# Reactive Power Generation Management for the Improvement of Power System Voltage Stability Margin<sup>\*</sup>

Yongchun Su, Shijie Cheng, Jinyu Wen

Institute of Electrical and Electronic Engineering HuaZhong University of Science and Technology Wuhan 430074, P.R China

## ychunsu@163.com

Abstract - Voltage stability margin (VSM) of the power system relates to the reactive power reserves in the network. This paper presents a method to improve the VSM by generator reactive power generation rescheduling. The management of the var generation formulated as an optimization problem and pseudo-gradient evolutionary programming (PGEP) was used to obtain the optimal solution. Modal analysis technique was used to guide the searching direction. Simulation results on the New England 39-bus system demonstrate that the proposed method is effective. Compared with the standard evolutionary programming (SEP), better solution can be obtained, and the convergence speed of the algorithm is improved also. The simulation results show that after the optimal reactive power rescheduling the reactive power reserves of the system is increased and the active/reactive power losses are decreased. The most important advantage is that, the voltage stability margin of power system can be improved without adding new var compensation equipment and changing the active power distribution.

### Index Terms – Power system, Voltage stability margin, Pseudogradient evolutionary programming, Modal analysis.

## I. INTRODUCTION

In recent years, much attention has been attracted to the problems of voltage stability for the electric power systems. Because of the continuous increase of power demands and large scale system interaction as well as the consideration of both the economic benefits and the environment protection, modern power systems are operated more and more close to their maximum operation conditions. Under such conditions, voltage instability of the systems likely occurs, which will, then, heavily threaten the stable operation of overall power systems. Power system voltage instability may be initiated by a disturbance or an accident under the conditions that are usually characterized by shortage of reactive power reserves. Hence, voltage stability of power systems has been closely linked with the reactive power reserves of the systems. As voltage collapse is associated with the fact that the reactive power demands is not able to met due to the limitations of the production and the transmission of the reactive power, the amount of reactive power reserves at generating stations can be used as a measure for the power system voltage stability. Over the years, system operators have relied on generator var reserves to gauge the voltage stability level of a power system.

Yonggao Zhang

School of Electrical and Electronics Engineering East China Jiaotong University Nanchang 330013, P.R China

The reactive power reserves of the key generators in the grid are also used by some practical equipment for online voltage stability monitoring [1].

The understanding of the power system voltage stability relating to dynamic var reserves has led to the following research efforts on this subject. The relationship of reactive power reserves and VSM is quantitatively analyzed in [1]. In [2] the voltage stability criteria is met by simultaneously changing the active and reactive power distribution, which will have great influence on the system operation scheme. In [3], the reactive generations are rescheduled to improve the voltage stability. However, in this research, the active power losses were not considered and obviously the solution can not be the optimum.

This paper proposed a new method for the management of the reactive power reserves. In the proposed method, the management of the reactive power reserves is processed as an optimization problem. The main objective of the optimization is to increase the reactive power reserves as well as to decrease the active power losses by rescheduling the reactive power injection of the generator units. As a result, the voltage stability margin will be improved with no negative impact on the active economical dispatch. The optimization problem is solved with a pseudo-gradient evolutionary programming algorithm, which incorporates the advantages of evolutionary programming in finding out the global optimal solution and the gradient method in increasing the convergence speed. The optimal solution is searched in the direction provided by the modal participation factors calculated for generator units. Simulation results show that the proposed method is effective. The system reactive power reserves as well as the voltage stability margin were improved and the active power losses were decreased.

## II. REACTIVE POWER RESERVES OPTIMIZATION

## A. Mathematic Model of the Objective Function

As has been mentioned in the previous section the voltage stability of a power system can be represented, in some way, by the voltage stability margin. The voltage stability margin is usually measured by the distance in MW or percentage between the current operating point and the maximum operating point (corresponding to the nose point of PV curve). Giving a load increasing mode and a generation distributing

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mode, the PV curve can be acquired by continuous load flow method. This method is used in the current research to calculate the PV curve and then the voltage stability margin.

The modal analysis on generators indicates that voltage stability can be improved by regulating the reactive power generations along the direction guided by the active participation factor (APF) of each generator in the power networks. According to this principle, the objective function of this optimization problem should contain two parts: maximizing the reactive power reserves (equal to minimize the var generations) and minimizing the active power losses. Therefore, the following objective function is obtained. It should be pointed out that in the objective function given in (1), maximizing the reactive power reserves has been changed to minimizing the reactive power generation of generator units

$$F = Min(W_1 \sum Q_{Gi} + W_2 P_{LOSS})$$
(1)

where  $Q_{Gi}$  is the reactive power generation of the *i*th generator, *i*=1,2,...,*N<sub>G</sub>*, *N<sub>G</sub>* is the number of generator units in the system;  $W_1, W_2$  are the weight factors;  $P_{LOSS}$  is the total active power loss in the network.

The constraints to the above mention optimization are given as following

$$\begin{cases} P_i = V_i \sum_{j=1}^{N} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \\ Q_i = V_i \sum_{j=1}^{N} V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \end{cases} \quad i \in N$$

$$(2)$$

 $\begin{bmatrix} Q_i = V_i \sum_{j=1}^{N} V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \\ V_{i\max} > V_i > V_{i\min} \quad i \in N \end{bmatrix}$ (3)

$$Q_{Gi\max} > Q_{Gi} > Q_{Gi\min} \quad i \in N_G \tag{4}$$

$$T_{i\max} > T_i > T_{i\min} \qquad i \in N_T \tag{5}$$

Equation (2) is the power flow equations, where  $P_i$  and  $Q_i$  are the active/reactive power injected into network at bus *i*;  $G_{ij}$  and  $B_{ij}$  represent the corresponding elements of the admittance matrices;  $\theta_{ij}$  is the voltage angle difference between bus *i* and *j*;  $V_i$  and  $V_j$  are the voltage amplitude at bus *i* and *j*; *N* is the number of buses in the network. Equation (3), (4) and (5) are the maximum and minimum constraints for the voltage at bus *i*, the reactive power injection of the ith generator and the tap position of the *i*th transformer. *N*,  $N_G$  and  $N_T$  are the number of the bus, the generator units and the tap changed transformers in the power network.

## B. Pseudo-gradient Evolutionary Programming

The objective function given in (1) shows that this is a mixed optimization problem in nature and the traditional gradient based optimal searching methods may not very efficient in processing such problem. However, recent research results show that application of evolutionary algorithms (EAs) provides an effective way to process the mixed optimization issues [4]. EAs are basically probabilistic searching methods and show excellent robustness in finding out the optimal solution. However, the standard evolutionary programming needs much calculating time for the convergence which greatly limits its application. On the other hand, the gradient based optimization methods have the

advantage of fast convergence speed, although the local optimal solution may be obtained when the multiple peak values exist for the objective functions. Recent research result indicates that both advantages of rapid convergence and the global optimal solution searching can be acquired by incorporating the SEP with the gradient based methods [5].

1) *Pseudo-Gradient*: For an *n*-dimensional parameter optimization problem, the conventional gradient  $\vec{g}$  of the objective function *f* is defined as an *n*-dimensional vector if the objective function is differentiable

$$\vec{g}(\vec{x}) = \nabla f(\vec{x}) = \left(\frac{\partial f}{x_1}, \frac{\partial f}{x_2}, \dots, \frac{\partial f}{x_n}\right)^T$$
(6)

Gradient  $\vec{g}$  gives the information about the direction along which the objective function will be changed with the fastest speed. This is a very important feature of the gradient methods. Using this feature, the pseudo-gradient  $\vec{g}_p$  is defined as following:

a) When  $f(\vec{x}_k) < f(\vec{x}_{k-1})$ , searching likes a "down hill" move and the direction of the move from  $\vec{x}_{k-1}$  to  $\vec{x}_k$  is defined as positive and the pseudo-gradient  $\vec{g}_p(\vec{x}_k)$  is defined as follow

$$\vec{g}_{p}(\vec{x}_{k}) = (dir(x_{k,1}), dir(x_{k,2}), ..., dir(x_{k,n}))^{T}$$
 (7)

Where

$$dir(x_{k,i}) = \begin{cases} 1 & x_{k,i} > x_{k-1,i} \\ 0 & x_{k,i} = x_{k-1,i} \\ -1 & x_{k,i} < x_{k-1,i} \end{cases} \quad i = 1, 2, ..., n$$
(8)

b) When  $f(\vec{x}_k) \ge f(\vec{x}_{k-1})$ , searching likes an "up hill" move and the direction of the move is defined as negative and the pseudo-gradient is defined as

$$\vec{g}_p(\vec{x}_k) = 0 \tag{9}$$

In the same way that the conventional gradient method yielding points toward a solution, the pseudo-gradient is able to identify a good search direction based on the latest two points in the search space. From the definition given above, we can see that if  $\vec{g}_p(\vec{x}_k) \neq 0$  a better solution of the minimization problem would be found at the next step by following the direction indicated by  $\vec{g}_p(\vec{x}_k)$ . Otherwise, the search direction at the point should be changed; in this situation, a randomly selected direction is used. The advantage of the pseudo-gradient is that it gives a good search direction without requiring the objective function to be differentiable. If  $\vec{g}_p$  is implemented in EA, an EA will still be problem-independent, which is an important feature of EA applications.

2) PGEP: In SEP, a "child" is generated from its "parent" by mutation. The mutation process is performed on each individual as follows:

$$\vec{x}_{(k+m),i}^{t} = \vec{x}_{k,i}^{t} + N(0, \sigma_{k}^{t}) \quad (i=1,...,n; k=1,...,m)$$
 (10)

Where  $\vec{x}_{k,i}^t$  is the *i*th element of the parent  $\vec{x}_k^t$ , the *k*th individual in the population of the *t*th generation, *m* is the population size,  $N(0, \sigma_k^t)$  is a Gaussian distribution variable

with mean 0 and variance  $\sigma_k^t$ . Using (7) and (9) by replacing  $\vec{x}_{k-1,i}$  and  $\vec{x}_{k,i}$  with  $\vec{x}_{k,i}^t$  and  $\vec{x}_{(k+m),i}^t$ , the pseudo-gradient of the child  $\vec{x}_{(k+m)}^t$  can be calculated from its parent and itself. To start the process, the pseudo-gradient for each individual in the initial population is given by modal analysis, which will be shown in next part *C*. The new mutation process is changed as the following:

$$\vec{x}_{(k+m),i}^{t} = \begin{cases} \vec{x}_{k,i}^{t} + dir(\vec{x}_{k,i}^{t}) | N(0, \sigma_{k}^{t}) | & \vec{g}_{p}(\vec{x}_{k}^{t}) \neq 0\\ \vec{x}_{k,i}^{t} + N(0, \sigma_{k}^{t}) & \vec{g}_{p}(\vec{x}_{k}^{t}) = 0 \end{cases}$$
(11)

The other operations of the PGEP are the same as those of the SEP. The algorithm convergence rate can increase dramatically by introducing the pseudo-gradient concept.

### C. Modal Analysis

The modal analysis technique provides voltage stability critical areas and gives information about the best corrective/preventive actions for improving system stability margins [6]. The basic principle of the modal analysis can be specified as follows. Equation (12) gives the relationship of the variations between  $\Delta P$  and  $\Delta Q$  with  $\Delta \theta$  and  $\Delta V$ .

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{P\theta} & J_{PV} \\ J_{Q\theta} & J_{QV} \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix}$$
(12)

Where  $\Delta P$  is active power variations of the buses;  $\Delta Q$  reactive power variations of the buses;  $\Delta \theta$  voltage angle variations of the buses;  $\Delta V$  voltage magnitude variations of the buses. *J* is actually the power system Jacobian matrix.

Considering the decoupled effects of active and reactive power variations on system voltage stability, given  $\Delta P=0$  and  $\Delta Q=0$  respectively, equation (12) is changed to the following:

$$\begin{cases} \Delta \theta = J_{RP\theta}^{-1} \Delta P \\ \Delta V = J_{RQV}^{-1} \Delta Q \end{cases}$$
(13)

Where  $J_{RP\theta}$  and  $J_{RQV}$  are the reduced active and reactive Jacobian matrix. Applying modal analysis on  $J_{RP\theta}$ , active participation factor *APF* is defined as

$$APF = \xi \bullet \eta \tag{14}$$

Where  $\xi$  and  $\eta$  are the left and right critical eigenvectors associated with the critical eigenvalue, which will become zero at the nose point of the PV curve and "•" means elementby-element production of  $\xi$  and  $\eta$ . The *APF*s corresponding to PV buses(*APF*<sub>PV</sub>) give the useful information about voltage stability. It has been shown in [7] that those generator units with small *APF*s are within the state of poor reactive power support areas. Therefore, those generator units should produce more reactive power in order to increase the voltage stability margin and vice versa.

According to the analysis above, the result of modal analysis is used in the paper to guide the searching direction of PGEP. The pseudo-gradient  $\vec{g}_p(\vec{x}_k^0)$  of the individual  $\vec{x}_k^0$  in the initial population is defined as the following:

$$dir(x_{k,i}^{0}) = \begin{cases} 1 & APF_{PVi} < \overline{APF_{PV}} \\ 0 & APF_{PVi} = \overline{APF_{PV}} \\ -1 & APF_{PVi} > \overline{APF_{PV}} \end{cases} \quad i = 1, 2, ..., N_{PV}$$
(15)

Where  $dir(x_{k,i}^0)$  is the *i*th element of  $\vec{g}_p(\vec{x}_k^0)$ ,  $APF_{PVi}$  the active participation factor of PV bus *i*,  $\overline{APF_{PV}}$  the average *APF* value of all PV buses,  $N_{PV}$  the total number of PV buses.

## III. CASE STUDY

In order to verify the effectiveness of the PGEP proposed in the previous section, it was used on the New England 10generator 39-bus power system. The configuration of the power system is shown in Fig.1. The objective of the current research is to increase the voltage stability margin. Parameters of the system and the generator units are given in [8].

The PV curves before and after optimizations are given in Fig.2. From this figure, it can be found that the voltage stability margin has been increased from 78% to 86% with approximately 8% increment. So the effect is rather significant since the optimization is applied without adding new equipment and the active power distribution not changed.



Fig.1 New England 10-generator 39-bus power system



The total reactive power generations of the generators in the system are shown in Fig.3 using PGEP and SEP method respectively. We can see that both the convergence speed and the optimal solution found of PGEP are better than those of SEP. The reason is that the direction provided by modal analysis technique and the conception of the conventional gradient method are introduced to PGEP. The reactive generation decreases about 10% which indirectly improves the voltage stability.

Fig.4 shows the system total active power losses variation vs. iteration steps, which indicates that the proposed method can effectively decreases the system losses as well. The total system reactive losses and the total transmission lines reactive power shunt generation are shown in Fig.5. It can be observed that the reactive losses decreases and the lines shunt generation increases after the optimization, all of which decrease the generators reactive generation and increase the dynamic var reserves.

### **IV. CONCLUSIONS**

Utilizing the understanding of voltage stability margin relating with power system reactive power reserves, this paper



Fig.4 Variation of system active power losses



Fig.5 Reactive power losses and lines shunt generations

presents a method to improve the system voltage stability margin by the generator reactive power generation rescheduling in the framework of short-term operation planning. The management of the var generation is processed as an optimization problem and pseudo-gradient evolutionary programming is used to search the optimal solution. Modal analysis technique is adopted to guide the searching direction. Simulation results on the New England 39-bus system demonstrate that the proposed method has the faster convergence speed and better capability in finding the optimal solution compared with the standard evolutionary programming. The reactive power reserves in the system increase and active and reactive power losses decrease after the optimization. The voltage stability margin has been improved without adding new equipment and changing the active power distribution.

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