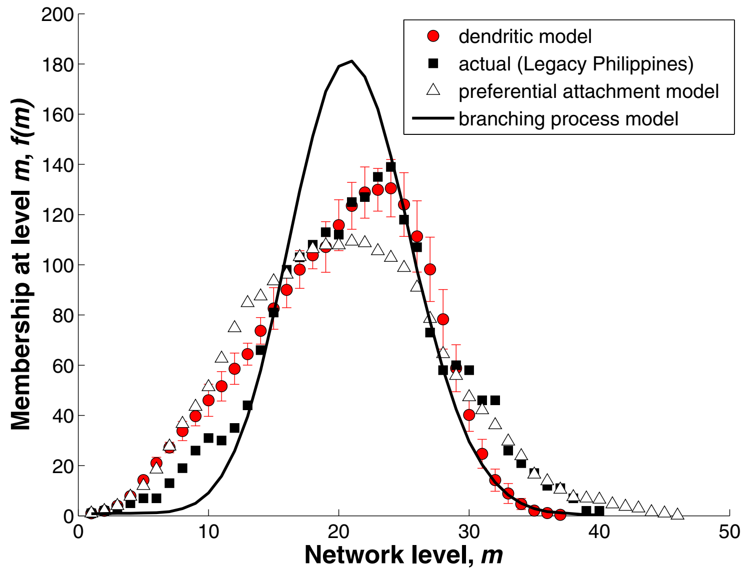



Figure 6: Network cross-section of a binary MLM structure. Filled circles represent mean data of 30 different realizations from simulation of a dendritic network with $bf$ value of 0.723. Simulation results from agent-based modeling with preferential attachment rule using $\beta = 0.0085$ (unfilled triangles) and analytic fit derived from branching process theory using $\lambda/N = 0.0068$ (solid line) are shown for comparison. An empirical data (filled squares) from a real-world MLM (Legacy Philippines) is also plotted.

Using existing data from a real MLM company, we statistically compare our model using Linfoot's criteria that measures the closeness of the empirical values $\{\Phi_{th}\}$ with the model results $\{\Phi_{model}\} = \{f(m)\}$ based on fidelity $F$, structural content $C$, and correlation quality $Q$ [14] given by

$$F = 1 - \frac{\langle(\Phi_{th} - \Phi_{model})^2\rangle}{\langle\Phi_{th}^2\rangle}, \quad (6)$$

$$C = \frac{\langle\Phi_{model}^2\rangle}{\langle\Phi_{th}^2\rangle}, \quad (7)$$

$$Q = \frac{\langle\Phi_{th}\Phi_{model}\rangle}{\langle\Phi_{th}^2\rangle}, \quad (8)$$

where $\langle\cdot\rangle$ is an average carried over the entire network level $m$. Statistically, $F$ measures the general similarity and is the normalized mean square error subtracted from 1, $C$ provides an estimate of the relative sharpness of the peaks, and $Q$ compares the alignment of peaks and troughs of $\{\Phi_{model}\}$ and $\{\Phi_{th}\}$. The model is identical to empirical data when exactly $F = C = Q = 1$. Note also that the relationship of the three quality measures is given by $F = 2Q - C$. Table 1 indicates that our fitted dendritic model is better aligned to the peaks and captures more precisely the spread of the actual data as compared to the branching process or agent based model results.

Table 1: *Quantitative comparison of different models. Comparison of the dendritic network model ($bf = 0.723$) with agent based preferential attachment model ($\beta = 0.0085$) and analytic branching process approximation ($\lambda/N = 0.0085$) using Linfoot's criteria.*

| | $F$ | $C$ | $Q$ |
|---|---|---|---|
| Dendritic model | 0.96 | 1.00 | 0.98 |
| Preferential attachment model | 0.95 | 0.94 | 0.94 |
| Analytic branching process approximation | 0.88 | 1.42 | 1.15 |

### *3.2. MLM income distribution*

The income of each member of a level is just the total income of the level divided equally among all members of the level assuming that each member sells the same volume of products. We plot in Fig. 7(a) the income distribution for $bf = 0.0$. The result shows a non-monotonically decreasing distribution where earnings of members belonging to the lowest levels become minimal as compared to the ones on top. This result corroborates with the shape of the membership distribution of the MLM network. We have shown that the number of members starts to decrease after reaching a peak level. This means that members belonging to levels after the peak level will have potentially fewer downlines. Since a member's earning is proportional to the number of members in his/her downlines, fewer downlines translate to lower income gain.

We also represent the income distribution among members by dividing the membership into five separate groups. Group membership will depend on the member index order such that the first group will comprise the first 20% of the membership with count starting from the first level; while the 2nd, 3rd, 4th, and 5th groups will comprise the second, third, fourth, and last 20% of the membership, respectively. The income of each member of the group is added and normalized with respect to the total income of the entire network to determine the normalized cumulative total income of each group.

By doing such a partition, equitability patterns are clearer. As an example, we show in Fig. 7(b) the partition scheme implemented for a network formed using $bf = 0.0$ that results to an almost uniform income distribution between groups.

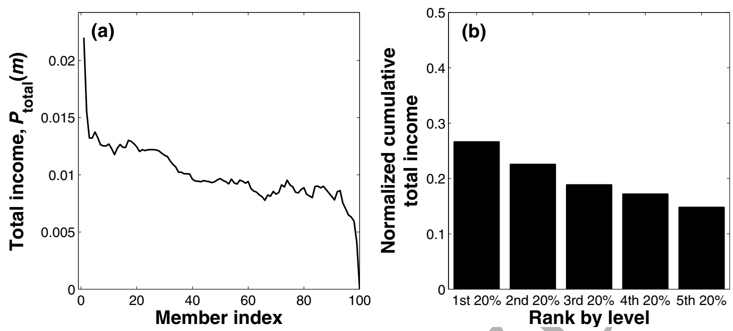



Figure 7: Income distribution among members for $bf = 0.0$. In (a), income of each member is indicated according to their level position in the network such that the member at $m = 1$ will be indexed 1 and so on. In (b), membership is divided into five groups such that the first group has the first 20% of the members ranked according to the member index and so on.

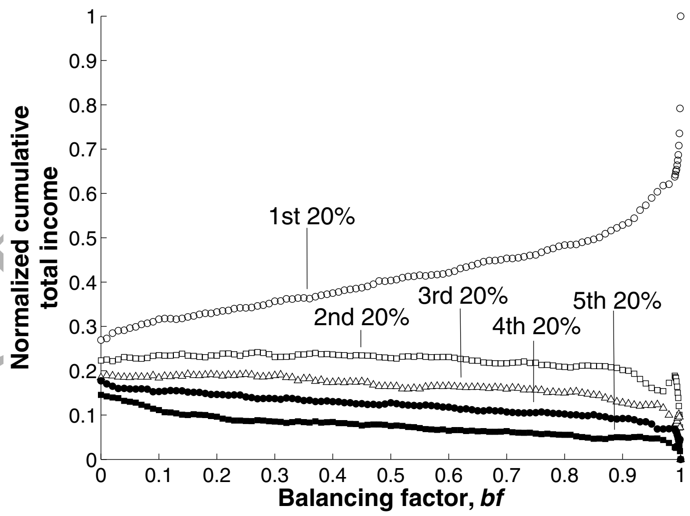



Figure 8: Earnings of the five clustered sub-groups. Each plotted with respect to the balancing factor $bf$.

The choice of clustering our members into five ranks allows direct verification of the 80-20 Pareto principle, a standard qualitative measure for income distribution equality [15]. The network follows the Pareto principle if 80% of the total income of the entire network is earned by 20% of the network members. We show in Fig. 8 that increasing $bf$ from 0.0 to 1.0 will change the landscape of income distribution in the network from nearly uniform to Pareto principle governed ($bf = 0.999$) to "winner-take-all" type ($bf = 1.0$). As the network evolves towards a perfectly stellar structure (all nodes connected to the root node), the income earned by the 1st 20% of the members of the network tends to increase until all the income is gained only by the root node. This is consistent with studies showing a great disparity in profit earnings between level groups of real MLM companies [16] and resembles the fraudulent earning potential in pyramid schemes [17].

### *3.3. Effect of random node removals*

We also characterize our system by finding the size of the giant component $S$, the largest number of nodes still connected, in the network upon random node removals [18, 19]. Figure 9 shows that the dendritic network can be made more robust to random node deletion by increasing $bf$. Node removal to an MLM network effectively models an agent's tendency to stop recruiting and/or quit from the enterprise, as observed in real sales companies [20]. We apply two rules to govern these dynamics: ($i$) the root node at level $m = 1$ cannot be removed; and ($ii$) removal of a node will nullify the contribution of the downlines to the total income. Note that the second rule is a simple approximation and may not perfectly adhere to all practices adopted by MLM companies. Fig. 10 shows that earnings stabilize to a fixed value after a certain fraction of nodes (in the representative figure, $\alpha \sim 0.4$) have been removed.

Consistent with what has been observed in Fig. 8, Fig. 11 shows that the top 20% members of the network will have a bigger income share as $bf$ increases. Interestingly, there exists a particular $bf$ value ($\sim 0.14$) that the Pareto principle is obeyed and this is observed over an extended portion in reference to the fraction of nodes removed ($0.5 < \alpha < 1.0$). This result shows that robust Pareto-governed income distribution exists that is deemed to be too unfair because the first 20% members will surely hold 80% of the network's total income even if members start to leave.

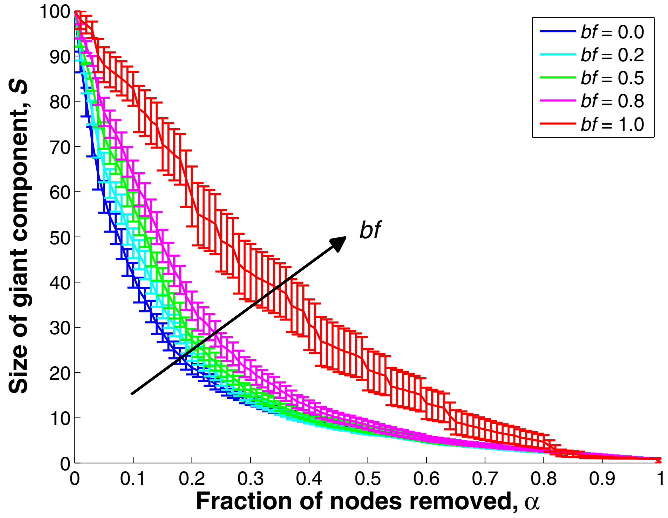



Figure 9: Size of giant component $S$ as a function of fraction of nodes removed $\alpha$. Data points are averages of 30 different realizations. Arrow points to increasing $bf$.

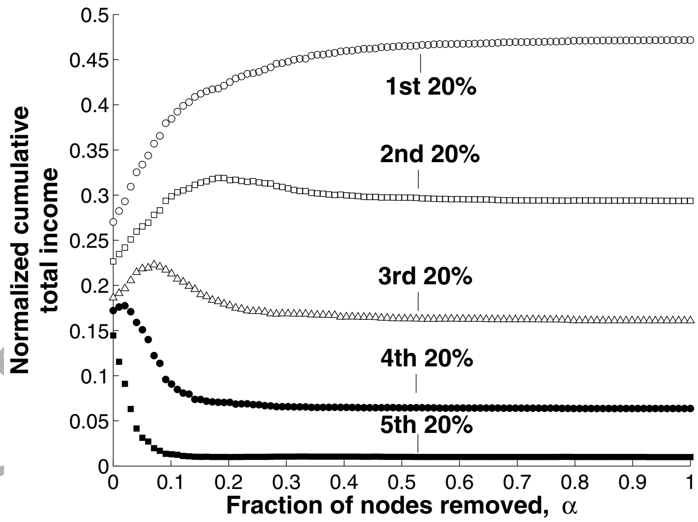



Figure 10: Earnings of the five sub-groups based on fraction of nodes removed. Income earned by each group is plotted with respect to the fraction of nodes removed in the network. Data points are averages of 30 different realizations.

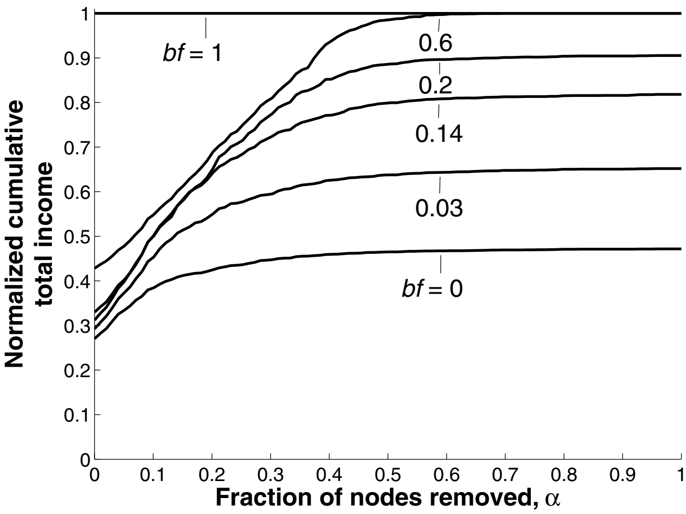



Figure 11: Earnings of the first 20% members as members start to leave. The earnings are plotted with respect to the fraction of nodes removed for various $bf$ values of 0.0, 0.03, 0.14, 0.20, 0.60 and 1.0. Data points are averages of 30 different realizations.

*3.4. Gini coefficient calculation*

The equitability of the network can be qualitatively compared using the 80-20 Pareto principle as shown in the previous subsections. However, the principle cannot quantitatively compare the income equality for different network structures depending on $bf$. We use a measurement for income equality based on the general Gini coefficient $G$ [21] defined as

$$G = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} |x_i - x_j|}{2n^2 \mu}, \tag{9}$$

where $x_k$ is the income of $k$, $n$ is the number of participants to which income equality is measured, and $\mu$ is the mean income of all participants. The Gini coefficient will have values between 0 and 1. A lower Gini coefficient indicates a more equal income distribution, with 0 deemed to be of perfect equality, while higher Gini coefficients indicate more unequal income distribution, with 1 corresponding to perfect inequality.

Following that of the five group network partition, the Gini coefficient is used to compare the income of each group. Note however that the upper bound Gini coefficient of $G = 1$ will only happen for comparison of very large

$n$. Since only five groups are compared ($n = 5$), the upper bound for perfect inequality can be easily derived to change from 1 to 0.8.

We show in Fig. 12 that $G$ increases as $bf$ increases consistent with Fig. 8 and our previous discussions. This further demonstrates that our generic parameter $bf$ provides a wide range of equitability, from an almost perfectly equal network to a perfectly unequal network. Moreover, the sustained value of $G$ as more nodes are removed as shown in Fig. 13 is in agreement with the observed income dynamics in Figs 10 and 11.

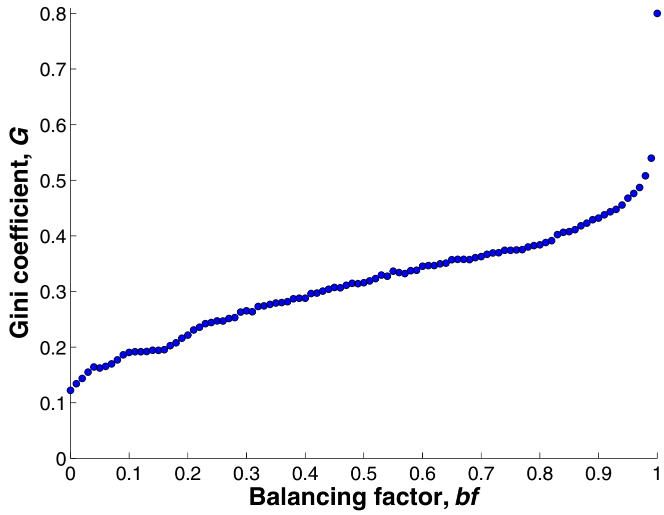



Figure 12: Gini coefficient $G$ for different balancing factors $bf$. The Gini coefficient is calculated for a wide range of $bf$ values from 0.0 to 1.0 preserving all nodes in the network (no node removal).

## 4. Conclusions

We have described a procedure that models a multilevel marketing scheme based on a dendritic growth mechanism. Our network evolves by following a branching process governed by a parameter $bf$ that balances the wiring and path length costs. We have shown that such dendritic dynamics can create network types with structures having similar morphological signature and capture the essential features of an MLM such as: ($i$) the *limited growth* distribution as exhibited by peak levels independent of network size and $bf$

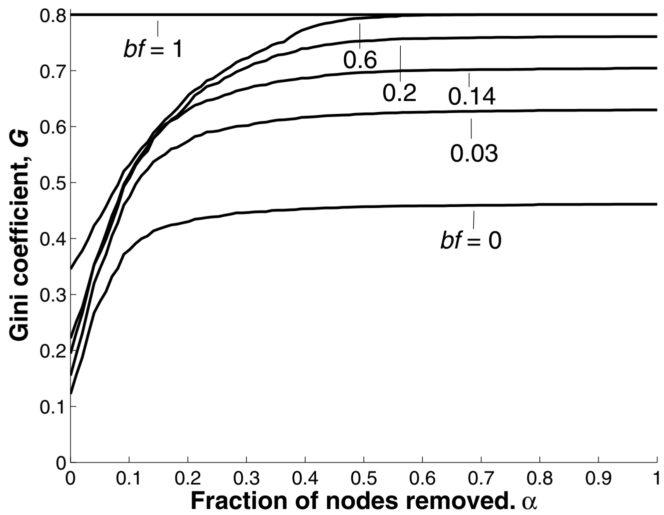



Figure 13: Gini coefficient $G$ as random members start to leave. The Gini coefficient is calculated for different $bf$ values of 0.0, 0.03, 0.14, 0.2, 0.6, and 1.0 with respect to the fraction of the network's nodes removed.

value; and (*ii*) a wide range of possible distribution of income from uniform to Pareto-governed.

Using an empirical data set taken from an actual MLM company in the Philippines, we have demonstrated that such procedure provides a better fit more accurate than the existing agent based preferential attachment and the analytic branching process models. We have also highlighted the rich type of network behavior that results from scanning the generic parameter $bf$, the sole driver of our network's dynamics. We have shown that higher $bf$ values translate to total income diverted to top members (members with ranks closer to the root node) of the network. However, we have to note that we have no firm interpretation of what the $bf$ parameter is in a real MLM company. Nonetheless, we have speculations of its implication such as the ability of potential recruits to choose which cost factor ($d$ or $pl$) has a dominant effect on their decision to join the network; or the management's ability to control the growth of the network. If the latter is true, we have shown in our analyses that the management can easily change $bf$ to manipulate the income distribution to their benefit. From another perspective, government agencies can potentially regulate an MLM company by ensuring that the management adjusts the network's $bf$ appropriately, so that the company maintains a fair income distribution among members. These are interesting insights that need to be verified in future researches.

Finally, we have shown that our dendritic network is robust to random node removals such that the signature income distributions of the different network types are sustained even with substantial stochastic agent removals. Other mechanisms for node removals such as targeted attacks can be explored in the future. Our work provides a groundwork in which the profitability and the equality of earnings among members of an MLM scheme can be evaluated using a biological paradigm.

**Acknowledgment**

CM is supported by Singapore A* SERC "Complex Systems Programme" research grant (1224504056).

**References**

[1] Muniz, R.T.. Biologically based neural network for mobile robot navigation. In: Mobile Robots XIII and Intelligent Transporation Systems, SPIE Proceedings. 1999, p. 285.

[2] Bergerson, K., Wunsch, D.. Neural Networks in Finance and Investing. Chicago: Probus Publishing Company; 1993.

[3] Greensmith, J.. The dendritic cell algorithm. Ph.D. thesis; School of Computer Science, University of Nottingham; 2007.

[4] Monterola, C., Lim, M., Garcia, J., Saloma, C.. Accurate forecasting of the undecided population in a public opinion poll. J of Forecasting 2002;21(6):435–49.

[5] Monterola, C., Saloma, C.. Solving the nonlinear Schrodinger equation with an unsupervised neural network. Opt Express 2001;9(2):72–84.

[6] Monterola, C., Saloma, C.. Solving the nonlinear Schrodinger equation: Estimation of error in solution. Opt Commun 2003;222:331–9.

[7] Monterola, C., Roxas, R., Carreon-Monterola, S.. Characterizing the effect of seating arrangement on classroom learning using neural networks. Complexity 2009;14(4):26–33.

[8] Koehn, D.. Ethical issues connected with multilevel marketing schemes. J Bus Ethics 2001;29:153–60.

[9] Albaum, G., Peterson, R.. Multilevel (network) marketing: An objective view. Market Rev 2011;11(4):347–61.

[10] Sparks, J., Schenk, J.. Socialization communication, organizational citizenship behaviors, and sales in a multilevel marketing organization. Journal of Personal Selling and Sales Management 2006;26(2):161–80.

[11] Legara, E., Monterola, C., Juanico, D., Litong-Palima, M., Saloma, C.. Earning potential in multilevel marketing enterprises. Physica A 2008;387(19-20):4889–95.

[12] Daquis, J., Castañeda, A., Sy, N., Abgona, R.. Profitability and growth topology analysis of unilevel-type of network marketing structures. The Philippine Statistician 2013;62(2):13–29.

[13] Cuntz, H., Forstner, F., Borst, A., Häusser, M.. One rule to grow them all: a general theory of neuronal branching and its practical application. PLoS Comput Biol 2010;6(8):e1000877.

[14] Huck, F., Fales, C., Halyo, N., Samms, R., Stacy, K.. Image gathering and processing: information and fidelity. J Opt Soc Am A 1985;2(10):1644–66.

[15] Anschuetz, N.. Profiting from the '80-20 rule of thumb'. J Adverstising Res 1997;37(6):51–6.

[16] Fitzpatrick, R.. The myth of MLM income opportunity. Available at: http://pyramidschemealert.org/PSAMain/resources/MythReport.html; 2008.

[17] Gastwirth, J., Bhattacharya, P.. Two probability models of pyramid or chain letter schemes demonstrating that their promotional claims are unreliable. Oper Res 1984;32(3):527–36.

[18] Cohen, R., Erez, K., ben Avraham, D., Havlin, S.. Resilience of the internet to random breakdowns. Phys Rev Lett 2000;85(21):4626–8.

[19] Callaway, D., Newman, M., Strogatz, S., Watts, D.. Network robustness and fragility: Percolation on random graphs. Phys Rev Lett 2000;85(25):5468–71.

[20] Verma, R., Aanchal, A.. A study of attrition rate among sales force of life insurance companies in Delhi. Researchers World 2012;3(4):90.

[21] Dixon, P., Weiner, J., Mitchell-Olds, T., Woodley, R.. Bootstrapping the Gini coefficient of inequality. Ecology 1987;68(5):1548–51.