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A two-step hybrid investment strategy for pension funds ☆

Bernardo K. Pagnoncelli [a,*], Arturo Cifuentes [b,c], Gabriela Denis [d]

[a] Escuela de Negocios, Universidad Adolfo Ibáñez, Diagonal las Torres 2640 oficina 533-C, Peñalolén, Santiago, Chile
[b] Columbia Business School, Finance Division, New York, United States
[c] CLAPES-UC, Santiago, Chile
[d] Master of Applied Economics UCLA, United States

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ABSTRACT

We propose a two-step hybrid investment strategy suitable for pension funds. Our method consists of an active component (an optimization-based approach to decide the asset allocation), followed by a passive strategy (an index-based approach). We test our strategy with data from the Chilean pension system using two different risk metrics and we show that our approach, in three out of five cases, yields results that are better than those generated by the Chilean fund administrators. In the two cases where our approach underperformed we show that it was the result of excessively tight constraints set up by the regulator.

1. Introduction

Pension funds are important actors in the global capital markets. For example, in the U.S., occupational pension funds (that is, funds sponsored by employers) manage assets equivalent to approximately 80% of the country's GDP (OECD, 2015). A study focused on sixteen major markets including the U.S., U.K., Japan and Canada, estimated that pension funds at the end of 2015 managed on average assets equal to 85% of their respective countries GDP (Watson, 2015). The same study indicated that such funds controlled approximately US$ 36 trillion. By way of comparison, the market capitalization of all U.S. listed companies is close to US$ 20 trillion (World Bank, 2014). Therefore, it is clear that in absolute and relative terms pension funds play a very influential role in the markets in which they operate.

Pension funds face three important challenges in the near and medium future: (1) a persistent low interest rates environment (this is particularly taxing for defined-benefits schemes); (2) a consistent longevity increase; and (3) an acute decline in fertility rates, a problem that affects mostly the pay-as-you-go (PAYG) systems.

From a structural viewpoint, most pension funds share certain common features. They all have mid- to long-term investment horizons; enjoy a rather stable inflow of funds; and invest primarily in equities and bonds. For example, at the end of 2014 U.S. pension funds had 63% of their portfolios invested directly in bonds and stocks (OECD, 2014). The total exposure to these assets was probably slightly higher since pension funds also invest in mutual funds which, in turn, hold these two types of assets. The exposure to alternative assets (such as mortgages, infrastructure loans, private equity, and structured products) was minor compared to the exposure to stocks and bonds. Although in general there has been a global trend to decrease the overall stocks-and-bonds holdings, and increase the positions in alternative assets, stocks and bonds remain

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* Corresponding author.
E-mail addresses: bernardo.pagnoncelli@uai.cl (B.K. Pagnoncelli), aocusa@gmail.com (A. Cifuentes), gaby.denis@gmail.com (G. Denis).

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dominant. Considering the U.S., U.K., Australia, Canada, Japan, the Netherlands, and Switzerland the aggregate exposure of pension funds to stocks and bonds was 73% in 2014 (in 1995 was 89%) (Watson, 2015). Additionally, most pension funds have maximum and minimum constraints in terms of domestic and foreign exposure as well as by asset class. All these considerations put pension funds in a category of their own, as they are quite different from hedge funds, retail-oriented funds, and speculative funds focused on venture capital, commodities and distressed debt.

Consequently, considering the challenges that pension funds face, coupled with their sizes compared to their respective economies, it is apparent that studying the investment strategies of pension funds is a problem of financial and political significance. With that as background, our aim is to present an easy-to-implement investment strategy suitable for pension funds (or funds with mid- to long-term horizons). The strategy is based on a two-step process. Step 1 (active) consists of selecting the appropriate asset allocation proportions; and Step 2 (passive) is based on following an index within each asset class. Rebalancing (changing the asset allocation percentages) is done once a year, and we adopt a data-driven methodology, that is, we work directly with historical data without any assumptions regarding the distribution that originated the observations.

The next section reviews some basic concepts and trends related to investment strategies and risk metrics; then, we present our investment method; and the following section tests the performance of our method using historic data from the Chilean pension system. The final section presents the conclusions of this study.

2. Investment strategies and risk metrics

2.1. Investment strategies

The development of successful investment strategies has preoccupied humans since ancient times. It was only recently however that a solid conceptual framework to study such a problem was articulated by Markowitz (1952). The merit of Markowitz's paper is twofold: first, it demonstrated the importance of diversification; and second, it provided the basis for constructing the best portfolio for a given risk-tolerance level. The shortcomings associated with Markowitz formulation (also known as mean–variance or MV portfolios) are mainly practical than theoretical. They are related to the difficulties encountered when estimating returns and correlations–the basic inputs for solving the MV problem–and the sensitivity of the optimal solution to the MV problem (assets weights) to small variations in the inputs (Kolm, Tütüncü, & Fabozzi, 2014; Michaud, 1989).

In the 90s, Black and Litterman (1991, 1992) proposed a method, the Black-Litterman (BL) model, which attempted to overcome some of the difficulties associated with Markowitz's formulation by incorporating the investor's views into the MV optimization problem. In essence, the BL approach estimates the future returns of the relevant securities by combining the investor's views and the returns implied by the CAPM model (market equilibrium).

Markowitz's approach, including all its variations, has been primarily applied to choosing an optimal portfolio within a specific asset class (e.g. U.S. stocks) rather than selecting the appropriate asset mix (asset allocation) for a given risk-level. Arguably, asset allocation can be more important than selecting the individual assets within a specific asset class when building a diversified portfolio, as stated in Gary, Brinson, Hood, and Beebower (1995) and Tokat (2005).

Another topic that has received considerable attention in the literature is the passive versus active investment conundrum. Active investment refers to a strategy aimed at selecting the winners among a certain asset class (e.g. U.S. investment-grade corporate bonds) to outperform that market. Passive investment refers to a strategy based on mimicking a market index representative of that asset class (e.g. the SPUSCIG index). Sharpe articulated, probably better than anybody, the argument against active management by showing in a clear and concise manner the difficulties associated with beating the market consistently (and after taking fees into account) plus the challenges associated with identifying those managers who can beat the market consistently (assuming they exist) (Gökçen & Yalçn, 2015; Sharpe, 1991). While definitely not all managers are alike, it is not an easy task to distinguish skill from luck (Fama et al., 2010). Furthermore, identifying managers that have done well in the past seems to be a poor predictor of future success. In any event, and despite the claims of professional active managers, the empirical evidence overwhelmingly favors the passive approach (e.g. Malkiel, 2003). On the other hand, the case for active management is typically argued based on "philosophical" considerations rather than hard data. A good–albeit extreme–example is the article by Ellis (2015).

In recent years, there has been a significant shift from active to passive low-fee alternatives (Brodbeck, 2013). For example, in the global stocks segment, the proportion of assets that are actively managed has declined from almost 80%–70% in the last six years. And the top 200 pension funds are increasingly investing in instruments such as indices (Olsen, 2012; Roose, 2013). Overall, active strategies that were preferred in the past are losing ground. A key reason behind this shift–in addition to the inability to outperform the market continuously–is fees. Investors are becoming more sensitive to fees, and active funds charge considerably higher fees simply because they have more expenses: they have bigger staffs, need to fund their research, and rotate their portfolio more often (higher trading costs, Smith, 2015). Moreover, beating the market or an index on a regular basis is very hard or almost impossible to achieve (Sharpe, 1991). Morningstar reported that in the first semester of 2016 only 19% of the managers that invested in large-caps beat the S&P 500, while only 6% of the managers focused on growth stocks outperformed their corresponding benchmarks (McCrumm, 2016). Not surprisingly, active stock funds have lost US$ 35 trillion to passive investment vehicles during the first semester of 2016. In the case of pension funds, which are essentially long-term investments, the current shift to passive strategies will probably be more pronounced in the upcoming years.

Two recent papers explored some ideas related to passive investment. In Walden (2015), the author considered state pension funds in the United States, and compared their performance with three fully passive strategies. The authors showed that, in terms of returns, the vast majorities of states would be better off by switching to low-fee alternatives. Nevertheless, the paper did not address any type of risk control. In Bessler, Opfer, and Wolff (2017), the authors studied the out-of-sample performance of the BL model against alternatives such as the equally-weighted scheme, and a Markowitz portfolio, among many others. Using indices as the investment universe, the overall risk was smaller for portfolios obtained with BL, and it had a better performance than other approaches when the Sharpe ratio was used as metric.

It is not our goal to review the extensive literature on quantitative investment techniques here since other authors have done this very well. Our goal is simply to outline some general ideas to put our method (described in Section 3) in the proper context. We recommend the paper by Kolm et al. (2014) which provides an excellent review of the state of the art in reference to portfolio optimization and quantitative investment techniques plus a comprehensive list of references. The book by Scherer (2012), parts 1 and 2, details recent advances in quantitative portfolio management, while the text by Cornuejols et al. (2006) is oriented to the use of optimization tools in the same context.

2.2. Risk metrics

The variance (of returns) enjoyed wide popularity for several decades as the favorite metric to describe risk. In part, this situation was due to the critical role that the variance played in Markowitz's framework. In the 90s, JPMorgan formally introduced the Value-at-Risk (VaR) concept, even though it had been in used in some circles before the JPMorgan publication (J.P. Morgan, 1996). The VaR is an estimate of the maximum amount (in monetary terms) that a given portfolio might lose within a specific time period. Because of its clear and intuitive interpretation, plus the fact that it is easy to compute when data are available, the VaR became quickly the risk metric of choice, embraced by both investment professionals and regulators; indeed, much of the upcoming Basel III and Solvency II guidelines are based on VaR-related estimates (Bank for International Settlements, 2010; European Commission, 2007). However, despite its popularity, the VaR presents some important practical and conceptual shortcomings. From a practical standpoint, in an optimization context, trying to minimize the VaR while keeping the returns above some threshold results in a nonconvex problem. Computationally, this is challenging for there is no assurance that the algorithm will be able to identify a global optimal solution. Conceptually, and perhaps more serious, the VaR does not satisfy the so-called subadditivity condition. In short, this means that the VaR might suggest that increasing the degree of diversification in a portfolio can actually increase its risk. This, of course, can lead to undesirable investment allocation decisions.

In 1999, Artzner et al. (1999) identified the properties that good risk measures should satisfy, and called the metrics that met these requirements coherent. One of these requirements is the subadditivity condition (violated by the VaR). This situation lead to the development of a better metric: the Conditional Value-at-Risk or CVaR (Rockafellar & Uryasev, 2000). The CVaR is simply the expected value of the losses that exceed the VaR. The CVaR has also one important advantage with respect to the VaR: when used to specify the objective function in a portfolio optimization setting, or in the constraints as a way of limiting extreme losses, the resulting problem is convex and can be efficiently solved with commercial software. Not surprisingly, the CVaR quickly gained acceptance among practitioners and academics, although regulators have been reluctant to incorporate it into risk-related guidelines.

3. Our strategy

Our strategy, whose target is pension funds and/or mid- to long-term investment vehicles, is based on two simple ideas. First, asset allocation matters, and to this end we employ an active approach based on Markowitz framework (with minor modifications) to select the asset allocation percentages. And second, recognizing the futility of beating the market within a given sector (or asset class), we rely on a passive strategy for each asset class, which is achieved by tracking indices.

Note that several authors who have studied active management strategies in portfolio selection have highlighted the importance of optimization as a tool for constructing passive portfolios (Oh, Kim, & Min, 2005; Sorensen, Miller, & Samak, 1998). They have also concluded that optimal allocation would include a substantial amount of index tracking.

Thus, our approach is a hybrid (active/passive) combination in which Step 1 (decision on asset allocation percentages) is followed by Step 2 (passive approach within each sector). Step 1 is done using market information (returns, correlations, etc.) from the previous three years, and rebalancing (recalibration of the asset allocation percentages) is carried out once a year.

More formally, the strategy can be described as follows: suppose we have N asset classes, and within each sector we have identified a representative index. We also assume that we have monthly data on the return of such indices, which will be represented by vector r, with components r_i, and mean values $\bar{r}_i, i = 1, \ldots, N$. The vector α, with components $\alpha_i, i = 1, \ldots, N$, represents the fraction of the portfolio allocated to each asset class; the weights must add to one, and be greater or equal than zero. Finally, the return R, which is the dot product of vectors r and α, denotes the estimated (future) portfolio return based on historic data (in the past three years in this paper). A first version of the approach can be stated as:

$$\max = \alpha_1 \bar{r}_1 + \ldots + \alpha_N \bar{r}_N$$

subject to

(i) asset allocation constraints on the weights imposed by the regulator (e.g. no more than 30% in foreign stocks; at least 20% in domestic investments); plus
(ii) Variance$(R) \leqslant V^*$, where V^* is a risk tolerance parameter. Variance (R) is estimated at each step of the optimization process based on the variance–covariance matrix of returns computed from the previous months data (last three years) and the current α, that is, Variance$(R) = \alpha^T C\alpha$ where C is the variance–covariance matrix. An alternative version of the approach can be formulated by substituting constraint (ii) above by
(iii) CVaR$(R) \leqslant$ CVaR*.

Again, CVaR* is a risk tolerance parameter (discussed in the next section); both V^* and CVaR* are real numbers estimated using historical data. In other words, the constraint on the variance limits the variability of the portfolio to the variability observed in past three years. Similarly, the CVaR constraint, controls the average of losses above the VaR level based on the previous three-year data. A few observations are in order:

- A common criticism to the conventional Markowitz framework is the uncertainties associated with estimating the assets return correlations, and also, the sensitivity of the optimization problem solution to small variations in the input parameters. By working with indices (instead of "primitive" assets such as individual stocks or specific bonds) we hope to mitigate this problem since in general indices are less volatile than individual assets.
- In all likelihood the set of potential asset classes (for example, domestic equities, foreign equities, emerging market sovereign bonds, etc.) will be rather small, at most ten or twelve different type of assets. This makes the optimization problem much more tractable and less prone to error propagation compared to the typical situation in which the MV approach is employed to decide on a universe of individual assets (for example, the 100 most liquid stocks in the US market). In addition, the problem of having very small positions in a high number of assets also disappears in this context.
- The one-year rebalancing period and the use of the last three years of return information to cast the optimization problem might seem rather arbitrary. However, there is some evidence that rebalancing more frequently does not result in significantly different outcomes, especially for minimum-variance portfolios (Chow, Kose, & Li, 2015). Also, since these previous observations have been made in reference to problems dealing with up to 1000 stocks–and we are dealing instead with indices–the one-year rebalancing period (at least as an initial starting point) seems reasonable. Similarly, most studies using Markowitz's framework rely on data from the previous one- to five-year periods (Chow et al., 2015; Bayraktar & Bilge, 2012), thus, the three-year period seems also adequate.

4. Application

4.1. The Chilean pension system

The Chilean pension system, a defined-contribution scheme that has been in place since 1980, constitutes an ideal test case in which to try our investment approach. The system was modified in 2002 with the introduction of five investment funds, known as A, B, C, D and E, each with a different risk profile. The A fund is supposed to be the riskiest and the E fund the most conservative. The regulator establishes minimum and maximum investment criteria–which can change from time to time–according to the fund risk profile. For example, the A fund could be invested up to 100% in foreign assets, whereas the E fund can only have, at most, 35% in foreign assets. Additionally, the A fund can have up to 80% of its holdings in equities whereas the corresponding percentage in the case of the E fund is just 5%. For all practical purposes, the funds are invested only in bonds and stocks in Chile and overseas, plus money market instruments. Exposure to alternative assets is essentially zero. Therefore, there are only six asset classes to consider: international equities; Chilean equities; international corporate bonds; Chilean corporate bonds; Chilean sovereign bonds; and domestic money market investments.

As of 2016 there were six private pension fund administrators (there were only five until 2010). These organizations, known as AFPs based on their Spanish acronym, can have only one business line by law: managing the savings of future retirees, they cannot engage in any other activity. All AFPs offer their version of the five basic funds (A, B, C, D and E) and they compete based on their management fees and customer service. Their overall performance is very similar, and strong herd behavior has been observed in Olivares (2008) and Raddatz et al. (2013). The chief reason is that the regulatory framework establishes a benchmark dictated by the average performance of the industry. Hence, since individual AFPs are compared against this benchmark (and penalized heavily if they underperform), the–predictable–result has been a great deal of similarity in terms of investment strategies among all funds, regardless of the administrator. (Incidentally, this phenomenon has also been detected in Poland, a country with a similar benchmarking scheme (Zbigniew W Kominek, 2006)). A more detailed description of the Chilean pension system can be found in Fernandez (2013).

Finally, the regulator provides on a regular basis detailed and reliable statistics on the performance of the system (broken down by AFP and by the type of fund). The performance is reported in inflation-adjusted Chilean pesos. All these considerations–five types of funds with different risk profiles plus reliable statistics over a reasonable long-time period–make the Chilean pension system a suitable case to test our approach. In this study, we focus on the 2003–2014 time period and we consider the five types of funds just described. For convenience, we rely on the industry average to characterize the performance of each of the five funds.

4.2. The data

For the purpose of this study we downloaded the data (industry-average monthly-returns, after fees), for the five funds mentioned, which are available from the Chilean pension regulator website (www.safp.cl). The monthly returns are reported in *unidades de fomento* (known as UF in Chile), that is, they correspond to actual returns (returns after adjusting for inflation). Based on historical information we were able to compute yearly returns, the corresponding variance–covariance matrices, and the risk parameters (Variance(R) and CVaR(R)). The period covered was from January 2003 to December 2014.

As mentioned in the previous section, the AFPs invest in six asset classes. In order to achieve a fair comparison with the AFP, the indices were selected to represent these asset classes as accurate as possible. Our choices are as follows: (1) international equities (MSCI All Country World Index); (2) Chilean equities (MSCI Chile Index); (3) international corporate bonds (Barclays US High Yield Index); (4) Chilean corporate bonds (DJLaTixx Chile Corporate Index); (5) Chilean sovereign bonds (DJLaTixx Chile Government Index); and (6) domestic money market investments (LVA Money Market Index).

All indices-related data were adjusted to express returns in inflation-adjusted (UF) Chilean pesos to make it consistent with the data reported by the regulator. The selection of the Barclays US High Yield Index might seem strange as a proxy for the international fixed income sector. However, given that the Chilean AFPs–at least during the period considered–concentrated all their international bond positions in the US high yield market exclusively, this index choice is appropriate. The remaining five indices reflect broadly the performance of their corresponding sectors without making any assumptions in terms of regional or industry-specific preferences.

4.3. A necessary feasibility study

Before actually testing our strategy it seems reasonable to investigate if–in theory–it would have been possible for the AFPs to achieve a better performance had they been forced to restrict themselves to invest in the six indices already identified. In other words, this exercise is really a feasibility study carried out with the benefit of hindsight. To this end, for each one of the five funds (A, B, C, D and E) and for every year (2003, ..., 2014) we solved the following optimization problem, where $\alpha_1, \ldots, \alpha_6$ refer to the six indices defined in Section 4.2:

$$\max = \alpha_1\bar{r}_1 + \alpha_2\bar{r}_2 + \alpha_3\bar{r}_3 + \alpha_4\bar{r}_4 + \alpha_5\bar{r}_5 + \alpha_6\bar{r}_6$$

subject to

(i) minimum and maximum constraints on the weights imposed by the regulator for each of the five funds. For instance, in the case of the E fund, at least 95% of the portfolio must be allocated to domestic fixed income securities (that is, $\alpha_4 + \alpha_5 + \alpha_6 \geqslant 0.95$), while for fund B at most 60% of the portfolio can be invested in equities ($\alpha_1 + \alpha_2 \leqslant 0.6$).

(ii) $\mathrm{Variance}(R) \leqslant V^*$.

The weights must add to one and be non-negative, as short-selling strategies are not permitted, and the r_i's represent the annual return of the indices based on 12 (monthly) data points per year. The left and right hand sides of the portfolio constraints were not changed over time, according to the information publicly available at the regulators web page (Superintendencia de Pensiones, 2013). Finally, V^* corresponds to the variance of the monthly returns observed during a given year, for the relevant fund (A, B, C, D and E), based on the AFP-funds' performance. In essence, what we are trying to accomplish is to find out if the AFPs could have obtained for each fund a better return than the return they actually obtained, but with similar or less risk (as measured by the variance of returns), had they invested only on the six indices already mentioned. In this formulation, it is assumed that rebalancing (changing the value of the weights) is done once a year only (at the beginning of the year). Table 1 summarizes the results obtained from solving the 5 (funds) $\times$ 12 (years, from 2003 until 2014) = 60 optimization problems. That is, the table compares the performance metrics between the actual AFP-managed funds (AFP) and the hypothetical results based on the index-based portfolio (Index), for the entire 2003–2014 period in an aggregate fashion.[1] Several observations are worth mentioning.

First, in all optimization problems it was possible to find a feasible solution; that is, a set of α's that defined a portfolio whose risk (as characterized by its variance of returns) was less or equal than that of the corresponding AFP-managed fund.

Second, in all five cases, in absolute terms (cumulative return), the index-based portfolio outperformed the AFPs by a wide margin. And third, on a risk-adjusted basis as measured by the Sharpe ratio, the index-based portfolio did better than the AFPs funds in four of the five cases. The exception was the E (most conservative) fund. More noticeably, for the three riskiest funds (A, B and C), the index-portfolio Sharpe's ratio is roughly twice that of the AFPs funds.

Finally, two words of caution. First, the above-mentioned results should not be taken as an indictment on the AFPs performance. Note that the AFPs are governed by a regulatory framework that encourages herd behavior and inhibits passive

[1] Throughout the paper we will present yearly results. Monthly data is available from the authors upon request.

Table 1
Comparison of performance metrics for two portfolios: AFPs and index-based from January 2003 to December 2014. Coeff.Var. stands for coefficient of variation, and (*) refers to the amount one would have had, at the end of December 2014, if one had invested 100 Chilean pesos at the beginning of January 2006 (expressed in Chilean pesos of 2006).

Fund	Type	Cum. return	Avg. yearly return (%)	Std dev. (%)	Sharpe Ratio	Coeff. var.	*
A	AFP	1.24	9.01	20.35	0.44	2.26	223.58
	Index	2.51	11.78	13.39	0.88	1.14	350.60
B	AFP	1.00	7.05	15.22	0.46	2.16	199.56
	Index	1.88	9.62	9.69	0.99	1.01	287.80
C	AFP	0.92	6.07	10.09	0.6	1.66	192.05
	Index	1.46	8.03	7.63	1.05	0.95	245.87
D	AFP	0.81	5.24	6.22	0.84	1.19	180.84
	Index	0.97	5.93	4.72	1.25	0.8	197.33
E	AFP	0.66	4.35	2.73	1.51	0.63	165.99
	Index	0.86	5.36	4.17	1.28	0.78	185.51

strategies based on indices. More specifically, if an AFP were to follow an index-based strategy, and in some specific year that strategy were to render inferior results compared to the industry average, that AFP would be exposed to regulatory fines. Second, and more important, these results do not correspond to a plausible investment strategy as the optimization problem was casted *a posteriori*, that is, with the benefit of "knowing the past". Nevertheless, the relevance of this feasibility exercise is to show that there was a better investment strategy, based on the six indices, than the alternative chosen by the AFPs. Therefore, with this result we are on a firm ground to test our investment strategy since we know that an index-based approach can indeed produce better outcomes. The challenge now is to see if our method can identify those index-based better-performing portfolios using only past data.

4.4. Investment strategy test

We tested our investment strategy starting on 2006 (that is, we select the α's in January 2006, based on the information available for the previous three years, and we examine the performance results at the end of the year, December 2006). We repeated this process for every year between 2006 and 2014. We also tested 24 and 48 month windows, and the results were qualitatively identical, and the differences in terms of Sharpe ratio were not significant. For this reason, all results reported are based on a 36-month time window. In short, we solve

$$\max = \alpha_1 \bar{r}_1 + \ldots + \alpha_6 \bar{r}_6$$

subject to the usual constraints on the α's plus those imposed by the regulator. In addition, we incorporate a risk-related condition based, first, on the variance of returns ($\text{Variance}(R) \leqslant V^*$), and then, based on the CVaR of returns ($\text{CVaR}(\text{R}) \leqslant \text{CVaR}^*$). The main difference here is that the returns r_i in the objective function are estimations based on 36 data points (from the previous 3 years). Unlike the feasibility exercise, we do not know the actual returns at the time the decision is made, so the best we can do is to optimize based on this information, and hope that returns will be satisfactory. Likewise, variance–covariance matrices are also estimated based on data from the previous three years. The return comparison done at the end of the year is obviously based on the actual performance exhibited by the index-based portfolio (specified by the α's obtained from solving the optimization problem) vis-à-vis those of the AFP funds.

4.4.1. First formulation: Variance-based risk characterization

This formulation is analogous to the conventional Markowitz's portfolio optimization problem. The value of V^* in the $\text{Variance}(\text{R}) \leqslant V^*$ constraint is calculated based on the actual variance of returns exhibited by the corresponding AFP funds during the previous three years. Table 2 displays the results.

In the case of the three riskiest funds (A, B and C) our strategy resulted in better Sharpe's ratios. In terms of absolute returns, our strategy also produced favorable results (better for funds A and B, and essentially identical for fund C). Somehow consistent with the fact that the index-based portfolios exhibited lower volatility than the corresponding AFP funds (as reflected by the lower coefficient of variation), is the fact that in the years in which the AFP funds showed extreme negative returns, the index-based portfolios experienced fewer negative returns. For instance, in 2008 the A fund (AFP) exhibited a -40.27% return while the index-based portfolio only exhibited a -25.49% return. Likewise, in 2011 the respective figures were -11.11% (AFP) versus -3.91% (Index). A similar situation is observed in relation to funds B and C.

Now let us see some of the allocations that resulted from our strategy. We will focus on the extreme cases, funds A and E, and we start displaying the regulatory constraints for each situation on Table 3. The constraints for the other funds are obtained by choosing intermediate values for the left and right hand sides in the constraints described in Table 3. Fund E has a very limited universe of investment possibilities, being essentially a bond portfolio highly concentrated in Chilean instruments, while fund A enjoys much greater flexibility. On Table 4 we show solutions for the years 2007, 2010 and 2014, and also the average investment on each index in the nine years considered:

Table 2
Comparison of performance metrics for two portfolios: AFPs and index-based from January 2006 to December 2014 and using a risk tolerance characterization specified by the variance of returns. Coeff.Var. stands for coefficient of variation, and (*) refers to the amount one would have had, at the end of December 2014, if one had invested 100 Chilean pesos at the beginning of January 2006 (expressed in Chilean pesos of 2006).

Fund	Type	Cum. return	Avg. Yearly return (%)	Std dev. (%)	Sharpe Ratio	Coeff. var.	*
A	AFP	0.41	6.42	21.49	0.30	3.35	141.07
	Index	0.45	5.42	15.73	0.34	2.90	144.71
B	AFP	0.45	5.66	16.47	0.34	2.91	145.30
	Index	0.51	5.54	12.92	0.43	2.91	151.08
C	AFP	0.53	5.43	11.03	0.49	2.03	152.62
	Index	0.52	5.19	9.45	0.55	1.82	151.67
D	AFP	0.51	4.92	6.75	0.73	1.37	151.13
	Index	0.44	4.37	6.70	0.65	1.53	144.01
E	AFP	0.51	4.72	2.86	1.65	0.61	150.95
	Index	0.45	4.34	5.05	0.86	1.16	145.01

Table 3
Regulatory constraints for funds A and E.

	Fund A	Fund E
Equity	$.4 \leqslant \alpha_1 + \alpha_2 \leqslant .8$	$0 \leqslant \alpha_1 + \alpha_2 \leqslant .05$
Fixed income	$.2 \leqslant \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 \leqslant .6$	$.95 \leqslant \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 \leqslant 1$
Foreign investment	$.45 \leqslant \alpha_1 + \alpha_3 \leqslant 1$	$.15 \leqslant \alpha_1 + \alpha_3 \leqslant .35$
Local investment	$0 \leqslant \alpha_2 + \alpha_4 + \alpha_6 \leqslant .55$	$.65 \leqslant \alpha_2 + \alpha_4 + \alpha_6 \leqslant .85$

Table 4
Some optimal portfolios obtained with our strategy, with a variance constraint.

	Fund A	Fund E
2007	(.286,.514,.164,.036,0,0)	(.039,.011,.111,.42,0,.419)
2010	(0,.55,.45,0,0,0)	(0,.019,.15,0,.42,.585,.247)
2014	(0,.4,.6,0,0,0)	(.026,0,.324,.368,.282,0)
Avg.	(.088,.479,.394,.018,.016,0)	(.012,.02,.185,.298,.238,.244)

For fund A, the portfolio starts concentrated among the first three indices, but as the average investment indicates, in the long run it is truly concentrated in the second and third indices. The binding constraints vary considerably over time, which indicates the difficulty in defining those values a priori: in 2017, we have essentially an 80/20 portfolio of assets and bonds, while in 2010 it is more balanced, and none of the first two constraints is binding, while the last two are. In 2014 the situation reverses in terms of binding constraints as the investment in domestic equities gives room to foreign equity.

For fund E, the portfolio exhibits great diversification, as all investments have positive values at some point in time. As forced by the constraints, bonds and risk-free instruments completely dominate the portfolio, with Chilean bonds featuring a higher concentration. Most of the time the solution is binding on the right hand side of the equity constraint, and the portfolio has all the other 95% invested in bonds. Finally, it is known that the bond market in Chile performed extremely well during the period under consideration, so the local investment constraint is binding for most years at the.85 level. It also explains the inferior performance of our strategy: while the bond indices have performed well, it was possible for the AFPs to pick certain local companies with extraordinary returns and beat the passive allocation. This behavior combined with the very limited exposure to equites seals the fate of essentially any passive strategy for the fund E.

In several instances the regulatory constraints were binding, suggesting that such constraints play a key role in determining the solution of the optimization problem, and therefore, a poorly chosen right-hand side limit imposed by the regulator could significantly impair the returns in the long run. It should be noted that the effects of constraints on the results of portfolio optimization problems is a topic that only recently began to receive a much-deserved consideration (Chow et al., 2015). These same authors have observed that including constraints in actively managed portfolios, can, in some cases, improve performance, but it is often at the expense of an increase in volatility.

Going back to Table 2, in the case of fund D the AFP fund showed a marginally better performance, while in the case of fund E, the AFP fund was definitively better. The high values of the Lagrange multipliers associated with some constraints, particular in funds D and E, suggest that a minor variation in the investment limit could translate into a significant improvement in terms of the portfolio returns. For instance, let us take the foreign investment constraint for fund *D*:

$\alpha_1 + \alpha_3 \geqslant .2.$

On Table 5 we show the excess return that would have been obtained if the constraint was relaxed, that is, if the right hand side was $.2 - \Delta$, where $\Delta = .01, .02, .05$ and $.10$. In some years the return would remain the same (that is, the associated Lagrange multiplier is zero), while in others the difference could be as high as 4.2%. Even with $\Delta = .01$, which is a very small change in the regulatory constraint, the damage on the returns in the long run can be significant. For the.10 level, the impact is notorious, with an average yearly loss of 1.44% in the years considered. Moreover, as we suspected, by choosing $\Delta = .10$ the cumulative return would have been 62.88%, with a Sharpe Ratio of 0.85, higher than the value of 0.73 obtained by the AFP with the original constraints in place, as shown in Table 2.

In summary, this analysis indicates that the limits set by the regulator should be chosen with extreme care since they could have detrimental effects on the returns of the funds. More precisely–and very relevant from a public policy viewpoint–constraint limits should be set taking into account the corresponding shadow prices, and monitored on a regular basis as market conditions evolve. If constraints are set too tight, they can have an important negative long-term effect on funds' performance, which is especially true for passive strategies with only a limited universe of indices, as reflected by Table 5.

We remind that, in order to achieve a fair comparison between the two approaches, we imposed portfolio constraints in the passive strategy model as well, on top of risk constraints via the Variance or the CVaR. Our findings suggest the possibility of eliminating portfolio constraints completely, and proposing an alternative method to control risk in pension fund problems. Albeit interesting, this extension is beyond the goals of the current work.

Finally, a practical aspect in the implementation of our strategy is that we used the observed variance of returns of the AFP funds to specify the right-hand side value of the risk constraint (V^*). A possible objection to our approach could be that in some cases such benchmark might not be available. In those instances, a potential solution would be to estimate a plausible set of V^* values, reflecting increasing levels of risk-aversion depending on the fund type (the highest V^* value could be associated with the A fund while the lowest with the E fund). The value of V^* could be estimated based on a combination of a relevant group of indices and their past performance, subject to the limitations by asset class imposed by the regulator.

4.4.2. Second formulation: CVaR-based risk characterization

This formulation is analogous to that of the previous case except that the risk-related constraint is expressed as $\text{CVaR}(R) \leqslant \text{CVaR}^*$. The CVaR can be computed in a parameter-free fashion, in the sense that no distribution assumption or variance–covariance matrix computation is needed to write down the risk measure. Therefore, in order to construct the optimization problem, only an estimation of returns is necessary in the objective function since we are maximizing their expected value. We estimated all the CVaR^* limits required, one for each type of fund, and using the 36 data points corresponding to the monthly returns of the relevant 3-year period, using a 90% confidence level. The results are shown in Table 6 and they are broadly consistent with those shown in Table 2.

Similar to Table 4, we show the corresponding portfolios obtained by our strategy with the CVaR constraint. The solutions, displayed in Table 7, are different from the case with the variance constraint. In fund A more weight was placed on first index

Table 5
Excess return (%) obtained by relaxing the foreign investment constraint for fund *D*.

Δ	2006	2007	2008	2009	2010	2011	2012	2013	2014	Avg.
0.01	0.00	0.25	0.00	0.22	0.15	0.26	0.42	0.00	0.00	0.14
0.02	0.00	0.50	0.00	0.43	0.30	0.53	0.84	0.00	0.00	0.29
0.05	0.00	1.24	0.00	1.08	0.74	1.32	2.10	0.00	0.00	0.72
0.10	0.00	2.49	0.00	2.17	1.48	2.63	4.20	0.00	0.00	1.44

Table 6
Comparison of performance metrics for two portfolios: AFPs and index-based from January 2006 to December 2014 and using a risk tolerance characterization specified by the CVaR of returns. Coeff.Var. stands for coefficient of variation, and (*) refers to the amount one would have had, at the end of December 2014, if one had invested 100 Chilean pesos at the beginning of January 2006 (expressed in Chilean pesos of 2006).

Fund	Type	Cum. return	Avg. yearl yreturn (%)	Std dev. (%)	Sharpe Ratio	Coeff. var.	*
A	AFP	0.41	6.42	21.49	0.30	3.35	141.07
	Index	0.51	5.97	15.92	0.37	2.67	151.31
B	AFP	0.45	5.66	16.47	0.34	2.91	145.30
	Index	0.48	5.29	13.03	0.41	2.47	147.70
C	AFP	0.53	5.43	11.03	0.49	2.03	152.62
	Index	0.50	5.10	9.67	0.53	1.90	150.17
D	AFP	0.51	4.92	6.75	0.73	1.37	151.13
	Index	0.40	4.03	6.77	0.60	1.68	139.83
E	AFP	0.51	4.72	2.86	1.65	0.61	150.95
	Index	0.45	4.34	5.06	0.85	1.17	144.67

Table 7
Some optimal portfolios obtained with our strategy, with a CVaR constraints.

	Fund A	Fund E
2007	(.25,.55,.2,0,0,0)	(.014,.036,.136,.420,0,.394)
2010	(0,.55,.45,0,0,0)	(0,.038,.150,.214,.412,.186)
2014	(.4,0,.6,0,0,0)	(0,0,.35,0,.65,0)
Avg.	(.148,.435,.379,.02,.016,0)	(.002,.028,.214,.257,.328,.169)

(MSCI ACWI), and an even higher concentration at Chilean sovereign bonds for fund E at the expense of the money market index α_6. In summary, our approach performs better in the case of funds A, B and C and worse for funds D and E.

5. Conclusions

We proposed a two-step hybrid investment strategy for pension funds, in which the asset allocation is decided via optimization (the active component). And within each asset class, we follow a passive investment approach based on a representative index. Rebalancing is done once a year based on information from the preceding three years. Also, we tested two metrics (Variance (R) and CVaR(R)) to control risk in the optimization problem. Part of the motivation for our strategy is the overwhelming evidence in favor of passive versus active investment, but also the recognition that asset allocation plays a fundamental role in selecting the portfolio composition. The proposed strategy is an attempt to combine these two key concepts.

The performance of our strategy was compared to the performance of each of the five funds that constitute the backbone of the Chilean pension system, a fairly well established (and highly regarded) defined-contributions scheme. The results indicated that our strategy did remarkably well. In three of the five funds (the three riskiest funds), our approach produced better absolute returns (compared to the returns experienced by the Chilean pension managers), higher Sharpe ratios, and lower volatilities. In the two cases in which our strategy was outperformed by that of the Chilean AFPs, we showed that portfolio allocations limits set by the regulator played a significant role. In fact, the large value associated with the Lagrange multipliers corresponding to those constraints suggested that a small relaxation of the constraints limits would result in a much better performing portfolio. And that was indeed the case. This conclusion is important for it implies that regulators should exercise extreme care when setting the portfolio allocation limits.

Our results are encouraging for two reasons. First, the implementation costs associated with our strategy, compared to those of actively managed funds, are substantially lower. The reason is that there is no need to maintain a staff of analysts to track market tendencies, macro-economic indicators, or central bank policies, and expensive business trips to visit the companies in which the fund managers have taken positions. Second, the increasing tendency of pension funds to migrate from active to passive strategies can benefit from our concept. For this reason, and given the magnitude and importance of pension fund investments, it is natural to expect an increase in the publications discussing passive investment strategies in the upcoming years. Our work presents one possibility out of many, and it certainly does not exhaust the universe of passive investment strategies.

As future work we plan to investigate the possibility of removing the regulatory constraints on asset allocation, and instead control risk directly using some appropriate risk measures. Furthermore, we plan to consider alternative investment strategies for pension funds using data-driven frameworks such as robust optimization.

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