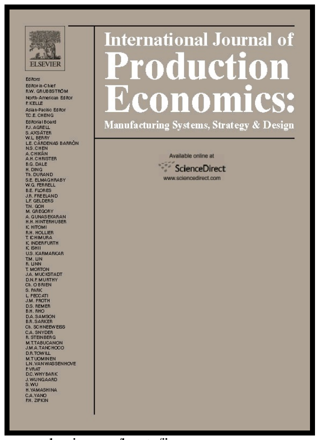
Author's Accepted Manuscript



www.elsevier.com/locate/ijpe

Order quantities for perishable inventory control with non-stationary demand and a fill rate constraint

Karin G.J. Pauls-Worm, Eligius M.T. Hendrix, Alejandro G. Alcoba, René Haijema

PII: S0925-5273(15)00390-4
DOI: http://dx.doi.org/10.1016/j.ijpe.2015.10.009
Reference: PROECO6259

To appear in: *Intern. Journal of Production Economics*

Received date: 30 October 2014
Revised date: 3 September 2015
Accepted date: 13 October 2015

Order quantities for perishable inventory control with non-stationary demand and a fill rate constraint

Order quantities for perishable inventory control with non-stationary demand and a fill rate constraint

Karin G.J. Pauls-Worm[*1], Eligius M.T. Hendrix[2], Alejandro G. Alcoba[2], René Haijema[1,3]
1 Wageningen University, Operations Research and Logistics Group, Wageningen, The Netherlands
2 Universidad de Málaga, Computer Architecture, Málaga, Spain
3 TI Food and Nutrition, Nieuwe Kanaal 9A, 6709 PA Wageningen, The Netherlands

Abstract

We study the practical decision problem of fresh food production with a long production lead time to decide every period (e.g. week) how many items to produce. When a batch is ready for use, its items have a fixed shelf life, after which the items become waste in the sense that they cannot be sold anymore. The demand for (fresh) food products is uncertain and highly fluctuating, mainly caused by price promotions of retail organisations. We focus on cases where a so-called cycle fill rate service level requirement applies. We investigate the generation of a production plan that fixes the timing and quantity of the production for a finite time horizon. To minimise waste, one issues the oldest items first, i.e. a FIFO issuing policy. In case of out-of-stock, sales are lost.

We model this decision problem as a Stochastic Programming (SP) model. The objective of our study is to find order quantities for the SP model, that approximately meet cycle fill rate service level requirements while keeping outdating low. To find approximate solutions for the SP model, an MILP model is developed. The MILP model is a deterministic approximation that generates feasible replenishment quantities in less than a second. With a scenario-based MINLP approach, optimal solutions are generated for a large sample of demand paths as a benchmark for the MILP solutions. We show that the MILP model is suitable for practical use if the setup cost is such that the replenishment cycles in the production plan are close to or of the same length as the maximum shelf life. In those cases, the expected total costs are close to the costs of the optimal solution and the average fill rate is close to the required one.

[*] Corresponding author. Tel.: +31 317 484729.
e-mail addresses: Karin.Pauls@wur.nl
Eligius.Hendrix@wur.nl
Alejandro G. Alcoba: gutialco@gmail.com
Rene.Haijema@wur.nl

Keywords:
production-inventory planning; long lead time; perishable product; non-stationary stochastic demand; fill rate.

1. Introduction

A producer of a fresh food product with a long lead time has to decide every period (e.g. week) how many items to produce. Multiple batches of the same product of different ages are in production, so there are multiple outstanding orders. When a batch is ready for use, the items get a ‘best before’ or a ‘use by’ date, resulting in a fixed shelf life for the product. Generally, producers have a contract with retail organisations about the minimum remaining shelf life of the items delivered. The time between ‘ready for use’ and the minimum remaining shelf life is the maximum internal shelf life the producer can use to organise production efficiently. After the maximum internal shelf life of several periods, the product becomes waste in the sense that it cannot be sold anymore with the aimed remaining shelf life. It still may be used for other purposes, so it may have a salvage value. These production characteristics can be found in e.g. the maturation of cheese, meat from breeding to slaughtering and crops from seed to harvesting. In the remainder of this paper we will use the shorter term ‘maximum shelf life’ to denote the maximum internal shelf life.
The demand for (fresh) food products is uncertain, so the production quantity is determined based on forecasts. A complicating factor is that demand is highly fluctuating, mainly due to price promotions of the retail organisations, i.e. demand is non-stationary. Competition in retail is very strong, so retail organisations are reluctant to share information about their promotional activities. Because of the long lead time, planned promotions sometimes become only known to the food producer after production of the items. However, retail organisations continuously work on improving their demand forecasts. Highly fluctuating demand is not necessarily highly uncertain. It is likely that the food producer has a contract with the retail organisation about the delivery of the product, with respect to remaining shelf life and fill rate. The fill rate indicates that a predefined percentage of the demand per replenishment cycle has to be fulfilled from stock, a so-called cycle fill rate. According to the food producer, demand that cannot be fulfilled from stock is lost. The food producer has control over the issuing of the items. In order to minimise waste due to outdating, the oldest items are issued first, so there is a first in – first out (FIFO) issuing policy. The food producer wants to make a

production plan minimising the expected total costs, indicating when to produce and how much, for a fixed time horizon of T weeks.

Having a long lead time, the inventory levels at the time of delivery are unknown at the time the replenishment quantity has to be determined. So the replenishment quantity should be decided on beforehand, independently of the inventory level I. Therefore, we consider a so-called static uncertainty policy denoted by (Y_t, Q_t), where Y_t denotes in which periods to deliver, and Q_t the corresponding replenishment quantities to deliver at the beginning of period t. The research question is whether it is possible to generate a production plan for T periods using existing solvers and for which instances the solution might be close to the optimal solution.

The rest of the paper is organised as follows. In Section 2, we explain how the paper contributes to literature by discussing the main characteristics of the problem. In Section 3, the problem is formulated as a Stochastic Programming (SP) model. Section 4 describes the steps towards a production plan that we use to formulate a deterministic MILP model that generates feasible production plans. The model determines the timing of deliveries, the replenishment cycle length and the replenishment or delivery quantity, in order to make a production plan for lead time L periods before t. This model is presented in Section 5. Section 6 investigates the applicability of the MILP model compared to 'optimal' solutions generated by a scenario-based MINLP approach. Section 7 concludes and provides topics for future research.

2. Literature

The lost sales inventory problem studied in this paper is to fix a production plan (and consequently replenishment plan) for a single perishable product with a long lead time, non-stationary demand and a fill rate constraint. The paper builds upon previous studies by Bookbinder and Tan (1988), Tarim and Kingsman (2004) and Pauls-Worm et al. (2014), which we will describe in more detail below. This paper contributes to these studies by moving from a zero lead time problem, where the current inventory levels can be taken into account, to a problem with long lead time. Moreover, motivated by a practical case, the problem includes a fill rate or β-service level constraint instead of an α-service level constraint. Consequently, also a different order policy is studied. In Pauls-Worm et al. (2014), the derived order policy is of the type (Y_t, S_t), that is, in period t, if $Y_t = 1$ the manager orders up to a level S_t. Due to the long lead time, in the present paper we consider an (Y_t, Q_t) policy;

if $Y_t = 1$ in period t, a fixed quantity Q_t will be delivered and consequently produced the lead time L periods before t.

Bookbinder and Tan (1988) formulated an SP model for a single-item inventory problem for a finite horizon, with a non-stationary demand, under an α-service level constraint. They distinguish two decision rules, a static uncertainty and a dynamic uncertainty strategy. In the static uncertainty strategy the timing and order sizes (Y_t, Q_t) are determined at the beginning of the time horizon, before demand is known. A dynamic uncertainty strategy bases decisions on new available information. Bookbinder and Tan (1988) combine the two strategies into a third static-dynamic uncertainty (Y_t, S_t) policy. Tarim and Kingsman (2004) formulated for a non-perishable product an MILP model for the static-dynamic uncertainty strategy resulting in an optimal (Y_t, S_t) policy. Rossi (2013) assessed the quality of a Constraint Programming solution procedure on a static-dynamic (Y_t, S_t) policy for perishable items. Pauls-Worm et al. (2014) extended the model of Tarim and Kingsman (2004) to perishable items, resulting in an approximate solution for the SP model for lead time zero and an α-service level.
For the static uncertainty (Y_t, Q_t) policy and non-perishable items with a non-stationary demand, Tempelmeier and Herpers (2011) formulated a Stochastic Single Item Uncapacitated Lot-Sizing Problem with a fill rate constraint. They assume that excess demand is backlogged and they found an optimal (Y_t, Q_t) solution with a modification of Dijkstra's shortest-path algorithm, as well as several heuristic approaches to solve the model.
Key characteristics of the lost sales inventory problem under study are perishability, non-stationary demand, fill rate constraints, and long lead time. In the remainder of this section, we discuss how these aspects are addressed in literature.

Perishability and non-stationary demand

Regarding perishability, we focus on products with a fixed shelf life. A recent review about perishable inventory, including products with a fixed life time, is due to Karaesmen et al. (2011). From their review, it becomes clear that inventory problems with a fixed shelf life, non-stationary demand, and a (long) lead time are relevant but challenging and understudied. Most studies on ordering products with a fixed shelf life, focus on stationary demand. Bijvank and Vis (2011) reviewed lost-sales inventory theory. For non-perishables, they conclude more research should focus on non-stationary demand. Tunc et al. (2011) discuss the use of stationary inventory policies when demand is non-stationary for non-perishable products. They conclude that in case of high demand variability, using a stationary policy can be very expensive. In case of high uncertainty, high setup cost and low penalty cost, using a stationary policy might be efficient. The above papers motivate the interest in policies for non-stationary demand.

A few articles have been published that deal with perishability and non-stationary demand. An exact method to solve the non-stationary problem is Stochastic Dynamic Programming (SDP). For a lead time of one period and a fixed shelf life of up to 7 periods, Haijema et al. (2007) and Haijema et al. (2009) solve the non-stationary ordering problem by SDP and discuss the near optimality of a periodic review order-up-to S_t policy. Their problem, however, lacks a service level constraint. Instead they apply a cost structure that includes a penalty for lost sales. In these studies, fixed setup cost can be included and order periods may be prefixed instead of being part of the optimisation. Some heuristic approaches are published, like the one of Broekmeulen and van Donselaar (2009). They develop a heuristic for a single store to determine a replenishment policy based on the estimated withdrawal and aging of the items in stock. Similar to Haijema et al. (2007), the weekly demand is stationary with a non-stationary demand pattern during the week. They do not consider fixed setup cost and use a lost sales cost to influence the fill rate.

Service levels

The inclusion of α-service level or fill rate constraints in an optimisation model such as SDP is complicated and subtle. Chen and Krass (2001) define the difference between mean service level constraints and minimal service level constraints. A mean service level constraint measures the service level over the time horizon, while a minimal service level constraint measures the service level in every period. We use a minimal service level criterion, a minimal fill rate per replenishment cycle. Food production companies often have service contracts with their retail customers requiring a certain fill rate service level. According to Chen and Krass (2001) a minimal service level criterion is preferred when the service level constraint is due to a contractual obligation or a company policy. Minner and Transchel (2010) determine a replenishment policy for perishable products in retail assuming a weekly demand pattern, negligible fixed cost, and positive but relatively short lead times. They apply an SDP model with marginal α-service-level and fill rate constraints. What they call marginal service levels are in terms of Chen and Krass (2001) minimal service levels as they should hold per (sub)period. Hendrix et al. (2012) also apply a minimal α-service level in an SDP approach for perishable products. Note that such an approach generates a $Q(I)$ policy (a dynamic uncertainty rule), which is not suitable in cases of long lead time. Pauls-Worm and

Hendrix (2015) show when considering service level constraints, SDP generates order policies that are not necessarily optimal. SDP meets a service level requirement that is conditional to each possible starting inventory level, no matter how small the chance of occurrence. This results in an overachievement of the service level constraint.

Lost sales and long lead time

Base stock policies are commonly studied policies for having appropriate structural properties and being close to optimal in many settings, especially in cases of backlogging and short lead times. Morton (1969) showed for non-perishables with a stationary demand that an order-up-to policy is not optimal for a lost-sales inventory model in case of a positive lead time. Van Donselaar et al. (1996) show for non-perishables, that compared to a replenishment policy with static order-up-to levels, it may be more efficient in a lost sales system to use dynamic order-up-to levels that dynamically meet fill rate constraints. Under dynamic order-up-to levels, the pattern of successive order sizes is smoother. The need for smoother order patterns is even stronger in case of long lead times, as shown in Goldberg et al. (2014). They show that as the lead time grows for non-perishables with a stationary demand, the constant-order policy is asymptotically optimal. The intuition of their approach is to select the constant-order policy that considers the inventory in the pipeline best. In case of non-stationary demand, the expected pipeline inventory is highly fluctuating per period because of the fluctuating demand. The accuracy of approximated pipeline inventory will be low. In our paper we deal with non-stationary demand of perishable items, meaning that at the end of the shelf life items will become waste. This results in an even less accurate approximation of the pipeline inventory. In the model for long lead times, presented in the next section, we thus study a policy with fixed replenishment quantities Q_t that are independent of the inventory available at the beginning of each replenishment period t.

3. Stochastic Programming Model

To determine a production plan for a perishable product with a long lead time under a fill rate constraint, we consider a single-product − single-echelon SP model, minimizing expected total costs. We focus on an (Y_t, Q_t) policy. Periods are of equal lengths and can be hours, days, weeks or months, whatever is applicable in the practical situation. The product has a fixed maximum integer (internal) shelf life $M \geq 2$ periods. Due to the findings of Goldberg et al. (2014), we leave lead time out of the model. The model solution is concerned with the delivery time t of a production batch, i.e. production or ordering should be done L periods before delivery. Demand is non-stationary independently distributed with a Normal distribution $d_t \sim N(\mu_t, (CV{\cdot}\mu_t)^2)$ in period t. We use a Normal distribution to keep fill rate calculations simple. Demand is never negative, food cannot be returned due to safety regulations. There are fixed and variable production costs, holding cost and cost of waste. Table 1 shows the list of symbols. Fill rate is defined as the proportion of demand per replenishment cycle that can be fulfilled directly from stock, being a cycle fill rate. The (maximum) available inventory is determined by the replenishment quantity, which should be such that the predetermined fill rate constraint can be met. Issuing is according to a FIFO policy in which the first delivered items are issued first. The age of the items is indexed by b = 1,..,M. The inventory level of age b at the end of period t is denoted by I_{bt}. Items delivered at the beginning of period t, have age b = 1 at the end of period t. Items of age M at the end of the period are considered waste and cannot be used in the next period. If demand in period t exceeds the available inventory of period $t - 1$ plus the delivered quantity Q_t at the beginning of t, demand is lost and there is a shortage of X_t. Replenishments can aim to cover demand from 1 up to M periods, so replenishment cycles have a varying length j.

Table 1 List of symbols

T	length of finite time horizon
t	period index , t = 1,..,T
M	fixed maximum (internal) shelf life
b	age index, b = 1,..,M
j	index denoting the length of the replenishment cycle
k	fixed setup cost for every replenishment
c	variable unit production cost
h	unit holding cost, for items that are carried over from one period to the next
w	unit disposal cost (w > 0) or salvage value (w < 0) for items becoming waste
β	target cycle fill rate
d_t	stochastic demand during period t, non-stationary for t = 1,..,T
Y_t	binary variable takes the value of 1 if there is a replenishment in period t, and 0 otherwise
Q_t	replenishment quantity for delivery at the beginning of period t
I_{bt}	inventory level of items with age b at the end of period t
I_{Mt}	inventory of age M at the end of period t is considered waste
X_t	number of items short at the end of period t
$E(TC)$	expected total costs over the time horizon

This problem is formulated as a stochastic programming model.

$$\mathrm{Min}\ E(TC) = \sum_{t=1}^{T}\left(kY_t + cQ_t + E\left(h\sum_{b=1}^{M-1} I_{bt} + wI_{Mt} \right)\right) \tag{1}$$

subject to

$$Y_t = \begin{cases} 1 & \text{if } Q_t > 0 \\ 0 & \text{otherwise} \end{cases} \qquad t = 1,..,T \qquad (2)$$

$$I_{bt} = \left(I_{b-1,t-1} - \left(d_t - \sum_{i=b}^{M-1} I_{i,t-1} \right)^+ \right)^+ \qquad t = 1,..,T;\ b = 2,..,M \qquad (3)$$

$$I_{1t} = \left(Q_t - \left(d_t - \sum_{b=1}^{M-1} I_{b,t-1} \right)^+ \right)^+ \qquad t = 1,..,T \qquad (4)$$

$$X_t = \left(d_t - \sum_{b=1}^{M-1} I_{b,t-1} - Q_t \right)^+ \qquad t = 1,..,T \qquad (5)$$

$$Y_t \cdot \sum_{j=1}^{\min\{M,\,T-t+1\}} \left(\frac{\sum_{i=t}^{t+j-1} E(X_i)}{\sum_{i=t}^{t+j-1} \mu_i} \cdot Y_{t+j} \cdot \prod_{i=t+1}^{j} (1-Y_i) \right) \le (1-\beta) \qquad t = 1,..,T \qquad (6)$$

$$Y_{T+1} = 1 \qquad (7)$$

$$I_{b0} = 0 \qquad b = 1,..,M \qquad (8)$$

$$I_{bt} \ge 0 \qquad t = 1,..,T;\ b = 1,..,M \qquad (9)$$

$$Y_t \in \{0,\ 1\},\ Q_t, X_t \ge 0 \qquad t = 1,..,T \qquad (10)$$

The model minimises the expected total costs (Eq. 1), consisting of fixed setup cost for every replenishment, production cost and the expected cost of holding inventory and of waste. Eqs. (3), (4) and (5) model the inventory levels of all ages and shortage in period t, under a FIFO issuing policy. To meet the fill rate requirement, the fraction of expected shortage over expected demand of a replenishment cycle should be less than or equal to $(1-\beta)$. Index j denotes the length of the replenishment cycle, which has a length of $j = 1$ to M periods. The first period after the replenishment cycle, i.e. period $t + j$, should have a delivery, and during the replenishment cycle no other delivery takes place. Eq. (6) models the fill rate requirement related to the length of the replenishment cycle, where $Y_t = Y_{t+j} = 1$, and $Y_i = 0$ with $t < i < t + j$. Eqs (7) to (10) are definition constraints. The model assumes that at the end of the time horizon T, also the last replenishment cycle ends, so in $T + 1$ a new replenishment arrives (Eq. 7). In the evaluations, the starting inventory level is zero (Eq. 8).

4. Towards a production plan

Eq. (6) describes the fill rate requirement $\frac{\sum_{i=t}^{t+j-1} E(X_i)}{\sum_{i=t}^{t+j-1} \mu_i} \leq (1-\beta)$ for all replenishment cycle lengths *j*. Consider the replenishment quantity Q_t for which this constraint holds when the inventory at delivery period *t* is zero, i.e. $\sum_{b=1}^{M-1} I_{b,t-1} = 0$. The aggregated demand $d = \sum_{i=t}^{t+j-1} d_i$ during the replenishment cycle of length *j* has a normal distribution with mean $\mu = \sum_{i=t}^{t+j-1} \mu_i$ and variance $\sigma^2 = \sum_{i=t}^{t+j-1} \sigma_i^2 = \sum_{i=t}^{t+j-1} (\mathrm{CV} \cdot \mu_i)^2$. The expected shortage $E(X) = \sum_{i=t}^{t+j-1} E(X_i)$ is the so-called loss function, expressing the expected shortage as a function of the delivered quantity Q_t. Let φ be the density function (pdf) and Φ the cumulative distribution function (cdf) of *d*. Then the loss function is

$$L(Q_t) = E(X) = E(d - Q_t)^+ = \int_Q^\infty (x - Q_t)\varphi(x)dx. \tag{11}$$

It is known that the loss function is convex in quantity Q_t and for the normal distribution can be expressed by (Chopra and Meindl, 2010)

$$L(Q_t) = \mu - Q_t + \sigma \cdot \left(\varphi\left(\tfrac{Q_t-\mu}{\sigma}\right) + \Phi\left(\tfrac{Q_t-\mu}{\sigma}\right) \cdot \tfrac{Q_t-\mu}{\sigma}\right). \tag{12}$$

The minimum replenishment quantity Q_t fulfilling the fill rate requirement can be found by solving $L(Q_t) = (1-\beta)\mu$, i.e.

$$Q_t - \sigma \cdot \varphi\left(\tfrac{Q_t-\mu}{\sigma}\right) - \Phi\left(\tfrac{Q_t-\mu}{\sigma}\right) \cdot (Q_t - \mu) = \beta\mu. \tag{13}$$

We solved Eq. (13) using a standard solver “fzero” of MATLAB. As shown in Alcoba et al. (2015), for a replenishment cycle of $j = 1$ period and the defined distribution, it is sufficient to solve Eq. (13) for all periods *t*. A replenishment cycle can have a length of $j = 1, 2,.., M$ periods. An $M \times T$ table called *LevelQ* can be generated with all possible replenishment quantity levels $LevelQ_{jt}$ for period *t* and replenishment cycle length *j* when the inventory is zero, i.e. $\sum_{b=1}^{M-1} I_{b,t-1} = 0$. Table 2 shows an example of the expected demand per period, and the corresponding $LevelQ_{jt}$ values.

Table 2 Expected demand μ_t and corresponding *LevelQ* values for CV = σ/μ = 0.25 and fill rate β = 95%.

t	1	2	3	4	5	6	7	8	9	10	11	12
μ_t	800	950	200	900	800	150	650	800	900	300	150	600
$LevelQ_{1t}$	899	1068	225	1011	899	169	731	899	1011	337	169	674
$LevelQ_{2t}$	1832	1243	1187	1779	1030	863	1518	1779	1280	475	807	0

$LevelQ_{3t}$	2011	2114	1958	1913	1652	1652	2390	2051	1414	1085	0	0

In the next section, we will show how the $LevelQ_{jt}$ values can be used to find an (Y_t, Q_t) policy with an MILP model.

5. MILP model

In Section 5.1, we formulate an MILP model to generate approximate solutions for the SP model. In Section 5.2, a numerical illustration of the MILP model is presented.

5.1 MILP model formulation

Besides the policy variables Y_t and Q_t, the other variables of the MILP model are denoted by their expected value variant, i.e. *EI* and *EX*, in contrast to the SP model. The objective function is given by

$$\text{Min } E(TC) = \sum_{t=1}^{T} \left\{ kY_t + cQ_t + h\sum_{b=1}^{M-1} EI_{bt} + wEI_{Mt} \right\} \tag{14}$$

and minimises the setup cost, the inventory holding cost over the on-hand inventory at the end of the period, the variable production cost and the cost of waste.

$$\sum_{j=1}^{M} Z_{jt} = Y_t \qquad t = 1,..,T-M+1 \tag{15}$$

$$\sum_{j=1}^{T-t+1} Z_{jt} = Y_t \qquad t = T-M+2,..,T \tag{16}$$

$$Z_{1t} \le Y_{t+1} \qquad t = 1,..,T-1 \tag{17}$$

$$j \cdot Z_{jt} \le \sum_{i=1}^{j-1} (1 - Y_{t+i}) + Y_{t+j} \qquad t = 1,..,T-j; \;\; j = 2,..,M \tag{18}$$

$$Y_{T+1} = 1 \tag{19}$$

Eqs. (15) – (19) are logical constraints to describe the order timing, thus linearising Eq. (6) of the SP model. Variable $Y_t = 1$ if there is a delivery in period *t*, and variable $Z_{jt} = 1$ denotes the replenishment cycle length *j* in period *t* aimed at fulfilling demand for *j* periods: for period *t* and the next *j* – 1 periods. In case of a delivery, eqs. (15) and (16) require a delivery for 1 or 2 or,.. up to *M* periods. Eq. (16) is valid at the end of the horizon. If there is no delivery in period *t*, then $Z_{jt} = 0$ for all *j*. On the other hand, a delivery in period *t* implies the choice of exactly one replenishment cycle length *j*, i.e. Z_{jt} has to be 1 for one value of *j*. For replenishment cycle length j (covering t to $t + j - 1$), a new replenishment takes place in period $t + j$ ($Y_t = 1$ and $Y_{t+j} = 1$) and in between there is no order (equations (17) – (19)) .

$$Y_1 = 1 \qquad (20)$$

$$\sum_{j=1}^{M} Y_{t+j-1} \geq 1 \qquad t = 1,..,T - M \qquad (21)$$

Eqs. (20) and (21) are constraints to ensure that at least in the first period and every M periods an order is delivered.

$$Q_t \geq \sum_{j=1}^{M} (Z_{jt} \cdot LevelQ_{jt}) \qquad t = 1,..,T - 1 \qquad (22)$$

Eq. (22) selects replenishment quantity Q_t for j periods from the table $LevelQ_{jt}$ (Section 4). The model ignores the pipeline inventory. When the replenishment cycle is of length M, the pipeline inventory is zero. In other cases the replenishment is greater than strictly necessary to meet the cycle fill rate requirement, causing an approximate solution of the formulated SP model. The size of the pipeline inventory when replenishment cycles are of length < M depends on the distribution of demand in successive periods and is therefore hard to approximate.

$$EI_{M-1,t-1} - Ed_t = EI_{Mt} - EA_{M-1,t} \qquad t = 1,..,T \qquad (23)$$

$$EI_{b,t-1} - EA_{b+1,t} = EI_{b+1,t} - EA_{bt} \qquad t = 1,..,T;\ b = 1,..,M - 2 \qquad (24)$$

$$Q_t - EA_{1t} = EI_{1t} - EX_t \qquad t = 1,..,T \qquad (25)$$

Eq. (23), (24) and (25) keep track of the age-distribution of the items in stock, under a FIFO-issuing policy. Let auxiliary variable EA_{bt} denote the shortage of inventory of age b with $b = 1,..,M - 1$ in period t to fulfil the demand of period t. If EA_{bt} has a positive value, then fresher inventory is used to fulfil demand. Eq. (23) imposes the oldest inventory to be used first to fulfil demand. What is left over has the maximum shelf life and will become waste, or shortage of the oldest inventory occurs. In the latter case, Eq. (24) is appropriate. The shortage has to be fulfilled by items of intermediate ages, until demand is fulfilled by the freshest items that have been delivered in the current period, according to Eq. (25). The FIFO constraints linearise constraints (3) – (5) of the SP model, causing an over- and underestimation of the different inventory levels as shown in (Pauls-Worm et al., 2014). That paper also shows that the FIFO constraints are necessary to meet FIFO issuing in the context of an MILP model.

$$\mathcal{M}_t \cdot BA_{bt} \geq EA_{bt} \qquad t = 1,..,T;\ b = 1,..,M - 1 \qquad (26)$$

$$\mathcal{M}_t \cdot (1 - BA_{bt}) \geq EI_{b+1,t} \qquad t = 1,..,T;\ b = 1,..,M - 1 \qquad (27)$$

$$\mathcal{M}_t \cdot BX_t \geq EX_t \qquad t = 1,..,T \qquad (28)$$

$$\mathcal{M}_t \cdot (1 - BX_t) \geq EI_{1t} \qquad t = 1,..,T \qquad (29)$$

At most one variable at the right-hand-sides of equations (23), (24) and (25) can have a positive value. The other variable needs to have a value of 0. Equations (26) − (29) take care of that, using the binary variables BA_{bt} and BX_t. $\mathcal{M}_t$ is a sufficiently large number, for instance $\mathcal{M}_t = LevelQ_{Mt}$.

$$EI_{b0} = 0 \qquad b = 1,..,M \qquad (30)$$

The starting inventory is zero (Eq.(30)).

$$EI_{bt},\ Q_t \geq 0 \qquad t = 1,..,T;\ b = 1,..,M \qquad (31)$$

$$EA_{bt} \geq 0 \qquad t = 1,..,T;\ b = 1,..,M-1 \qquad (32)$$

$$EX_t \geq 0 \qquad t = 1,..,T \qquad (33)$$

$$Y_t,\ Z_{jt} \in \{0, 1\} \qquad t = 1,..,T;\ \ j = 1,..,M \qquad (34)$$

$$BA_{bt},\ BX_t \in \{0, 1\} \qquad t = 1,..,T;\ b = 1,..,M-1 \qquad (35)$$

Eqs. (31) to (35) are definition constraints.

5.2 Numerical illustration of the MILP model

We consider a base case for the MILP model with a fixed setup cost k = 500, inventory holding cost h = 0.5 over the on-hand inventory at the end of the period, variable production cost c = 2, cost of waste w = 0, CV = σ/μ = 0.25 and required fill rate β = 95%. The maximum shelf life is M = 3. Cost of waste w = 0 implies that for wasted items, there is no extra cost of disposal, nor a salvage value. However, the production cost and holding cost during $M - 1$ periods are still imposed on these items. Expected demand (repeated in the first row of Table 3) and $LevelQ_{jt}$ is given in Table 2. The solution of this model is given in Table 3. The shaded row shows the replenishment quantities with $Y_t = 1$ if $Q_t > 0$. The production plan prescribes deliveries in periods 1, 4, 7, 9 and 12, resulting in 5 replenishment cycles of respectively length 3, 3, 2, 3 and 1. The replenishment quantities are equal to the corresponding values of $LevelQ_{jt}$ in Table 2. The expected inventory levels of all ages of period 9 in the table showcase well that the available older inventory is used before the fresh items. The MILP solution is evaluated in a simulation based on 10,000 samples to measure the expected total costs and the fill rate. The expected total costs of the simulation are 8.4% higher than the costs of the MILP solution. The final row of Table 3 contains the simulated fill rate β_{sim}. In period

11 the fill rate is higher than required, because the replenishment cycle starting in period 9 has on-hand inventory at the start of the period. The levels of *LevelQ* are based on no on-hand inventory at the beginning of the period. The fill rate of other replenishment cycles is close to the required value. These observations fit with the design of the MILP model and the definition of $LevelQ_{jt}$.

Table 3 Numerical illustration of the base case: $E(TC)_{\text{MILP}}$ = 19846; $E(TC)_{\text{Sim}}$ = 20013; Avg β_{sim} 95.44%

t	1	2	3	4	5	6	7	8	9	10	11	12
μ_t	800	950	200	900	800	150	650	800	900	300	150	600
Y_t	1	0	0	1	0	0	1	0	1	0	0	1
Q_t	2011	0	0	1913	0	0	1518	0	1414	0	0	674
EI_{1t}	1211	0	0	1013	0	0	868	0	582	0	0	74
EI_{2t}	0	261	0	0	213	0	0	68	0	282	0	0
EI_{3t}	0	0	61	0	0	63	0	0	0	0	132	0
EX_t	0	0	0	0	0	0	0	0	0	0	0	0
β_{sim}	0	0	95.07	0	0	95.01	0	95.06	0	0	97.02	95.04

6. Results of the MILP model

To investigate the sensitivity and applicability of the MILP model for different parameter values, a design of experiments is set up, reported in Section 6.1. Section 6.2 describes the benchmark with the scenario-based MINLP approach we use. In Section 6.3 a comparison of the MILP solutions is made with solutions of a MINLP scenario-based approach.

6.1 Design of Experiments

The experimental design is similar to the one in Pauls-Worm et al. (2014). For the setup cost k we consider values 0, 500, and 1000. The time horizon is T = 12 periods. The inventory holding cost is h = 0.5 over the on-hand inventory at the end of the period, variable production cost is c = 2, and the maximum shelf life is M = 2, 3 or 4. This setting is based on values used in practice. We learned from Pauls-Worm et al. (2014) that M = 3 is the most interesting case to study. Table 4 shows the design of experiments. The design varies the parameter values systematically for setup cost k, cost of waste w, fill rate β and Coefficient of Variation CV. Negative waste cost represents a salvage value for the wasted items, whereas positive waste cost implies disposal cost. This results in 83 experiments using the same erratic demand pattern due to promotion activities of the customer of the producer, the retail organisation. In variation of the base case, the MILP model is also tested with three other demand patterns, being an erratic variant, a highly erratic demand pattern and a stationary demand (Pauls-Worm et al., 2014) as depicted in Figure 1. The total expected demand of all patterns is 7200.

Table 4 Design of Experiments

Experiment	Demand	*k*	*w*	Fill rate β (%)	CV	*M*
Base	Erratic	500	0	95	0.25	3
1 – 9	Erratic	0	-0.5, 0, 0.5	90, 95, 98	0.10	3
10 – 18	Erratic	0	-0.5, 0, 0.5	90, 95, 98	0.25	3
19 – 27	Erratic	0	-0.5, 0, 0.5	90, 95, 98	0.33	3
28 – 36	Erratic	500	-0.5, 0, 0.5	90, 95, 98	0.10	3
*37 – 45	Erratic	500	-0.5, 0, 0.5	90, 95, 98	0.25	3
46 – 54	Erratic	500	-0.5, 0, 0.5	90, 95, 98	0.33	3
55 – 63	Erratic	1000	-0.5, 0, 0.5	90, 95, 98	0.10	3
64 – 72	Erratic	1000	-0.5, 0, 0.5	90, 95, 98	0.25	3
73 – 81	Erratic	1000	-0.5, 0, 0.5	90, 95, 98	0.33	3
82 – 83	Erratic	500	0	95	0.25	2, 4
84	Err Variant	500	0	95	0.25	3
85	Highly Err	500	0	95	0.25	3
86	Stationary	500	0	95	0.25	3

* including the base case

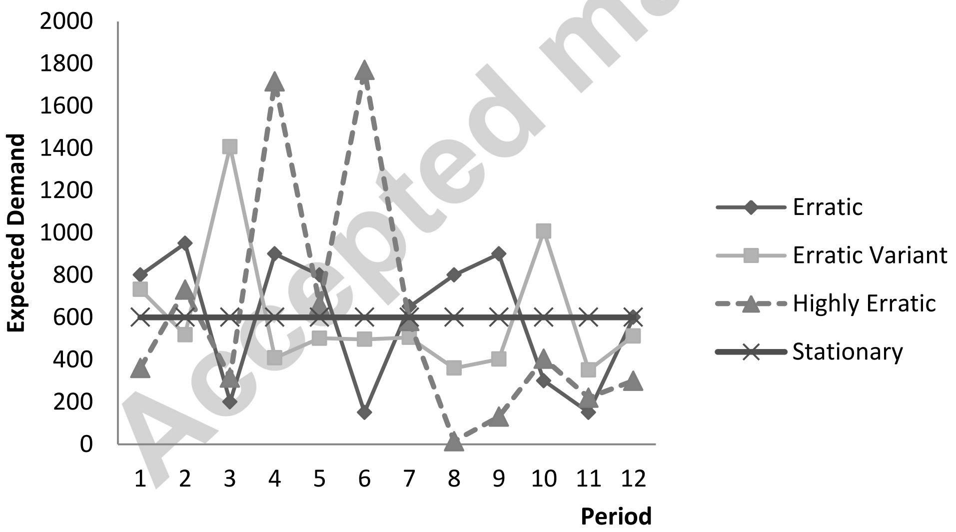


Fig. 1 Demand patterns

6.2 Benchmark with a scenario-based approach

The MILP model is a deterministic approach that generates feasible production plans in less than a second for the performed experiments. The question is whether the approach is suitable to use in practice. To investigate this question, an alternative MINLP approach was implemented similar to the approach discussed by Alcoba et al. (2015). For each (integer) feasible timing $Y = (Y_1,.., Y_T)$ the best (continuous) replenishment quantities Q_t are generated by nonlinear programming for scenarios consisting of 50,000 sample demand paths. The objective function and fill rate of a quantity vector Q are approximated simulating the inventory development using the 50,000 demand runs. The quantity vector with the lowest average total costs fulfilling the required fill rate is considered to be the optimal (Y_t, Q_t) policy for the given scenarios. In contrast to the MILP approach, it may be clear that due to the enumeration of delivery timings this approach is not tractable as the number of timing vectors grows exponentially in the number of periods. Although that is not a problem for the executed experiments, it may be a problem when the same model described here is applied to other practical cases.

6.3 Experiments

Table 5 shows the results of the experiments. For each experiment of Table 4, the expected total costs of the MILP model ($E(TC)_{MILP}$), the simulated expected total costs ($E(TC)_{sim}$) of the MILP policy, the number of orders of the MILP policy (NrO MILP) and the average fill rate of the MILP policy in the simulation (Avg β_{sim}) are given. These are compared with the expected total costs of the MINLP approach ($E(TC)_{MINLP}$), the number of orders (NrO MINLP) and the average fill rate (Avg β_{MINLP}) of the MINLP policy. The last column shows $E(TC)_{sim}$ relative to $E(TC)_{MINLP}$ (x100%), to show the cost increase if an MILP policy is used instead of the MINLP policy. The average fill rate is the average over the fill rates per replenishment cycle. The base case has a grey shade.

Table 5 Overview of the results

Exp	k	CV	β	w	$E(TC)_{\text{MILP}}$	$E(TC)_{\text{sim}}$	NrO MILP	Avg β_{sim}	$E(TC)_{\text{MINLP}}$	NrO MINLP	Avg β_{MINLP}	$\frac{E(TC)_{\text{sim}}}{E(TC)_{\text{MINLP}}}$
1	0	0.10	90	-0.5	13124	13195	12	91.29	13050	12	90.17	101.11
2			90	0	13124	13195	12	91.29	13050	12	90.17	101.11
3			90	0.5	13124	13195	12	91.29	13050	12	90.17	101.11
4			95	-0.5	14142	14553	12	97.27	13980	12	95.10	104.10
5			95	0	14142	14553	12	97.27	13980	12	95.10	104.10
6			95	0.5	14142	14553	12	97.27	13980	12	95.10	104.10
7			98	-0.5	16042.5	16112	8	98.78	14934	12	98.05	107.89
8			98	0	16104.5	16238	10	99.16	14935	12	98.05	108.72
9			98	0.5	16165.5	16319	10	99.16	14936	12	98.05	109.26
10		0.25	90	-0.5	14400	15758	12	95.59	13912	12	90.07	113.27
11			90	0	14400	15807	12	95.59	13923	12	90.07	113.53
12			90	0.5	14400	15855	12	95.59	13934	12	90.07	113.79
13			95	-0.5	17154.5	17326	7	96.56	15572	12	95.09	111.26
14			95	0	17344.5	17497	6	95.95	15613	12	95.09	112.07

Exp	k	CV	β	w	$E(TC)_{\mathrm{MILP}}$	$E(TC)_{\mathrm{sim}}$	NrO MILP	Avg β_{sim}	$E(TC)_{\mathrm{MINLP}}$	NrO MINLP	Avg β_{MINLP}	$E(TC)_{\mathrm{sim}}$ $E(TC)_{\mathrm{MINLP}}$
15			95	0.5	17474	17792	5	95.44	15654	12	95.09	113.66
16			98	-0.5	19185	19171	6	98.34	17334	11	98.07	110.60
17			98	0	19630	19669	6	98.34	17474	12	98.08	112.56
18			98	0.5	20075	20167	6	98.34	17585	12	98.08	114.68
19		0.33	90	-0.5	15923	16479	8	93.85	14604	12	90.07	112.84
20			90	0	15923	16670	8	93.85	14648	12	90.07	113.80
21			90	0.5	15923	16861	8	93.85	14691	12	90.07	114.77
22			95	-0.5	18356.5	18375	5	95.53	16546	12	95.10	111.05
23			95	0	18666	18832	5	95.53	16657	12	95.10	113.06
24			95	0.5	18975.5	19289	5	95.53	16768	12	95.10	115.03
25			98	-0.5	20734	20724	6	98.37	18595	11	98.09	111.45
26			98	0	21430.5	21472	6	98.37	18835	11	98.09	114.00
27			98	0.5	22054	22141	5	98.24	19069	12	98.08	116.11
28	500	0.10	90	-0.5	16875.5	16933	5	90.11	16925	5	90.07	100.05
29			90	0	16875.5	16939	5	90.11	16930	5	90.07	100.05
30			90	0.5	16875.5	16945	5	90.11	16936	5	90.07	100.05
31			95	-0.5	18088	18109	5	95.17	18062	6	95.03	100.26
32			95	0	18088	18145	5	95.17	18088	6	95.03	100.32
33			95	0.5	18088	18181	5	95.17	18114	6	95.03	100.37
34			98	-0.5	19188	19180	5	98.19	19043	6	98.03	100.72
35			98	0	19240	19292	5	98.19	19123	6	98.03	100.88
36			98	0.5	19292	19404	5	98.19	19203	6	98.03	101.05
37		0.25	90	-0.5	17734.5	17860	5	90.44	17736	5	90.04	100.70
38			90	0	17734.5	17988	5	90.44	17853	5	90.04	100.76
39			90	0.5	17734.5	18116	5	90.44	17970	5	90.04	100.81
40			95	-0.5	19718	19735	5	95.44	19505	6	95.05	101.18
41			95	0	19846	20013	5	95.44	19704	6	95.05	101.57
42			95	0.5	19974	20292	5	95.44	19903	6	95.05	101.95
43			98	-0.5	21691	21693	5	98.30	21299	6	98.05	101.85
44			98	0	22144	22203	5	98.30	21664	6	98.05	102.49
45			98	0.5	22597	22712	5	98.30	21926	7	98.07	103.58
46		0.33	90	-0.5	18528	18636	5	90.60	18419	5	90.04	101.18
47			90	0	18528	18878	5	90.60	18640	6	90.06	101.28
48			90	0.5	18528	19120	5	90.60	18813	6	90.06	101.63
49			95	-0.5	20856.5	20875	5	95.53	20511	6	95.06	101.77
50			95	0	21166	21332	5	95.53	20837	6	95.06	102.38
51			95	0.5	21475.5	21789	5	95.53	21036	7	95.10	103.58
52			98	-0.5	23052.5	22995	4	98.04	22626	6	98.06	101.63
53			98	0	23743	23757	4	98.04	23040	7	98.09	103.11
54			98	0.5	24433.5	24519	4	98.04	23398	8	98.07	104.79
55	1000	0.10	90	-0.5	19375.5	19433	5	90.11	19425	5	90.07	100.04
56			90	0	19375.5	19439	5	90.11	19430	5	90.07	100.05
57			90	0.5	19375.5	19445	5	90.11	19436	5	90.07	100.05
58			95	-0.5	20588	20609	5	95.17	20568	5	95.03	100.20
59			95	0	20588	20645	5	95.17	20601	5	95.03	100.21
60			95	0.5	20588	20681	5	95.17	20634	5	95.03	100.23
61			98	-0.5	21688	21680	5	98.19	21585	5	98.02	100.44
62			98	0	21740	21792	5	98.19	21684	5	98.02	100.50
63			98	0.5	21792	21904	5	98.19	21784	5	98.02	100.55
64		0.25	90	-0.5	20223	20186	4	90.03	20186	4	90.02	100.00
65			90	0	20223	20317	4	90.03	20317	4	90.02	100.00
66			90	0.5	20223	20448	4	90.03	20448	4	90.02	100.00

Exp	k	CV	β	w	$E(TC)_{MILP}$	$E(TC)_{sim}$	NrO MILP	Avg β_{sim}	$E(TC)_{MINLP}$	NrO MINLP	Avg β_{MINLP}	$E(TC)_{sim}$ $E(TC)_{MINLP}$
67			95	-0.5	22097.5	21955	4	95.02	21955	4	95.02	100.00
68			95	0	22197	22235	4	95.02	22235	4	95.02	100.00
69			95	0.5	22296.5	22515	4	95.02	22515	4	95.02	100.00
70			98	-0.5	23780	23719	4	98.03	23719	4	98.03	100.00
71			98	0	24216	24227	4	98.03	24227	4	98.03	100.00
72			98	0.5	24652	24735	4	98.03	24735	4	98.03	100.00
73		0.33	90	-0.5	20910	20771	4	90.03	20771	4	90.02	100.00
74			90	0	20910	21016	4	90.03	21016	4	90.02	100.00
75			90	0.5	20910	21261	4	90.03	21261	4	90.02	100.00
76			95	-0.5	22990	22855	4	95.03	22855	4	95.03	100.00
77			95	0	23268	23313	4	95.03	23313	4	95.03	100.00
78			95	0.5	23546	23771	4	95.03	23771	4	95.03	100.00
79			98	-0.5	25052.5	24995	4	98.04	24995	4	98.04	100.00
80			98	0	25743	25757	4	98.04	25757	4	98.04	100.00
81			98	0.5	26433.5	26519	4	98.04	26519	4	98.04	100.00
82[1]	500	0.25	95	0	20385	20395	6	95.05	20120	7	95.05	101.37
83[2]			95	0	20282	20535	4	95.48	19626	5	95.10	104.63
84			95	0	20037	20277	5	96.19	19495	5	95.06	104.01
85			95	0	19909.5	20137	5	95.67	19486	6	95.07	103.34
86			95	0	20348	20346	4	95.00	19728	6	95.05	103.13

[1] $M = 2$; [2] $M = 4$.

The MILP model is considered to be suitable for use in practise if the expected total costs are close to the costs of the optimal solution, and the average fill rate is close to the requirement. The perception of 'close' has to be determined by the producer in practise. Table 5 shows that the MILP model performs best with setup cost $k = 500$ or 1000, when the number of deliveries is limited to 4 or 5 times during the 12 period time horizon. In those cases the expected total costs are less than 5% higher than those of the optimal policy. Interesting is that for setup cost $k = 0$, the MILP policy prescribes for more than half of the experiments not to deliver in each period. Due to aggregation and the fill rate requirement, the total replenishment quantity can be lower when delivering for more periods. If the holding cost is lower than the cost of production of extra items, one delivers for multiple periods. In all experiments but one, the MINLP approach prescribes to deliver in every period. The MINLP approach has lower expected total costs, because it takes the pipeline inventory into account that, apart from period 1, is nonzero, in determining the replenishment quantity. The MILP model determines the replenishment quantity ignoring the availability of pipeline inventory. The evaluated MILP approach goes for certain as the level of the pipeline inventory is highly uncertain due to the long production time. By less replenishments, MILP lowers the total replenishment quantity during the time horizon. With setup cost $k = 1000$ and CV = 0.25 or

0.33, the MILP policy prescribes to deliver every 3 periods. In these cases there is no pipeline inventory and the MILP policy coincides with the optimal policy.

A higher coefficient of variation leads to fewer or the same number of deliveries. The same holds for a higher fill rate. The effect of varying the cost of waste is less clear. The MILP solution is insensitive to varying the tested values of w when $k > 0$. When $k = 0$, varying w leads to an increase, a decrease or no change in the number of deliveries. For the MINLP solution, varying w leads to an increase or no change in the number of deliveries. Varying the maximum shelf life M confirms the earlier findings with less orders in the MILP solution than optimal and slightly higher costs. The other tested erratic demand patterns have the same number of deliveries than the base demand pattern, while the highly erratic demand pattern performs better on the expected total costs than the erratic variant. The stationary demand requires only four deliveries.

The MILP approach appears especially suitable if the cost structure is such that one does not deliver every period. In that case the pipeline inventory is relatively small and consequently the approach performs well with respect to costs. In case the replenishment cycle is equal to the maximum shelf life, the pipeline inventory is zero, so the MILP approach gives the optimal solution. This analysis is also true for other values of the maximum shelf life.

7 Conclusions

We studied the practical problem to determine a production plan for a perishable product with a long lead time and a fixed time horizon under a cycle fill rate constraint. Demand is non-stationary. In case of out-of-stock, demand is lost. Issuing is according to a FIFO policy. We focus on an (Y_t, Q_t) policy, where Y_t denotes in which periods to deliver, and Q_t the corresponding replenishment quantities to deliver. We investigated whether it is possible to construct practical solutions using existing solvers. We considered a single-product – single-echelon SP model, minimizing the expected total costs. To find approximate solutions for the SP model, an MILP model has been formulated. The MILP model is a deterministic approach that generates feasible production plans in less than a second for the performed 86 experiments. With a scenario-based MINLP approach, optimal solutions with respect to a large sample of demand paths are generated as a benchmark for the MILP solutions. The results are data-dependent, but from the performed experiments can be concluded that if the setup cost is low, the MILP model solutions have fill rates higher than required and expected total costs higher than in the optimal solution. Finding a reasonably good approximation for the pipeline inventory in case of non-stationary demand could solve this problem. If the setup cost is higher, such that the replenishment cycle lengths are equal or close to the length of the internal shelf life, the influence of the starting inventory is less. The MILP model generates production plans with fill rates close to the required values and expected total costs are close to optimal. Given the results and the short solver time, the MILP model is suitable for use in practise.

Acknowledgement

This research has been supported and partly funded by (1) TI Food and Nutrition (project RE002), The Netherlands, and (2) The Spanish state (project TIN2012-37483-C03-01) and Junta de Andalucía (P11-TIC-7176), in part financed by the European Regional Development Fund (ERDF).

References

Alcoba, A.G., Hendrix, E.M.T., García, I., Ortega, G., Pauls-Worm, K.G.J., Haijema, R., 2015. On Computing Order Quantities for Perishable Inventory Control with Non-stationary Demand, in: Gervasi, O., Murgante, B., Misra, S., Gavrilova, M.L., Rocha, A.M.A.C., Torre, C., Taniar, D., Apduhan, B.O. (Eds.), Computational Science and Its Applications -- ICCSA 2015. Springer International Publishing, pp. 429-444.

Bijvank, M., Vis, I.F.A., 2011. Lost-sales inventory theory: A review. Eur. J. Oper. Res. 215, 1-13.

Bookbinder, J.H., Tan, J.Y., 1988. Strategies for the probabilistic lot-sizing problem with service-level constraints. Manage. Sci. 34, 1096-1108.

Broekmeulen, R., van Donselaar, K.H., 2009. A heuristic to manage perishable inventory with batch ordering, positive lead-times, and time-varying demand. Comput. Oper. Res. 36, 3013-3018.

Chen, F.Y., Krass, D., 2001. Inventory models with minimal service level constraints. Eur. J. Oper. Res. 134, 120-140.

Chopra, S., Meindl, P., 2010. Supply Chain Management: Strategy, Planning, and Operation, Fourth Edition ed. Pearson Education, Inc., New Yersey.

Goldberg, D.A., Katz, D.A., Lu, Y., Sharma, M., Squillante, M.S., 2014. Asymptotic Optimality of Constant-Order Policies for Lost Sales Inventory Models with Large Lead Times. arXiv preprint arXiv:1211.4063v2 (2014).

Haijema, R., van der Wal, J., van Dijk, N.M., 2007. Blood platelet production: Optimization by dynamic programming and simulation. Comput. Oper. Res. 34, 760-779.

Haijema, R., van Dijk, N., van der Wal, J., Sibinga, C.S., 2009. Blood platelet production with breaks: optimization by SDP and simulation. Int. J. Prod. Econ. 121, 464-473.

Hendrix, E.M.T., Haijema, R., Rossi, R., Pauls-Worm, K.G.J., 2012. On Solving a Stochastic Programming Model for Perishable Inventory Control, in: Murgante, B., Gervasi, O., Misra, S., Nedjah, N., Rocha, A.C., Taniar, D., Apduhan, B. (Eds.), Computational Science and Its Applications – ICCSA 2012. Springer Berlin Heidelberg, pp. 45-56.

Karaesmen, I.Z., Scheller-Wolf, A., Deniz, B., 2011. Managing Perishable and Aging Inventories: Review and Future Research Directions, in: Kempf, K.G., Keskinocak, P., Uzsoy, R. (Eds.), Planning Production and Inventories in the Extended Enterprise Springer, pp. 393-436.

Minner, S., Transchel, S., 2010. Periodic review inventory-control for perishable products under service-level constraints. OR Spectrum 32, 979-996.
Morton, T.E., 1969. Bounds on the Solution of the Lagged Optimal Inventory Equation with no Demand Backlogging and Proportional Costs. SIAM Review 11, 572-596.
Pauls-Worm, Karin G.J., Hendrix, E.M.T., 2015. SDP in Inventory Control: Non-stationary Demand and Service Level Constraints, in: Gervasi, O., Murgante, B., Misra, S., Gavrilova, M.L., Rocha, A.M.A.C., Torre, C., Taniar, D., Apduhan, B.O. (Eds.), Computational Science and Its Applications -- ICCSA 2015. Springer International Publishing, pp. 397-412.
Pauls-Worm, K.G.J., Hendrix, E.M.T., Haijema, R., van der Vorst, J.G.A.J., 2014. An MILP approximation for ordering perishable products with non-stationary demand and service level constraints. Int. J. Prod. Econ. 157, 133-146.
Rossi, R., 2013. Periodic review for a perishable item under non stationary stochastic demand, pp. 2021-2026.
Tarim, S.A., Kingsman, B.G., 2004. The stochastic dynamic production/inventory lot-sizing problem with service-level constraints. Int. J. Prod. Econ. 88, 105-119.
Tempelmeier, H., Herpers, S., 2011. Dynamic uncapacitated lot sizing with random demand under a fillrate constraint. Eur. J. Oper. Res. 212, 497-507.
Tunc, H., Kilic, O.A., Tarim, S.A., Eksioglu, B., 2011. The cost of using stationary inventory policies when demand is non-stationary. Omega 39, 410-415.
Van Donselaar, K., de Kok, T., Rutten, W., 1996. Two replenishment strategies for the lost sales inventory model: A comparison. Int. J. Prod. Econ. 46–47, 285-295.

Highlights

- Practical inventory control for a perishable product with non-stationary demand and long lead time
- An MILP model is constructed with a fill rate constraint per replenishment cycle and a FIFO issuing policy
- MILP uses standard software to specify when to deliver an how much, while meeting the cycle fill rate requirement
- The MILP model is suitable for practical use in case of significant setup cost