

Research Article **Terminal Sliding Mode Control with Adaptive Feedback Control in a Class of Chaotic Systems**

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Received 1 May 2014; Accepted 22 June 2014; Published 13 July 2014

Academic Editor: Chuandong Li

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This paper analyzes semifinite time stability for a general chaotic system. By cooperating methods terminal sliding mode (TSM) with adaptive feedback control (AFC), a controller based on the two methods is derived to achieve semifinite time stability. The theoretical analysis employs the theories of linear matrix inequalities and Lyapunov functional method. Finally, numerical simulation is given to illustrate the derived theoretical results.

1. Introduction

Chaos phenomenon can be found in many physics and engineering systems in practice. However, to improve the system's performance, it is often desirable to avoid chaos, and various methods are proposed. Due to different emphases, controllers have different merits and drawbacks. For example, TSM establishes terminal sliding mode surface to couple system variables and control them to reach equilibrium points. Its control is effective, but it can only control system states coupled in the sliding mode surface; readers are referred to [1–7] for more detailed information. As for impulse control [8-16], they add impulse effects to continuous differential equation and, by constructing comparison system, establish relationships between parameters of system and impulse. Their controllers are effective, but design processes of their controllers are too much complex. For adaptive feedback control as in [17–21], similar to TSM, they have unified the format with different parameters. AFC has a wide range of applications in various fields, but its dynamic is not as good as the first two.

For the system's structure constructed in this paper, we design controllers from methods TSM and AFC, both of which have unified formats. If TSM is used only, we should design several TSM surfaces. If AFC is used only, its controller is very simple and flexible, but it can only achieve asymptotical stability. Combining their merits and drawbacks, a cooperative controller is proposed in the paper. TSM method finite-timely controls system states, which are coupled in TSM surface, as in [4–7]. Simple AFC is introduced as a supplementary control into the remaining states of the system, controlling system states which are outside TSM surface and making them asymptotically stable [18, 20], and, finally, the overall system tends to be semifinitetime stable [5].

This design scheme can control main elements of system finite-time stability, firstly, then use AFC method to ensure that other dimensions are asymptotically stable, and finally realize the overall system's semifinite time stability. Compared with TSM only, this design can greatly reduce the control input and simplify the design process of controller; compared with AFC only, it has obvious advantages in time sequence.

The rest of the paper is organized as follows. In Section 2, a general chaotic system model and some preliminaries are presented. In Section 3, we will show theoretical analysis, establish several sufficient conditions for SFTS, and formulate controller. In Section 4, numerical simulation is presented to verify the validity of theoretical results. Finally, the conclusions are drawn in Section 5.

2. Problem Statement and Preliminaries

We use the following differential equation to describe general dynamic chaos system:

$$\dot{y} = f(y). \tag{1}$$

In this paper, we are committed to solve the stability analysis of chaotic systems which can be organized into the following form:

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2$$
$$\dot{x}_2 = A_{21}x_1 + A_{22}x_2 + f_2(x_1, x_2, x_3)$$
(2)

$$\dot{x}_3 = f_3(x_1, x_2, x_3),$$

where $x_1 = (y_1, ..., y_r)^T x_2 = (y_{r+1}, ..., y_s)^T x_3 = (y_{s+1}, ..., y_t)^T$, $f_2 f_3$, are nonlinear part.

Definition 1 (finite-time stability [22, 23]). Consider that $x(t) \in \mathbb{R}^n$ are system states. If there exists constant T > 0, such that $\lim_{t \to T} ||x|| = 0$ and if $t \ge T$, ||x(t)|| = 0, then the system realizes finite-time stability.

Definition 2 (semifinite-time stability [5]). Take dynamic system (2), for example; after adding controllers, if the states x_1 and x_2 are finite-time stable and the states x_3 are asymptotically stable, then we call the system semifinite-time stable.

Lemma 3 (see [23]). Assume that a continuous, positive-definite function V(t) satisfies the following differential inequality:

$$\dot{V}(t) \le -cV^{\tau}(t), \quad \forall t \ge t_0, \ V(t_0) \ge 0, \tag{3}$$

where c > 0, $0 < \tau \le 1$ are two constants. Then, for any given t_0 , V(t) satisfies the following results. $V(t) \equiv 0$, $\forall t \ge t_1$ with t_1 being given by

$$t_1 = t_0 + \frac{V^{1-\tau}(t_0)}{c(1-\tau)}.$$
(4)

Assumption 4. Assume that there exist positive constants l_k (k = 1, ..., m), f_2 , f_3 satisfying

$$\|f(x)\| \le \sum_{k=1}^{m} l_k \|x\|^k, \quad \|x\| = \max |y_i|, \ (i = s + 1, \dots, t).$$

(5)

3. Main Results

In this section, controller is designed from TSM and AFC separately with detailed theoretical analysis. TSM portion is used to derive x_1x_2 finite-time stability, and AFC is for asymptotical stability of x_3 . Finally, system (2) tends to semifinite time stability.

3.1. Terminal Sliding Mode Portion Design. Take the system's first two parts of the states and add controlling part:

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2$$

$$\dot{x}_2 = A_{21}x_1 + A_{22}x_2 + f_2(x_1, x_2, x_3) + u_1.$$
(6)

Generally speaking, the process of terminal sliding mode control can be divided into two stages: the first stage is to establish the nonsingular terminal sliding model surface; the second is to design TSM controller, which can make the system variables reach and maintain the TSM surface within finite time.

So in this paper, a nonsingular TSM surface is introduced as follows:

$$s = C_1 x_1 + C_2 x_2 + C_3 x_1^{q/p}, \tag{7}$$

where parameters $C_1, C_2, C_3, 0 < q < p$ are odd matrices determined by the designer.

Theorem 5. *In the system* (6), *we add the following controller:*

$$u_{1} = -\frac{1}{C_{2}} \left[\left(C_{1} + C_{3} \frac{q}{p} x_{1}^{(q-p)/p} \right) (A_{11}x_{1} + A_{12}x_{2}) \right] - \left(A_{21}x_{1} + A_{22}x_{2} + f_{2} \left(x_{1}, x_{2}, x_{3} \right) \right) - \frac{\psi s}{C_{2}}$$
(8)
$$\psi = \mu + \eta \|s\|^{\xi-1},$$

where $\mu > 0$, $\eta > 0$, and $0 < \xi < 1$. Then the system states slide on the switch surface s = 0.

Proof. Introduce the following Lyapunov function:

$$V_1 = \frac{1}{2}s^T s. (9)$$

Differentiating the function along the trajectories of *s*, we will obtain

$$\begin{split} \dot{V}_{1} &= s^{T} \dot{s} \\ &= s^{T} \left(C_{1} + C_{3} \frac{q}{p} x_{1}^{(q-p)/p} \right) \left(A_{11} x_{1} + A_{12} x_{2} \right) \\ &+ s^{T} C_{2} \left(A_{21} x_{1} + A_{22} x_{2} + f_{2} \left(x_{1}, x_{2}, x_{3} \right) \right) + s^{T} u_{1} \\ &= -s^{T} \psi s \\ &= -\mu s^{T} s - \eta \| s \|^{\xi+1} \\ &\leq -\eta \| s \|^{\xi+1} \\ &\leq -2^{(\xi+1)/2} \eta V_{1}^{(\xi+1)/2}. \end{split}$$

$$(10)$$

From Lemma 3 and Definition 1, TSM surface could reach the equilibrium point within finite time.

When the system states slide on the switch surface s = 0, they satisfy the following equations:

$$s = 0,$$
 (11)
 $\dot{s} = 0$

Then

$$x_2 = -C_2^{-1} \left(C_1 x_1 + C_3 x_1^{q/p} \right).$$
 (12)

Thus, we can get the following sliding mode dynamics:

$$\dot{x}_1 = \left(A_{11} - A_{12}C_1C_2^{-1}\right)x_1 - A_{12}C_3C_2^{-1}x_1^{q/p}.$$
 (13)

Theorem 6. For differential equation (13), if there exist a positive symmetric matrix P and matrices C_1C_2 , introduced in (7), satisfying the following conditions:

$$P\left(A_{11} - A_{12}C_{1}C_{2}^{-1}\right) + \left(A_{11} - A_{12}C_{1}C_{2}^{-1}\right)^{T}P < 0$$

$$PA_{12}C_{3}C_{2}^{-1} = \operatorname{diag}\left\{b_{i}\right\}, \quad b_{i} > 0, \ i = 1, \dots, r, \qquad (14)$$

$$b = \min\left\{b_{i}\right\},$$

then the system (6) is finite-time stable.

Proof. Introduce the following Lyapunov function:

$$V_2 = x_1^T P x_1, (15)$$

where *P* is a symmetric positive definite matrix satisfying the conditions in (14).

It can be proved easily that

$$\|x_1\| \ge \sqrt{\frac{V(x_1)}{\lambda}}, \quad \lambda = \lambda_{\max}(P),$$
 (16)

where $\lambda_{\max}(P)$ is the maximum eigenvalue of *P*.

Calculate its derivative along the solution of system (13) and we can obtain

$$\begin{split} \dot{V}_{2} &= 2x_{1}^{T}P\dot{x}_{1} \\ &= 2x_{1}^{T}P\left[\left(A_{11} - A_{12}C_{1}C_{2}^{-1}\right)x_{1} - A_{12}C_{3}C_{2}^{-1}x_{1}^{q/p}\right] \\ &= x_{1}^{T}\left[P\left(A_{11} - A_{12}C_{1}C_{2}^{-1}\right) + \left(A_{11} - A_{12}C_{1}C_{2}^{-1}\right)^{T}P\right]x_{1} \\ &\quad - 2x_{1}^{T}PA_{12}C_{3}C_{2}^{-1}x_{1}^{q/p} \\ &\leq -2\sum_{i=1}^{r}b_{i}\left(x_{1i}^{2}\right)^{(q+p)/2p} \\ &\leq -2b\|x_{1}\|^{(p+q)/p} \\ &\leq -\frac{2b}{\lambda^{(q+p)/2p}}V(x_{1})^{(q+p)/2p}. \end{split}$$
(17)

Then differential equation (13) is exponentially finite-time stable; state x_1 tends to zero within finite time. By (12), x_2 also tends to zero; then, the desired objective, finite-time stability of system (6), is achieved.

3.2. Adaptive Feedback Control Portion Design. After the finite-time convergence of system (6), system (4) transforms into

$$\dot{x}_3 = f\left(x_3\right). \tag{18}$$

Show each state of x_3 as the following form:

$$\dot{x} = f(x) \,. \tag{19}$$

Theorem 7. AFC controller u_2 is designed as follows and added to the system (19) on the right side:

$$u_2 = \sum_{k=1}^m \beta_k x^{2k-1},$$
 (20)

where β_k (k = 1,...,m) are adaptive parameters whose adaptive laws are

$$\dot{\beta}_k = -\sum_{j=1}^n \gamma_{kj} x^{2(j+k)-2}.$$
(21)

Then, the controlled system changes into the following form:

$$\dot{x} = f(x) + u_2 \tag{22}$$

and it is asymptotically stable.

Proof. We introduce the following Lyapunov function:

$$V_3 = \frac{1}{2} \sum_{j=1}^n \frac{1}{j} x^{2j} + \frac{1}{2} \sum_{k=1}^m \frac{1}{\gamma_k} (\beta_k + L)^2,$$
(23)

where *L* is a constant bigger than the maximum of

$$\begin{bmatrix} \frac{m}{2} \end{bmatrix} + l_{2k-1} \quad \left(k = 1, 2, \dots, \left[\frac{m+1}{2}\right]\right),$$

$$\begin{bmatrix} \frac{m}{2} \end{bmatrix} + \frac{nl_{2k}^2}{4} \quad \left(k = 1, 2, \dots, \left[\frac{m}{2}\right]\right),$$
(24)

where [m/2] denotes the largest integer, no more than m/2. The derivative of V_3 along trajectories of (23) is

$$\begin{split} \dot{V}_{3} &= \sum_{j=1}^{n} x^{2j-1} \dot{x} + \sum_{k=1}^{m} \frac{1}{\gamma_{k}} \left(\beta_{k} + L\right) \dot{\beta}_{k} \\ &\leq \sum_{j=1}^{n} |x|^{2j-1} \sum_{k=1}^{m} l_{k} \|x\|^{k} - L \sum_{k=1}^{m} \sum_{j=1}^{n} |x|^{2(j+k)-2} \\ &= \sum_{j=1}^{n} \sum_{k=1}^{\left[(m+1)/2\right]} |x|^{2j-1} l_{2k-1} \|x\|^{2k-1} + \sum_{j=1}^{n} \sum_{k=1}^{\left[m/2\right]} |x|^{2j-1} l_{2k} \|x\|^{2k} \\ &- L \sum_{j=1}^{n} \sum_{k=1}^{m} |x|^{2(j+k)-2} \\ &\leq \sum_{j=1}^{n} \sum_{k=1}^{\left[(m+1)/2\right]} l_{2k-1} |x|^{2(j+k)-2} + \sum_{j=1}^{n} \sum_{k=1}^{\left[m/2\right]} |x|^{4j-2} \\ &+ \sum_{j=1}^{n} \sum_{k=1}^{\left[m/2\right]} \frac{l_{2k}^{2}}{4} |x|^{4k} - L \sum_{j=1}^{n} \sum_{k=1}^{m} |x|^{2(j+k)-2} \end{split}$$

$$\leq \sum_{j=1}^{n} \sum_{k=1}^{[(m+1)/2]} l_{2k-1} |x|^{2(j+k)-2} + \left[\frac{m}{2}\right] \sum_{j=1}^{n} \sum_{k=1}^{m} |x|^{2(j+k)-2} + \frac{nl_{2k}^2}{4} \sum_{j=1}^{n} \sum_{k=1}^{[m/2]} |x|^{2(j+k)-2} - L \sum_{j=1}^{n} \sum_{k=1}^{m} |x|^{2(j+k)-2} \leq - \left[\sum_{k=1}^{[(m+1)/2]} \left(L - \left[\frac{m}{2}\right] - l_{2k-1}\right) \sum_{j=1}^{m} |x|^{2(j+k)-2} - \left[\sum_{k=1}^{[m/2]} \left(L - \left[\frac{m}{2}\right] - \frac{nl_{2k}^2}{4}\right) \sum_{j=1}^{m} |x|^{2(j+k)-2} \leq 0.$$
(25)

So if and only if x = 0, $\dot{V}_3 = 0$, then asymptotical stability of (22) is proved.

Remark 8. Add related terms of *i* to prove course; introduce Lyapunov function:

$$V = \frac{1}{2} \sum_{i=s+1}^{t} \sum_{j=1}^{n} \frac{1}{j} y_i^{2j} + \frac{1}{2\gamma_{ki}} \sum_{i=s+1}^{t} \sum_{k=1}^{m} (\beta_{ki} + L)^2$$
(26)

by the same method and almost the same process; we can prove the globally asymptotical stability of system state (18) with AFC controller. So AFC acts as a single-state control and also can be designed globally.

4. Simulation Results

After the theoretical analyses are investigated, we discuss the following numerical example to illustrate the derived theoretical results. From [13], we can obtain the system model of Chua's Oscillator:

$$\dot{x}_{1} = \alpha \left(-x_{1} + x_{2} - f(x_{1}) \right),$$

$$\dot{x}_{2} = x_{1} - x_{2} + x_{3},$$

$$\dot{x}_{3} = -\beta x_{2} - \gamma x_{3},$$

(27)

where $f(x_1) = bx_1 + ((a - b)/2)(|x_1 + 1| - |x_1 - 1|)$.

The Chua oscillator is illustrated in Figures 1 and 2. For simulation, the system parameters are used as $\alpha = 8.72$, $\beta = 14.29$, $\gamma = 0.01$, a = -8/7, b = -5/7, with initial conditions $(x_1(t_0), x_2(t_0), x_3(t_0))^T = (-2.1213, -0.0662, 2.8811)^T$.

With constraint condition (14), we get $C_1 = 2$, $C_2 = C_3 = 1$, introduced from (7); we take $\mu = \eta = 10$, $\xi = 1/2$, in (9); via (8), obtain u_1 , added to system (6). Its effectiveness is shown in Figure 3. After the convergence of x_1 , x_2 , there is only $\dot{x}_3 = -0.01x_3$ in (18). So we let m = 1, n = 2, and $\gamma_{kj} = 1$; in (20) and (21), get u_2 , added to system (22). Its effectiveness is shown in Figure 4.



FIGURE 1: The time series of uncontrolled Chua oscillator.



FIGURE 2: Chaotic attractor of uncontrolled Chua oscillator.

According to simulation results, it is easy to find that, the TSM method can effectively control the first two of system states and realize their finite-time stability. Comparing Figures 3 and 4, we can find that AFC controller effectively controls the remaining one of system state and makes it asymptotically stable. Finally, system states globally tend to semifinite time stable and we complete the expected objectives.

5. Conclusions

A controller, cooperating TSM with AFC, is proposed to control a class of chaotic system as described above in this paper. Two methods are complementary in the procedure and finally achieve good effectiveness. Complex TSM method controls main elements of chaotic system to finite-time stability; then, simple AFC method controls dimension elements of chaotic system to asymptotic stability and finally the overall system goes to semifinite time stability. This design scheme can not only guarantee the system's convergence but also



FIGURE 3: Time series of Chua oscillator with TSM only.



FIGURE 4: Time series of Chua oscillator with both TSM and AFC.

reduce the system's control-input spending and also further improve their applications in chaos control.

In this paper, effective performance of the simulation results proves the feasibility of this design scheme. The proposed method can be applied in many famous chaotic systems such as Lorenz, liu chaotic system, and Chua's circuit.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The authors are greatly indebted to the anonymous referees for their constructive comments. The work described in this paper was partially supported by the National Natural Science Foundation of China (no. 10971240), the Natural Science Foundation Project of CQ CSTC (nos. cstc2012jjA40052, cstc2013jcyjA0973, and cstc2013jcyjA80013), Applying Basic Research Program of Chongqing Education Committee (nos. KJ120615, KJ120630, KJ130611, and KJ1400505), the Foundation Project of Chongqing Normal University (no. 13XLZ01), and the Program of Chongqing Innovation Team Project in University under Grant no. KJTD201308.

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