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# State contingent and conventional banking: The optimal banking choice model

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## A R T I C L E I N F O

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## ABSTRACT

This paper compares and contrasts the optimality of debt based banking and state contingent banking. We show that the advantage each of these banking types holds over the other might not be universal; rather it may be an outcome of the informational and institutional environment in which they operate. In our model, banks optimize both the riskiness of the project and moral hazard concerns to identify the most profitable banking model. We find that state contingent banking is more profitable where projects are riskier, and debt abased conventional banking is adopted for relatively lower risk projects. Our model also suggests that state contingent banking would be the optimal choice in cases where there exist greater moral hazard concerns. We explore the empirical implications of our model and find that state contingent banking would be more suitable for small firms, emerging markets, community and Islamic banking.

#### 1. Introduction

This paper compares and contrasts the optimality of debt based banking and state contingent banking<sup>2</sup>. We argue that the advantage each of these banking types holds over the other might not be universal; rather it may be an outcome of the informational and institutional environment in which they operate. Assuming that the primary purpose of a bank is to manage the tradeoffs between neutralizing asymmetric information, minimizing risk and maximizing profitability, we build a model that identifies the conditions under which each banking type could become more optimal than the other. The efficiency and optimality of debt and state contingent contracts are widely debated topics in the literature. In the presence of costly state verification, debt is argued to be more optimal (e.g., Townsend, 1979; Gale and Hellwig, 1985; Williamson, 1987). The returns on a debt contract are determined ex-ante. They are independent of the outcome faced by the borrower, whether it's the profitability of the underlying business or the income earned by an individual. This neutralizes the moral hazard concerns of the lender, making the debt contract much more efficient. Interestingly, this non-state contingent nature of debt has come under severe criticism in some of the recent literature. The pre-determined rate of return exposes the contract to multiple externalities, which can result in inefficient borrowing. Mian et al. (2017)

explain the externality of debt by taking an exogenous view of the business cycle along with assuming myopia amongst borrowers and lenders. During the boom period, when the economy is doing well, the debt contracts should seem more optimal for both the lenders and the borrowers. This is because during an upturn, the defaults are low, resulting in a relatively secured return for the lenders while the borrower (particularly the borrowing firm) can enjoy the significant upside which the high growth period offers in the form of greater profits. During the downturn, when the economy underperforms, debt contracts should be less optimal as the possibility of defaults can end up imposing a cost on all parties. Ignoring the possibility of a downturn (when making decisions during an upturn) could be a possible cause of the debt externality. This externality can be neutralized by state contingent contracts. During an upturn or downturn, borrowers would have no incentive to over or under borrow in a state contingent contract.

Another stream of literature explains the externality of a debt contract by highlighting that it can make the borrower more risk averse. Mostly, the literature assumes risk preference to be exogenous to the investor's decision. Fischer (2013) argues that the inherent focus on returning the principal means that the borrowers would be risk averse in their decisions, making risk preference endogenous. Fischer (2013) explains the presence of this externality in the microfinance

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<sup>2</sup> State contingent banking is one in which a bank has a share in returns from the projects it has financed. Therefore, the return to the bank is cotingent on the state of the project. If the project earns more, the bank gets more and vice versa. On the other hand, conventional banking in this paper is defined as one in which a bank has a fixed, predetermined share in the projects' proceeds.

http://dx.doi.org/10.1016/j.econmod.2017.07.008 Received 13 July 2017; Accepted 13 July 2017 0264-9993/ © 2017 Elsevier B.V. All rights reserved. industry, where he argues that most microfinance finance ventures fail to become big businesses, owing to the fact that the debt contract makes the micro-borrowers inherently risk averse. Azmat et al. (2014) explain this externality by showing that returning the principal, which is an integral part of the debt contract, increases the riskiness of the decision in situations where the underlying projects are inherently risky. The challenge with these streams of literature is that they approach the question of debt externality from a social planner or spectator's perspective. In the moment when the decision regarding the optimality of debt has to be taken by the investor, given the informational and institutional environment, debt contract remains the most optimal contract. The state contingent contract, owing to costly state verification, remains a less viable contract. The literature on debt externality therefore proposes an external intervention by the regulator, in the form of incentive, to ensure its viability for the investor. In this paper we take a different approach from what is otherwise proposed in the literature, and try to show that the state contingent contract remains a viable, beneficial and profitable contract for certain type of borrowing while debt remains the profitable option for others. We reframe the debate regarding the superiority of each type of financing by moving away from the idea that each type is universally better than the other from a profitability perspective, towards identifying conditions where each might be better than the other. The important contribution of our paper is that it depicts state contingent banking and debt not as a universal choice between the two for all informational and institutional environments, but goes on to identify the regions where each become more optimal than the other.

We start with building a basic model involving a bank and a continuum of firms. The firms have initiated a project requiring financing from the bank. The returns from the projects are uncertain. We assume that the bank has the market power and after taking into account the firm's riskiness, decides which contract, either state contingent or debt, would more be more optimal for the bank. In case of debt contract, the bank would charge a fixed interest rate on the loan advanced. For state contingent contracts, the bank would receive a proportion of the returns from the underlying project. The rates offered on the debt contract and the proportion of profit on the state contingent contract are endogenous in the model. The firm, based on the type of the contract and the rates offered, makes a decision whether to implement a good or a bad project. The moral hazard concern can emanate from either the firm shirking in its efforts or siphoning the funds to a risker project. In our model these moral hazard concerns directly affect the bank's profitability. Our model shows that the contract type, whether state contingent or debt, affects the firm's moral hazard behavior. The bank optimizes the risk and return of the project in the presence of these moral hazard concerns. Our results show that highly risky projects, those with high variance, would have less moral hazard concerns and great profitability in the presence of state contingent banking. For projects with lower risk, debt contracts would be more profitable for the banks. We show these results by plotting isoprofit lines, and identifying regions where each type of banking would become more profitable than the other. We also discuss the empirical implications of our results for different informational and institutional environments. We argue that state contingent banking is the optimal banking model in emerging economies which are characterized by higher riskiness of projects and greater moral hazard concerns. We also make a case that for small firms, community banks and Islamic banks, state contingent banking should be more optimal than debt based banking

Our paper contributes to two streams of literature. Firstly, the emerging literature on the externality of debt (Mian et al., 2017; Fischer, 2013), has been critical of the nature of the debt contract and supports state contingent banking as a more welfare enhancing alternative. However, they approach the discussion from a welfare perspective and ignore the viability of state contingent contracts. In this paper we have focused on the conditions and cases when state

contingent banking becomes more profitable than debt. We also contribute to the costly state verification literature. Our model is related to Ueda (2004), which focuses on the monitoring role of venture capitals and compares conventional banking with the venture capitals. Our paper adopts a similar approach. Unlike Ueda (2004), however, our paper focusses on moral hazard, which is the driving force behind the optimal choice of a project<sup>3</sup>. The remaining part of the paper is structured as follows. The environment is described in Section 2. Section 3 discusses the state contingent banking model. The conventional banking model with debt is explained in Section 4. Section 5 compares the two models. The empirical implications of the model are analyzed in Section 6. Section 7 finally concludes the paper.

#### 2. Environment

In this section we discuss the economic environment in which our banking model operates. We consider an economy with three types of agents: financiers (also called depositors), entreprenuers and a bank. Time is discrete and lasts two periods. In the first period, the bank operating as a monopoly on the loan side with aggregate deposits of 1 unit lends to a continuum of perfectly competitive entreprenuers. Investors can also invest in a riskless storage technology which generates a gross return  $R_d$ . Entreprenuers are agents who have ideas but no wealth of their own. To convert their ideas into projects they need to borrow from the bank. They have an aggregate demand of 1 unit for investment in their projects. In the second period, the projects generate random return x. The return to entreprenuers cannot be less than zero because of limited liability. The entreprenuers have a choice between implementing good projects or bad projects. Returns from good projects have a distribution with pdf g(x) with continuous support  $x \in [0, \infty)$ , mean  $\mu_{\sigma}$  and variance  $\sigma_{\sigma}^2$ . Bad projects give return according to distribution b(x) in the second period, with continuous support  $x \in [0, \infty)$ , mean  $\mu_{b}$  and variance  $\sigma_{b}^{2}$ . The distributions of returns are assumed to be such that  $\mu_b < R_d < \mu_e$  and  $\sigma_b^2 > \sigma_e^2$ . Thus the distribution of returns of the good projects g(x) first order stochastically dominate the distribution of returns of bad projects b(x). Bad projects are attractive to the entreprenuers because by choosing bad projects, they get a private benefit S. The bank is perfectly able to monitor the returns of the projects, but is unable to monitor whether agents are choosing good projects or bad projects.

The bank operating as a monopoly can choose one of the two banking models: state contingent banking or conventional banking.

A conventional bank lends money to entreprenuers and charges them a constant gross interest rate R on the amount lent. If a project's return is less than R then the bank takes away all of the project's return. On the other hand if the project's return is greater than R, then the entreprenuers pay only a constant interest payment R to the bank and keep the rest. Thus the return to the bank from a project will be min[x, R]. A state contingent bank, on the other hand, lends money to entreprenuers and charges them a proportion  $\alpha$  on the return of their

<sup>&</sup>lt;sup>3</sup> Our model differs from costly state verification models used by Townsend (1979), Gale and Hellwig (1985) and Williamson (1987). These models are based on the revelation principle (Myerson, 1979), where the mechanism is designed in such a way that it is always in the interest of the borrower to report truthfully. We also follow this approach in the paper. However, due to limited liability that we assume throughout the paper, the banks cannot punish the entrepreneurs by taking more than what they received from the project. This means that upon reporting returns less than the agreed returns, the banks undertake an audit and get all the returns from the project. The cost of audit is a lump-sum cost, and since it does not enter marginal decisions, we have ignored this cost in our model. Our model also differs from Innes (1990) and their adoption of moral hazard. Our model borrows the concept that entrepreneurs exert a level of effort depending on the returns demanded by the financiers. However, our model differs from Innes (1990) with regard to the payments to the financiers by the entrepreneurs. Our model assumes a range of payments, depending on the outcome of the project, whereas Innes (1990) consider a payment of 0 in extreme case by entrepreneurs for taking good projects - a rather impractical solution.

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projects. These banks do not seize assets/returns from projects whose return is very low. Therefore, profitability of the bank directly depends upon the returns from the projects.

Since investors also have access to storage technology, the bank has to pay at least  $R_d$  on deposit to attract depositors. This may refer to the rate that the government offers on its debt (e.g., interest rate on Tbills), or it could be the rate on alternate sources of investments like direct financing (bonds). Thus  $R_d$  is determined exogeneously.

In the sections that follow, we will first consider a state contingent intermediation model in which banks share the profits of the project with entreprenuers. We the consider a conventional model and compare the two.

#### 3. State contingent bank model

#### 3.1. Bank profitability

The model is based on Stiglitz and Weiss (1981). A state contingent bank pays an exogenously determined interest rate  $R_d$  on deposits and charges return sharing rate  $\alpha$  on loans to entreprenuers. We can express the profit of a state contingent bank,  $\pi_{\rm S}$  as

$$\pi_{\rm S} = \alpha \int_0^\infty x dG(x) - R_d \tag{1}$$

where  $\alpha$  is the share of the bank in the project's return and  $\int_0^\infty x dG(x)$  is the expected return from a good project. We have the following theorem regarding the profitability of a state contingent bank:

**Proposition 1.** There exists a unique cut off value for the sharing rate charged by a state contingent bank,  $\overline{\alpha}$ , after which the bank earns a profit.

**Proof.** Consider a plane with expected return to a bank from projects on y-axis and  $\alpha$  on x-axis. In this plane  $R_d$  is a straight line with slope zero because  $R_d$  is exogenous and does not depend on  $\alpha$  and on the expected return from projects. Since bank's expected return increases as  $\alpha$  increases, expected return to a bank is upward sloping and monotonic in this plane and since  $\mu_g > R_d$ , it intersects  $R_d$  at the cut off value. Hence the cut off value of  $\alpha \in (0, 1)$  exists, and because of monotonicity, it is unique. QED.

Notice that at  $\alpha = 0$  the bank's expected return from projects is zero but the bank still has to pay  $R_d$ . This requires that  $\alpha$  should at least cover the cost of deposits for banks to be profitable.

The bank's participation constraint requires that the profit of the bank exceeds zero i.e.

$$\alpha \int_0^\infty x dG(x) - R_d \ge 0 \tag{2}$$

$$\alpha \ge \frac{R_d}{\mu_g} \tag{3}$$

State contingent banking will exist for the above mentioned values of  $\alpha$ . We get the cut off value of sharing rate  $\overline{\alpha}$  when Eq. (3) binds.  $\Box$ 

#### 3.2. Moral hazard for the borrowing entrepreneur

A state contingent bank would like to set  $\alpha$  as big as possible in order to increase its profitability but it cannot increase  $\alpha$  unconditionally. The following proposition formalizes this:

**Proposition 2.** There exists a unique cut-off value  $\hat{\alpha} = 1 - \frac{s}{[\mu_e - \mu_b]}$  for the sharing rate of the state contingent bank. For sharing rates bigger than  $\hat{\alpha}$ , entreprenuers implement bad projects.

**Proof.** At a particular value of  $\alpha$  the expected return to entreprenuers from good projects should be greater than their expected return from bad projects. Hence the moral hazard condition of the entreprenuer can be written as:

$$(1-\alpha)\int_0^\infty x dG(x) \ge S + (1-\alpha)\int_0^\infty x dB(x)$$
(4)

In the above equation  $(1 - \alpha)$  is the share of return of an entreprenuer and  $\int_0^\infty x dB(x)$  is the expected return from a bad project. From Eq. (4) we get

$$\alpha \le 1 - \frac{S}{[\mu_g - \mu_b]} \tag{5}$$

where  $\mu_g = \int_0^\infty x dG(x)$  and  $\mu_b = \int_0^\infty x dB(x)$ The solution to the above expression exists because  $\mu_G > \mu_B$ . Since  $\alpha$ cannot be negative  $S \leq [\mu_{g} - \mu_{b}]$ . This means that if private benefits of taking bad projects are big enough, then state contingent banking cannot exists. Since banks cannot monitor the projects taken ex-ante, entreprenuers have an incentive to take bad projects if their private benefits are large.

When Eq. (5) binds we get the cut off value of share rate:

$$\hat{\alpha} = 1 - \frac{S}{[\mu_g - \mu_b]} \tag{6}$$

Uniqueness follows from the fact that all S,  $\mu_{\rm g}$  and  $\mu_{\rm b}$  are constants. QED.

For  $\alpha > \hat{\alpha}$ , profitable deviations exist for entreprenuers<sup>4</sup> to choose bad projects.

If the banks increase  $\alpha$  to some large value  $\alpha > \overline{\alpha}$ , then the entrepreneurs have no incentive to take good projects. At large values of  $\alpha$ , entrepreneurs have to transfer most of the projects' return to the bank, so they are attracted towards bad projects where they get at least a private benefit S. The bank would set  $\alpha$  low enough to keep entrepreneurs incentive compatible.

#### 3.3. Feasible region of a state contingent bank

From the previous discussion we have the following result:

Result:. According to profitability and moral hazard conditions, equilibrium exists in the range  $\overline{\alpha} \leq \alpha \leq \hat{\alpha}$  where both the bank is profitable and entreprenuers take up good projects. Therefore the set of feasible banking region is  $\Omega_{S} = \{\alpha : \overline{\alpha} \leq \alpha \leq \widehat{\alpha}\}$ 

Using the result we can write the following:

$$\hat{\alpha}$$

Putting the value of  $\overline{\alpha}$  and  $\hat{\alpha}$  in the above expression and simplifying it for *S* we get:

(7)

$$S \le \mu_g - \mu_b - R_d \left[ 1 - \frac{\mu_b}{\mu_g} \right] \tag{8}$$

The term  $R_d \left[ 1 - \frac{\mu_b}{\mu_g} \right]$  is a further adjustment to the feasible value of benefit S for state contingent banking to exist, because of the cost that the bank incurs in attracting depositors.

#### 3.4. Comparative statics

 $\Rightarrow \overline{\alpha} \leq$ 

Before discussing comparative statics, let us discuss feasible range for the cost of deposit,  $R_d$ . Intuitively, if the cost of deposits is very high relative to the average return on good projects, the intermediation breaks down as the banks are unable to earn sufficient returns on their advances to compensate depositors. Since in our environment, average return on good projects is higher than the cost of deposits<sup>5</sup>, we will lok

<sup>&</sup>lt;sup>4</sup> If  $\alpha > \hat{\alpha}$  then the entreprenuers will go for bad projects and the bank profit expression will become  $\pi_{SC} = \alpha \int_0^\infty x dB(x) - R_d$ , which has to be greater than zero. This gives the value  $\alpha = \frac{R_d}{\mu_p}$ . However, since  $R_d > \mu_B$ , the incentive compatible value of  $\alpha$ becomes greater than 1  $^{\mu_{R}}_{\text{which}}$  is not possible. Therefore, equilibrium where banks earn profits does not exist for values  $\alpha > \hat{\alpha}$ .

<sup>&</sup>lt;sup>5</sup> This is the bare minimum requirement for intermediation to exist

more carefully at the moral hazard constraint. From Eq. (8), for a given level of private benefits,  $R_d \leq \mu_g - \frac{\mu_g}{\mu_g - \mu_b}S$ . This expression gives us an upper bound on the cost of deposits. If the moral hazard in an economy increases, the cost of deposit should decrease so the banks remain profitable while satisfying the moral hazard constraint of the entrepreneurs.

The lower bound on acceptable share rate,  $\overline{\alpha}$  increases in the deposit rate and decreases in the expected return from good projects. Both these results are intuituve. If the deporsit rate increases, banks need to get a bigger share from projects to remain profitable. On the other hand, if the expected return from projects increases, banks may retain a lesser proportion of the returns from all projects to cover their cost of funds. The cut-off profitability share rate does not depend on the variance of project returns, as long as the good project distribution first order stochastically dominates the distribution of returns from bad projects. The cut-off share rate also does not depend on private benefit *S* and the expected returns from bad projects,  $\mu_{\rm b}$ .

Comparative statics of the upper bound on acceptable share rate,  $\hat{\alpha}$  can be studied by rewriting Eq. (6) as

$$W(\alpha, \mu_g, \mu_b, S) \equiv 1 - \alpha - \frac{S}{\mu_g - \mu_b}$$
(9)

At the cut-off value, we know that  $W(\hat{\alpha}, \mu_g, \mu_g, S) = 0$ . Totally differentiating W(.), we get

$$dW = -d\hat{\alpha} - \frac{dS}{\mu_g - \mu_b} + \frac{S}{(\mu_g - \mu_b)^2} d\mu_g - \frac{S}{(\mu_g - \mu_b)^2} d\mu_b$$
(10)

If any of the parameters changes in *W*, the cut-off value  $\hat{\alpha}$  also changes so as to maintain the equality  $W(\hat{\alpha'}, \mu'_g, \mu'_b, S') = 0$  (this is the definition of  $\hat{\alpha}$ ), where the prime sign with each variable indicates a possibly changed value of the parameter. Therefore, *dW* must equal 0 for the cut-off value of  $\hat{\alpha}$ . To study the comparative statics, we first change a single parameter and study its effects on  $\hat{\alpha}$ . Doing so gives us the following results:

$$\frac{d\hat{\alpha}}{dS} = \frac{-1}{\mu_g - \mu_b} < 0 \tag{11}$$

$$\frac{d\hat{\alpha}}{d\mu_g} = \frac{S}{(\mu_g - \mu_b)^2} > 0 \tag{12}$$

$$\frac{d\hat{a}}{d\mu_b} = \frac{-S}{(\mu_g - \mu_b)^2} < 0$$
(13)

If the private benefits from taking up bad projects increase, then the upper limit on  $\alpha$  decreases as now the banks have to give more share to the entreprenuers to entice them not to take bad projects. If the expected value of returns from good projects increases, then banks can keep a bigger share with themsleves without violating the incentive compatible condition of the entreprenuers. If the expected return on good projects increases, entreprenuers get more from the project. Therefore,  $\hat{\alpha}$  should increase (entreprenuers' share should decrease) to maintain the strict equality for cut-off rate. Intuition for bad projects is same, except in the reverse direction.

The upper bound on feasible values of private benefits increases in  $\mu_{\rm g}$  and decreases in  $\mu_{\rm b}$  and  $R_d$ . If  $\mu_{\rm g}$  increases, then entreprenuers need to have a higher private benefit to forego higher expected returns in favor of taking bad projects and settling for lower expected returns in addition to the private benefits. The intuition for decreasing upper bound with  $\mu_{\rm b}$  is similar: if expected return on bad projects increases, entreprenuers need a smaller private benefit to take up bad projects. The cause of decrease in upper bound of *S* because of increasing  $R_d$  is a result of banks increases, entreprenuers get lesser share in profits, and hence the upper bound of *S* decreases.

#### 4. Conventional banking model

#### 4.1. Bank profitability

A conventional bank charges a constant interest rate *R* to entreprenuers on amounts lent for their projects. All the entreprenuers whose return on projects is greater than the interest rate charged by a bank pay only the agreed rate *R* to the bank. Entreprenuers whose return from their projects is less than *R* pay only the projects' return to the bank because of limited liability. The profit of the conventional bank  $\pi_C$  can be expressed as

$$\pi_C = \int_0^R \min[x, R] dG(x) - R_d \tag{14}$$

The above expression can be simplified as

$$\pi_{C} = \int_{0}^{R} dG(x) \frac{\int_{0}^{K} x dG(x)}{\int_{0}^{R} dG(x)} + R \int_{R}^{\infty} dG(x) - R_{d}$$
(15)

In the equation above  $\frac{\int_0^R x dG(x)}{\int_0^R dG(x)}$  is the conditional expected return<sup>6</sup>

from a good project given the project return is less than R and  $\int_0^R dG(x)$  is the probability of good projects having return less than R (which is also the measure of projects having return less than R) and  $\int_R^\infty dG(x)$  is the measure of good projects with return greater than R.

Bank's participation constraint is:

$$\int_{0}^{R} dG(x) \frac{\int_{0}^{R} x dG(x)}{\int_{0}^{R} dG(x)} + R \int_{R}^{\infty} dG(x) - R_{d} \ge 0$$
(16)

In the above equation  $\int_0^R dG(x) \frac{\int_0^R x dG(x)}{\int_0^R dG(x)} + R \int_R^\infty dG(x)$  is the revenue of a bank from good projects.

Eq. (16) can be written as:

$$R \ge R_d + \int_0^R G(x) dx \tag{17}$$

In the above equation the first term on the R.H.S. is the cost of deposits and the second term is the cost of serving entreprenuers having ex-post return less than *R*. To remain profitable an interest rate charged by a conventional bank should cover these two costs.

**Proposition 3.** There exists a unique cut off value for interest rate on loans  $\overline{R}$  below which bank is not profitable.

**Proof.** As Eq. (17) binds we get the cut off value of interest rate  $\overline{R}$  for coventional bank profitability as:

$$\overline{R} = R_d + \int_0^{\overline{R}} G(x) dx$$
(18)

Consider a plane having expected return to the bank on y-axis and the interest rate charged by the bank on the x-axis. In this plane  $R_d$  is a straight line with slope zero because  $R_d$  is exogenous and does not depend on either of the returns. However as the interest rate increases, the bank's expected revenue increases. The revenue function of the bank is upward sloping and monotonic. This can be seen by the slope of the revenue function of the bank which is:

$$1 - G(R_L) \tag{19}$$

Since the above expression is always positive, the slope of the bank's return on loans is positive and thus a unique  $\overline{R}$  exists and  $\overline{R} = R_d + \int_0^R G(x) dx$  for sufficiently small values of  $R_d$ .

<sup>&</sup>lt;sup>6</sup> For bank the terms revenue and return are interchangeable as the measure of loans is 1.

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The second derivative of the expression (7) is -g(R) which shows that the bank's revenue function is concave.

The cut-off value for bank profitability  $\overline{R}$  is increasing in  $R_d$  and  $\sigma_g$ . If the cost of funds,  $R_d$ , increases, banks naturally have to increase R to remain profitable. If the variance of project returns increases, more projects now give returns less than the interest rate,  $\overline{R}$ . Therefore, banks have to increase  $\overline{R}$  to remain profitable.

#### 4.2. Moral hazard equation for the borrowing entrepreneur

Since the profit function of a conventional bank is increasing in R, a conventional bank naturally would try to increase its profit by increasing the interest rate R. However the conventional bank cannot increase R to a very large value, as then it would incentivize the entreprenuers to take bad projects. Very large values of interest rate R induce entreprenuers to implement bad projects by giving them an additional private benefit which they get from shirking and becoming careless. Furthermore entreprenuers get attracted to invest in riskier projects having higher upside returns so that their residual returns are greater. Thus the bank is limited by moral hazard on the part of entreprenuers in charging a very high interest rate. At a particular value of R, the expected return to entreprenuers from good projects should be greater than their expected return from bad projects. We have

$$0. \frac{\int_{0}^{R} xdG(x)}{\int_{0}^{R} dG(x)} \int_{0}^{R} dG(x) + \frac{\int_{R}^{\infty} (x - R)dG(x)}{\int_{R}^{\infty} dG(x)} \int_{R}^{\infty} dG(x)$$
  

$$\geq S + 0. \frac{\int_{0}^{R} xdB(x)}{\int_{0}^{R} dB(x)} \int_{0}^{R} dB(x) + \frac{\int_{R}^{\infty} (x - R)dB(x)}{\int_{R}^{\infty} dB(x)} \int_{R}^{\infty} dB(x)$$
(20)

In the equation above  $\frac{\int_0^R xdG(x)}{\int_0^R dG(x)}$  (resp.  $\frac{\int_0^R xdB(x)}{\int_0^R dB(x)}$ ) is the conditional expected return of the entreprenuer who implements a good project (resp. bad project), but his project generates a return less than R, and  $\int_0^R dG(x)$  (resp.  $\int_0^R dB(x)$ ) is the probability that the return is less than R, and  $\frac{\int_{R}^{\infty} (x-R)dG(x)}{\int_{R}^{\infty} dG(x)}$  (resp.  $\frac{\int_{R}^{\infty} (x-R)dB(x)}{\int_{R}^{\infty} dB(x)}$ ) is the conditional residual return of the entreprenuer who implements a good project (bad project) and his project generates a return greater than R, and  $\int_{R}^{\infty} dG(x)$  (resp.  $\frac{\int_{R}^{\infty} (x-R)dB(x)}{\int_{R}^{\infty} dB(x)}$ ) is the probability that the return is begin and his project generates a return greater than R, and  $\int_{R}^{\infty} dG(x)$  (resp.  $\int_{R}^{\infty} dB(x)$ ) is the probability that the return from the project is bigger than R.

**Proposition 4.** There exists a unique cut off value of interest rate  $\hat{R}$  charged by a conventional bank on loan after which entreprenuers have an incentive to implement bad projects.

Proof. From Eq. (20) we get,

$$\mu_{g} - \int_{0}^{R} [1 - G(x)] dx \ge S + \mu_{b} - \int_{0}^{R} [1 - B(x)] dx$$

$$\Box$$
(21)

When R = 0, the left hand side equals  $\mu_g$  whereas the right hand side equals  $S + \mu_b$ . For sufficiently low values of S (we can also observe from the profitability condition of state contingent banking that  $\mu_g > \mu_b + S$ ), the right hand side is bigger than the left hand side, which means that the inequality is satisfied and agents take up good projects for R = 0. Consider the case when  $R \to \infty$ . The left hand side is 0, whereas the right hand side is S. Therefore, the above inequality is satisfied for some  $\hat{R}$  between 0 and  $\infty$ . Uniqueness follows from the monotonicity of both the sides of the inequality<sup>7</sup>. QED.

#### 4.3. Feasible conventional banking region

Previous discussion gives the following result:

Equilibrium exists in the range  $\overline{R} < R < \widehat{R}$  where the banks are profitable and the entrpreneurs take up good projects. Therefore, the set of feasible conventional banking region is  $\Omega_C = \{R: \overline{R} < R < \widehat{R}\}$ .

A conventional bank can charge any interest rate within this range.

#### 4.4. Comparative statics

Cut-off values of both the profitability interest rate  $\overline{R}$  and incentive compatible interest rate for entreprenuers  $\hat{R}$  are sensitive to changes in various paramters in the model. In this section, we will go through some of the comparative statics for both these cut-off values, beginning with profitability interest rate  $\overline{R}$ .

With the abuse of notation (W has already been used above), let

$$W(\overline{R}, R_d, \mu_g, \sigma_g) \equiv \overline{R} - R_d - \int_0^{\overline{R}} G(x) dx = \mu_g - R_d$$
$$- \int_{\overline{R}}^{\infty} [1 - G(x)] dx = 0$$
(22)

Total differentiation gives us

$$dW(.) = d\mu_g + [1 - G(\overline{R})]d\overline{R} - dR_d - \frac{\partial}{\partial\sigma} \left[\int_{\overline{R}}^{\infty} [1 - G(x)]dx\right]d\sigma$$
(23)

Since  $dW(\overline{R}, .) = 0$  (by definition), we get the following comparative statics:

$$\frac{d\overline{R}}{d\mu_g} = \frac{-1}{1 - G(\overline{R})} < 0 \tag{24}$$

$$\frac{d\overline{R}}{dR_d} = \frac{1}{1 - G(\overline{R})} = 1 + \frac{G(\overline{R})}{1 - G(\overline{R})} > 0$$
(25)

$$\frac{d\overline{R}}{d\sigma} = \frac{\partial}{\partial\sigma} \left[ \int_{\overline{R}}^{\infty} \left[ 1 - G(x) \right] dx \right] = \frac{\partial}{\partial\sigma} \left[ \mu_g - \overline{R} + \int_0^{\overline{R}} G(x) dx \right] > 0$$
(26)

If the expected return on projects increases, then with the variance unchanged, there are more projects that give returns more than  $\overline{R}$ . Therefore, the bank can charge a lower interest rate to be profitable, reducing  $\overline{R}$ . The second equation is intuitive: as the deposit rate increases, the bank needs to charge a higher interest rate to be profitable. The change in  $\overline{R}$  is more than 1 because increasing the interest rate also increases the measure of projects giving return less than the charged interest rate. The second term,  $\frac{G(\overline{R})}{1-G(\overline{R})}$  is the adjustment because of this increase in measure of projects below the cut-off interest rate. Increased variance of the returns from good projects less than the initial  $\overline{R}$  increases, causing  $\overline{R}$  to increase to make the banks profitable.

Now lets consider the upper bound of the interest rate  $\hat{R}$  that the bank can charge on their loans to entreprenuers. Let (with the abuse of notation)  $W(.) = \mu_g - \int_0^{\hat{R}} [1 - G(x)] dx - S - \mu_b + \int_0^{\hat{R}} [1 - B(x)] dx$ . Total differentiation gives

$$dW(.) = d\mu_g - dS - d\mu_b + \left[\frac{\partial}{\partial \hat{R}} \int_0^{\hat{R}} G(x) dx\right] d\hat{R} - \left[\frac{\partial}{\partial \hat{R}} \int_0^{\hat{R}} B(x) dx\right] d\hat{R} + \left[\frac{\partial}{\partial \sigma_g} \int_0^{\hat{R}} G(x) dx\right] d\sigma_g - \left[\frac{\partial}{\partial \sigma_g} \int_0^{\hat{R}} B(x) dx\right] d\sigma_g$$
(27)

Lets first consider  $\mu_{\rm g}$ . We know that by definition,  $dW(\widehat{R}_{,.}) = 0$ . Therefore, we obtain the following equations (assuming change in parameters that are not of interest equal to 0)

$$\frac{d\widehat{R}}{d\mu_g} = \frac{1}{B(\widehat{R}) - G(\widehat{R})} > 0$$
(28)

$$\frac{d\widehat{R}}{d\mu_b} = \frac{-1}{B(\widehat{R}) - G(\widehat{R})} < 0 \tag{29}$$

<sup>&</sup>lt;sup>7</sup> Consider LHS. To check monotonicity,  $\frac{\partial [\mu_g - \int_0^R [1 - G(x)] dx]}{\partial R} = -[1 - G(R)] < 0$  for all  $R \in [0, \infty)$ 

$$\frac{d\hat{R}}{dS} = \frac{-1}{B(\hat{R}) - G(\hat{R})} < 0 \tag{30}$$

$$\frac{d\widehat{R}}{d\sigma_g} = \frac{\frac{\partial}{\partial \sigma_g} \int_0^{\widehat{R}} G(x) dx}{B(\widehat{R}) - G(\widehat{R})} > 0$$
(31)

If the expected return from good projects increases, then the upside for the entreprenuers also increases. Therefore, the entreprenuers' incentive constraint can support a bigger  $\hat{R}$ , as now their stake in projects increases because of the increase in the expected return from good projects. If the expected return from bad projects increases, then  $\hat{R}$  decreases, as now taking up bad projects becomes more attractive to the entreprenuers. To counter this effect,  $\hat{R}$  decreases. Increases in private benefit *S* has a similar effect and intuition. Increase in variance of good projects increases  $\hat{R}$ . As the variance increases, the measure of projects giving higher returns also increases. Therefore, the stake of entreprenuers in good projects also increases, causing an increase in the cut-off value  $\hat{R}$ .

#### 5. Banking regions

Given the analysis in the previous sections, we can identify the regions of the banking industry where any specific banking model will function Fig. 1.

In Fig. 2, sharing rate of the state contingent bank is on y axis and interest rate of the conventional banks is on x axis. The value of  $\overline{\alpha}$  and  $\hat{\alpha}$  do not change with *R*. Thus we get horizontal lines for  $\overline{\alpha}$  and  $\hat{\alpha}$  in this plane. Similarly  $\overline{R}$  and  $\hat{R}$  do not change with  $\alpha$ . Thus we get the vertical lines for  $\overline{R}$  and  $\hat{R}$ .

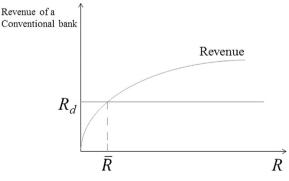
In Fig. 2, three types of regions are identified on the basis of individual rationality (profitability constraint) of a bank and the incentive compatability (moral hazard constraint) of an entreprenuer.

- The region in dark grey represents the ones where neither the state contingent bank nor the conventional bank operate i.e. Ω<sub>S</sub> = Ω<sub>C</sub> = φ.
- In the light grey regions, only one type of bank operates i.e. either
   Ω<sub>C</sub> = φ or Ω<sub>S</sub> = φ.
- Both types of banks are feasible in the white regions i.e.,  $\Omega_S \neq \phi$  and  $\Omega_C \neq \phi$ .

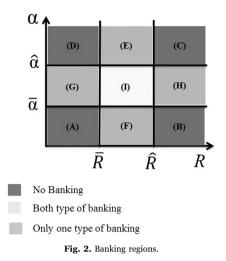
Table 1 illustrates banking regions in detail.

Any change in the private benefit of the entreprenuers shifts  $\hat{\alpha}$  and  $\hat{R}$ . In Fig. 2, an increase in private benefit of entreprenuers shifts  $\hat{\alpha}$  downwards and  $\hat{R}$  leftwards. If the private benefit becomes too large such that  $\hat{\alpha} < \overline{\alpha}$  and  $\hat{R} < \overline{R}$  then it is not profitable for the state contingent bank and the conventional bank to operate respectively.

In Fig. 3, a large increase in the private benefit of the entreprenuers of the conventional bank has shifted  $\hat{R}$  below  $\overline{R}$ . Therefore it is never feasible for the conventional bank to operate. In the lower dark grey regions the state contingent bank does not exist because participation







constraint is violated. In the upper dark grey regions, the state contingent bank does not exist as its entreprenuers face moral hazard. Thus in dark grey regions, both types of banks are not feasible. The white regions in Fig. 3 are the areas where state contingent bank operates as it satisfies both the individual rationality and the incentive compatibility constraints. Hence in this situation, a monopoly bank adopts state contingent banking as it is the only feasible type of banking.

Similarly Fig. 4 shows an increase in the private benefit of the entreprenuers who borrowed from a state contingent bank. A large increase in the private benefit shifts  $\hat{\alpha}$  below  $\overline{\alpha}$ . It means that it is never feasible for the state contingent bank to operate either due to moral hazard of entreprenuers or due to violation of participation constraint. In the dark grey regions to the left conventional banking is not profitable. In the dark grey regions to the right of Fig. 4, the incentive compatability constraint of the conventional bank's enreprenuers is not satisfied. Therefore no banking type is feasible in these regions. In the white regions of Fig. 4 only the conventional bank operates.

#### 5.1. Isoprofit curve

#### Definition

The curve which plots the values of *R* and a where the profit of the conventional bank equals the profit of the state contingent bank is known as isoprofit curve.

The expression for the isoprofit curve is obtained by equating  $\pi_S$  and  $\pi_C$  which gives:

$$\alpha \int_{0}^{\infty} x dG(x) - R_{d} = \int_{0}^{R} dG(x) \frac{\int_{0}^{R} x dG(x)}{\int_{0}^{R} dG(x)} + R \int_{R}^{\infty} dG(x) - R_{d}$$
(32)

Simplifying the above equation we get:

$$\alpha = \frac{R - \int_0^R G(x) dx}{\mu_G}$$
(33)

The derivative of  $\alpha$  with respect to *R* is:

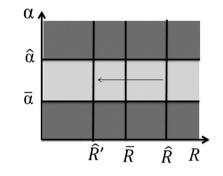
$$\frac{d\alpha}{dR} = \frac{1 - G(R)}{\mu_G} \tag{34}$$

The above expression is always positive which means the curve is upward sloping and monotonic. The slope of this curve is positive because when the conventional bank increases R its profits increase. To remain equally profitable the state contingent bank also needs to increase  $\alpha$ .

The second derivative of Eq. (33) is  $\frac{-g(R)}{\mu_g}$  which means the isoprofit

Table 1 Banking regions.

Region		Conventional banking		State contingent banking		Feasible model
		Individual rationality constraint satisfied	Incentive compatibility constraint satisfied	Individual rationality constraint satisfied	Incentive compatibility constraint satisfied	—
Dark Grey	Α	No	No	No	No	None
	В	No	No	No	No	None
	С	No	No	No	No	None
	D	No	No	No	No	None
Light Grey	Е	Yes	Yes	Yes	No	Conventional Banking
	F	Yes	Yes	No	Yes	Conventional Banking
	G	No	Yes	Yes	Yes	State Contingent
						Banking
	Н	Yes	No	Yes	Yes	State Contingent
						Banking
White	I	Yes	Yes	Yes	Yes	Both



No Banking State Contingent Banking

Fig. 3. Change in private benefit for a conventional bank.

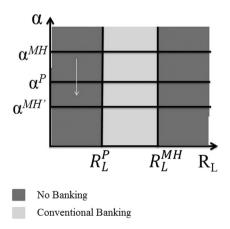


Fig. 4. Change in private benefit for a state contingent bank.

curve is concave.

The elasticity of  $\alpha$  with respect to  $R_L$  is:

$$\frac{\frac{\Delta \alpha}{\alpha}}{\frac{\Delta R}{R}} = \frac{\Delta \alpha}{\Delta R} \frac{R}{\alpha}$$

$$\epsilon_{\alpha} = \frac{R - RG(R)}{\alpha} < 1$$
(35)

$$x = \frac{1}{R - \int_0^R G(x) dx} < 1$$
(36)

The elasticity of  $\alpha$  with respect to R is less than 1. The concavity and elasticity of the isoprofit curve imply that 1% increase in R by the conventional bank induces less than 1% increase in  $\alpha$  of the state contingent bank to stay on the isoprofit curve. This is because by

increasing *R* the conventional bank increases only its fixed return and does not have any claims on returns higher than *R*. On the other hand the state contingent bank gets higher returns from all the projects by increasing  $\alpha$ .

#### Result:

The origin of isoprofit curve is the intersection of  $\overline{R}$  and  $\overline{\alpha}$ . At  $\overline{\alpha}$  the profit of state contingent bank is zero and at  $\overline{R}$  the profit of conventional bank is zero. Therefore the isoprofit curve starts from the intersection point of  $\overline{R}$  and  $\overline{\alpha}$  lines.

#### 5.1.1. Comprative statics

We can get comparative statics of  $\alpha$  by differentiating with respect to the parameters at given levels of interest rate, *R*.

$$\frac{\partial \alpha}{\partial \mu_g} = -\frac{R - \int_0^{\kappa} G(x) dx}{(\mu_g)^2} < 0$$
(37)

$$\frac{\partial \alpha}{\partial \sigma_g} = -\frac{\frac{\partial}{\partial \sigma_g} \int_0^\kappa G(x) dx}{\mu_g} < 0$$
(38)

As the expected return on projects increases, the state contingent bank may reduce its share of profits to be just as profitable as the conventional bank charging R. Therefore, the isoprofit line rotates downwards with an increase in expected return from good projects. The share of profits of the state contingent bank is also decreasing in variance of good projects. If the variance of good projects increases, the profitability of the conventional bank decreases at a given R, as now there are more projects that give a return below R, resulting in a downward rotation and a lower  $\alpha$  on the isoprofit line.

#### 5.2. Intersection of isoprofit line with moral hazard upper bounds

Isoprofit curve may intersect  $\hat{R}$  at a point where  $\alpha = \hat{\alpha}$  or  $\alpha < \hat{\alpha}$ . Similarly isoprofit curve may intersect  $\hat{\alpha}$  at a point where  $R < \hat{R}$ . For a detailed mathematical treatment, see A and B.

#### 5.2.1. Case I

Isoprofit curve intersects  $\hat{R}$  at a point where  $\alpha$  equals  $\hat{\alpha}$ . Evaluating  $\alpha$  at  $\hat{R}$  by putting  $\hat{R}$  in the isoprofit curve we get:

$$\hat{\alpha} = \frac{\hat{R} - \int_0^R G(x) dx}{\mu_g}$$
(39)

Using Eq. (4) the above expression becomes:

$$\widehat{R} - \int_{0}^{R} G(x) dx = \mu_{g} - \frac{\mu_{g}}{\mu_{g} - \mu_{b}} S$$
(40)

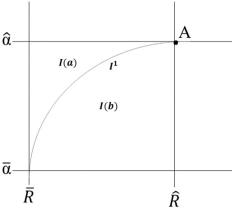


Fig. 5. Isoprofit curve-case I.

If the above condition is satisfied then the isoprofit curve intersects  $\hat{R}$  line at  $\hat{\alpha}$  (point A in Fig. 5). The isoprofit curve divides the bank profitability region in Fig. 5 into two sub-regions I(a) and I(b). In the region I(a) the profit of the state contingent bank is greater than the conventional bank. In the region I(b) the profit of the state contingent bank is less than the profit of the conventional bank. On the isoprofit curve  $I^1$  a bank is indifferent between state contingent banking and conventional banking. To maximize profit a monopoly bank would like to charge enterpreneurs at the cut off value of moral hazard (point A).

#### 5.2.2. Case II

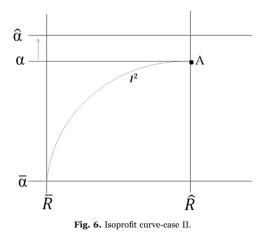
If isoprofit curve intersects  $\widehat{R}$  line at a point where  $\alpha$  is less than  $\widehat{\alpha}$ , then:

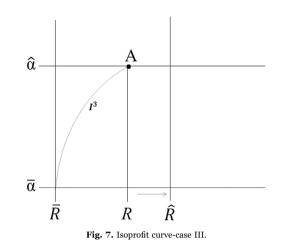
$$\widehat{R} - \int_{0}^{\widehat{R}} G(x) dx < \mu_{g} - \frac{\mu_{g}}{\mu_{g} - \mu_{b}} S$$
(41)

In this case, isoprofit curve  $I^2$  is plotted in Fig. 6. The curve  $I^2$  intersects  $\hat{R}$  at the point A where  $\alpha < \hat{\alpha}$ . By adopting conventional banking, the bank cannot charge entrepreneur higher than  $\hat{R}$ , staying point A. If it charges higher, then entrepreneurs take bad projects. Notice that at this point  $\alpha < \hat{\alpha}$ , therefore the monopoly bank can choose state contingent banking to maximize its profits by charging  $\alpha = \hat{\alpha}$ . Thus a monopoly bank prefers state contingent banking as opposed to conventioanl banking.

#### 5.2.3. Case III

If isoprofit curve intersects  $\hat{\alpha}$  line at a point where *R* is less than  $\hat{R}$  then  $\hat{\alpha} = \frac{R - \int_0^{R} G(x) dx}{\mu g}$  for  $R < \hat{R}$ . Putting the value of  $\hat{\alpha}$ , the expression becomes as shown below:







The curve  $I^3$  depicts this scenario in Fig. 7. In this case a monopoly bank prefers conventional banking to state contingent banking because the curve intersects  $\hat{\alpha}$  at a point where  $R < \hat{R}$ . At point A the state contingent bank cannot increase  $\alpha$  but the conventional bank has the capacity to increase R as it is less than  $\hat{R}$  and by doing so, a monopoly bank can maximize its profits. Therefore a monopoly bank prefers conventional banking.

#### 6. Empirical implications

This section discusses the empirical implications of our model to explain the case of small firms, financing in the emerging economies and the challenges of community and Islamic banking.

#### 6.1. Small firms

Small firms are characterized by limited access to collateralized assets that can be pledged for financing. With limited pledged assets, moral hazard becomes a challenge as entrepreneurs have less 'skin in the game': It increases financial frictions. As financial frictions are higher, the moral hazard cut-off lines shift leftwards (for conventional banking) and downwards for state contingent banking. This results in a contraction of the feasible region where the banks can exist. However, due to concavity of the iso-profit line, it can be observed that the increased financial friction makes the moral hazard problem more severe. This should result in state contingent banking becoming more profitable in the case of small firms.

#### 6.2. Emerging markets

Our model can be applied to financing in emerging markets. Firms in the emerging markets are more credit constrained compared to their counterparts in the developed world. This is because information asymmetry concerns are higher in these countries, possibly because of weaker legal systems. Conventional banks are less willing to lend. Our model shows that credit supply can increase in these countries by using state contingent banking. The model predicts that profitable financing region for state contingent banking would be bigger relative to the conventional banking region for more severe moral hazard conditions. In emerging markets state contingent may prove to be more suitable than their conventional counter part.

#### 6.3. Community banking

Community banking is a depository institution that normally serves the need of local communities. These banks have better knowledge of the

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local requirements, so they can monitor lending activities more effectively. This monitoring may make the moral hazard problem less severe. If these banks adopt a state contingent banking model, they would be in a better position to finance riskier projects. As projects in small communities are generally riskier, a state contingent banking model would be better able to finance them compared to conventional banks.

#### 6.4. Islamic banking

Our model suggests that Islamic banks can also benefit from state contingent banking. Given that debt contracts involving a predetermined return are prohibited in Islam, state contingent banking can be used by Islamic banks to finance new ventures and riskier projects.

#### 7. Conclusion

This paper compares state contingent banking with conventional debt based banking. We show the neither form of banking is universally superior to the other; rather their optimality depends on the riskiness of the underlying projects and moral hazard concerns. We find that state contingent banking is more profitable where projects are riskier, and debt based conventional banking is adopted for relatively lower risk projects. Our model also suggests that state contingent banking would be the optimal choice in cases where there exist greater moral hazard concerns. In our model, banks optimize both the riskiness of the project and moral hazard concerns to identify the most profitable banking model. We explore the empirical implications of our model and find that state contingent banking would be more suitable for small firms, emerging markets, community and Islamic banking.

While the previous literature had identified issues related to the externality of debt and proposed state contingent banking as a more welfare enhancing alternative, the viability and profitability of this form of banking was largely ignored. We have tried to fill this gap by identifying the informational and institutional environments where state contingent banking may become the more profitable banking model.

Our findings should help regulators, policy makers and banks to better implement state contingent banking. For regulators and policy makers, our paper has shown that this banking model is not simply about increasing societal welfare but remains a viable and incentive compatible banking model. For the banks, our results indicate that by adopting state contingent banking, they can make use of profitable opportunities which otherwise would seem too risky to undertake.

#### Appendix A

Before cotinuing, let us define  $E_g$  and  $E_b$  as

$$E_g = \frac{\int_R^{\infty} x dG(x)}{\int_{R_L}^{\infty} dG(x)}$$
$$E_b = \frac{\int_R^{\infty} x dB(x)}{\int_{R_L}^{\infty} dB(x)}$$

The terms  $E_g$  and  $E_b$  are expected values of returns, conditional on return being bigger than  $R_L$ .

**Proof.** Consider a plane with conditional expected return from a project on y axis and bank interest rate on loans on x axis. According to assumption of the model, at R = 0,  $E_g > E_b$ . As R increases, eventually there comes a point when the increase in conditional expected return from good projects becomes lower than the increase in the conditional expected return from bad projects i.e.  $E_g' < E_b'$ . Thus, conditional expected return from bad projects intersects conditional expected return from a good projects at a cut off value  $\hat{R}$  where  $E_g = E_b + S$ . At  $R < \hat{R}$  entreprenuers implement good projects. We check whether the condition  $E_{g'} < E_{b'}$  for the existence of  $\hat{R}$ , holds.<sup>8</sup>

$$E_{g} = \frac{\int_{R} x dG(x)}{\int_{R_{L}}^{\infty} dG(x)}$$

$$E_{g} = \lim_{U \to \infty} \left[ \frac{\left[ -x \left( 1 - G(R) \right) \right]_{R}^{U} + \int_{R}^{U} \left( 1 - G(R) \right) dx}{1 - G(R)} \right]$$

$$E_{g} = \lim_{U \to \infty} \left[ \frac{\left[ -U.0 + R(1 - G(R)) + \int_{R}^{U} \left( 1 - G(R) \right) dx}{1 - G(R)} \right]$$

8

li

$$\begin{split} \lim_{U \to \infty} \left[ \int_{R}^{U} xg(x)dx + \int_{R}^{U} G(x)dx \right] &= \lim_{U \to \infty} \left[ [xG(x)]_{R}^{U} \\ m_{U \to \infty} \int_{R}^{U} xg(x)dx = \lim_{U \to \infty} \left[ [xG(x)]_{R}^{U} - \lim_{U \to \infty} \int_{R}^{U} G(x)dx \\ \text{dd and subtract } \lim_{U \to \infty} [x]_{R}^{U} \\ m_{U \to \infty} \int_{R}^{U} xg(x)dx = \lim_{U \to \infty} [xG(x) - x]_{R}^{U} + \lim_{U \to \infty} \int_{R}^{U} (1 - G(x))dx \\ \text{nd} \\ m_{U \to \infty} \int_{R}^{U} (1 - G(x))dx = \lim_{U \to \infty} \int_{R}^{U} xg(x)dx - \lim_{U \to \infty} [xG(x) - x]_{R}^{U} \end{split}$$

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Now compute slope by differentiating with respect to R:

$$\begin{aligned} \frac{d}{dR}E_g &= \frac{(1-G(R))[1-G(R)-g(R)+\frac{d}{R}\lim_{U\to\infty}\int_R^U(1-G(R))dx]}{[1-G(R)]^2} + \frac{[R(1-G(R))+\lim_{U\to\infty}\int_R^U(1-G(R))dx]g(R)}{[1-G(R)]^2} \\ \frac{d}{dR}E_g &= \frac{(1-G(R_L))[1-G(R_L)-g(R_L)-1+G(R_L)]}{[1-G(R_L)]^2} + \frac{[R_L(1-G(R_L))+\lim_{U\to\infty}\int_R^U(1-G(R_L))dx]g(R_L)}{[1-G(R_L)]^2} \\ \frac{d}{dR}E_g &= \frac{-(1-G(R))[g(R)(R)]+g(R)R(1-G(R))}{[1-G(R)]^2} + \frac{g(R)\lim_{U\to\infty}\int_R^U(1-G(R))dx}{[1-G(R)]^2} \\ \frac{d}{dR}E_g &= \frac{g(R)\lim_{U\to\infty}\int_R^U(1-G(R))dx}{[1-G(R)]^2} \\ \frac{d}{dR}E_g &= \frac{g(R)}{1-G(R)}\frac{\lim_{U\to\infty}\int_R^U(1-G(R))dx}{[1-G(R)]} \\ \frac{d}{dR}E_g &= \frac{g(R)}{1-G(R)}\frac{\lim_{U\to\infty}\int_R^U(1-G(R))dx}{[1-G(R)]} \\ \frac{d}{dR}E_g &= \frac{g(R)}{1-G(R)}\frac{\lim_{U\to\infty}\int_R^U(1-G(R))dx}{[1-G(R)]} \\ \frac{d}{dR}E_g &= h_g[\frac{-R(1-G(R))}{[1-G(R)]} + \frac{\lim_{U\to\infty}\int_R^Uxg(x)dx}{1-G(R)} \end{aligned}$$

Where  $h_g$  is the hazard function of the good projects and is equal to  $\frac{g(R)}{[1 - G(R)]}$ . Now for  $\hat{R}$  to exist the following condition needs to be satisfied at some value of R

$$\begin{aligned} &\frac{d}{dR}E_g < \frac{d}{dR}E_b \\ &h_g[E_g - R] < h_b[E_b - R] \\ &R < \frac{h_bE_b - h_gE_g}{(h_b - h_g)} \end{aligned}$$

The above expression does not exist for the values of *R* where  $h_g = h_b$ . The moral hazard exists for all the other values of *R* which satisfy the above expression. Above expression can also be written as:

$$R + \frac{h_g}{h_b}(E_g - R) < E_b$$

The above expression shows that for the moral hazard to exist the conditional expected return from the bad projects  $E_b$  has to be sufficiently large.

#### Appendix B

• Comparison of cut off value of moral hazard with monitoring and cut off value of moral hazard without monitoring for a conventional bank. Whether cut off value of interest rate with monitoring lies before or after the cut off value of interest rate without monitoring depends on the magnitude of  $E_b$  and  $\widehat{R}^m$ . If

$$\widehat{R^m} < E_h \Longrightarrow \widehat{R^m} < \widehat{R}$$

Since the need of monitoring arose only because  $\hat{R} < \overline{R}$  so the cut off value of interest rate with monitoring cannot be less than  $\hat{R}$  because in that case banking will not be profitable.

If

$$\widehat{R^m} = E_h \Longrightarrow \widehat{R^m} = \widehat{R}$$

The equality of the two cut of values of interest rate is also not possible because then the banking with monitoring will also become unprofitable. If

$$\widehat{R^m} > E_b \Longrightarrow \widehat{R^m} > \widehat{R}$$

This is the only possible sloution for the cut off value of interest rate with monitoring. It has to be greater the cut off value of interest rate without monitoring so that the moral hazard does not arise.

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