Economic Modelling 30 (2013) 825–834

Contents lists available at SciVerse ScienceDirect

Economic Modelling

journal homepage: www.elsevier.com/locate/ecmod

Economic growth and polluting resources: Market equilibrium and optimal policies

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ARTICLE INFO

Article history:
Accepted 23 October 2012

JEL classification:
O13
O31
O44
Q58

Keywords:
Natural resources
Pollution
Economic growth
Environmental policy

ABSTRACT

This paper develops an endogenous growth model to study the decentralized equilibrium and the optimum conditions in an economy which uses polluting resources. The model includes two policy instruments, a subsidy to final consumption and an emissions tax. It also considers two forms of endogenous technical change, pollution-reducing knowledge and horizontal innovation. We show that, if the efficiency of knowledge to reduce emissions is sufficiently high, a higher output is compatible with lower emissions in both levels and growth rates. Additionally, if the two instruments are used together the economy may achieve a higher output and lower emissions since the subsidy may offset, at least partially, the negative tax effects.

1. Introduction

In this article we aim to analyze the compatibility between economic growth and a cleaner environment in a framework where production requires polluting resources and there is environmental policy. Our endogenous growth model assumes two forms of technical change: horizontal innovation in the natural resource sector and pollution-reducing knowledge accumulation in the final-goods sector. We start by analyzing the decentralized steady-state equilibrium. Then, we explore the policy implications when the government uses two policy tools, a tax on emissions and a subsidy to final consumption. After that, we derive the policy conditions under which the decentralized equilibrium is optimal. Finally, we perform a simple numerical exercise.

The set-up of our model follows Grimaud and Tournemaine (2007) (hereafter GT), but we depart from their model in several aspects, including our main focus. Firstly, in GT, growth is sustained by human capital accumulation and no natural resources are considered. The authors found that a tighter environmental policy promotes growth since it enhances the willingness of individuals to acquire education. We analyze an alternative path to harmonize the economy and the environment and, for that, we adapt the final-goods production function to use only natural resources which generate emissions.[1] Secondly, we assume horizontal innovation in the natural resources sector in an attempt to include something new in the literature. Throughout time scientists have found ways to use resources that were not usable before. For instance, uranium was not particularly useful before the development of the nuclear fission technology. These innovations increase the variety of usable natural resources. This type of differentiation, in line with Barro and Sala-i-Martin (2004, Ch. 6), among other, implies that when new resource varieties are discovered or made usable old ones do not become obsolete. Finally, we depart from GT by assuming that final-goods producers invest a given amount of their own product (instead of human capital) to generate knowledge, i.e., our model is lab-equipment and not knowledge driven (e.g., Rivera-Batiz and Romer, 1991).

Our model shows that, in the decentralized equilibrium, if the efficiency of knowledge to reduce emissions is sufficiently high, higher output is compatible with lower emissions both in their steady-state

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[1] Apart from the endogenous growth debate on the consideration of human capital accumulation and physical capital, we do not consider these production factors as mainly instrumental for the isolation of the effects of natural resources on economic growth and environment. For the same reason, we also abstract from the labor market (as, e.g., Grimaud and Tournemaine, 2007; Schou, 2002).There is no doubt that considering additional production factors would increase realism in our model, notwithstanding, the analysis would be more complicated at the risk of losing our main focus: the role of natural resources.

0264-9993/$ – see front matter
http://dx.doi.org/10.1016/j.econmod.2012.10.014

levels and their growth rates. Additionally, if the government uses the two instruments together it may achieve a higher output and lower emissions since the subsidy may offset, at least partially, the negative output effects of the tax. The derivation of the economic optimum gives the conditions to impose on public policies in order to achieve an optimal equilibrium. Our empirical application shows that the catching-up process between a developed and a developing country is faster in the central planner (optimum) situation than in the decentralized equilibrium.

The economic growth literature dealing with natural resources has often focused on the conditions of growth under scarcity, but has at times ignored a key aspect: resource use generates pollution (e.g., Barbier, 1999; Garg and Sweeney, 1978; Grimaud and Rougé, 2003; Scholz and Ziemes, 1999). Fossil fuels combustion and mineral resources are, in fact, responsible for a large share of anthropogenic pollution, and policies have been conducted worldwide in an attempt to reduce environmental problems (e.g., Halicioglu, 2009; Sadorsky, 2009; Soytas and Sari, 2009). If, to produce, firms use polluting resources, it is crucial to know how environmental policy affects economic growth (and consumption levels). The approach to this problem has differed among studies. Authors who include pollution often consider polluting resources as necessary but non-essential to production as they may be substituted by non-polluting resources or innovations (e.g., Bretschger and Smulders, 2007; Gradus and Smulders, 1993; Grimaud and Rougé, 2003). In particular, Bretschger and Smulders (2007) considered the possible substitution between polluting resources (energy) and non-polluting ones (labor and capital) but did not include the role of policy intervention in the harmonization of economic growth and the environment. Some models considered the role of innovation in overcoming resource scarcities, but modeled innovation as exogenous. In contrast, endogenous growth theory often ignored the contribution of natural resources to growth (Barbier, 1999). We consider two forms of technical change: final-goods producers run research activities to generate emission-reducing knowledge and resource firms run R&D to increase the variety of usable natural resources.

Authors who have found compatibility between a cleaner environment and economic growth commonly consider resource scarcity (e.g., Grimaud and Rougé, 2005; Schou, 2000, 2002). As Schou (2002) pointed out, if resources are scarce, in the long run the need to save them will necessarily reduce pollution. To avoid this problem we ignore resource scarcity exploring a new trail to match the economy and the environment. Implicitly we are assuming that the economy can extract as much resources as it needs to satisfy production.

Other authors study the relationship between economic growth and environmental quality, but do not include natural resources. In this case, we find, for example, Xapapadeas (2005) who found compatibility between a growing economy and a cleaner environment if some economic wealth was devoted to environmental protection or pollution abatement activities. In the same line, Gupta and Barman (2009) analyzed the problem in a dynamic perspective using an endogenous growth model. The authors focused on the interaction between public expenditure and environmental pollution when government allocated its tax revenue between pollution abatement and productive expenditure. They also examined the characteristic of the optimal fiscal policy in a dynamic perspective. Among other interesting findings, the article found no conflict between the social welfare maximizing solution and the growth rate maximizing solution in steady-state.

Finally, some authors consider a broader question, not only the relationship between economic growth and pollution, but the relationship between economic decisions and pollution dynamics. In this case, we find Saltari and Travaglini (2011) who analyze the effects of environmental policy on the value of the firm and investment decisions. These authors include pollution uncertainty and investment irreversibility and focus on two types of policy instruments: taxes on polluting inputs and subsidies to reduce the costs of abatement capital. They found that an increase in the tax may decrease the value of the firm and therefore decrease investment in abatement capital. The effect of the subsidy on the firm's value is undetermined.

The remainder of the paper is organized as follows. Section 2 presents the model set-up. Section 3 shows the market equilibrium conditions in the balanced growth path, highlighting its major properties. Section 4 sums up the environmental policy implications. Section 5 characterizes the optimum. Section 6 performs a numerical exercise. Finally, Section 7 concludes the paper.

2. Model set-up

We consider a model in continuous time with differentiated final-goods, and a natural resource sector. In this section we present the several sectors.

2.1. Final-goods producers

The differentiated final-goods are produced by an exogenous number of firms ($n=1,\dots,N$). These goods are sold in imperfectly competitive markets and produced using natural resources, R:

$$Y_{n,t}=A\sum_{j=1}^{J}R_{j,n,t} \tag{1}$$

where t represents the time, $Y_{n,t}$ is the output of firm n, J is the number of usable resources varieties, $R_{j,n,t}$ is the amount of the jth type of natural resources used by firm n, $A>1$ represents the overall productivity or efficiency of the economy.

Final-good producers run indoor research activities to generate pollution-reducing knowledge, Z. As in GT, the knowledge stock at each time is composed by a continuum of pieces. A piece of knowledge is an indivisible, infinitely-lived, differentiated, public good. In this specific case it refers to techniques which allow having less pollution for a given level of resources consumed, for instance, carbon capture and sequestration technologies or new production processes.

Each firm spends $\zeta_{n,t}$ units of its own output to produce new pieces of knowledge. $Z_{n,t}$ is the knowledge stock of firm n at time t.[2] New pieces of knowledge are produced with the technology:

$$\dot{Z}_{n,t}=\delta\zeta_{n,t} \tag{2}$$

where $\delta>0$ is a productivity parameter. This knowledge accumulation function implies that the more firms spend on research activities, the more knowledge they generate (e.g., Buonanno et al., 2003; Goulder and Schneider, 1999).

Knowledge is used to reduce pollution (e.g., Bovenberg and Smulders, 1995; Grimaud and Rougé, 2003; GT). The emissions flow is:

$$E_{n,t}=\sum_{j=1}^{J}R_{j,n,t}Z_t^{-\beta} \tag{3}$$

where $\beta>0$ measures the efficiency of knowledge to reduce pollution. Emissions increase with natural resources consumption (e.g., Bovenberg and Smulders, 1995; Schou, 2002), since it is the fossil fuels combustion and mineral resources use that generates most emissions/pollution in the production process. We treat emissions as a flow instead of a stock. In reality, many environmental issues last for several decades, but by considering pollution as a flow we simplify the analysis and reach similar results as we do treating emissions as a stock (e.g., Gradus and Smulders, 1993; Stokey, 1998).[3]

[2] Several models consider research conducted using only labor (e.g., Grimaud and Rougé, 2003, 2005; Schou, 2002). In line with lab-equipment growth models (e.g., Rivera-Batiz and Romer, 1991), we modify this view by considering firms spending a given amount of resources to conduct research.

[3] For a deeper discussion of this issue see, for example, GT and (Grimaud and Rougé, 2005).

2.2. Consumers

There is a mass [0,1] of identical individuals owning the economy's assets. We assume no population growth so that all aggregate variables can be interpreted as *per capita* quantities (e.g. Gupta and Barman, 2009). In fact, environmental problems emerge, to a certain extent, from population growth. Existing individuals consume more and, additionally, new individuals increase consumption even further.[4] Specifically, the inclusion of population growth would increase the discount rate, ρ (Acemoglu, 2009), but the qualitative results would remain unchanged.

Individuals value a clean environment, i.e., their utility increases with consumption and decreases with pollution. Their instantaneous utility function is:

$$U = \ln\left[\sum_{n=1}^{N} c_{n,t}{}^{\mu}\right]^{\frac{1}{\mu}} - \omega \ln E_t \tag{4}$$

where $0<\mu<1$ represents the elasticity of consumption, $\omega>0$ reflects the strength of environmental preferences, $0<\rho<1$ is the time preference rate, and $E_t = \sum_{n=1}^{N} E_{n,t}$ is the total emissions flow. This specification, chosen for simplicity, is in line with a great part of the literature (e.g., Gradus and Smulders, 1993; Grimaud and Rougé, 2005; Schou, 2002; GT) while other authors (e.g., Bovenberg and Smulders, 1995; Gupta and Barman, 2009; Saltari and Travaglini, 2011; Schou, 2000) consider that emissions also negatively affect productivity.

2.3. Resources sector

Innovation in the resources sector follows the prominent works of Romer (1990), Rivera-Batiz and Romer (1991), Grossman and Helpman (1991, Ch. 3) and Barro and Sala-i-Martin (2004, Ch. 6), among others. We consider technical progress through horizontal R&D. The specification reflects that, over the years, scientists have found ways to use resources which were not useful before. Since there are no quality improvements, no varieties ever become obsolete and research firms remain leaders since they are granted a permanent patent. Thus, monopolistic firms sell natural resources to final-good producers and, simultaneously, conduct R&D activities to increase the varieties of usable resources. This endogenous technical progress increases J. Natural resources production can be interpreted as oil extraction/refinery or wood treatment to make it usable by final-good producers.

2.4. Government

We first analyze the decentralized equilibrium with government intervention. The imperfect competition in final-goods production and the existence of pollution create distortions in this decentralized equilibrium. To deal with those distortions we assume, following GT, two policy instruments: a tax on emissions, τ_t, and a subsidy to final-goods consumption, $\sigma_{n,t}$ (which by assumption is $0<\sigma_{n,t}<1$).

3. Equilibrium

This section presents the decentralized long-run equilibrium conditions. The balanced growth path or steady-state is characterized by constant growth rates of all variables and clearance in all markets. Next, we describe the long-run equilibrium features and the agents' behavior in more detail. Since many derivations are common to GT, we have put them in Appendix A and focused here in the main results.

[4] Apart from the debate on the consideration of population growth, we abstract from that point, in order to isolate the effects of natural resources on economic growth and the environment.

3.1. Symmetric equilibrium and steady-state

In the symmetric equilibrium all final-good firms produce the same, use the same resources amount, create the same knowledge stock, emit the same pollution, charge the same price and make the same profit. Similarly, all resource firms produce the same, charge the same price, and make the same profit. These results are proved in the deduction of the market equilibrium. Furthermore, all output is used for consumption, knowledge investment in both the final-goods sector and in the resources sector extraction/production:

$$Y_t = C_t + \zeta_t + \eta_R \dot{J}_t + NJ_t R_{j,n,t} \tag{5}$$

As usual in the literature, we assume that, in steady-state, consumption is a fixed proportion (γ) of output. As in GT, the steady-state existence requires the term $\tau_t Z_t{}^{-\beta}$ to be constant over time. The government chooses a tax growth path such that $g_\tau = \beta g_z$, i.e., $\tau_t Z_t{}^{-\beta} = \tau_0 Z_0{}^{-\beta}$ where τ_0 and Z_0 are the initial (base year) values for the tax and the knowledge stock. We designate $x = \tau_0 Z_0{}^{-\beta}$, which represents environmental policy. Additionally, the final-goods subsidy is equal for all goods and constant overtime in steady-state (e.g., Grimaud and Rougé, 2003). The government budget is always balanced, i.e., tax revenues are used to finance the subsidy and a lump-sum transfer to individuals, T_t.

Knowledge is a public good, which raises difficulties in a decentralized equilibrium. Notwithstanding, the imperfect competition in the final-goods sector solves this problem. Other authors, e.g., Grimaud and Rougé (2005), consider public funded R&D. In our model, final-good firms compete "a la Cournot" and have free entry. They sell the differentiated goods at a price greater than the marginal production cost and spend the rest of their profits on knowledge. Since there is free market entry final profits are zero.

There are no verification, exclusion, and information problems in the knowledge creation process. Firm n's profit without payment of knowledge is denoted by $\tilde{\pi}_{n,t}$, total profit is $\pi_{n,t}$. Firm's Willingness-to-Pay (WTP) to use a piece of knowledge at time t, $v_{n,t}$, is $\frac{\partial \tilde{\pi}_{n,t}}{\partial z_{n,t}}$. The value of a piece of knowledge for firm n (or the price paid to use a piece of knowledge forever) is $\Theta_{n,t} = \int_t^\infty v_{n,s} e^{-\int_t^s r_u du} ds$, where r_u denotes the interest rate. The total (aggregated) value of a piece of knowledge is $\Theta_t = \int_t^\infty v_s e^{-\int_t^s r_u du} ds$, where $\Theta_t = \sum_{n=1}^{N} \Theta_{n,t}$ and $v_t = \sum_{n=1}^{N} v_{n,t}$. Differentiating the expression of Θ_t with respect to time we obtain:

$$r_t = \frac{v_t}{\Theta_t} + g_{\Theta_t} \tag{6}$$

where g_κ is the growth rate of any variable κ. This expression shows that the interest rate is equal to the present gain of knowledge investment (surplus value of each piece of knowledge) plus the increase in the knowledge value (a type of a capital gain). Hence, the return on investments is determined by the knowledge value and its growth.

3.2. Agents' behavior

3.2.1. Individuals

The representative individual maximizes utility (1), subject to the budget constraint $\dot{B}_t = r_t B_t - \sum_{n=1}^{N} (1-\sigma_{n,t}) p_{n,t} c_{n,t} - T_t$, where $p_{n,t}$, is the price of each final-good, and chooses plans for each final-good consumption, $c_{n,t}$, and wealth, B_t. The consumers' maximization problem gives final-goods demand (complete deduction in Appendix A):

$$p_{n,t} = \left(\frac{D_t}{\gamma Y_{n,t}}\right)^{1-\mu} * \frac{1}{\left(1-\sigma_{n,t}\right)} \tag{7}$$

where D_t is aggregate demand. The final-goods' price increases with aggregate demand and with the subsidy to final consumption, but decreases with the consumption of its own good. The Keynes–Ramsey rule, in equilibrium, simplifies to:

$$r_t = g_{Y_{n,t}} + \rho \tag{8}$$

This condition summarizes the consumers' decisions. As usual in the literature (e.g., Bovenberg and Smulders, 1995), individuals postpone consumption if saving (i.e. earning the return rate) compensates for the rate of time-preference and the consumption's marginal value change.

3.2.2. Final-goods sector

Final-good producers use all available natural resources varieties to produce. Simultaneously, they invest in knowledge to reduce emissions. The maximization problem in this sector allows to obtain the demand of each resource variety, the value of a piece of knowledge, and firm's WTP to use a piece of knowledge, respectively (complete deduction in Appendix A):

$$R_{j,n,t} = \frac{D_t}{\gamma A J_t}\left[\frac{\mu(A-x)}{\psi_{j,t}(1-\sigma_{n,t})}\right]^{\frac{1}{1-\mu}} \tag{9}$$

$$\Theta = \frac{1}{\delta} \tag{10}$$

$$v_{n,t} = \frac{\beta x {D_t}^{1-\mu}\left(J_t R_{j,n,t}\right)\mu}{\left(\gamma A\right)1-\mu\left(1-\sigma_{n,t}\right)Z_t} \tag{11}$$

Eq. (10) shows that the value of a piece of knowledge is constant in steady-state, i.e., $g_\Theta = 0$. From Eq. (11) we see that the WTP for knowledge increases with the environmental policy term, the number of resource varieties and the consumption of each variety (in sum with pollution and the tax paid for it), and decreases with the knowledge stock.

Replacing the consumption of each resource variety into the final-goods production function, we obtain the production (supply) of each final-good:

$$Y_{n,t} = \frac{D_t}{\gamma}\left[\frac{\mu\left(A-x\right)}{\psi_{j,t}\left(1-\sigma_{n,t}\right)}\right]^{\frac{1}{1-\mu}} \tag{12}$$

Final production increases with the subsidy to final consumption and with aggregate demand, but decreases with the environmental policy and the resources price.

3.2.3. Natural resource sector

In this sector there is monopolistic competition and, following Barro and Sala-i-Martin (2004, Ch. 6), there is horizontal R&D, i.e., technical progress expands the number of usable resource varieties.[5] Once a new variety has been invented the firm receives a perpetual monopoly rent. Each firm faces a two-step decision process: firstly, it decides whether or not to invest in the discovery of new resource varieties and it invests if the net present value of future expected profits covers investment costs; secondly, it determines the optimal price for the newly invented resource variety.

The process is solved backwards. Initially, the optimal price is derived, assuming that the new variety has been discovered. Then, the present value of profits is calculated and compared with R&D costs. If the latter are lower, the firm will undertake R&D expenditures.

Stage 2: the present value of discovering the jth variety is given by:

$$V_t = \int_t^\infty \Lambda_{j,\vartheta} e^{-\bar{r}_{j,\vartheta}(t,\vartheta).(\vartheta-t)} d\vartheta \tag{13}$$

where $\Lambda_{j,\vartheta}$ is the profit flow at date ϑ, and $\bar{r}_{j,\vartheta} \equiv \frac{1}{(\vartheta-t)}$. The average interest rate between t and ϑ is $\int_t^\vartheta r_\omega d\omega$. In equilibrium, the interest rate is constant, and the present value factor simplifies to $e^{-r.(\vartheta-t)}$. The firm's profit flow equals revenues less production costs. We assume, as usual (e.g., Barro and Sala-i-Martin, 2004, Ch. 6) that once discovered, the new resource variety costs one unit of Y to produce, i.e., marginal and average production costs are constant and normalized to 1.

Stage 1: The firm decides whether or not to invest in R&D. As in Barro and Sala-i-Martin (2004, Ch. 6) a new variety discovery cost is fixed, being η_R units of Y. In equilibrium, R&D profits equal costs:

$$V_t = \eta_R, \forall t \tag{14}$$

Differentiating Eq. (13) with respect to time, we have:

$$r_t = \frac{\Lambda_{j,t}}{V_t} + \frac{\dot{V}_t}{V_t}. \tag{15}$$

The profit flow is given by:

$$\Lambda_{j,t} = \left(\psi_{j,t} - 1\right) R_{j,t} \tag{16}$$

where $R_{j,t} = NR_{j,n,t}$. The maximization problem of resource firms gives each resource variety supply (or price), which in turn allows to obtain the final expression for the consumption of each resource variety, the production of each final-good, and the price of each final-good, respectively (complete deduction in Appendix A):

$$\psi_{j,t} = \frac{1}{\mu} > 1 \tag{17}$$

$$R_{j,n,t} = \frac{D_t}{\gamma A J_t}\left[\frac{\mu^2\left(A-x\right)}{\left(1-\sigma_{n,t}\right)}\right]^{\frac{1}{1-\mu}} \tag{18}$$

$$Y_{n,t} = \frac{D_t}{\gamma}\left[\frac{\mu^2\left(A-x\right)}{\left(1-\sigma_{n,t}\right)}\right]^{\frac{1}{1-\mu}} \tag{19}$$

$$p_{n,t} = \frac{1}{(A-x)\mu^2} \tag{20}$$

From Eq. (17) $g_{\psi_{j,t}} = 0$, i.e., all resource varieties have the same price which is constant in time. Eq. (18) shows that consumption of each resource variety (equal for all varieties) increases with final-output aggregate demand and with the subsidy to final consumption. On the other hand, it decreases with the number of resource varieties available and the environmental policy (x). We have already analyzed the effects in final production, Eq. (19). Finally, Eq. (20) shows that final-goods' price is equal for all final-goods and increases with the environmental policy, but is constant in time, i.e., $g_{p_{n,t}} = 0$. Each resources firm has the following profit:

$$\Lambda_{j,t} = \left(\frac{1-\mu}{\mu}\right)\frac{ND_t}{\gamma A J_t}\left[\frac{\mu^2\left(A-x\right)}{\left(1-\sigma_{n,t}\right)}\right]^{\frac{1}{1-\mu}} \tag{21}$$

This expression is useful for later calculations.

[5] Hence, firms invent a new variety and become monopolists of that variety by selling it to final-good producers. Similar results would be obtained if we assumed that those R&D activities were developed in a different firm and the two firms shared a royalty.

3.2.4. Financial market

The financial market is perfectly competitive (e.g., Grimaud, 1999; Grimaud and Rougé, 2003, 2005; GT). It is possible to derive the interest rate for the final-goods sector, and the interest rate for the resources sector, respectively (see Appendix A):

$$r_t = \frac{\delta\beta x {D_t}^{1-\mu} N\left(J_t R_{j,n,t}\right)^{\mu}}{(\gamma A)^{1-\mu}\left(1-\sigma_{n,t}\right)Z_t} \tag{22}$$

$$r_t = \frac{\Lambda_{j,t}}{\eta_R} = \frac{\left(1-\mu\right)NR_{j,n,t}}{\mu\eta_R} \tag{23}$$

Imposing the equality between the rates, one obtains the ratio:

$$\frac{J_t}{Z_t} = \frac{\left(1-\mu\right)\mu\left(A-x\right)}{\beta x\delta\eta_R} \tag{24}$$

which implies that $g_{J_t} = g_{Z_t}$, i.e., in each t, resource varieties and final-goods' knowledge grow at the same rate. Higher discovery costs and higher environmental policy imply a lower ratio $\frac{J_t}{Z_t}$. Additionally, if $(A-x)>0$, which is the condition necessary for the ratio to be positive, the number of varieties and the knowledge stock always move in the same direction, i.e., they are complementary instead of substitutes. This is because both technology options are driven by final-goods consumption and a higher number of resources varieties (higher resource consumption) generates more emissions which is an incentive to pollution-reducing knowledge creation. The condition $(A-x)>0$ means that the environmental policy (represented by x), which depends on governmental choices, should be lower than the general economic efficiency (A), that is, there is an upper limit for the environmental policy.

3.3. General properties of the steady-state equilibrium

The free entry condition allows deducing the share of output devoted to knowledge creation, $\alpha = \frac{1-\mu}{\mu^2 A}$, which implies that $g_{\zeta_{n,t}} = g_{Y_{n,t}}$ and (complete deduction in Appendix A):

$$\zeta_{n,t} = \left(\frac{1-\mu}{\mu^2 A}\right)\frac{D_t}{\gamma}\left[\frac{\mu^2\left(A-x\right)}{\left(1-\sigma_{n,t}\right)}\right]^{\frac{1}{1-\mu}} \tag{25}$$

Investment in knowledge creation is driven by final-output production. From Eq. (8) and the interest rate in the resources sector Eq. (23), it is possible to conclude that, in steady-state, $g_N = -g_{R_{j,nt}} = 0$. Consequently, Eqs. (18), (19), (24), and (25) show that the growth rate is the same for the aggregate demand, the number of resource varieties, the output of each final-good firm, aggregate final-good, the knowledge stock, and the amount invested in knowledge accumulation, i.e., $g_{D_t} = g_{J_t} = g_{Y_{n,t}} = g_{Y_t} = g_{Z_t} = g_{\zeta_t}$. Therefore, the WTP for knowledge $\left[v_{n,t} = \frac{\beta x {D_t}^{1-\mu}\left(J_t R_{j,n,t}\right)^{\mu}}{(\gamma A)^{1-\mu}\left(1-\sigma_{n,t}\right)Z_t}\right]$ and the interest rate are constant overtime, i.e., all prices are constant in steady-state: $g_{p_{n,t}} = g_{\psi_{j,t}} = g_{v_{n,t}} = g_{r_t} = g_{\Theta_{n,t}} = 0$. We can also obtain the expression for the number of resource varieties (complete deduction in the Appendix A):

$$J_t = \left[\left(\frac{1-\mu}{\mu\eta_R A}\right) - 1 + \gamma + \left(\frac{1-\mu}{\mu^2 A}\right) + \frac{1}{A}\right]\frac{Y_t}{\rho} \tag{26}$$

The number of resource varieties increases with final output.
Replacing Eq. (26) in Eq. (25), we obtain the knowledge stock:

$$Z_t = \frac{\beta x\delta\mu\eta_R}{(1-\mu)\mu^2(A-x)}\left[\left(\frac{1-\mu}{\mu\eta_R A}\right) - 1 + \gamma + \left(\frac{1-\mu}{\mu^2 A}\right) + \frac{1}{A}\right]\frac{Y_t}{\rho} \tag{27}$$

Finally, emissions are given by:

$$E_t = \frac{{Y_t}^{1-\beta}}{\frac{1}{A}\left\{\frac{\beta x\delta\mu\eta_R}{(1-\mu)\mu^2(A-x)}\left[\left(\frac{1-\mu}{\mu\eta_R A}\right) - 1 + \gamma + \left(\frac{1-\mu}{\mu^2 A}\right) + \frac{1}{A}\right]\frac{1}{\rho}\right\}^{\beta}} \tag{28}$$

Eqs. (27) and (28) will be useful in the next section. From Eqs. (8) and (23):

$$g_{Y_{n,t}} = g_{Y_t} = \frac{(1-\mu)NR_{j,n,t}}{\mu\eta_R} - \rho \tag{29}$$

where $Y_t = \sum_{n=1}^{N} Y_{n,t}$.The higher the consumption of each resource variety (which is driven by the factors referred before), the higher the economic growth rate. From Eq. (28), if everything else remains unchanged, higher present output does not increase present emissions if $\beta>1$. Additionally, $g_{E_t} = (1-\beta)g_{Y_t}$, i.e., it is possible to have growing output and decreasing emissions with the same condition ($\beta>1$). In conclusion, if the efficiency of knowledge to decrease pollution is sufficiently high (higher than one), there is compatibility between economic growth and a cleaner environment both in the present and in the future.

4. Policy effects

Now, we analyze the main effects of a change in the subsidy level and in the emissions tax rate. The complete deduction is in Appendix B.

4.1. Subsidy

A higher subsidy to final consumption increases the output of each final-goods firm, the number of resource varieties and the knowledge stock. Notwithstanding, it does not affect the consumption of each resource variety. The mechanism is as follows. A higher subsidy stimulates demand, and in order to produce more, final-good firms demand more natural resources. This, in turn, has three effects: (i) higher resource consumption generates more pollution and consequently firms pay more tax; (ii) to minimize tax payment, final-good firms invest in more pollution-reducing knowledge increasing the knowledge stock in steady-state; (iii) a higher resources demand stimulates the invention of new resource varieties leaving the consumption of each variety unchanged since the demand pressure is satisfied with a higher number of resource varieties.

The net effect of a higher subsidy on emissions depends on which of two opposite effects dominate: higher output which increases emissions or higher a knowledge stock which decreases emissions. The second effect dominates if the efficiency of knowledge to reduce pollution is sufficiently high. The subsidy has only real effects on this economy, i.e., prices are not affected. The output and emissions growth rates also remain unchanged. Table 1 summarizes the results.

4.2. Tax on emissions (environmental policy)

A higher tax on emissions decreases the output of each final-goods firm, the number of resource varieties and the knowledge stock (for a given interval of the environmental policy). As for the subsidy, a stronger environmental policy does not affect the consumption of each resources variety. The intuition is the same as before, a lower final output, decreases the incentive to invest in knowledge if the environmental policy is sufficiently low, i.e., $x<A(1-\mu)$, leading to a

Table 1
Main effects of a change in the subsidy to final consumption.

$\frac{\partial Y_{n,t}}{\partial\sigma_{n,t}} > 0$	$\frac{\partial Z_t}{\partial\sigma_{n,t}} > 0$	$\frac{\partial J_t}{\partial\sigma_{n,t}} > 0$	$\frac{\partial R_{j,n,t}}{\partial\sigma_{n,t}} = 0$	$\frac{\partial E_t}{\partial\sigma_{n,t}} < 0_{if\ \beta>1}$
$\frac{\partial p_{n,t}}{\partial\sigma_{n,t}} = 0$	$\frac{\partial r_t}{\partial\sigma_{n,t}} = 0$	$\frac{\partial\psi_{j,t}}{\partial\sigma_{n,t}} = 0$	$\frac{\partial g_Y}{\partial\sigma_{n,t}} = 0$	$\frac{\partial g_E}{\partial\sigma_{n,t}} = 0$

lower knowledge stock. At the same time, the lower demand pressure on resources decreases the incentive to invest in varieties creation (since each variety is consumed in the same amount).

The net effect on emissions depends on which of the two effects dominate: lower output which generates lower emissions or lower knowledge stock which increases emissions. In our model, the first effect dominates if the efficiency of knowledge to reduce pollution is sufficiently high ($\beta > 1$), and if the environmental policy is not too strong $\left(x < \frac{A\beta(1-\mu)}{\beta-1}\right)$. The last condition may indicate that if the environmental policy is too strong there could be incentive for tax evasion with increasing emissions.

A stronger environmental policy increases final-goods price, but does not affect any other price in the economy. The output and emissions growth rates also remain unchanged. Table 2 summarizes the results.

If the two policy instruments are used together, under certain conditions, the government may be able to decrease emissions without harming output since the subsidy offsets, at least partially, the negative output effects of a stronger environmental policy.

Table 2
Main effects of a change in the environmental policy.

$\frac{\partial Y_{n,t}}{\partial x} < 0$	$\frac{\partial Z_t}{\partial x} < 0$ *if* $x < A(1-\mu)$	$\frac{\partial J_t}{\partial x} < 0$	$\frac{\partial R_{j,n,t}}{\partial x} = 0$	$\frac{\partial E_t}{\partial x} < 0$ *if* $x < \frac{A\beta(1-\mu)}{\beta-1}$ *and* $\beta > 1$
$\frac{\partial p_{n,t}}{\partial x} > 0$	$\frac{\partial r_t}{\partial x} = 0$	$\frac{\partial \psi_{j,t}}{\partial x} = 0$	$\frac{\partial g_Y}{\partial x} = 0$	$\frac{\partial g_E}{\partial \tau} = 0$

5. Optimum

In the optimum, the social planner maximizes aggregate utility, subject to the aggregate production process, the aggregate knowledge accumulation, the aggregate emissions flow, and the general equilibrium constraint. In Appendix C, we deduce the optimal conditions. We demonstrate that $\sigma_{n,t} = 1 - x\left(\frac{A}{A-1}\right)\left[\frac{D_t N\omega}{(A-1)\gamma}\right]^{1-\mu}$. This relationship shows that, in the optimum, the two instruments are not independent and that when one instrument increases the other decreases.

With certain values for the policy intervention, in particular, $x = \frac{1}{A}$ and $\sigma_{n,t} = (D_t N\omega)^{1-\mu}(A-1)^{\mu-2}$, the government guarantees that the decentralized equilibrium is optimal.

6. Empirical analysis

In this section we perform a simple empirical exercise in which we compare the situations of two different countries, one developed (country 1) and one developing (country 2). We assume that the differences of development of the two countries are reflected in some parameters or variable levels. The calibration and its sources are described in Table 3. In the case of $x = \frac{\tau_0}{Z_0^\beta}$, we considered small values since it is reasonable to assume that the knowledge stock is considerably higher than the tax level. Additionally, the ratio is lower in country 1 since the knowledge stock is expected to be much higher than in country 2, while the tax level cannot be so much higher.

With the values presented on Table 3 we obtained the relationship between some important variables in the two countries both for the decentralized equilibrium (DE) and the optimum (*). Given the expressions deduced for the model, the analysis of the decentralized equilibrium is more complete. We found, in this decentralized equilibrium, the ratio $\frac{Y_1^{DE}}{Y_2^{DE}} = 2.7$, which indicates, as expected, that country 1 has a higher output than country 2. In terms of emissions, the ratio $\frac{E_1^{DE}}{E_2^{DE}}$ is 3.6; i.e., in line with Ferreira-Lopes et al. (in press), the developed country has higher emissions.

Table 3
Parameter values and exogenous variables.

Parameter values/ exogenous variables	In line with existing literature and/or data	And in line with the theoretical assumption(s)
$A_1 = 1.20$	Afonso (2012)	$A_1 > 1$
$A_2 = 1.10$		$A_1 > A_2 > 1$, since expected productivity is higher in the developed country
$\delta_1 = 1.20$		$\delta_1 > 0$
$\delta_2 = 1.10$		$\delta_1 > \delta_2 > 0$, since expected productivity in knowledge accumulation is higher in the developed country
$\rho_1 = \rho_2 = 0.03$		$0 < \rho_1, \rho_2 < 1$
$\eta_{R1} = \eta_{R2} = 6.00$	Afonso and Alves (2009)	$\eta_{R1} > 0, \eta_{R2} > 0$
$\sigma_1 = \sigma_2 = 0.70$	Grimaud et al. (2011)	$0 \leq \sigma_1, \sigma_2 < 1$
$\alpha_1 = 0.15$ $\alpha_2 = 0.20$	Barro and Sala-i-Martin (1997)	$1 > \alpha_2 > \alpha_1 > 0$; since country 2 is in a catching up process
$\mu_1 = \mu_2 = 0.70$	Lewis (2006)	$0 < \mu_1, \mu_2 < 1$
$\gamma_1 = 0.55 \Rightarrow \omega_1 < 0.68$ $\omega_1 = 0.60$	Ferreira-Lopes et al. (in press) and	$0 < \gamma_1 < 1$, $\omega_1 < (A_1 - 1)/A_1 * \gamma_1$
$\gamma_2 = 0.4 \Rightarrow \omega_2 < 0.23$ $\omega_2 = 0.17$	World Bank data	$0 < \gamma_2 < 1$, $\omega_2 < (A_2 - 1)/A_2{}^* \gamma_2$
$\beta_1 = \beta_2 = 0.50$		$\beta_1 > 0, \beta_2 > 0$
$N_1 = N_2 \Rightarrow N_1/N_2 = 1$	Barro and Sala-i-Martin (1997)	$N_1 = N_2 > 0$
$x_1 = 0.02 < x_2 = 0.03$	World Bank data	$\tau_1/Z_1^\beta < \tau_2/Z_2^\beta$, since the knowledge stock is higher in country 1.

The comparison of the decentralized equilibrium and the optimum can be done using some important ratios and the growth rates of the most important variables. We keep in mind that $g_Y^{DE} = g_Z^{DE} = g_J^{DE}$ and that $g_Y{}^* = g_Z{}^* = g_J{}^*$. We compared the intensity of resource varieties on output, $\frac{J}{Z}$, and the intensity of knowledge on output, $\frac{Z}{Y}$. We concluded that, in the decentralized equilibrium, the intensity of resource varieties and the intensity of knowledge are higher in country 2, $\frac{J_1^{DE}/Y_1^{DE}}{J_2^{DE}/Y_2^{DE}} = 0.65$ and $\frac{Z_1^{DE}/Y_1^{DE}}{Z_2^{DE}/Y_2^{DE}} = 0.43$, which indicates the effort done by this country to achieve the catching-up process. In the optimum, the intensity of resource varieties is still higher in country 2, $\frac{J_1^*/Y_1^*}{J_2^*/Y_2^*} = 0.7$, but the intensity of knowledge is lower, $\frac{Z_1^*/Y_1^*}{Z_2^*/Y_2^*} = 2.5$. This last result derives from the fact that emissions are lower in country 2 and thus the internalization of this external effect has stronger outcomes in the country where emissions are higher. Finally, in the optimum, the catching-up process is faster since the difference between the variables growth rates, $\frac{g_{Y_1}}{g_{Y_2}} = \frac{g_{J1}}{g_{J_2}} = \frac{g_{Z_1}}{g_{Z_2}} = \frac{g_{E_1}}{g_{E_2}}$, is lower than in the decentralized equilibrium.

7. Conclusions

In this paper we considered an endogenous growth model with horizontal innovation in the natural resource sector and pollution-reducing knowledge accumulation in the final-goods sector. Firstly, we studied the decentralized steady-state equilibrium, highlighting its most important features. We found that, if the efficiency of knowledge to reduce emissions is sufficiently high, higher output is compatible with lower emissions both in their steady-state levels and their growth rates. Furthermore, we found that the two technical change forms are complementary instead of substitutes.

Secondly, we analyzed the main effects of changes in the two policy instruments: a subsidy to final consumption and a tax on emissions. If the government uses the two instruments together it may achieve a higher output and lower emissions since the subsidy may offset, at least partially, the negative output effects of the tax. Under certain conditions both instruments reduce steady-state emissions.

Then, we derived the policy conditions under which the decentralized equilibrium is optimal which will be useful in future research. The empirical application showed that the catching-up process between a developed and a developing country was faster in the optimum situation than in the decentralized equilibrium. We intend to conduct further research, performing a numerical exercise to compare the decentralized equilibrium conditions with the optimal ones and quantifying welfare gains. The comparison between different countries and inclusion of other policy instruments, such as a subsidy to knowledge investment, would also be of great interest in our further research.

Appendix A

Individuals maximization problem

The Current Value Hamiltonian (CVH) for this problem is:

$$CVH = \left(\ln\left(\left[\sum_{n=1}^{N} c_{n,t}{}^{\mu}\right]^{1/\mu}\right) - \omega \ln E_t\right) + \lambda_t\left[r_t B_t - \sum_{n=1}^{N}\left(1-\sigma_{n,t}\right)p_{n,t}c_{n,t} - T_t\right]$$

where λ_t is the dynamic multiplier of the number of resource varieties. The first order conditions are: $\frac{\partial CVH}{\partial c_{n,t}} = 0$; $\frac{\partial CVH}{\partial B_t} = -\dot{\lambda}_t + \lambda_t \rho$. The transversality condition is: $\lim_{t\to\infty}\lambda_t B_t e^{-\rho t} = 0$. The first order conditions give the following conditions

$$c_{n,t}{}^{\mu-1} = \lambda_t\left(1-\sigma_{n,t}\right)p_{n,t}\sum_{n=1}^{N} c_{n,t}{}^{\mu} \tag{30}$$

$$\frac{\dot{\lambda}_t}{\lambda_t} = \rho - r_t \tag{31}$$

We can now perform an algebraic exercise to obtain the demand for each good. Knowing that $\lambda_t = \frac{(1-\sigma_{n,t})p_{n,t}\sum_{n=1}^{N} c_{n,t}{}^{\mu}}{c_{n,t}{}^{\mu-1}}$, we may replace this expression in Eq. (30) and manipulate to obtain: $c_{n,t} = \frac{c_{n,t}(1-\sigma_{n,t})p_{n,t}}{[(1-\sigma_{n,t})p_{n,t}]^{\frac{\mu}{\mu-1}}}\left[(1-\sigma_{n,t})p_{n,t}\right]^{\frac{1}{\mu-1}}$, if we aggregate the first part for all final-goods and call $D_t = \frac{\sum_{k=1}^{N}(1-\sigma_{k,t})p_{k,t}c_{k,t}}{\sum_{k=1}^{N}[(1-\sigma_{k,t})p_{k,t}]^{\frac{\mu}{\mu-1}}}$, the consumption of each final-good may be written as: $c_{n,t} = D_t\left[(1-\sigma_{n,t})p_{n,t}\right]^{\frac{1}{\mu-1}}$. The inverse demand function is then given by $p_{n,t} = \left(\frac{c_{n,t}}{D_t}\right)^{\mu-1} * \frac{1}{(1-\sigma_{n,t})}$. In equilibrium, consumption is a given proportion of output ($c_{n,t} = \gamma Y_{n,t}$), therefore:

$$p_{n,t} = \left(\frac{\gamma Y_{n,t}}{D_t}\right)^{\mu-1} * \frac{1}{\left(1-\sigma_{n,t}\right)} \tag{32}$$

Combining Eqs. (30) and (31) one derives the Keynes–Ramsey rule:

$$r_t = (1-\mu)g_{c_{n,t}} + g_{\Omega_t} + g_{(1-\sigma_{n,t})} + g_{p_{n,t}} + \rho$$

where $\Omega_t = \sum_{n=1}^{N} c_{n,t}{}^{\mu}$. In steady-state, since $g_{\Omega_t} = \mu g_{c_{n,t}} = \mu g_{Y_{n,t}}$, the price and the subsidy are constant, the rule simplifies to:

$$r_t = g_{Y_{n,t}} + \rho \tag{33}$$

Final-good firms maximization problem

Each final-good firm maximizes:

$$\tilde{\pi}_{n,t} = p_{n,t}Y_{n,t} - \sum_{j=1}^{J}\psi_{j,t}R_{j,n,t} - \tau_t p_{n,t}E_{n,t} + \Theta_t \dot{Z}_{n,t} - \zeta_{n,t}$$

subject to the production technology Eq. (1), knowledge accumulation Eq. (2), the emissions flow Eq. (3), and the inverse demand function Eq. (7). In the symmetric equilibrium, all resources are used in the same amount, have the same price, and $c_{n,t} = \gamma Y_{n,t}$. After substitutions, we have:

$$\tilde{\pi}_{n,t} = \left(\frac{D_t}{\gamma A J_t R_{j,n,t}}\right)^{1-\mu}\frac{A J_t R_{j,n,t}}{\left(1-\sigma_{n,t}\right)} - \psi_{j,t}J_t R_{j,n,t} - \left(\frac{D_t}{\gamma A J_t R_{j,n,t}}\right)^{1-\mu}\frac{J_t R_{j,n,t}}{\left(1-\sigma_{n,t}\right)}\tau_t Z_t{}^{-\beta} + (\Theta_t\delta - 1)\zeta_{n,t}$$

The first order conditions, $\frac{\partial\tilde{\pi}_{n,t}}{\partial R_{j,n,t}} = 0, \frac{\partial\tilde{\pi}_{n,t}}{\partial\zeta_{n,t}} = 0, \frac{\partial\tilde{\pi}_{n,t}}{\partial Z_{n,t}} = v_{n,t}$ give, respectively, the demand for each resources variety, the value of a piece of knowledge, and the firm's WTP to use a piece of knowledge:

$$R_{j,n,t} = \frac{D_t}{\gamma A J_t}\left[\frac{\mu\left(A-x\right)}{\psi_{j,t}\left(1-\sigma_{n,t}\right)}\right]^{\frac{1}{1-\mu}} \tag{34}$$

$$\Theta_t\delta = 1 \Leftrightarrow \Theta = \frac{1}{\delta} \tag{35}$$

$$v_{n,t} = \frac{\beta x D_t{}^{1-\mu}\left(J_t R_{j,n,t}\right)^{\mu}}{(\gamma A)^{1-\mu}\left(1-\sigma_{n,t}\right)Z_t} \tag{36}$$

From Eq. (35), $g_{\Theta_t} = 0$, i.e., a piece of knowledge's value is constant over time.

A higher tax rate leads to a higher WTP for knowledge. From the market free entry condition $\left(\pi_{n,t} = \tilde{\pi}_{n,t} - \Theta_{n,t}\dot{Z}_t = 0, \forall n\right)$ and $\zeta_{n,t} = \alpha Y_{n,t}$ (in equilibrium, the knowledge investment is a given proportion, α, of output), one gets:

$$Y_{n,t} = \frac{D_t}{\gamma}\left[\frac{A-x}{\left(\psi_{j,t} + \alpha A\right)\left(1-\sigma_{n,t}\right)}\right]^{\frac{1}{1-\mu}} \tag{37}$$

Natural resources firms maximization problem

The profit flow is given by:

$$\Lambda_{j,t} = \left[\psi_{j,t} - 1\right]R_{j,t} \tag{38}$$

where $R_{j,t} = NR_{j,n,t}$. Using Eqs. (1) and (9), one obtains the maximization problem:

$$\max \Lambda_{j,t} = \left[\psi_{j,t} - 1\right]\frac{ND_t}{\gamma A J_t}\left[\frac{\mu\left(A-x\right)}{\psi_{j,t}\left(1-\sigma_{n,t}\right)}\right]^{\frac{1}{1-\mu}}$$

The first order condition, $\frac{\partial\Lambda_{j,t}}{\partial\psi_{j,t}} = 0$, gives:

$$\psi_{j,t} = \frac{1}{\mu} \tag{39}$$

Replacing the expression of ψ in the resource demand:

$$R_{j,n,t}=\frac{D_t}{\gamma AJ_t}\left[\frac{\mu^2(A-x)}{(1-\sigma_{n,t})}\right]^{\frac{1}{1-\mu}} \tag{40}$$

The production of each final-good is $Y_{n,t}=AJ_tR_{j,n,t}$:

$$Y_{n,t}=\frac{D_t}{\gamma}\left[\frac{\mu^2(A-x)}{(1-\sigma_{n,t})}\right]^{\frac{1}{1-\mu}} \tag{41}$$

Replacing this expression into the final-goods demand one has:

$$p_{n,t}=\frac{1}{(A-x)\mu^2} \tag{42}$$

Each resource firms has the profit $\Lambda_{j,t}=(\psi_{j,t}-1)NR_{j,n,t}$:

$$\Lambda_{j,t}=\left(\frac{1-\mu}{\mu}\right)\frac{ND_t}{\gamma AJ_t}\left[\frac{\mu^2(A-x)}{(1-\sigma_{n,t})}\right]^{\frac{1}{1-\mu}} \tag{43}$$

From Eqs. (37),(39) and (41) it is possible to obtain the expression $\alpha=\frac{1-\mu}{\mu^2A}$, therefore:

$$\zeta_{n,t}=\left(\frac{1-\mu}{\mu^2A}\right)\frac{D_t}{\gamma}\left[\frac{\mu^2(A-x)}{(1-\sigma_{n,t})}\right]^{\frac{1}{1-\mu}} \tag{44}$$

Financial market

From Eqs. (6), (10), (11) and since $v_t=\sum_{n=1}^{N}v_{n,t}=Nv_{n,t}$, one derives the final-goods sector interest rate:

$$r_t=\frac{\delta\beta xD_t^{1-\mu}N(J_tR_{j,n,t})^{\mu}}{(\gamma A)^{1-\mu}(1-\sigma_{n,t})Z_t} \tag{45}$$

Using Eqs. (1), (14), (15) and (16) one obtains the resources sector interest rate:

$$r_t=\frac{\Lambda_{j,t}}{\eta_R}=\frac{(1-\mu)NR_{j,n,t}}{\mu\eta_R} \tag{46}$$

which may be interpreted as the technology curve and summarizes the production side decisions. There is only one interest rate in the economy (the financial market is perfectly competitive), therefore Eqs. (45) and (46) give:

$$Z_t=\frac{\beta x\delta\mu\eta_RD_t^{1-\mu}J_t^{\mu}}{(\gamma A)^{1-\mu}(1-\sigma_{n,t})(1-\mu)R_{j,n,t}^{1-\mu}}$$

Replacing the expression of $R_{j,n,t}$ into this equation, one has:

$$Z_t=\frac{\beta x\delta\eta_R}{(1-\mu)\mu(A-x)}J_t \tag{47}$$

For that ratio to be positive the condition $(A-x)>0$ must hold.

Finally, the general equilibrium condition $\left(Y_t=C_t+\eta_R\dot{J}_t+\zeta_t+NJ_tR_{j,n,t}\right)$ may be developed (considering that $C_t=\gamma Y_t,\zeta_t=\left(\frac{1-\mu}{\mu^2A}\right)Y_t,\dot{J}_t=g_{J_t}*J_t,g_{J_t}=g_{Y_t}$, and $g_{Y_t}=\frac{(1-\mu)NR_{j,n,t}}{\mu\eta_R}-\rho$) to give:

$$J_t\left[\frac{(1-\mu)NR_{j,n,t}}{\mu\eta_R}-\rho\right]=\left(1-\gamma-\frac{1-\mu}{\mu^2A}-\frac{1}{A}\right)Y_t \tag{48}$$

Replacing the expression of $R_{j,n,t}$ and developing, one gets:

$$J_t=\left[\left(\frac{1-\mu}{\mu\eta_RA}\right)-1+\gamma+\left(\frac{1-\mu}{\mu^2A}\right)+\frac{1}{A}\right]\frac{Y_t}{\rho} \tag{49}$$

Appendix B

For the policy analysis we use the expressions found in the derivation after substitution:

$$Y_{n,t}=\frac{D_t}{\gamma}\left[\frac{\mu^2(A-x)}{(1-\sigma_{n,t})}\right]^{\frac{1}{1-\mu}}$$

$$J_t=\left[\left(\frac{1-\mu}{\mu\eta_RA}\right)-1+\gamma+\left(\frac{1-\mu}{\mu^2A}\right)+\frac{1}{A}\right]\frac{D_t}{\rho\gamma}\left[\frac{\mu^2(A-x)}{(1-\sigma_{n,t})}\right]^{\frac{1}{1-\mu}}$$

$$Z_t=\frac{\beta x\delta\mu\eta_RD_t}{(1-\mu)\mu^2(A-x)\rho\gamma}\left[\left(\frac{1-\mu}{\mu\eta_RA}\right)-1+\gamma+\left(\frac{1-\mu}{\mu^2A}\right)+\frac{1}{A}\right]\left[\frac{\mu^2(A-x)}{(1-\sigma_{n,t})}\right]^{\frac{1}{1-\mu}}$$

$$R_{j,n,t}=\frac{\rho}{A\left[\left(\frac{1-\mu}{\mu\eta_RA}\right)-1+\gamma+\left(\frac{1-\mu}{\mu^2A}\right)+\frac{1}{A}\right]}$$

$$p_{n,t}=\frac{1}{(A-x)\mu^2}$$

$$\psi_{j,t}=\frac{1}{\mu}$$

$$r_t=\frac{(1-\mu)N\rho}{\mu\eta_RA\left[\left(\frac{1-\mu}{\mu\eta_RA}\right)-1+\gamma+\left(\frac{1-\mu}{\mu^2A}\right)+\frac{1}{A}\right]}$$

$$E_t=\frac{ND_t\left[\frac{\mu^2(A-x)}{(1-\sigma_{n,t})}\right]^{\frac{1-\beta}{1-\mu}}}{\gamma A\left\{\frac{\beta x\delta\mu\eta_RD_t}{(1-\mu)\mu^2(A-x)\rho\gamma}\left[\left(\frac{1-\mu}{\mu\eta_RA}\right)-1+\gamma+\left(\frac{1-\mu}{\mu^2A}\right)+\frac{1}{A}\right]\right\}^{\beta}}$$

$$g_{Y_t}=\frac{(1-\mu)N\rho}{\mu\eta_RA\left[\left(\frac{1-\mu}{\mu\eta_RA}\right)-1+\gamma+\left(\frac{1-\mu}{\mu^2A}\right)+\frac{1}{A}\right]}-\rho$$

$$g_{E_t}=(1-\beta)\left\{\frac{(1-\mu)N\rho}{\mu\eta_RA\left[\left(\frac{1-\mu}{\mu\eta_RA}\right)-1+\gamma+\left(\frac{1-\mu}{\mu^2A}\right)+\frac{1}{A}\right]}-\rho\right\}$$

Policy effects

We analyze qualitatively the main effects of a higher subsidy to final consumption and a higher environmental policy (reflected in the term x). The sign of some derivates is straightforward, but we put the expressions for convenience.

Subsidy

In the derivations we omit the time subscripts for simplicity.

$\frac{\partial Y_{n,t}}{\partial \sigma}$	>0
$\frac{\partial Z_t}{\partial \sigma}$	>0
$\frac{\partial J_t}{\partial \sigma}$	>0
$\frac{\partial R_{j,n,t}}{\partial \sigma}$	$=0$
$\frac{\partial E_t}{\partial \sigma} = -\frac{D_t N(\beta-1)}{A\gamma(1-\mu)(1-\sigma_{n,t})} * \left[\frac{\mu^2(A-x)}{(1-\sigma_{n,t})}\right]^{\frac{1}{1-\mu}} * \left\{\frac{A\gamma(1-\mu)(1-\sigma_{n,t})\mu\rho}{D_t x\beta\delta\left[\frac{\mu^2(A-x)}{(1-\sigma_{n,t})}\right]^{\frac{1}{1-\mu}}\{\eta_R[\mu^2(A\gamma-A+1)-\mu+1]+(1-\mu)\mu\}}\right\}^{\beta}$	<0 if $\beta>1$[a]
$\frac{\partial p_{n,t}}{\partial \sigma}$	$=0$
$\frac{\partial \psi_{j,t}}{\partial \sigma}$	$=0$
$\frac{\partial r_t}{\partial \sigma}$	$=0$
$\frac{\partial g_{Y_t}}{\partial \sigma}$	$=0$
$\frac{\partial g_{E_t}}{\partial \sigma}$	$=0$

[a] We can guarantee this since we below show that $\{\eta_R[\mu^2(A\gamma-A+1)-\mu+1]+(1-\mu)\mu\}>0$.

Emission tax

In steady-state $\tau_t {Z_t}^{-\beta}$ is constant overtime and equal to $\tau_0 {Z_0}^{-\beta}$ (x). We perform the derivatives with respect to x since their sign is equal to the derivatives with respect to τ_0.

$\frac{\partial Y_{n,t}}{\partial x} = -\frac{D_t\left[\frac{\mu^2(A-x)}{(1-\sigma_{n,t})}\right]^{\frac{1}{1-\mu}}}{(A-x)(1-\mu)\gamma}$	<0
$\frac{\partial J_t}{\partial x} = -\frac{D_t\left[\frac{\mu^2(A-x)}{(1-\sigma_{n,t})}\right]^{\frac{\mu}{1-\mu}}\{\eta_R[\mu^2(A\gamma-A+1)-\mu+1]+(1-\mu)\mu\}}{A\gamma\eta_R(1-\sigma_{n,t})\mu(1-\mu)}$	<0

Since J_t is only affected by the negative impact of x on $Y_{n,t}$, we know that $\frac{\partial J_t}{\partial x}$ has to be negative. This, in turn, implies that $\{\eta_R[\mu^2(A\gamma-A+1)-\mu+1]+(1-\mu)\mu\}>0$, which will be useful in the next derivations.

$\frac{\partial Z_t}{\partial x} = \frac{D_t\beta\delta[A(1-\mu)-x]\left[\frac{\mu^2(A-x)}{(1-\sigma_{n,t})}\right]^{\frac{1}{1-\mu}}\{\eta_R[\mu^2(A\gamma-A+1)-\mu+1]+(1-\mu)\mu\}}{A\gamma(\mu-1)^2\mu^3\rho(A-x)^2}$	<0 if $x>A(1-\mu)$
$\frac{\partial R_{j,n,t}}{\partial x}$	$=0$
$\frac{\partial E_t}{\partial x} = -\frac{D_t N[A\beta(1-\mu)-x(\beta-1)]\left[\frac{\mu^2(A-x)}{(1-\sigma_{n,t})}\right]^{\frac{1}{1-\mu}}}{Ax\gamma(1-\mu)(A-x)\left\{\frac{D_t x\beta\delta\left[\frac{\mu^2(A-x)}{(1-\sigma_{n,t})}\right]^{\frac{\mu}{1-\mu}}\{\eta_R[\mu^2(A\gamma-A+1)-\mu+1]+(1-\mu)\mu\}}{A\gamma(1-\mu)(1-\sigma_{n,t})\mu\rho}\right\}^{\beta}}$	<0 if $x<\frac{A\beta(1-\mu)}{\beta-1}$ and $\beta>1$
$\frac{\partial p_{n,t}}{\partial x} = \frac{1}{(A-x)^2\mu^2}$	>0
$\frac{\partial \psi_{j,t}}{\partial x}$	$=0$
$\frac{\partial r_t}{\partial x}$	$=0$
$\frac{\partial g_{Y_t}}{\partial x}$	$=0$
$\frac{\partial g_{E_t}}{\partial x}$	$=0$

Appendix C

In the optimum, the social planner maximizes aggregate utility, Eq. (4), subject to the aggregate production process ($Y_t = \sum_{n=1}^{N} Y_{n,t} = AN\sum_{j=1}^{J} R_{j,n,t} = AJ_t R_{j,t}$, where we call $R_{j,t} = NR_{j,n,t}$), the aggregate knowledge accumulation ($\dot{Z}_t = \delta\zeta_t$ where $\zeta(t) = \sum_{n=1}^{N}\zeta_{n,t}$), the emissions flow ($E_t = \sum_{n=1}^{N} E_{n,t} = J_t R_{j,t} {Z_t}^{-\beta}$, and the general equilibrium constraint, Eq. (5). The CVH for this problem is:

$$CVH = \ln(c_t) - \omega\ln\left(J_t R_{j,t} {Z_t}^{-\beta}\right) + \nu_t \frac{1}{\eta_R}\left[AJ_t R_{j,t} - c_t - \zeta_t - J_t R_{j,t}\right] + \xi_t\delta\zeta_t$$

where ν_t and ξ_t are co-estate variables. The first order conditions are: $\frac{\partial CVH}{\partial c_t} = 0$; $\frac{\partial CVH}{\partial \zeta_t} = 0$; $\frac{\partial CVH}{\partial R_{j,t}} = 0$; $\frac{\partial CVH}{\partial J_t} = -\dot{\nu}_t + \nu_t\rho$; $\frac{\partial CVH}{\partial Z_t} = -\dot{\xi}_t + \xi_t\rho$. The transversality conditions are: $\lim_{t\to\infty}\nu_t J_t e^{-\rho t} = 0$ and $\lim_{t\to\infty}\xi_t Z_t e^{-\rho t} = 0$.

The first order conditions give, respectively:

$$\nu_t = \frac{\eta_R}{c_t} \tag{50}$$

$$\xi_t = \frac{\nu_t}{\delta\eta_R} \tag{51}$$

$$R_{j,t} = \frac{\omega\eta_R}{\nu_t J_t(A-1)} \tag{52}$$

$$\frac{\nu_t R_{j,t}(A-1)}{\eta_R} - \frac{\omega}{J_t} = -\dot{\nu}_t + \nu_t\rho \tag{53}$$

$$\frac{\omega\beta}{Z_t} = -\dot{\xi}_t + \xi_t\rho \tag{54}$$

Eqs. (50) and (51) give $g_{\nu_t} = g_{\xi_t} = -g_{c_t}$ and we know that $g_{c_t} = g_{Y_t}$. Eq. (53) can be divided by ν_t and written:

$$\frac{Y_t}{J_t}\left[\frac{(A-1)}{A\eta_R} - \frac{\omega\gamma}{\eta_R}\right] = g_{Y_t} + \rho \tag{55}$$

which in steady-state, implies that $g_{Y_t} = g_{J_t}$ as in the decentralized equilibrium. Dividing Eq. (54) by ξ_t, one gets:

$$\frac{\omega\beta\gamma Y_t\delta}{Z_t} = g_{Y_t} + \rho \tag{56}$$

which in steady-state, implies that $g_{Y_t} = g_{Z_t}$, as in the decentralized equilibrium. The equality of Eqs. (55) and (56) gives:

$$\frac{J_t}{Z_t} = \frac{(A-1)-A\omega\gamma}{\omega\beta\gamma\delta A\eta_R} \tag{57}$$

Comparing this equation with Eq. (24), we see that, for the decentralized equilibrium to be optimal, the condition $\frac{(1-\mu)\mu(A-x)}{\beta x\delta\eta_R} = \frac{(A-1)-A\omega\gamma}{\omega\beta\gamma\delta A\eta_R}$ must hold. Therefore, for the decentralized equilibrium to be optimal:

$$x = \frac{(1-\mu)\mu A}{A-1-A\omega\gamma+(1-\mu)\mu} \tag{58}$$

Dividing Eq. (52) for Eq. (50) and using Eq. (1), one gets: $\gamma = \frac{A-1}{A\omega}$. Replacing this expression into Eq. (58) the environmental policy simplifies to $x = \frac{1}{A}$.

From Eqs. (8) and (22): $g_{Y_t} = \frac{\delta\beta x {D_t}^{1-\mu} N(J_t R_{j,n,t})^{\mu}}{(\gamma A)^{1-\mu}(1-\sigma_{n,t})Z_t}$. Combining this expression with Eq. (56), $\gamma = \frac{A-1}{A\omega}$,we obtain the relationship between the two policy instruments: $\sigma_{n,t} = 1 - x\left(\frac{A}{A-1}\right)\left[\frac{D_t N\omega}{(A-1)\gamma}\right]^{1-\mu}$. This relationship shows that when one instrument increases the other decreases in the optimum. With $x = \frac{1}{A}$, we obtain the subsidy to final consumption which guarantees that the decentralized equilibrium is optimal: $\sigma_{n,t} = (D_t N\omega)^{1-\mu}(A-1)^{\mu-2}$.

References

Acemoglu, D., 2009. Introduction to Modern Economic Growth. Princeton University Press, Princeton, NJ.

Afonso, O., 2012. Scale-independent North-South trade effects on the technological knowledge bias and on wage inequality. Review of World Economics 148, 181–207.

Afonso, O., Alves, R., 2009. Endogenous Growth and European Fiscal Rules. Applied Economics 41 (7), 849–858.

Barbier, E., 1999. Endogenous Growth and Natural Resource Scarcity. Environmental and Resource Economics 14, 51–74.

Barro, R., Sala-i-Martin, X., 1997. Technological diffusion, convergence, and growth. Journal of Economic Growth 2 (1), 1–26.

Barro, R.J., Sala-i-Martin, X., 2004. Economic growth, 2nd edition. The MIT Press, Cambridge, Massachusetts, London, England.

Bovenberg, A.L., Smulders, S., 1995. Environmental quality and pollution-augmenting technological change in a two-sector endogenous growth model. Journal of Public Economics 57, 369–391.

Bretschger, L., Smulders, S., 2007. Sustainable resources use and economic dynamics. Environmental and Resource Economics 36, 1–13.

Buonanno, P., Carraro, C., Galeotti, M., 2003. Endogenous induced technical change and the costs of Kyoto. Resources and Energy Economics 25, 11–34.

Ferreira-Lopes, A., Sequeira, T., Roseta-Palma, C., in press. On the effect of technological progress on pollution: an overlooked distortion in endogenous growth. Oxford Economic Papers. http://dx.doi.org/10.1093/oep/gps019.

Garg, P.C., Sweeney, J.L., 1978. Optimal growth with depletable resources. Resources and Energy 1, 43–56.

Goulder, L.H., Schneider, S.H., 1999. Induced technological change and the attractiveness of CO_2 abatement policies. Resource and Energy Economics 21, 211–253.

Gradus, R., Smulders, S., 1993. The trade-off between environmental care and long-term growth — pollution in three prototype growth models. Journal of Economics 58 (1), 25–51.

Grimaud, A., 1999. Pollution permits and sustainable growth in a Schumpeterian model. Journal of Environmental Economics and Management 38, 249–266.

Grimaud, A., Rougé, L., 2003. Non-renewable resources and growth with vertical innovations: optimum, equilibrium and economic policies. Journal of Environmental Economics and Management 45, 433–453.

Grimaud, A., Rougé, L., 2005. Polluting non-renewable resources, innovation and growth: welfare and environmental policy. Resource and Energy Economics 27, 109–129.

Grimaud, A., Tournemaine, F., 2007. Why can an environmental policy tax promote growth through the channel of education? Ecological Economics 62, 27–36.

Grimaud, A., Lafforgue, G., Magné, B., 2011. Climate change mitigation options and directed technical change: a decentralized equilibrium analysis. Resource and Energy Economics 33, 938–962.

Grossman, G.M., Helpman, E., 1991. Innovation and growth in the global economy. MIT Press, Cambridge, Massachusetts.

Gupta, M., Barman, T., 2009. Fiscal policies, environmental pollution and economic growth. Economic Modelling 26, 1018–1028.

Halicioglu, F., 2009. An econometric study of CO_2 emissions, energy consumption, income and foreign trade in Turkey. Energy Policy 37, 1156–1164.

Lewis, V., 2006. Macroeconomic Fluctuations and Firm Entry: Theory and Evidence, Working Paper Research, no 103. National Bank of Belgium.

Rivera-Batiz, L.A., Romer, P.M., 1991. Economic Integration and Endogenous Growth. Quarterly Journal of Economics 106 (2), 531–555 (May).

Romer, P.M., 1990. Endogenous technological change. Journal of Political Economy 98 (5), S71–S102 (October).

Sadorsky, P., 2009. Renewable energy consumption, CO_2 emissions and oil prices in the G7 countries. Energy Economics 31, 456–462.

Saltari, E., Travaglini, G., 2011. The effects of environmental policies on the abatement investment decisions of a green firm. Resource and Energy Economics 33, 666–685.

Scholz, C., Ziemes, G., 1999. Exhaustible resources, monopolistic competition, and endogenous growth. Environmental and Resource Economics 13, 169–185.

Schou, P., 2000. Polluting non-renewable resources and growth. Environmental and Resource Economics 16, 211–227.

Schou, P., 2002. When environmental policy is superfluous: growth and polluting resources. Scandinavian Journal of Economics 104 (4), 605–620.

Soytas, U., Sari, R., 2009. Energy consumption, economic growth, and carbon emissions: challenges faced by an EU candidate member. Ecological Economics 68, 1667–1675.

Stokey, N.L., 1998. Are there limits to growth? International Economic Review 39 (1), 1–39.

Xapapadeas, A., 2005. Economic Growth and the Environment. Handbook of Environmental Economics, vol. 3, pp. 1219–1271. Chapter 23.