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Observer-based adaptive fuzzy control for SISO nonlinear systems

Shaocheng Tong[a,*], Han-Xiong Li[b], Wei Wang[c]

[a] Department of Basic Mathematics, Liaoning Institute of Technology, JinZhou, Liaoning 121001, PR China
[b] Department of Manufacturing Engineering and Engineering Management, City University of Hong Kong, Hong Kong
[c] Research Center of Information and Control, DaLian University of Technology, DaLian 116024, PR China

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Abstract

The observer-based indirect and direct adaptive fuzzy controllers are developed for a class of SISO uncertain nonlinear systems. The proposed approaches do not need the availability of the state variables. By designing the state observer, the adaptive fuzzy systems, which are used to model the unknown functions, can be constructed using the state estimations. Thus, a new hybrid adaptive fuzzy control method is proposed by combining the above adaptive fuzzy system with the H^∞ control technique. Based on Lyapunov stability theorem, the proposed adaptive fuzzy control system can guarantee the stability of the whole closed-loop systems and obtain good tracking performance as well. The proposed methods are applied to an inverted pendulum system and a chaotic system and achieve satisfactory simulation results.

Keywords: Fuzzy control; Nonlinear systems; Adaptive control; Observer; Stability

1. Introduction

Adaptive control schemes for nonlinear systems via feedback linearization concept have been employed for decades. The ideas of feedback linearization approaches are to transform a nonlinear dynamic system into a linear system through state feedback mechanisms. With such transformations, those well-explored linear control skills can then be applied to meet desired control specifications. Several primitive results and parameter adaptive control schemes have been reported in [9,10]. The major deficiency of those approaches is that their good performances are largely relied on exact cancellation of nonlinear terms, or restricted to conditions that the unknown parameters of nonlinear

* Corresponding author.
E-mail address: jztsc@eyou.com (S. Tong).

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systems are linear. If there exist uncertainties in those nonlinear terms, or the nonlinear terms are completely unknown, the performance may be awful due to non-exact cancellation. In this study, we intend to apply fuzzy modeling techniques to cope with the unknowns while employing adaptive linearization control schemes.

Since Zadeh [19] introduced the fuzzy set theory in 1965, it has received much attention from various fields and has also demonstrated nice performance in various applications. One of those successful fuzzy applications is to model unknown nonlinear systems by a set of fuzzy rules. One important property of fuzzy modeling approaches is that they are universal approximators [17]. In other words, fuzzy systems can be used to model virtually any nonlinear systems within a required accuracy provided that enough rules are given. Based on the universal approximation theorem and by incorporating fuzzy systems into adaptive control schemes, the stable direct and indirect fuzzy adaptive controllers are first proposed by Wang [15,16]. Afterwards, various adaptive fuzzy control approaches for nonlinear systems have been developed [1,2,8,11–13,18].

Generally, the direct and indirect adaptive fuzzy control approaches can have nice performance. However, such approaches are based on the assumption that the state variables of the system are available for feedback. As pointed out in [3,4,6], for most of the nonlinear systems, state variables are often unavailable in practice, the above requirement may be too restrictive or does not hold since frequently only an input–output model is available from on-line observations. In this situation, observer-based fuzzy adaptive controllers are more appealing. Using the state observer, an adaptive fuzzy-neural controller was proposed by Leu and Wang [7], this approach is only belongs to the category of indirect adaptive controllers and lacks in the complete stability of the whole closed-loop system. Furthermore, a prescribed tracking performance cannot be guaranteed. Another type of the observer-based fuzzy controller was introduced by Tong et al. [14]. Although this kind of adaptive fuzzy controller can ensure the stability of the closed-loop system and achieve a prescribed tracking performance, it sometimes exhibits a peaking phenomenon in the transient behavior due to the high gain.

The goal of the paper is to present new direct and indirect adaptive fuzzy controllers for a class of SISO uncertain nonlinear systems, which are called observer-based adaptive fuzzy controllers in this paper. Like [7,14], our approaches do not need the availability of the state variables, but design the state observer to estimate them. The Lyapunov stability theorem is used to derive controllers parameters update laws, which ensure the stability of the closed-loop system and plant output to achieve the H^∞ tracking performance.

The paper is organized as follows. First, the control problems and fuzzy system are introduced in Section 2. The observer-based indirect adaptive fuzzy controller and stability are proposed in Section 3. The observer-based direct adaptive fuzzy controller and stability are given in Section 4. In Section 5, simulation results are illustrated to confirm the feasibility of the proposed methods. Finally, conclusion remarks are presented in Section 6.

2. Problem formulation and fuzzy systems

Consider a class of the following SISO nth-order nonlinear system of the form [2,16]:

$$x^{(n)} = f(x, \dot{x}, \ldots, x^{(n-1)}) + g(x, \dot{x}, \ldots, x^{(n-1)})u,$$
$$y = x, \tag{1}$$

where f and g are unknown continuous functions, $\underline{x}=(x,\dot{x},\dots,x^{(n-1)})\in R^n$ is the state vector of the system, $u\in R$ and $y\in R$ are the input and output of the system, respectively.

It is assumed that $g(\underline{x})\neq 0$ for $\underline{x}$ in the certain controllability region $U_{\underline{x}}\subset R$. Without loss of generality, it is assumed that $g(\underline{x})>0$. Rewriting (1) in the following form:

$$\dot{\underline{x}} = A\underline{x} + B(f(\underline{x}) + g(\underline{x})u),$$
$$y = C^{\mathrm{T}}\underline{x}, \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}.$$

Let y_m be a bounded reference signal, $e=y_m-y$ the output tracking error, and $\hat{\underline{x}}$ the estimate of $\underline{x}$. Denote

$$\underline{y}_m = [y_m, \dot{y}_m, \dots, y_m^{(n-1)}]^{\mathrm{T}}, \quad \underline{e} = \underline{y}_m - \underline{x} = [e, \dot{e}, \dots, e^{(n-1)}]^{\mathrm{T}},$$

$$\hat{\underline{e}} = \underline{y}_m - \hat{\underline{x}} = [\hat{e}, \dot{\hat{e}}, \dots, \hat{e}^{(n-1)}]^{\mathrm{T}}, \quad \tilde{\underline{e}} = \underline{e} - \hat{\underline{e}}.$$

Control objectives: Utilizing fuzzy systems, reference signal y_m and the output of the system y to determine a fuzzy controller and an update law for adjusting the parameter vectors such that the following conditions are satisfied:

(i) The closed-loop system is stable and the tracking error converges to zero, i.e., $\lim_{t\to\infty} e(t)=0$.
(ii) For a prescribed attenuation level $\rho>0$, the H^{∞} tracking performance is achieved as below:

$$\int_0^T E^{\mathrm{T}} Q E \,\mathrm{d}t \leqslant E^{\mathrm{T}}(0) P E(0) + \frac{1}{r}\tilde{\underline{\theta}}^{\mathrm{T}}(0)\tilde{\underline{\theta}}(0) + \rho^2 \int_0^T w^2 \,\mathrm{d}t \tag{3}$$

where $Q=Q^{\mathrm{T}}\geqslant 0$, $P=P^{\mathrm{T}}\geqslant 0$ are weighing matrixes, $E^{\mathrm{T}}=[\hat{\underline{e}}^{\mathrm{T}},\tilde{\underline{e}}^{\mathrm{T}}]$, $r>0$ is an adaptive gain, and ρ a prescribed attenuation level.

A fuzzy system consists of four main components: fuzzy rule base, fuzzy inference engine, fuzzifier and defuzzifier [16]. The fuzzy rule base is composed of a collection of IF–THEN inference rules:

$$R^l\text{: If } x_1 \text{ is } F_1^l \text{ and } x_2 \text{ is } F_1^l \text{ and} \cdots \text{and } x_n \text{ is } F_n^l \text{ Then } y \text{ is } G^l, \tag{4}$$

where $\underline{x}=(x_1,\dots,x_n)^{\mathrm{T}}\in R^n$ and $y\in R$ are the input and output of the fuzzy system, respectively, F_i^l and G^l are fuzzy sets in R, $l=1,2,\dots,M$. The fuzzy inference engine performs a mapping from fuzzy sets in R^n to fuzzy set in R based on the IF–THEN rules in the fuzzy rule base and the compositional rule of inference. The fuzzifier maps a crisp point $\underline{x}=(x_1,\dots,x_n)^{\mathrm{T}}\in R^n$ into a fuzzy set A_x in R. The defuzzifier maps a fuzzy set in R to a crisp point in R. Since the strategy of singleton fuzzification, center-average defuzzification and product inference is used, the output of the fuzzy system can be formulated as

$$y = \frac{\sum_{j=1}^{M} y^j \left(\prod_{i=1}^{n} \mu_{F_i^j}(x_i) \right)}{\sum_{j=1}^{M} \prod_{i=1}^{n} \mu_{F_i^j}(x_i)}, \tag{5}$$

where y^j is the point at which fuzzy membership function $\mu_{G^j}(y^j)$ achieves its maximum value, and we assume that $\mu_{G^j}(y^j) = 1$. Eq. (5) can be rewritten as

$$y(\underline{x}) = \underline{\theta}^{\mathrm{T}} \psi(\underline{x}) \tag{6}$$

$\theta = [y^1, y^2, \ldots, y^M]^{\mathrm{T}}$ is a parameter vector, and $\psi(\underline{x}) = [\xi_1(\underline{x}), \ldots, \xi_M(\underline{x})]^{\mathrm{T}}$ is a regressive vector with the regressor $\xi_l(\underline{x})$, which is defined as fuzzy basis function

$$\xi_l = \frac{\prod_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i) \right)}. \tag{7}$$

Two main reasons arise for using the fuzzy system (6) as basic building block of adaptive fuzzy controllers. First, the fuzzy systems in the form of (6) are proven in [16] to be universal approximators, i.e., for any given real continuous function f on the compact set U, there exists a fuzzy system (6) such that it can uniformly approximate f over U to arbitrary accuracy. Therefore, the fuzzy systems (6) are qualified for modeling nonlinear systems. Second, the fuzzy systems (6) are constructed from the fuzzy IF–THEN rules of (3) using some specific fuzzy inference, fuzzification, and defuzzification strategies. Therefore, linguistic information from a human expert can be directly incorporated into the controller.

3. Observer-based indirect adaptive fuzzy control

According to the definition in [15], adaptive fuzzy approaches can be classified as indirect fuzzy controller and direct fuzzy controller. An indirect fuzzy controller uses fuzzy systems to model the system plant and a suitable controller is developed for the estimated system. A direct adaptive fuzzy controller uses fuzzy systems as controllers, it incorporates linguistic fuzzy control rules directly into the controllers.

The observer-based indirect adaptive fuzzy controller and its stability are discussed in this section.

Suppose the state variable $\underline{x}$ is known, i.e., it is available for feedback control. Suppose the fuzzy logic systems $\hat{f}(\underline{x}|\underline{\theta}_f)$ and $\hat{g}(\underline{x}|\underline{\theta}_g)$ are in the form of (6), i.e.,

$$\hat{f}(\underline{x}|\underline{\theta}_f) = \underline{\theta}_f^{\mathrm{T}} \psi(\underline{x}), \quad \hat{g}(\underline{x}|\underline{\theta}_g) = \underline{\theta}_g^{\mathrm{T}} \psi(\underline{x}). \tag{8}$$

Using $\hat{f}(\underline{x}|\underline{\theta}_f)$ and $\hat{g}(\underline{x}|\underline{\theta}_g)$ to approximate the unknown functions $f(\cdot)$ and $g(\cdot)$, respectively, then according to the work in paper [2], the following indirect adaptive fuzzy controller is suggested:

$$u = \frac{1}{\hat{g}(\underline{x}|\underline{\theta}_g)} [-\hat{f}(\underline{x}|\underline{\theta}_f) + y_m^{(n)} + K_c^{\mathrm{T}} \underline{e} + \frac{1}{r} B^{\mathrm{T}} P \underline{e}^{\mathrm{T}}] \tag{9}$$

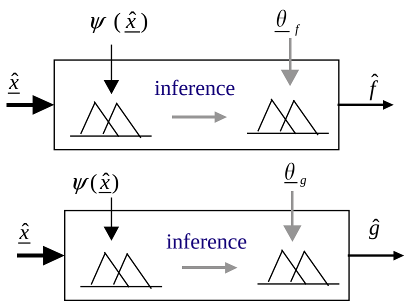

Fig. 1. Structure of fuzzy systems.

with the update law for the parameters

$$\dot{\underline{\theta}}_f = -\gamma_1 \underline{e}^{\mathrm{T}} PB\psi(\underline{x}), \tag{10}$$

$$\dot{\underline{\theta}}_g = -\gamma_2 \underline{e}^{\mathrm{T}} PB\psi(\underline{x})u, \tag{11}$$

where $K_c^{\mathrm{T}} = [k_n^c, k_{n-1}^c, \ldots, k_1^c]^{\mathrm{T}}$ is the feedback gain vector to make sure that the characteristic polynomial of $A - BK_c^{\mathrm{T}}$ is Hurwitz; r is a positive scalar value and $P = P^{\mathrm{T}} > 0$ is the solution of the following Riccati-like equation:

$$(A - BK_c)^{\mathrm{T}} P + P(A - BK_c) - PB\left(\frac{2}{r} - \frac{1}{\rho^2}\right) B^{\mathrm{T}} P = -Q. \tag{12}$$

It is proved in paper [2] that the closed-loop system is stable and the H^∞ tracking performance (3) is achieved with fuzzy controller (9) and the parameter update laws (10) and (11).

It is noted that Eqs. (9)–(11) contain the variable state vector $\underline{x}$, so if $\underline{x}$ is unknown, then controller (9), update laws (10) and (11) cannot be used to control nonlinear systems (1). In this situation, we must first design a state observer and obtain the state estimation $\underline{\hat{x}}$.

To begin with, taking $\underline{\hat{x}} = (\hat{x}_1\ \hat{x}_2 \cdots \hat{x}_n)^{\mathrm{T}}$ as the input of fuzzy system, and fuzzy rule base is

$$R^l: \text{If } \hat{x}_1 \text{ is } F_1^l \text{ and } \hat{x}_2 \text{ is } F_2^l \text{ and } \cdots \text{ and } \hat{x}_n \text{ is } F_n^l$$
$$\text{Then } y \text{ is } G^l \quad (l = 1, 2, \ldots, M). \tag{13}$$

Using singleton fuzzification, center-average defuzzification and product inference, we can obtain fuzzy systems $\hat{f}(\underline{\hat{x}}|\underline{\theta}_f)$ and $\hat{g}(\underline{\hat{x}}|\underline{\theta}_g)$ in the form

$$\hat{f}(\underline{\hat{x}}|\underline{\theta}_f) = \underline{\theta}_f^{\mathrm{T}}\psi(\underline{\hat{x}}) \quad \hat{g}(\underline{\hat{x}}|\underline{\theta}_g) = \underline{\theta}_g^{\mathrm{T}}\psi(\underline{\hat{x}}). \tag{14}$$

The structures of fuzzy systems $\hat{f}(\underline{\hat{x}}|\underline{\theta}_f)$ and $\hat{g}(\underline{\hat{x}}|\underline{\theta}_g)$ are illustrated by Fig. 1. Design the control law as

$$u = \frac{1}{\hat{g}(\underline{\hat{x}}|\underline{\theta}_g)}[-\hat{f}(\underline{\hat{x}}|\underline{\theta}_f) + y_m^{(n)} + K_c^{\mathrm{T}}\underline{\hat{e}} - u_a - u_s]. \tag{15}$$

where u_a is a H^∞ robust control term, which is used to compensate for the fuzzy approximation errors, u_s a linear combination of the error estimates, which ensures the stability of the whole closed-loop system. u_a and u_s will be designed later.

Substituting (15) into (2), after some manipulations, we obtain

$$\begin{aligned}\dot{\underline{e}} &= A\underline{e} - BK_c^{\mathrm{T}}\hat{\underline{e}} + Bu_a + Bu_s + B[(\hat{f}(\hat{\underline{x}}|\underline{\theta}_f) - f(\underline{x})) \\ &\quad + (\hat{g}(\hat{\underline{x}}|\underline{\theta}_g) - g(\underline{x}))u], \\ e &= C^{\mathrm{T}}\underline{e}. \end{aligned} \tag{16}$$

Design the error observer as follows:

$$\begin{aligned}\dot{\hat{\underline{e}}} &= A\hat{\underline{e}} - BK_c^{\mathrm{T}}\hat{\underline{e}} + K_0(e - \hat{e}), \\ \hat{e} &= C^{\mathrm{T}}\hat{\underline{e}}, \end{aligned} \tag{17}$$

where $K_0^{\mathrm{T}} = [k_1^0, k_2^0, \ldots, k_n^0]$ is the observer gain vector, which is selected to make sure that the characteristic polynomial of $A - K_0C^{\mathrm{T}}$ is Hurwitz.

Define the observation error $\tilde{\underline{e}} = \underline{e} - \hat{\underline{e}}$. Subtracting (17) from (16) yields

$$\begin{aligned}\dot{\tilde{\underline{e}}} &= (A - K_0C^{\mathrm{T}})\tilde{\underline{e}} + Bu_a + Bu_s + B[\hat{f}(\hat{\underline{x}}|\underline{\theta}_f) - f(\underline{x}) + (\hat{g}(\hat{\underline{x}})|\underline{\theta}_g) - g(\underline{x}))u], \\ \tilde{e} &= C^{\mathrm{T}}\tilde{\underline{e}}. \end{aligned} \tag{18}$$

Define the optimal parameter vector $\underline{\theta}_f^*$, $\underline{\theta}_g^*$ as follows

$$\begin{aligned}\underline{\theta}_f^* &= \arg\min_{\underline{\theta}_f \in \Omega_1}\left\{\sup_{\underline{x}\in U_1, \hat{\underline{x}}\in U_2} |f(\underline{x}) - \hat{f}(\hat{\underline{x}}|\underline{\theta}_f)|\right\}, \\ \underline{\theta}_g^* &= \arg\min_{\underline{\theta}_g \in \Omega_2}\left\{\sup_{\underline{x}\in U_1, \hat{\underline{x}}\in U_2} |g(\underline{x}) - \hat{g}(\hat{\underline{x}}|\underline{\theta}_g)|\right\}, \end{aligned} \tag{19}$$

where Ω_1, Ω_2, U_1 and U_2 denote the sets of suitable bounds on $\underline{\theta}_f$, $\underline{\theta}_g$, $\underline{x}$ and $\hat{x}$, respectively. We assume that $\underline{\theta}_f$, $\underline{\theta}_g$, $\underline{x}$ and $\hat{\underline{x}}$ never reach the boundary of Ω_1, Ω_2, U_1, and U_2. Also the minimum approximation error is defined as

$$\begin{aligned} w &= \left(\hat{f}\left(\hat{\underline{x}}|\underline{\theta}_f^*\right) - \hat{f}\left(\underline{x}|\underline{\theta}_f^*\right)\right) + \left(\hat{f}\left(\underline{x}|\underline{\theta}_f^*\right) - f(\underline{x})\right) \\ &\quad + \left[(\hat{g}(\hat{\underline{x}}|\underline{\theta}_g^*) - \hat{g}(\underline{x}|\underline{\theta}_g^*)) + (\hat{g}(\underline{x}|\underline{\theta}_g^*) - g(\underline{x}))\right] u. \end{aligned} \tag{20}$$

Then the observation error dynamic equation (18) can be rewritten as

$$\begin{aligned}\dot{\tilde{\underline{e}}} &= \left(A - K_0C^{\mathrm{T}}\right)\tilde{\underline{e}} + Bu_a + Bu_s + B\left[(\hat{f}(\hat{\underline{x}}|\underline{\theta}_f) - \hat{f}(\hat{\underline{x}}|\underline{\theta}_f^*))\right. \\ &\quad \left. + ((\hat{g}(\hat{\underline{x}}|\underline{\theta}_g) - \hat{g}(\hat{\underline{x}}|\underline{\theta}_g^*))u + w\right], \\ \tilde{e} &= C^{\mathrm{T}}\tilde{\underline{e}}. \end{aligned} \tag{21}$$

From (14) and (20), (21) can be rewritten as

$$\dot{\underline{\tilde{e}}} = (A - K_0C^{\mathrm{T}})\underline{\tilde{e}} + B\left[\underline{\tilde{\theta}}_f^{\mathrm{T}}\psi(\underline{\hat{x}}) + \underline{\tilde{\theta}}_g^{\mathrm{T}}\psi(\underline{\hat{x}})u + u_a + u_s + w\right],$$

$$\tilde{e} = C^{\mathrm{T}}\underline{\tilde{e}}, \tag{22}$$

where $\underline{\tilde{\theta}}_f = \underline{\theta}_f - \underline{\theta}_f^*$ and $\underline{\tilde{\theta}}_g = \underline{\theta}_g - \underline{\theta}_g^*$ are the parameter errors.

The output error dynamics of (22) can be given as

$$\tilde{e} = H(s)\left[\underline{\tilde{\theta}}_f^{\mathrm{T}}\psi(\underline{\hat{x}}) + \underline{\tilde{\theta}}_g^{\mathrm{T}}\psi(\underline{\hat{x}})u + u_a + u_s + w\right], \tag{23}$$

where

$$H(s) = C^{\mathrm{T}}(sI - (A - K_0C^{\mathrm{T}}))^{-1}B$$

is a known stable transfer function. In order to use the SPR-Lyapunov design approach [5,7], Eq. (23) is rewritten as

$$\tilde{e} = H(s)L(s)\left[\underline{\tilde{\theta}}_f^{\mathrm{T}}\psi_1(\underline{\hat{x}}) + \underline{\tilde{\theta}}_g^{\mathrm{T}}\psi_1(\underline{\hat{x}})u + u_{a1} + u_{s1} + w_1\right], \tag{24}$$

where $\psi_1(\underline{\hat{x}}) = L^{-1}(s)\psi(\underline{\hat{x}})$, $u_{a1} = L^{-1}(s)u_a$, $u_{s1} = L^{-1}(s)u_s$ and $w_1 = L^{-1}(s)w$. $L(s)$ is chosen so that $L^{-1}(s)$ is a proper stable transfer function and $H(s)L(s)$ is a proper strictly-positive-real (SPR) transfer function. Let

$$L(s) = s^m + b_1s^{m-1} + \cdots + b_m \quad (m = n-1).$$

The state-space realization of (24) can be written as

$$\dot{\underline{\tilde{e}}}_s = A_s\underline{\tilde{e}}_s + B_s\left[\underline{\tilde{\theta}}_f^{\mathrm{T}}\psi_1(\underline{\hat{x}}) + \underline{\tilde{\theta}}_g^{\mathrm{T}}\psi_1(\underline{\hat{x}})u + u_{a1} + u_{s1} + w_1\right],$$

$$\tilde{e} = C_s^{\mathrm{T}}\underline{\tilde{e}}_s, \tag{25}$$

where

$$\underline{\tilde{e}}_s = \left[\tilde{e}\ \dot{\tilde{e}}\ \cdots\ \tilde{e}^{(n-1)}\right]^{\mathrm{T}}, \quad A_s = A - K_0C^{\mathrm{T}},$$

$$B_s = [1\ b_1\ \cdots\ b_m]^{\mathrm{T}}, \quad C_s = [1\ 0\ \cdots\ 0]^{\mathrm{T}}.$$

Assume that P_1 and P_2 are positive definite solutions of the following matrix equations, respectively:

$$(A - BK_c^{\mathrm{T}})^{\mathrm{T}}P_1 + P_1(A - BK_c^{\mathrm{T}}) = -Q_1, \tag{26}$$

$$A_s^{\mathrm{T}}P_2 + P_2A_s = -Q_2,$$

$$P_2B_s = C_s. \tag{27}$$

In Eqs. (26) and (27), Q_1 and Q_2 are the given semi-definite positive matrices.

From Eq. (27), we know that $\underline{\tilde{e}}^{\mathrm{T}}P_2B_s = C_s^{\mathrm{T}}\underline{\tilde{e}} = \tilde{e}$, while $\tilde{e} = y_m - y - \hat{e}$ is available for feedback control, so we define the compensation control terms u_{a1}, u_{s1} and the parameter update laws as

$$u_{a1} = -\frac{1}{r}B_s^{\mathrm{T}}P_2\underline{\tilde{e}}_s = -\frac{1}{r}\tilde{e}, \tag{28}$$

$$u_{s1} = -K_0^{\mathrm{T}} P_1 \underline{\hat{e}}, \tag{29}$$

$$\dot{\underline{\theta}}_f = -\gamma_1 \underline{\tilde{e}}_s^{\mathrm{T}} P_2 B_s \psi_1(\underline{\hat{x}}) = -\gamma_1 \tilde{e} \psi_1(\underline{\hat{x}}), \tag{30}$$

$$\dot{\underline{\theta}}_g = -\gamma_2 \underline{\tilde{e}}_s^{\mathrm{T}} P_2 B_s \psi_1(\underline{\hat{x}}) u = -\gamma_2 \tilde{e} \psi_1(\underline{\hat{x}}) u, \tag{31}$$

where r is a positive scalar value satisfying $r = 2\rho^2$.

The main result of the observer-based indirect adaptive fuzzy control scheme is summarized in the following theorem.

Theorem 1. *Consider system* (1), *the adaptive fuzzy control scheme is chosen as* (15), (28)–(31). *If* $\int_0^\infty w_1^2(t)\,\mathrm{d}t < \infty$, *then the whole adaptive fuzzy control scheme guarantees the following properties*:

(i) $\underline{\hat{x}}$, $\underline{x}$, $\underline{e}$, $\underline{\hat{e}} \in L_\infty$, $\lim_{t\to\infty} e = 0$ *and* $\lim_{t\to\infty} \underline{\tilde{e}} = 0$;
(ii) *for a prescribed attenuation level* ρ, *the* H^∞ *tracking performance* (3) *is achieved.*

Proof. Consider the Lyapunov function candidate

$$V = \frac{1}{2}\underline{\hat{e}}^{\mathrm{T}} P_1 \underline{\hat{e}} + \frac{1}{2}\underline{\tilde{e}}_s^{\mathrm{T}} P_2 \underline{\tilde{e}}_s + \frac{1}{2\gamma_1}\underline{\tilde{\theta}}_f^{\mathrm{T}} \underline{\tilde{\theta}}_f + \frac{1}{2\gamma_2}\underline{\tilde{\theta}}_g^{\mathrm{T}} \underline{\tilde{\theta}}_g. \tag{32}$$

The time derivative of V is

$$\begin{aligned} \dot{V} = {} & \frac{1}{2}\dot{\underline{\hat{e}}}^{\mathrm{T}} P_1 \underline{\hat{e}} + \frac{1}{2}\underline{\hat{e}}^{\mathrm{T}} P_1 \dot{\underline{\hat{e}}} + \frac{1}{2}\dot{\underline{\tilde{e}}}_s^{\mathrm{T}} P_2 \underline{\tilde{e}}_s + \frac{1}{2}\underline{\tilde{e}}_s^{\mathrm{T}} P_2 \dot{\underline{\tilde{e}}}_s \\ & + \frac{1}{\gamma_1}\dot{\underline{\tilde{\theta}}}_f^{\mathrm{T}} \underline{\tilde{\theta}}_f + \frac{1}{\gamma_2}\dot{\underline{\tilde{\theta}}}_g^{\mathrm{T}} \underline{\tilde{\theta}}_g. \end{aligned} \tag{33}$$

From (17), (25) and by the fact $\dot{\underline{\tilde{\theta}}}_f = \dot{\underline{\theta}}_f$, $\dot{\underline{\tilde{\theta}}}_g = \dot{\underline{\theta}}_g$, the above equation becomes

$$\begin{aligned} \dot{V} = {} & \frac{1}{2}\underline{\hat{e}}^{\mathrm{T}} \left[(A - BK_C^{\mathrm{T}})^{\mathrm{T}} P_1 + P_1(A - BK_C^{\mathrm{T}})\right] \underline{\hat{e}} + \left(\underline{\hat{e}}^{\mathrm{T}} P_1 K_0 C^{\mathrm{T}} \underline{\tilde{e}}_s + \underline{\tilde{e}}_s^{\mathrm{T}} P_2 B_s u_{s1}\right) \\ & + \frac{1}{2}\underline{\tilde{e}}_s^{\mathrm{T}} (A_s^{\mathrm{T}} P_2 + P_2 A_s^{\mathrm{T}}) \underline{\tilde{e}}_s + \underline{\tilde{e}}_s^{\mathrm{T}} P_2 B_s u_{a1} + \underline{\tilde{e}}_s^{\mathrm{T}} P_2 B_s w_1 \\ & + \left(\underline{\tilde{e}}_s^{\mathrm{T}} P_2 B_s \underline{\tilde{\theta}}_f^{\mathrm{T}} \psi_1(\underline{\hat{x}}) + \frac{1}{\gamma_1}\dot{\underline{\theta}}_f^{\mathrm{T}} \underline{\tilde{\theta}}_f\right) \\ & + \left(\underline{\tilde{e}}_s^{\mathrm{T}} P_2 B_s \underline{\tilde{\theta}}_f^{\mathrm{T}} \psi_1(\underline{\hat{x}}) u + \frac{1}{\gamma_2}\dot{\underline{\theta}}_g^{\mathrm{T}} \underline{\tilde{\theta}}_g\right). \end{aligned} \tag{34}$$

From $u_{s1} = -K_0^{\mathrm{T}} P_1 \underline{\hat{e}}$ and $P_2 B_s = C_s$, we have

$$\underline{\tilde{e}}_s^{\mathrm{T}} P_2 B_s u_{s1} = -\underline{\tilde{e}}_s^{\mathrm{T}} P_2 B_s K_0^{\mathrm{T}} P_1 \underline{\hat{e}} = -\underline{\tilde{e}}_s^{\mathrm{T}} C_s K_0^{\mathrm{T}} P_1 \underline{\hat{e}} = -\underline{\hat{e}}^{\mathrm{T}} P_1 K_0 C_s^{\mathrm{T}} \underline{\tilde{e}}_s. \tag{35}$$

Applying (28), (30), (31) and (35) to (34) yields

$$\dot{V} \leqslant \frac{1}{2} \underline{\hat{e}}^{\mathrm{T}} \left[(A - BK_C^{\mathrm{T}})^{\mathrm{T}} P_1 + P_1 (A - BK_C^{\mathrm{T}})\right] \underline{\hat{e}} + \frac{1}{2} \underline{\tilde{e}}_s^{\mathrm{T}} (A_s^{\mathrm{T}} P_2 + P_2 A_s^{\mathrm{T}} - \frac{2}{r} P_2 B_s B_s^{\mathrm{T}} P_2) \underline{\tilde{e}}_s + \underline{\tilde{e}}_s^{\mathrm{T}} P_2 B_s w_1. \tag{36}$$

Since (A, B) is controllable, and Eq. (25) is SPR, positive definite solutions P_1 and P_2 exist for matrix equations (26) and (27). Letting $r = 2\rho^2$, we obtain

$$\begin{aligned}\dot{V} &\leqslant -\frac{1}{2} \underline{\hat{e}}^{\mathrm{T}} Q_1 \underline{\hat{e}} - \frac{1}{2} \underline{\tilde{e}}_s^{\mathrm{T}} Q_2 \underline{\tilde{e}}_s - \frac{1}{2\rho^2} \underline{\tilde{e}}_s^{\mathrm{T}} P_2 B_s B_s^{\mathrm{T}} P_2 \underline{\tilde{e}}_s + \frac{1}{2} (w_1^{\mathrm{T}} B_s^{\mathrm{T}} P \underline{\tilde{e}}_s + \underline{\tilde{e}}_s^{\mathrm{T}} P_2 B_s^{\mathrm{T}} w_1) \\ &= -\frac{1}{2} \underline{\hat{e}}^{\mathrm{T}} Q_1 \underline{\hat{e}} - \frac{1}{2} \underline{\tilde{e}}_s^{\mathrm{T}} Q_2 \underline{\tilde{e}}_s + \frac{1}{2} \rho^2 w_1^2 - \frac{1}{2} \left(\frac{1}{\rho} B_s^{\mathrm{T}} P_2 \underline{\tilde{e}}_s - \rho w_1\right)^{\mathrm{T}} \left(\frac{1}{\rho} B_s^{\mathrm{T}} P_2 \underline{\tilde{e}}_s - \rho w_1\right) \\ &\leqslant -\frac{1}{2} \underline{\hat{e}}^{\mathrm{T}} Q_1 \underline{\hat{e}} - \frac{1}{2} \underline{\tilde{e}}_s^{\mathrm{T}} Q_2 \underline{\tilde{e}}_s + \frac{1}{2} \rho^2 w_1^2.\end{aligned} \tag{37}$$

Denoting $Q = diag[Q_1, Q_2]$, $E^{\mathrm{T}} = [\underline{\hat{e}}^{\mathrm{T}}, \underline{\tilde{e}}_s^{\mathrm{T}}]$, then the above equation becomes

$$\dot{V} \leqslant -\frac{1}{2} E^{\mathrm{T}} Q E + \frac{1}{2} \rho^2 w_1^2. \tag{38}$$

Since $w_1 \in L_2$ and via the same argument as [15], we establish that $\underline{e}$, $\underline{\hat{e}}$, $\underline{\hat{x}}$, $\underline{x}$, $u \in L_\infty$, $\lim_{t\to\infty} \underline{\hat{e}} = 0$ and $\lim_{t\to\infty} \underline{\tilde{e}}_2 = 0$. Therefore, we conclude that $\lim_{t\to\infty} e = 0$ and $\lim_{t\to\infty} \underline{\tilde{e}} = 0$.

Integrating the above equation from $t = 0$ to T yields

$$V(T) - V(0) \leqslant -\frac{1}{2} \int_0^{\mathrm{T}} E^{\mathrm{T}} Q E \,\mathrm{d}t + \frac{1}{2} \rho^2 \int_0^{\mathrm{T}} w_1^2 \,\mathrm{d}t. \tag{39}$$

Denoting $P = diag(P_1, P_2)$, since $V(T) \geqslant 0$, (39) implies the following:

$$\frac{1}{2}\int_0^T E^{\mathrm{T}} Q E \,\mathrm{d}t \leqslant \frac{1}{2} E^{\mathrm{T}}(0) P E(0) + \frac{1}{2\gamma_1}\tilde{\underline{\theta}}_f^{\mathrm{T}}(0)\tilde{\underline{\theta}}_f(0) + \frac{1}{2\gamma_2}\tilde{\underline{\theta}}_g^{\mathrm{T}}(0)\tilde{\underline{\theta}}_g(0) + \frac{1}{2}\rho^2 \int_0^{\mathrm{T}} w_1^2 \,\mathrm{d}t \tag{40}$$

Therefore, the H^∞ tracking performance (3) is achieved. □

4. Observer-based direct adaptive fuzzy control

If $g(\underline{x})$ is a known constant, and the state variables are available for feedback control, according to the works in [2], the following direct adaptive fuzzy controller is suggested:

$$u = \hat{u}(\underline{x}|\underline{\theta}) - \frac{1}{b} u_a, \tag{41}$$

where $\hat{u}(\underline{x}|\underline{\theta}) = \underline{\theta}^{\mathrm{T}}\psi(\underline{x})$ is the fuzzy system to directly approximate the following control law:

$$u(\underline{x}) = \frac{1}{b}\left[-f(\underline{x}) + y_m^{(n)} + K_c^{\mathrm{T}}\underline{e}\right], \tag{42}$$

$u_a = -(1/r)B^{\mathrm{T}}P\underline{e}$ is the H^∞ robust term to compensate the fuzzy approximation error.

If the parameter update law is chosen as

$$\dot{\underline{\theta}} = \gamma \underline{e}^{\mathrm{T}} P B \psi(\underline{x}). \tag{43}$$

Then the whole adaptive fuzzy control scheme (41) and (43) can guarantee the stability of the closed-loop system and achieve the H^∞ tracking performance (3).

If the state vector $\underline{x}$ is not available for feedback control, in this situation, we design direct adaptive fuzzy controller as

$$u = \hat{u}(\hat{\underline{x}}|\theta) - u_a - u_s, \tag{44}$$

where $\hat{u}(\hat{\underline{x}}|\underline{\theta})$ is a fuzzy system, which is in the form

$$\hat{u}(\hat{\underline{x}}|\underline{\theta}) = \underline{\theta}^{\mathrm{T}}\psi(\hat{\underline{x}}) \tag{45}$$

and u_a is a H^∞ robust control term to compensate the fuzzy approximation errors, u_s is a linear combination of error estimations to ensure the stability of the whole closed-loop system.

Substituting (44) and (45) into (2), after some manipulation, we have

$$\begin{aligned} x^{(n)} &= f(\underline{x}) + b(\hat{u}(\hat{\underline{x}}|\underline{\theta}) - u_a - u_s) - bu^*(\underline{x}) + bu^*(\underline{x}), \\ &= y_m^{(n)} + K_c^{\mathrm{T}}\underline{e} - bu_a - bu_s + b(\hat{u}(\hat{\underline{x}}|\underline{\theta}) - u^*(\underline{x})) \end{aligned} \tag{46}$$

or, equivalently

$$\dot{\underline{e}} = A\underline{e} - BK_c^{\mathrm{T}}\hat{\underline{e}} + Bb\left[u_a + u_s + (u^*(\underline{x}) - \hat{u}(\hat{\underline{x}}|\underline{\theta}))\right],$$
$$e = C^{\mathrm{T}}\underline{e}, \tag{47}$$

where $u^*(\underline{x}) = (1/b)[-f(\underline{x}) + y_m^{(n)} + K_c^{\mathrm{T}}\hat{\underline{e}}]$.

Design the error state observer as

$$\dot{\hat{\underline{e}}} = A\hat{\underline{e}} - BK_c^{\mathrm{T}}\hat{\underline{e}} + K_0(e - \hat{e}),$$
$$\hat{e} = C^{\mathrm{T}}\hat{\underline{e}}, \tag{48}$$

where $K_0^{\mathrm{T}} = [k_1^0, k_2^0, \ldots, k_n^0]$ is the observer gain vector, chosen such that the characteristic polynomial of $A - K_0C^{\mathrm{T}}$ is a Hurwitz.

Define the observation error $\tilde{\underline{e}} = \underline{e} - \hat{\underline{e}}$, and subtracting (48) from (47), yields

$$\dot{\tilde{\underline{e}}} = (A - K_0C^{\mathrm{T}})\tilde{\underline{e}} + Bb[u_a + u_s + (u^*(\underline{x}) - u(\underline{x}|\underline{\theta}))],$$
$$\tilde{e} = C^{\mathrm{T}}\tilde{\underline{e}}. \tag{49}$$

Define the optimal parameter vector $\underline{\theta}^*$ and fuzzy approximation error w as follows:

$$\underline{\theta}^* = \arg\min_{\underline{\theta}\in\Omega}\left\{\sup_{\underline{x}\in U_1,\hat{\underline{x}}\in U_2} |u^*(\underline{x}) - u(\hat{\underline{x}}|\underline{\theta})|\right\}, \tag{50}$$

$$w = (u^*(x) - \hat{u}(\underline{x}|\underline{\theta}^*)) + (\hat{u}(\underline{x}|\underline{\theta}^*) - (\hat{u}(\hat{\underline{x}}|\underline{\theta}^*)). \tag{51}$$

Substituting (45) and (51) into (49), we have

$$\dot{\tilde{\underline{e}}} = (A - K_0C^{\mathrm{T}})\tilde{\underline{e}} + B[b\tilde{\underline{\theta}}^{\mathrm{T}}\psi(\hat{\underline{x}}) + bu_a + bu_s + bw],$$
$$\tilde{e} = C^{\mathrm{T}}\tilde{\underline{e}}, \tag{52}$$

where $\tilde{\underline{\theta}} = \underline{\theta}^* - \underline{\theta}$ is the parameter error vector.

Repeating the same manipulations as the above section, Eq. (52) can be rewritten as

$$\dot{\tilde{\underline{e}}}_s = A_s\tilde{\underline{e}}_s + B_s[b\tilde{\underline{\theta}}^{\mathrm{T}}\psi_1(\hat{\underline{x}}) + bu_{a1} + bu_{s1} + bw_1],$$
$$\tilde{e} = C_s^{\mathrm{T}}\tilde{\underline{e}}_s, \tag{53}$$

where

$$\tilde{\underline{e}}_s = [\tilde{e}\ \dot{\tilde{e}}\ \cdots\ \tilde{e}^{(n-1)}]^{\mathrm{T}} \quad A_s = A - K_0C^{\mathrm{T}},$$
$$B_s = [1\ b_1\ \cdots\ b_m]^{\mathrm{T}}, \quad C_s = [1\ 0\ \cdots\ 0]^{\mathrm{T}}$$

and $\psi_1(\hat{\underline{x}}) = L^{-1}(s)\psi(\hat{\underline{x}})$, $w_1 = L^{-1}(s)w$; $u_{a1} = L^{-1}(s)u_a$, $u_{s1} = L^{-1}(s)u$.

Assume that P_1 and P_2 are positive definite solutions of the following matrix equations, respectively:

$$(A - BK_c^{\mathrm{T}})^{\mathrm{T}}P_1 + P_1(A - BK_c^{\mathrm{T}}) = -Q_1, \tag{54}$$

$$A_s^{\mathrm{T}} P_2 + P_2 A_s^{\mathrm{T}} = -bQ_2,$$

$$P_2 B_s = C_s. \tag{55}$$

Since $\underline{\tilde{e}}^{\mathrm{T}} P_2 B_s = C_s^{\mathrm{T}} \underline{\tilde{e}} = \tilde{e}$, while $\tilde{e} = y_m - y - \hat{e}$ is available for feedback control, design u_{a1}, u_{s1} and the parameter update law as

$$u_{a1} = -\frac{1}{r} B_c^{\mathrm{T}} P_2 \underline{\tilde{e}}_s = -\frac{1}{r}\tilde{e} \tag{56}$$

$$u_{s1} = -K_0^{\mathrm{T}} P_1 \underline{\hat{e}} \tag{57}$$

$$\underline{\dot{\theta}} = \gamma \underline{\tilde{e}}_s^{\mathrm{T}} P_2 B_s \psi_1(\underline{\hat{x}}) = \gamma \tilde{e} \psi_1(\underline{\hat{x}}), \tag{58}$$

where r is a positive scalar value satisfying $r = 2\rho^2$.

The main result of the observer-based direct adaptive fuzzy control scheme is summarized in the following theorem.

Theorem 2. *In the nonlinear system* (1), *if the following direct adaptive fuzzy control law is selected according to* (44) *and* (56)–(58), *and* $\int_0^\infty w_1^2\,\mathrm{d}t < \infty$, *then the whole control scheme guarantees the following properties*:

(i) $\underline{\hat{x}}$, $\underline{x}$, $\underline{e}$, $\underline{\hat{e}} \in L_\infty$, $\lim_{t\to\infty} \tilde{e} = 0$, $\lim_{t\to\infty} \underline{\tilde{e}} = 0$;
(ii) *for a prescribed attenuation level* ρ, *the* H^∞ *tracking performance* (3) *is achieved.*

Proof. Consider the Lyapunov function candidate

$$V = \frac{1}{2}\underline{\hat{e}}^{\mathrm{T}} P_1 \underline{\hat{e}} + \frac{1}{2b}\underline{\tilde{e}}_s^{\mathrm{T}} P_2 \underline{\tilde{e}}_s + \frac{1}{2\gamma}\underline{\tilde{\theta}}^{\mathrm{T}} \underline{\tilde{\theta}}. \tag{59}$$

The following proof is similar to that of Theorem 1. □

5. Simulation results

In this paper, two examples are used to verify the performance of the proposed controllers; one is an inverted pendulum system and the other is a Duffing forced oscillation system.

Example 1. Indirect adaptive fuzzy control approach. The inverted pendulum system is defined as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (f + gu + d),$$

$$y = [1 \;\; 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \tag{60}$$

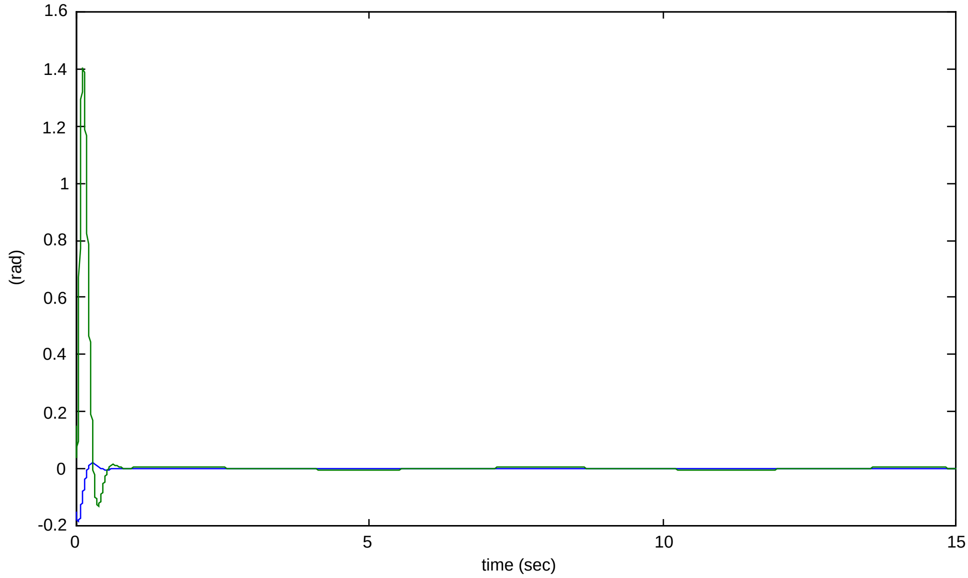

Fig. 2. Trajectories of the state estimation errors $\hat{e}_1$ (solid line) and $\hat{e}_2$ (dash–dotted) with $d=0$ and $\rho=0.01$.

where

$$f=\frac{mlx_2\sin x_1\cos x_1-(M+m)g\sin x_1}{ml\cos^2 x_1-\frac{4}{3}l(M+m)},\quad g=\frac{-\cos x_1}{ml\cos^2 x_1-\frac{4}{3}l(M+m)},$$

$g=9.8\,\mathrm{m/s^2}$, $m=0.1\,\mathrm{kg}$, $M=1\,\mathrm{kg}$, $l=0.5\,\mathrm{m}$. The external disturbance d is a square wave with the amplitude ± 1 and the period π(s). The used reference is $y_m=(\pi/30)\sin t$.

In the implementation, seven fuzzy sets are defined over interval $[-\pi/3,\pi/3]$ for both x_1 and x_2, with labels N_1, N_2, N_1, Z, P_1, P_2 and P_3, and their membership functions are $\mu_{N_1}(x_i)=1/(1+\exp(5(x_i+0.6)))$, $\mu_{N_2}(x_i)=\exp(-(x_i+0.4)^2)$, $\mu_{N_3}(x_i)=\exp(-(x_i+0.2)^2)$, $\mu_Z(x_i)=\exp(-x_i^2)$, $\mu_{P_1}(x_i)=\exp(-(x_i-0.2)^2)$, $\mu_{P_2}(x_i)=\exp(-(x_i-0.4)^2)$ and $\mu_{P_3}(x_i)=1/(1+\exp(-5(x_i-0.6)))$. Select $L(s)=1/(s+2)$. Given the positive matrices $Q_1=Q_2=diag[10,10]$, feedback and observer gain vector are chosen as $K_c^{\mathrm{T}}=[100.10]$ and $K_0^{\mathrm{T}}=[40.700]$, solving the matrix equation (26) and the first equation of (27), we obtain the positive matrices as

$$P_1=\begin{bmatrix}51 & 0.05\\ 0.05 & 0.504\end{bmatrix},\quad P_2=\begin{bmatrix}74 & -5\\ -5 & 0.46\end{bmatrix}.$$

The initial values are chosen as $x_1(0)=x_2(0)=0.2$; $\hat{x}_2(0)=\hat{x}_1(0)=1.5$; $\underline{\theta}_f(0)=\underline{0}, \underline{\theta}_g(0)=0.2I_{7\times 1}$, and $\gamma_1=70$, $\gamma_2=0.5$.

In the first case, Figs. 2–4 show the results of indirect adaptive output feedback fuzzy control approach with the external disturbance $d=0$ and the prescribed attenuation level $\rho=0.01$. From Figs. 2 and 3, we can see that state estimation errors $\hat{e}_1(\hat{e}_2)$ and the tracking errors $e_1(e_2)$ converge to zero after a few seconds. In the second case, the square wave external disturbance enters system

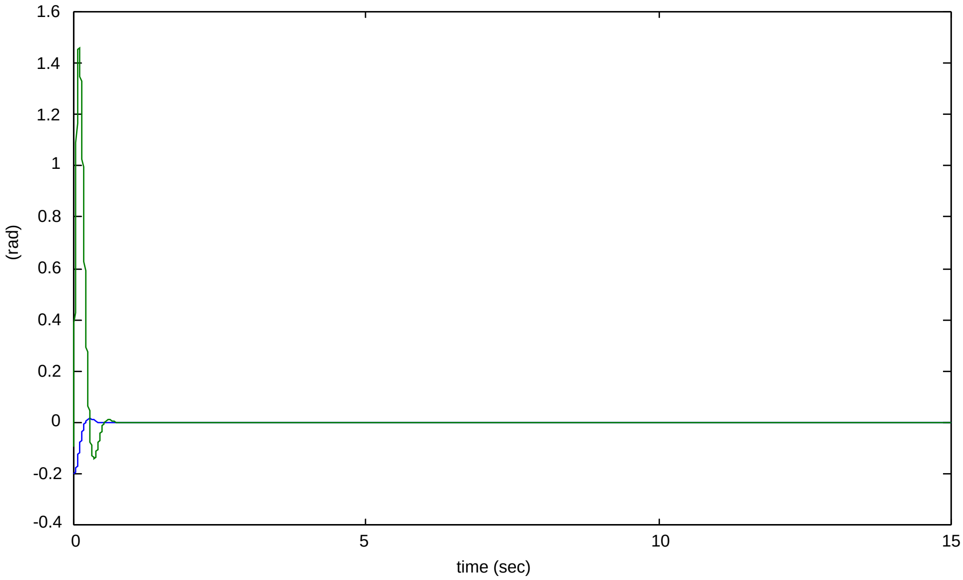

Fig. 3. Trajectories of the tracking errors e_1 (solid line) and e_2 (dash–dotted) with $d=0$ and $\rho=0.01$.

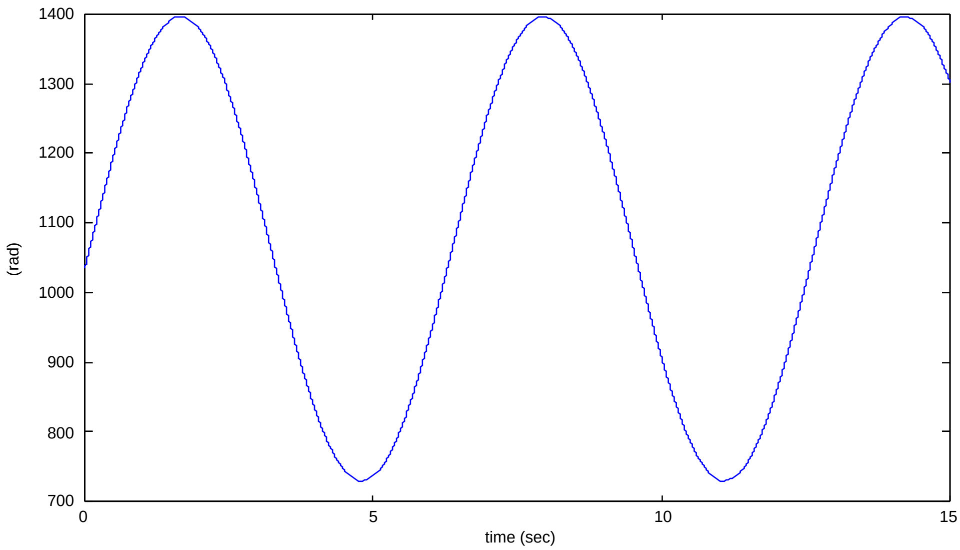

Fig. 4. Control input u with $d=0$ and $\rho=0.01$.

(60), Figs. 5–7 show the results of indirect adaptive output feedback fuzzy control approach. Compared Figs. 2 and 3 with 5 and 6, respectively, we can see that the external disturbance affects the converging performances of state estimation and output error tracking.

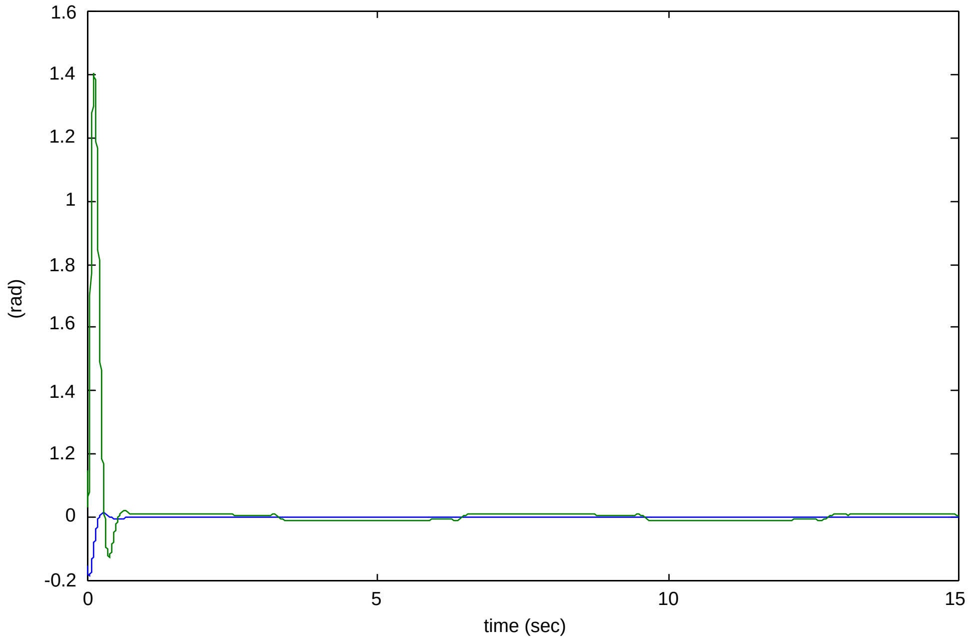

Fig. 5. Trajectories of the state estimation errors $\hat{e}_1$ (solid line) and $\hat{e}_2$ (dash–dotted) with $d \neq 0$ and $\rho = 0.01$.

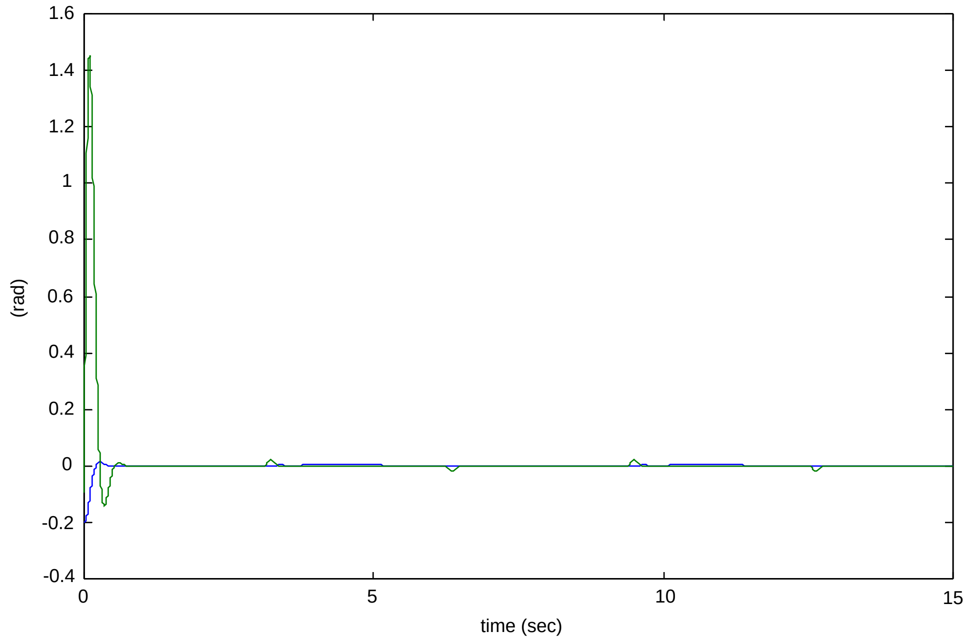

Fig. 6. Trajectories of the tracking errors e_1 (solid line) and e_2 (dash–dotted) with $d \neq 0$ and $\rho = 0.01$.

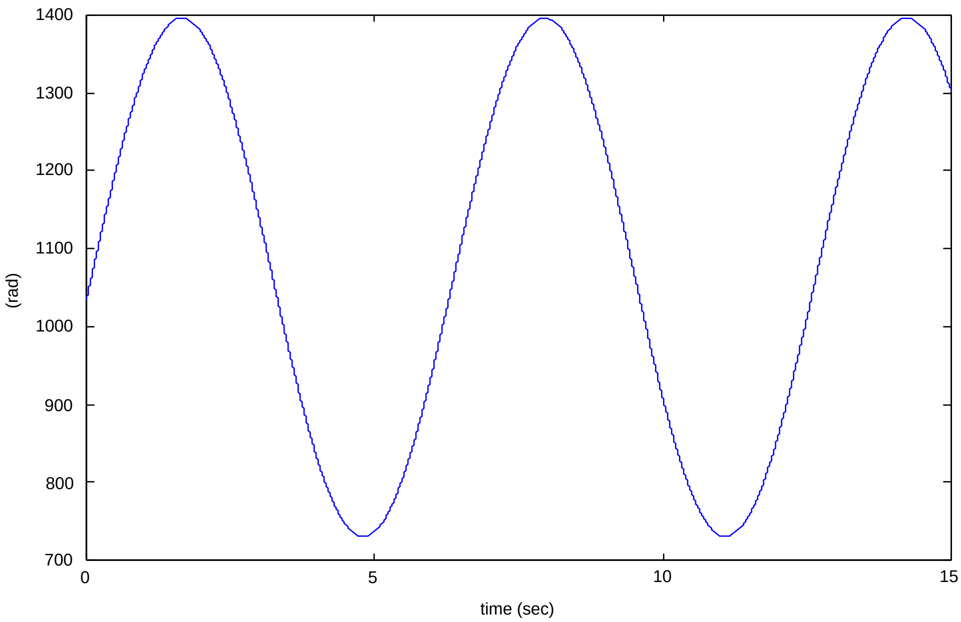

Fig. 7. Control input u with $d \neq 0$ and $\rho = 0.01$.

To illustrate the H^{∞} tracking performance of the proposed design algorithm, increase the prescribed attenuation level, i.e., $\rho = 0.05$, the results are given in Figs. 8–10. From Figs. 4 to 10, we can conclude that the state estimation errors and the tracking errors can be decreased under the larger prescribed attenuation level. As expected, Figs. 7 and 10 indicate that the control effort at a higher attenuation is observed to be larger than that at low ones.

Example 2. Direct adaptive fuzzy control approach. The Duffing forced oscillation system used in [2] is

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -0.1x_2 - x_1^3 + 12\cos t + u + d,$$
$$y = x_1, \tag{61}$$

where the external disturbance d is a square wave with the amplitude ± 1 and the period π(s). The reference signal is defined as $y_m = \sin t$.

In this example, the seven fuzzy labels shown in Example 1 are also used for both x_1 and x_2. Given the positive matrices $Q_1 = Q_2 = diag[10, 10]$, feedback and observer gain vector are chosen as $K_c^{\mathrm{T}} = [2, 1]$ and $K_0^{\mathrm{T}} = [40, 700]$, solutions of the matrix equation (54) and the first

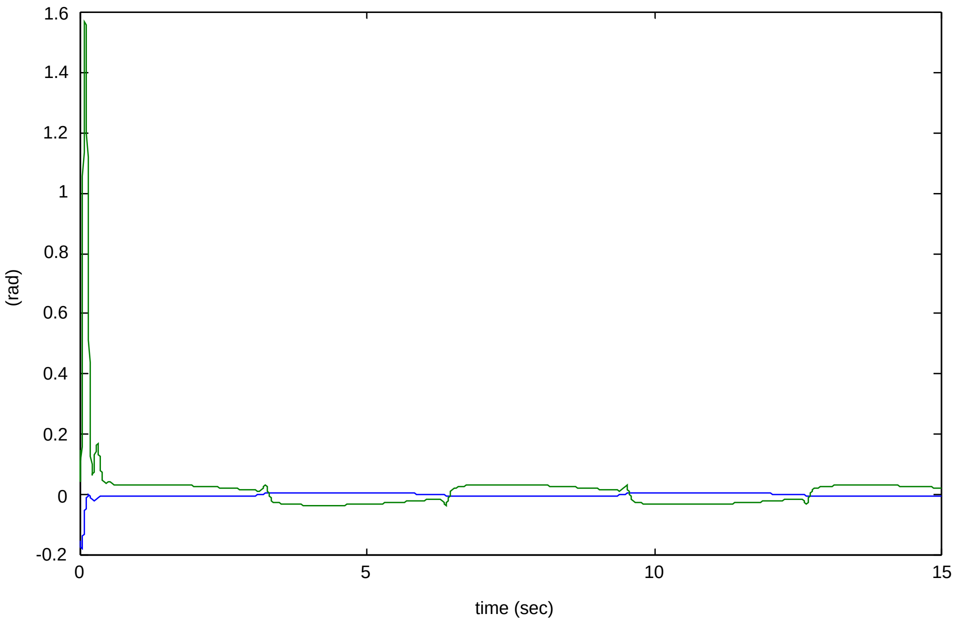

Fig. 8. Trajectories of the state estimation errors $\hat{e}_1$ (solid line) and $\hat{e}_2$ (dash–dotted) with $d \neq 0$ and $\rho = 0.05$.

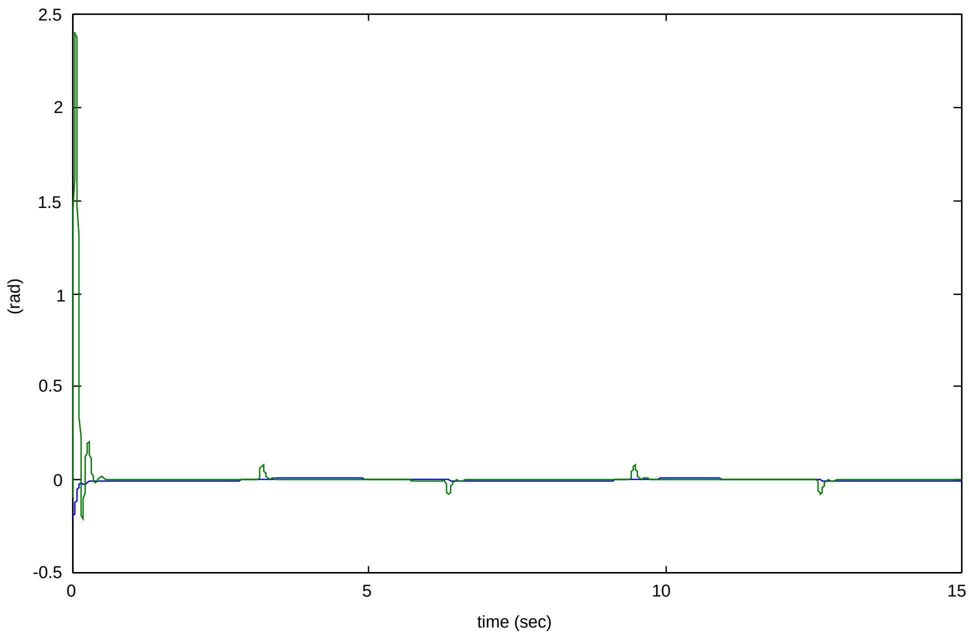

Fig. 9. Trajectories of the tracking errors e_1 (solid line) and e_2 (dash–dotted) with $d \neq 0$ and $\rho = 0.05$.

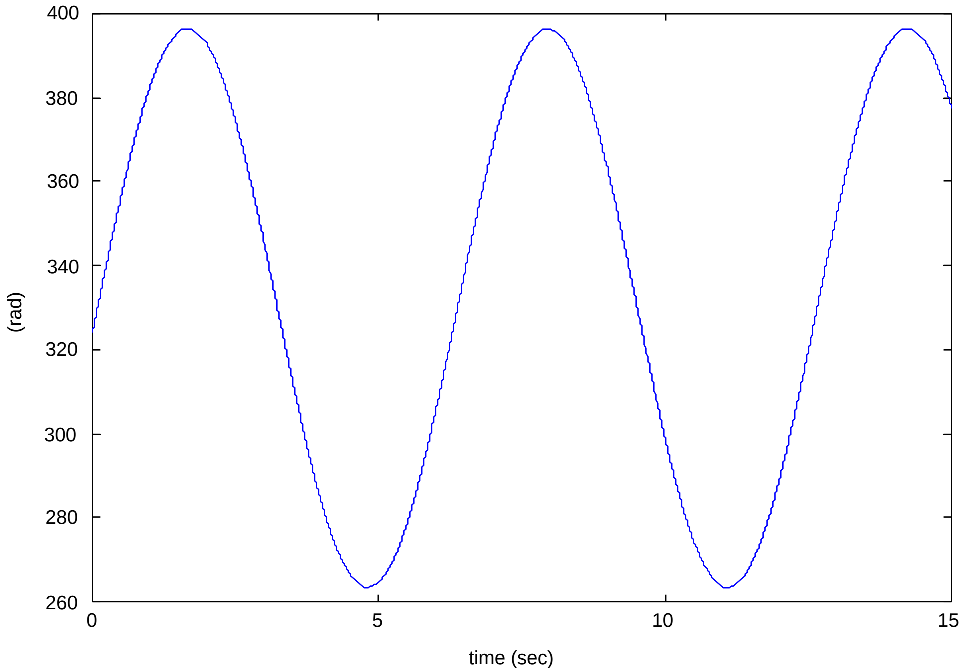

Fig. 10. Control input u with $d \neq 0$ and $\rho = 0.05$.

equation of (55) are

$$P_1 = \begin{bmatrix} 17.4 & 2.5 \\ 2.5 & 7.5 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 74 & -5 \\ -5 & 0.46 \end{bmatrix}.$$

Take $\gamma = 0.5$, initial conditions are chosen as $x_1(0) = x_2(0) = 0.2$, $\hat{x}_1(0) = \hat{x}_2(0) = 1.5$, $\underline{\theta}(0) = \underline{0}$.

In the simulation, we take the prescribed attenuation level $\rho = 0.01$ and the square wave. Figs. 11–13 show the results of direct adaptive output feedback fuzzy control approach. Figs. 11 and 12 show that state estimation errors $\hat{e}_1(\hat{e}_2)$ and the tracking errors $e_1(e_2)$ converge which occurs at the starting point within a small bound after a few second. To illustrate H^∞ tracking performance, we increase the prescribed attenuation level $\rho = 0.05$; the results are given in Figs. 14–16. Comparing Figs. 11 and 12 with 14 and 15, respectively, we can see that the state estimation errors and the tracking errors can be decreased under the larger prescribed attenuation level. As expected, Figs. 13 and 16 indicate that the control effort at a higher attenuation is observed to be larger than that at low ones.

6. Conclusion

Two adaptive fuzzy controllers, called observer-based direct and indirect adaptive fuzzy controllers are proposed in this paper. Since the state variables of nonlinear systems are assumed to be unknown,

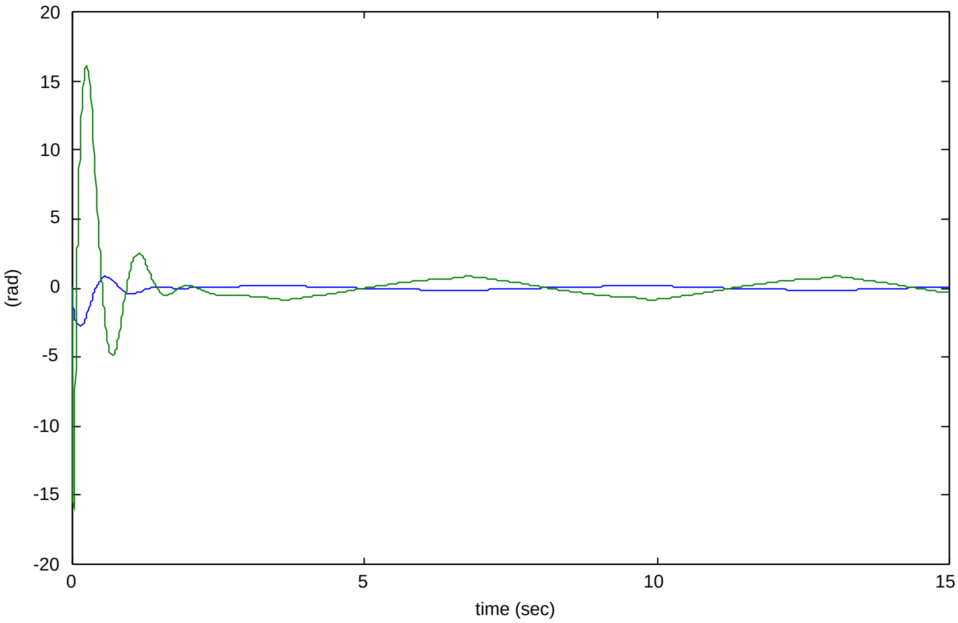

Fig. 11. Trajectories of the state estimation errors $\hat{e}_1$ (solid line) and $\hat{e}_2$ (dash–dotted) with $d \neq 0$ and $\rho = 0.01$.

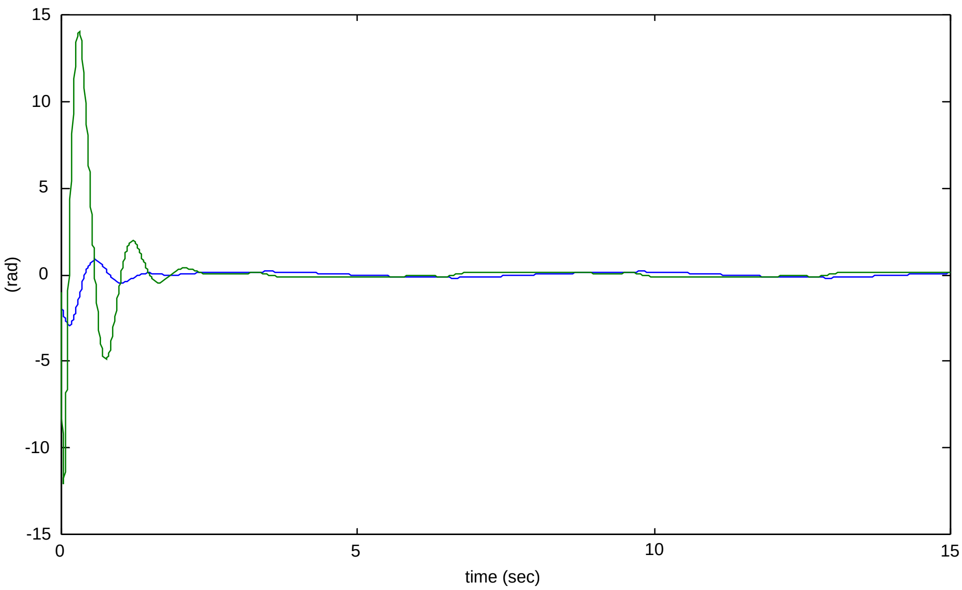

Fig. 12. Trajectories of the tracking errors e_1 (solid line) and e_2 (dash–dotted) with $d \neq 0$ and $\rho = 0.01$.

the state observer is first designed to estimate state variables, via which fuzzy control schemes are formulated. Based on the Lyapunov stability theorem, it is rigorously proved that the stability of the closed-loop system is assured and the tracking performance is achieved. Application of the

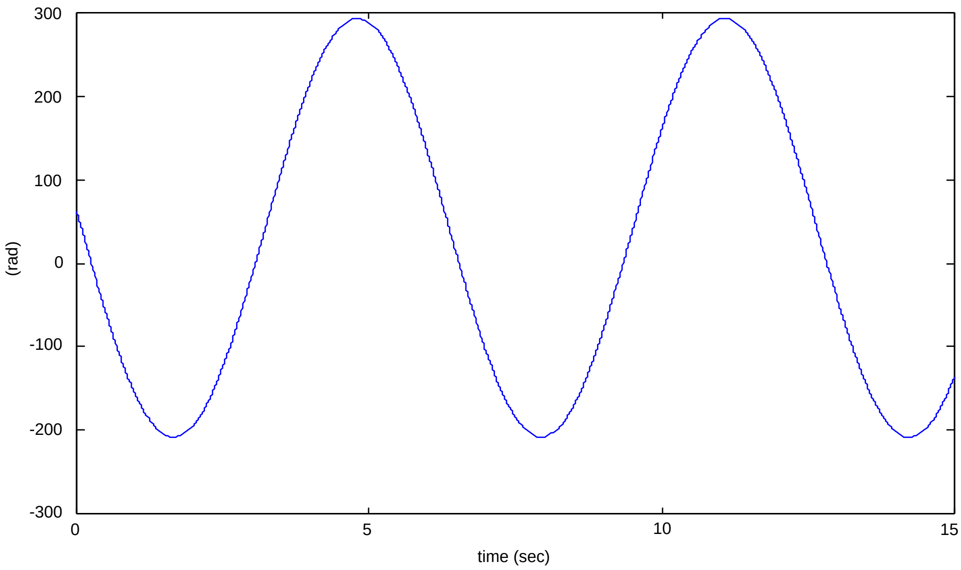

Fig. 13. Control input u with $d \neq 0$ and $\rho = 0.01$.

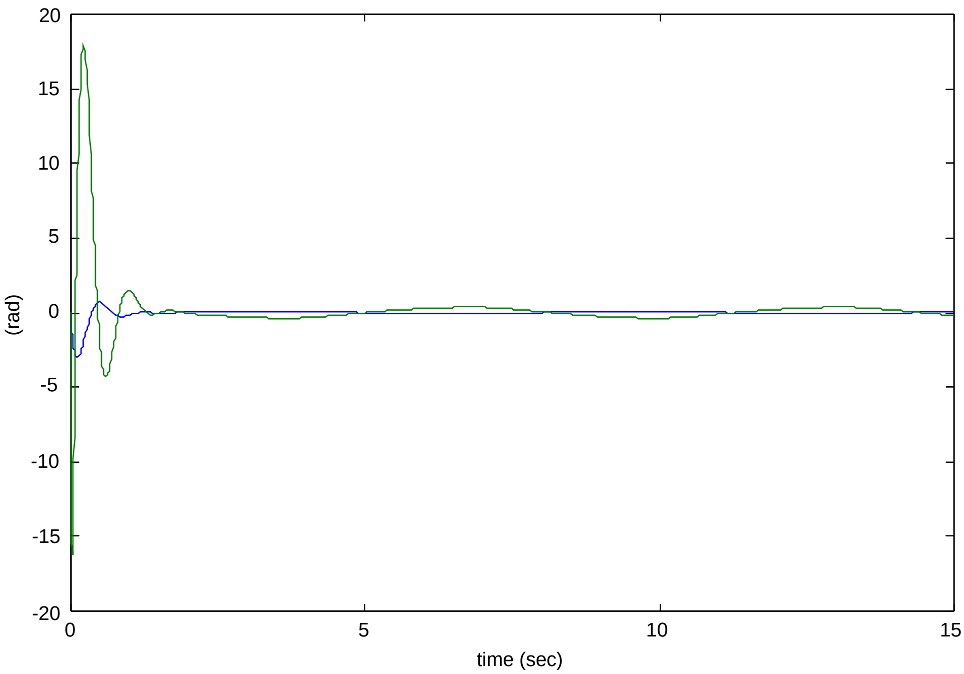

Fig. 14. Trajectories of the state estimation errors $\hat{e}_1$ (solid line) and $\hat{e}_2$ (dash–dotted) with $d \neq 0$ and $\rho = 0.05$.

proposed approaches to an inverted pendulum system and a chaotic system show a very satisfactory performance. Compared to the previous approach [2], our approach can achieve the desired H^{∞} tracking performance without the assumption of the known state variables.

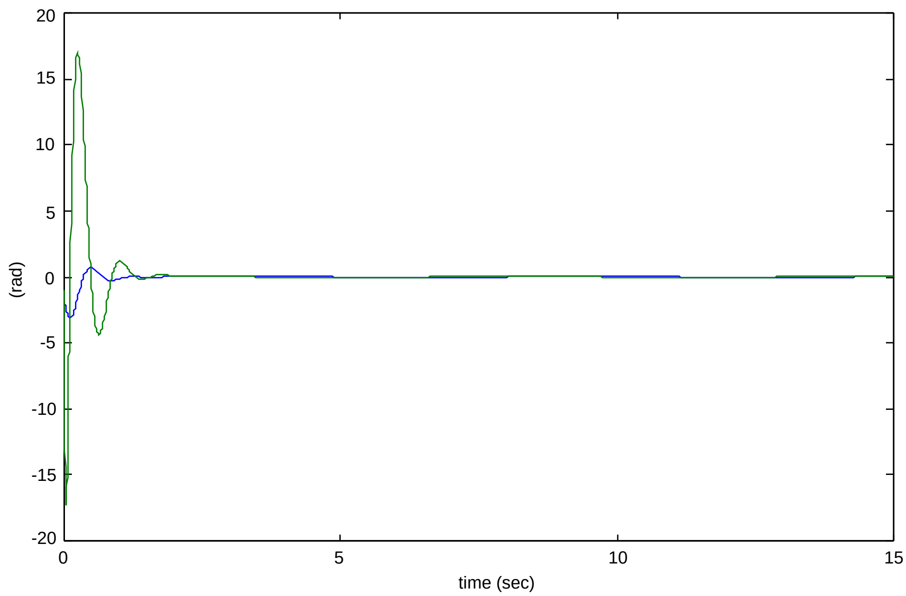

Fig. 15. Trajectories of the tracking errors e_1 (solid line) and e_2 (dash–dotted) with $d \neq 0$ and $\rho = 0.05$.

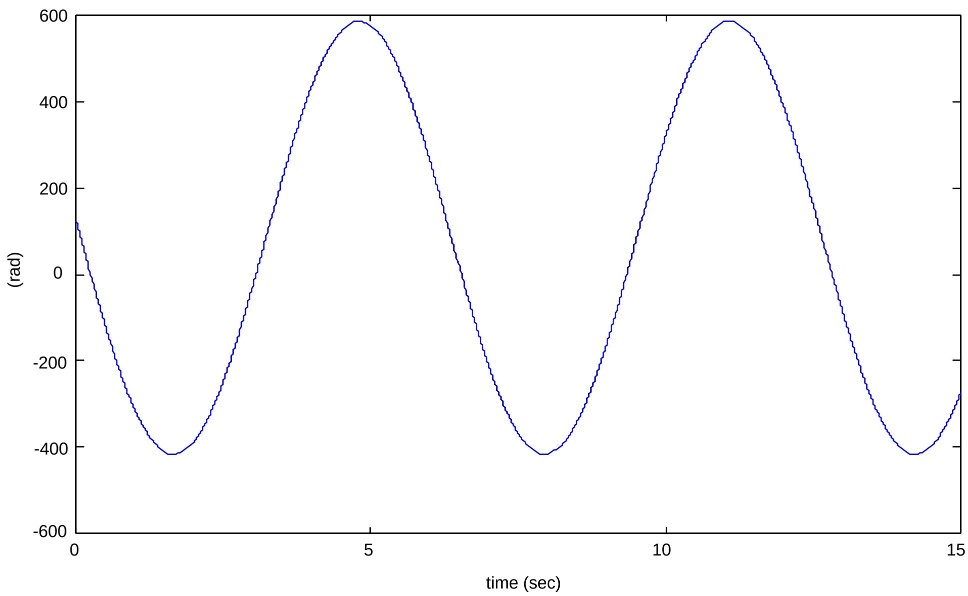

Fig. 16. Control input u with $d \neq 0$ and $\rho = 0.05$.

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