Reliability analysis of offshore wind turbine support structures under extreme ocean environmental loads

Dong Hyawn Kim*, Sang Geun Lee

Department of Ocean Science & Engineering, Kunsan National University, 558 Daehangro, Kunsan, Jeonbuk, 573701, Republic of Korea

**Abstract**

Reliability analysis of jacket type offshore wind turbine (OWT) support structure under extreme ocean environmental loads was performed. Limit state function (LSF) of OWT support structure is defined by using structural dynamic response at mud-line. Then, the dynamic response is expressed as the static response multiplied by peak response factor (PRF). Probabilistic distribution of PRF is found from response time history under design significant wave load. Band limited beta distribution is used for internal friction angle of ground soil. Wind load is obtained in the form of thrust force from commercial code called Bladed and then, applied to tower hub as random load. In numerical example, response surface method (RSM) is used to express LSF of jacket type support structure for 5 MW OWT. Reliability index is found using first order reliability method (FORM).

© 2014 Published by Elsevier Ltd.

1. Introduction

To assure the safety of offshore wind turbine (OWT) support structures under risky environment, it is required to evaluate probability of failure using reliability analysis [1,2]. If a limit state equation in reliability problem is formulated based on static response, it is quite simple and straightforward to evaluate probability of failure or reliability index. However, reliability analysis of support structure, whose response should be obtained from dynamic analysis, is not that easy in terms of analysis time. Basically the dynamics of support structure is coupled with irregular wave, turbulent wind, and nonlinear ground soil. It takes a lot of time in obtaining a set of dynamic response of OWT with long pile-foundation. In addition, the number of dynamic analysis in a reliability analysis is proportional to the square of the number of random variables.

Therefore, most of previous studies have proposed algorithms to reduce the number of simulation time in reliability analysis. Sometimes, algorithms with small number of random variables have been proposed. Peak-Over-Threshold (POT) is a representative approach to reliability analysis of OWT [3]. Only peak values exceeding a threshold are extracted from response time history. Then extreme value distribution is estimated by using the peak values. Block maxima approach is another widely used one [4]. A long dynamic response history is divided into lots of blocks. Then, maximum values are chosen from each block and used to estimate extreme value distribution. However, randomness of design variables such as ground soil properties and structural parameters were still not considered. If the randomness of such variables is considered, extreme value distribution should be calculated for the every single variable at every step of iteration. Then, total simulation time increases geometrically. Therefore, variations only in wind and wave are considered in those studies.

In this study, a new approach to reliability analysis of OWT support structure under dynamic load is proposed. Dynamic peak response is estimated by using static response and a factor accounting dynamic amplification. Since the static response is used during reliability analysis, much less computational cost is required than using dynamic response. Jacket type support structure for 5 MW OWT is used for numerical example of the approach.

2. Reliability analysis of support structures

2.1. Reliability analysis using peak response factor

Dynamic response of a support structure is dependent on such design variables as mechanical properties of support structures, ground materials, and even design loads. They all should be treated as random variables in reliability analysis of support structures. Then, a limit state equation for support structure can be defined as
where $R_{\text{all}}$ is the allowable response of the structure; $R_p$ is the maximum peak response due to design wave and wind load; $X$ is the design random variable. PDF of $R_p$ is dependent upon design variables ($X$) such as ground soil stiffness, wind speed, etc. Therefore, different PDFs for each $X$ can be drawn as in Fig. 1.

The joint PDF for $R_p$ and $X$ can be found by multiplying them as

$$f_{R_p,X}(r_p,X) = f_{R_p|X}(r_p|X)f_X(X)$$

(2)

Fig. 2 shows the contour for $f_{R_p,X}(r_p,X)$. The hatched area is the region where the limit state function becomes negative, which means failure in the reliability analysis. Using the failure region, probability of failure can be calculated as

$$P_f = \int_{g < 0} f_{R_p,X}(r_p,X)dr_pdx = \int_{-\infty}^{R_{\text{all}}} \int_{-\infty}^{g(r_p,X)} f_{R_p|X}(r_p|X)f_X(X)dr_pdx$$

(3)

As can be seen in Fig. 2, calculation of $P_f$ is quite difficult since the failure region is skewed to the PDF. To get $P_f$ easier, a new random variable called peak response factor (PRF) is introduced as follows.

$$R_0 = \frac{R_p}{R_{\text{all}}}$$

(4)

where $R_0$ is the static response under design condition. Of course, $R_0$ is the variable dependent on such parameters as ground properties and wind, wave load. Eq. (4) is introduced in this study to utilize the idea that dynamic peak response might be proportional to static response if the dynamic properties of support structure doesn’t change that much. Using eq. (4), the limit state equation can be rewritten as

$$g(X) = R_{\text{all}} - R_pR_0(X)$$

(5)

$R_0$ is a function forcing frequencies and the natural frequencies. But for small change of natural frequencies, it can be assumed to be constant with acceptable error. Then, $R_0$ can be treated as independent of $X$, and level II type of reliability analysis such as FORM can be easily applied. Fig. 3 shows the flowchart of the approach combining a special purpose code such as Bladed to find thrust force distribution and a general structural analysis code for reliability analysis.

### 2.2. Distribution of peak response

This section describes how to find distribution function for peak response of support structure. There are numerous peak responses during wave and wind loading. Among them, it is important to take significant ones from a structural point of view to form distribution function. There are several approaches to obtain distribution for the peak response. Widely used is block maxima method. In this method, long time history response is divided into several block and those peak responses exceeding a threshold value are gathered to estimate PDF. The other method is the so-called Peak-Over-Threshold (POT) method. In POT method, all peak responses exceeding a pre-defined threshold value are taken to find distribution. Fig. 4 shows extreme value sampling example by the two methods. In block maxima, some significant peak responses might be lost during sampling from each block. This can be made up for by decreasing the unit block size. Distribution function from POT method is dependent on the threshold value. But there exists some useful criteria to pre-set relevant value for the threshold [5].

### 2.3. Design wave load

Using Morison equation, the dynamic fluid force on moving cylinder can be formulated as

$$f_W = \frac{1}{2} \rho_w CD \left(U - \dot{r} \right)(U - \dot{r}) + C_M \rho_w A \frac{\partial U}{\partial t}$$

(6)

where $CD$ and $CM$ denote the drag coefficient and inertia coefficient, respectively; $\rho_w$ denotes water density; $A$ projected area normal to the cylinder axis per unit length; $D$ the effective diameter of circular cylindrical member including marine growth, $U$ the component of the velocity vector of the water normal to the axis of the member.

To obtain the static response of support structure used in eq. (4), virtual static force should be defined. In static analysis, the structural motion like $\dot{r}$ in eq. (6) cannot be used. Therefore, the structural velocity term in eq. (6) is neglected. In addition, the maximum water particle motion can be obtained by applying Airy’s theory with sine and cosine is set to below [6]

$$\sin \theta_0 = \pm \frac{\pi DC_M}{HC_D} \frac{2 \sin h^2/(kd)}{2kd + \sin h(2kd)}$$

(7)
\[
\cos \theta_0 = \pm \sqrt{1 - \sin^2 \theta_0} \tag{8}
\]

Then, the static force which should be used in reliability analysis can be calculated using eqs. (6)-(8) by neglecting structural motion. The static load is a virtual load to support structure to get reference static response, \( R_{st} \).

2.4. \( P-y \) curves for ground soils

Ground soil exerts nonlinear reaction force to foundation structure under dynamic loading and usually modeled by soil spring using the so called \( P-y \) curves. API suggests how to make soil springs as follows [7]

\[
p_u = \min \left( \frac{p_{us}}{p_{0}}, \frac{p_{ud}}{p_{0}} \right)
\tag{9}
\]

where \( p_u \) denotes the ultimate strength of ground soil; \( s \) and \( d \) denote shallow and deep ground, respectively; \( C_i \)'s are coefficients from API code; \( p_0 \) the effective overburden pressure; \( D \) the diameter of foundation; \( \gamma \) the depth below mud-line. Nonlinear \( P-y \) relation is modeled as follows in API.

\[
P = A p_u \tan h \left( \frac{k \gamma}{A p_u} y \right)
\tag{10}
\]

where \( A \) is the factor for cyclic or static loading; \( k \) the initial modulus of subgrade reaction, which is a function of internal friction angle; and \( y \) the lateral displacement.
2.5. Response surface method

To find reliability index of a limit state, dependent variables in the form of implicit function of random variables, such as \( R_n(X) \) in eq. (5), should be expressed as explicit form. This can be done by using Response Surface Method (RSM) which was suggested by Scheuller et al. [8]. In obtaining response surface, it is required to accomplish structural analysis as little as possible with acceptable error margin. Design points at which a structural analysis is done is determined by following equation [9]:

\[
X_i = X_i^f \pm h_i \sigma_{X_i} \tag{11}
\]

where \( X_i^f \) is the center point at each iteration step; \( \sigma_{X_i} \) is the standard deviation of \( X_i \); \( h_i \) is the width; \( i \) is the scattering index; \( i \) is the variable number. \( f \) determines where to do structural analysis around \( X_i^f \). Fig. 5 shows design points for structural analysis when saturated design scheme is used.

3. Numerical example

3.1. Verification using SDOF model

To verify the proposed approach, reliability analysis of SDOF model is shown first. The mass is assumed to be 1085.156 ton which is the same as full model of OWT, and the stiffness of the verification model was tuned to be and 3.059 MN/m in order to get natural frequency of 0.267 Hz. Damping ratio was assumed to be 1%. Randomness is assumed only for the stiffness which is normal distribution with COV of 0.1. Significant wave height is 7.9 m, peak wave period is 15.19 sec. Irregular waves generated by using Jonswap spectrum were applied to SDOF turbine model. Then, dynamic response was analyzed by Morison equation. Verification was done by comparing the results from FORM based proposed approach and MCS based conventional approach. Limit state equation of each approach is as follows.

\[
\xi_{\text{FORM}} = D_{\text{all}} - R_n R_d \tag{12}
\]

\[
\xi_{\text{MCS}} = D_{\text{all}} - R_n R_d \tag{13}
\]

Allowable displacements at mud-line (\( D_{\text{all}} \)) in the both limit state equations are 0.038 m. In MCS, peak responses are used to count failure cases in which response exceeds \( D_{\text{all}} \). Six thousand of 10-min dynamic analyses were done on MCS. But in the proposed approach, only six times of 10-min transient analyses were done to get distribution for \( D_n \) whose scale and shape parameters are 0.8033 and 2.8279, respectively. Then, static analysis based FORM is applied to find reliability index. Comparison results of the two approaches are listed in Table 1. There seems small difference in reliability index which may include intrinsic error of FORM and the error caused by the approximation of peak response to the multiplication of static response by PRF. Though there is small error, the proposed approach gives good promise in calculation time spent on reliability analysis of offshore wind turbine.

3.2. Application to full scale 5 MW wind turbine model

3.2.1. Support structure and ground condition

For numerical example, a jacket type support structure for 5 MW offshore wind turbine is used. Fig. 6 shows geometry of the jacket structure.

It has four legs and is planned to be installed into ground using pin piles. It was originally designed for a wind farm at Southwest coast of Korea. Each mass for RNA [10], tower, and support structure is 350 ton, 391 ton and 265 ton, respectively. Offshore ground is composed of three layers as shown in Fig. 7 and corresponding P-y curves are obtained by using eqs. (9), (10). From eigen value analysis using finite element code [11], it was found that the first two natural frequencies of OWT with support structure are 0.267 Hz and 1.577 Hz, respectively.

3.2.2. Environmental loads

To get wind load distribution, design wind of 42.5 m/s and turbulence intensity of 0.12, which is CLASS II of IEC 61400-1, is used [5]. Using Bladed [12], one hour thrust force to hub was obtained as Fig. 8.

Thrust force is modeled as normal distribution with mean of 0.129 MN and coefficient of variance (COV) of 0.22 MN. To calculate wind load to tower part, wind velocity at which thrust force to hub is calculated can be obtained by using the cumulative distribution functions (CDFs) of both wind velocity and thrust force as follows

\[
v = F_{V}^{-1}(F_{h}) \tag{14}
\]

where \( F_V \) is the CDF for thrust force and \( F_h \) is CDF for wind speed.

Wind force on the tower at the elevation of \( z \) above sea level was calculated by using the following equation.

\[
f_t(z) = \frac{1}{2} \rho_{\text{air}} C_D v^2(z) D(z) \tag{15}
\]

where \( \rho_{\text{air}} \) denotes the air density; \( C_D \) the drag coefficient for tower; \( z \) the elevation from mean sea level. And the other design environmental conditions for the OWT are listed in Table 2.

3.2.3. Dynamic analysis and distribution of peak response

Dynamic analysis of OWT under wind and wave load at design condition was done. Fig. 9 shows a time history for support structure displacement at mud-line during one hour. It takes 13 h and

<table>
<thead>
<tr>
<th>Method</th>
<th>Reliability index, ( \beta )</th>
<th>CPU time (hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed (FORM)</td>
<td>2.563</td>
<td>0.0075</td>
</tr>
<tr>
<td>Conventional (MCS)</td>
<td>2.6139</td>
<td>45.01</td>
</tr>
</tbody>
</table>

Table 1. Reliability analysis results for SDOF model.
30 min to get the one-hour time history on Windows 7 64bit with
CPU Quad Core 3.4 GHz platform. Therefore, it is practically
impossible to find probability of failure by considering all the
random variables used from direct dynamic response of OWT.

Using block maxima method with block size of one minute,
lognormal distribution of peak response was estimated as in Fig. 10.
Fig. 11 shows PDFs for PRF under different environments around
mean design variable. Though design values, \( X \), are changed,
the PDFs remains almost the same. Of course, the peak dynamic
response under bigger wind and smaller ground stiffness will in-
crease. But, the static response will also increase. Therefore,
the change in PRF becomes small. Table 3 shows mean and stan-
dard deviation for the lognormal distributions of Fig. 11 where \( \lambda \) and \( \zeta \)
are the mean and standard deviation of lognormal distribution,
respectively. PRF has very close distribution though the design
variables such as wind load and ground soil parameters are
different.

### 3.2.4. Reliability analysis using FORM

Random variables used in reliability analysis are listed in Table 4.
PRF is assumed to be lognormal. Normal distribution is used for
thrust force and unit weight of ground soils. Especially, beta
distribution with lower and upper limit is used for internal friction
angles of soils [14].

Limit state function is defined as eq. (16) by using allowable
displacement of support structure at sea bed.

\[
g = R_{all} - R_aR_{st}(f_1, \gamma_1, \gamma_2, \gamma_3, \phi_1, \phi_2, \phi_3)
\]  
(16)

\( R_{all} \) was calculated to be 38 mm from AASHTO LRFD Bridge Design
Specification which suggests horizontal foundation displacement
[15].

Using FORM, reliability index (\( \beta \)) was found as in Fig. 12. Since
the beta distributions with upper and lower limit are used for in-
ternal friction angle, convergence is relatively slow. But, after 40
iterations, \( \beta \) converged to 6.0493.

Table 5 shows sensitivity factors and most probable failure
points for the random variables. From the table, one easily knows

![Fig. 7. Soil properties.](image1)

![Fig. 8. Thrust force time history.](image2)

![Fig. 9. Dynamic response of support structure at mud-line.](image3)

![Fig. 10. Lognormal distribution of peak response.](image4)

![Fig. 11. PDFs of normalized peak response under different environmental load.](image5)
the PRF is the most decisive one. Next important variable was wind load. The unit weights of soil and internal friction angles were found to be little sensitive ones except the surface layer friction angle, $\phi_3$.

### 3.3. Limitation

$R_n$ is assumed to be constant for OWT support structure in eq. (5). This is shown numerically in Fig. 11. However, it is still not proven for the structure with high nonlinear property. In that case, right tail of PDF for $R_n$ may be longer and peak of PDF would be smaller due to nonlinear response. Therefore, this approach should be carefully applied after identifying the level of nonlinearity of the structure concerned.

### 4. Conclusions

A static analysis based reliability analysis of OWT support structure under dynamic loads was proposed. The proposed approach defines limit state function by using dynamic response of OWT. However, the dynamic response is not directly analyzed but estimated by multiplying static response and PRF during reliability analysis. Then, total computational effort can be shortened very much. This is because that the distribution of PRF remains almost the same with little but acceptable error though other environmental conditions change. In numerical example, the proposed approach is applied to 5 MW OWF support structure. Using the random variables such as unit weight of soil, internal friction angle of soil, wind and wave load, reliability index for the limit state of horizontal displacement of foundation was found.

### Acknowledgment

This work was supported by the Human Resources Development of the Korea Institute of Energy Technology Evaluation and Planning (KETEP) grant funded by the Korea government Ministry of Knowledge Economy (No. 20144030200590).

### References


