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A diameter distribution model for even-aged beech in Denmark

Thomas Nord-Larsen [a,*], Quang V. Cao [b]

[a] Royal Veterinary and Agricultural University, Forest & Landscape, Hørsholm Kongevej 11, DK-2970 Hørsholm, Denmark
[b] School of Renewable Natural Resources, Louisiana State University, Agricultural Center, Baton Rouge, LA 70803, USA

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Abstract

We developed a diameter distribution model for even-aged stands of European beech in Denmark using the Weibull distribution. The model parameters were estimated using a large dataset from permanent sample plots covering a wide range of different treatments. Parameters of the model were estimated by fitting the cumulative density function using a non-linear least squares procedure. Further, a model constrained to yield estimates consistent with observed basal area was also developed. Predicted distributions confirmed the expected development of diameter distributions in even-aged beech stands. Due to large differences in initial stem-numbers care should be taken when the model is applied to young stands (<40 years).

Keywords: Weibull; CDF-regression; Diameter distribution; Stand structure; *Fagus sylvatica*

1. Introduction

Forest growth modelling has been an intrinsic part of forest management planning and research for more than two centuries. The majority of models operate at the stand-level and predict stand-level variables such as basal area or dominant height to provide information needed to estimate harvesting costs, expected yield, financial results, etc. (e.g. Møller, 1933; Schober, 1972). Although such models have proved invaluable for forest managers they remain crude simplifications of reality. Recent advances in forest growth modelling have resulted in increasingly complex models operating at the individual-tree level, explicitly modelling complex interactions between trees and their surrounding environment (e.g. Pretzsch et al., 2002). Although such models represent a significant leap forward in our understanding of the processes of tree growth, they may prove to be of little practical value to forest managers because the detailed measurements required for the implementation of these models are complicated and costly to obtain.

Bridging the gap between crude stand-level simplifications and complex individual tree models, size distribution models are potent tools for providing more detailed knowledge on the forest structure, product mix, product value, and forest operations costs for forest managers and researchers, without additional inventory costs. Empirical size distributions are given in some yield tables, providing the expected stand structure development or expected size distribution of thinned volumes for the specific silvicultural regime underlying the table (Carbonnier, 1971). Such distributions may enhance the level of detail provided by the table but are of limited value when the practised silvicultural regime diverges from that of the yield table. More flexible size distribution models are often developed using the parameter recovery approach, in which the parameters of a desired family of distributions are related to stand-level characteristics such as age, site index and stand density (Clutter and Bennett, 1965; Bailey, 1980). In practical application, overall stand attributes may then be disaggregated into more detailed resolutions to provide the forest manager with more detailed information.

From a practical perspective it is desirable that the same family of functions can be used throughout a stand's life and only the parameters need to be changed regardless of initial spacing or differing thinning practices. This necessitates a flexible function but at the same time it is the desirable that the function is both parsimonious and easy to estimate. In one of the first studies on size distributions de Liocourt (1898)

* Corresponding author. Tel.: +45 35281758; fax: +45 35281517.
E-mail addresses: tnl@kvl.dk (T. Nord-Larsen), qcao@lsu.edu (Q.V. Cao).

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suggested that the diameter distribution of natural forests may be described by an inverse J-shaped distribution, which was later formulated as a negative exponential function (Meyer, 1933).

In even-aged stands most modellers have recognized that size distributions are non-normal. However, an example of a flexible application of the normal distribution is the Gram–Charlier distribution which consist of an, in principle, infinite series of normal distributions (Prodan, 1953). The simplest, yet more flexible, alternative to the normal distribution is the three-parameter log-normal which is described completely by the mean and variance of the sample, when the origin is known or assumed a priori (Bliss and Reinker, 1964). Adding to the flexibility but also to the complexity of the size distribution model, the Gamma-distribution and the Pearl-Reed growth curve have been fitted to loblolly pine data (Nelson, 1964).

In the first attempt to develop a size distribution model, where diameter distributions were predicted directly from stand attributes, the beta distribution was fitted to slash pine data by Clutter and Bennett (1965). Keeping the location and range parameters fixed, they estimated the two shape parameters from age, stem number and site index. The beta distribution was later used to characterize observed distributions in beech (Kennel, 1972). The simpler, yet highly flexible two- and three-parameter Weibull distributions are probably the most widely applied functions for modelling tree size distributions and were first used for this purpose by Bailey and Dell (1973). They have been used for predicting the size distribution of Douglas fir (Knowe and Stein, 1995), eastern cottonwood (Knowe et al., 1994), Scots pine (von Gadow, 1984; Sarkkola et al., 2005), black spruce (Newton et al., 2005), slash pine (Schreuder et al., 1979), loblolly pine (Borders and Patterson, 1990; Cao, 2004; Matney and Sullivan, 1982), jack pine (Bailey and Dell, 1973) and different species mixtures (Siipilehto, 1999; Chen, 2004). Even more flexible, but also more complex, the four-parameter Johnson's S_B distribution has been used to model the distributions of Norway spruce (Tham, 1988), Sitka spruce (Skovsgaard, 1997), Changbai larch (Rennolls and Wang, 2005), and loblolly pine (Hafley and Buford, 1985; Scolforo et al., 2003).

Despite the evident efficacy of size distribution models, no such model is presently available for beech (*Fagus sylvatica* L.) forest management planning or research in Denmark. Hence, the objective of this study was to develop a diameter distribution model for even-aged beech based on a large permanent sample plot data.

2. Materials

Data were collected from 1872 to 2005 from 69 permanent spacing, species and thinning experiments in European beech, totalling 204 individual plots. Plot sizes varied between 0.07 and 2.65 ha with an average of 0.40 ha. Plots were measured at every thinning, identifying crop trees as well as trees to be thinned. The number of measurement occasions total 1539. The experiments were located in most parts of Denmark and covered a wide range of site and growth conditions.

All plots were essentially even-aged and mono-specific, covering a wide range of different treatments in terms of initial spacing and thinning regimes. In the thinning experiments, treatments ranged from unthinned controls to heavily thinned shelterwoods. Some plots were managed according to specific thinning strategies, such as group- or selection-thinning, and others were managed according to the thinning strategy typical at the time. Although thinning intervals ranged from 1 to 35 years, the majority of plots were thinned every 4–6 years.

In most of the sample plots, trees were numbered, marked permanently at breast height (1.3 m) and recorded individually. On 451 measurement occasions carried out before 1930 and in some young stands with high stem numbers, trees were recorded in tally lists to 1-cm diameter classes (or 1-in. diameter classes before 1901). In 13 very young planted stands with high stem numbers, only a subset of stems was measured, e.g. every 5th or 10th row. Breast height diameters were generally obtained by averaging two perpendicular calliper readings. Observations also included records on whether the tree was alive or dead at the time of measurement. Height measurements were typically obtained from about 30 trees per plot.

Based on the paired observations of diameter and height, diameter–height equations were developed for each plot and measurement combination using a modified Näslund-equation (Näslund, 1936; Johannsen, 2002):

$$h = 1.3 + \left(\frac{d}{\alpha + \beta d}\right)^3 \tag{1}$$

where d is diameter at breast height, h tree height and α and β are parameters to be estimated. The equations were used to estimate the height of trees not measured for height. Dominant height, H_{100} (m), defined as the mean height of the 100 thickest trees per hectare, was calculated for each plot and measurement combination. Where stem numbers were less than 100 per hectare, H_{100} was estimated as the mean height.

Stem numbers, N (ha^{-1}), were calculated as the number of individual trees taller than 1.3 m. When trees forked below 1.3 m, each stem was measured individually, but multiple stems from the same root were counted as one tree. Understorey trees were identified as smaller trees growing entirely beneath the forest canopy. Understorey trees were measured less intensively and were therefore not included in this study.

Stand basal area, G (m^2 ha^{-1}), of each plot was estimated by summation of individual tree basal areas calculated from the diameter measurements. When trees were recorded in tally lists, mean class values were used as an estimate of the diameter of all trees in that class. Diameter corresponding to mean basal area, D_g (cm), was derived from the estimates of N and G. The data represent a wide range of stand ages and stand values in terms of H_{100}, G, D_g and N (Fig. 1).

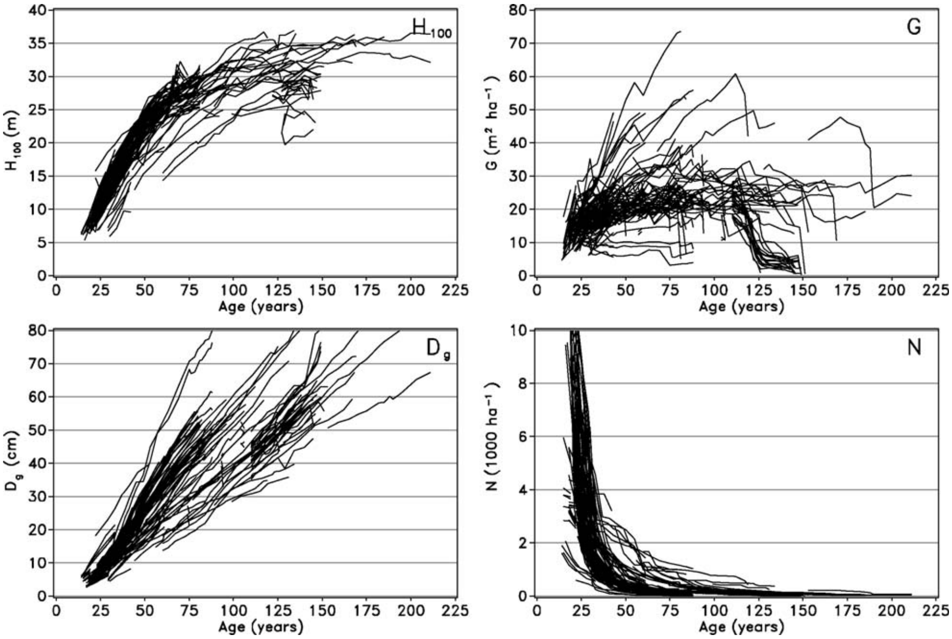

Fig. 1. Stand-level values of H_{100}, G, D_g, and N. The lines represent repeated measurements on each individual plot.

3. Methods

The Weibull distribution (Weibull, 1951):

$$f(x) = \frac{c}{b}\left(\frac{x-a}{b}\right)^{c-1} \mathrm{e}^{-((x-a)/b)^c}; \quad x \geq a, \ b > 0, \ c > 0. \tag{2}$$

covers most of the desired shapes for a diameter distribution model. It describes the inverse J-shape for $c < 1$ and the exponential distribution for $c = 1$. For $1 < c < 3.6$ the density function is mound shaped and positively skewed and for $c = 3.6$ the density function becomes approximately normal. If $c > 3.6$ the density function becomes increasingly negatively skewed. Contrasting the flexibility of the Weibull distribution, mathematical derivations are simple and allow for simple solutions in simulation studies (Weibull, 1951). Motivated by a comparison of skewness and kurtosis observed on the individual measurement occasions and the possible combinations of the Weibull distribution (Fig. 2) as well as the simplicity of the Weibull distribution and its well-described properties, this function was used for modelling the diameter distribution of European beech in Denmark.

3.1. Model estimation

Parameters of the Weibull distribution may be estimated using various kinds of transformations to linearize the function and subsequent estimation by (weighted) linear regression, or by moment (Burk and Newberry, 1984) or percentile estimation (Bailey and Burgan, 1989; Borders and Patterson, 1990). Estimation of the parameters by maximum likelihood has been found to produce consistently better goodness-of-fit statistics compared to the previous methods, but also put the largest demands on the computational resources (Cao, 2004). Recently, parameters of the Weibull distribution were iteratively searched to minimize the squared deviations between the observed and predicted cumulative distribution function (cdf) (CDF-regression; Cao, 2004). CDF-regression was found to yield the best goodness-of-fit statistics among the methods tested in this study.

The cumulative distribution function of the Weibull distribution is:

$$F(x_{ij}) = 1 - \exp\left(-\left(\frac{x_{ij}-a}{b}\right)^c\right) \tag{3}$$

where F_{ij} is the cumulative probability for diameter at breast height (x_{ij}) of the ith tree in the jth plot and age combination. In this study, the parameters of the Weibull distribution were initially estimated for each age and plot combination using

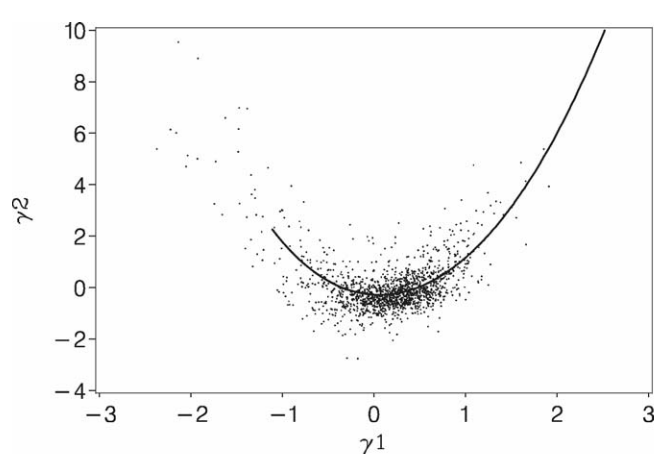

Fig. 2. Skewness (γ_1) and kurtosis (γ_2) of observed diameter distributions compared with the possible solutions of the Weibull distribution (full line).

CDF-regression. The parameters were iteratively searched to minimize:

$$\sum_{j=1}^{p}\sum_{i=1}^{n_j}\frac{(F_{ij}-\hat{F}_{ij})^2}{n_i} \tag{4}$$

Due to high levels of correlations between parameters, estimation of the individual cdf's in this preliminary analysis frequently failed to converge. We therefore used 0.5 times the observed minimum diameters ($D_{\min}$) of the different measurement occasions as an a priori estimate of the location parameter a.

The a priori estimates of a were plotted against various stand-level variables. Based on the observed relationships, a set of candidate functions for modelling a were selected and fitted to the data. Based on a statistical analysis of model fit and the parsimony of the candidate functions, a was estimated from a Chapman–Richards function (Richards, 1959) of D_g, where the asymptote is D_g and the other parameters were estimated from D_g, H_{100}, and N:

$$\begin{aligned} a_j &= D_{g,j}(1-\exp(-a_1 D_{g,j}))^{a_2}+\varepsilon_{a,j},\\ a_1 &= a_{01}D_{g,j}H_{100,j}\\ a_2 &= a_{02}+a_{03}\ln(N_j) \end{aligned} \tag{5}$$

where a_{01}– a_{03} are parameters to be estimated, and $\varepsilon_{a,j}\sim N(0,\sigma^2_{a,j})$ is the error term. Subscript denotes the jth plot–age combination.

Using the a priori estimates of a, the scale (b) and shape (c) parameters were subsequently estimated for each age and plot combination and plotted against various stand variables. Based on the observed relationships, a set of candidate functions for modelling b and c were selected and fitted to the data.

Based on a statistical analysis of model fit and model parsimony of the candidate functions, the scale parameter was estimated from a saturation growth-rate type model of D_g:

$$b_j = \frac{b_{01}D_{g,j}}{b_{02}+D_{g,j}}+\varepsilon_{b,j} \tag{6}$$

where b_{01} and b_{02} are parameters to be estimated and $\varepsilon_{b,j}\sim N(0,\sigma^2_{b,j})$ is the error term. Based on a similar analysis, the shape parameter was estimated from a logistic function of D_g where the parameters were estimated from H_{100}, and N:

$$\begin{aligned} c_j &= \frac{c_1}{1+c_2\exp(-c_3 D_{g,j})}+\varepsilon_{c,j},\\ c_1 &= c_{01}+c_{02}N_j D^2_{g,j}\\ c_2 &= c_{03}\\ c_3 &= c_{04}\frac{D_{g,j}}{H_{100,j}} \end{aligned} \tag{7}$$

where c_{01}– c_{04} are parameters to be estimated and $\varepsilon_{c,j}\sim N(0,\sigma^2_{c,j})$ is the error term.

Based on the initial analyses, the parameters in Eqs. (5)–(7) were estimated simultaneously using CDF-regression as described by Cao (2004). The diameter observations were assumed to be independent, random observations.

3.2. Constrained estimation

When the distribution parameters of the Weibull distribution are known for a stand with N trees per hectare, the basal area may be calculated as:

$$\begin{aligned} G_j &= \frac{\pi}{4}N_j c_j b_j^{-1}\int_a^\infty x_{ij}^2\left(\frac{x_{ij}-a_j}{b_j}\right)^{c_j-1} e^{-(x_{ij}-a_j/b_j)^{c_j}}\,dx\\ &= \frac{\pi}{40,000}N_j(a_j^2+2a_jb_j\Gamma_1+b^2\Gamma_2) \end{aligned} \tag{8}$$

where $\Gamma_1=\Gamma(1+1/c_j)$, $\Gamma_2=\Gamma(1+2/c_j)$, and $\Gamma(.)$ is the complete Gamma-function (Arfken, 1985). Solving Eq. (8) and substituting for either of the parameters constrain the Weibull distribution to yield estimates consistent with the observed or predicted basal area of a particular stand. In addition to the un-constrained model, we estimated the parameters of Eqs. (5) and (7), where b_j was constrained to yield estimates of the diameter distribution function, consistent with observed or predicted basal area as:

$$b_j = \frac{-a_j\Gamma_1+\sqrt{a_j^2(\Gamma_1^2-\Gamma_2)+\Gamma_2 D^2_{g,j}}}{\Gamma_2} \tag{9}$$

3.3. Model evaluation

Estimated diameter distributions were evaluated by statistical tests that included t-tests of predicted mean and χ^2, Kolmogorov–Smirnoff (KS), and Anderson–Darling (AD) goodness-of-fit tests. Although the KS and AD goodness-of-fit tests may be useful when comparing different families of distributions in the early stages of model building, one important caveat applies to the use of these formal tests when evaluating diameter distribution models (Reynolds et al., 1988). Theoretically, the tests only apply to the case where the distribution function is completely specified. This is seldom the case and critical values have been provided for various cases where the parameters must be estimated (Stephens, 1977). However, no critical values have been calculated for the case where parameters are estimated by CDF-regression. We therefore conducted the tests, ignoring the fact that the distribution was not completely specified.

In addition to the goodness-of-fit statistics, the unconstrained diameter distribution model was evaluated by comparing observed basal area to basal area predicted by the diameter distribution model. This comparison was motivated by the importance of basal area in forest applications, and the fact that errors among the large and thereby more valuable trees have more weight.

Finally, we conducted a leave-one-out cross-validation for both the constrained and un-constrained model, where entire experiments were left out one at a time during the estimation procedure. Estimated models were subsequently applied to the left-out experiment and goodness-of-fit statistics were calculated. The stability of estimates was evaluated by comparing the number of rejected distributions to the numbers obtained in the original estimation.

Table 1
Parameter estimates and their approximate standard errors of the model fitted by the CDF-method

Parameter	Not constrained estimates			Constrained estimates		
	Estimate	S.E.	*t*-Value	Estimate	S.E.	*t*-Value
a_{01}	3.00095E−8	1.25E−9	23.94	4.38261E−7	2.17E−8	20.22
a_{02}	0.33278	3.82E−3	87.09	0.18282	4.84E−3	37.76
a_{03}	−0.00165	1.74E−4	−9.49	0.00321	4.81E−4	6.68
b_{01}	283.13622	5.56	50.93	–	–	–
b_{02}	268.60650	5.18	51.85	–	–	–
c_{01}	9.30607	3.51E−2	267.81	5.10905	2.12E−2	240.47
c_{02}	−5.36769E−6	4.26E−8	−125.88	−3.28411E−6	3.11E−8	−105.74
c_{03}	3.27888	1.46E−2	224.92	1.71800	1.12E−2	152.83
c_{04}	0.07880	2.86E−4	275.24	0.10277	6.84E−4	150.32

Estimates are provided for both the un-constrained model and for the model constrained to yield estimates consistent with observed basal area. All parameter estimates were highly significant ($P < 0.0001$).

Table 2
Test statistics and the number of predicted distributions significantly different from the observed ($P \leq 0.05$)

Statistic	Not constrained			Constrained			Cross-validation		
	Mean	Rejected	%	Mean	Rejected	%	Mean	Rejected	%
Mean	0.242	42	2.7	0.021	25	1.6	0.242	27	2.7
χ^2	56.480	701	47.3	59.035	674	45.5	57.851	708	47.8
K–S	0.138	536	34.8	0.135	511	33.2	0.139	546	35.5
A–D	8.186	609	39.6	8.338	566	36.8	8.446	615	40.0

Test statistics are provided for both the un-constrained and the constrained model as well as for the cross-validation procedure.

4. Results

Fitting of the cumulative distribution function accounted for 94.1% of the variation (R^2) and resulted in all parameter estimates significantly different from 0 ($P \leq 0.05$; Table 1). When the model was constrained to yield estimates consistent with the observed basal area, the model accounted for 93.3% of the observed variation.

The basal area predicted by the unrestricted model deviated less than 10% from the observed (Fig. 3), but the model seemed to underestimate basal area systematically.

The null hypothesis, that the estimated mean diameter is similar to the observed mean diameter was rejected for 1.6–2.7% of the 1539 distributions for the un-constrained model (Table 2). Also for the un-constrained model, 34.8–47.3% of predicted distributions differed significantly from the observed, depending on the applied goodness-of-fit statistic. The number of rejected distributions was similar for the constrained model, although this model consistently rejected fewer of the predicted distributions. The leave-one-out cross-validation showed only a small increase in the number of rejected distributions for both the constrained and un-constrained models, which indicated a large stability of the model.

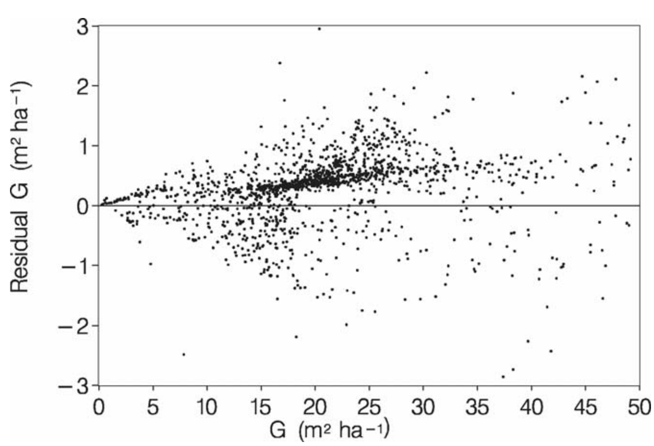

Fig. 3. Residual basal area of the unrestricted diameter distribution model.

5. Discussion

Judging from the model predictions, the diameter distribution is typically right skewed in young, even-aged beech stands, probably as a result of self thinning among the smallest trees (Fig. 4). As the stand matures, the peakedness of the distribution is reduced and it becomes less skewed, but the variation increases. Classical thinning from below causes a relative reduction in variation, peakedness and skewness of the distribution. In old beech stands (>100 years) skewness and kurtosis of the diameter distribution begins to increase again. Further, the diameter distribution becomes increasingly left skewed, possibly due to harvesting of the largest trees. The observed pattern of the diameter distribution is concordant with the findings of a study on even-aged beech stands in Germany (Kennel, 1972) and with findings from southern Sweden (Carbonnier, 1971).

Despite the apparent agreement between expected development of the diameter distribution over time and the model

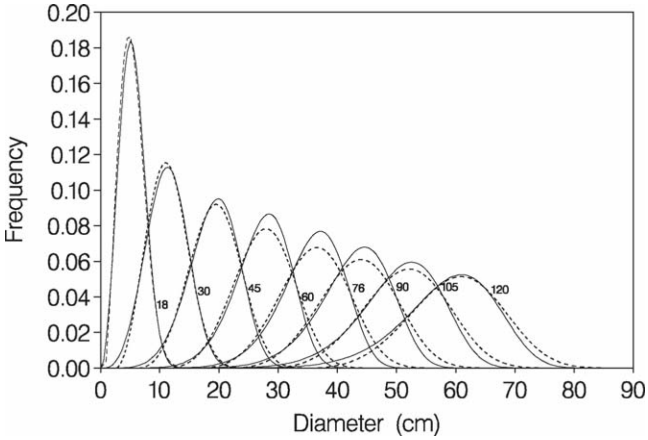

Fig. 4. Simulated development of the diameter distribution for the un-constrained (full line) and constrained model (dashed line). Simulations are based on stand-level variables (H_{100}, G and N) obtained from the most commonly used yield table for beech in Denmark (Møller, 1933). Numbers indicate stand ages.

predictions, 30–45% of the predicted distributions differed from the observed distribution according to our criteria at the 5% significance level (for χ^2, KS and AD-statistics). The seeming weakness of the model is to be expected when fitting a smooth curve to diameter distributions from relatively small plots in managed stands. Size distributions are affected by both spatial structure and the chosen plot size. Considering the relatively small plot size (0.40 ha on average) it is probably not realistic to sample the actual, smooth distribution that can be fitted by the hypothesized distribution function. Further, thinnings in managed stands are often targeted at specific social classes of trees, or at obtaining specific assortments or a specific stand structure. In the typical Danish beech stand management, also practised on the permanent sample plots, thinnings are often targeted at class III-trees according to Kraft Crown Classification System (Kraft, 1884). This causes the diameter distribution to be irregular as specific diameter classes are often almost entirely removed (example in Fig. 5). Such diameter distributions would not be successfully fitted by any smooth statistical distribution. This probably explains why stand table projection methods are considered superior to distribution prediction methods when predicting future diameter distributions (Pienaar and Harrison, 1988; Borders and Patterson, 1990; Nepal and Somers, 1992; Cao and Baldwin, 1999).

In a number of plots where observed distributions differed from the predicted, visual inspection showed that observed distributions tended to be bi-modal, possibly because the initial exclusion of understorey trees was not always successful (see Section 2). The Weibull distribution is uni-modal and is thus unsuited to model such distributions. Bi-modal diameter distributions have been modelled by finite mixtures of various distributions (for a forestry related application, see Skovsgaard, 1997; Liu et al., 2002). However, in this study we decided not to model the distribution of understorey trees for two reasons. Firstly, the understorey trees were measured less intensively and it was uncertain if all understorey trees were in fact measured in the historical data. Secondly, understorey trees have been treated very differently across the individual plots, but their treatment has little effect on stand-level variables such as basal area or dominant height. Hence, the diameter distribution model cannot be expected to adequately model the distribution of understorey trees based on stand-level variables.

Further analyses showed that the frequency of predicted distributions that differed significantly from the observed was

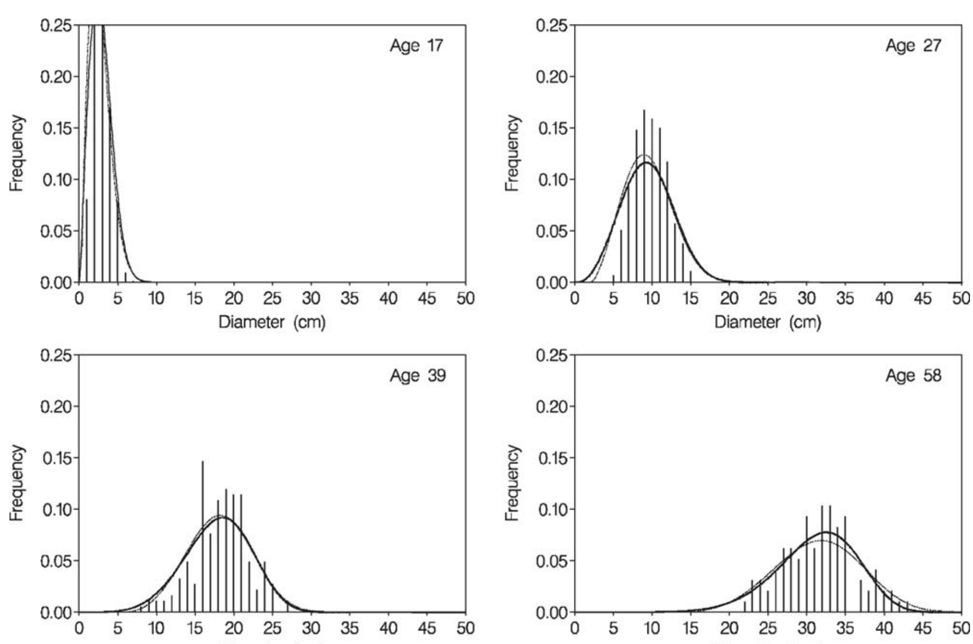

Fig. 5. An example of simulated diameter distributions (experiment EC, plot 01, C-grade thinning) for the un-constrained (full line) and constrained model (dashed line) compared with the observed diameter distribution.

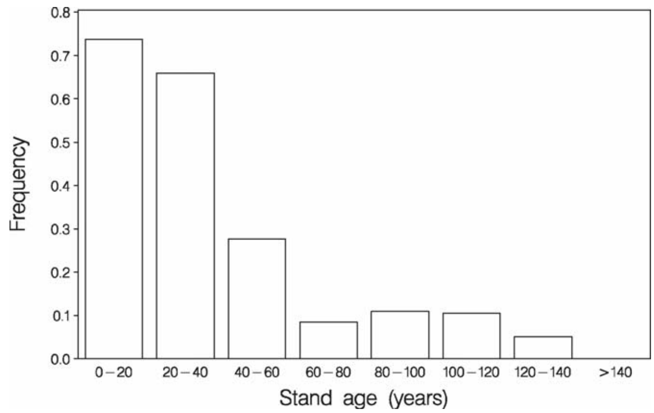

Fig. 6. Frequency of experiments in different age-groups where the Kolmogorov–Smirnoff–test resulted in rejection of the hypothesis that observed and predicted distributions are the same.

highest among young age classes (Fig. 6). This reflects a general problem of all diameter distribution models: that prior treatment influences the diameter distribution, but is not entailed in the model since such information is generally not available for its application. In other words, we model the diameter distribution based on the assumption that all information needed is expressed by the stand variables, although these may reflect many different thinning strategies that result in different diameter distributions. This becomes especially evident for young stands since initial conditions (i.e. stem numbers) differ considerably between, for example a naturally regenerated stand with several hundred thousand plants per hectare and a planted stand with less than 5000 plants per hectare. Such stands are not likely to have similar diameter distributions, even if they are thinned to approximately the same stem number or basal area. Later, the frequency of failed estimates is reduced because multiple thinnings according to a similar strategy even out initial differences but probably also because stem numbers are reduced and the estimated distribution therefore becomes harder to reject.

6. Conclusion

The diameter distribution model was successfully estimated using CDF-estimation and predicted distributions confirmed the expected development of diameter distributions in even-aged beech stands. However, predicted distributions deviated significantly from the observed in 60–80% of stands younger than 40 years, probably due to large differences in initial stem-numbers. Hence, the diameter distribution model may be used for predicting distributions from observed or predicted stand-level values of stem numbers and basal area, but care should be taken when the model is applied to young stands (<40 years).

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