Fault Tolerant Controller Design for T–S Fuzzy Systems With Time-Varying Delay and Actuator Faults: A *K*-Step Fault-Estimation Approach

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***Abstract*—This paper is concerned with the problem of robust fault estimation and fault-tolerant control for a class of Takagi–Sugeno (T–S) fuzzy systems with time-varying state delay and actuator faults. Based on the $(k-1)$th fault estimation information, a novel k-step fault-estimation observer is proposed to construct the kth fault error dynamics. The obtained fault estimates via k-step fault-estimation can practically better depict the size and shape of the faults. Then, based on the information of online k-step fault-estimation, a dynamic output feedback fault tolerant controller is designed to compensate the fault effects on the closed-loop fuzzy system. Furthermore, some less conservative delay dependent sufficient conditions for the existence of fault estimation observers and fault tolerant controllers are given in terms of solution to a set of linear matrix inequalities. Finally, simulation results of two numerical examples are presented to show the effectiveness and merits of the proposed methods.**

***Index Terms*—Dynamic output feedback control, fault tolerant control (FTC), k-step-fault-estimation, linear matrix inequalities (LMIs), Takagi–Sugeno (T–S) fuzzy systems, time-varying delay.**

I. Introduction

Since the Takagi–Sugeno (T–S) [41] fuzzy model can provide an effective representation of complex nonlinear systems in terms of fuzzy sets and fuzzy reasoning applied to a set of linear input–output submodels, it has become a popular and effective approach to control complex and ill-defined systems. This method is feasible since in many situations, human experts can provide linguistic descriptions of subsystems in terms of IF–THEN rules. So far, the problem of robust control, robust H_∞ control, fault tolerant control (FTC), and other stability analysis and stabilization of nonlinear systems through T–S fuzzy models has been extensively studied and a number of significant results on these issues, especially via the linear matrix inequality (LMI) approach, [3], [22] have been reported; see e.g., [2], [5]–[10], [14], [18]–[24], [28], [39], [42], [49], and [50].

On the other hand, due to the finite speed of information processing, time varying delays (state or/and input delays) are frequently the sources of instability and commonly exist in various engineering, biological, and economical systems for which the application of conventional controllers is infeasible. So far, during the last decade, the problem of robust control, robust H_∞ control, FTC, and other stability analysis and stabilization of nonlinear systems with time varying delays (state or/and input delays) has been extensively studied. Many criteria for checking the stability of systems with time delays have been derived from [2], [5], [6], [11], [13], [14], [19], [20], [23], [24], [26], [30], [34], [39], [42], and [48] and references therein. However, in [11], [14], [20], [23], [24], and [34], some useful integral terms in the derivatives of Lyapunov functionals were ignored, which leads to conservativeness of the existing delay dependent stability conditions. Furthermore, to our knowledge, the fault estimation for T–S fuzzy systems with time-varying state delay and actuator faults has not been fully investigated, which motivates the current work of this paper.

In the previously stated issues, especially in control systems, fault detection and isolation (FDI) and FTC have been the subjects of intensive investigations over the past two decades [1], [4], [12], [15], [17], [18], [21], [25]–[27], [29], [32], [36], [37], [40], [43], [45]–[48], [51]. However, in practical engineering, as a result of unexpected model uncertainties, time delays, disturbances, perturbations and noises may occur in the fault systems, it is quite difficult to obtain the accurate size of the fault from an FDI scheme only [21]. Fortunately, fault estimation can depict the size and shape of the fault and can thus automatically perform the required fault detection. The problem of fault estimation has stirred renewed research interest, and a variety of fault estimation approaches have been developed in the literatures; see, for instance, [12], [15], [21], [30], [31], [33], [38], [44], [50], and references therein. In [40] and [43], a reliable fault-tolerant controller for T–S fuzzy models against actuator faults via passive FTC idea was designed, while issues of fault detection and estimation were not involved. Robust fault detection for T–S fuzzy systems was investigated in [18], [29], and [32], but the issue of fault estimation was not involved. Under a restrictive assumption on the faults, i.e., $f(t) \in L_2[0, \infty)$, Nguang *et al.* [30] studied the problem of robust fault estimation

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for T–S fuzzy models with time-varying delay. A sliding-mode observer and an adaptive observer were proposed to achieve fault estimation in [3], [15], and [21], but their design needed very restrictive conditions to be satisfied. Recently, Zhang *et al.* [50] relaxed some restrictive conditions in these issues and dealt with robust fault estimation for T–S fuzzy models, but time-varying state delay was not included. In the existing one-step fault-estimation approach [50], the effect of input disturbances from the derivatives of actuator faults was ignored, which may result in that the faults cannot be well estimated by using the one-step fault-estimation approach. Therefore, a new method should be considered to deal with input disturbances from the derivatives of actuator faults so that the faults can be better estimated, which leads to challenge and interest and also motivates the current research.

This paper mainly studies the problem of robust fault estimation and FTC for a class of T–S fuzzy systems with time-varying state delay and actuator faults. A k-step fault-estimation method is proposed, which in practice relaxes the existing one in [50] through applying the initial derivatives of fault estimates to weaken the effect of input disturbance from the derivatives of actuator faults in the error dynamics. In this way, the obtained fault estimates can be practically close to the faults. Then, based on the information of online k-step fault estimation, a dynamic output feedback fault tolerant controller is designed to compensate the fault effects on the closed-loop fuzzy system. The obtained delay dependent sufficient conditions for the existence of k-step fault-estimation observers and fault tolerant controllers for T–S fuzzy systems with time-varying state delay and actuator faults are given in terms of solution to a set of linear matrix inequalities (LMIs). Finally, simulation results of two numerical examples are presented to demonstrate the effectiveness and advantages of the proposed methods.

Throughout the paper, R^n denotes the n-dimensional real Euclidean space; I denotes the identity matrix; the superscripts "T" and "-1" stand for the matrix transpose and inverse, respectively; notation $X > 0$ ($X \geq 0$) means that the matrix X is real symmetric positive definite (positive semidefinite); and $\|\cdot\|$ is the spectral norm. If not explicitly stated, all matrices are assumed to have compatible dimensions for algebraic operations. The symbol "$*$" in a matrix stands for the transposed elements in the symmetric positions.

II. Problem Formulation

Consider a continuous-time fuzzy system with time-varying state delay and actuator faults, which is represented by a T–S fuzzy model composed of a set of fuzzy implications. Each implication is expressed by a linear time-delay system and the ith rule of the T–S fuzzy model is written as follows:
Plant rule i:
IF $\xi_1(t)$ is M_{i1} and ... and $\xi_p(t)$ is M_{ip} THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{\tau i} x(t-\tau(t)) \\ \qquad\qquad + B_{ui}[u(t)+f(t)] + B_{wi} w(t), t \geq 0 \\ y(t) = C_i x(t) + C_{\tau i} x(t-\tau(t)) + D_{wi} w(t) \\ x(t) = \phi_i(t), t \in [-\tau, 0], i = 1, 2, \ldots, r \end{cases} \tag{1}$$

where M_{ij}, $(i = 1, 2, \ldots, r, j = 1, 2, \ldots, p)$ are fuzzy sets, $x(t) \in R^n$ is the state vector, $u(t) \in R^q$ is the input vector, $w(t) \in R^m$ is the exogenous disturbance input that belongs to $L_2[0, \infty)$, $y(t) \in R^l$ is the output, and $f(t) \in R^q$ represents the additive actuator fault. $A_i, A_{\tau i}, B_{ui}, B_{wi}, C_i, C_{\tau i}$, and D_{wi} are constant real matrices of appropriate dimensions. It is supposed that matrices B_{ui} are of full column rank, the pairs (A_i, B_{ui}) are controllable, and the pairs (A_i, C_i) are observable, where $i = 1, 2, \ldots, r$, and r is the number of IF–THEN rules. In addition, $\xi_1(t), \ldots, \xi_p(t)$ are the premise variables. It is assumed in this paper that the premise variables do not depend on the input variables; $\tau(t)$ is the time-varying state delay satisfying $\tau(t) \leq \tau$ ($\dot{\tau}(t) \leq \tau_D$ or unknown). $\phi_i(t)$ is a vector-valued initial continuous function defined on the interval $[-\tau, 0]$.

Then, the overall opened-loop T–S fuzzy system with time-varying state delay and actuator faults is inferred as follows:

$$\begin{cases} \dot{x}(t) = \mathscr{A}(\xi) x(t) + \mathscr{A}_\tau(\xi) x(t-\tau(t)) \\ \qquad\qquad + \mathscr{B}_u(\xi)[u(t)+f(t)] + \mathscr{B}_w(\xi) w(t), t \geq 0 \\ y(t) = \mathscr{C}(\xi) x(t) + \mathscr{C}_\tau(\xi) x(t-\tau(t)) + \mathscr{D}_w(\xi) w(t) \\ x(t) = \phi(t), \forall t \in [-\tau, 0]. \end{cases} \tag{2}$$

where

$$\mathscr{A}(\xi) = \sum_{i=1}^{r} \mu_i(\xi(t)) A_i, \mathscr{A}_\tau(\xi) = \sum_{i=1}^{r} \mu_i(\xi(t)) A_{\tau i}$$

$$\mathscr{B}_u(\xi) = \sum_{i=1}^{r} \mu_i(\xi(t)) B_{ui}, \mathscr{B}_w(\xi) = \sum_{i=1}^{r} \mu_i(\xi(t)) B_{wi}$$

$$\mathscr{C}(\xi) = \sum_{i=1}^{r} \mu_i(\xi(t)) C_i, \mathscr{C}_\tau(\xi) = \sum_{i=1}^{r} \mu_i(\xi(t)) C_{\tau i}$$

$$\mathscr{D}_w(\xi) = \sum_{i=1}^{r} \mu_i(\xi(t)) D_{wi}, \phi(t) = \sum_{i=1}^{r} \mu_i(\xi(t)) \phi_i(t)$$

and $\xi(t) = (\xi_1(t), \xi_2(t), \ldots, \xi_p(t)), \mu_i(\xi(t)) = \beta_i(\xi(t))/\sum_{j=1}^{r} \beta_j(\xi(t)), \beta_i(\xi(t)) = \prod_{i=1}^{p} M_{ij}(\xi(t))$, and $\xi_i(t)$ are the premise variables. $M_{ij}(\xi_j(t))$ is the grade of membership of $\xi_j(t)$ in M_{ij}. It is easy to find that $\forall t$: $\beta_i(\xi(t)) \geq 0, (i = 1, 2, \ldots, r), \sum_{j=1}^{r} \beta_j(\xi(t)) > 0$. Therefore, $\mu_i(\xi(t)) \geq 0$, for $i = 1, 2, \ldots, r$ and $\sum_{j=1}^{r} \mu_j(\xi(t)) = 1, \forall t$.

III. Main Results

In the following, the main results are to be expressed for the T–S fuzzy system with time-varying state delay and additive actuator faults. First, a novel k-step fault-estimation approach is proposed to detect and estimate the actuator faults.

A. *Actuator Fault Estimation: A K-Step Fault-Estimation Approach*

1) One-Step Fault Estimation: An one-step fault-estimation observer similar to the existing one in [50] (with no state delay) is constructed as follows:

$$\begin{cases} \dot{\hat{x}}_1(t) = \mathscr{A}(\xi)\hat{x}_1(t) + \mathscr{A}_\tau(\xi)\hat{x}_1(t-\tau(t)) \\ \qquad + \mathscr{B}_u(\xi)[u(t) + \hat{f}_1(t)] - \mathscr{L}(\xi)(\hat{y}_1(t) - y(t)) \\ \hat{y}_1(t) = \mathscr{C}(\xi)\hat{x}_1(t) + \mathscr{C}_\tau(\xi)\hat{x}_1(t-\tau(t)) \\ \dot{\hat{f}}_1(t) = -\mathscr{G}(\xi)(\hat{y}_1(t) - y(t)) \end{cases} \tag{3}$$

where $\hat{x}_1(t) \in R^n$ is the observer state, $\hat{y}_1(t) \in R^l$ is the observer output, and $\hat{f}_1(t) \in R^q$ is an initial estimate of $f(t)$. $\mathscr{L}(\xi)$ and $\mathscr{G}(\xi)$ are the gain matrices of appropriate dimensions to be designed: $\mathscr{L}(\xi) = \sum_{i=1}^r \mu_i(\xi(t))L_i, \mathscr{G}(\xi) = \sum_{i=1}^r \mu_i(\xi(t))G_i$. If we denote $e_{x1}(t) = \hat{x}_1(t) - x(t), e_{y1}(t) = \hat{y}_1(t) - y(t)$, $e_{f1}(t) = \hat{f}_1(t) - f(t)$, and $e_1^T(t) = [e_{x1}^T(t), e_{f1}^T(t)], \omega_1^T(t) = [w^T(t), \dot{f}^T(t)]$, then, combining (2), the first error dynamics is obtained by

$$\begin{cases} \dot{e}_1(t) = [\mathcal{A}(\xi) - \mathcal{L}_{\mathscr{G}}(\xi)\mathcal{C}(\xi)]e_1(t) \\ \qquad + [\mathcal{A}_\tau(\xi) - \mathcal{L}_{\mathscr{G}}(\xi)\mathcal{C}_\tau(\xi)]e_1(t-\tau(t)) \\ \qquad + [\mathcal{L}_{\mathscr{G}}(\xi)\mathcal{D}_w(\xi) - \mathcal{B}_w(\xi)]\omega_1(t) \\ e_{y1}(t) = \mathcal{C}(\xi)e_1(t) + \mathcal{C}_\tau(\xi)e_1(t-\tau(t)) - \mathcal{D}_w(\xi)\omega_1(t) \end{cases} \tag{4}$$

where

$$\mathcal{A}(\xi) = \begin{bmatrix} \mathscr{A}(\xi) & \mathscr{B}_u(\xi) \\ 0 & 0 \end{bmatrix}, \quad \mathcal{A}_\tau(\xi) = \begin{bmatrix} \mathscr{A}_\tau(\xi) & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathcal{L}_{\mathscr{G}}(\xi) = \begin{bmatrix} \mathscr{L}(\xi) \\ \mathscr{G}(\xi) \end{bmatrix}, \quad \mathcal{B}_w(\xi) = \begin{bmatrix} \mathscr{B}_w(\xi) & 0 \\ 0 & I_q \end{bmatrix}$$

$$\mathcal{C}(\xi) = [\,\mathscr{C}(\xi) \quad 0\,], \quad \mathcal{C}_\tau(\xi) = [\,\mathscr{C}_\tau(\xi) \quad 0\,]$$

$$\mathcal{D}_w(\xi) = [\,\mathscr{D}_w(\xi) \quad 0\,].$$

Assumption 1: $\dot{f}(t)$ belongs to $L_2[0, \infty)$.

Remark 1: In [29] and [31], the fault estimation filter is designed under the assumption $f(t) \in L_2[0, \infty)$. In general, sliding-mode observer-based fault estimation requires the preliminary knowledge of the upper bound of $f(t)$ [3], [21], [35]. However, as described in [50], in many practical systems, there is a transient period during which the fault establishes itself, after which, it remains more or less constant, meaning that the derivatives of the faults are energy-bounded, i.e., $\dot{f}(t) \in L_2[0, \infty)$. This is stated by Assumption 1, which is more general than those used in the aforementioned design methods.

Remark 2: If there is no state delay, then (3) reduces to the existing one in [50], which implies that one-step fault-estimation observer with state delay is more challenging than the existing one in [50] since time varying delays are frequently the sources of instability and commonly exist in various control systems. However, one-step fault-estimation method is generally conservative for estimating actuator faults. One can see from the first error dynamics that $\dot{f}(t)$ is straightforward considered as an input disturbance while the effect of it on the system ignored. Therefore, for the error dynamics (4), a new method should be proposed to constrain the effect of input disturbance from $\dot{f}(t)$, which leads to challenge and interest and also motivates us to propose the following k-step fault-estimation approach.

2) Two-Step Fault-Estimation: To constrain the effect of input disturbance from $\dot{f}(t)$, the information of $\dot{\hat{f}}_1(t)$ obtained from the first step fault-estimation will be applied to reconstruct a two-step fault-estimation observer as follows:

$$\begin{cases} \dot{\hat{x}}_2(t) = \mathscr{A}(\xi)\hat{x}_2(t) + \mathscr{A}_\tau(\xi)\hat{x}_2(t-\tau(t)) \\ \qquad + \mathscr{B}_u(\xi)[u(t) + \hat{f}_2(t)] - \mathscr{L}(\xi)(\hat{y}_2(t) - y(t)) \\ \hat{y}_2(t) = \mathscr{C}(\xi)\hat{x}_2(t) + \mathscr{C}_\tau(\xi)\hat{x}_2(t-\tau(t)) \\ \dot{\hat{f}}_2(t) = -\mathscr{G}(\xi)(\hat{y}_2(t) - y(t)) + \dot{\hat{f}}_1(t) \end{cases} \tag{3'}$$

where $\hat{x}_2(t) \in R^n$ is the observer state, $\hat{y}_2(t) \in R^l$ is the observer output, and $\hat{f}_2(t) \in R^q$ is the second estimate of $f(t)$. $\mathscr{L}(\xi)$ and $\mathscr{G}(\xi)$ are defined as in the first-step estimation. Denote $e_{x2}(t) = \hat{x}_2(t) - x(t), e_{y2}(t) = \hat{y}_2(t) - y(t)$, $e_{f2}(t) = \hat{f}_2(t) - f(t)$, and $e_2^T(t) = [e_{x2}^T(t), e_{f2}^T(t)], \omega_2^T(t) = [w^T(t), \dot{f}^T(t) - \dot{\hat{f}}_1^T(t)]$; then, combining (2), the second-error dynamics is obtained by

$$\begin{cases} \dot{e}_2(t) = [\mathcal{A}(\xi) - \mathcal{L}_{\mathscr{G}}(\xi)\mathcal{C}(\xi)]e_2(t) \\ \qquad + [\mathcal{A}_\tau(\xi) - \mathcal{L}_{\mathscr{G}}(\xi)\mathcal{C}_\tau(\xi)]e_2(t-\tau(t)) \\ \qquad + [\mathcal{L}_{\mathscr{G}}(\xi)\mathcal{D}_w(\xi) - \mathcal{B}_w(\xi)]\omega_2(t) \\ e_{y2}(t) = \mathcal{C}(\xi)e_2(t) + \mathcal{C}_\tau(\xi)e_2(t-\tau(t)) - \mathcal{D}_w(\xi)\omega_2(t) \end{cases} \tag{4'}$$

where $\mathcal{A}(\xi), \mathcal{A}_\tau(\xi), \mathcal{L}_{\mathscr{G}}(\xi), \mathcal{B}_w(\xi), \mathcal{C}(\xi), \mathcal{C}_\tau(\xi)$, and $\mathcal{D}_w(\xi)$ are defined as in the first-step estimation.

3) K-Step Fault Estimation: Similar to the two-step fault-estimation approach, using the information of $\dot{\hat{f}}_{k-1}(t)$ obtained from the $(k-1)$-step fault-estimation ($k \in \{2, 3, \ldots\}$), a k-step fault-estimation observer is constructed as follows:

$$\begin{cases} \dot{\hat{x}}_k(t) = \mathscr{A}(\xi)\hat{x}_k(t) + \mathscr{A}_\tau(\xi)\hat{x}_k(t-\tau(t)) \\ \qquad + \mathscr{B}_u(\xi)[u(t) + \hat{f}_k(t)] - \mathscr{L}(\xi)(\hat{y}_k(t) - y(t)) \\ \hat{y}_k(t) = \mathscr{C}(\xi)\hat{x}_k(t) + \mathscr{C}_\tau(\xi)\hat{x}_k(t-\tau(t)) \\ \dot{\hat{f}}_k(t) = -\mathscr{G}(\xi)(\hat{y}_k(t) - y(t)) + \dot{\hat{f}}_{k-1}(t) \end{cases} \tag{3''}$$

where $\hat{x}_k(t) \in R^n$ is the observer state, $\hat{y}_k(t) \in R^l$ is the observer output, and $\hat{f}_k(t) \in R^q$ is the kth estimate of $f(t)$. $\mathscr{L}(\xi)$ and $\mathscr{G}(\xi)$ are defined as in the first-step estimation. Denote $e_{xk}(t) = \hat{x}_k(t) - x(t), e_{yk}(t) = \hat{y}_k(t) - y(t)$, $e_{fk}(t) = \hat{f}_k(t) - f(t)$, and $e_k^T(t) = [e_{xk}^T(t), e_{fk}^T(t)], \omega_k^T(t) = [w^T(t), \dot{f}^T(t) - \dot{\hat{f}}_{k-1}^T(t)]$; then, combining (2), the kth error dynamics is obtained by

$$\begin{cases} \dot{e}_k(t) = [\mathcal{A}(\xi) - \mathcal{L}_{\mathscr{G}}(\xi)\mathcal{C}(\xi)]e_k(t) \\ \qquad + [\mathcal{A}_\tau(\xi) - \mathcal{L}_{\mathscr{G}}(\xi)\mathcal{C}_\tau(\xi)]e_k(t-\tau(t)) \\ \qquad + [\mathcal{L}_{\mathscr{G}}(\xi)\mathcal{D}_w(\xi) - \mathcal{B}_w(\xi)]\omega_k(t) \\ e_{yk}(t) = \mathcal{C}(\xi)e_k(t) + \mathcal{C}_\tau(\xi)e_k(t-\tau(t)) - \mathcal{D}_w(\xi)\omega_k(t) \end{cases} \tag{4''}$$

where $\mathcal{A}(\xi), \mathcal{A}_\tau(\xi), \mathcal{L}_{\mathscr{G}}(\xi), \mathcal{B}_w(\xi), \mathcal{C}(\xi), \mathcal{C}_\tau(\xi)$, and $\mathcal{D}_w(\xi)$ are defined as in the first-step estimation.

Remark 3: In the k-step fault-estimation procedure ($k \geq 2$), one can see that the input disturbance $\dot{f}(t)$ is converted to $\dot{f}(t) - \dot{\hat{f}}_i(t), i = 1, 2, \ldots k-1$. Therefore, by using k-step

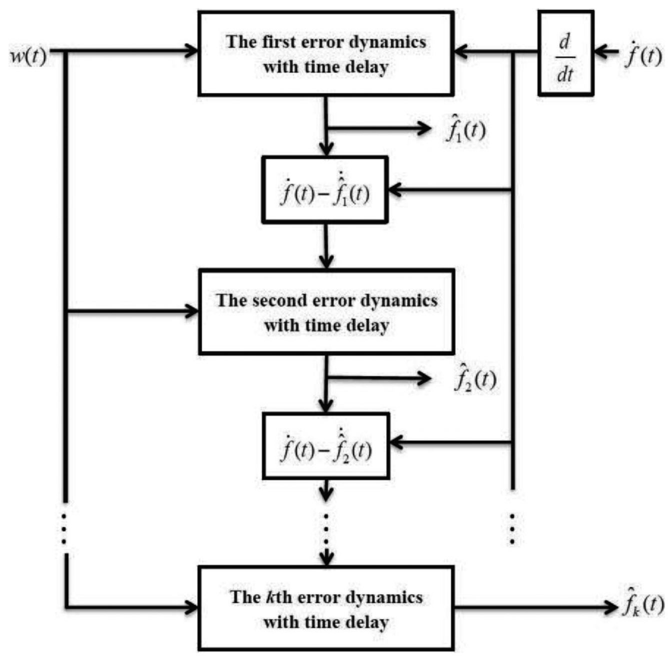

Fig. 1. Flowchart of k-step fault-estimation.

fault-estimation approach, a group of fault estimates $\hat{f}_i(t), i = 1, 2, \ldots, k-1$ are obtained. Since $\hat{f}_i(t)$ is the estimate of $f(t)$, then $-\dot{\hat{f}}_i(t)$ can more or less increasingly weaken the effect intensive of input disturbance from $\dot{f}(t)$ in the error dynamics. Therefore, after k-step fault estimating, $\hat{f}_k(t)$ can be practically close well to the fault $f(t)$. The simulation examples illustrate the effectiveness and merits. Furthermore, the flowchart of our k-step fault-estimation approach is shown in Fig. 1 .

So far, the aim of k-step fault-estimation includes one to convert disturbances from the derivatives of $f(t)$ to that from the derivatives of $f(t) - \hat{f}(t)$, the other is to design the gain matrices such that

1) the kth ($k \in \{1, 2, \ldots\}$) error dynamics ($4''$) with time-varying state delay is asymptotically stable (with $\omega_k(t) = 0$);
2) the following H_∞ performance is satisfied:

$$\int_0^L \|e_{fk}(t)\|^2 dt \leq \gamma_k^2 \int_0^L \|\omega_k(t)\|^2 dt, k \in \{1, 2, \ldots\} \tag{5}$$

for all $L > 0$ and $\omega_k(t) \in L_2[0, \infty)$ under zero initial conditions.

Remark 4: In [50], the gain matrices was designed to made the eigenvalues of state matrices in the error dynamics belong to a circular region $D(a, r)$ with center $a + j0$ and radius r, and the error dynamics satisfying a given H_∞ performance. However, in this paper, the design objective is converted to design the gain matrices such that the error dynamics satisfies 1) and 2).

Lemma 1: For the given positive scalars: τ, τ_D, δ, and γ_k, the kth error dynamics ($4''$) with time-varying state delay is asymptotically stable (with $\omega_k(t) = 0$) while satisfying a prescribed H_∞ performance (5) ($k \in \{1, 2, \ldots\}$) if there exist matrices $P > 0, Q_1 > 0, Q_2 > 0, R > 0, L_i, G_i, i = 1, 2, \ldots, r$ and free weighting matrices M_i, N_i of appropriate dimensions, $i = 1, 2, \ldots, r$ such that the following inequalities hold:

$$\begin{bmatrix} \tilde{\Omega}(\xi) & \tau M(\xi) \\ * & -\tau R \end{bmatrix} < 0 \tag{6}$$

and,

$$\begin{bmatrix} \tilde{\Omega}(\xi) & \tau N(\xi) \\ * & -\tau R \end{bmatrix} < 0 \tag{7}$$

where

$$\begin{aligned} \tilde{\Omega}(\xi) = &\begin{bmatrix} \Psi_1(\xi) & \Psi_2(\xi) & 0 & \Psi_3(\xi) \\ * & -\varepsilon(1-\tau_D)Q_1 & 0 & 0 \\ * & * & -Q_2 & 0 \\ * & * & * & -\gamma_k^2 I \end{bmatrix} \\ &+ \tau\Gamma^T(\xi)P[2\delta P - \delta^2 R]^{-1}P\Gamma(\xi) \\ &- M(\xi)(\mathbb{E}_1 - \mathbb{E}_2) - (\mathbb{E}_1 - \mathbb{E}_2)^T M^T(\xi) \\ &- N(\xi)(\mathbb{E}_2 - \mathbb{E}_3) - (\mathbb{E}_2 - \mathbb{E}_3)^T N^T(\xi) \end{aligned} \tag{8}$$

where

$$\begin{aligned} \Psi_1(\xi) &= P(\mathcal{A}(\xi) - \mathcal{L}_{\mathcal{G}}(\xi)\mathcal{C}(\xi)) + (\mathcal{A}(\xi) - \mathcal{L}_{\mathcal{G}}(\xi)\mathcal{C}(\xi))^T P \\ &\quad + \varepsilon Q_1 + Q_2 + \tilde{I}_q \tilde{I}_q^T, \quad \tilde{I}_q^T = [0 \quad I_q] \\ \Psi_2(\xi) &= P(\mathcal{A}_\tau(\xi) - \mathcal{L}_{\mathcal{G}}(\xi)\mathcal{C}_\tau(\xi)) \\ \Psi_3(\xi) &= P(\mathcal{L}_{\mathcal{G}}(\xi)\mathcal{D}_w(\xi) - \mathcal{B}_w(\xi)) \\ \Gamma(\xi) &= [\mathcal{A}(\xi) - \mathcal{L}_{\mathcal{G}}(\xi)\mathcal{C}(\xi), \quad \mathcal{A}_\tau(\xi) - \mathcal{L}_{\mathcal{G}}(\xi)\mathcal{C}_\tau(\xi), \quad 0 \\ &\quad \mathcal{L}_{\mathcal{G}}(\xi)\mathcal{D}_w(\xi) - \mathcal{B}_w(\xi)] \\ M(\xi) &= \sum_{i=1}^r \mu_i(\xi(t))M_i, N(\xi) = \sum_{i=1}^r \mu_i(\xi(t))N_i \\ \mathbb{E}_1 &= [I, 0, 0, 0], \mathbb{E}_2 = [0, I, 0, 0], \ldots, \mathbb{E}_4 = [0, 0, 0, I]. \end{aligned}$$

Proof: In this paper, the Lyapunov function candidate is constructed as follows: for $k \in \{1, 2, \ldots\}$

$$V_k(t) = e_k^T(t)Pe_k(t) + V_1 + V_2 \tag{9}$$

where

$$V_1 = \varepsilon \cdot \int_{t-\tau(t)}^t e_k^T(s)Q_1 e_k(s)ds + \int_{t-\tau}^t e_k^T(s)Q_2 e_k(s)ds, \tag{10}$$

$$V_2 = \int_{-\tau}^0 \int_{t+\theta}^t \dot{e}_k^T(s)R\dot{e}_k(s)dsd\theta \tag{11}$$

where $P > 0, \varepsilon \geq 0, Q_1 > 0, Q_2 > 0, R > 0$. Then, the time derivatives of $V_k(t)$, along the trajectories of the error dynamics (4) satisfy

$$\begin{aligned} \dot{V}_k(t) = &\ e_k^T(t)P(\mathcal{A}(\xi) - \mathcal{L}_{\mathcal{G}}(\xi)\mathcal{C}(\xi))e_k(t) \\ &+ e_k^T(t)(\mathcal{A}(\xi) - \mathcal{L}_{\mathcal{G}}(\xi)\mathcal{C}(\xi))^T Pe_k(t) \\ &+ e_k^T(t)(\varepsilon Q_1 + Q_2)e_k(t) \\ &+ 2e_k^T(t)P(\mathcal{A}_\tau(\xi) - \mathcal{L}_{\mathcal{G}}(\xi)\mathcal{C}_\tau(\xi))e_k(t-\tau(t)) \\ &+ 2e_k^T(t)P(\mathcal{L}_{\mathcal{G}}(\xi)\mathcal{D}_w(\xi) - \mathcal{B}_w(\xi))\omega_k(t) \end{aligned}$$

$$\begin{aligned}&-\varepsilon(1-\dot{\tau}(t))e_k^T(t-\tau(t))Q_1e_k(t-\tau(t))\\&-e_k^T(t-\tau)Q_2e_k(t-\tau)\\&+\tau\dot{e}_k^T(t)R\dot{e}_k(t)-\int_{t-\tau}^{t}\dot{e}_k^T(s)R\dot{e}_k(s)ds.\end{aligned}$$

Furthermore, from the Newton–Leibniz formula, a straightforward computation gives

$$\begin{aligned}&\dot{V}_k(t)+e_{fk}^T(t)e_{fk}(t)-\gamma_k^2\omega_k^T(t)\omega_k(t)\\&=\dot{V}_k(t)+e_k^T(t)\tilde{I}_q\tilde{I}_q^Te_k(t)-\gamma_k^2\omega_k^T(t)\omega_k(t)\\&\quad-2\zeta^T(t)M(\xi)\left[e_k(t)-e_k(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{e}_k(s)ds\right]\\&\quad-2\zeta^T(t)N(\xi)\left[e_k(t-\tau(t))-e_k(t-\tau)-\int_{t-\tau}^{t-\tau(t)}\dot{e}_k(s)ds\right]\\&\le\frac{1}{\tau}\int_{t-\tau(t)}^{t}\begin{bmatrix}\zeta(t)\\\dot{e}_k(s)\end{bmatrix}^T\begin{bmatrix}\tilde{\Omega}(\xi)&\tau M(\xi)\\ *&-\tau R\end{bmatrix}\begin{bmatrix}\zeta(t)\\\dot{e}_k(s)\end{bmatrix}\\&\quad\times\frac{1}{\tau}\int_{t-\tau}^{t-\tau(t)}\begin{bmatrix}\zeta(t)\\\dot{e}_k(s)\end{bmatrix}^T\begin{bmatrix}\tilde{\Omega}(\xi)&\tau N(\xi)\\ *&-\tau R\end{bmatrix}\begin{bmatrix}\zeta(t)\\\dot{e}_k(s)\end{bmatrix}.\end{aligned}$$

where $\zeta^T(t)=[e_k^T(t),e_k^T(t-\tau(t)),e_k^T(t-\tau),\omega_k^T(t)]$. On the other hand, if (6) and (7) hold, one has

$$\dot{V}_k(t)+e_{fk}^T(t)e_{fk}(t)-\gamma_k^2\omega_k^T(t)\omega_k(t)<0.$$

It follows from (1) that $x(t)=0,\forall t\in[-\tau,0]$ when $\phi_i(t)=0,i=1,2,\ldots,r$. It further follows that $e_k(t)=0$ and $\dot{e}_k(t)=0,\forall t\in[-\tau,0]$. Therefore, for $e_k(0)=0$, one has

$$\begin{aligned}V_k(t)|_{t=0}&=e_k^T(0)Pe_k(0)+\varepsilon\cdot\int_{-\tau(t)}^{0}e_k^T(s)Q_1e_k(s)ds\\&\quad+\int_{-\tau}^{0}e_k^T(s)Q_2e_k(s)ds\\&\quad+\int_{-\tau}^{0}\int_{\theta}^{0}\dot{e}_k^T(s)R\dot{e}_k(s)dsd\theta=0.\end{aligned}$$

Consequently, it follows from $V_k(t)|_{t=L}\ge 0$ that

$$\int_0^L(\|e_{fk}(t)\|^2-\gamma_k^2\|\omega_k(t)\|^2)dt+V_k(t)|_{t=L}-V_k(t)|_{t=0}\le 0$$

which implies that (5) holds. Therefore, the H_∞ performance is verified.

In addition, if (6) and (7) hold, then the time derivatives of $V_k(t)$ along the solution of (4′′) when $\omega_k(t)=0$ satisfies $\dot{V}_k(t)<0$. As a result, the asymptotic stability of error dynamics (4′′) follows immediately when $\omega_k(t)=0$ for $k\in\{1,2,\ldots\}$. The proof is thus completed. ■

Remark 5: In the estimation of upper bounds of time delay terms, for the Lyapunov functional candidate always involves the integral term $\int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{\eta}^T(s)R\dot{\eta}(s)dsd\theta$, the derivatives of the term was always estimated as $\tau\dot{\eta}^T(t)R\dot{\eta}(t)-\int_{t-\tau(t)}^{t}\dot{\eta}^T(s)R\dot{\eta}(s)ds$, and the term $-\int_{t-\tau}^{t-\tau(t)}\dot{\eta}^T(s)R\dot{\eta}(s)ds$ was ignored [14], [19], [20], or some useful negative integral terms lost; see, e.g., [11], [14], [19], [20], [23], [24], and [34]. However, in Lemma 1, an improved integral inequality method without ignoring any integral term is applied to convert the integral inequalities to matrix inequalities, which theoretically leads to less conservativeness than the existing ones in [11], [14], [19], [20], [23], [24], and [34], and the relevant results and proofs can be seen in Appendix. Furthermore, the introduction of ε ($\varepsilon\ge 0$) indicates that Lemma 1 can be suitable for the time-varying delay $\tau(t)$ being or not differentiable, that is, in the case of time-varying delay $\tau(t)$ being not differentiable, then one can set $\varepsilon=0$ in the result.

Theorem 1: For the given positive scalars τ,τ_D,δ, and γ_k, the kth error dynamics (4′′) with time-varying state delay is asymptotically stable (with $\omega_k(t)=0$) while satisfying a prescribed H_∞ performance (5) ($k\in\{1,2,\ldots\}$) if there exist matrices $P>0,Q_1>0,Q_2>0,R>0,\mathcal{Y}_i,i=1,2,\ldots,r$ and free weighting matrices M_i,N_i of appropriate dimensions, $i=1,2,\ldots,r$, such that the following LMIs hold:

$$\Xi_{ii}<0,\quad i=1,2,\ldots,r \tag{12}$$

$$\Xi_{ij}+\Xi_{ji}\le 0,\quad 1\le i<j\le r \tag{13}$$

$$\Pi_{ii}<0,\quad i=1,2,\ldots,r \tag{14}$$

$$\Pi_{ij}+\Pi_{ji}\le 0,\quad 1\le i<j\le r \tag{15}$$

where

$$\Xi_{ij}=\begin{bmatrix}\Omega_{ij}&\tau M_i&\sqrt{\tau}\Gamma_{ij}^T\\ *&-\tau R&0\\ *&*&-2\delta P+\delta^2R\end{bmatrix} \tag{16}$$

and

$$\Pi_{ij}=\begin{bmatrix}\Omega_{ij}&\tau N_i&\sqrt{\tau}\Gamma_{ij}^T\\ *&-\tau R&0\\ *&*&-2\delta P+\delta^2R\end{bmatrix} \tag{17}$$

where $M_i^T=[M_{1i}^T,M_{2i}^T,M_{3i}^T,U_{1i}^T]$, $N_i^T=[N_{1i}^T,N_{2i}^T,N_{3i}^T,U_{2i}^T]$, $\Gamma_{ij}=[P\mathcal{A}_i-\mathcal{Y}_i\mathcal{C}_j,P\mathcal{A}_{\tau i}-\mathcal{Y}_i\mathcal{C}_{\tau j},0,\mathcal{Y}_i\mathcal{D}_{wj}-P\mathcal{B}_{wi}]$, and

$$\begin{aligned}\Omega_{ij}=&\begin{bmatrix}\Psi_{1ij}&\Psi_{2ij}&0&\Psi_{3ij}\\ *&-\varepsilon(1-\tau_D)Q_1&0&0\\ *&*&-Q_2&0\\ *&*&*&-\gamma_k^2I\end{bmatrix}\\&-M_i(\mathbb{E}_1-\mathbb{E}_2)-(\mathbb{E}_1-\mathbb{E}_2)^TM_i^T\\&-N_i(\mathbb{E}_2-\mathbb{E}_3)-(\mathbb{E}_2-\mathbb{E}_3)^TN_i^T.\end{aligned} \tag{18}$$

where

$\Psi_{1ij}=P\mathcal{A}_i-\mathcal{Y}_i\mathcal{C}_j+(P\mathcal{A}_i-\mathcal{Y}_i\mathcal{C}_j)^T+\varepsilon Q_1+Q_2+\tilde{I}_q\tilde{I}_q^T$
$\Psi_{2ij}=P\mathcal{A}_{\tau i}-\mathcal{Y}_i\mathcal{C}_{\tau j}$
$\Psi_{3ij}=\mathcal{Y}_i\mathcal{D}_{wj}-P\mathcal{B}_{wi}$
$\mathbb{E}_1=[I,0,0,0],\mathbb{E}_2=[0,I,0,0],\ldots,\mathbb{E}_4=[0,0,0,I]$.

Then, the gain matrices can be obtained as follows:

$$L_{\mathscr{G}i}=\begin{bmatrix}L_i\\G_i\end{bmatrix}=P^{-1}\mathcal{Y}_i,\quad i=1,2,\ldots,r. \tag{19}$$

Proof: If (12)–(15) hold, then by the Schur complement, one has

$$\tilde{\Xi}_{ii} < 0, \quad i = 1, 2, \ldots, r \tag{20}$$

$$\tilde{\Xi}_{ij} + \tilde{\Xi}_{ji} \leq 0, \quad 1 \leq i < j \leq r \tag{21}$$

$$\tilde{\Pi}_{ii} < 0, \quad i = 1, 2, \ldots, r \tag{22}$$

$$\tilde{\Pi}_{ij} + \tilde{\Pi}_{ji} \leq 0, \quad 1 \leq i < j \leq r \tag{23}$$

where

$$\tilde{\Xi}_{ij} = \begin{bmatrix} \tilde{\Omega}_{ij} & \tau M_i \\ * & -\tau R \end{bmatrix} \tag{24}$$

and

$$\tilde{\Pi}_{ij} = \begin{bmatrix} \tilde{\Omega}_{ij} & \tau N_i \\ * & -\tau R \end{bmatrix} \tag{25}$$

where

$$\begin{aligned} \tilde{\Omega}_{ij} = & \begin{bmatrix} \Psi_{1ij} & \Psi_{2ij} & 0 & \Psi_{3ij} \\ * & -\varepsilon(1-\tau_D)Q_1 & 0 & 0 \\ * & * & -Q_2 & 0 \\ * & * & * & -\gamma_k^2 I \end{bmatrix} \\ & + \tau \Gamma_{ij}^T P[2\delta P - \delta^2 R]^{-1} P \Gamma_{ij} \\ & - M_i(\mathbb{E}_1 - \mathbb{E}_2) - (\mathbb{E}_1 - \mathbb{E}_2)^T M_i^T \\ & - N_i(\mathbb{E}_2 - \mathbb{E}_3) - (\mathbb{E}_2 - \mathbb{E}_3)^T N_i^T \end{aligned} \tag{26}$$

with the variable changing $\mathcal{Y}_i = PL_{\mathscr{G}i}$. Denote $\mu_i = \mu_i(\xi(t))$. Therefore, if (20)–(23) hold, then

$$\sum_{i=1}^{r} \mu_i^2 \tilde{\Xi}_{ii} + \sum_{i=1}^{r} \sum_{i<j}^{r} \mu_i \mu_j (\tilde{\Xi}_{ij} + \tilde{\Xi}_{ji}) < 0 \tag{27}$$

and

$$\sum_{i=1}^{r} \mu_i^2 \tilde{\Pi}_{ii} + \sum_{i=1}^{r} \sum_{i<j}^{r} \mu_i \mu_j (\tilde{\Pi}_{ij} + \tilde{\Pi}_{ji}) < 0 \tag{28}$$

which imply that (6) and (7) hold. Then, by Lemma 2, the kth error dynamics (4″) with time-varying state delay is asymptotically stable (with $\omega_k(t) = 0$) while satisfying a prescribed H_∞ performance (5). Thus, the proof is completed. ■

Remark 6: It is noted that the conditions in Lemma 1 are nonconvex by adding slack variables, Theorem 1 presents the convex conditions. In fact, from our k-step fault-estimation approach, one can see that each error dynamics possesses identical system matrices, which implies that the obtained sufficient conditions in Theorem 1 are suitable for each step fault estimation but independent on the scalar k. However, the kth error dynamics is dependent on the kth input disturbance $\omega_k(t)$ which involves the term $\dot{f}(t) - \dot{\hat{f}}_{k-1}(t)$.

B. Dynamic Output Feedback Controllers Design

Based on the obtained online kth fault-estimation information of $\hat{f}_k(t)$, a fault-tolerant controller via dynamic output feedback to guarantee the stability in the presence of faults is to be designed in the following. Similar to [8] and [22], the dynamic output feedback fault tolerant controller for T–S fuzzy models is constructed as

$$\begin{cases} \dot{\eta}(t) = \mathscr{A}_f(\xi, \xi)\eta(t) + \mathscr{A}_{\tau f}(\xi, \xi)\eta(t - \tau(t)) + \mathscr{B}_f(\xi)y(t) \\ u(t) = \mathscr{C}_f(\xi)\eta(t) + \mathscr{C}_{\tau f}(\xi)\eta(t - \tau(t)) + D_f y(t) - \hat{f}_k(t) \\ \eta(t) = \phi(t), \forall t \in [-\tau, 0] \end{cases} \tag{29}$$

where $\eta(t) \in R^n$ is the state, and $\mathscr{A}_f(\xi, \xi), \mathscr{A}_{\tau f}(\xi, \xi), \mathscr{B}_f(\xi)$, $\mathscr{C}_f(\xi), \mathscr{C}_{\tau f}(\xi)$, and $\mathscr{D}_f(\xi)$ are of appropriate dimensions to be designed with similar forms as those in (2), e.g., $\mathscr{A}_f(\xi, \xi) = \sum_{i,j=1}^{r} \mu_i(\xi(t))\mu_j(\xi) A_{fij}$. $\phi(t)$ is defined as in (2).

Denote $\tilde{x}^T(t) = [x^T(t), \eta^T(t)]$ and $\tilde{\omega}^T(t) = [w^T(t), e_f^T(t)]$, where $e_f^T(t) = f(t) - \hat{f}_k(t)$. Then, combining (2), one can obtain

$$\begin{cases} \dot{\tilde{x}}(t) = \mathcal{A}(\xi, \xi)\tilde{x}(t) + \mathcal{A}_\tau(\xi, \xi)\tilde{x}(t - \tau(t)) + \mathcal{B}_w(\xi, \xi)\tilde{\omega}(t) \\ y(t) = \mathcal{C}(\xi)\tilde{x}(t) + \mathcal{C}_\tau(\xi)\tilde{x}(t - \tau(t)) + \mathcal{D}_\omega(\xi)\tilde{\omega}(t) \end{cases} \tag{30}$$

where

$$\mathcal{A}(\xi, \xi) = \begin{bmatrix} \mathscr{A}(\xi) + \mathscr{B}_u(\xi)D_f\mathscr{C}(\xi) & \mathscr{B}_u(\xi)\mathscr{C}_f(\xi) \\ \mathscr{B}_f(\xi)\mathscr{C}(\xi) & \mathscr{A}_f(\xi, \xi) \end{bmatrix}$$

$$\mathcal{A}_\tau(\xi, \xi) = \begin{bmatrix} \mathscr{A}_\tau(\xi) + \mathscr{B}_u(\xi)D_f\mathscr{C}_\tau(\xi) & \mathscr{B}_u(\xi)\mathscr{C}_{\tau f}(\xi) \\ \mathscr{B}_f(\xi)\mathscr{C}_\tau(\xi) & \mathscr{A}_{\tau f}(\xi, \xi) \end{bmatrix}$$

$$\mathcal{B}_w(\xi, \xi) = \begin{bmatrix} \mathscr{B}_u(\xi)D_f\mathscr{D}_w(\xi) + \mathscr{B}_w(\xi) & -\mathscr{B}_u(\xi) \\ \mathscr{B}_f(\xi)\mathscr{D}_w(\xi) & 0 \end{bmatrix}$$

$$\mathcal{C}(\xi) = [\,\mathscr{C}(\xi) \quad 0\,], \mathcal{C}_\tau(\xi) = [\,\mathscr{C}_\tau(\xi) \quad 0\,]$$

$$\mathcal{D}_w(\xi) = [\,\mathscr{D}_w(\xi) \quad 0\,].$$

So far, the problem of dynamic output feedback control for the closed-loop fuzzy system is to design the gain matrices such that

1) the closed-loop fuzzy system (30) with time-varying state delay is asymptotically stable (when $\tilde{\omega}(t) = 0$);
2) the following H_∞ performance is satisfied for $\tilde{x}(0) = 0$, and $\phi(t) = 0, \forall t \in [-\tau, 0]$:

$$\int_0^L \|y(t)\|^2 dt \leq \gamma^2 \int_0^L \|\tilde{\omega}(t)\|^2 dt \tag{31}$$

for all $L > 0$ and $\tilde{\omega}(t) \in L_2[0, \infty)$.

Lemma 2: For the given positive scalars τ, τ_D, δ, and γ, the closed-loop fuzzy system (30) with time-varying state delay is asymptotically stable (with $\tilde{\omega}(t) = 0$) while satisfying a prescribed H_∞ performance (31) if there exist matrices $X > 0$, $Y > 0, Q_1 > 0, Q_2 > 0, R > 0, A_{fij}, A_{\tau fij}, B_{fi}, C_{fi}, C_{\tau fi}$, D_f, and $i = 1, 2, \ldots, r$ and free weighting matrices M_i, N_i of appropriate dimensions, $i = 1, 2, \ldots, r$ such that the following inequalities hold:

$$\begin{bmatrix} \tilde{\Omega}(\xi, \xi) & \tau M(\xi) \\ * & -\tau R \end{bmatrix} < 0 \tag{32}$$

and

$$\begin{bmatrix} \tilde{\Omega}(\xi, \xi) & \tau N(\xi) \\ * & -\tau R \end{bmatrix} < 0 \tag{33}$$

where

$$\tilde{\Omega}(\xi,\xi) = \begin{bmatrix} \Psi_1(\xi,\xi) & \Psi_2(\xi,\xi) & 0 & \Psi_3(\xi,\xi) \\ * & -\varepsilon(1-\tau_D)Q_1 & 0 & 0 \\ * & * & -Q_2 & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix}$$
$$+ \tau\Gamma^T(\xi,\xi)P[2\delta P-\delta^2 R]^{-1}P\Gamma(\xi,\xi)+\Upsilon^T(\xi)\Upsilon(\xi)$$
$$- M(\xi)(\mathbb{E}_1-\mathbb{E}_2)-(\mathbb{E}_1-\mathbb{E}_2)^T M^T(\xi)$$
$$- N(\xi)(\mathbb{E}_2-\mathbb{E}_3)-(\mathbb{E}_2-\mathbb{E}_3)^T N^T(\xi) \quad (34)$$

where

$$\Psi_1(\xi,\xi) = P\mathcal{A}(\xi,\xi)+\mathcal{A}^T(\xi,\xi)P+\varepsilon Q_1+Q_2$$
$$\Psi_2(\xi,\xi) = P\mathcal{A}_\tau(\xi,\xi), \Psi_3(\xi) = P\mathcal{B}_\omega(\xi,\xi)$$
$$\Gamma(\xi,\xi) = [\mathcal{A}(\xi,\xi), \quad \mathcal{A}_\tau(\xi,\xi), \quad 0, \quad \mathcal{B}_\omega(\xi,\xi)]$$
$$\Upsilon(\xi) = [\mathcal{C}(\xi), \quad \mathcal{C}_\tau(\xi), \quad 0, \quad \mathcal{D}_\omega(\xi)]$$
$$M(\xi) = \sum_{i=1}^r \mu_i(\xi(t))M_i, N(\xi) = \sum_{i=1}^r \mu_i(\xi(t))N_i$$
$$\mathbb{E}_1 = [I,0,0,0], \mathbb{E}_2 = [0,I,0,0], \ldots, \mathbb{E}_4 = [0,0,0,I].$$

Proof: In this paper, the Lyapunov function candidate is constructed as follows:

$$V(t) = \tilde{x}^T(t)P\tilde{x}(t)+V_1+V_2 \quad (35)$$

where

$$V_1 = \varepsilon\cdot\int_{t-\tau(t)}^t \tilde{x}^T(s)Q_1\tilde{x}(s)ds+\int_{t-\tau}^t \tilde{x}^T(s)Q_2\tilde{x}(s)ds \quad (36)$$

$$V_2 = \int_{-\tau}^0\int_{t+\theta}^t \dot{\tilde{x}}^T(s)R\dot{\tilde{x}}(s)dsd\theta. \quad (37)$$

where $P>0, \varepsilon\geq 0, Q_1>0, Q_2>0$, and $R>0$. Then, the time derivatives of $V(t)$, along the trajectories of the closed-loop fuzzy system (30), satisfy

$$\dot{V}(t) = \tilde{x}^T(t)[P\mathcal{A}(\xi,\xi)+\mathcal{A}^T(\xi,\xi)P+\varepsilon Q_1+Q_2]\tilde{x}(t)$$
$$+ 2\tilde{x}^T(t)P\mathcal{A}_\tau(\xi,\xi)\tilde{x}(t-\tau(t))$$
$$+ 2\tilde{x}^T(t)P\mathcal{B}_\omega(\xi,\xi)\tilde{\omega}(t)$$
$$- \varepsilon(1-\dot{\tau}(t))\tilde{x}^T(t-\tau(t))Q_1\tilde{x}(t-\tau(t))$$
$$- \tilde{x}^T(t-\tau)Q_2\tilde{x}(t-\tau)+\tau\dot{\tilde{x}}^T(t)R\dot{\tilde{x}}(t)$$
$$- \int_{t-\tau}^t \dot{\tilde{x}}^T(s)R\dot{\tilde{x}}(s)ds.$$

Furthermore, from the Newton–Leibniz formula, a straightforward computation gives

$$\dot{V}(t)+y^T(t)y(t)-\gamma^2\tilde{\omega}^T(t)\tilde{\omega}(t)$$
$$= \dot{V}(t)+y^T(t)y(t)-\gamma^2\tilde{\omega}^T(t)\tilde{\omega}(t)$$
$$- 2\zeta^T(t)M(\xi)\left[\tilde{x}(t)-\tilde{x}(t-\tau(t))-\int_{t-\tau(t)}^t \dot{\tilde{x}}(s)ds\right]$$
$$- 2\zeta^T(t)N(\xi)\left[\tilde{x}(t-\tau(t))-\tilde{x}(t-\tau)-\int_{t-\tau}^{t-\tau(t)} \dot{\tilde{x}}(s)ds\right]$$
$$\leq \frac{1}{\tau}\int_{t-\tau(t)}^t \begin{bmatrix}\zeta(t)\\ \dot{\tilde{x}}(s)\end{bmatrix}^T \begin{bmatrix}\tilde{\Omega}(\xi,\xi) & \tau M(\xi)\\ * & -\tau R\end{bmatrix}\begin{bmatrix}\zeta(t)\\ \dot{\tilde{x}}(s)\end{bmatrix}$$
$$\frac{1}{\tau}\int_{t-\tau}^{t-\tau(t)} \begin{bmatrix}\zeta(t)\\ \dot{\tilde{x}}(s)\end{bmatrix}^T \begin{bmatrix}\tilde{\Omega}(\xi,\xi) & \tau N(\xi)\\ * & -\tau R\end{bmatrix}\begin{bmatrix}\zeta(t)\\ \dot{\tilde{x}}(s)\end{bmatrix}$$

where $\zeta^T(t) = [\tilde{x}^T(t), \ \tilde{x}^T(t-\tau(t)), \ \tilde{x}^T(t-\tau), \ \tilde{\omega}^T(t)]$. Therefore, if (32) and (33) hold, then one has

$$\dot{V}(t)+y^T(t)y(t)-\gamma^2\tilde{\omega}^T(t)\tilde{\omega}(t)<0.$$

It follows from (1) and (29) that $x(t)=0$ and $\eta(t)=0, \forall t\in[-\tau,0]$ when $\phi(t)=0$. It further follows that $\tilde{x}(t)=0$ and $\dot{\tilde{x}}(t)=0, \forall t\in[-\tau,0]$. Consequently, it follows from $V(t)|_{t=0}=0$ for $\tilde{x}(0)=0$ and $V(t)|_{t=L}\geq 0$ that

$$\int_0^L (\|y(t)\|^2-\gamma^2\|\tilde{\omega}(t)\|^2)dt+V(t)|_{t=L}-V(t)|_{t=0}\leq 0$$

which implies that (31) holds. Therefore, the H_∞ performance is verified.

In addition, if (32) and (33) hold, then the time derivatives of $V(t)$ along the solution of (30) when $\tilde{\omega}(t)=0$ satisfies $\dot{V}(t)<0$. As a result, the asymptotic stability of a closed-loop fuzzy system (30) follows immediately when $\tilde{\omega}(t)=0$. The proof is thus completed. ■

Remark 7: In Lemma 2, the integral inequality method same as in Lemma 1 is applied to obtain the relevant results. The obtained delay dependent results as discussed in Remark 5 is also less conservative than the existing ones in [11], [14], [19], [20], [23], [24], and [34].

Theorem 2: For the given positive scalars τ, τ_D, δ, and γ, the closed-loop fuzzy system (30) with time-varying state delay is asymptotically stable (with $\tilde{\omega}(t)=0$) while satisfying a prescribed H_∞ performance (31) if there exist matrices $X>0, Y>0, \hat{Q}_1>0, \hat{Q}_2>0, \hat{R}>0, X_1, Y_1, \hat{A}_{fij}, \hat{A}_{\tau fij}, \hat{B}_{fi}, \hat{C}_{fi}, \hat{C}_{\tau fi}, \hat{D}_f, i=1,2,\ldots,r$ and free weighting matrices $\hat{M}_i, \hat{N}_i$ of appropriate dimensions, $i=1,2,\ldots,r$ such that the following inequalities hold:

$$\Xi_{ii}<0, \quad i=1,2,\ldots,r \quad (38)$$
$$\Xi_{ij}+\Xi_{ji}\leq 0, \quad 1\leq i<j\leq r \quad (39)$$
$$\Pi_{ii}<0, \quad i=1,2,\ldots,r \quad (40)$$
$$\Pi_{ij}+\Pi_{ji}\leq 0, \quad 1\leq i<j\leq r \quad (41)$$

where

$$\Xi_{ij} = \begin{bmatrix}\Omega_{ij} & \tau\hat{M}_i & \sqrt{\tau}\Gamma_{ij}^T & \Upsilon_i^T\\ * & -\tau\hat{R} & 0 & 0\\ * & * & -2\delta\phi+\delta^2\hat{R} & 0\\ * & * & * & -I\end{bmatrix} \quad (42)$$

and

$$\Pi_{ij} = \begin{bmatrix} \Omega_{ij} & \tau\hat{N}_i & \sqrt{\tau}\Gamma_{ij}^T & \Upsilon_i^T \\ * & -\tau\hat{R} & 0 & 0 \\ * & * & -2\delta\phi+\delta^2\hat{R} & 0 \\ * & * & * & -I \end{bmatrix} \quad (43)$$

where

$$\Omega_{ij} = \begin{bmatrix} \Psi_{ij} & \psi_{2ij} & 0 & \psi_{3ij} \\ * & -\varepsilon(1-\tau_D)\hat{Q}_1 & 0 & 0 \\ * & * & -\hat{Q}_2 & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} - \hat{M}_i(\mathbb{E}_1-\mathbb{E}_2) - (\mathbb{E}_1-\mathbb{E}_2)^T\hat{M}_i^T - \hat{N}_i(\mathbb{E}_2-\mathbb{E}_3) - (\mathbb{E}_2-\mathbb{E}_3)^T\hat{N}_i^T \quad (44)$$

where

$$\Psi_{ij} = \psi_{1ij} + \psi_{1ij}^T + \varepsilon\hat{Q}_1 + \hat{Q}_2$$

$$\psi_{1ij} = \begin{bmatrix} A_iX + B_{ui}\hat{C}_{fj} & A_i + B_{ui}\hat{D}_fC_j \\ \hat{A}_{fij} & YA_i + \hat{B}_{fj}C_i \end{bmatrix}$$

$$\psi_{2ij} = \begin{bmatrix} A_{\tau i}X + B_{ui}\hat{C}_{\tau fj} & A_{\tau i} + B_{ui}\hat{D}_fC_{\tau j} \\ \hat{A}_{\tau fij} & YA_{\tau i} + \hat{B}_{fj}C_{\tau i} \end{bmatrix}$$

$$\psi_{3ij} = \begin{bmatrix} B_{wi} + B_{ui}\hat{D}_fD_{wj} & -B_{ui} \\ YB_{wi} + \hat{B}_{fj}D_{wi} & -YB_{ui} \end{bmatrix}$$

$$\phi = \begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0, \Gamma_{ij} = [\psi_{1ij}, \quad \psi_{2ij}, \quad 0, \quad \psi_{3ij}]$$

$$\Upsilon_i = [\psi_{4i}, \quad \psi_{5i}, \quad 0, \quad \psi_{6i}], \psi_{4i} = [XC_i \quad C_i]$$

$$\psi_{5i} = [XC_{\tau i} \quad C_{\tau i}], \psi_{6i} = [D_{wi} \quad 0]$$

$$\hat{M}_i^T = [\hat{M}_{1i}^T, \hat{M}_{2i}^T, \hat{M}_{3i}^T, \hat{U}_{1i}^T], \hat{N}_i^T = [\hat{N}_{1i}^T, \hat{N}_{2i}^T, \hat{N}_{3i}^T, \hat{U}_{2i}^T]$$

$$\mathbb{E}_1 = [I,0,0,0], \mathbb{E}_2 = [0,I,0,0], \ldots, \mathbb{E}_4 = [0,0,0,I].$$

Then, the gain matrices of the dynamic output feedback fault tolerant controller are given by

$$D_f = \hat{D}_f$$

$$C_{fi} = (\hat{C}_{fi} - D_fC_iX)X_1^{-T}$$

$$C_{\tau fi} = (\hat{C}_{\tau fi} - D_fC_{\tau i}X)X_1^{-T}$$

$$B_{fi} = Y_1^{-1}(\hat{B}_{fi} - YB_{ui}D_f)$$

$$A_{fij} = Y_1^{-1}(\hat{A}_{fij} - Y(A_i + B_{ui}D_fC_j)X)X_1^{-T} - B_{fj}C_iXX_1^{-T} - Y_1^{-1}YB_{ui}C_{fj}$$

$$A_{\tau fij} = Y_1^{-1}(\hat{A}_{\tau fij} - Y(A_{\tau i} + B_{ui}D_fC_{\tau j})X)X_1^{-T} - B_{fj}C_{\tau i}XX_1^{-T} - Y_1^{-1}YB_{ui}C_{\tau fj}$$

where X_1, Y_1 satisfy $X_1Y_1^T = I - XY$.

Proof: By the Schur complement, (38)–(41) are equivalent to the following inequalities, respectively:

$$\tilde{\Xi}_{ii} < 0, \quad i = 1,2,\ldots,r$$

$$\tilde{\Xi}_{ij} + \tilde{\Xi}_{ji} \le 0, \quad 1 \le i < j \le r$$

$$\tilde{\Pi}_{ii} < 0, \quad i = 1,2,\ldots,r$$

$$\tilde{\Pi}_{ij} + \tilde{\Pi}_{ji} \le 0, \quad 1 \le i < j \le r$$

where

$$\tilde{\Xi}_{ij} = \begin{bmatrix} \tilde{\Omega}_{ij} & \tau\hat{M}_i \\ * & -\tau\hat{R} \end{bmatrix}, \tilde{\Pi}_{ij} = \begin{bmatrix} \tilde{\Omega}_{ij} & \tau\hat{N}_i \\ * & -\tau\hat{R} \end{bmatrix}$$

where

$$\tilde{\Omega}_{ij} = \begin{bmatrix} \Psi_{ij} & \psi_{2ij} & 0 & \psi_{3ij} \\ * & -\varepsilon(1-\tau_D)\hat{Q}_1 & 0 & 0 \\ * & * & -\hat{Q}_2 & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} + \tau\Gamma_{ij}^T(-2\delta\phi+\delta^2\hat{R})^{-1}\Gamma_{ij} + \Upsilon_i^T\Upsilon_i - \hat{M}_i(\mathbb{E}_1-\mathbb{E}_2) - (\mathbb{E}_1-\mathbb{E}_2)^T\hat{M}_i^T - \hat{N}_i(\mathbb{E}_2-\mathbb{E}_3) - (\mathbb{E}_2-\mathbb{E}_3)^T\hat{N}_i^T.$$

Denote $\mu_i = \mu_i(\xi(t))$. It further follows that

$$\sum_{i=1}^{r}\mu_i^2\tilde{\Xi}_{ii} + \sum_{i=1}^{r}\sum_{i<j}^{r}\mu_i\mu_j(\tilde{\Xi}_{ij}+\tilde{\Xi}_{ji}) < 0 \quad (45)$$

and

$$\sum_{i=1}^{r}\mu_i^2\tilde{\Pi}_{ii} + \sum_{i=1}^{r}\sum_{i<j}^{r}\mu_i\mu_j(\tilde{\Pi}_{ij}+\tilde{\Pi}_{ji}) < 0. \quad (46)$$

Similar to [50], we express the symmetric positive definite matrix P and its inverse matrix P^{-1} as

$$P = \begin{bmatrix} Y & Y_1 \\ * & W \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} X & X_1 \\ * & Z \end{bmatrix}.$$

Due to $PP^{-1} = I$, one has

$$P\begin{bmatrix} X \\ X_1^T \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix}, \quad P\begin{bmatrix} X & I \\ X_1^T & 0 \end{bmatrix} = \begin{bmatrix} I & Y \\ 0 & Y_1^T \end{bmatrix}$$

respectively. If we denote

$$F_1 = \begin{bmatrix} X & I \\ X_1^T & 0 \end{bmatrix}, \quad F_2 = \begin{bmatrix} I & Y \\ 0 & Y_1^T \end{bmatrix}$$

then it follows that $PF_1 = F_2$. Pre and post multiplying (45) and (46) by diag$\{F_1^{-T}, F_1^{-T}, F_1^{-T}, I, F_1^{-T}\}$ and its transpose produces (32) and (33), respectively. Then, by Lemma 2, the closed-loop fuzzy system (30) with time-varying state delay is asymptotically stable (with $\tilde{\omega}(t) = 0$) while satisfying a prescribed H_∞ performance (31). Thus, the proof is completed. ■

Remark 8: It is noted that the conditions in Lemma 2 are also nonconvex by adding slack variables, Theorem 2 presents the convex conditions. As in [50], from $X > 0, Y > 0$, and $\phi > 0$ in (42) and (43), one can obtained that $I - XY$ is nonsingular. Therefore, we can always find nonsingular matrices X_1 and Y_1 satisfying $X_1Y_1^T = I - XY$, which can be computed by using the *qr* function of the MATLAB toolbox.

IV. Simulation Results

In this section, two numerical examples are presented to illustrate the effectiveness of the proposed results in this paper and to compare with the existing results in [50] to show the advantages of our design method.

Example 1: This example shows the problem of balancing and the swing-up of an inverted pendulum on a cart.

Consider the problem of balancing and swing-up of an inverted pendulum on a cart [10]. The considered model can be represented by using a two-rule T–S fuzzy model; see [50]. Then, the overall fuzzy system can be written as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \mu_i [A_i x(t) + B_{ui}(u(t) + f(t)) + B_{wi} w(t)] \\ y(t) = Cx(t) + D_w w(t) \\ x(t) = \sum_{i=1}^{2} \mu_i \phi_i(t) \end{cases}$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ \frac{g}{4l/3 - aml} & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 \\ \frac{2g}{\pi(4l/3 - aml\beta^2)} & 0 \end{bmatrix}$$

$$B_{u1} = \begin{bmatrix} 0 \\ -\frac{a}{4l/3 - aml} \end{bmatrix}, B_{u2} = \begin{bmatrix} 0 \\ -\frac{a\beta}{4l/3 - aml\beta^2} \end{bmatrix}$$

$$B_{w1} = B_{w2} = \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix}, C = [1 \quad 0], D_w = 0.001$$

where $g = 9.8, m = M = 2.0, a = 1/(m + M)$, and $l = 0.5, \beta = \cos(88^\circ)$.

We first estimate the actuator fault. Setting $\tau = 0.01, \delta = 10, \varepsilon = 0$ (τ_D unknown), and computing matrix inequalities in (12)–(15) in Theorem 1 gives the minimum attenuation value $\gamma = 0.9985$ and a feasible solution as

$$L_1 = \begin{bmatrix} 7.6246 \\ 54.3220 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 4.4041 \\ 31.1735 \end{bmatrix}$$

$$G_1 = -128.1721, \quad G_2 = -67.9022.$$

Next, we design the dynamic output feedback fault tolerant controller. Setting $\tau = 0.01, \delta = 10$, and $\varepsilon = 0$ (τ_D unknown), computing matrix inequalities in (38)–(41) in Theorem 2 gives the minimum attenuation value $\gamma = 0.185$ and a feasible solution as $D_f = -16.4734$

$$A_{f11} = \begin{bmatrix} -10.5683 & -1.5074 \\ 172.8762 & -2.3885 \end{bmatrix}$$

$$A_{f12} = \begin{bmatrix} -10.6536 & -1.5022 \\ 171.4555 & -2.3018 \end{bmatrix}$$

$$A_{f21} = \begin{bmatrix} -10.2176 & -1.4197 \\ 178.4193 & -0.9866 \end{bmatrix}$$

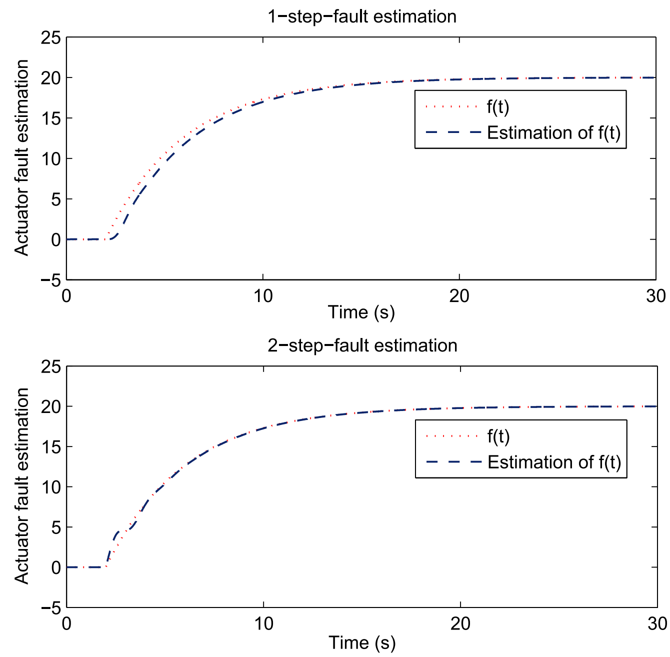

Fig. 2. Response curves of $f(t)$ and its estimates for $a = 0, b = 20$.

$$A_{f22} = \begin{bmatrix} -10.3029 & -1.4145 \\ 176.9986 & -0.9000 \end{bmatrix}$$

$$B_{f1} = \begin{bmatrix} -0.0522 \\ 6.7990 \end{bmatrix}, C_{f1} = [-205.0965 \quad 384.9619]$$

$$B_{f2} = \begin{bmatrix} -0.0571 \\ 6.7186 \end{bmatrix}, C_{f2} = [-205.0965 \quad 384.9619].$$

It is assumed that the actuator fault $f(t)$ is created as

$$f(t) = \begin{cases} 0, & 0 \le t < 2 \\ (a \sin(t-2) + b)(1 - e^{\frac{-t+2}{4}}), & 2 \le t < 30. \end{cases}$$

If we let $a = 0, b \neq 0$, then $f(t)$ is a constant actuator fault. It is supposed that $w(t)$ is band-limited white noise with power 0.001 and sampling time 0.01s. For simulation purposes, here we choose $\mu_1(\xi(t)) = 1/(1 + \exp(x_1 + 0.5))$ and $\mu_2(\xi(t)) = 1 - \mu_1(\xi(t))$.

First, we consider the constant actuator fault; let $a = 0$, and let $b = 20$. The simulation results run for the closed-loop fuzzy system (4) and (4″) by one-step fault-estimation (which, in fact, is the method proposed in [50]) and our k-step fault estimation, respectively. Fig. 2 shows the actuator fault estimation simulation results under one and two-step fault estimation, respectively. Obviously, it can be seen from Fig. 2 that the value of $\hat{f}(t)$ under two-step fault-estimation is closer to $f(t)$ than that under one-step fault estimation. Furthermore, Fig. 3 shows the response curves of state errors under $k = 1, 2$, respectively. It can be seen from Figs. 2 and 3 that two-step fault-estimation really weaken the effects from the input disturbance of $\dot{f}(t)$ but is not clear for constant faults.

Next, we consider the time-varying actuator fault; let $a = 5$, and let $b = 0$. The simulation results run for the closed-loop fuzzy system (4) and (4″) by one-step-fault-estimation and our k-step fault-estimation, respectively. Fig. 4 shows the actuator

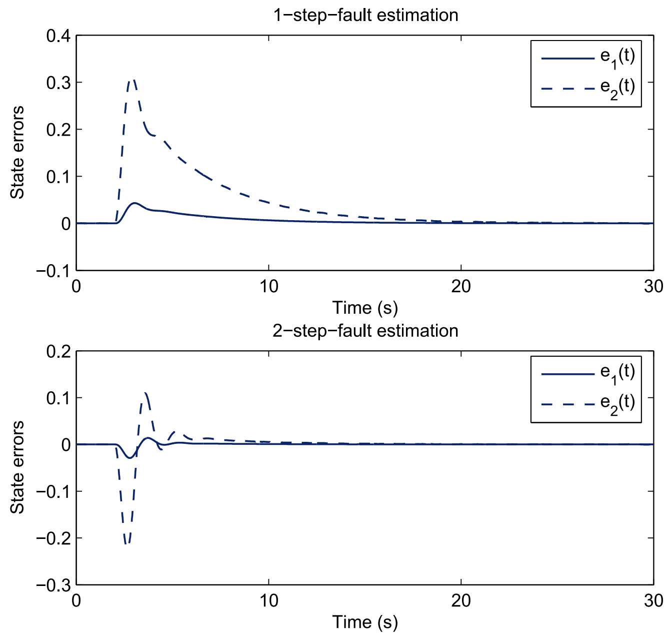

Fig. 3. Response curves of state errors for $a = 0, b = 20$.

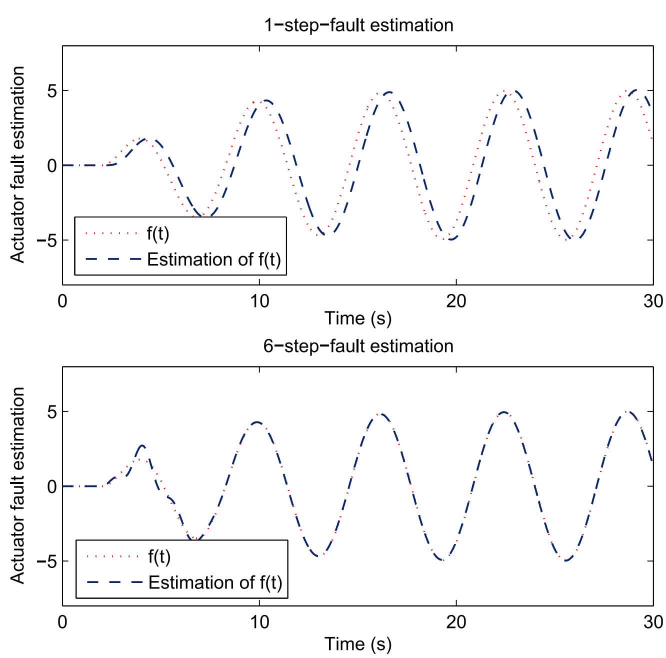

Fig. 4. Response curves of $f(t)$ and its estimates for $a = 5, b = 0$.

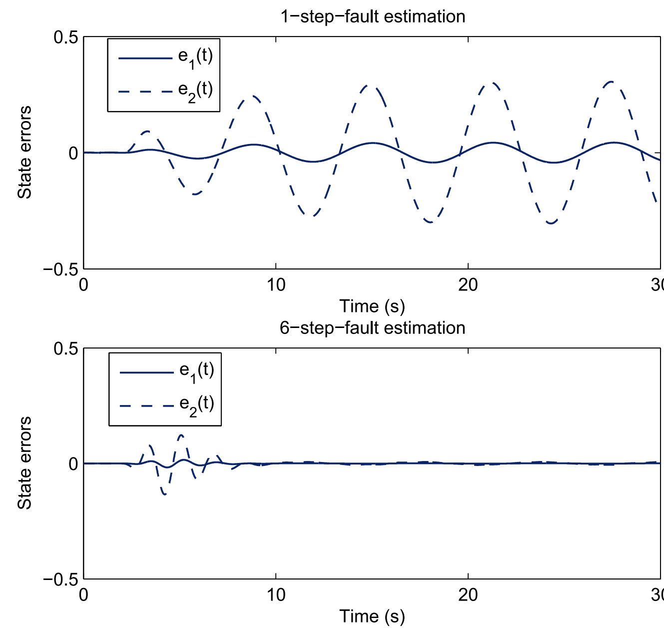

Fig. 5. Response curves of state errors for $a = 5, b = 0$.

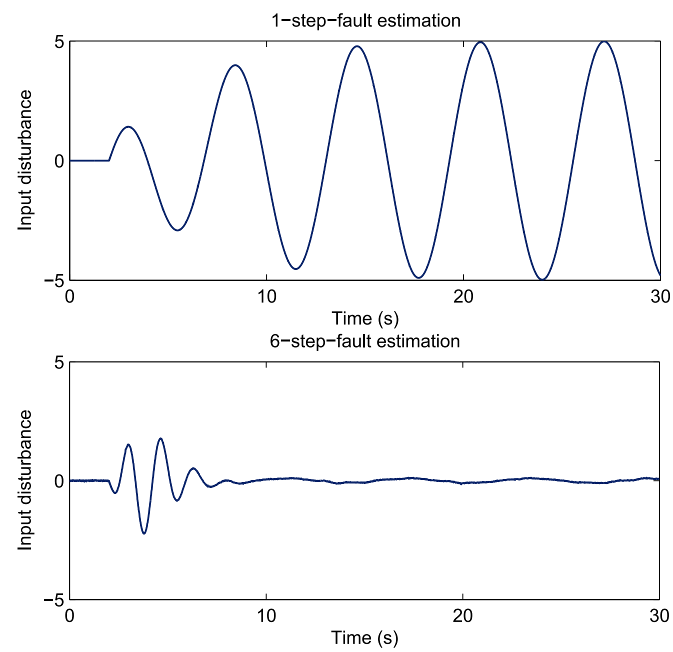

Fig. 6. Response curves of input disturbances $\dot{f}(t)$ and $\dot{f}(t) - \dot{\hat{f}}_5(t)$ by one and six-step fault estimation for $a = 5, b = 0$, respectively.

fault estimation simulation results under one and six-step fault estimation, respectively. Obviously, it can be seen from Fig. 4 that the value of $\hat{f}(t)$ under six-step fault estimation is close to $f(t)$. Furthermore, Fig. 5 shows the response curves of state errors under $k = 1$ and 6, respectively. It can be seen from Fig. 5 that when $t > 10$ s, the state errors go nearly to zero under $k = 6$, while those in one-step fault estimation cannot. Furthermore, Fig. 6 shows the response curves of $\dot{f}(t)$ and $\dot{f}(t) - \dot{\hat{f}}_5(t)$ under $k = 1$ and 6, respectively. Obviously, $\dot{f}(t) - \dot{\hat{f}}_5(t)$ goes nearly to zero under six-step fault estimation, which clearly demonstrates that our proposed k-step fault-estimation approach really weakens the effects of input disturbance from $\dot{f}(t)$ in this example for time-varying faults. Under initial condition $\phi(t) = [\pi/3 \quad 0]^T$, simulation results for response curves of state (top) and output (bottom) for the closed-loop fuzzy system (30) based on dynamic output feedback fault tolerant controllers are illustrated in Fig. 7.

In Example 1, there is no time-varying state delay in the fuzzy models. Therefore, to further illustrate how $-\dot{\hat{f}}_k(t)$ weakening the input disturbances from $\dot{f}(t)$ also increasingly weaken the

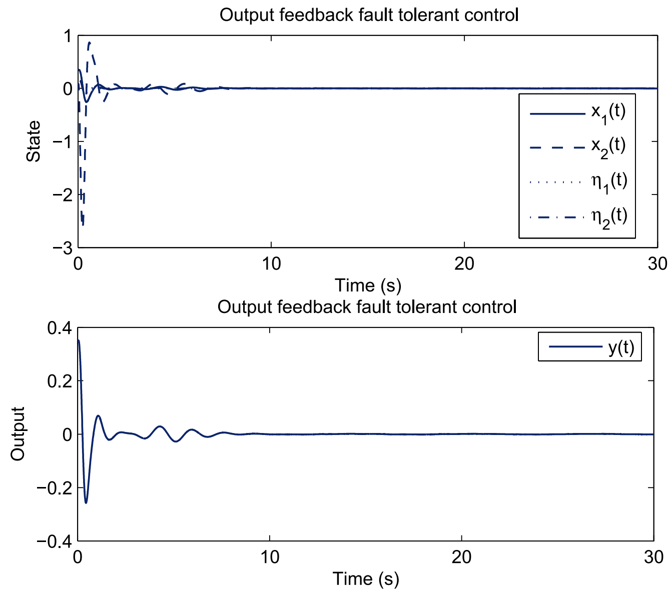

Fig. 7. Response curves of state and output $y(t)$ under initial condition $\phi(t) = [\pi/3 \quad 0]^T$ for $a = 5, b = 0$.

effect from time-varying delay, we consider the following system with actuator fault and time-varying state delays under input disturbance $w(t) \equiv 0$.

Example 2: This example considers the truck trailer system with time delays.

Consider the truck trailer system with time-varying state delay cited from [5]:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \mu_i(\xi(t))[A_i x(t) + A_{\tau i} x(t-\tau(t))] \\ \qquad + \sum_{i=1}^{2} \mu_i(\xi(t))[B_{ui}(u(t) + f(t))], t \geq 0 \\ y(t) = Cx(t) + C_\tau x(t-\tau(t)) \\ x(t) = \sum_{i=1}^{2} \mu_i(\xi(t))\phi_i(t) \end{cases}$$

where

$$A_1 = \begin{bmatrix} -a\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{v^2\bar{t}^2}{2Lt_0} & \dfrac{v\bar{t}}{t_0} & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -a\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{dv^2\bar{t}^2}{2Lt_0} & \dfrac{dv\bar{t}}{t_0} & 0 \end{bmatrix}$$

$$A_{\tau 1} = \begin{bmatrix} -(1-a)\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{v^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}, B_{u1} = \begin{bmatrix} \dfrac{v\bar{t}}{lt_0} \\ 0 \\ 0 \end{bmatrix}$$

$$A_{\tau 2} = \begin{bmatrix} -(1-a)\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{dv^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}, B_{u2} = \begin{bmatrix} \dfrac{v\bar{t}}{lt_0} \\ 0 \\ 0 \end{bmatrix}$$

$$C = [-0.2 \quad 0.05 \quad -0.15], C_\tau = (1-a)C$$

with $l = 2.8,\ L = 5.5,\ v = -1.0,\ \bar{t} = 2.0,\ t_0 = 0.5,\ d = 10 * t_0/\pi$, and $f(t) = 5(\sin 2t + \cos t), t > 0$. For simulation purposes, here, we choose $\mu_1(\xi(t)) = 1/(1 + \exp(x_1 + 0.5))$ and $\mu_2(\xi(t)) = 1 - \mu_1(\xi(t))$ with initial condition $[0.5\pi \quad 0.75\pi \quad -5]^T$.

Setting $\tau = 0.5, \delta = 10$ and $\varepsilon = 0$ (τ_D unknown), computing matrix inequalities in (12)–(15) in Theorem 1 gives the minimum attenuation value $\gamma = 0.3579$ and a feasible solution as

$$L_1 = \begin{bmatrix} -62.3063 \\ 2.6251 \\ -1.6312 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -62.1415 \\ 2.6863 \\ -2.0412 \end{bmatrix}$$

$$G_1 = 393.2601 \quad G_2 = 394.2996.$$

Setting $\tau = 0.5, \delta = 10$, and $\varepsilon = 0$ (τ_D unknown), computing matrix inequalities in (38)–(41) in Theorem 2 gives the minimum attenuation value $\gamma = 0.1225$ and a feasible solution as $D_f = -43.5658$

$$A_{f11} = A_{f12} = \begin{bmatrix} -17.8437 & -6.0271 & -6.3009 \\ 9.3718 & 0.3873 & -0.8603 \\ -10.9258 & -1.9598 & -2.0074 \end{bmatrix}$$

$$A_{f21} = A_{f22} = \begin{bmatrix} -18.4911 & -6.9820 & -5.4074 \\ 9.6042 & 0.7296 & -1.1812 \\ -11.1525 & -2.2921 & -1.6940 \end{bmatrix}$$

$$A_{\tau f11} = A_{\tau f12} = \begin{bmatrix} -1.5754 & -1.1623 & -1.3045 \\ 0.8546 & 0.3000 & 0.1906 \\ -0.9886 & -0.6009 & -0.6176 \end{bmatrix}$$

$$A_{\tau f21} = A_{\tau f22} = \begin{bmatrix} -1.5949 & -1.0534 & -1.1280 \\ 0.8611 & 0.2614 & 0.1282 \\ -0.9943 & -0.5639 & -0.5581 \end{bmatrix}$$

$$C_{f1} = C_{f2} = [\,234.0786 \quad 148.9879 \quad -214.2511\,]$$

$$B_{f1} = B_{f2} = \begin{bmatrix} 1.7163 \\ -0.8802 \\ 1.0570 \end{bmatrix}$$

$$C_{\tau f1} = C_{\tau f2} = [\,31.3575 \quad -42.8518 \quad -78.2211\,].$$

The simulation results run for the closed-loop fuzzy system (4) and (4″) by our k-step fault-estimation. Fig. 8 shows the actuator fault estimation simulation results under one and four-step fault estimation, respectively. Obviously, it can be seen that the value of $\hat{f}(t)$ under four-step fault estimation has been close to $f(t)$. Fig. 9 shows the response curves of input

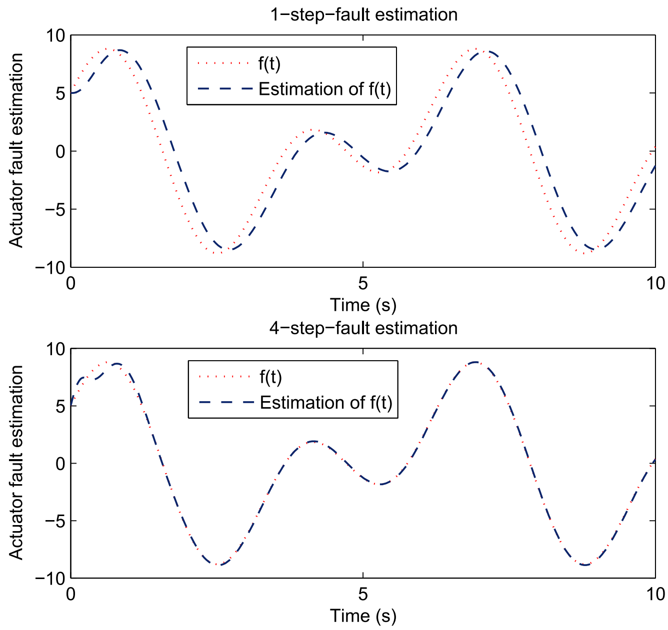

Fig. 8. Response curves of $f(t)$ and its estimates.

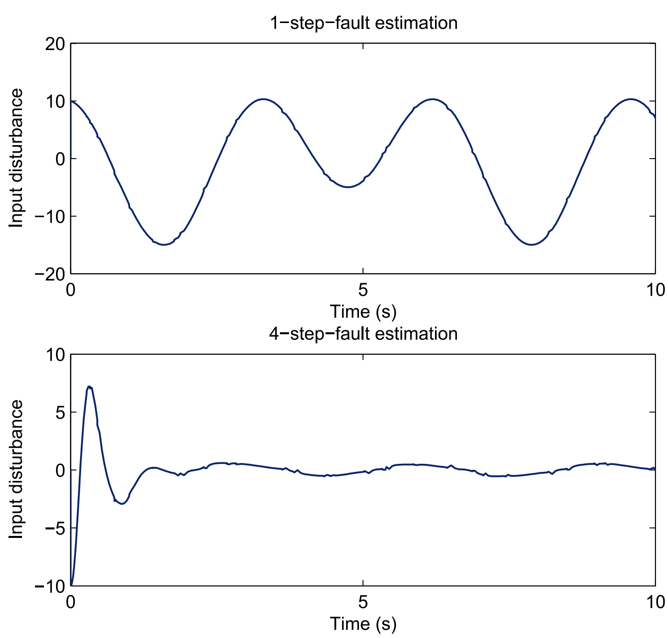

Fig. 9. Response curves of input disturbance from $\dot{f}(t)$ and $\dot{f}(t) - \dot{\hat{f}}_3(t)$, respectively.

disturbances from $\dot{f}(t)$ and $\dot{f}(t) - \dot{\hat{f}}_3(t)$, respectively. It can be seen from Fig. 9 that $\dot{f}(t) - \dot{\hat{f}}_3(t)$ has been close to zero, which implies that $-\dot{\hat{f}}_3(t)$ practically weakens $\dot{f}(t)$. Fig. 10 shows the response curves of state errors under one and four-step fault estimation, respectively. In Fig. 10, the state errors under four-step fault estimation have gone nearly to zero, which also implies that $-\dot{\hat{f}}_3(t)$ increasingly weakening the effect from $\dot{f}(t)$ also weakens the effect from time-varying delay. Finally, under initial condition $\phi(t) = [0.5\pi \quad 0.75\pi \quad -5]^T$, simulation results for the response curve of output for the closed-loop fuzzy sys-

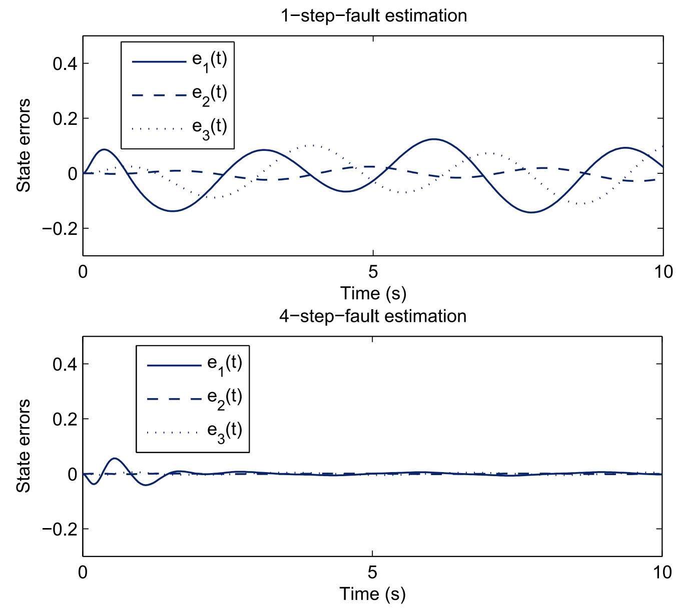

Fig. 10. Response curves of state errors.

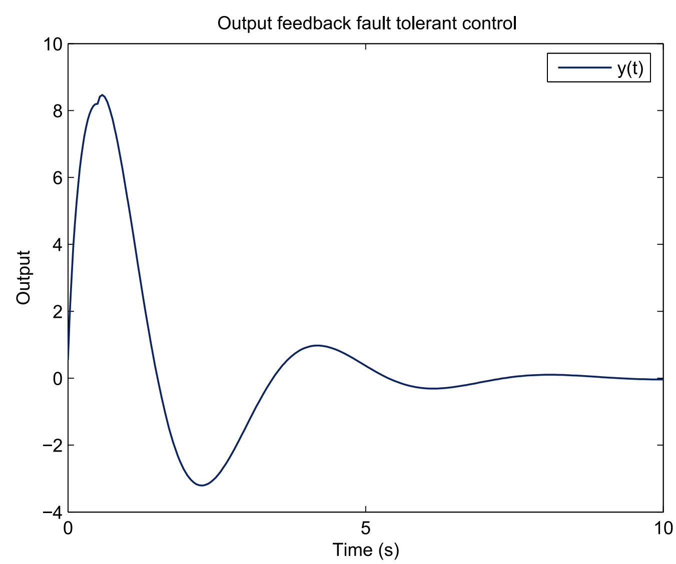

Fig. 11. Response curve of output $y(t)$ under initial condition $\phi(t) = [0.5\pi \quad 0.75\pi \quad -5]^T$.

tem (30) with state delay based on dynamic output feedback fault tolerant controllers are illustrated in Fig. 11.

V. Conclusion

This paper has studied the problem of the FTC design for T–S fuzzy systems with time-varying state delay, actuator faults, and external disturbances. A novel k-step fault-estimation detailed design framework for observer-based robust fault estimation and FTC is proposed for a class of nonlinear systems with time-varying state delay described by T–S fuzzy models. The main contribution of this paper is that 1) a novel k-step fault-estimation approach is proposed for T–S fuzzy systems with time-varying state delay and actuator faults. The fault estimates via this method can practically better depict the size and shape of the faults than that via the existing one. 2) By using an improved integral inequality method without ignoring any useful integral terms in the derivatives of Lyapunov functionals, some less conservative delay dependent stability conditions for the existence of k-step fault-estimation observers and fault-tolerant controllers for T–S fuzzy systems with time-varying state delay and actuator faults are given in terms of solution to a set of linear matrix inequalities (LMIs). Simulation results of two numerical examples demonstrate the effectiveness and merits of the proposed methods. The proposed k-step fault-estimation approach being applied to the model with actuator and sensor faults, as well as to the systems with state and input delays against actuator or/and sensor faults, will be our next challenge.

APPENDIX

In fact, to obtain the relevant delay dependent sufficient stability conditions, Lemmas 1 and 2 and the methods in [11], [14], [19], [20], [23], [24], and [34], mainly focus on how to convert the integral inequality (47) to matrix inequalities:

$$\zeta^T(t)Q\zeta(t) - \int_{t-\tau}^{t} \dot{x}^T(s)R\dot{x}(s)ds \le 0 \tag{47}$$

where $\tau > 0$, and we assume that $\zeta^T(t) = [x^T(t), x^T(t-\tau(t)), x^T(t-\tau), w(t)], 0 \le \tau(t) \le \tau, R > 0$, and that Q is a symmetric matrix of appropriate dimensions. Denote $\xi^T(t) = [\zeta^T(t), \dot{x}^T(s)]$ and

$$\Psi = \begin{bmatrix} M_1 & N_1 - M_1 & -N_1 & 0 \\ M_2 & N_2 - M_2 & -N_2 & 0 \\ M_3 & N_3 - M_3 & -N_3 & 0 \\ M_4 & N_4 - M_4 & -N_4 & 0 \end{bmatrix}$$

where M_i and N_i are of appropriate dimensions $i = 1, 2, 3, 4$.

The following result summarizes the integral inequality method in [14], [19], [20], [23], and [34].

Lemma 3: If we assume that $\tau > 0, R > 0$, and that Q is a symmetric matrix of appropriate dimensions, then (47) holds if there exist free weighting matrices $M = [M_1^T, M_2^T, M_3^T, M_4^T]^T$ and $N = [N_1^T, N_2^T, N_3^T, N_4^T]^T$ of appropriate dimensions such that

$$Q - \Psi - \Psi^T + \tau M R^{-1} M^T + \tau N R^{-1} N^T \le 0. \tag{48}$$

Proof: For $\tau > 0$ and $0 \le \tau(t) \le \tau$, by the Leibniz–Newton formula, one has

$$\begin{aligned}
&\zeta^T(t)Q\zeta(t) - \int_{t-\tau}^{t} \dot{x}^T(s)R\dot{x}(s)ds \\
&= \zeta^T(t)Q\zeta(t) \\
&\quad - 2\zeta^T(t)M[x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{x}(s)ds] \\
&\quad - 2\zeta^T(t)N[x(t-\tau(t)) - x(t-\tau) - \int_{t-\tau}^{t-\tau(t)} \dot{x}(s)ds] \\
&\quad - \int_{t-\tau(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds - \int_{t-\tau}^{t-\tau(t)} \dot{x}^T(s)R\dot{x}(s)ds \\
&\le \zeta^T(t)[Q - \Psi - \Psi^T + \tau M R^{-1} M^T + \tau N R^{-1} N^T]\zeta(t) \\
&\quad - \int_{t-\tau(t)}^{t} \xi^T(t) \begin{bmatrix} MR^{-1}M^T & -M \\ * & R \end{bmatrix} \xi(t)ds \\
&\quad - \int_{t-\tau}^{t-\tau(t)} \xi^T(t) \begin{bmatrix} NR^{-1}N^T & -N \\ * & R \end{bmatrix} \xi(t)ds \\
&\le \zeta^T(t)[Q - \Psi - \Psi^T + \tau M R^{-1} M^T + \tau N R^{-1} N^T]\zeta(t).
\end{aligned}$$

Hence, if (48) holds, then (47) holds. The proof is completed. ■

However, in the proof of Lemma 3, some useful integral terms are ignored, and $\tau(t), \tau - \tau(t)$ are enlarged as τ. Therefore, Lemma 3 may be conservative in this way. Recently, some existing papers, e.g., [11] and [24], improved the method which can be summarized as the following result.

Lemma 4: If we assume that $\tau > 0, R > 0$, and that Q is a symmetric matrix of appropriate dimensions, then (47) holds if there exist free weighting matrices M, N defined as in (48) and $X > 0$ satisfying $X \ge MR^{-1}M^T, X \ge NR^{-1}N^T$ such that

$$Q - \Psi - \Psi^T + \tau X \le 0. \tag{49}$$

Proof: For $\tau > 0$ and $0 \le \tau(t) \le \tau$, similar to the proof of Lemma 3, by the Leibniz–Newton formula, if $X \ge MR^{-1}M^T, X \ge NR^{-1}N^T$, one has

$$\begin{aligned}
&\zeta^T(t)Q\zeta(t) - \int_{t-\tau}^{t} \dot{x}^T(s)R\dot{x}(s)ds \\
&= \zeta^T(t)[Q - \Psi - \Psi^T + \tau X]\zeta(t) \\
&\quad - \int_{t-\tau(t)}^{t} \xi^T(t) \begin{bmatrix} X & -M \\ * & R \end{bmatrix} \xi(t)ds \\
&\quad - \int_{t-\tau}^{t-\tau(t)} \xi^T(t) \begin{bmatrix} X & -N \\ * & R \end{bmatrix} \xi^T(t)ds \\
&\le \zeta^T(t)[Q - \Psi - \Psi^T + \tau X]\zeta(t).
\end{aligned}$$

Therefore, if (49) holds, then (47) holds. The proof is completed. ■

In fact, in the proof of Lemma 4, some useful integral terms are still ignored. However, by using an improved integral inequality method without ignoring any useful integral term, Lemmas 3 and 4 can be improved as following Lemma 5, which has been applied to Lemmas 1 and 2 in this paper.

Lemma 5: If we assume that $\tau > 0, R > 0$, and that Q is a symmetric matrix of appropriate dimensions, then (47) holds if there exist free weighting matrices M, N defined as in (48) such that

$$\begin{cases} Q - \Psi - \Psi^T + \tau M R^{-1} M^T \le 0 \\ Q - \Psi - \Psi^T + \tau N R^{-1} N^T \le 0. \end{cases} \tag{50}$$

Proof: For $\tau > 0$ and $0 \leq \tau(t) \leq \tau$, similar to the proof of Lemma 3, by the Leibniz–Newton formula, one has

$$\zeta^T(t)Q\zeta(t) - \int_{t-\tau}^{t} \dot{x}^T(s)R\dot{x}(s)ds$$
$$= \frac{1}{\tau}\int_{t-\tau(t)}^{t} \xi^T(t)\begin{bmatrix} Q-\Psi-\Psi^T & \tau M \\ * & -\tau R\end{bmatrix}\xi(t)ds$$
$$+ \frac{1}{\tau}\int_{t-\tau}^{t-\tau(t)} \xi^T(t)\begin{bmatrix} Q-\Psi-\Psi^T & \tau N \\ * & -\tau R\end{bmatrix}\xi(t)ds.$$

On the other hand, by the Schur complement, (50) is equivalent to

$$\begin{bmatrix} Q-\Psi-\Psi^T & \tau M \\ * & -\tau R\end{bmatrix} \leq 0$$

and

$$\begin{bmatrix} Q-\Psi-\Psi^T & \tau N \\ * & -\tau R\end{bmatrix} \leq 0.$$

Therefore, if (50) holds, then (47) holds. The proof is thus completed. ■

The following result theorematically demonstrates that Lemma 5 is less conservative than Lemmas 3 and 4.

Theorem 3: Under the same conditions, (48)⇒(50), and (49)⇒(50).

Proof: Under the same conditions, (48)⇒(50) is obvious. From $X \geq MR^{-1}M^T$ and $X \geq NR^{-1}N^T$, then one has

$$Q - \Psi - \Psi^T + \tau MR^{-1}M^T \leq Q - \Psi - \Psi^T + \tau X$$

and

$$Q - \Psi - \Psi^T + \tau NR^{-1}N^T \leq Q - \Psi - \Psi^T + \tau X.$$

It further follows that (49)⇒(50). The proof is completed. ■

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