Framework of Group Decision Making With Intuitionistic Fuzzy Preference Information

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***Abstract*—Group decision making is an essential activity in various fields of operations research and management science. This paper focuses on the intuitionistic fuzzy group decision making problem in which all the experts use the intuitionistic fuzzy preference relations (IFPRs) to express their preferences. To start our discussion, we first propose the novel framework of intuitionistic fuzzy group decision making and clarify the difficulties in deriving the final result which is accepted by all individuals in the group. Next, a consistency checking method, which is based on the multiplicative consistency, is developed to check the consistency of each IFPR furnished by the group of experts. For those IFPRs that do not have the acceptable consistency, an iterative procedure is proposed to improve the consistency. Furthermore, after introducing a novel consensus measure, an interesting consensus-reaching procedure is developed to help the group to find a solution which is accepted by most members in the group. Finally, in order to make our approaches more applicable, a step-by-step algorithm is given. A numerical example concerning the selection of outstanding Ph.D. students for the China Scholarship Council is given to illustrate and validate the proposed approaches.**

***Index Terms*—Acceptable multiplicative consistency, consensus, group decision making, intuitionistic fuzzy preference relation, multiplicative consistency.**

I. Introduction

GROUP decision making takes place widely in various fields of operations research and management science. This paper focuses on a group decision making problem where a group of individuals or a committee collectively shares the responsibility for making a choice among a set of alternatives for action. Let $A = \{A_1, A_2, \ldots, A_n\}$ be the set of alternatives under consideration and $E = \{e_1, e_2, \ldots, e_s\}$ be the set of experts who are invited to evaluate the alternatives. In many cases, a group decision making problem is very complicated and consists of different criteria, which makes experts sometimes difficult or unable to assess accurately all the aspects of the candidate alternatives. As a result, the experts often are only able to express their opinions roughly and subjectively. In such a situation, how to describe experts' opinions is very important, and it influences the final result directly. Preference relation, which is constructed via pairwise comparisons over the alternatives, can express the preferences of the experts deeply. With the preference relation, there is no need for experts to determine the crisp utility values of alternatives over each criterion, and they can express their judgments subjectively according to their cognition. In addition, in many cases, it might be very difficult for experts to determine crisp preference degrees over the alternatives especially when some experts are not very familiar with the group decision making problem or there contains some incomplete information of the alternatives. In such situations, experts would prefer to express their opinions of preference relation over the alternatives from three aspects, which are "preferred," "not preferred," and "indeterminate." The intuitionistic fuzzy preference relation (IFPR) [1], [2], whose elements are intuitionistic fuzzy numbers (IFNs) [3], can be used to depict such preference information. An IFN is a triplet that can be denoted as $r_{ij} = (\mu_{ij}, v_{ij}, \pi_{ij})$ with $\mu_{ij} \in [0, 1], v_{ij} \in [0, 1], \quad \pi_{ij} \in [0, 1], \mu_{ij} + v_{ij} \leq 1$. μ_{ij} denotes the preference degree of alternative A_i to A_j; v_{ij} indicates the nonpreference degree of alternative A_i to A_j; and $\pi_{ij} = 1 - \mu_{ij} - v_{ij}$ is interpreted as an indeterminacy degree or a hesitancy degree. In practical applications, when evaluating some candidate alternatives, experts may not be able to express their preferences accurately due to the fact that they may not grasp sufficient knowledge of the alternatives or they are unable or unwilling to discriminate explicitly the degree to which the alternative is better than others. In other words, there is a certain degree of hesitation. In such cases, experts may provide their preferences for the alternatives to a certain degree, but it is possible that they are not sure about it. Thus, it is suitable to express their preference information in IFNs. There are also some other structures which can be used to represent the experts' preference information, such as the interval fuzzy set [4], [5] and the incomplete fuzzy preference relation [6]. In this paper, we only focus on IFPR.

Up to now, there are many achievements on decision making with IFPR. Szmit and Kacprzyk [1] first proposed the concept of IFPR. Later, Xu [2] gave a simple notion of it. For the case that some preference values were missing in an IFPR, Xu *et al*. [7] proposed the concept of incomplete IFPR and developed some algorithms for estimating the missing elements of the incomplete

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IFPR. Since a key step in decision making with IFPR is to derive the priority vector of the IFPR, Xu [8] introduced some simple linear programming models to obtain the weights of alternatives. Gong *et al*. [9] also put forward a least squares model and a goal programming model to yield the priority vector of an IFPR. Both Xu's [8] and Gong *et al.*'s [9] models were based on the transformation function between the IFPR and its corresponding interval-valued fuzzy preference relation, which was not direct. Thus, Wang [10] introduced a linear goal programming model, which calculates the membership degrees and the nonmembership degrees of the IFNs directly, to obtain the normalized intuitionistic fuzzy priorities of an IFPR. The underlying theoretical foundation for the models proposed by Xu [8], Gong *et al*. [9], and Wang [10] are the distinct definitions of additive consistency of an IFPR. However, additive consistency is sometimes unreasonable because it might be in conflict with the $[0, 1]$ scale used for providing the preference values [11]. Xu *et al*. [7], Gong *et al*. [12], and Liao and Xu [11] further proposed some different models, which are based on the multiplicative consistency of an IFPR, to derive the underlying priorities of an IFPR. Xu [13] introduced an error-analysis-based method, which is quite different from all the above methods, to generate the weights of the alternatives, and compared with Xu's [8] and Gong *et al.*'s [9] methods. In order to handle more complex decision making problems, Xu and Liao [14] extended the classical AHP method to the intuitionistic fuzzy AHP (IFAHP), and applied the IFAHP method to global supplier management. Recently, Liao and Xu [15] gave the detailed steps of the PROMETHEE method for decision making within the context of intuitionistic fuzzy preference circumstances.

Although IFPR has been applied successfully in many decision making activities, much work still needs to be done on group decision making with IFPRs. Group decision making with IFPRs is more complex but much closer to the practical decision making situation. Generally, group decision making with preference information is faced with three processes.

1) *Consistency checking process of each IFPR:* This process guarantees that the experts' preferences yield no contradiction.
2) *Consensus checking process of the group:* Consensus makes it possible for a group to reach a final decision that all group members can support despite their differing opinions.
3) *Selection process:* The selection process is to find the final result that is accepted by most individuals.

Based on the multiplicative consistency, Xu and Xia [16] proposed some iterative procedures for improving the consistency of IFPRs in a group, but they did not pay much attention to the consensus process, which means that they did not consider Step 2. Meanwhile, the multiplicative consistency they introduced was somehow not reasonable [11]. The consistency improving procedures proposed in [16] were based on the assumption that all the experts were willing to change their preference. This, in general, may not match the practical situation. Sometimes, the experts might not agree to change their opinions or the analyst cannot find the experts who are willing to modify their IFPRs. In order to find out the ranking of alternatives within the context of group decision making with IFPRs, Xu [2] employed the intuitionistic fuzzy arithmetic averaging operator to aggregate all individual IFPRs into an overall IFPR, and then used the intuitionistic fuzzy weighted arithmetic averaging operator to obtain the score value of each alternative. Based on another definition of multiplicative consistency, Liao and Xu [17] built several algorithms for group decision making with IFPRs, which can be divided into two sorts, i.e., aggregate individual priority vectors and aggregate individual IFPRs. The drawback of Xu's [2] and Liao and Xu's [17] approaches was that they did not consider the consistency and consensus checking processes, which is to say, they only considered the third step but ignore Steps 1 and 2. After introducing a new form of similarity measure of IFNs, Xu and Yager [18] made some consensus analysis in group decision making with IFPRs. Szmidt and Kacprzyk [19] also investigated the consensus of IFPR by extending the idea of a fuzzy analysis of consensus from the distance point of view. Both Xu and Yager [18] and Szmidt and Kacprzyk [19] only focused on Step 2 but did not consider Steps 1 and 3 in group decision making with IFPRs. In addition, all of them only focused on how to measure the consensus of a group but proposed no approach to help the group to reach consensus.

To circumvent all the above drawbacks, in this paper, we propose a novel and reasonable framework for group decision making with intuitionistic fuzzy preference information. All the three steps, i.e., the consistency checking process, the consensus checking process, and the selection process, are discussed in details. The novelties of this paper can be highlighted as follows.

1) We first propose a framework for group decision making with IFPRs and point out the difficulties which would take place in the process of group decision making.
2) We use a new method to check the consistency of IFPRs. As to those IFPRs that are not of acceptable consistency, we develop a novel method to help the experts to repair them until they are of acceptable consistency.
3) We introduce a distinguished method to measure the consensus of a group and propose a novel consensus reaching process.
4) We give a complete algorithm for group decision making with IFPRs. This algorithm is quite interactive and, thus, is flexible and can match the practical group decision making situation perfectly.

The rest of this paper is organized as follows: Section II reviews some basic concepts about intuitionistic fuzzy set (IFS) and describes the intuitionistic fuzzy group decision making problem in details. This section sets out the framework of group decision making with IFPRs and clarifies the difficulties in intuitionistic fuzzy group decision making as well. The consistency checking and inconsistency repairing method is then proposed in Section III, and a novel consensus reaching process is given in Section IV. Section V gives a brief discussion on the selection process. A step-by-step algorithm for group decision making with intuitionistic fuzzy preference information is given in Section VI. A numerical example concerning the selection of outstanding Ph.D. students for the China Scholarship Council (CSC) is given to validate the proposed algorithm in Section VII. This paper ends with some concluding remarks in Section VIII.

II. Preliminaries

To facilitate our analysis, some basic concepts that will be used in the future discussion are introduced first.

A. Intuitionistic Fuzzy Set

Let a crisp set X be fixed, and $A \subset X$ be a fixed set. An IFS [20], [21] $\tilde{A}$ in X is an object with the following form:

$$\tilde{A} = \{(x, \mu_A(x), v_A(x)) | x \in X\} \quad (1)$$

where functions $\mu_A : X \to [0, 1]$ and $v_A : X \to [0, 1]$ define the degree of membership and the degree of nonmembership of the element $x \in X$ to set A, respectively, and for every $x \in X$, $0 \leq \mu_A + v_A \leq 1$ holds. For each IFS $\tilde{A}$ on X

$$\pi_A(x) = 1 - \mu_A(x) - v_A(x) \quad (2)$$

is called the degree of nondeterminacy (uncertainty) of the membership of element $x \in X$ to the set A. In the case of ordinary fuzzy sets, $\pi_A(x) = 0$ for every $x \in X$. For convenience, Xu [3] called $\alpha = (\mu_\alpha,\ v_\alpha)$ an IFN.

Szmidt and Kacprzyk [22] justified that $\pi_A(x)$ cannot be omitted when calculating the distance between two IFSs. For two IFNs $\alpha = (\mu_\alpha,\ v_\alpha, \pi_\alpha)$ and $\beta = (\mu_\beta, v_\beta, \pi_\beta)$, the normalized Hamming distance they defined is as follows:

$$d(\alpha, \beta) = \frac{1}{2}(|\mu_\alpha - \mu_\beta| + |v_\alpha - v_\beta| + |\pi_\alpha - \pi_\beta|) \quad (3)$$

and it satisfies $0 \leq d(\alpha, \beta) \leq 1$.

Remark: There are many different forms of distance measures for IFNs (for more information, see [22] and [23]). Generally, the distance measure should be selected according to the practical measure selected to measure the similarity of two IFNs. Here, we just use the normalized Hamming distance as a representation.

In the process of decision making with intuitionistic fuzzy information, the order relation among the IFNs should be fixed, and that plays a crucial role for the obtained results [24]. There are many different procedures for ranking IFNs [2], [24]–[26]. Liao and Xu [11] provided an indepth comparison between these different types of ranking methods from both empirical and theoretical points of view and showed that Zhang and Xu's [26] method is the most appropriate and reasonable one in ranking the IFNs. The ranking scheme of Zhang and Xu is based on the similarity function

$$L(\alpha) = \frac{1 - v_\alpha}{1 + \pi_\alpha} \quad (4)$$

and the accuracy function

$$H(\alpha) = \mu_\alpha + v_\alpha \quad (5)$$

Scheme 1:

For two IFNs $\alpha = (\mu_\alpha, v_\alpha, \pi_\alpha)$ and $\beta = (\mu_\beta, v_\beta, \pi_\beta)$,

1) if $L(\alpha) > L(\beta)$, then α is greater than β, denoted as $\alpha > \beta$;
2) if $L(\alpha) = L(\beta)$, then
   a) if $H(\alpha) > H(\beta)$, then α is greater than β, denoted as $\alpha > \beta$;
   b) if $H(\alpha) = H(\beta)$, then α and β represent the same information, denoted as $\alpha = \beta$.

B. Group Decision Making Problem With Intuitionistic Fuzzy Preference Relations

As we have presented in the Introduction, when a group of experts are asked to evaluate several candidate alternatives, it is straightforward for them to provide their preferences through pairwise comparisons, and use IFNs to express their fuzzy and imprecise cognition from the affirmative, negative, and hesitative points of view in the case that they are not able to give crisp membership degrees of their preferences over alternatives because of vague information and incomplete knowledge about the preference degrees between any pair of alternatives. For instance, suppose that a group of experts are evaluating the performance of two ERP systems, 60% experts may consider that the first one is better than the second one, 30% experts deem that the first one is worse than the second one, and 10% experts do not provide any information. In this case, the preference degree of the performance of the first EPR system against the second one can be represented by an IFN $(0.6, 0.3)$. Once all the intuitionistic fuzzy preference information is stored in a matrix, an IFPR is established.

Definition 1 (see [2]): An IFPR on the set $A = \{A_1, A_2, \ldots, A_n\}$ is represented by a matrix $\tilde{R} = (\tilde{r}_{ij})_{n\times n}$, where $\tilde{r}_{ij} = (\mu(A_i, A_j), v(A_i, A_j), \pi(A_i, A_j))$ for all $i, j = 1, 2, \ldots, n$. For convenience, we let $\tilde{r}_{ij} = (\mu_{ij}, v_{ij}, \pi_{ij})$, where μ_{ij} denotes the degree to which the object A_i is preferred to the object A_j, v_{ij} indicates the degree to which the object A_i is not preferred to the object A_j, and $\pi_{ij} = 1 - \mu_{ij} - v_{ij}$ is interpreted as an indeterminacy degree or a hesitancy degree, with the conditions

$$\mu_{ij}, \nu_{ij} \in [0, 1], \mu_{ij} + v_{ij} \leq 1, \mu_{ij} = v_{ji}, \mu_{ii} = v_{ii} = 0.5$$
$$\pi_{ij} = 1 - \mu_{ij} - v_{ij},\ \text{for all } i, j = 1, 2, \ldots, n. \quad (6)$$

For a group decision making problem, let $A = \{A_1, A_2, \ldots, A_n\}$ be the set of alternatives under consideration, and $E = \{E_1, E_2, \ldots, E_s\}$ be the set of experts, who are invited to evaluate the alternatives. The weighting vector of the experts $E_l (l = 1, 2, \ldots, s)$ is $\lambda = (\lambda_1,\ \lambda_2,\ \ldots, \lambda_s)^T$, where $\lambda_l > 0$, $l = 1, 2, \ldots, s$, and $\sum_{l=1}^{s} \lambda_l = 1$, which can be determined subjectively or objectively according to the experts' expertise, experience, judgment quality, and related knowledge. In general, they can be assigned equal importance if there is no evidence to show significant differences among the experts or specific preference on some experts. In this paper, we assume that the weights of experts can always be given.

Suppose that expert E_l provides his/her preference values for alternative A_i against alternative A_j as $\tilde{r}_{ij}^{(l)} = (\mu_{ij}^{(l)}, v_{ij}^{(l)}, \pi_{ij}^{(l)})$, $(i, j = 1, 2, \ldots, n,\ l = 1, 2, \ldots, s)$ in which $\mu_{ij}^{(l)}$ denotes the degree to which the object A_i is preferred to the object A_j, $v_{ij}^{(l)}$ indicates the degree to which the object A_i is not preferred to the object A_j, and $\pi_{ij}^{(l)} = 1 - \mu_{ij}^{(l)} - v_{ij}^{(l)}$ is interpreted as an indeterminacy degree or a hesitancy degree, subject to $\mu_{ij}^{(l)},\ v_{ij}^{(l)} \in [0, 1]$, $\mu_{ij}^{(l)} + v_{ij}^{(l)} \le 1$, $\mu_{ij}^{(l)} = v_{ji}^{(l)}$, $\mu_{ii}^{(l)} = v_{ii}^{(l)} = 0.5$, for all $i, j = 1, 2, \ldots, n$ and $l = 1, 2, \ldots, s$. The IFPR $\tilde{R}^{(l)} = (\tilde{r}_{ij}^{(l)})_{n\times n}$ for the lth expert can be written as

$$\tilde{R}^{(l)} = \begin{pmatrix} \tilde{r}_{11}^{(l)} & \tilde{r}_{12}^{(l)} & \cdots & \tilde{r}_{1n}^{(l)} \\ \tilde{r}_{21}^{(l)} & \tilde{r}_{22}^{(l)} & \cdots & \tilde{r}_{2n}^{(l)} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{r}_{n1}^{(l)} & \tilde{r}_{n2}^{(l)} & \cdots & \tilde{r}_{nn}^{(l)} \end{pmatrix}. \tag{7}$$

For any a group decision making problem with s experts, we can obtain s individual IFPRs in the form of (7).

C. Difficulties in Intuitionistic Fuzzy Group Decision Making

Group decision making with intuitionistic fuzzy preference information is a very complicated problem owing to the complexity introduced by the conflicting opinions from the experts. These conflicting opinions take place not only among the experts but in the experts themselves as well.

For an individual expert, when he/she furnishes his/her preferences, he/she may give some self-contradict judgments. Naturally, these judgments should be picked out and modified in order not to mislead the final result. To do so, we should know how to judge whether the preferences are contradictory or not. The transitivity properties were proposed to depict the consistency relationship among the preference judgments [2]. Based on the distinct forms of transitivity condition, many different consistency definitions of IFPR were proposed, which mainly involve two sorts: the additive consistency of IFPR [8]–[10] and the multiplicative consistency of IFPR [7], [11], [12], [14], [16], [17]. Liao and Xu [11] compared these different consistency definitions and pointed out that the multiplicative consistency definition they proposed can be taken as a standard definition for the consistent IFPR. Consistency checking process is highly important as it makes sure that all the preferences are consistent. Without this process, some unreasonable results may be produced. Generally, in practice, it is very difficult or impossible for the experts to provide the perfect consistent IFPRs due to the limited cognition of humans, especially when the number of alternatives is very large. As for the inconsistent IFPRs, we can return them to the experts to reevaluate and construct new ones until the consistency is reached or accepted [27], or repair the inconsistent IFPRs automatically by some iterative algorithms. Thus, some hard questions have arisen here: How do we check whether the IFPRs are of consistency or acceptable consistency or not; how do we repair the inconsistent IFPRs to be of acceptable consistency? In Section IV, we will solve these two problems.

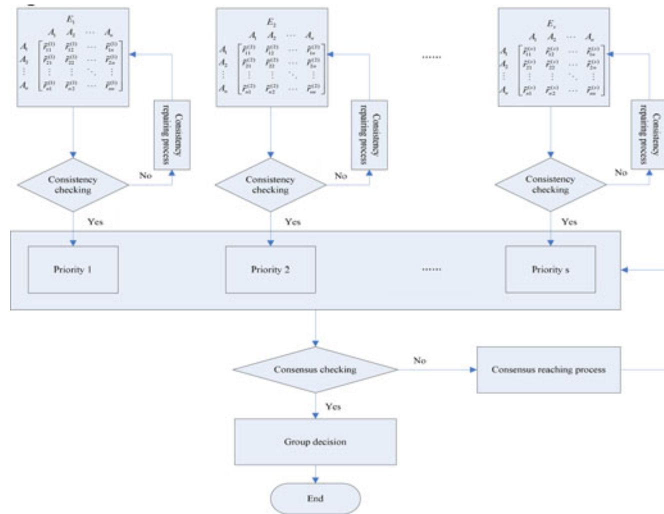

Fig. 1. Framework of group decision making with intuitionistic fuzzy preference information.

After checking the consistency of each IFPR, the following tasks we should do is to find a decision which is agreed by all the experts. However, in practice, this is a great challenge because the experts have their own inherent value systems and consideration, and thus, the disagreement among the experts is inevitable. In such a case, consensus turns out to be very important in group decision making. Strictly speaking, consensus is meant as a full and unanimous agreement. However, such a strict concept of consensus often is a utopia [17]. Therefore, Ness and Hoffman [28] softened the concept of consensus as "*a decision that has been reached when most members of the team agree on a clear option and the few who oppose it think they have had a reasonable opportunity to influence that choice. All team members agree to support the decision*." Consensus is a pathway to a true group decision because it can guarantee that the final result should be supported by all the group members despite their different opinions. To find such a consensus solution, we should first check the consensus and, then, find some ways to help the experts to reach group consensus. How to measure the consensus degree of the group of experts and how to reach a high intensity of consensus are both difficult questions to be solved. In Section V, we will address this issue.

Once all the experts in a group reach a consensus, it is easy to find the final result. The framework of group decision making with an IFPR can be illustrated in Fig. 1.

It should be pointed out that it is not appropriate to aggregate all the experts' intuitionistic fuzzy preference information together directly with some aggregation operators. Before doing so, we should make sure that each individual IFPR is of acceptable consistency. Furthermore, if the decision maker really wants to aggregate all the IFPRs together, in the paper by Liao and Xu [27], it was proved that only the simple intuitionistic fuzzy weighted geometric operator they proposed is reasonable to synthesize the individual IFPRs because only this kind of aggregation method can guarantee that the fused IFPR is still of acceptable consistency when all individual IFPRs are of acceptable consistency.

III. Consistency Checking and Inconsistency Repairing Process for Intuitionistic Fuzzy Preference Relations

This section focuses on the first issue: how to check the consistency of IFPRs and how to repair the inconsistent IFPRs.

A. Consistency Checking Process

Consistency is a critical issue in decision making with preference relations owing to the lack of consistency in preference relations may lead to unreasonable conclusions. With respect to IFPR, several different forms of consistency have been proposed, which mainly involve two sorts: the additive consistency [8]–[10] and the multiplicative consistency [7], [11], [12], [14]. Liao and Xu [11] made an in-depth review over these distinct forms of consistency and then proposed a more general definition of multiplicative consistency for IFPR, which is more reasonable yet easy to use and, thus, can be taken as a standard measurement for the consistency of an IFPR.

Definition 2 (see [11]): An IFPR $\tilde{R} = (\tilde{r}_{ij})_{n\times n}$ with $\tilde{r}_{ij} = (\mu_{ij}, v_{ij}, \pi_{ij})$ is called multiplicative consistent if the following multiplicative transitivity is satisfied:

$$\mu_{ij} \cdot \mu_{jk} \cdot \mu_{ki} = v_{ij} \cdot v_{jk} \cdot v_{ki} \text{ for all } i, j, k = 1, 2, \ldots, n. \tag{8}$$

The consistency condition in the above definition uses the membership degree and the nonmembership degree of each IFN directly without any transformation, and thus, it is very easy to use. In addition, when the IFPR reduces to a fuzzy preference relation, (8) is equivalent to the consistence condition of a fuzzy preference relation [29]. Hence, in general, Definition 2 is sufficient and suitable to measure the multiplicative consistency for an IFPR.

Let $\tilde{\omega} = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)^T = ((\omega_1^\mu, \omega_1^v), (\omega_2^\mu, \omega_2^v), \ldots, (\omega_n^\mu, \omega_n^v))^T$ be a underlying intuitionistic fuzzy priority vector of the IFPR $\tilde{R} = (\tilde{r}_{ij})_{n\times n}$, where $\tilde{\omega}_i = (\omega_i^\mu, \omega_i^v)$ $(i = 1, 2, \ldots, n)$ is an IFN, which satisfies $\omega_i^\mu, \omega_i^v \in [0, 1]$ and $\omega_i^\mu + \omega_i^v \le 1$. ω_i^μ and ω_i^v indicate the membership and nonmembership degrees of alternative A_i as per a fuzzy concept of "importance," respectively. $\tilde{\omega}$ is said to be normalized if it satisfies the following conditions [11]:

$$\sum_{j=1, j\neq i}^{n} \omega_j^\mu \le \omega_i^v, \;\; \omega_i^\mu + n - 2 \ge \sum_{j=1, j\neq i}^{n} \omega_j^v \quad \text{for all } i = 1, 2, \ldots, n. \tag{9}$$

With the underlying normalized intuitionistic fuzzy priority vector $\tilde{\omega} = (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n)^T$, a multiplicative consistent IFPR $\tilde{R}^* = (\tilde{r}_{ij}^*)_{n\times n}$ can be established [11]

$$\tilde{r}_{ij}^* = (\mu_{ij}^*, v_{ij}^*) = \begin{cases} (0.5, 0.5), & \text{if } i = j \\ \left(\dfrac{2\omega_i^\mu}{\omega_i^\mu - \omega_i^v + \omega_j^\mu - \omega_j^v + 2}, \dfrac{2\omega_j^\mu}{\omega_i^\mu - \omega_i^v + \omega_j^\mu - \omega_j^v + 2}\right) & \text{if } i \ne j \end{cases} \tag{10}$$

where $\omega_i^\mu, \omega_i^v \in [0, 1]$, $\omega_i^\mu + \omega_i^v \le 1$, $\sum_{j=1, j\neq i}^{n} \omega_j^\mu \le \omega_i^v$, and $\omega_i^\mu + n - 2 \ge \sum_{j=1, j\neq i}^{n} \omega_j^v$ for all $i = 1, 2, \ldots, n$.

Due to the complexity of the decision making problem and the limited knowledge of the experts, to furnish perfect multiplicative consistent IFPRs is somehow too strict for the experts, especially when the number of objects is too large. Thus, Liao and Xu [27] defined the acceptable multiplicative consistent IFPR.

Definition 3 (see [27]): Let $\tilde{R} = (\tilde{r}_{ij})_{n\times n}$ be an IFPR with $\tilde{r}_{ij} = (\mu_{ij}, v_{ij}, \pi_{ij})$, $i, j = 1, 2, \ldots, n$; then, we call R an acceptable multiplicative consistent IFPR if

$$d(\tilde{R}, \tilde{R}^*) \le \xi \tag{11}$$

where $d(\tilde{R}, \tilde{R}^*)$ is the distance measure between the given IFPR $\tilde{R}$ and its corresponding underlying multiplicative consistent IFPR $\tilde{R}^*$, which can be calculated by

$$d(\tilde{R}, \tilde{R}^*) = \frac{1}{(n-1)(n-2)} \sum_{1\le i<j<n}^{n} \times (|\mu_{ij} - \mu_{ij}^*| + |v_{ij} - v_{ij}^*| + |\pi_{ij} - \pi_{ij}^*|) \tag{12}$$

and ξ is the consistency threshold.

Based on the above definition, we can introduce an index to measure the consistency degree of an IFPR.

Definition 4: Let $\tilde{R} = (\tilde{r}_{ij})_{n\times n}$ be an IFPR with $\tilde{r}_{ij} = (\mu_{ij}, v_{ij}, \pi_{ij})$, $i, j = 1, 2, \ldots, n$. The consistency measure of $\tilde{R}$ is function $C_{\tilde{R}} : \tilde{R} \to [0, 1]$, satisfying

$$C_{\tilde{R}} = 1 - \frac{1}{(n-1)(n-2)} \sum_{1\le i<j<n}^{n} \times (|\mu_{ij} - \mu_{ij}^*| + |v_{ij} - v_{ij}^*| + |\pi_{ij} - \pi_{ij}^*|) \tag{13}$$

where $\tilde{R}^* = (\tilde{r}_{ij}^*)_{n\times n}$, with $\tilde{r}_{ij}^* = (\mu_{ij}^*, v_{ij}^*, \pi_{ij}^*)$ being the corresponding multiplicative consistent IFPR of $\tilde{R}$ derived by (10).

To calculate $C_{\tilde{R}}$, we should first establish the corresponding multiplicative consistent IFPR $\tilde{R}^*$ for $\tilde{R}$. Generally, we do not know the underlying normalized intuitionistic fuzzy priority vector $\tilde{\omega}$. Hence, in order to check the consistency of an IFPR, we should first determine the underlying intuitionistic fuzzy priority vector $\tilde{\omega}$. Liao and Xu [11] presented a fractional programming model to derive the underlying normalized intuitionistic fuzzy priority vector $\tilde{\omega}$. The main idea of their model is to minimize the deviation between $\tilde{R}$ and $\tilde{R}^*$. The model is shown as follows:

Model 1: Min $Z = \sum_{i=1}^{n-1}\sum_{j=i+1}^{n}(\varepsilon_{ij}^{+}+\varepsilon_{ij}^{-}+\xi_{ij}^{+}+\xi_{ij}^{-})$

$$s.t.\begin{cases}\dfrac{2\omega_i^\mu}{\omega_i^\mu-\omega_i^v+\omega_j^\mu-\omega_j^v+2}-\mu_{ij}-\varepsilon_{ij}^{+}+\varepsilon_{ij}^{-}=0\\ \qquad i=1,2,\ldots,n-1;j=i+1,\ldots,n\\ \dfrac{2\omega_j^\mu}{\omega_i^\mu-\omega_i^v+\omega_j^\mu-\omega_j^v+2}-v_{ij}-\xi_{ij}^{+}+\xi_{ij}^{-}=0\\ \qquad i=1,2,\ldots,n-1;j=i+1,\ldots,n\\ \omega_i^\mu,\omega_i^v\in[0,1],\omega_i^\mu+\omega_i^v\le 1,\qquad i=1,2,\ldots,n-1\\ \sum_{j=1,j\ne i}^{n}\omega_j^\mu\le\omega_i^v,\omega_i^\mu+n-2\ge\sum_{j=1,j\ne i}^{n}\omega_j^v\\ \qquad i=1,2,\ldots,n-1\\ \varepsilon_{ij}^{+}\ge0,\varepsilon_{ij}^{-}\ge0,\xi_{ij}^{+}\ge0,\xi_{ij}^{-}\ge0,\varepsilon_{ij}^{+}\cdot\varepsilon_{ij}^{-}=0\\ \xi_{ij}^{+}\cdot\xi_{ij}^{-}=0,\qquad i=1,2,\ldots,n-1;j=i+1,\ldots,n\end{cases}$$

where

$$\varepsilon_{ij} = \frac{2\omega_i^\mu}{\omega_i^\mu-\omega_i^v+\omega_j^\mu-\omega_j^v+2}-\mu_{ij}$$

$$\xi_{ij} = \frac{2\omega_j^\mu}{\omega_i^\mu-\omega_i^v+\omega_j^\mu-\omega_j^v+2}-v_{ij}\ \ \varepsilon_{ij}^{+}=\frac{|\varepsilon_{ij}|+\varepsilon_{ij}}{2}$$

$$\varepsilon_{ij}^{-} = \frac{|\varepsilon_{ij}|-\varepsilon_{ij}}{2}\ \ \xi_{ij}^{+}=\frac{|\xi_{ij}|+\xi_{ij}}{2}$$

$$\xi_{ij}^{-} = \frac{|\xi_{ij}|-\xi_{ij}}{2},\quad i,j=1,2,\ldots,n;\quad i\ne j.$$

If the minimal value $Z^*=0$, then $\tilde{R}$ is equal to $\tilde{R}^*$, which implies that $\tilde{R}$ is of perfect multiplicative consistency. However, in normal cases, Z^* should be not equal to zero. With Model 1, we can always yield the underlying normalized intuitionistic fuzzy priority vector $\tilde{\omega}$ for an IFPR $\tilde{R}$. Thus, with (10) and (13), we can calculate the consistency measure for the IFPR $\tilde{R}$.

B. Inconsistency Repairing Process

In practical group decision making process, there usually is a set of experts who give their evaluations over the alternatives, and there is an analyst who analyses the data determined by the experts. In such a decision support system (DSS), the experts establish the input data for the system, while the analyst produces the output data and the final result for the group decision making problem. In the first step, when the experts input their preference information in terms of IFPRs into the DSS, it is highly possible that these IFPRs may not be of perfect consistency. To avoid this, the analyst should first ask the experts the threshold ξ for the consistency degree of the IFPRs they give and then check whether the IFPRs are of acceptable consistency or not. If yes, the analyst can go to further investigate; if not, the analyst would return the inconsistent IFPRs to the experts immediately and asks them to repair it. If the experts refuse to modify the inconsistent IFPR, they should be deleted from the expert group since their preference values are self-contradict. It should be stated that the consistency threshold ξ is determined by the experts in accordance with their knowledge and requirement over the specific decision making problem. There is no theoretical methodology to establish such a value. Within the context of multiplicative preference relation, Saaty [30] pointed out that if the consistency is less than 0.1, the multiplicative preference relation is of acceptable consistency. Generally, we always set $\xi = 0.1$; however, it is not constant. More research will be done on this topic in the future.

When the DSS reminds the experts to modify their inconsistent IFPRs, it is intelligent for the analyst to give some suggestions to help the experts to find the acceptable IFPRs. Generally, the inconsistency repairing process is an iterative process. Suppose that p is the number of iteration and η is the step size which satisfies $0\le p\eta\le 1$. For the inconsistency IFPR $\tilde{R}^p=(\tilde{r}_{ij}^p)_{n\times n}$, which satisfies $C_{\tilde{R}^p}<\xi$, we can modify it into $\tilde{R}^{p+1}=(\tilde{r}_{ij}^{p+1})_{n\times n}$ by the following iterative formulas:

$$\mu_{ij}^{p+1} = (\mu_{ij}^p)^{1-p\eta}\cdot(\mu_{ij}^{p*})^{p\eta},\quad i,j=1,2,\ldots,n \quad (14)$$

$$v_{ij}^{p+1} = (v_{ij}^p)^{1-p\eta}\cdot(v_{ij}^{p*})^{p\eta},\quad i,j=1,2,\ldots,n \quad (15)$$

where $\tilde{R}^*=(\tilde{r}_{ij}^*)_{n\times n}$, with $\tilde{r}_{ij}^*=(\mu_{ij}^*,v_{ij}^*,\pi_{ij}^*)$ being the corresponding multiplicative consistent IFPR of $\tilde{R}^p$ derived by (10).

Theorem 1: The iteration process with (14) and (15) is convergent.

Proof: The iteration ends when $C_{\tilde{R}^p}\ge\xi$, i.e., the repaired IFPR $\tilde{R}^p$ is of acceptable consistency. Since $0\le p\eta\le 1$, there exists a $\delta\in[0,1]$ such that $\delta=p\eta$. Let N be a given maximum iteration number. If we set the iteration step $\eta=1/N$, then after $p=N$ iterations of calculation, with (14) and (15), we can obtain $\mu_{ij}^{p+1}=\mu_{ij}^{p*}$ and $v_{ij}^{p+1}=v_{ij}^{p*}$, i.e., $\tilde{R}^{p+1}=\tilde{R}^{p*}$. Since we have proven that $\tilde{R}^{p*}$ is multiplicative consistent, $\tilde{R}^{p+1}$ should be multiplicative consistent as well. Thus, the iteration process is convergent. ■

In fact, if we only need to yield the acceptable consistency with the consistency level of ξ, we can find the acceptable consistent IFPR even more quickly. The illustrative example for the inconsistency repairing process is given in Section VIII (see Step 4).

IV. Consensus Reaching Process for Intuitionistic Fuzzy Group Decision Making

After the consistency checking process and the inconsistency repairing process, the analyst collects a set of IFPRs which are all of acceptable consistency with the level of ξ. The following task we should do is to find a solution which is accepted by all the experts. With each IFPR $\tilde{R}^{(l)}$ determined by the expert E_l, we can derive an underlying normalized intuitionistic fuzzy priority vector $\tilde{\omega}$ via Model 1. Then, the ranking of the alternatives is easy to obtain by the comparison Scheme 1. However, different experts would produce different rankings, and thus, it is hard to derive a solution agreed by all the members. In such a case, most scholars have solved this problem by using some aggregation operators to fuse the different individual preferences of the experts together into overall preferences and, then, yielded the final ranking through the overall IFPR [2], [16], [17]. Basically, the outcome determined this way is very much dependent on the chosen aggregation procedure [31]. Simply aggregating all the individual IFPRs together by some aggregation operators is somehow not reasonable because it averages all the preferences without considering the consensus among the experts. Consensus is not to be enforced or obtained through a negotiation or bargaining process, but it is expected to emerge after the exchanges of opinions among the experts [32]. Consensus is essential in group decision making. In this section, we would give a novel method to find a solution for group decision making with IFPRs with an interactive consensus-reaching process.

Normally, in group decision making, maximizing the group consensus is a critical aspect in judging the stability of the final result. In practical decision making circumstances, however, we always determine a minimum consensus degree γ in advance, and once the actual consensus degree is smaller than γ, at least one expert would be advised to modify his/her preference values.

Suppose that the priority vector of the expert E_l derived from the IFPR $\tilde{R}^{(l)}$ is $\tilde{\omega}^{(l)} = (\tilde{\omega}_1^{(l)}, \tilde{\omega}_2^{(l)}, \ldots, \tilde{\omega}_n^{(l)})^T$. Then, we can introduce a distance measure between the experts E_l and E_m as

$$\begin{aligned} d(E_l, E_m) &= \frac{1}{n}\sum_{i=1}^{n} d(\tilde{\omega}_i^{(l)}, \tilde{\omega}_i^{(m)}) \\ &= \frac{1}{2n}\sum_{i=1}^{n}(|\tilde{\omega}_i^{\mu(l)} - \tilde{\omega}_i^{\mu(m)}| + |\tilde{\omega}_i^{v(l)} - \tilde{\omega}_i^{v(m)}| \\ &\quad + |\tilde{\omega}_i^{\pi(l)} - \tilde{\omega}_i^{\pi(m)}|) \end{aligned} \quad (16)$$

where $d(\tilde{\omega}_i^{(l)}, \tilde{\omega}_i^{(m)})$ is the normalized Hamming distance between the IFNs $\tilde{\omega}_i^{(l)}$ and $\tilde{\omega}_i^{(m)}$. Since $0 \le d(\tilde{\omega}_i^{(l)}, \tilde{\omega}_i^{(m)}) \le 1$, it follows that $0 \le d(E_l, E_m) \le 1$.

Based on the distance measure between any two experts, a consensus degree for a group of experts with IFPRs can be defined as follows.

Definition 5: For a group decision making problem, suppose that the expert E_l provides an IFPR $\tilde{R}^{(l)} = (\tilde{r}_{ij}^{(l)})_{n\times n}$, whose underlying priority vector is $\tilde{\omega}^{(l)} = (\tilde{\omega}_1^{(l)}, \tilde{\omega}_2^{(l)}, \ldots, \tilde{\omega}_n^{(l)})^T$. Then, the consensus degree of such a group can be defined as

$$\Gamma = 1 - \max_{l,m=1,2,\cdots s}\{d(E_l, E_m)\} \quad (17)$$

where $d(E_l, E_m)$ is the distance between the experts E_l and E_m defined as (16).

As $0 \le d(E_l, E_m) \le 1$, then the consensus degree $\Gamma \in [0, 1]$. For the given minimum consensus degree γ, if $\Gamma \ge \gamma$, then we can say the group reaches consensus. Otherwise, we should propose a procedure to reach a higher consensus degree. In such a situation, the experts first make some discussion and each expert may persuade others to adopt his/her opinion. If, consequently, some experts modify the preference values, then, a new consensus degree is obtained, and we can check whether it satisfies the minimum requirement or not. If all the experts do not agree to change their mind or if the final consensus degree Γ is still less than γ, then, as an analyst, he/she should find out the expert who should change his/her preferences in order to reach a higher consensus.

The consensus reaching process is done iteratively over the experts one by one. The main idea of our consensus reaching procedure is as follows: First, we pick out the expert who should adjust his/her preferences in order to reach a higher group consensus. Then, we would ask the expert whether he/she agrees to change him/her preferences or not. If not, we can exclude him/her from the group because his/her preference information is quite different from the group. This is reasonable and applicable in practical group decision making process. After excluding the first expert, the other experts then form a new group, and then, we can calculate a new consensus degree and check whether the new group reaches consensus or not. If not, we can do the same process as before.

The critical step of this consensus reaching process is to find out the expert who should change his/her opinions. Definitely, he/she should be the person who has a furthest distance with the other experts, which can be represented mathematically in terms of

$$\begin{aligned} E_{l^*} \in \overleftrightarrow{E} \triangleq \{E_l \in E | \exists E_m, s.t.\ d(E_l, E_m) \\ = \max_{l,m=1,2,\ldots,n}\{d(E_l, E_m)\}\}. \end{aligned} \quad (18)$$

In general, there are at least two experts in $\overleftrightarrow{E}$ because the distance measure is reflexive. Suppose that E_l and E_m attain the maximum distance $d(E_l, E_m)$; then, how to judge whether E_l or E_m should be selected to change his/her assessments? The optimal one should be an expert who satisfies the condition that if we pick out him/her, the reminder of the group reaches a higher consensus. Mathematically, E_{l^*} can be depicted as

$$E_{l^*} = \arg\min_{E_q \in \overleftrightarrow{E}} \max\{d(E_l, E_m) | E_l, E_m \in E \backslash E_q\}. \quad (19)$$

If there are two experts satisfying (18) and (19) simultaneously (in fact, this is not the usual case since it is too special), then the analyst asks any one of then to adjust his/her assessments. If the selected expert refuses to change his/her mind, then he/she should be excluded from the group.

Theoretically, the above procedure can yield a higher consensus. However, we have to admit that the convergence to consensus depends on the willingness of the experts to compromise. If all the experts are not willing to change their mind, then such a group decision making process is meaningless. Hence, the majority degree should be also introduced to measure the percentage of experts who are in the final consensus group. Suppose that t experts have been excluded from the group. The majority degree of the final group decision is defined as $\Psi = (s - t)/s$. A minimum majority degree φ should also be determined in advance. This index is indispensable in many practical group decision making process. For example, when the congress discusses a new Act, it should be approved by a certain percentage of the congress members before the Act can be implemented.

It is stated that there are some alternatives to obtain the consensus of intuitionistic fuzzy group decision making. For example, by extending the idea of fuzzy consensus analysis based on α-cuts, Szmidt and Kacprzyk [19] used an interval to describe the consensus degree of a group. After that, Xu and Yager [18] proposed a consensus analysis method for group decision making with IFPRs based on the similarity measure between IFPRs. Comparing with these two consensus methods, our iterative consensus reaching process is somehow more appropriate. First, the iterative consensus reaching process is much closer to practical decision making process as it takes the experts' feedback into account. Second, both the methods in [18] and [19] only develop the methods to measure the consensus of a group, but not introduced any procedure to improve the consensus of a group, while the iterative consensus reaching process solves this problem perfectly.

V. Selection Process

Suppose that there are s^* experts E_l $(l = 1, 2, \ldots, s^*)$ who reach the final consensus $\Gamma^* \geq \gamma$ with the majority degree $\Psi^* = s^*/s \geq \varphi$ after the consensus reaching process. The underlying priority vectors of the experts E_l $(l = 1, 2, \ldots, s^*)$ are $\tilde{\omega}^{(l)} = (\tilde{\omega}_1^{(l)}, \tilde{\omega}_2^{(l)}, \ldots, \tilde{\omega}_n^{(l)})^T$, $l = 1, 2, \ldots, s^*$. In order to select the best alternative(s), we aggregate the priority vectors together by some aggregation operators, such as the IFWA operator [3]. Then, for each alternative A_i, the final weighted priority value is

$$\tilde{\omega}_i = (\omega_i^\mu, \omega_i^v) = \left(1 - \prod_{l=1}^{s^*} (1 - \omega_i^{\mu(l)})^{\lambda_j}, \prod_{l=1}^{s^*} (\omega_i^{v(l)})^{\lambda_j}\right). \tag{20}$$

Using Scheme 1, we can rank the alternatives and the best alternative A^* is selected such that

$$A^* = \arg \max_{i=1,2,\cdots,n} \{A_i\}. \tag{21}$$

VI. Procedure for Group Decision Making With Intuitionistic Fuzzy Preference Information

A. Algorithm Description

Based on all the above analysis, for the convenience of application, a step by step procedure for group decision making with intuitionistic fuzzy preference information can be given as follows.

Algorithm:

Step 1: A group of experts $E = \{E_1, E_2, \ldots E_l, \ldots, E_s\}$, whose weighting vector is established by the decision maker as $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_s)^T$ with $\lambda_l > 0$, $(l = 1, 2, \ldots, s)$, and $\sum_{l=1}^s \lambda_l = 1$, are invited to evaluate the alternatives A_i $(i = 1, 2, \ldots, n)$, and each expert E_l expresses his/her opinions by the IFPR $\tilde{R}^{(l)} = (\tilde{r}_{ij}^{(l)})_{n\times n}$ in the form of (7). To find a consistency and consensus solution, the following parameters are established in advance by the committee of experts, as well as the decision maker: the consistency threshold ξ, the minimum consensus degree of the group γ, and the minimum majority degree φ. Then, let $\tilde{R}^{(p)(l)} = \tilde{R}^{(l)}$, and set $p = 1$. Go to the next step.

Step 2: For each individual IFPR $\tilde{R}^{(p)(l)} = (\tilde{r}_{ij}^{(p)(l)})_{n\times n}$, calculate the underlying normalized intuitionistic fuzzy priority vector $\tilde{\omega}^{(p)(l)} = (\tilde{\omega}_1^{(p)(l)}, \tilde{\omega}_2^{(p)(l)}, \ldots, \tilde{\omega}_n^{(p)(l)})^T$ for the IFPR $\tilde{R}^{(p)(l)}$ by Model 1, and then construct the corresponding multiplicative consistent IFPR $\tilde{R}^{(p)(l)*} = (\tilde{r}_{ij}^{(p)(l)*})_{n\times n}$, by (10). Furthermore, we compute the consistency degree $C_{\tilde{R}^{(p)(l)}}$ for each IFPR $\tilde{R}^{(p)(l)}$ via (13). Go to the next step.

Step 3: Determine whether the IFPR $\tilde{R}^{(p)(l)}$ is of acceptable consistency or not. If $C_{\tilde{R}^{(p)(l)}} \geq \xi$, then let $\bar{\tilde{R}}^{(l)} = \tilde{R}^{(p)(l)}$, and go to Step 5; otherwise, ask the expert E_l to adjust his/her preferences. If the expert E_l refuses to change his/her preferences, then his/her IFPR should be excluded from further investigation and he/she should also be excluded from the group. Thus, let $E = E - \{E_l\}$; if the expert is willing to change his/her judgments, go to the next step.

Step 4: Determine the iterative step $\eta \in [0, 1]$ and adjust $\tilde{R}^{(p)(l)}$ to $\tilde{R}^{(p+1)(l)} = (\tilde{r}_{ij}^{(p+1)(l)})_{n\times n}$ with $\tilde{r}_{ij}^{(p+1)(l)} = (\mu_{ij}^{(p+1)(l)}, v_{ij}^{(p+1)(l)}, \pi_{ij}^{(p+1)(l)})$, where

$$\mu_{ij}^{(p+1)(l)} = (\mu_{ij}^{(p)(l)})^{1-p\eta} \cdot (\mu_{ij}^{(p)(l)*})^{p\eta}, \quad i, j = 1, 2, \ldots, n \tag{22}$$

$$v_{ij}^{(p+1)(l)} = (v_{ij}^{(p)(l)})^{1-p\eta} \cdot (v_{ij}^{(p)(l)*})^{p\eta}, \quad i, j = 1, 2, \ldots, n. \tag{23}$$

Let $p = p + 1$. Go to Step 2.

Step 5: Calculate the majority degree of the group $\Psi = s'/s$, where s' is the number of experts after the first round iteration. If $\Psi < \varphi$, the algorithm ends, and there is no any consensus solution for this group decision making problem; otherwise, we calculate the distance $d(E_l, E_m)$ between any pair of the experts (E_l, E_m), $l, m = 1, 2, \ldots, s'$, according to (16). Furthermore, we obtain the consensus degree Γ of the group through (17); then, go to the next step.

Step 6: Judge whether the consensus degree of the group is acceptable or not. If $\Gamma \geq \gamma$, then, go to Step 9; otherwise, go to the next step.

Step 7: Ask the experts to make some communication with each other and modify their preferences through reevaluation. If expert E_l is persuaded to modify his/her IFPR $\bar{\tilde{R}}^{(l)}$ into a new one $R\tilde{(l)}'$, then let $\tilde{R}^{(l)} = R\tilde{(l)}'$, and go back to Step 2; if no expert agrees to change the preferences at this time, then go to the next step.

Step 8: Find out expert E_{l^*} who should be requested to adjust his preferences in order to reach a higher group consensus according to (18) and (19). If expert E_{l^*} refuses to change his/her opinions, then exclude him/her from the group, and let $s' = s' - 1$, then go back to Step 6. If expert E_{l^*} provides a new IFPR $R\tilde{(l^*)}'$, then let $\tilde{R}^{(l)} = R\tilde{(l^*)}'$, and go back to Step 2.

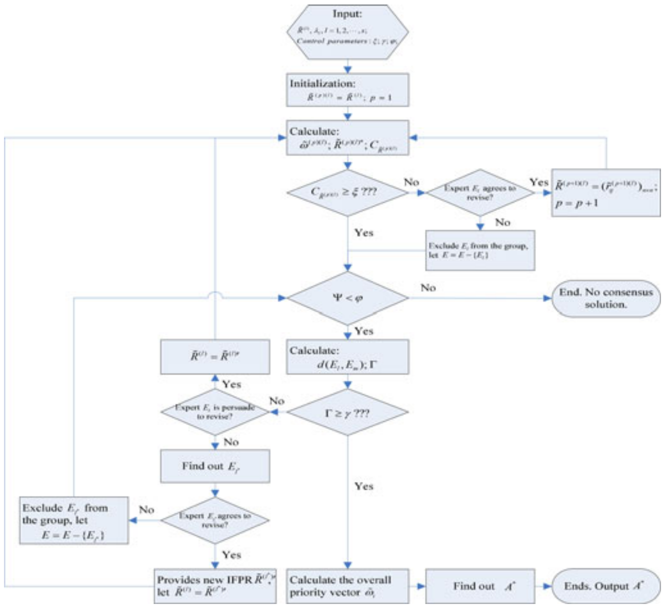

Fig. 2. Schematic diagram of the algorithm for intuitionistic fuzzy group decision making.

Step 9: Aggregate the overall weighted priorities for the alternatives according to (20), and then, find the best alternative A^* via (21).

Step 10: End.

To help the readers and potential practitioners to understand our algorithm thoroughly, let us give more explanation step by step (the schematic diagram of this algorithm is provided in Fig. 2). In the above algorithm, the first step is to establish the input data of a complicated group decision making problem with intuitionistic fuzzy preference information. Steps 2–4 make up of the consistency checking process and the inconsistency repairing process. After Step 4, the DSS would derive s' IFPRs, which have the acceptable consistency within the consistency threshold ξ. Meanwhile, there are $(s - s')$ experts who have been excluded for further decision process. Steps 5 and 6 compose the consensus checking process, while the consensus reaching process consists of Steps 7 and 8. With Step 9, the final result is produced by aggregating the individual priority vectors together and ranking the score of alternatives.

In this algorithm, three parameters, ξ, γ, and φ, should be given by the experts. These parameters are essential to the final decision result. ξ is used to control the consistency level of each IFPR given by the experts, γ is used to control the consensus degree of the group, and φ is indispensable for maintaining the majority degree of the group. In fact, these parameters have already been used explicitly or implicitly in the process of practical decision making. In this algorithm, if the majority degree of the group Ψ is less than the minimum majority degree φ, which implies that the percentage of members in the final decision committee is lower than the minimum requirement, the procedure should be end, and there is no feasible consensus solution. After all the IFPRs are acceptably consistent and the group consensus is attained, we can find the final solution, and then, the algorithm ends.

B. Comparison With the Other Intuitionistic Fuzzy Group Decision Making Methods

The following comparison analysis shows that our algorithm has many advantages against the existing methodologies for intuitionistic fuzzy group decision making.

1) Comparing With Xu's [2] Method: The main idea of Xu's method is as follows: First, obtain the overall preference value of each alternative with respect to each expert by the intuitionistic fuzzy arithmetic aggregation (IFAA) operator; then, aggregate these overall preference values of alternatives over the experts into the collective preference values of the alternatives by the intuitionistic fuzzy weighted arithmetic aggregation (IFWAA) operator; finally, derive the rank of these alternatives according to the collective preference values of the alternatives. The drawbacks of Xu's method are obvious. For one thing, Xu's method is based on the assumption that all the IFPRs given by the experts are consistent. However, this does not match the real situation. In many cases, especially when the number of alternatives is very large, the inconsistency is inevitable. If one of the IFPRs is not consistent, then the result derived by Xu's method should be not convincing. For another thing, in Xu's method, the priorities of the alternatives are obtained by simply aggregating the preference values of the alternatives given by different experts. It should be noted that when we aggregate different values together, these values should be independent; otherwise, the synthesis does not make sense. However, in each IFPR, the preference values are definitely correlated with each other. Hence, it is not very reasonable to derive the priorities of the alternatives by simply aggregating the preferences together with the IFAA operator.

As to our approach, the group decision making process is divided into three parts, i.e., the consistency checking and inconsistency improving process, the consensus checking and consensus reaching process, and the selection process. Detailed discussion over the consistency of each IFPR is given, which makes our method much more efficient and applicable than Xu's method. In addition, in our approach, the priorities of the alternatives with respect to the experts are derived by solving the constructed fractional programming models, which is quite different from Xu's method. Moreover, in our approach, much attention is paid on the consensus checking and reaching processes. However, this was not mentioned in Xu's method.

2) Comparing With Xu and Xia's [16] Method: In [16], the consistency of all IFPRs established by the group of members has been discussed. It also proposed an algorithm to help experts to improve the inconsistent IFPRs. However, there are also some drawbacks in contrast with our method. First, the theoretical foundation of Xu and Xia's consistency checking method is the definition of multiplicative consistency which was introduced in [7]. However, as recently pointed out by Liao and Xu [11], such definition of multiplicative consistency for IFPR presented in [7] is somehow not reasonable. Hence, the consistency checking method used by Xu and Xia is not reasonable as well. Second, the consistency improving procedure proposed by Xu and Xia is quite automatic. It does not interact with the experts. The preferences of the experts whose IFPRs are inconsistent are forced to be modified by the analyst without consulting the experts. This is not reasonable. Furthermore, the priorities of the alternatives are also derived by the IFWW operator, which is also not convincing. Finally, in Xu and Xia's method, the consensus of the group is not included, which consequently makes the final result might not be supported by all the members in the group.

In contrast, the consistency measure used in our approach is based on a much more reasonable definition of multiplicative consistency for IFPR, which is more general than the definition used in Xu and Xia's method [11]. In addition, the inconsistency repairing procedure proposed in this approach is an interactive procedure. Thus, it is much more attractive and easier to be accepted by more practitioners. Last, but not the least, our algorithm gives a quite novel and interesting consensus reaching procedure, which can guarantee that the final result is supported by all the experts despite their differing opinions.

3) Comparing With Liao and Xu's [17] Method: Recently, Liao and Xu studied the intuitionistic fuzzy group decision making problem and proposed a different and interesting method to yield the result for a group. In their approach, the priorities were derived by the programming models rather than aggregation operators. The reasonable multiplicative consistency measure was also used in their approach. However, there are still some flaws. They did not consider the inconsistency improving process. Meanwhile, they did not contain any consensus issue as well.

VII. Numerical Example and Some Discussions

A. Numerical Example on Selecting Outstanding Ph.D. Students for China Scholarship Council

In what follows, we present a numerical example to illustrate the proposed decision analysis process and validate the proposed algorithm in aiding group decision making.

Example 1: Since China has become the world's second largest economy, the Chinese government has paid more and more attention to the education for Chinese citizens. To service this purpose, CSC was set up as a nonprofit institution affiliated with the Ministry of Education of China. It is entrusted by the Chinese Government with the responsibilities of managing the State Scholarship Fund and other related affairs. It sponsors Chinese citizens to pursue study abroad and international students to study in China. There is a scholarship under the State Scholarship Fund which awards the best Ph.D. students to go to some top oversea universities to study as joint Ph.D.'s. The rigid academia evaluation process for those candidate Ph.D. students is organized by the CSC. The CSC constructs many committees to evaluate the Ph.D. candidates all over the country. The criteria to show the applicants' profile are established by CSC in each application form, which involves academic achievement (ς_1), the quality of research proposal (ς_2), english skills (ς_3), the reputation of the university (ς_4), and the reputation of the supervisors inside and outside (ς_5). The evaluation process is not very easy because the members in a committee are professors gathered from different universities in China, and they are not very familiar with the candidate Ph.D. students whom they are evaluating. In such a case, it is straightforward for the experts to provide their evaluations over the Ph.D. students through pairwise comparison according to the performance of the applicants with respect to the criteria. The IFPR is an efficient tool to represent the experts' preference information. To simplify the presentation, suppose that there are five experts E_1, E_2, E_3, E_4, and E_5 who are evaluating four candidate Ph.D. students P_1, P_2, P_3, and P_4. Since there is no significant difference among the experts, the weights of them are set to be equal. The minimum majority degree φ is assumed to be $3/5$, which means that at least three out of five experts in the committee are needed in order to make a decision. Since the award are very attractive to Ph.D. students, the group of experts want to be unanimous in choosing the candidates and requires the minimum consensus degree $\gamma = 0.9$. Since there are four candidates, it is a little difficult for an expert to furnish a consistent IFPR. Thus, the consistency threshold ξ is set to be 0.85. After doing some pairwise comparisons, the experts construct five IFPRs, which are shown as

$$\tilde{R}^{(1)} = \begin{pmatrix} (0.5,0.5) & (0.5,0.2) & (0.7,0.1) & (0.5,0.3) \\ (0.2,0.5) & (0.5,0.5) & (0.6,0.2) & (0.3,0.6) \\ (0.1,0.7) & (0.2,0.6) & (0.5,0.5) & (0.3,0.6) \\ (0.3,0.5) & (0.6,0.3) & (0.6,0.3) & (0.5,0.5) \end{pmatrix}$$

$$\tilde{R}^{(2)} = \begin{pmatrix} (0.5,0.5) & (0.6,0.2) & (0.8,0.2) & (0.6,0.3) \\ (0.2,0.6) & (0.5,0.5) & (0.5,0.3) & (0.3,0.5) \\ (0.2,0.8) & (0.3,0.5) & (0.5,0.5) & (0.3,0.6) \\ (0.3,0.6) & (0.5,0.3) & (0.6,0.4) & (0.5,0.5) \end{pmatrix}$$

$$\tilde{R}^{(3)} = \begin{pmatrix} (0.5,0.5) & (0.6,0.2) & (0.8,0.1) & (0.6,0.2) \\ (0.2,0.6) & (0.5,0.5) & (0.6,0.3) & (0.3,0.4) \\ (0.1,0.8) & (0.3,0.6) & (0.5,0.5) & (0.2,0.5) \\ (0.2,0.6) & (0.4,0.3) & (0.5,0.2) & (0.5,0.5) \end{pmatrix}$$

$$\tilde{R}^{(4)} = \begin{pmatrix} (0.5,0.5) & (0.6,0.2) & (0.7,0.2) & (0.6,0.3) \\ (0.2,0.6) & (0.5,0.5) & (0.5,0.3) & (0.3,0.4) \\ (0.2,0.7) & (0.3,0.5) & (0.5,0.5) & (0.3,0.6) \\ (0.3,0.6) & (0.4,0.3) & (0.6,0.3) & (0.5,0.5) \end{pmatrix}$$

$$\tilde{R}^{(5)} = \begin{pmatrix} (0.5,0.5) & (0.7,0.1) & (0.6,0.2) & (0.4,0.3) \\ (0.1,0.7) & (0.5,0.5) & (0.6,0.1) & (0.3,0.6) \\ (0.2,0.6) & (0.1,0.6) & (0.5,0.5) & (0.3,0.5) \\ (0.3,0.4) & (0.6,0.3) & (0.5,0.3) & (0.5,0.5) \end{pmatrix}.$$

Below, let us use the proposed intuitionistic fuzzy group decision making algorithm to solve this problem. Step 1 has been done above; therefore, we departure the calculation process from Step 2.

Step 2: Construct five fractional programming models (we omit the models here) with respect to the five IFPRs according to Model 1. Then, use the optimization software package such as Lingo to solve these models and obtain the underlying intuitionistic fuzzy weighting vectors for these five individual IFPRs:

$$\tilde{\omega}^{(1)} = ((0.3951, 0.4221), (0.1354, 0.8397), (0.0451, 0.8894) \\ (0.2370, 0.6298))^T$$

$$\tilde{\omega}^{(2)} = ((0.4260, 0.5385), (0.1420, 0.7278), (0.0888, 0.9112) (0.2130, 0.6805))^T$$

$$\tilde{\omega}^{(3)} = ((0.4763, 0.3835), (0.1237, 0.8351), (0.0619, 0.9381) (0.1567, 0.6619))^T$$

$$\tilde{\omega}^{(4)} = ((0.4105, 0.4842), (0.1368, 0.6947), (0.0947, 0.9053) (0.1895, 0.7474))^T$$

$$\tilde{\omega}^{(5)} = ((0.3874, 0.4548), (0.1076, 0.8924), (0.1291, 0.7704) (0.2152, 0.7130))^T.$$

By (10), the corresponding multiplicative consistent IFPRs can be generated as

$$\tilde{R}^{(1)*} = \begin{pmatrix} (0.5000, 0.5000) & (0.6228, 0.2134) & (0.7001, 0.0799) & (0.5001, 0.3000) \\ (0.2134, 0.6228) & (0.5000, 0.5000) & (0.5999, 0.1998) & (0.2999, 0.5250) \\ (0.0799, 0.7001) & (0.1998, 0.5999) & (0.5000, 0.5000) & (0.1182, 0.6213) \\ (0.3000, 0.5001) & (0.5250, 0.2999) & (0.6213, 0.1182) & (0.5000, 0.5000) \end{pmatrix}$$

$$\tilde{R}^{(2)*} = \begin{pmatrix} (0.5000, 0.5000) & (0.6545, 0.2182) & (0.7999, 0.1667) & (0.6000, 0.3000) \\ (0.2182, 0.6545) & (0.5000, 0.5000) & (0.4799, 0.3001) & (0.3000, 0.4500) \\ (0.1667, 0.7999) & (0.3001, 0.4799) & (0.5000, 0.5000) & (0.2501, 0.5999) \\ (0.3000, 0.6000) & (0.4500, 0.3000) & (0.5999, 0.2501) & (0.5000, 0.5000) \end{pmatrix}$$

$$\tilde{R}^{(3)*} = \begin{pmatrix} (0.5000, 0.5000) & (0.6870, 0.1794) & (0.7803, 0.1020) & (0.5976, 0.1977) \\ (0.1794, 0.6870) & (0.5000, 0.5000) & (0.5999, 0.3002) & (0.3158, 0.4001) \\ (0.1020, 0.7803) & (0.3002, 0.5999) & (0.5000, 0.5000) & (0.2001, 0.5066) \\ (0.1977, 0.5976) & (0.4001, 0.3158) & (0.5066, 0.2001) & (0.5000, 0.5000) \end{pmatrix}$$

$$\tilde{R}^{(4)*} = \begin{pmatrix} (0.5000, 0.5000) & (0.6000, 0.1999) & (0.7359, 0.1698) & (0.6000, 0.2770) \\ (0.1999, 0.6000) & (0.5000, 0.5000) & (0.4333, 0.2999) & (0.3094, 0.4286) \\ (0.1698, 0.7359) & (0.2999, 0.4333) & (0.5000, 0.5000) & (0.2999, 0.6002) \\ (0.2770, 0.6000) & (0.4286, 0.3094) & (0.6002, 0.2999) & (0.5000, 0.5000) \end{pmatrix}$$

$$\tilde{R}^{(5)*} = \begin{pmatrix} (0.5000, 0.5000) & (0.6750, 0.1875) & (0.6000, 0.2000) & (0.5400, 0.3000) \\ (0.1875, 0.6750) & (0.5000, 0.5000) & (0.3750, 0.4499) & (0.3000, 0.5999) \\ (0.2000, 0.6000) & (0.4499, 0.3750) & (0.5000, 0.5000) & (0.2999, 0.4999) \\ (0.3000, 0.5400) & (0.5999, 0.3000) & (0.4999, 0.2999) & (0.5000, 0.5000) \end{pmatrix}.$$

According to (13), the consistency degree of these five IFPRs given by the experts are calculated as $C_{\tilde{R}^{(1)}} = 0.8621$, $C_{\tilde{R}^{(2)}} = 0.9246$, $C_{\tilde{R}^{(3)}} = 0.9553$, $C_{\tilde{R}^{(4)}} = 0.9453$, and $C_{\tilde{R}^{(5)}} = 0.8074$.

Step 3: Since only $C_{\tilde{R}^{(5)}}$ is less than the consistency threshold $\xi = 0.85$, we ask the fifth expert to modify his/her preferences because his/her IFPR $\tilde{R}^{(5)}$ is inconsistent.

Step 4: Based on the initial IFPR $\tilde{R}^{(5)}$ given by the fifth expert, to save his/her time and utilize the maximum information in $\tilde{R}^{(5)}$, we can give some advice to the fifth expert with an revised IFPR, which is derived by (22) and (23) with $\eta = 0.2$. Let $p = 1$ and $\tilde{R}^{(1)(5)} = \tilde{R}^{(5)}$; then, we have

$$\tilde{R}^{(2)(5)} = \begin{pmatrix} (0.5000, 0.5000) & (0.6949, 0.1134) & (0.6000, 0.2000) & (0.4247, 0.3000) \\ (0.1134, 0.6949) & (0.5000, 0.5000) & (0.5462, 0.1351) & (0.3000, 0.6000) \\ (0.2000, 0.6000) & (0.1351, 0.5462) & (0.5000, 0.5000) & (0.3000, 0.5000) \\ (0.3000, 0.4247) & (0.6000, 0.3000) & (0.5000, 0.3000) & (0.5000, 0.5000) \end{pmatrix}.$$

The underlying intuitionistic fuzzy weights of $\tilde{R}^{(2)(5)}$ is derived by Model 1

$$\tilde{\omega}^{(2)(5)} = ((0.3874, 0.4560), (0.1074, 0.8926) (0.1288, 0.7710), (0.2147, 0.7137))^T.$$

Thus, the corresponding multiplicative consistent IFPR for $\tilde{R}^{(2)(5)}$ is

$$\tilde{R}^{(2)(5)*} = \begin{pmatrix} (0.5000, 0.5000) & (0.6760, 0.1874) & (0.6010, 0.1998) & (0.5409, 0.2998) \\ (0.1874, 0.6760) & (0.5000, 0.5000) & (0.3751, 0.4499) & (0.3001, 0.5999) \\ (0.1998, 0.6010) & (0.4499, 0.3751) & (0.5000, 0.5000) & (0.3000, 0.5000) \\ (0.2998, 0.5409) & (0.5999, 0.3001) & (0.5000, 0.3000) & (0.5000, 0.5000) \end{pmatrix}.$$

With (13), the consistency degree of $\tilde{R}^{(2)(5)}$ is calculated as $C_{\tilde{R}^{(2)(5)}} = 0.8313 < \xi$. Hence, we have to iterate $\tilde{R}^{(2)(5)}$ again.

TABLE I
PAIRWISE DISTANCES OF THE FIVE EXPERTS

	E_1	E_2	E_3	E_4	E_5
E_1		0.0938	0.0602	0.1014	0.0714
E_2	0.0938		0.0910	0.0452	0.1157*
E_3	0.0602	0.0910		0.0980	0.1049
E_4	0.1014	0.0452	0.0980		0.1052
E_5	0.0714	0.1157	0.1049	0.1052	

Let $p = 2$; according to (22) and (23), we have

$$\tilde{R}^{(3)(5)} = \begin{pmatrix} (0.5000, 0.5000) & (0.6873, 0.1386) & (0.6004, 0.1999) & (0.4678, 0.2999) \\ (0.1386, 0.6873) & (0.5000, 0.5000) & (0.4700, 0.2186) & (0.3000, 0.6000) \\ (0.1999, 0.6004) & (0.2186, 0.4700) & (0.5000, 0.5000) & (0.3000, 0.5000) \\ (0.2999, 0.4687) & (0.6000, 0.3000) & (0.5000, 0.3000) & (0.5000, 0.5000) \end{pmatrix}.$$

Analogously, we can calculate that

$$\tilde{\omega}^{(3)(5)} = ((0.3889, 0.4532), (0.1079, 0.8921), (0.1295, 0.7698), (0.2158, 0.7122))^T$$

and the corresponding multiplicative consistent IFPR for $\tilde{R}^{(3)(5)}$

$$\tilde{R}^{(3)(5)*} = \begin{pmatrix} (0.5000, 0.5000) & (0.6755, 0.1874) & (0.6004, 0.1999) & (0.5404, 0.2999) \\ (0.1874, 0.6755) & (0.5000, 0.5000) & (0.3750, 0.4500) & (0.3000, 0.5999) \\ (0.1999, 0.6004) & (0.4500, 0.3750) & (0.5000, 0.5000) & (0.3000, 0.5000) \\ (0.2999, 0.5404) & (0.5999, 0.3000) & (0.5000, 0.3000) & (0.5000, 0.5000) \end{pmatrix}.$$

Thus, the consistency degree of $\tilde{R}^{(3)(5)}$ is $C_{\tilde{R}^{(3)(5)}} = 0.8824 > \xi$, which implies that $\tilde{R}^{(3)(5)}$ is of acceptable consistency. We, then, give this IFPR $\tilde{R}^{(3)(5)}$ to the fifth expert as a suggestion. If he/she does not accept $\tilde{R}^{(3)(5)}$, he/she should give another IFPR referring to $\tilde{R}^{(3)(5)}$ and make sure the new IFPR being given is of acceptable consistency; otherwise, we would check the consistency of the new IFPR again in the same way. Suppose that the expert accepts our advice and gives an updated IFPR as $\tilde{R}^{(3)(5)}$; then, we reach to the stage where all the IFPRs established by the experts are of acceptable consistency.

Step 5: Since there is no expert being excluded from the group, the majority degree of the group $\Psi = 1 > \varphi$. Then, we calculate all the pairwise distances $d(E_l, E_m)$ between the experts by (16), as shown in Table I.

Step 6: Since the maximum $d(E_l, E_m)$ is $d(E_2, E_5) = 0.1157$, according to Definition 5, the consensus degree of this group is $\Gamma = 0.8843 < \gamma$. Hence, we should go into the sensus-reaching process.

TABLE II
PAIRWISE DISTANCES WITHOUT THE EXPERT E_2

	E_1	E_3	E_4	E_5
E_1		0.0602	0.1014	0.0714
E_3	0.0602		0.0980	0.1049*
E_4	0.1014	0.0980		0.1052
E_5	0.0714	0.1049	0.1052	

TABLE III
PAIRWISE DISTANCES WITHOUT THE EXPERT E_5

	E_1	E_2	E_3	E_4
E_1		0.0938	0.0602	0.1014*
E_2	0.0938		0.0910	0.0452
E_3	0.0602	0.0910		0.0980
E_4	0.1014	0.0452	0.0980	

TABLE IV
PAIRWISE DISTANCES WITHOUT THE EXPERTS E_5 AND E_1

	E_2	E_3	E_4
E_2		0.0910	0.0452
E_3	0.0910		0.0980*
E_4	0.0452	0.0980	

TABLE V
PAIRWISE DISTANCES WITHOUT THE EXPERTS E_5 AND E_4

	E_1	E_2	E_3
E_1		0.0938*	0.0602
E_2	0.0938		0.0910
E_3	0.0602	0.0910	

Step 7: First, ask if some of the experts would like to modify their preferences. Here, we suppose that no expert agrees to change their mind with no tips.

Step 8: Since $d(E_2, E_5)$ is the furthest distance, one of them should be asked to change his/her preferences. Remove E_2 and E_5 from the group, respectively; then, we get the corresponding maximum distance of the reminder group as $d(E_3, E_5) = 0.1049$, $d(E_1, E_4) = 0.1014$, respectively (See Tables II and III). Since $d(E_1, E_4) < d(E_3, E_5)$, the fifth expert should be asked to change his preferences.

Suppose that expert E_5 claims that he would not change his mind any more. Hence, the analyst or the decision maker excludes E_5 from the group, and the rest of the experts then form a new group and try to reach an acceptable consensus. In this case, $d(E_1, E_4)$ then turns out to be the furthest distance and $\Gamma = 1 - 0.1014 = 0.8986 < \gamma$, which implies that new group still does not reach an acceptable consensus. In the same way illustrated as above, we can find out that E_4 is required to revise his preferences (For details, see Tables IV and V).

Suppose that E_4 is cooperative and willing to follow our advice and furnishes a new IFPR referring to the other experts'

TABLE VI
PAIRWISE DISTANCES OF THE EXPERTS IN THE NEW GROUP

	E_1	E_2	E_3	E_4
E_1		0.0938*	0.0602	0.0319*
E_2	0.0938		0.0910	0.0697
E_3	0.0602	0.0910		0.0503
E_4	0.0319	0.0697	0.0503	

persuasion and suggestion, as follows:

$$\tilde{R}^{(4)} = \begin{pmatrix} (0.5, 0.5) & (0.6, 0.2) & (0.7, 0.2) & (0.5, 0.3) \\ (0.2, 0.6) & (0.5, 0.5) & (0.6, 0.3) & (0.3, 0.5) \\ (0.2, 0.7) & (0.3, 0.6) & (0.5, 0.5) & (0.3, 0.6) \\ (0.3, 0.5) & (0.5, 0.3) & (0.6, 0.3) & (0.5, 0.5) \end{pmatrix}.$$

Its underlying intuitionistic fuzzy weighting vector can be calculated and obtained as follows:

$$\tilde{\omega}^{(4)} = ((0.3793, 0.4492), (0.1449, 0.8156)$$
$$(0.0768, 0.9232), (0.2276, 0.6405))^T$$

and the corresponding multiplicative consistent IFPR is

$$\tilde{R}^{(4)*} = \begin{pmatrix} (0.5000, 0.5000) & (0.6024, 0.2301) & (0.7000, 0.1417) & (0.5000, 0.3000) \\ (0.2301, 0.6024) & (0.5000, 0.5000) & (0.6001, 0.3181) & (0.3162, 0.4967) \\ (0.1417, 0.7000) & (0.3181, 0.6001) & (0.5000, 0.5000) & (0.2074, 0.6146) \\ (0.3000, 0.5000) & (0.4967, 0.3162) & (0.6146, 0.2074) & (0.5000, 0.5000) \end{pmatrix}.$$

Thus, the consistency degree of this new IFPR is calculated, such as $C_{\tilde{R}^{(4)}} = 0.9274 > \xi$, which implies that the new IFPR given by the expert E_4 is of acceptable consistency.

By (16), the pairwise distances over these four experts can be derived, as shown in Table VI.

From Table VI, we can find that the $d(E_1, E_2)$ is the furthest distance, and thus, $\Gamma = 1 - 0.0938 = 0.9062 > \gamma$, which implies that new group reaches the group consensus.

Step 9: Aggregate the underlying weighting vectors of the experts E_1, E_2, E_3, and E_4 into an overall priority vector. As all the professors in the expert committee are treated equally, they should have equal weights. Using the IFWG aggregation operator shown as (20), we can obtain the overall weights of the Ph.D. students

$$\tilde{\omega} = ((0.4196, 0.4448), (0.1365, 0.8032)$$
$$(0.0683, 0.9153), (0.2092, 0.6529))^T.$$

Using (4), we have $L(\tilde{\omega}_1) = 0.4889$, $L(\tilde{\omega}_2) = 0.1856$, $L(\tilde{\omega}_3) = 0.0833$, and $L(\tilde{\omega}_4) = 0.3050$. Since $L(\tilde{\omega}_1) > L(\tilde{\omega}_4) > L(\tilde{\omega}_2) > L(\tilde{\omega}_3)$, according to Scheme 1, the ranking of the Ph.D. students should be $P_1 \succ P_4 \succ P_2 \succ P_3$, which is to say, the first Ph.D. student is the most outstanding student according to the experts' evaluation and, thus, should be supported by the CSC for overseas study.

B. Some Discussion

In Section VI-B, some indepth comparison analysis between our new algorithm and the existing methods for intuitionistic fuzzy group decision making is given. In order to further demonstrate the advantages of our iterative approach against these existing methodologies, below, we also use these methods, respectively, to solve the problem.

1) Using Xu's [2] Method:

Step 1: Derive the averaged intuitionistic fuzzy preference value for each Ph.D. candidate with respect to each expert by the IFAA operator

$$r_1^{(1)} = (0.5599, 0.2340), \quad r_2^{(1)} = (0.4215, 0.4162)$$
$$r_3^{(1)} = (0.2915, 0.5958), \quad r_4^{(1)} = (0.5135, 0.3873)$$
$$r_1^{(2)} = (0.6443, 0.2783), \quad r_2^{(2)} = (0.3883, 0.4606)$$
$$r_3^{(2)} = (0.3346, 0.5886), \quad r_4^{(2)} = (0.4856, 0.4356)$$
$$r_1^{(3)} = (0.6443, 0.2115), \quad r_2^{(3)} = (0.4215, 0.4356)$$
$$r_3^{(3)} = (0.2915, 0.5886), \quad r_4^{(3)} = (0.4114, 0.3663)$$
$$r_1^{(4)} = (0.6064, 0.2783), \quad r_2^{(4)} = (0.3883, 0.4356)$$
$$r_3^{(4)} = (0.3346, 0.5692), \quad r_4^{(4)} = (0.4616, 0.4054)$$
$$r_1^{(5)} = (0.5644, 0.2340), \quad r_2^{(5)} = (0.4042, 0.3807)$$
$$r_3^{(5)} = (0.2915, 0.5477), \quad r_4^{(5)} = (0.4856, 0.3663).$$

Step 2: Calculate the collective preference values for Ph.D. students by the IFWAA operator

$$r_1 = (0.6056, 0.2458), \quad r_2 = (0.4049, 0.4249)$$
$$r_3 = (0.3091, 0.5777), \quad r_4 = (0.4726, 0.3913).$$

Step 3: Rank the Ph.D. students according to the collective preference values. As the score values of these preference values are $\Delta(r_1) = 0.3598$, $\Delta(r_2) = -0.0200$, $\Delta(r_3) = -0.2686$, and $\Delta(r_4) = 0.0813$, the final ranking of these four Ph.D. students is $P_1 \succ P_4 \succ P_2 \succ P_3$, which is the same as that derived by our approach.

2) Using Xu and Xia's [16] Method: By [16, Alg. 1], the multiplicative consistent IFPRs $\bar{\tilde{R}}^{(l)}$ of $\tilde{R}^{(l)}$ $(l = 1, 2, 3, 4, 5)$

are constructed as follows:

$$\tilde{\bar{R}}^{(1)} = \begin{pmatrix} (0.5000, 0.5000) & (0.5000, 0.2000) & (0.6000, 0.0588) & (0.3913, 0.0857) \\ (0.2000, 0.5000) & (0.5000, 0.5000) & (0.6000, 0.2000) & (0.3913, 0.2727) \\ (0.0588, 0.6000) & (0.2000, 0.6000) & (0.5000, 0.5000) & (0.3000, 0.6000) \\ (0.0857, 0.3913) & (0.2727, 0.3913) & (0.6000, 0.3000) & (0.5000, 0.5000) \end{pmatrix}$$

$$\tilde{\bar{R}}^{(2)} = \begin{pmatrix} (0.5000, 0.5000) & (0.6000, 0.2000) & (0.6000, 0.0968) & (0.3913, 0.1385) \\ (0.2000, 0.6000) & (0.5000, 0.5000) & (0.5000, 0.3000) & (0.3000, 0.3913) \\ (0.0968, 0.6000) & (0.3000, 0.5000) & (0.5000, 0.5000) & (0.3000, 0.6000) \\ (0.1385, 0.3913) & (0.3913, 0.3000) & (0.6000, 0.3000) & (0.5000, 0.5000) \end{pmatrix}$$

$$\tilde{\bar{R}}^{(3)} = \begin{pmatrix} (0.5000, 0.5000) & (0.6000, 0.2000) & (0.6923, 0.0968) & (0.3600, 0.0968) \\ (0.2000, 0.6000) & (0.5000, 0.5000) & (0.6000, 0.3000) & (0.2727, 0.3000) \\ (0.0968, 0.6923) & (0.3000, 0.6000) & (0.5000, 0.5000) & (0.2000, 0.5000) \\ (0.0968, 0.3600) & (0.3000, 0.2727) & (0.5000, 0.2000) & (0.5000, 0.5000) \end{pmatrix}$$

$$\tilde{\bar{R}}^{(4)} = \begin{pmatrix} (0.5000, 0.5000) & (0.6000, 0.2000) & (0.6000, 0.0968) & (0.3913, 0.1385) \\ (0.2000, 0.6000) & (0.5000, 0.5000) & (0.5000, 0.3000) & (0.3000, 0.3913) \\ (0.0968, 0.6000) & (0.3000, 0.5000) & (0.5000, 0.5000) & (0.3000, 0.6000) \\ (0.1385, 0.3913) & (0.3913, 0.3000) & (0.6000, 0.3000) & (0.5000, 0.5000) \end{pmatrix}$$

$$\tilde{\bar{R}}^{(5)} = \begin{pmatrix} (0.5000, 0.5000) & (0.7000, 0.1000) & (0.7778, 0.0122) & (0.6000, 0.0122) \\ (0.1000, 0.7000) & (0.5000, 0.5000) & (0.6000, 0.1000) & (0.3913, 0.1000) \\ (0.0122, 0.7778) & (0.1000, 0.6000) & (0.5000, 0.5000) & (0.3000, 0.5000) \\ (0.0122, 0.6000) & (0.1000, 0.3913) & (0.5000, 0.3000) & (0.5000, 0.5000) \end{pmatrix}.$$

Aggregate all the individual multiplicative consistent IFPRs $\tilde{\bar{R}}^{(l)}$ $(l = 1, 2, 3, 4, 5)$ into a group IFPR $\bar{\tilde{R}}$ by [16, eq. (40)]

$$\bar{\tilde{R}} = \begin{pmatrix} (0.5000, 0.5000) & (0.6017, 0.1753) & (0.6584, 0.0588) & (0.4258, 0.0738) \\ (0.1753, 0.6017) & (0.5000, 0.5000) & (0.5605, 0.2270) & (0.3292, 0.2725) \\ (0.0588, 0.6584) & (0.2270, 0.5605) & (0.5000, 0.5000) & (0.2779, 0.5605) \\ (0.0738, 0.4258) & (0.2725, 0.3292) & (0.5605, 0.2779) & (0.5000, 0.5000) \end{pmatrix}.$$

The deviations between each individual IFPR $\tilde{\bar{R}}^{(l)}$ and the group IFPR $\bar{\tilde{R}}$ are calculated by [16, eq. (41)], which are $d_2(\tilde{\bar{R}}^{(1)}, \bar{\tilde{R}}) = 0.0351$, $d_2(\tilde{\bar{R}}^{(2)}, \bar{\tilde{R}}) = 0.0471$, $d_2(\tilde{\bar{R}}^{(3)}, \bar{\tilde{R}}) = 0.0427$, $d_2(\tilde{\bar{R}}^{(4)}, \bar{\tilde{R}}) = 0.0471$, and $d_2(\tilde{\bar{R}}^{(5)}, \bar{\tilde{R}}) = 0.0883$. Since all $d_2(\tilde{\bar{R}}^{(l)}, \bar{\tilde{R}}) < 0.1$, $l = 1, 2, 3, 4, 5$, then according to the Xu and Xia multiplicative consistency definition, all the above five individual IFPRs are consistent. Thus, we use the SIFA operator defined as in [16, eq. (44)] to aggregate all the intuitionistic fuzzy preference values in $\bar{\tilde{R}}$ into the overall preference values

$$r_1 = (0.5408, 0.1528), \quad r_2 = (0.3765, 0.3897)$$
$$r_3 = (0.2247, 0.5709), \quad r_4 = (0.3064, 0.3796).$$

By [16, eq. (45)], we obtain $s(r_1) = 0.2999$, $s(r_2) = 0.3846$, $s(r_3) = 0.4669$, and $s(r_4) = 0.4557$. Since $s(r_3) > s(r_4) > s(r_2) > s(r_1)$, then the ranking of the Ph.D. students is $P_1 \succ P_2 \succ P_4 \succ P_3$, which is slightly different from the results derived by our approach, as well as the Xu [2] method as the positions of the Ph.D. students P_2 and P_4 are changed.

3) Using Liao and Xu's [17] Method: According to Model 5 in [17], a fractional programming model is constructed in Model 2, shown at the bottom of the next page.

Solving this model with the Lingo software, it follows that the objective function value is $f^* = 0.2777$, and the optimal intuitionistic fuzzy weights are

$$\tilde{\omega} = ((0.4106, 0.4525), (0.1460, 0.7354), (0.0913, 0.9087), (0.2129, 0.6520))^T.$$

By (5), it follows that $L(\tilde{\omega}_1) = 0.4816$, $L(\tilde{\omega}_2) = 0.2365$, $L(\tilde{\omega}_3) = 0.0913$, and $L(\tilde{\omega}_4) = 0.3066$; thus, $L(\tilde{\omega}_1) > L(\tilde{\omega}_4) > L(\tilde{\omega}_2) > L(\tilde{\omega}_3)$. According to Scheme 1, the ranking of the Ph.D. students is $P_1 \succ P_4 \succ P_2 \succ P_3$, which is the same as that derived by our approach and also Xu's [2] method.

The above numerical example shows that Xu's, Liao and Xu's, and our approaches produce the same ranking order $P_1 \succ P_4 \succ P_2 \succ P_3$, while Xu and Xia's method derives a slightly different order in which the positions of P_2 and P_4 are changed. Although these methods can generate the same result or slightly different ones, we still can find out some significant drawbacks of the previously approaches. According to Step 4 of our approach, we can see that the fifth IFPR $\tilde{R}_5$ is inconsistent; thus, it should be repaired. However, in Xu's approach, the consistency of IFPRs is not taken into account; thus, the result is in fact not theoretically convincing. Even though Xu and Xia's method considers the multiplicative consistency, the definition they used does not generally hold as we see that via their definition, all of those five IFPRs are consistent, which is not in consistent with our result. Most importantly, from the above example, we can see that all of the Xu's, Xu and Xia's, and Liao and Xu's methods do not consider the consensus reaching process. As consensus is critical to make sure that the final result is accepted by all the individuals, our approach proposed in this paper is much more convincing and applicable.

Roughly speaking, Liao and Xu's method is quite simple as it neither considers the consistency of each IFPR nor takes the consensus of the group into account; meanwhile, there is no need to do some aggregation calculation as well. Xu's method is also very easy because with this method only some aggregation operations are needed. In contrast, our approach seems to be much more complicated. However, our method is much more reasonable and closer to the real decision practice due to the fact that it contains many interactive actions. The first interactive activity takes place in the inconsistency repairing process in which we ask the fifth expert E_5 whose IFPR $\tilde{R}_5$ is of unacceptable consistency to reevaluate the Ph.D. students and reconstruct a new IFPR with higher consistency. During this process, generally, some experts who are not willing to cooperate are excluded from further decision making process. Another interactive action happens in the consensus reaching process. When the group does not reach the minimum consensus required by the decision maker (for example, in Step 6, $\Gamma = 0.8843 < \gamma$), then we would first ask the experts to have some discussion and adjust their preferences. As no one agrees to change his/her opinions, we then find out an expert E_5 who should modify his/her judgments and asks the expert E_5 to change the inappropriate preferences. As he/she is not willing to do so, then we exclude him/her from the final group in order to find a solution which is agreed by most individuals. This is done over the experts one by one. These interactive activities make our algorithm very flexible. Most importantly, it fits well to the practical decision making process.

Model 2:

$$\begin{aligned}\text{Min } f = {} & (\bar{\varepsilon}_{12}^{+} + \bar{\varepsilon}_{12}^{-} + \bar{\xi}_{12}^{+} + \bar{\xi}_{12}^{-}) + (\bar{\varepsilon}_{13}^{+} + \bar{\varepsilon}_{13}^{-} + \bar{\xi}_{13}^{+} + \bar{\xi}_{13}^{-}) \\ & + (\bar{\varepsilon}_{14}^{+} + \bar{\varepsilon}_{14}^{-} + \bar{\xi}_{14}^{+} + \bar{\xi}_{14}^{-}) + (\bar{\varepsilon}_{23}^{+} + \bar{\varepsilon}_{23}^{-} + \bar{\xi}_{23}^{+} + \bar{\xi}_{23}^{-}) \\ & + (\bar{\varepsilon}_{24}^{+} + \bar{\varepsilon}_{24}^{-} + \bar{\xi}_{24}^{+} + \bar{\xi}_{24}^{-}) + (\bar{\varepsilon}_{34}^{+} + \bar{\varepsilon}_{34}^{-} + \bar{\xi}_{34}^{+} + \bar{\xi}_{34}^{-})\end{aligned}$$

$$\text{s.t.}\quad \begin{cases} \dfrac{2\omega_1^\mu}{\omega_1^\mu - \omega_1^v + \omega_2^\mu - \omega_2^v + 2} - 0.6 - \bar{\varepsilon}_{12}^{+} + \bar{\varepsilon}_{12}^{-} = 0; \quad \dfrac{2\omega_1^\mu}{\omega_1^\mu - \omega_1^v + \omega_3^\mu - \omega_3^v + 2} - 0.72 - \bar{\varepsilon}_{13}^{+} + \bar{\varepsilon}_{13}^{-} = 0 \\ \dfrac{2\omega_1^\mu}{\omega_1^\mu - \omega_1^v + \omega_4^\mu - \omega_4^v + 2} - 0.54 - \bar{\varepsilon}_{14}^{+} + \bar{\varepsilon}_{14}^{-} = 0; \quad \dfrac{2\omega_2^\mu}{\omega_2^\mu - \omega_2^v + \omega_3^\mu - \omega_3^v + 2} - 0.56 - \bar{\varepsilon}_{23}^{+} + \bar{\varepsilon}_{23}^{-} = 0 \\ \dfrac{2\omega_2^\mu}{\omega_2^\mu - \omega_2^v + \omega_4^\mu - \omega_4^v + 2} - 0.3 - \bar{\varepsilon}_{24}^{+} + \bar{\varepsilon}_{24}^{-} = 0; \quad \dfrac{2\omega_3^\mu}{\omega_3^\mu - \omega_3^v + \omega_4^\mu - \omega_4^v + 2} - 0.28 - \bar{\varepsilon}_{34}^{+} + \bar{\varepsilon}_{34}^{-} = 0 \\ \dfrac{2\omega_2^\mu}{\omega_1^\mu - \omega_1^v + \omega_2^\mu - \omega_2^v + 2} - 0.18 - \bar{\xi}_{12}^{+} + \bar{\xi}_{12}^{-} = 0; \quad \dfrac{2\omega_3^\mu}{\omega_1^\mu - \omega_1^v + \omega_3^\mu - \omega_3^v + 2} - 0.16 - \bar{\xi}_{13}^{+} + \bar{\xi}_{13}^{-} = 0 \\ \dfrac{2\omega_4^\mu}{\omega_1^\mu - \omega_1^v + \omega_4^\mu - \omega_4^v + 2} - 0.28 - \bar{\xi}_{14}^{+} + \bar{\xi}_{14}^{-} = 0; \quad \dfrac{2\omega_3^\mu}{\omega_2^\mu - \omega_2^v + \omega_3^\mu - \omega_3^v + 2} - 0.24 - \bar{\xi}_{23}^{+} + \bar{\xi}_{23}^{-} = 0 \\ \dfrac{2\omega_4^\mu}{\omega_2^\mu - \omega_2^v + \omega_4^\mu - \omega_4^v + 2} - 0.5 - \bar{\xi}_{24}^{+} + \bar{\xi}_{24}^{-} = 0; \quad \dfrac{2\omega_4^\mu}{\omega_3^\mu - \omega_3^v + \omega_4^\mu - \omega_4^v + 2} - 0.56 - \bar{\xi}_{34}^{+} + \bar{\xi}_{34}^{-} = 0 \\ 0 \le \omega_1^\mu \le 1,\quad 0 \le \omega_2^\mu \le 1,\ 0 \le \omega_3^\mu \le 1,\quad 0 \le \omega_4^\mu \le 1,\ 0 \le \omega_1^v \le 1,\ 0 \le \omega_2^v \le 1,\ 0 \le \omega_3^v \le 1,\ 0 \le \omega_4^v \le 1, \\ \omega_1^\mu + \omega_1^v \le 1,\quad \omega_2^\mu + \omega_2^v \le 1,\quad \omega_3^\mu + \omega_3^v \le 1,\quad \omega_4^\mu + \omega_4^v \le 1, \\ \omega_2^\mu + \omega_3^\mu + \omega_4^\mu \le \omega_1^v,\quad \omega_1^\mu + \omega_3^\mu + \omega_4^\mu \le \omega_2^v,\quad \omega_1^\mu + \omega_2^\mu + \omega_4^\mu \le \omega_3^v,\quad \omega_1^\mu + \omega_2^\mu + \omega_3^\mu \le \omega_4^v, \\ \omega_1^\mu + 2 \ge \omega_2^v + \omega_3^v + \omega_4^v,\quad \omega_2^\mu + 2 \ge \omega_1^v + \omega_3^v + \omega_4^v,\quad \omega_3^\mu + 2 \ge \omega_1^v + \omega_2^v + \omega_4^v,\quad \omega_4^\mu + 2 \ge \omega_1^v + \omega_2^v + \omega_3^v, \\ \bar{\varepsilon}_{ij}^{+} \ge 0,\quad \bar{\varepsilon}_{ij}^{-} \ge 0,\quad \bar{\xi}_{ij}^{+} \ge 0,\quad \bar{\xi}_{ij}^{-} \ge 0,\quad \text{for } i = 1,2,3;\quad j = 2,3,4 \\ \bar{\varepsilon}_{ij}^{+} \cdot \bar{\varepsilon}_{ij}^{-} = 0,\quad \bar{\xi}_{ij}^{+} \cdot \bar{\xi}_{ij}^{-} = 0,\qquad \text{for } i = 1,2,3;\quad j = 2,3,4. \end{cases}$$

VIII. Conclusion

In this paper, we have investigated the intuitionistic fuzzy group decision making problem in which all the experts' preference information is represented by IFPRs. A systematical framework for intuitionistic fuzzy group decision making has been proposed. This complex group decision making problem has been divided into three subproblems, which are the consistency checking and inconsistency repairing process, the consensus checking and reaching processes and the selection process. We then have introduced a useful consistency checking method, which is based on the multiplicative consistency of IFPRs. A novel inconsistency repairing method has been developed. Furthermore, after giving the consensus measure, an interesting consensus reaching process has been suggested. A step-by-step algorithm for group decision making with intuitionistic fuzzy preference information has been given for application. The numerical example concerning the selection of outstanding Ph.D. students for CSC has shown that our algorithm is valid and efficient. Finally, we have made some comparison analysis to show the advantages of our approach.

Based on the theoretical and numerical analysis between our approach and the existing intuitionistic fuzzy group decision making methodologies in the literature, we can find that our method is the most comprehensive and convincing one among them as it takes the integral framework of intuitionistic fuzzy group decision making into account, including the consistency checking and inconsistency repairing process, the consensus reaching process, and the selection process, while all the existing methods only focus on one or two process(es). Moreover, our approach contains many interactive actions; thus, it is very flexible and can match the practical group decision making situation perfectly. In the future, we will apply the consensus reaching method to multiple criteria group decision making with intuitionistic fuzzy judgment information.

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