Available online at www.sciencedirect.com

SCIENCE DIRECT®

Mechanics of Materials 36 (2004) 849–864

MECHANICS OF MATERIALS

www.elsevier.com/locate/mechmat

Triaxial testing and stress relaxation of asphalt concrete

J. Murali Krishnan, K.R. Rajagopal *

Department of Mechanical Engineering, Texas A&M University, College Station, TX 77843, USA

Received 25 February 2003

Abstract

This study is concerned with the constitutive modeling of asphalt concrete mixtures. The constitutive model derived is based on a thermodynamic framework that recognizes the fact that materials can exist in multiple stress free states. We derive the constitutive relations by assuming that asphalt concrete is a constrained two constituent mixture of asphalt mastic and aggregate matrix. Assumptions are made concerning the manner in which the body stores and dissipates energy and the constitutive relations for the stress is deduced from such an assumption. The triaxial and stress relaxation tests conducted by Monismith and Secor [Viscoelastic behavior of asphalt concrete pavements. Report, Institute of Transportation and Traffic Engineering, University of California, Berkeley, 1962] are used as benchmarks to delineate the efficacy of the model in predicting the mechanical response characteristics of asphalt concrete over a wide range of temperatures and confining pressures.

Keywords: Asphalt concrete; Triaxial test; Stress relaxation; Multiple natural configurations; Viscoelasticity

1. Introduction

Few materials have been put to the range of diverse uses as asphalt, a material that was used in the Paleolithic era to fix flint stones to wooden handles and which finds over 250 distinct uses currently (Whiteoak, 1990), and thus it is not surprising that a great deal of effort has been expended in trying to describe its response. Maxwell (1970) recognized that asphalt is a viscoelastic material and in fact used it to describe what is meant by a viscoelastic material "*Thus a block of pitch may be so hard that you cannot make a dent in it by striking it with your knuckles; and yet it will in the course of time, flatten itself by its own weight, and glide down hill like a stream of water*". It was also well recognized that asphalt exhibited non-linear response characteristics. Asphalt concrete, a mixture of continuously graded aggregates, filler and asphalt, is far more complex in its response characteristics, and though it has been studied intensely over the last several decades, there is much that needs to be understood with regards to its response. In this paper, we discuss a model for asphalt concrete that is quite robust over a wide range of temperatures. We assess the efficacy of the model by comparing its predictions with experiments concerning triaxial deformations of asphalt concrete specimens.

* Corresponding author. Tel.: +1-979-862-4552/845-1251; fax: +1-979-845-3081.

E-mail addresses: murali_j97@hotmail.com (J. Murali Krishnan), krajagopal@mengr.tamu.edu (K.R. Rajagopal).

0167-6636/$ - see front matter © 2003 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mechmat.2003.08.003

One of the important testing procedures adopted for testing soils and bituminous mixtures is the triaxial test. In this test, the stresses acting on the specimen in the laboratory closely approaches the stress conditions existing in the field. One of the earliest conceptualization of the simultaneous measurement of vertical and lateral pressure is due to Jamieson (1904) in an apparatus that he called a 'manometer'. This was used by him for measuring the lateral grain pressure in deep bins loaded vertically by weights while the bins were being emptied. At about the same time, Goodrich (1904) while investigating the lateral pressure against retaining walls devised a cylinder for measuring the ratio of lateral and vertical pressure. The present form in which the triaxial test is used owes a lot to the ideas of Buisman (1934) who used it for testing sand and clay (see also Division of Tests, 1939 for a brief history of the triaxial tests). This test was used to characterize bituminous pavement materials by Stanton and Hveem (1934).

The design of triaxial test appeals to a very simple notion: "*if a cylinder of material were confined laterally while the material was distorted by a vertical load, a very simple relationship must exist between vertical and horizontal pressures*" (Endersby, 1951). Terminologies such as '*stability*' were employed in the testing protocol of the triaxial test to differentiate between a fluid (the confining pressure equal to the vertical load for a fluid to be in equilibrium) and solid (confining pressure equal to zero for complete, i.e., '100% stability'). The Mohr diagram (Mohr, 1914) was employed by Nijboer (1948) to interpret the results of the triaxial cell when testing bituminous mixtures. Nijboer recognized the multiconstituent nature of asphalt mixtures and specifically the role of bitumen in the deformation characteristics of the material. Using the ideas of the Mohr–Coulomb criterion for the yielding of granular materials, he assumed that the resistance to 'plastic flow' of bituminous mixtures comprised of (a) friction between aggregate particles (ascribed by him to contact friction irrespective of the influence of bitumen), (b) interlocking resistance which is a function of the aggregate angularity and temperature and (c) viscous resistance due to the mixture of bitumen and filler particles (Nijboer, 1948).

Making a simplistic assumption that "*bituminous mixtures are aggregates modified by the presence of bitumen and that the aggregate qualities are major determining factors in service*", the Mohr envelope was used to analyze the results of the triaxial test (Endersby, 1951). Endersby made some interesting observations related to the Mohr envelope. Experiments on bituminous materials carried out in a triaxial cell indicated that the Mohr envelope was not a straight line but curved. This essentially means that, while for a pure granular material, a straight Mohr envelope indicates that the shear resisting properties do not change with progressive deformation, such is not the case for bituminous materials. Endersby conjectured that it may be due to one of the following reasons: friction decreasing with load while cohesion is constant, friction increasing while cohesion decreases, friction being constant while cohesion increases and finally the presence of 'structural resistance' similar to 'arching' in granular materials. Interesting observations were also made by Hveem and Davis (1951) concerning the triaxial compression of asphalt mixtures. For instance, they discounted the idea of using Mohr's envelope (which essentially underlie the use of the Mohr–Coulomb criterion for failure) as the correct one for characterizing the failure of bituminous mixtures. These observations were confirmed by Housel when conducting triaxial compression tests on granular mixtures (Housel, 1951) (see also Burmister, 1951). An extensive laboratory program was conducted by Oppenlander (1957) to test the viability of using the triaxial test in characterizing bituminous mixtures and the validity of the Mohr–Coulomb equation as a failure criterion. Two types of aggregate gradation named as 'One-size' and 'Open-graded' were used in this study. While both these gradations performed well in the field, the use of Mohr–Coulomb specifications predicted the failure of one-size mixtures.

The one-size mixture is porous in nature due to the gradations employed and hence there is a tendency for the material to densify and readjust during the course of the testing. This was confirmed by Oppenlander as follows: "*It logically may be reasoned that the one-size mixture develops increasing shearing strength with increases in the* *confining pressure and with increases in the degree of specimen deformation*''. Various modifications were suggested to the triaxial test in terms of applying variable lateral pressure. Also suggestions related to the influence of height to diameter ratio with regard to the possible failure of the Mohr–Coulomb equation for modeling deformation characteristics of bituminous mixtures were considered. It was also suggested that for the material to follow the Mohr–Coulomb equation, 'shear planes' must be allowed to develop within the samples without restraint (Goetz and Schaub, 1959). Use of the triaxial test in designing the pavement thickness was later investigated by different authors by making the assumption that the Mohr envelope is linear. The entire thrust of most of these studies was in specifying satisfactory 'stability' values based on the 'cohesion' and 'angle of internal friction' of different trial mixtures. McLeod (1950, 1951) investigated the consequences of a variable vertical load that simulates the effect of tire pressure on the pavement, the development of shearing resistance at the interface of the pavement and tire and between the pavement and the base course layers (see also Smith, 1944; Smith, 1951; State Highway Commission, 1947; Barber, 1946).

Within the purview of triaxial testing and Mohr envelope, Endersby (1940) studied void changes in asphalt mixtures in an attempt to separate the respective deformation mechanisms in the aggregate and binder. He assumed that asphalt mixtures can be characterized as 'linear-plastic' bodies and used combinations of the shear box and triaxial tests to quantify the different mechanisms which come into play other than the 'apparent cohesion' and 'apparent friction' in the deformation resistance of asphalt mixtures. The change of the internal structure of the material as it is deformed leading to aggregate interlock, reduction of air voids etc., was not clearly captured by the assumption of the Mohr–Coulomb criterion as is clear from the remarks of Endersby ''*... and it is hard to imagine that a material which is being polished or degraded under progressive deformation should suddenly become rough and coarse again, which would be the explanation of the sudden reconstitution of resistance seen in several cases*''. Investigations concerning the influence of chemical stabilization of asphalt on natural sand were carried out within the context of triaxial tests by Carpenter (1961). Addition of even 2% of bitumen markedly changed the test measurements. However, the specific internal structure changes evident in normal bituminous mixtures such as air voids change, aggregate movement and interlock were absent since uniform sized sand particles were used in this study.

The potential usefulness of the triaxial test not withstanding, constitutive equations to explain the response characteristics of the material are necessary if useful information is to be gleaned from these tests. This point cannot be overemphasized and the inherent difficulty in understanding the stress distribution in a typical triaxial test has perplexed many researchers. One of the earliest to study this problem was Filon (1902) and he considered ''*a cylinder of moderate length, which is compressed between two rough rigid planes in such a way that the terminal cross-sections are constrained to remain plane, but are not allowed to expand, their perimeter being kept fixed*'' (Filon, 1902). However, he was unable to find a solution for this specific boundary value problem. Pickett (1944) attempted to solve this problem by using multiple Fourier technique. D́Appolonia and Newmark used a lattice analogy method to solve the above problem (D́Appolonia and Newmark, 1951) but again their solution did not meet all the boundary conditions. Assuming different boundary conditions, Balla (1960) solved a similar problem for a linearized elastic material subjected to triaxial stress conditions. He introduced additional boundary conditions related to the roughness of the loading plate and assumed that the radial displacement on the periphery varied inversely with the friction factor characterizing the roughness of the loading plate. All these studies considered a linearized elastic material with Poisson's ratio ranging from 1/3 to 1/4 (see Saada and Townsend, 1981 for a discussion of the different approaches concerning triaxial tests of solid circular cylinders). In practice, the hypothesis postulated by Haar and von Kármán are used intuitively in the triaxial testing, i.e., the intermediate principal stress is either equal to the major principal stress or the minor principal stress (Haar and von Kármán, 1909). For instance, in the triaxial cell, while lateral pressures are usually applied by means of some enveloping fluid, the axial thrust is applied through the end plates. The entire literature on the triaxial test assumes that, apart from end effects ascribed to the equipment and specimen geometry, a homogeneous stress distribution exists in the specimen (see Bishop and Henkel, 1964 for more details). This is made possible by invoking the ideas of Haar and von Kármán which were based on the testing of rocks in similar conditions. However, Haythornthwaite (1960) showed that there are an infinity of lateral to axial pressure ratios that are possible which satisfy the Haar-von Kármán hypothesis and unless a typical stress–strain relation is assumed, it is difficult to calculate an 'unique failure load'.

The above discussion was meant to highlight the difficulties associated with testing of materials in a triaxial test and the necessity of having a constitutive model which will at least help one in understanding and interpreting the test results. For a material like asphalt concrete, in which there are changes taking place in the internal structure during every phase of testing, it becomes even more complicated. However, our main interest here is not in proposing any 'failure criterion' as in most studies related to the triaxial test, but rather to use the output of the triaxial test such as displacement vs time to check the validity of the constitutive model which we propose in this work.

While the use of the triaxial test in Geotechnical engineering has tremendously increased in recent years, the same cannot be said in the case of bituminous mixtures. The initial improvements and refinements of the triaxial test were carried out by researchers in the pavement engineering field who used this test for characterizing granular materials used in the base course construction as well as bituminous mixtures for the pavement. Research along this direction seems to have been discontinued due to the complexity of the analysis and the peculiarities one encounters when testing asphalt mixtures. Similar sentiments have been expressed by Fwa et al. (2001), however, their suggestion of a link between Marshall's test, an empirical test used in bituminous mixture design with the triaxial test, is not well-taken (see also Kiryukhin, 2000).

Monismith and his co-workers conducted an elaborate study of triaxial testing of asphalt concrete specifically with an intent to take into account the viscoelastic nature of asphalt mixtures (Secor and Monismith, 1961; Monismith and Secor, 1962; Secor and Monismith, 1964; Monismith et al., 1966). These studies make a significant departure from other studies which were conducted during that time in which asphalt concrete was assumed to behave like a granular material with a failure criterion similar to the Mohr–Coulomb criterion. Secor and Monismith (1961) examined the validity of using linearized elasticity theory and concluded that "*Inasmuch as the theory fails to account completely for the effect of lateral pressure on such specimens, there appears little to be gained from the additional effort required for its use*". In our investigation here, experimental data on asphalt concrete tested at different temperatures and lateral pressures for creep and stress relaxation by Monismith and his co-workers are used to compare with our theoretical predictions.

There have been few careful studies conducted on stress relaxation of asphalt concrete. It is important to understand this aspect to the response of asphalt concrete as it can give one insight into the healing mechanisms of asphalt mixtures. Davis et al. (1963) assumed that asphalt was a linear viscoelastic material and that the addition of mineral aggregates imparts non-linear viscoelastic characteristics to it. However, assumption of time-temperature superposition, use of an arbitrary correction factor to correlate experimental results made in this study calls the model into question (Davis et al., 1963). Most of the present day models for asphalt concrete borrow heavily from the ideas of Schapery (1984) in which a correspondence principle is postulated between linearized elasticity and linearized viscoelasticity, under special conditions. Using such an assumption, Shields et al., discuss stress relaxation tests on asphalt concrete (Shields et al., 1998). As asphalt concrete responds in a non-linear fashion, it does not seem reasonable to appeal to the correspondence principle in general.

At this point of time, it is difficult to quantify from experiments the role of the different constituents especially the asphalt mastic on the relaxation characteristics of asphalt concrete. Also, the mechanics behind the stress relaxation behavior of asphalt concrete can be markedly different depending on whether constant compressive strain or constant tensile strain is applied.

In this paper, we propose a methodology for deriving constitutive equations for asphalt concrete. We make no attempt here to review all the attempts on constitutive modeling of asphalt concrete. Interested readers are referred to Murali Krishnan and Rajagopal (2003) where a detailed discussion of the same can be found. For the purpose of constitutive modeling of asphalt concrete, we use the continuum theory based on the concept of 'multiple natural configurations'. Eckart (1948) was one of the earliest to recognize that materials can possess multiple stress free states. A detailed discussion of the role of 'natural configuration' on modeling different types of dissipative processes can be found in Rajagopal (1995). This framework recognizes that the underlying natural configuration changes as the body dissipates energy. In the case of the classical elastic body, there is a tacit assumption that there is only one stress-free configuration of the body, modulo rigid body motions. As this elastic body is deformed, its underlying stress-free configuration does not change. However, most real materials can exist in a variety of stress free configurations and these configurations are not related to each other by rigid body motions. Such a framework is particularly well suited for a material like asphalt concrete wherein the micro-mechanisms change due to deformation. This approach has been used to describe the material response of a large class of materials under a unifying framework: plasticity due to slip (Rajagopal and Srinivasa, 1998a,b), twinning (Rajagopal, 1995; Rajagopal and Srinivasa, 1995), solid to solid phase transition (Rajagopal and Srinivasa, 1999), inelastic response of multinetwork polymers (Wineman and Rajagopal, 1990; Rajagopal and Wineman, 1992), viscoelastic response of materials (Rajagopal and Srinivasa, 2000; Murali Krishnan and Rajagopal, in press) and crystallization of polymers (Rao and Rajagopal, 2000; Rao and Rajagopal, 2001; Rao and Rajagopal, 2002; Kannan et al., 2002). We first discuss the thermodynamic framework used for modeling asphalt concrete. Assuming asphalt concrete to be a constrained mixture of asphalt mastic and aggregate matrix, constitutive models for asphalt concrete are derived. Experimental studies carried out by Monismith and Secor (1962) are used for determining the efficacy of the model. We find that the theory is capable of describing experiments on triaxial creep and stress relaxation for different confining pressure and temperatures.

2. Preliminaries

Consider a body $\mathscr{B}$ in a configuration $\kappa_R(\mathscr{B})$. We shall, for the ease of notation refer to the configuration as κ_R. Let $\mathbf{X}$ denote a typical position of a material point in κ_R. Let κ_t be the configuration at a time t, then the motion χ_{κ_R} assigns to each particle in configuration κ_R a particle in the configuration κ_t at time t, i.e.,

$$\mathbf{x} = \chi_{\kappa_R}(\mathbf{X}, t). \tag{1}$$

The deformation gradient $\mathbf{F}_{\kappa_R}$ is defined through

$$\mathbf{F}_{\kappa_R} \equiv \frac{\partial \chi_{\kappa_R}}{\partial \mathbf{X}}. \tag{2}$$

The left and right Cauchy–Green stretch tensors $\mathbf{B}_{\kappa_R}$ and $\mathbf{C}_{\kappa_R}$ are defined through

$$\mathbf{B}_{\kappa_R} \equiv \mathbf{F}_{\kappa_R}\mathbf{F}^T_{\kappa_R}, \tag{3}$$

$$\mathbf{C}_{\kappa_R} \equiv \mathbf{F}^T_{\kappa_R}\mathbf{F}_{\kappa_R}. \tag{4}$$

Any acceptable process has to satisfy the appropriate balance laws. For the specific problem at hand, the appropriate balance equations are the conservation of mass, linear and angular momentum and energy. We assume asphalt concrete to be incompressible and that the density of the asphalt mastic and the aggregate matrix are the same. This assumption simplifies the problem at hand while in actual practice density changes do occur in the bulk of the material. These density changes are mainly due to the air voids reduction of asphalt concrete (Smith, 1951).

Now the conservation of mass is given by

$$\dot{\rho} + \rho \operatorname{div} \mathbf{v} = 0, \tag{5}$$

where ρ is the density and $\mathbf{v}$ is the velocity. In the light of the assumption of incompressibility, the conservation of mass reduces to

$$\operatorname{div} \mathbf{v} = 0. \tag{6}$$

The conservation of linear momentum is

$$\rho \left[\frac{\partial \mathbf{v}}{\partial \mathbf{t}} + (\nabla \mathbf{v})\mathbf{v} \right] = \operatorname{div} \mathbf{T} + \rho \mathbf{g}, \tag{7}$$

where $\mathbf{T}$ is the Cauchy stress and $\mathbf{g}$ is the acceleration due to gravity. For an incompressible material, the Cauchy stress tensor $\mathbf{T}$ reduces to

$$\mathbf{T} = -p\mathbf{I} + \mathbf{T}^{\mathrm{E}}, \tag{8}$$

where p is the Lagrange multiplier due to the constraint of incompressibility and $\mathbf{T}^{\mathrm{E}}$ is the constitutively determined extra stress. The balance of angular momentum for a body in the absence of internal couples implies that the stress tensor is symmetric. The conservation of energy takes the form,

$$\rho \dot{\epsilon} + \operatorname{div} \mathbf{q} = \mathbf{T} \cdot \mathbf{L} + \rho r, \tag{9}$$

where ϵ is the specific internal energy, $\mathbf{q}$ is the heat flux vector, $\mathbf{L}$ is the velocity gradient and r is the radiant working. The second law of thermodynamics has been commonly used in Continuum Mechanics in the form of the Clausius–Duhem inequality (Truesdell and Noll, 1992). In this work, however, we interpret the second law in the form adopted by Green and Naghdi (1977) and Rajagopal and Srinivasa (1998a), that is we require that:

$$\rho \dot{\zeta} + \operatorname{div}\left(\frac{\mathbf{q}}{\theta} \right) = \rho \frac{r}{\theta} + \rho \Xi, \quad \Xi \geqslant 0, \tag{10}$$

where ζ is the entropy, θ is the absolute temperature and Ξ is the rate of entropy production. Combining the balance of energy, Eq. (9), and the balance of entropy, Eq. (10), gives the reduced energy-dissipation equation (Green and Naghdi, 1977; Rajagopal and Srinivasa, 1998a).

The reduced energy-dissipation equation is given by

$$\mathbf{T} \cdot \mathbf{L} - \rho \dot{\psi} - \rho \zeta \dot{\theta} - \frac{\mathbf{q} \cdot \operatorname{grad} \theta}{\theta} = \rho \theta \Xi := \xi \geqslant 0, \tag{11}$$

where ψ is the Helmholtz potential and is given by $\psi = \epsilon - \theta \zeta$ and ξ is the rate of dissipation. Also, it is usually assumed that the rate of dissipation can be split into two parts, the first pertaining to heat conduction and the second related to the rate of work converted into thermal energy. Assuming that the rate of dissipation due to heat conduction is given by

$$\xi_c = -\frac{\mathbf{q} \cdot \operatorname{grad} \theta}{\theta} \geqslant 0, \tag{12}$$

we can rewrite Eq. (11) as follows,

$$\mathbf{T} \cdot \mathbf{L} - \rho \dot{\psi} - \rho \zeta \dot{\theta} = \xi_d \geqslant 0. \tag{13}$$

In this work, we use the above reduced energy-dissipation equation to place restrictions on the constitutive equations.

3. Modeling of asphalt concrete

Asphalt concrete consists of aggregates of different sizes and percentage mixed together with asphalt. The specific influence of different sizes and shapes of the aggregates on the overall mechanical behavior of asphalt concrete is still an unresolved issue. Hence, at this point of time, it would suffice to assume that the overall mechanical behavior of asphalt concrete is essentially made up of the mechanical behavior of asphalt mastic (asphalt and filler particles) and aggregate matrix. In the present development, we neglect the influence of air voids and their reduction. Hence the modeling attempt consists in proposing constitutive equations for a constrained mixture of aggregate matrix and asphalt mastic each with different natural configurations.

Taking into account the influence of loading and temperature, the material can have two natural configurations associated with aggregate matrix and the asphalt mastic, respectively, corresponding to the current deformed configuration. While the first configuration corresponds to that of aggregate matrix with a very small relaxation time, the second configuration corresponds to that of the asphalt mastic with relatively large relaxation time. Each constituent is modeled as a rate type model and the derivation of the constitutive

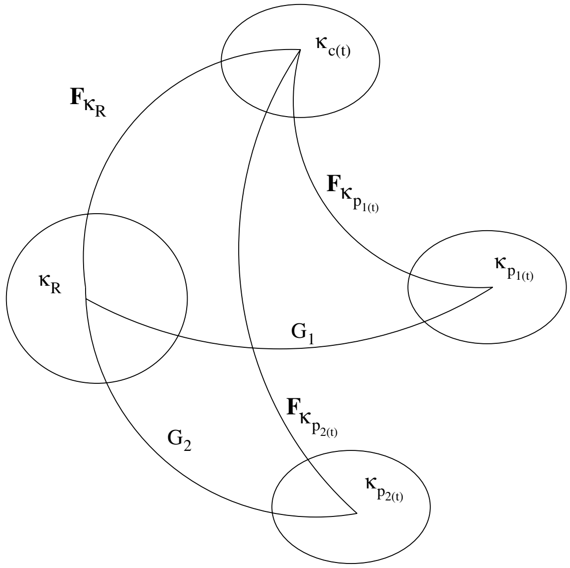

Fig. 1. Natural configurations associated with the asphalt concrete mixture.

equations follows the work of Rajagopal and Srinivasa (2000). Referring to Fig. 1, κ_R is a reference configuration and $\kappa_{c(t)}$ is the configuration currently occupied by the material. $\kappa_{p_1(t)}$ and $\kappa_{p_2(t)}$ refer to the two natural configurations associated with the material, that is the configurations pertaining to the state when the tractions on $\kappa_{c(t)}$ are removed.

The deformation gradient $\mathbf{F}_{\kappa_R}$ denotes the mapping between the tangent space associated with κ_R, at the material point, to the tangent space associated with the same material point in $\kappa_{c(t)}$. Let $\mathbf{F}_{\kappa_{p_1(t)}}$ denote the mapping between the tangent space associated at a material point in the configuration $\kappa_{p_1(t)}$ to the tangent space associated with the material point in the current configuration $\kappa_{c(t)}$ and similarly $\mathbf{F}_{\kappa_{p_2(t)}}$ is the mapping between the tangent space at a material point in the configuration $\kappa_{p_2(t)}$ to the tangent space associated with the same material point at the current configuration $\kappa_{c(t)}$. This kind of framework gives us flexibility in taking into account the influence of different aggregate types on the mechanical behavior of asphalt concrete. Also, different kinds of asphalt mixtures such as stone matrix asphalt, sand asphalt, sheet asphalt or any of the currently used dense mixtures can be modeled by assuming appropriate relaxation mechanisms.

The natural configurations $\kappa_{p_1(t)}$ and $\kappa_{p_2(t)}$, are not fixed as in an elastic solid but evolve as the material is deformed. The evolution of these configurations depends upon the dissipative response of the aggregate matrix and the asphalt mastic. We treat the mixture of aggregate matrix and the asphalt mastic as a constrained mixture, that is we allow co-occupancy of the constituents in an averaged sense as is done in traditional mixture theory (Truesdell, 1957; Bowen, 1975; Atkin and Craine, 1976; Rajagopal and Tao, 1996), but allow the constituents to move together. Let us define $\mathbf{G}_1$ and $\mathbf{G}_2$ to be the appropriate mapping between appropriate tangent spaces at points belonging to κ_R and the natural configurations $\kappa_{p_1(t)}$ and $\kappa_{p_2(t)}$, respectively, i.e.,

$$\mathbf{G}_i \equiv \mathbf{F}_{\kappa_R \to \kappa_{p(t)}} = \mathbf{F}^{-1}_{\kappa_{p_i(t)}} \mathbf{F}_{\kappa_R}, \quad i = 1, 2. \tag{14}$$

The velocity gradients, $\mathbf{L}_{\kappa_{p_i(t)}}$ and the symmetric part of $\mathbf{L}_{\kappa_{p_i(t)}}$, $\mathbf{D}_{\kappa_{p_i(t)}}$ are defined as

$$\mathbf{L}_{\kappa_{p_i(t)}} = \dot{\mathbf{G}}_i \mathbf{G}_i^{-1},$$
$$\mathbf{D}_{\kappa_{p_i(t)}} = \frac{1}{2}(\mathbf{L}_{\kappa_{p_i(t)}} + \mathbf{L}^T_{\kappa_{p_i(t)}}), \quad i = 1, 2. \tag{15}$$

Also it can be easily shown that (Rajagopal and Srinivasa, 2000)

$$\overset{\triangledown}{\mathbf{B}}_{\kappa_{p_i(t)}} \equiv \dot{\mathbf{B}}_{\kappa_{p_i(t)}} - \mathbf{L}\mathbf{B}_{\kappa_{p_i(t)}} - \mathbf{B}_{\kappa_{p_i(t)}}\mathbf{L}^{\mathbf{T}}$$
$$= -2\mathbf{F}_{\kappa_{p_i(t)}} \mathbf{D}_{\kappa_{p_i(t)}} \mathbf{F}^T_{\kappa_{p_i(t)}}, \quad i = 1, 2, \tag{16}$$

where the inverted triangle is the 'upper convected' Oldroyd derivative and the superposed dot signifies the material time derivative. As the material is assumed to be incompressible, we shall assume that the motions associated with these natural configurations are isochoric, i.e.,

$$\mathrm{tr}(\mathbf{D}_{\kappa_{p_i(t)}}) = 0, \quad i = 1, 2. \tag{17}$$

We assume that the internal energy and the entropy of the mixture depend on the temperature and $\mathbf{F}_{\kappa_{p_i(t)}}$, i.e.,

$$\epsilon_i = \epsilon(\theta, \mathbf{F}_{\kappa_{p_i(t)}}), \quad i = 1, 2, \tag{18}$$

$$\zeta_i = \zeta(\theta, \mathbf{F}_{\kappa_{p_i(t)}}), \quad i = 1,2. \tag{19}$$

Here, we assume that the aggregate matrix and the asphalt mastic have the same temperature and hence associate a single temperature θ for the whole asphalt concrete mixture. The forms chosen above can be further simplified by assuming that the asphalt concrete mixture is isotropic and incompressible, and hence the internal energy and entropy depend on $\mathbf{F}_{\kappa_{p_i(t)}}$ through the first two invariants of $\mathbf{C}_{\kappa_{p_i(t)}}$, i.e.,

$$\mathbf{I}_i = \mathrm{tr}(\mathbf{C}_{\kappa_{p_i(t)}}), \quad \mathbf{II}_i = \mathrm{tr}(\mathbf{C}^2_{\kappa_{p_i(t)}}), \quad i = 1,2. \tag{20}$$

Hence the internal energy and the entropy have the form,

$$\epsilon_i = \epsilon_i(\theta, \mathbf{I}_i, \mathbf{II}_i), \quad i = 1,2, \tag{21}$$

$$\zeta_i = \zeta_i(\theta, \mathbf{I}_i, \mathbf{II}_i), \quad i = 1,2, \tag{22}$$

and hence the Helmholtz potential has the following form

$$\psi_i = \psi_i(\theta, \mathbf{I}_i, \mathbf{II}_i), \quad i = 1,2. \tag{23}$$

We also assume the following form for the rate of dissipation in the mixture of aggregate matrix and the asphalt mastic,

$$\xi_i = \xi_i(\theta, \mathbf{B}_{\kappa_{p_i(t)}}, \mathbf{D}_{\kappa_{p_i(t)}}), \quad i = 1,2, \tag{24}$$

where ξ_i is the rate of dissipation. While we expect the asphalt mastic to be dissipative in nature due to the presence of asphalt, the use of a dissipation function for the aggregate matrix is due to the presence of the asphalt as the binder at the contact point between the aggregate particles. In the case of stone matrix asphalt wherein there is a direct stone-to-stone contact with little asphalt, there is a possibility that the total dissipation in such a case may be only due to the filler particles held in the interstices, but for the present case, we rule out that possibility. For extremely rapid loading, the underlying natural configuration does not change and therefore the mixture behaves like an elastic solid and in that case, $\mathbf{D}_{\kappa_{p_i(t)}}$ is the null tensor. For such processes we expect

$$\xi_i(\cdot,\cdot,\mathbf{0}) = \mathbf{0}, \quad i = 1,2. \tag{25}$$

Since we shall assume that the material is isotropic, we can choose without any loss of generality configurations $\kappa_{p_1(t)}$ and $\kappa_{p_2(t)}$ rotated appropriately such that

$$\mathbf{F}_{\kappa_{p_i(t)}} = \mathbf{V}_{\kappa_{p_i(t)}}, \quad i = 1,2, \tag{26}$$

where $\mathbf{V}_{\kappa_{p_i(t)}}$, $i = 1, 2$ are the right stretch tensor in the polar decomposition. Substituting these forms into the reduced energy-dissipation equation, Eq. (13), and using Eqs. (16), (23) and (24) we get

$$\begin{aligned}
&\left[\mathbf{T} - \sum_{i=1}^{2} 2\rho\left(\frac{\partial\psi_i}{\partial I_i}\mathbf{B}_{\kappa_{p_i(t)}} + 2\frac{\partial\psi_i}{\partial II_i}\mathbf{B}^2_{\kappa_{p_i(t)}}\right)\right]\mathbf{D} \\
&\quad + \sum_{i=1}^{2}\left[2\rho\left(\frac{\partial\psi_i}{\partial I_i}\mathbf{B}_{\kappa_{p_i(t)}} + 2\frac{\partial\psi_i}{\partial II_i}\mathbf{B}^2_{\kappa_{p_i(t)}}\right)\mathbf{D}_{\kappa_{p_i(t)}}\right] \\
&\quad - \sum_{i=1}^{2}\left(\frac{\partial\psi_i}{\partial\theta} + \zeta_i\right)\dot{\theta} \\
&\quad = \sum_{i=1}^{2}\widehat{\xi}_i\left[\theta, \mathbf{B}_{\kappa_{p_i(t)}}, \mathbf{D}_{\kappa_{p_i(t)}}\right] \geqslant 0.
\end{aligned} \tag{27}$$

Since the second and the third term on the left-hand side and the term on the right-hand side of Eq. (27) are independent of $\mathbf{D}$ and since only isochoric motions are possible, it is reasonable to assume the following form for the total stress,

$$\mathbf{T} = -p\mathbf{1} + \sum_{i=1}^{2} 2\rho\left(\frac{\partial\psi_i}{\partial I_i}\mathbf{B}_{\kappa_{p_i(t)}} + 2\frac{\partial\psi_i}{\partial II_i}\mathbf{B}^2_{\kappa_{p_i(t)}}\right), \tag{28}$$

and the Helmholtz potential and the entropy are related through

$$\frac{\partial\psi_i}{\partial\theta} = -\zeta_i, \quad i = 1,2. \tag{29}$$

This assumption is sufficient to ensure that for all motions for which natural configurations do not change, the material responds elastically. Also Eq. (29) is equivalent to the following relationship between internal energy and entropy,

$$\frac{\partial\epsilon_i}{\partial\theta} = \theta\frac{\partial\zeta_i}{\partial\theta}, \quad i = 1,2. \tag{30}$$

Substituting Eqs. (28) and (29) into Eq. (27), we get

$$\mathbf{T}_i \cdot \mathbf{D}_{\kappa_{p_i(t)}} = \xi_i, \quad i = 1,2. \tag{31}$$

The above equation places restrictions on the tensors $\mathbf{D}_{\kappa_{p_i(t)}}$ that are achievable. To elaborate, the actual values of $\mathbf{D}_{\kappa_{p_i(t)}}$ are the ones satisfying the constraints given by Eqs. (31) and (17) and also corresponding to a maximum of the rate of dissipation (Rajagopal and Srinivasa, 2000). This is enforced by using the method of Lagrange multipliers by extremizing Eq. (24) subject to the constraints given by Eqs. (31) and (17) and we obtain

$$\mathbf{T} - \lambda_{1_i} \frac{\partial \xi_i}{\partial \mathbf{D}_{\kappa_{p_i(t)}}} - \lambda_{2_i} \mathbf{1} = 0, \quad i = 1, 2, \tag{32}$$

where $\lambda_{1_i} = (1 + \hat{\lambda}_{1_i})/\hat{\lambda}_{1_i}$ and $\lambda_{2_i} = \hat{\lambda}_{2_i}/\hat{\lambda}_{1_i}$. Here $\hat{\lambda}_{1_i}$ and $\hat{\lambda}_{2_i}$ are the Lagrange multipliers whose values are obtained by the satisfaction of the constraints (31) and (17). Using Eqs. (32) and (28), we can obtain an equation for $\mathbf{D}_{\kappa_{p_i(t)}}$ which contains information about the evolution of the natural configuration.

For the current problem, we make the following constitutive assumptions for the internal energy and entropy in the temperature range of interest. For the internal energy we assume that, it is a linear function of temperature and for entropy we assume a form that is similar to that for a neo-Hookean material. With the above assumptions, we get the following form for internal energy

$$\epsilon_i = C\theta + A_i, \quad i = 1, 2, \tag{33}$$

and entropy,

$$\zeta_i = C \ln(\theta) + B_i - \mu_i(\mathbf{B}_{\kappa_{p_i(t)}})(I_i - 3), \quad i = 1, 2, \tag{34}$$

where A, B are constants, C is the specific heat of the asphalt concrete and μ is the material function related to the shear modulus. We assume the specific heat of the aggregate matrix and the asphalt mastic to be the same. The above forms chosen for the internal energy and entropy satisfy Eq. (29). Substituting Eqs. (33) and (34) into Eq. (28), we get the following equation for the stress,

$$\mathbf{T} = -p\mathbf{I} + \mu_1(\mathbf{B}_{\kappa_{p_1(t)}})\mathbf{B}_{\kappa_{p_1(t)}} + \mu_2(\mathbf{B}_{\kappa_{p_2(t)}})\mathbf{B}_{\kappa_{p_2(t)}}. \tag{35}$$

The form for the stress given by Eq. (35) is a generalization of that for a neo-Hookean material. The rate of dissipation is assumed to have the following form,

$$\xi_i = (\eta_i \mathbf{D}_{\kappa_{p_i(t)}} \cdot \mathbf{B}_{\kappa_{p_i(t)}} \mathbf{D}_{\kappa_{p_i(t)}})^q, \quad i = 1, 2. \tag{36}$$

Here q can be referred to as a power law exponent and η_i is the viscosity and can depend in general on both the temperature and the deformation through the tensor $\mathbf{B}_{\kappa_{p_i(t)}}$, i.e.,

$$\eta_i = \eta_i(\theta, \mathbf{B}_{\kappa_{p_i(t)}}). \tag{37}$$

Substituting Eq. (36) into Eq. (32) and eliminating λ_{1_i} by using Eqs. (17) and (31), we obtain,

$$\mathbf{T}_i = 2\eta_i \mathbf{B}_{\kappa_{p_i(t)}} \mathbf{D}_{\kappa_{p_i(t)}} + \lambda_{2_i} \mathbf{1}, \quad i = 1, 2. \tag{38}$$

Using Eqs. (35), (38), (26), (16) and (17) we get

$$-\frac{1}{2} \overset{\triangledown}{\mathbf{B}}_{\kappa_{p_i(t)}} = \left(\frac{\mu_i(\mathbf{B}_{\kappa_{p_i(t)}})}{\eta_i} \right)^{\left[\frac{-1}{1-2q}\right]} \times \left(\mathrm{tr}\mathbf{B}_{\kappa_{p_i(t)}} - 3\Phi \right)^{\left[\frac{q-1}{1-2q}\right]} (\mathbf{B}_{\kappa_{p_i(t)}} - \Phi \mathbf{1}), \quad i = 1, 2, \tag{39}$$

where Φ is given by $3/\mathrm{tr}(\mathbf{B}^{-1}_{\kappa_{p_i(t)}})$.

The specific form for the viscosity function that we use in this work is given by

$$\eta_1 = \bar{\eta}_1(N(\mathrm{tr}(\mathbf{B}_{\kappa_{p_i(t)}}) - 3)^m + 1), \tag{40}$$

and

$$\eta_2 = \bar{\eta}_2. \tag{41}$$

We pick the following form for the shear modulus function (Knowles, 1977)

$$\mu_i = \bar{\mu}_i \left(1 + \frac{b_i}{n_i} (\mathrm{tr}(\mathbf{B}_{\kappa_{p_i(t)}}) - 3) \right)^{(n_i - 1)}. \tag{42}$$

Setting $n_i = 1$ in Eq. (42) and $q = 1$ in Eq. (39), we get a generalization of the Burger's model that has been used within a one-dimensional setting to model asphalt concrete. This completes the development of the model.

4. Application

4.1. Kinematics related to constant traction loading

We study the deformation due to the application of a constant traction. We assume that the deformation is homogeneous and seek a deformation of the following form, in cylindrical polar coordinates:

$$r = \frac{1}{\sqrt{\Lambda(t)}}R, \quad \theta = \frac{1}{\sqrt{\Lambda(t)}}\Theta, \quad z = \Lambda(t)Z, \tag{43}$$

where R, Θ and Z are the coordinates in the undeformed configuration; r, θ and z are the coordinates in the deformed state, and $\Lambda(t)$ is the time dependent stretch in the direction of the applied force. The deformation gradient for this motion is given by

$$\mathbf{F}_{\kappa_R} = \mathrm{diag}\left(\frac{1}{\sqrt{\Lambda}}, \frac{1}{\sqrt{\Lambda}}, \Lambda\right), \tag{44}$$

and the velocity gradient for this motion is given by

$$\mathbf{L} = \mathrm{diag}\left(-\frac{1}{2}\frac{\dot{\Lambda}}{\Lambda}, -\frac{1}{2}\frac{\dot{\Lambda}}{\Lambda}, \frac{\dot{\Lambda}}{\Lambda}\right). \tag{45}$$

The components of $\mathbf{B}_{\kappa_{p_i(t)}}$ are assumed to be diag$(\frac{1}{B(t)}, \frac{1}{B(t)}, B^2(t))$. This assumption is consistent with the stipulation that the stress-free state for the material is achieved via a motion of the form given by Eq. (43). The constitutive equation can now be written as

$$T_{rr} = -p + \mu_1 B_{1_{rr}} + \mu_2 B_{2_{rr}}, \tag{46}$$

$$T_{\theta\theta} = -p + \mu_1 B_{1_{\theta\theta}} + \mu_2 B_{2_{\theta\theta}}, \tag{47}$$

and

$$T_{zz} = -p + \mu_1 B_{1_{zz}} + \mu_2 B_{2_{zz}}. \tag{48}$$

For triaxial loading, with the lateral pressure given by k, we can rewrite the above Eqs. (46) and (47) as follows,

$$k = -p + \mu_1 B_{1_{rr}} + \mu_2 B_{2_{rr}}, \tag{49}$$

$$k = -p + \mu_1 B_{1_{\theta\theta}} + \mu_2 B_{2_{\theta\theta}}. \tag{50}$$

Eliminating the term p by substituting Eq. (49) to Eq. (48), we get

$$T_{zz} = k + \mu_1(B_{1_{zz}} - B_{1_{rr}}) + \mu_2(B_{2_{zz}} - B_{2_{rr}}). \tag{51}$$

The evolution equation for $\mathbf{B}_{\kappa_{p_i(t)}}$ (Eq. (39)) can be written in the following form along the z-direction as

$$\frac{1}{2}\left[\frac{\partial B_{i_{zz}}}{\partial t} + v_{i_{zz}}\frac{\partial B_{i_{zz}}}{\partial z} - 2L_{i_{zz}}B_{i_{zz}}\right] = \left(\frac{\bar{\mu}_i}{\bar{\eta}_i}\right)^{\left[\frac{-1}{1-2q}\right]}\left\{(B_{i_{zz}} + B_{i_{rr}} + B_{i_{\theta\theta}}) - \frac{9B_{i_{zz}}B_{i_{rr}}}{2B_{i_{zz}} + B_{i_{rr}}}\right\}^{\left[\frac{q-1}{1-2q}\right]}\left[\frac{2B_{i_{zz}}(B_{i_{rr}} - B_{i_{zz}})}{2B_{i_{zz}} + B_{i_{rr}}}\right], \quad i = 1, 2, \tag{52}$$

and along the r direction as

$$\frac{1}{2}\left[\frac{\partial B_{i_{rr}}}{\partial t} + v_{i_{rr}}\frac{\partial B_{i_{rr}}}{\partial r} - 2L_{i_{rr}}B_{i_{rr}}\right] = \left(\frac{\bar{\mu}_i}{\bar{\eta}_i}\right)^{\left[\frac{-1}{1-2q}\right]}\left\{(B_{i_{zz}} + B_{i_{rr}} + B_{i_{\theta\theta}}) - \frac{9B_{i_{zz}}B_{i_{rr}}}{2B_{i_{zz}} + B_{i_{rr}}}\right\}^{\left[\frac{q-1}{1-2q}\right]}\left[\frac{B_{i_{rr}}(B_{i_{rr}} - B_{i_{zz}})}{2B_{i_{zz}} + B_{i_{rr}}}\right], \quad i = 1, 2. \tag{53}$$

Next, we make the following choice for material parameters $\bar{\mu}_i$ and $\bar{\eta}_i$ that reflects the fact that viscosities and elastic moduli are dependent on the deformation

$$\bar{\eta}_i = \eta_i(N(\mathrm{tr}(\mathbf{B}_{\kappa_{p_i(t)}}) - 3)^m + 1), \tag{54}$$

and

$$\bar{\mu}_i = \mu_i\left(1 + \frac{1}{n_i}(\mathrm{tr}(\mathbf{B}_{\kappa_{p_i(t)}}) - 3)\right)^{(n_i-1)}. \tag{55}$$

The initial conditions are given by

$$B_{i_{zz}} = \Lambda(0)^2, \tag{56}$$

$$B_{i_{rr}} = B_{i_{\theta\theta}} = \frac{1}{\Lambda(0)}. \tag{57}$$

Here we arrive at the appropriate initial conditions by recognizing that a sudden application of the force elicits an instantaneous elastic response from the material with the value of stretch at time $t = 0$ being $\Lambda(0)$. Ignoring the inertial terms, Eq. (51) in conjunction with Eqs. (52) and (53) are solved numerically with the initial conditions (Eqs. (56) and (57)).

In the case of stress relaxation, the shear strain is constant and it corresponds to the initially applied deformation, and thus the velocity gradient **L** vanishes identically for all $t > 0$. Also, due to the instantaneous elasticity of the material, during the instantaneous elastic response, the stress is given by Eq. (35) with κ_p being the stress free natural configuration. At subsequent times, the stress is calculated by solving the differential equation (39) together with Eq. (16) with $\mathbf{L} = 0$.

4.2. Numerical results and comparison with experimental data

Monismith and Secor (1962) conducted an extensive laboratory testing program on asphalt concrete. Cylindrical specimens of diameter 2.8 in and height 6.5 in were fabricated. Creep and stress relaxation experiments were conducted at three different temperatures, 40, 77 and 140°F with lateral pressures of 0, 43.8 and 250 psi. More details on the specimen fabrication and testing protocol employed can be found in Monismith and Secor (1962). Figs. 2–5 show a comparison of the experimental results with the predictions of the theory for the creep response during compression and Figs. 6–11 provide the same comparison for the stress response during stress relaxation. The predictions are quite satisfactory considering the fact that we are able to account for response for different temperatures and different lateral pressures.

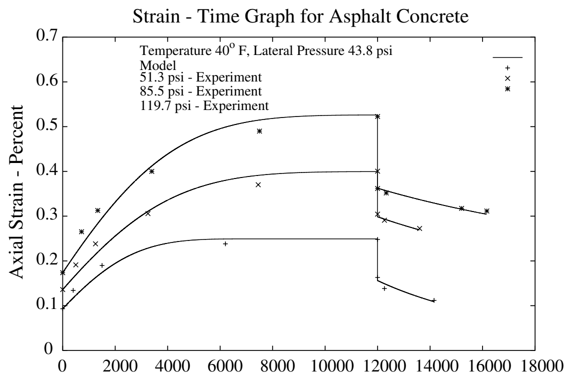

Fig. 2. Triaxial creep test at a temperature of 40°F and lateral pressure of 43.8 psi. Comparison of the predictions of the model with the experimental results of Monismith and Secor (1962). Material parameters used are: $\mu_0 = 185.1$ MPa, $\eta = 30\,000$ MPa s, $n = 3$, $m = -0.85$, $N = 2$ and $q = 1.05$.

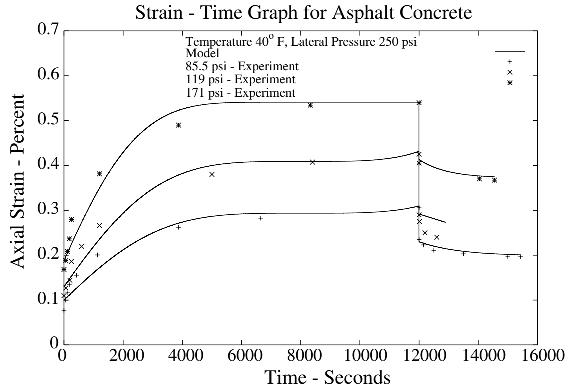

Fig. 3. Triaxial creep test at a temperature of 40°F and lateral pressure of 250 psi. Comparison of the predictions of the model with the experimental results of Monismith and Secor (1962). Material parameters used are: $\mu_0 = 185.1$ MPa, $\eta = 30\,000$ MPa s, $n = 3$, $m = -0.85$, $N = 2$ and $q = 1.04$.

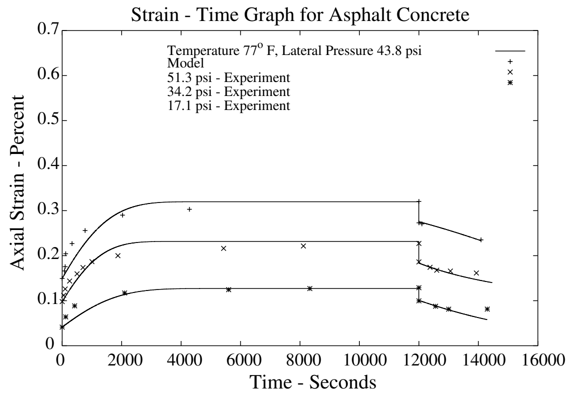

Fig. 4. Triaxial creep test at a temperature of 77°F and lateral pressure of 43.8 psi. Comparison of the predictions of the model with the experimental results of Monismith and Secor (1962). Material parameters used are: $\mu_0 = 185.1$ MPa, $\eta = 26\,000$ MPa s, $n = 2.5$, $m = -0.80$, $N = 1.9$ and $q = 1.07$.

5. Conclusion

We briefly recapitulate the framework we have used and its importance in modeling bituminous mixtures. The model developed here reflects the

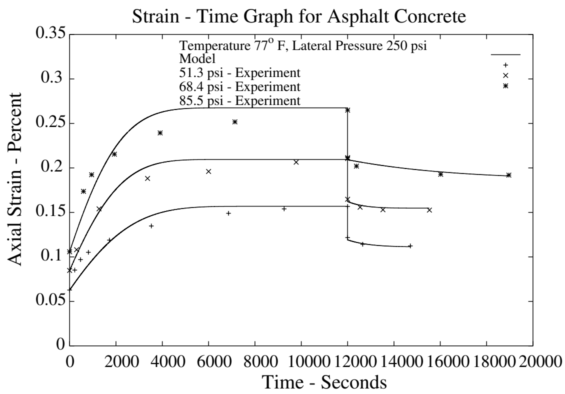

Fig. 5. Triaxial creep test at a temperature of 77°F and lateral pressure of 250 psi. Comparison of the predictions of the model with the experimental results of Monismith and Secor (1962). Material parameters used are: $\mu_0 = 185.1$ MPa, $\eta = 26\,000$ MPa s, $n = 3$, $m = -0.80$, $N = 2$ and $q = 1.07$.

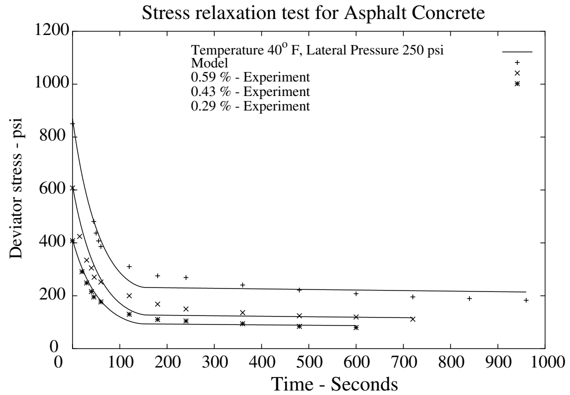

Fig. 7. Stress relaxation test at a temperature of 40°F and lateral pressure of 250 psi. Comparison of the predictions of the model with the experimental results of Monismith and Secor (1962). Material parameters used are: $\mu_0 = 333$ MPa, $\eta = 28\,000$ MPa s, $n = 1.8$, $m = -1.28$, $N = 2$ and $q = 1.53$.

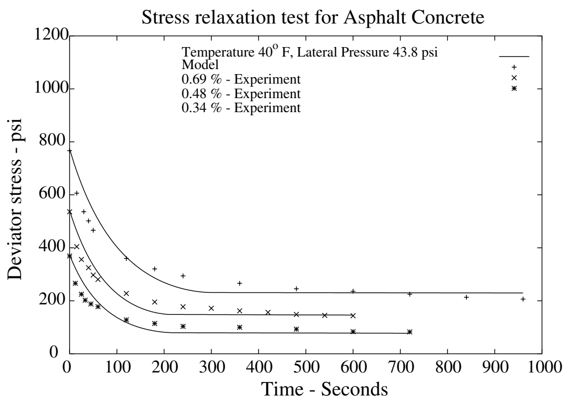

Fig. 6. Stress relaxation test at a temperature of 40°F and lateral pressure of 43.8 psi. Comparison of the predictions of the model with the experimental results of Monismith and Secor (1962). Material parameters used are: $\mu_0 = 255$ MPa, $\eta = 25\,000$ MPa s, $n = 2.2$, $m = -1.35$, $N = 2$ and $q = 1.54$.

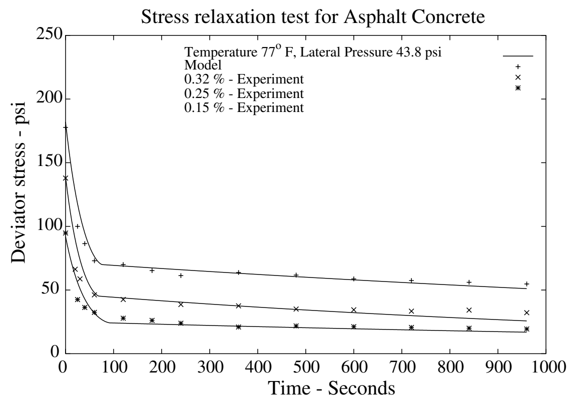

Fig. 8. Stress relaxation test at a temperature of 77°F and lateral pressure of 43.8 psi. Comparison of the predictions of the model with the experimental results of Monismith and Secor (1962). Material parameters used are: $\mu_0 = 130$ MPa, $\eta = 22\,000$ MPa s, $n = 1.25$, $m = -1.275$, $N = 2$ and $q = 1.58$.

fact that the response of a body is defined through a set of response functions from various natural configurations in which the body can exist. As the material is subjected to deformation, the underlying natural configurations and their response functions change. We use a thermodynamic criterion namely the maximization of the rate of dissipation (entropy production in general) to find the evolution of the natural configurations. Making choices for the Helmholtz potential and the rate of dissipation, we derive the constitutive relations for the Cauchy stress. The different micro-structural changes which take place when a material like asphalt concrete is deformed are many and till today, there has not been a framework which can deal with them in the appropriate manner for a

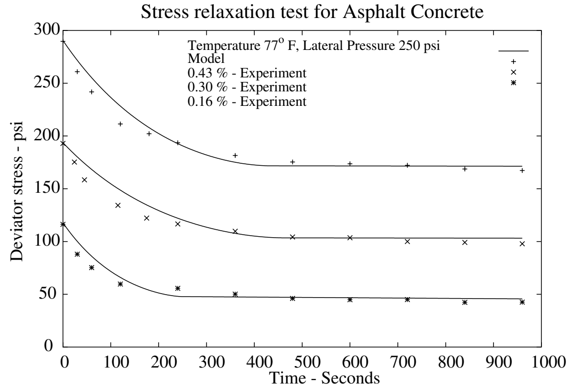

Fig. 9. Stress relaxation test at a temperature of 77°F and lateral pressure of 250 psi. Comparison of the predictions of the model with the experimental results of Monismith and Secor (1962). Material parameters used are: $\mu_0 = 155$ MPa, $\eta = 22000$ MPa s, $n = 2.15$, $m = -1.22$, $N = 2$ and $q = 1.488$.

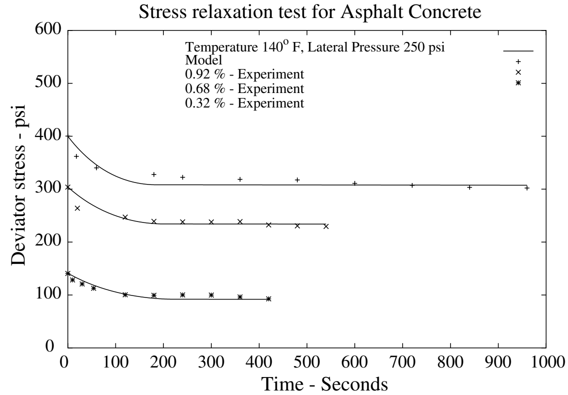

Fig. 11. Stress relaxation test at a temperature of 140°F and lateral pressure of 250 psi. Comparison of the predictions of the model with the experimental results of Monismith and Secor (1962). Material parameters used are: $\mu_0 = 100$ MPa, $\eta = 12000$ MPa s, $n = 4$, $m = -1.4$, $N = 2$ and $q = 1.565$.

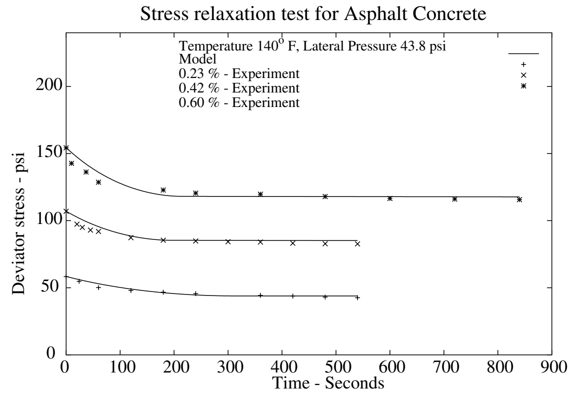

Fig. 10. Stress relaxation test at a temperature of 140°F and lateral pressure of 43.8 psi. Comparison of the predictions of the model with the experimental results of Monismith and Secor (1962). Material parameters used are: $\mu_0 = 59.2$ MPa, $\eta = 12000$ MPa s, $n = 3.5$, $m = -1.36$, $N = 2$ and $q = 1.54$.

wide range of temperatures and deformations. The framework we have used here comes in handy for handling many of the complex issues related to asphalt concrete's internal structural change such as air voids change, aggregate movement, aging of asphalt mastic to name a few. While we have not specifically looked into all these issues here, such internal structural parameters can be incorporated into the theory.

An important issue, both from the theoretical perspective and from the point of view of an asphalt pavement technologist, concerns modeling the asphalt concrete pavements that are are compacted during laying. It is not easy to model this problem as there are many difficulties related to modeling a complex material like asphalt concrete especially when it changes from a material exhibiting viscoelastic fluid-like behavior to one having viscoelastic solid-like behavior. Any small insight into the mechanics of this process can help immensely in constructing durable pavements as many asphalt pavement technologists believe that cracks introduced during the compaction process considerably affect the durability of the pavement. Needless to say, the entire compaction process is more a craft than science and depends heavily on the skills of the pavement construction crew. Recent studies have pointed out that most of the compacting equipment and the field procedures developed and used now pertain mostly to compacting granular layers, a mixture of granular materials, water and air voids, as compared to asphalt concrete, a mixture of aggregate particles, asphalt and air voids (see Murali Krishnan and Rajagopal, 2003 for more details). Clearly the role of asphalt in the compaction process of asphalt concrete is not equivalent to that of the role of water in the compaction process of granular layers. The thermodynamics involved in modeling this process can be quite complicated but within the framework proposed here, it is possible to model the main features of such processes.

Acknowledgements

We thank the National Science Foundation for its support of this work.

References

Atkin, R.J., Craine, R.E., 1976. Continuum theories of mixtures: basic theory and historical development. Quarterly Journal of Mechanics and Applied Mathematics XXIX, 209–244.

Balla, A., 1960. Stress conditions in triaxial compression. Journal of the Soil Mechanics and Foundations Division SM6 (December), 57–84.

Barber, E.S., 1946. Application of triaxial compression test results to the calculation of flexible pavement thickness. Proceedings of the Highway Research Board 26, 26–39.

Bishop, A.W., Henkel, D.J., 1964. The Measurement of Soil Properties in the Triaxial Test, second ed. Edward Arnold (Publishers) Ltd., London, Reprint of the Second Edition.

Bowen, R.M., 1975. Theory of mixtures. In: Eringen, A.C. (Ed.), Continuum Physics, vol. III. Academic Press, New York.

Buisman, A.S., 1934. Proefondervindelijke bepaling van de grens van inwendig evenwicht van een grondmassa. De Ingenieur 26 (June), B83–B88.

Burmister, D.M., 1951. The importance of natural controlling conditions upon triaxial compression test conditions. In: Triaxial Testing of Soils and Bituminous Mixtures, ASTM Special Technical Publication 106. American Society for Testing and Materials, Philadelphia, pp. 248–266.

Carpenter, J., 1961. Stress deformation characteristics of sand-bituminous mixtures. In: American Society for Testing and Materials, vol. 294. American Society for Testing and Materials, Philadelphia, pp. 3–18.

ĎAppolonia, E., Newmark, N.M., 1951. A method for the solution of the restrained cylinder under compression. In: Proceedings of the First US National Congress on Applied Mechanics. American Society of Mechanical Engineers, New York, pp. 217–226.

Davis, E.F., Krokosky, E.M., Tons, E., 1963. Stress relaxation of bituminous concrete in tension. Report R 63-40, School of Engineering, Massachusetts Institute of Technology, Cambridge.

Division of Tests, United States Public Roads Administration. Public Roads, 1939. Essential features of Triaxial Shear tests, 20, 133–144, 154.

Eckart, C., 1948. The thermodynamics of irreversible processes: Iv, the theory of elasticity and anelasticity. Physical Review 73, 373–382.

Endersby, V.A., 1940. The mechanics of granular and granular-plastic materials with special reference to bituminous road materials and subsoils. Proceedings of the 43rd Annual Meeting of American Society for Testing and Materials 40, 1154–1173.

Endersby, V.A., 1951. The history and theory of triaxial testing, and the preparation of realistic test speciments—a report of the triaxial institute. In: Triaxial Testing of Soils and Bituminous mixtures. ASTM Special Technical Publication, vol. 106. American Society for Testing and Materials, Philadelphia, pp. 5–20.

Filon, L.N.G., 1902. On the elastic equilibrium of circular cylinders under certain practical systems of load. Philosophical Transactions of the Royal Society of London, Series A, Containing papers of a Mathematical or Physical Character 198, 147–233.

Fwa, T.F., Tan, S.A., Zhu, L.Y., 2001. Reexamining C-ϕ concept for asphalt paving mix design. Journal of Transportation Engineering 127, 67–73.

Goetz, W.H., Schaub, J.H., 1959. Triaxial testing of bituminous mixtures. Technical Report 6, Purdue University, Lafayette, Indiana.

Goodrich, E.P., 1904. Lateral earth pressure and related phenomena. Transactions of the American Society of Civil Engineers LIII (December), 272–321.

Green, A.E., Naghdi, P.M., 1977. On thermodynamics and the nature of the second law. Proceedings of the Royal Society of London A357, 253–270.

Haar, M.A., von Kármán, T., 1909. Zur Theorie der spannungszustände in plastischen und sandartigen Medien. Nachrichten der K. Gesellschaft der Wissenschaften zu Göttingen Mathematisch—physikalische Klasse K1, 204–218.

Haythornthwaite, R.M., 1960. Mechanics of the triaxial test for soils. Journal of the Soil Mechanics and Foundations Division SM5 (October), 35–62.

Housel, W.S., 1951. Interpretation of triaxial compression tests on granular mixtures. In: Triaxial Testing of Soils and Bituminous Mixtures, ASTM Special Technical Publication 106. American Society for Testing and Materials, Philadelphia, pp. 267–276.

Hveem, F.N., Davis, H.E., 1951. Some concepts concerning triaxial compression testing of asphaltic paving mixtures and subgrade materials. In: Triaxial Testing of Soils and Bituminous Mixtures, ASTM Special Technical Publication 106. American Society for Testing and Materials, Philadelphia, pp. 25–45.

Jamieson, J.A., 1904. Grain pressures in deep bins. Engineering News 51, 236–243.

Kannan, K., Rao, I.J., Rajagopal, K.R., 2002. A thermomechanical framework for the glass transition phenomenon in

certain polymers and its application to fiber spinning. Journal of Rheology 46, 977–999.
Kiryukhin, G.N., 2000. Designing of asphalt pavements on the ultimately permissible irreversible deformation. In: 24th International Baltic Road Conference. Risa, Latvia, pp. 1–8.
Knowles, J.K., 1977. The finite anti-plane shear field near the tip of a crack for a class of incompressible elastic solids. International Journal of Fracture 13, 611–639.
Maxwell, J.C., 1970. Theory of Heat. Greenwood Press, Westport, Connecticut, third ed., Reprint of the 1872 edition.
McLeod, N.W., 1950. A rational method of design for bituminous paving mixtures. Proceedings of the Association of Asphalt Paving Technologists 19, 82–224.
McLeod, N.W., 1951. Application of triaxial testing to the design of bituminous pavements. In: Triaxial Testing of Soils and Bituminous mixtures, ASTM Special Technical Publication 106. American Society for Testing and Materials, Philadelphia, pp. 79–111.
Mohr, O., 1914. Obhandlungen Aus dem Geliete der Technischen Mechanik. Ernst, Berlin.
Monismith, C.L., Alexander, R.L., Secor, K.E., 1966. Rheologic behavior of asphalt concrete. Proceedings of the Association of the Asphalt Paving Technologists 35, 400–450.
Monismith, C.L., Secor, K.E., 1962. Viscoelastic behavior of asphalt concrete pavements. Report, Institute of Transportation and Traffic Engineering, University of California, Berkeley.
Murali Krishnan, J., Rajagopal, K.R., 2003. Review of the uses and modeling of bitumen from ancient to modern times. Applied Mechanics Review 56, 149–214.
Murali Krishnan, J., Rajagopal, K.R., in press. A thermodynamic framework for constitutive modeling of asphalt concrete—theory and applications. ASCE Journal of Materials in Civil Engineering.
Nijboer, L.W., 1948. Plasticity as a factor in the design of dense bituminous road carpets. Elsevier, Amsterdam.
Oppenlander, J.C., 1957. Triaxial testing of bituminous mixtures at high confining pressures. Technical Report 14, Purdue University, Lafayette, Indiana, June.
Pickett, G., 1944. Application of the fourier method to the solution of certain boundary problems in the theory of elasticity. Journal of Applied Mechanics 11, A176–A182.
Rajagopal, K.R., 1995. Multiple configurations in continuum mechanics. Report Volume 6, Institute for Computational and Applied Mechanics, University of Pittsburgh, Pittsburgh, PA.
Rajagopal, K.R., Srinivasa, A.R., 1995. On the inelastic behavior of solids—Part I: Twinning. International Journal of Plasticity 6, 653–678.
Rajagopal, K.R., Srinivasa, A.R., 1998a. Mechanics of the inelastic behavior of materials—Part II: Inelastic response. International Journal of Plasticity 14, 969–995.
Rajagopal, K.R., Srinivasa, A.R., 1998b. Mechanics of the inelastic behavior of materials—Part I: Theoretical underpinnings. International Journal of Plasticity 14, 945–967.
Rajagopal, K.R., Srinivasa, A.R., 1999. On the thermodynamics of shape memory wires. ZAMP 50, 459–496.
Rajagopal, K.R., Srinivasa, A.R., 2000. A thermodynamic framework for rate type fluid models. Journal of Non-Newtonian Fluid Mechanics 88, 207–227.
Rajagopal, K.R., Tao, L., 1996. Mechanics of Mixtures. World Scientific, Singapore.
Rajagopal, K.R., Wineman, A.S., 1992. A constitutive equation for nonlinear solids which undergo deformation induced micro-structural changes. International Journal of Plasticity 8, 385–395.
Rao, I.J., Rajagopal, K.R., 2000. Phenomenological modeling of polymer crystallization using the notion of multiple natural configurations. Interfaces and Free Boundaries 2, 73–94.
Rao, I.J., Rajagopal, K.R., 2001. A study of strain-induced crystallization of polymers. International Journal of Solids and Structures 38, 1149–1167.
Rao, I.J., Rajagopal, K.R., 2002. A thermodynamic framework for the study of crystallization in polymers. ZAMP 53, 365–406.
Saada, A.S., Townsend, F.C., 1981. State of the art: Laboratory strength testing of soils. In: Yong, R.N., Townsend, F.C. (Eds.), Laboratory Shear Strength of Soil, ASTM Special Technical Publication 740. American Society for Testing and Materials, Philadelphia, pp. 7–77.
Schapery, R.A., 1984. Correspondence principles and a generalized *j* integral for large deformation and fracture analysis of viscoelastic media. International Journal of Fracture 25, 195–223.
Secor, K.E., Monismith, C.L., 1961. Analysis of triaxial test data on asphalt concrete using viscoelastic principles. Proceedings of the 40th Annual Meeting of the Highway Research Board, pp. 295–314.
Secor, K.E., Monismith, C.L., 1964. Analysis and interrelation of stress–strain–time data for asphalt concrete. Transactions of the Society of Rheology 8, 19–32.
Shields, D.H., Zeng, M., Kwok, R., 1998. Nonlinear viscoelastic behavior of asphalt concrete in stress relaxation. Proceedings of the Association of Asphalt Paving Technologists 67, 358–400.
Smith, V.R., 1944. Flexible pavement foundation design. Journal of Asphalt Technology 3 (March–April), 1–25.
Smith, V.R, 1951. Application of the triaxial test to bituminous mixtures california research corporation method. In: Triaxial Testing of Soils and Bituminous Mixtures, ASTM Special Technical Publication 106. American Society for Testing and Materials, Philadelphia, pp. 55–72.
Stanton, T.E., Hveem, F.N., 1934. Role of laboratory in the preliminary investigation and control of materials for low cost bituminous pavements. In: Crum, Roy W., (Ed.), Proceedings of the Highway Research Board, Part II,

14:14–54, A Symposium on Research Features of Flexible-type Bituminous Roads.
State Highway Commission, Kansas. Highway Research Board, 1947. Design of flexible pavements using the triaxial compression test, Bulletin No. 8, 1–63.
Truesdell, C., 1957. Sulle basi della thermomeccanica. Rend. Accad. Naz. Linc. 22, 33–38.
Truesdell, C., Noll, W., 1992. The Non-Linear Field Theories of Mechanics, second ed. Springer, Berlin.
Whiteoak, D., 1990. The Shell Bitumen Handbook. Shell Bitumen, United Kingdom.
Wineman, A.S., Rajagopal, K.R., 1990. On a constitutive theory for materials undergoing micro-structural changes. Archives of Mechanics 42, 53–75.