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Distribution Uncertainty and Expected Stock Returns*

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Abstract

We investigate the significance of differences of the return distribution (distribution uncertainty) in the cross-sectional pricing of stocks. Our parsimonious proxies for distribution uncertainty measure the difference of distributions between an individual stock return and the market return. We find that stocks with higher distribution uncertainty exhibit higher returns, and the difference between the returns on the portfolios with the highest and lowest distribution uncertainty is significantly positive. We investigate the robustness of our empirical results and find that the impact of distribution uncertainty persists after accounting for firm characteristics.

Keywords: Distribution Uncertainty; Expected Stock Returns; Differences of Return Distribution;

JEL: C52, D81, E21, E44, G12

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I. Introduction

In one of the most seminal papers in financial economics, Markowitz (1952a) argues that there exists two stages when we select a portfolio: firstly forming beliefs about asset returns and secondly optimizing our portfolio based upon those beliefs.[1] Traditional asset pricing models like the CAPM overlook the first stage and are constructed based on the optimization in the second stage. Those models assume that investors already have beliefs about asset returns and know the form of distribution. For example, the CAPM assumes that asset returns follow a multivariate normal distribution or investors have quadratic utility function and that investors are mean-variance optimizing. However, empirical evidence confirms that portfolio returns are not normally distributed (Fama, 1965; Rosenberg, 1974), and even vague agreement about a specific stock return distribution does not exist (Tsay, 2010). That is, empirical evidence seems to suggest that investors do not know the distributional form of future stock returns.[2] In particular, a recent paper by Kacperczyk and Damien (2011) assumes that the form of the distribution of returns is not known, and proposes a novel method to incorporate "*distribution uncertainty*", uncertainty about the type of return distribution, to obtain an optimal portfolio. While the apparent difficulties of understanding the form of return distribution are generally recognized, surprisingly little is known about whether the uncertainty about

[1] Markowitz (1952a) says that *"The process of selecting a portfolio may be divided into two stages. The first stage starts with observation and experience and ends with beliefs about the future performances of available securities. The second stage starts with the relevant beliefs about future performances and ends with the choice of portfolio. This paper is concerned with the second stage."*

[2] For example, Pastor and Stambaugh (2003), Ang and Bekaert (2004), and Guidolin and Timmermann (2007) show that distributions of assets returns tend to switch between different regimes. Liu et al. (2003) and Liu et al. (2005) suggest that the presence of rare events may perturb beliefs about the form of distributions. Welch (2000) argues that investors tend to differ in their assessment of future returns.

the return distribution affects empirical phenomena in finance such as the cross-sectional difference of asset returns. Therefore, in this paper, we empirically investigate whether there exists a significant relation between distribution uncertainty and expected stock returns.

We define a proxy for distribution uncertainty of each stock return, which reflects an economic agent's difficulty in obtaining information about stock return distribution. We measure distribution uncertainty about the shape of individual stock return distribution compared to that of a benchmark portfolio. For example, Fox and Tversky (1995) propose the comparative ignorance hypothesis, which contends that uncertainty aversion is produced by a comparison with less uncertain events or with more knowledgeable individuals. Motivated by the literature, we suggest proxies for distribution uncertainty of a stock return, the Kolmogorov-Smirnov (KS), the Cramer-von Mises (CM), and the Kuiper (K) statistics, which non-parametrically measure differences between empirical return distributions of an individual stock and a benchmark portfolio. We use the market portfolio as the benchmark portfolio, since the market portfolio is probably the most important and well-known asset to every agent in an economy. For example, based upon the two-fund separation theorem[3], agents can optimize their portfolio by investing in the market portfolio and the risk-free asset. Also, various finance periodicals such as the Wall Street Journal or Financial Times report returns of market proxies such as S&P500 or Wilshire 5000, a proxy of the market portfolio. Therefore, we believe that we can measure relative distribution uncertainty of a stock return by comparing the return of the

[3] See Markowitz (1952b), Tobin (1958), Ross (1978), Cass and Stiglitz (1970) and others for more discussion about mutual fund separation theorem.

market portfolio.

We examine the role of distribution uncertainty in the cross-sectional pricing of stocks. To do this, we first sort stocks by the degree of distribution uncertainty over the previous 12 months and examine the monthly returns on the resulting portfolios over the period of 1965 to 2012. The results show that stocks with severe distribution uncertainty exhibit high returns on average, and the difference between returns on the portfolios with highest and lowest distribution uncertainty is almost 2% per month. The corresponding four-factor alphas from high-minus-low KS, CM, and K-sorted portfolios are 1.87% to 2.62% a month. We extensively investigate the robustness of our empirical results and find that the impact of distribution uncertainty persists after accounting for firm characteristics, such as beta, size, book-to-market ratio, momentum, short-term reversal, and illiquidity.

The results of distribution uncertainty may be related to ambiguity. Knight (1921) says, with ambiguity, the location and shape of the distribution is open to question. In Ellsberg (1961) and Camerer and Weber (1992), ambiguity is generally defined as uncertainty about distribution. If an investor has to optimize her portfolio without knowing the distribution of stock returns, she is much like an agent in Ellsberg's paradox. In Ellsberg's paradox (Ellsberg, 1961), an agent almost always prefers a game with known probability distribution (for example, a game with an urn containing 50 red balls and 50 black balls) to one without a specific probability distribution (for instance, a game with an urn containing 100 balls of unknown number of red and black balls)[4]. This preference for the existence of a specific distribution is referred to as ambiguity aversion (Ellsberg, 1961;

[4] For more thorough discussion about Ellsberg type example, see Ellsberg (1961), Epstein and Schneider (2008), and others.

Sherman, 1974; and etc.). Furthermore, Becker and Brownson (1964) argue that "*ambiguity is defined by any distribution of probabilities other than a point estimate.*" Based on the literature, we may infer that an agent in an economy is more uncertain about an individual stock return if its empirical distribution is far from the most popular benchmark distribution of stock returns.

Another interpretation of our empirical results is that investors do not want to hold stocks with distributions different from the average. Our measures estimate the difference of distributions between individual stock returns and the market portfolio return. Therefore, if investors want to keep up with the Joneses, they hesitate to hold stocks with different distributions and ask premium to hold these. This explanation is also perfectly corresponding to our empirical findings.

The rest of this paper is organized as follows. In Section II, we describe our dataset and construct variables of the distribution uncertainty. Section III reports our empirical results. We conclude in Section IV.

II. Data and Construction of Variables

The sample data include returns from the Center for Research on Security Prices (CRSP) Daily Stock File and book value from the Compustat of all stocks listed in NYSE, Amex, and NASDAQ. CRSP is used to obtain prices, daily return, market returns, shares outstanding, trading volume, etc. We also obtain balance sheet information including assets, liabilities, and total equity from Compustat. We use stock prices and shares outstanding to calculate market capitalization, and use daily returns to calculate distribution uncertainty for each firm in each month as well as beta, idiosyncratic volatility, skewness, and kurtosis. The market portfolio return is the value weighted index return in the CRSP Daily Stock File. The sample period spans from January 1965 to December 2012. To be included in the final sample for a given month, at least 100 daily returns must exist in the previous 12 months.

We measure how different the empirical return distribution of a stock is from that of the benchmark portfolio. Using daily returns of each company and the market portfolio in the previous year, for each month we estimate three statistics that non-parametrically measure the distribution uncertainty: the Kolmogorov-Smirnov (KS), the Cramer-von Mises (CM), and the Kuiper (K) statistics. Before we estimate each statistic, we demean returns of each stock and the market portfolio by subtracting average returns estimated from the data of the previous year in order to control the effect of expected returns on our results. Since we measure distribution uncertainty by the difference of the return distribution of a stock from that of the market portfolio, if we do not demean returns of each stock and the market portfolio, our proxy for distribution uncertainty merely catches the difference of expected returns of a stock and the market portfolio, not reflecting the degree of difficulty in understanding underlying distributions. Therefore, by demeaning returns, our KS, CM, and K statistics can compute the degree of difference in shapes of a stock return and the market portfolio return distributions other than the location of distributions. Since we control the size of mean for each stock return to construct KS, CM, and K, if we observe larger return for a portfolio sorted by KS, CM, or K, it is from the difference of distribution, not from the difference of expected returns of the portfolio or risk of the portfolio.

Three statistics of KS, CM, and K measure difference among several empirical distributions or between a given distribution and empirical distributions. In this section, we briefly introduce the definition of these three statistics adjusted for our case; a comparison between two empirical distributions.

The Kolmogorov-Smirnov (KS) statistic is used in the KS test to investigate the difference of distributions of two samples. Suppose that a first sample $x_1, \ldots, x_n$ has distribution with its cumulative distribution function $F_1(x)$ and the second sample $y_1, \ldots, y_n$ has distribution with cumulative distribution function $F_2(x)$. Then, the KS test investigate whether $F_1 = F_2$. If $F_{1n}(x)$ and $F_{2n}(x)$ are corresponding empirical cumulative distribution functions, then the KS statistic is defined as follows.

$$\mathrm{KS} = \max_j \left|F_1\left(x_j\right) - F_2\left(x_j\right)\right| \ where\ j = 1,2, \ldots, n$$

In short, the KS statistic for two samples is the maximum distance between two empirical cumulative distribution functions.

The Cramer-von Mises statistic also measures how different two empirical distributions are. It is defined as follows:

$$\mathrm{CM} = \frac{1}{n^2} \sum_i \left(n_i \sum_{j=1}^{p} t_j (F_i\left(x_j\right) - F\left(x_j\right))^2 \right)$$

where $F(x) = \frac{1}{n}\sum_i(n_i F_i(x))$, $n = n_1 + n_2$, n_i is the number of observation of class i, t_j is the number of ties at the jth distinct value, and p is the number of distinct values.

The Kuiper statistic is closely related to the Kolmogorov-Smirnov statistic. The Kuiper statistic uses not only the information of maximum distance between two empirical distributions as in the Kolmogorov-Smirnov statistic, but also the information of minimum distance between two empirical distributions. The exact formula of the Kuiper statistic is as follows.

$$K = \max_j(F_1(x_j) - F_2(x_j)) - \min_j(F_1(x_j) - F_2(x_j)) \text{ where } j = 1, 2, \ldots, n$$

For further information about these three statistics, see Gibbons and Chakraborti (2010).

III. Empirical Results

The first empirical investigation is whether distribution uncertainty can explain the cross-sectional variation of expected stock returns. Table I reports time series averages (AR) and holding period returns (HPR) of decile portfolios formed on each of the three distribution uncertainty measures. To construct this table, we first calculate these measures for each sample firm over the previous month. Each month we sort stocks into 10 equal-weighted portfolios using our measures for distribution uncertainty (KS, CM, and K). AR represents average daily returns in percentage multiplied by 21, and HPR is the holding period return of a decile portfolio rebalanced each month from 1965 to 2012. The portfolios sorted on three distribution uncertainty measures demonstrate strong variation in mean return, as shown in Table I. The results show that the average returns (AR) on the decile portfolios sorted by distribution uncertainty increase monotonically in portfolio rank. The bottom decile portfolio (S) by KS has 1.13% of expected return per month on average and the top decile does 3.75%. The B-S spread shows 2.62% of expected return per month and t-statistic of 9.81. When we form decile portfolios by K, stocks (S) with the least distribution uncertainty provide 1.12% of expected return per month on average and the stocks (B) with the most distribution uncertainty do 3.59%. Further, the top decile of portfolio by CM seems to demonstrate considerably higher returns than the bottom decile portfolio. Since the cross-sectional dispersion of returns is most striking between group 9 and group B, we calculate the return spread of 9-S for a robustness check. Our results still sustain with large expected returns more than 2%. Overall, we find significant evidence that stocks with the most distribution uncertainty have higher expected return than do stocks with the least distribution uncertainty. It implies that since investors need to spend more resources to understand unfamiliar distributions of a stock compared to that of the benchmark portfolio, they may require a premium to hold the stock. Our results show the evidence of a positive premium for bearing distribution uncertainty.

We examine the relation between distribution uncertainty and future stock returns after controlling for firm characteristics. For example, stocks with high distribution uncertainty tend to be small and illiquid.[5] To ensure that the effect of distribution uncertainty is not driven by these characteristics, we investigate the profitability of portfolios sorted by distribution uncertainty after controlling for firm characteristics, such as beta, size, book-to-market ratio, momentum, short-term reversal, and illiquidity. The beta of a stock for a month (BETA) is estimated by regressing the daily stock return on the value weighted index return using a previous year sample. SIZE is the natural logarithm of the market value of equity of the company (in thousands of dollars) measured by times series average of a firm's market capitalization for the most recent 12 months. Book-to-

[5] In particular, following Olsen and Troughton (2000), 84% of respondents agreed that estimates of future stock return distributions are more unreliable for small than large firms.

market ratio (BM) is the book value of equity divided by its market value at the end of the last fiscal year. MOM is the cumulative stock return over the previous 11 months starting two months ago to isolate momentum from the short-term reversal effect. We measure short-term reversal (REV) for each stock in month t as the return on the stock over the previous month. Following Amihud (2002), stock illiquidity (ILLIQ) is defined as the ratio of the absolute monthly stock return to its dollar trading volume.

Table II shows monthly returns averaged across the portfolios formed by two-way sorts on a stock return's distribution uncertainty and firm characteristics, following Bali et al. (2011) and Baltussen et al. (2013). First, stocks are categorized into 10 groups by firm characteristics. Then, within each decile portfolio, we further sort stocks into decile portfolios ranked based upon our KS, K, and CM statistics, which results in a total of 100 portfolios. Next, we average each of the distribution uncertainty portfolios across the firm characteristic deciles. As Baltussen et al. (2013) argue, we can control for each firm characteristic without assuming a parametric form about the relationship between distribution uncertainty and future stock returns. For each of these portfolios, we calculate average equal-weighted returns over the following month.

The first column of Panel A in Table II reports returns averaged across the ten beta deciles to produce decile portfolios with dispersion in KS. Since we average across beta deciles, the produced decile portfolios sorted by KS will include all betas. The portfolio returns for each month are calculated as an equal-weighted average of returns from strategies initiated at the end of the past month. After controlling for beta, the average return difference between the low and high KS portfolios is about 2.425% per month with a *t*-statistic of 28.61. It suggests that the positive relation between distribution uncertainty and future stock returns is not affected by beta. The results in Panel A show that the highest distribution uncertainty firms earn an average of 2.204%, compared to 1.441% for the smallest distribution uncertainty firms, when we control for size. The return differential between these two deciles (B-S) is 0.764% and significant (t=7.84). When controlling for book-to market ratio (BM), the return differentials between B and S are also positive and significant. When stocks are sorted based on momentum, the average return of the big-small portfolio is 1.972%, with a *t*-statistic of 23.28. Subsequently, the average excess return of the B-S portfolio equals 1.887% per month when controlling for short-term reversal. Finally, we see whether the illiquidity explains the higher returns for the highest distribution uncertainty stocks relative to the smallest distribution uncertainty stocks. The average return of the B-S portfolio is 1.497% per month with a *t*-statistic of 14.61. These results suggest that a positive distribution uncertainty premium remains and firm characteristics do not explain the positive relation between distribution uncertainty and futures stock returns. Panel B of Table II presents average monthly returns to portfolios formed by two-way sorts on CM and firm characteristics. We find similar, confirmatory evidence in Panel B with CM as a proxy for distribution uncertainty. In Panel C, we examine the performance of K-sorted portfolios after controlling firm characteristics. The results with K are also similar to those in Panel A and Panel B. Overall, the results from these robustness tests using alternative measures of distribution uncertainty still support our hypothesis.[6]

[6] These results are also robust when controlling for return distribution characteristics, such as firms' idiosyncratic return volatility, skewness, kurtosis, and maximum daily return.

Next, we examine whether a rational risk-based approach can explain our result that the degree of distribution uncertainty provides premium. Table III presents the equal-weighted portfolios' postranking alphas estimated under three different factor specifications, the capital asset pricing model (CAPM), the three factors proposed in Fama and French (1993), and the four-factor proposed in Carhart (1997). The results in Panel A show that our measures for distribution uncertainty are highly correlated with alphas estimated from three different factor specifications. The magnitude of the alpha is positively related to the level of distribution uncertainty, which implies that the high distribution uncertainty portfolios earn more positive abnormal returns. All three alphas of the B-S spread are significantly positive. The CAPM alpha is 2.56% per month (t=15.10), the three-factor alpha is 2.51% per month (t=15.81), and the four-factor alpha is 2.62% per month (t=13.09). A trading strategy with a short position in the low distribution uncertainty firms and a long position in high distribution uncertainty firms generates a monthly abnormal return of 2.62% after controlling for the market, size, value, and momentum effects. This pattern of alphas from the three different factor specifications implies that the abnormal returns of B-S portfolios are not specific to an asset pricing models and confirms our hypothesis of distribution uncertainty premium. The results of positive alphas are also robust across various distribution uncertainty proxies.

IV. Conclusion

This paper investigates the significance of uncertainty about the return distribution (distribution uncertainty) in the cross-sectional pricing of stocks. We suggest proxies for distribution uncertainty of a stock return, the Kolmogorov-Smirnov (KS), Cramer-von Mises (CM), and Kuiper (K) statistics, which non-parametrically measure difference between empirical return distributions of an individual stock and a benchmark portfolio.

Our results show that stocks with severe distribution uncertainty exhibit high returns on average, and the difference between returns on the portfolios with highest and lowest distribution uncertainty is almost 2% per month. The corresponding four-factor alphas from high-minus-low KS, CM, and K-sorted portfolios are 1.87% to 2.62% a month. We extensively investigate the robustness of our empirical results and find that the impact of distribution uncertainty persists after accounting for firm characteristics, such as beta, size, book-to-market ratio, momentum, short-term reversal, and illiquidity.

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Table I
Portfolio Returns Sorted on Distribution Uncertainty

This table presents equal-weighted average returns (AR) and holding period returns (HPR) for portfolios formed on each distribution uncertainty proxy within a month. We multiply daily returns by 21 to obtain monthly returns. All figures are expressed in percentage terms. The decile portfolios updated each month are formed by the sizes of Kolmogorov-Smirnov (KS), Cramer-von Mises (CM), and Kuiper (K) statistics estimated using daily demeaned individual stock return and value weighted index return over previous 12 months. These statistics of KS, CM, and K non-parametrically measure the difference of distributions between demeaned individual stock return and demeaned market. Portfolio 'S' is the portfolio of stocks with the lowest distribution uncertainty measures, Portfolio 'B' is the portfolio of stocks with the highest distribution uncertainty measures, 'S-B' is their difference in monthly returns, and *t*-statistics are reported in parentheses. ***, **, * correspond to statistical significance at 1, 5, and 10%, respectively. The sample includes all firms listed in NYSE, AMEX, and NASDAQ from 1965 to 2012.

	KS			CM			K		
	KS	AR	HPR	CM	AR	HPR	K	AR	HPR
S	0.0767	1.13	626.89	0.0020	1.12	626.12	0.2765	1.12	622.02
2	0.1001	1.25	710.61	0.0036	1.23	742.55	0.3649	1.24	701.19
3	0.1153	1.32	745.49	0.0048	1.34	833.72	0.4223	1.30	735.28
4	0.1281	1.39	795.17	0.0059	1.48	867.92	0.4695	1.45	814.85
5	0.1398	1.60	853.47	0.0070	1.55	909.58	0.5113	1.50	834.08
6	0.1518	1.70	889.66	0.0082	1.76	987.56	0.5514	1.67	904.73
7	0.1657	1.76	934.05	0.0099	1.84	1021.37	0.5930	1.85	956.33
8	0.1836	1.97	1000.51	0.0124	2.04	1095.99	0.6421	1.96	1045.76
9	0.2071	2.23	1211.66	0.0165	2.21	1209.64	0.7072	2.35	1260.71
B	0.2524	3.75	2097.63	0.0266	3.29	1283.50	0.8055	3.59	1908.18
9-S		1.09	584.77		1.23	638.69		1.09	583.52
t(9-S)		(4.31)***	(3.97)***		(4.79)***	(4.32)***		(4.26)***	(3.92)***
B-S		2.62	1470.73		2.17	657.38		2.46	1286.16
t(B-S)		(9.81)***	(9.34)***		(8.20)***	(4.12)***		(9.33)***	(8.36)***

Table II
Portfolios Returns Sorted on Distribution Uncertainty and Firm Characteristics

This table reports average returns (AR) for portfolios based on distribution uncertainty proxies and firm characteristics. We multiply daily returns by 21 to obtain monthly returns and report the monthly returns in percent. In each case, we first sort the stocks into deciles using the firm characteristic. Within each characteristic decile, we sort stocks into ten additional portfolios based on distribution uncertainty proxy (KS, CM, K) and compute the returns on the corresponding portfolios over the subsequent month. These statistics of KS, CM, and K non-parametrically measure the difference of distributions between demeaned individual stock return and demeaned market. This table presents average returns across the firm characteristic deciles. Portfolio 'S' is the portfolio of stocks with the lowest distribution uncertainty measures, Portfolio 'B' is the portfolio of stocks with the highest distribution uncertainty measures, 'S-B' is their difference in monthly returns, and *t*-statistics are reported in parentheses. ***, **, * correspond to statistical significance at 1, 5, and 10%, respectively. The sample includes all firms listed in NYSE, AMEX, and NASDAQ from 1965 to 2012.

Panel A. KS

	BETA	SIZE	BM	MOM	REV	ILLIQ
S	1.123	1.441	1.176	1.145	1.173	1.184
2	1.188	1.519	1.257	1.284	1.276	1.295
3	1.291	1.594	1.254	1.374	1.347	1.371
4	1.311	1.591	1.338	1.471	1.478	1.480
5	1.420	1.707	1.406	1.574	1.510	1.646
6	1.527	1.712	1.493	1.632	1.721	1.745
7	1.642	1.687	1.718	1.776	1.760	1.852
8	1.876	1.794	1.922	1.916	1.889	2.032
9	2.186	1.845	2.144	2.036	2.088	2.109
B	3.548	2.204	3.396	3.117	3.060	2.682
B-S	2.425	0.764	2.220	1.972	1.887	1.497
t(B-S)	(28.61)***	(7.84)***	(25.35)***	(23.28)***	(22.24)***	(14.61)***

Panel B. CM

	BETA	SIZE	BM	MOM	REV	ILLIQ
S	1.129	1.504	1.157	1.164	1.169	1.195
2	1.247	1.572	1.216	1.339	1.272	1.347
3	1.298	1.624	1.300	1.411	1.334	1.426
4	1.340	1.674	1.356	1.562	1.528	1.501
5	1.432	1.714	1.462	1.563	1.541	1.618
6	1.552	1.730	1.552	1.617	1.679	1.733
7	1.678	1.722	1.709	1.759	1.771	1.871
8	1.934	1.726	1.995	1.875	1.944	1.899
9	2.218	1.816	2.220	2.013	2.122	2.148
B	3.283	2.010	3.133	3.008	2.940	2.656
B-S	2.154	0.506	1.977	1.844	1.771	1.461
t(B-S)	(25.94)***	(5.15)***	(22.65)***	(21.55)***	(20.89)***	(14.00)***

Panel C. K

	BETA	SIZE	BM	MOM	REV	ILLIQ
S	1.108	1.378	1.162	1.130	1.154	1.118

2	1.194	1.458	1.213	1.265	1.289	1.261
3	1.268	1.596	1.273	1.356	1.340	1.371
4	1.300	1.601	1.334	1.417	1.453	1.501
5	1.432	1.704	1.409	1.546	1.552	1.596
6	1.498	1.785	1.573	1.671	1.669	1.752
7	1.646	1.744	1.730	1.814	1.822	1.884
8	1.850	1.820	1.965	1.930	1.903	1.978
9	2.268	1.909	2.177	2.099	2.073	2.198
B	3.534	2.100	3.259	3.084	3.035	2.731
B-S	2.426	0.723	2.097	1.955	1.881	1.614
t(B-S)	(28.82)***	(7.40)***	(23.89)***	(23.17)***	(22.25)***	(15.48)***

Table III
Alphas of Portfolios Sorted on Distribution Uncertainty

This table reports the alphas of the CAPM, the Fama-French 3-factor model, and the Carhart (1997) 4-factor models for 10 portfolios based on three proxies for distribution uncertainty. The decile portfolios updated each month are formed by the sizes of Kolmogorov-Smirnov (KS), Cramer-von Mises (CM), and Kuiper (K) statistics estimated using daily demeaned individual stock return and value weighted index return over previous 12 months. These statistics of KS, CM, and K non-parametrically measure the difference of distributions between demeaned individual stock return and demeaned market. Alphas are from a time series regression of the daily returns on daily Rm-Rf, SMB, HML, and UMD as in Fama and French (1993) and Carhart (1997). We multiply daily alphas by 21 to obtain monthly alphas and report the monthly alphas in percent. Portfolio 'S' is the portfolio of stocks with the lowest distribution uncertainty measure, Portfolio 'B' is the portfolio of stocks with the highest distribution uncertainty measure, 'S-B' is their difference in monthly returns, and *t*-statistics are reported in parentheses. ***, **, * correspond to statistical significance at 1, 5, and 10%, respectively. The sample includes all firms listed in NYSE, AMEX, and NASDAQ from 1965 to 2012.

Panel A. Kolmogorov-Smirnov (KS) Statistic

	CAPM		Fama-French 3 Factor		Carhart 4 Factor	
	Alpha	Adj Rsq	Alpha	Adj Rsq	Alpha	Adj Rsq
S	0.4408	0.8357	0.3052	0.8793	0.3053	0.8793
2	0.5290	0.8125	0.3478	0.8951	0.3648	0.8954
3	0.5809	0.7761	0.3620	0.8962	0.3914	0.8970
4	0.6252	0.7559	0.3744	0.9060	0.4218	0.9076
5	0.8023	0.7423	0.5328	0.9087	0.5990	0.9114
6	0.8788	0.7269	0.5988	0.9130	0.6849	0.9169
7	0.9337	0.6925	0.6577	0.8964	0.7751	0.9031
8	1.1572	0.6057	0.8798	0.8259	1.0397	0.8376
9	1.4417	0.5053	1.1759	0.7203	1.3516	0.7340
B	3.0036	0.3571	2.8189	0.5366	2.9301	0.5414
B-S	2.5628		2.5137		2.6247	
t(B-S)	(15.10)***		(15.81)***		(13.09)***	

Panel B. Cramer-Mises (CM) Statistic

	CAPM		Fama-French 3 Factor		Carhart 4 Factor	
	Alpha	Adj Rsq	Alpha	Adj Rsq	Alpha	Adj Rsq
S	0.4485	0.8175	0.3077	0.8693	0.3096	0.8693
2	0.5175	0.8005	0.3396	0.8816	0.3527	0.8817
3	0.6035	0.7691	0.3854	0.8875	0.4178	0.8884
4	0.7193	0.7424	0.4651	0.8933	0.5150	0.8951
5	0.7613	0.7323	0.4912	0.8978	0.5570	0.9005
6	0.9388	0.7257	0.6575	0.9054	0.7431	0.9093
7	1.0055	0.6994	0.7303	0.8958	0.8349	0.9010
8	1.2218	0.6258	0.9453	0.8416	1.0773	0.8495
9	1.4151	0.5218	1.1525	0.7368	1.3174	0.7482
B	2.5369	0.3733	2.3444	0.5503	2.5162	0.5617
B-S	2.0883		2.0367		2.2066	
t(B-S)	(12.40)***		(13.57)***		(10.78)***	

Panel C. Kuiper (K) Statistic

	CAPM		Fama-French 3 Factor		Carhart 4 Factor	
	Alpha	Adj Rsq	Alpha	Adj Rsq	Alpha	Adj Rsq
S	0.4421	0.8229	0.3292	0.8684	0.3346	0.8685
2	0.4907	0.8142	0.3463	0.8885	0.3591	0.8887
3	0.5113	0.7880	0.3377	0.8966	0.3617	0.8972
4	0.6006	0.7671	0.3995	0.9053	0.4413	0.9066
5	0.5868	0.7529	0.3687	0.9128	0.4279	0.9151
6	0.7023	0.7362	0.4769	0.9113	0.5509	0.9144
7	0.8224	0.7047	0.5962	0.8973	0.6913	0.9018
8	0.8897	0.6456	0.6785	0.8474	0.7994	0.8541
9	1.2331	0.5375	1.0079	0.7424	1.1526	0.7516
B	2.2613	0.3797	2.0860	0.5399	2.2093	0.5459
B-S	1.8192		1.7568		1.8747	
t(B-S)	(12.81)***		(13.73)***		(11.44)***	