Aggregated Wind Park Models for Analyzing Power System Dynamics

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Abstract—The increasing amount of wind power generation in European power systems requires stability analysis considering interaction between wind-farms and transmission systems.

Dynamics introduced by dispersed wind generators at the distribution level can usually be neglected. However, large on- and off-shore wind farms have a considerable influence to power system dynamics and must definitely be considered for analyzing power system dynamics.

Compared to conventional power stations, wind power plants consist of a large number of generators of small size. Therefore, representing every wind generator individually increases the calculation time of dynamic simulations considerably. Therefore, model aggregation techniques should be applied for reducing calculation times.

This paper presents aggregated models for wind parks consisting of fixed or variable speed wind generators.

Index Terms—Wind Generation, off-shore wind power, power system stability, model aggregation

I. INTRODUCTION

The growing importance of wind power, which can be observed in Europe and the United States, requires more and more detailed analysis of the impact of wind power on transmission systems.

At the end of 2002, there were wind turbines with a total rated power of around 12000MW installed in Germany. Further increase of wind power can mainly be expected off-shore, if this technology can be proved to be reliable and cost-efficient. Estimates for off-shore wind power installations in Germany are in the range of 2000MW to 3000MW in the medium term (until 2010) and up to 25000MW in the long term (until 2030) [2].

If these wind resources shall be exploited, transmission networks need to be considerably extended for transporting the generated wind-power to the main load centers.

Also with regard to the operation of wind generators, a change in mind could be observed. In former times, when the amount of wind generation was very small compared to the overall installed generation capacity, connection conditions usually required wind generators to disconnect under any disturbance, even on small voltage dips, for not doing anything "bad" to the network. However, this practice led to substantial problems with regard to active and reactive power control in some cases. Newer connection conditions usually require wind generators to participate in reactive power control and to "ride-through" network faults (e.g. [3]), or at least reconnect shortly after disturbances.

Future off-shore installations as well as changed operation practices require network planning and operation analysis studies to assess the impact of wind generation on power system stability and system control.

However, wind generators are much smaller than conventional power generators and their number is therefore much larger why model aggregation techniques have to be applied for being able to carry out dynamic simulations within reasonable calculation time.

This paper presents an approach for aggregated wind park models representing an entire wind park by one equivalent wind generator.

Various wind generator technologies will be covered:

• Fixed speed, induction machine (stall and pitch controlled)
• Variable speed, doubly-fed induction machine.
• Variable speed, converter-driven synchronous machine.

The paper starts by introducing dynamic models of single wind generators based on the above listed technologies. The models are designed for representing correctly the reaction of wind generators to network faults and wind fluctuations. Because the presented models are primarily used for dynamic analysis of entire power systems, the wind generator’s output at the connection point to the transmission network is in the center of interest.

Dynamic models of all wind generator components including turbine, generator, power electronics converters and controllers are presented. A model for simulating wind turbulence considering coherence between turbulences at different wind turbines in a wind farm is described. The paper continues by presenting model simplification and aggregation techniques. For validating the aggregated wind-farm models, they are benchmarked against detailed, not-aggregated models.

II. GENERATOR MODELING

There are many different generator concepts for wind-power applications in use today. The main distinction can be made between fixed-speed and variable-speed wind-generator concepts.

A fixed-speed wind-generator is usually equipped with a squirrel-cage induction generator whose speed variations are only very limited (see figure 1). Power can here only be controlled through pitch-angle variations. Because the efficiency of wind-turbines (expressed by the power coefficient $c_p$, see also section III) depends on the tip-speed ratio $\lambda = \omega R/v_w$, the power of a fixed-speed wind generator varies directly with the wind speed.

Since induction machines have no reactive power control capabilities, fixed or variable power factor correction systems are usually required for compensating the reactive power demand of the generator.

In contrast to this, variable speed concepts allow operating the wind turbine at the optimum tip-speed ratio $\lambda$ and hence at the optimum power-coefficient $c_p$ for a wide wind-speed range. Varying the generator’s speed requires frequency converters that increase investment costs.

The two most-widely used variable-speed wind-generator concepts are the doubly-fed induction generator (figure 2) and the converter-driven synchronous generator (figure 4 and figure 5).

Active power of a variable-speed generator is controlled electronically by fast power electronics converters, which reduces the impact of wind-fluctuations to the grid. Additionally, frequency converters that are in use today (self-commutated PWM-converters) allow for reactive power control, why no additional reactive power compensation is required.

A. Induction Generator

Induction generator models for power system stability studies usually consider rotor-flux transients and mechanics by differential equations but neglect stator-flux transients by reducing stator-voltage equations to arithmetic equations [4].

For representing current displacement effects in the rotor, a double cage model can be applied that models the rotor impedance by two or more parallel R-L-ladder circuits.
A doubly-fed induction machine is basically a standard, wound-rotor induction machine with a frequency-converter connected to the slip-rings of the rotor.

In modern DFIG designs, the frequency converter is usually built by two self-commutated PWM converters with an intermediate DC-voltage circuit (see Figure 3).

The converter connected to the rotor controls the total active and reactive power of the DFIG. The inner, faster control loop consists of a q-d current controller operating in a stator-flux oriented reference system. Hence the q-axis current component represents active current and the d-axis component reactive current.

An overcurrent protection system protects the rotor-side converter against high current. When the maximum current limit is exceeded, the rotor-side converter is blocked and bypassed (‘crow-bar protection’).

The stator-side converter usually regulates DC-voltage and reactive power. Likewise the rotor-side converter, the inner control loop regulates active and reactive currents. However, the stator-side controller operates in a stator-voltage oriented reference frame, why d-axis represents the active component and q-axis the reactive component.

In contrast to standard, single-fed induction machines, the level of detail required for a stability model of a DFIG is still subject to discussions (see also [14], [15], [13]).

Generally, it is accepted that stator-flux derivatives can be neglected, analogously to the single-fed induction machine model.

With regard to the representation of the converters and their controls (Figure 3), it is not as clear what the actual requirements are. Generally, the model according to Figure 3 can be reduced by assuming that fast controls, like the d-q-current controllers can be assumed to work ideally, which means that the controlled quantities are always equal to their reference values (e.g. \(i_d = i_d^{ref}\)).

In [13], DFIG-models of different order are presented and benchmarked against each other. In this paper, we come to the conclusion that reducing the stator-side current controller and the DC-voltage/Q-controller is appropriate. When doing so, the grid-side converter behaves to the AC-side like a d-q-current-source and to the DC-side like an ideal voltage-source. Reducing the rotor-side controller however removes the criterion for inserting the rotor-side over-current protection (‘crow-bar-protection’), why it is recommended not to reduce any of the rotor-side controllers if faults close to DFIGs are simulated.

C. Converter-Driven Generator

Figure 4 and figure 5 show two typical concepts using a frequency converter in series to the generator.

Generally, the generator can be an induction or a synchronous generator. In most modern designs, a synchronous generator or a permanent magnet generator is used.

In contrast to the DFIG, the total power flows through the converter why it’s size must be larger (and it’s cost higher) than in case of a DFIG. Figure 5 shows a direct drive wind-turbine that works without any gear box. This concept requires a slowly rotating synchronous generator with a lot of pole-pairs.

The two typically applied frequency converter concepts are shown in figures 6 and 7. The converter according to figure 6 is very similar...
to the DFIG-converter. The grid-side converter regulates $P$ and $Q$, the generator-side converter $V_{dc}$ and $Q_{gen}$. This controller concept can be applied to electrically excited synchronous machines and permanent magnet generators.

The converter according to figure 7 uses an uncontrolled diode-rectifier at the generator-side and a voltage-source PWM converter at the grid-side. For boosting the DC-voltage, a DC/DC converter is required. The typical control scheme of this configuration consists of a P-Q-controller at the grid-side and DC-voltage controller connected to the booster. Reactive power at the generator-side cannot be controlled; it simply results from the reactive power demand of the diode-bridge.

In [16] converter-driven synchronous machine models are analyzed in detail. It is shown in this paper, that reducing the generator-side converters and assuming a constant DC-voltage behind the grid-side PWM-converter provides sufficient accuracy for stability studies. This reduced-order model works for both, the concept according to figure 6 and the concept according to figure 7. In the reduced-order model, the synchronous machine is just represented by its inertia (first order model).

A further model reduction can be applied by assuming an ideal grid-side current-controller. This works very well in many cases (better than a reduction of the DFIG rotor-side current controller) but in case of faults very near to the wind-generator, numerical problems can be observed.

**D. Shaft Model**

When applications are limited to the impact of wind fluctuations, it is usually sufficient to consider just a single-mass shaft model for variable-speed wind turbines, because shaft oscillations of variable speed wind generators are not reflected to the electrical grid due to fast active power controllers. [7]. However it’s different with fixed-speed induction machines: Because there is no decoupling between wind-turbine and electrical grid, shaft oscillation can directly be observed in the electrical power.

In stability analysis, when the system response to heavy disturbances is analyzed, a detailed, mult-mass shaft model is required for all types of wind generators. The shaft is here usually approximated by at least a two mass model. One mass represents the turbine inertia, the other mass is equivalent to the generator inertia. Equations for second order shaft models are given in e.g. [13] and [16].

**III. Wind Turbine Model**

Dynamic models of wind turbines for dynamic simulations must represent with sufficient accuracy the turbine’s reaction to:
- mechanical speed variations
- wind-speed variations, including wind-turbulence
- Pitch-angle variations (if the turbine is pitch-controlled)

The components to be modeled are turbine and pitch-controller as shown in figure 8.

**A. Aerodynamics**

The kinetic power of an air flow with air-density $\rho$ and speed $v_w$ through an area $A$ can be expressed by:

$$P_{w0} = \frac{1}{2} \rho A v_w^3$$

(1)
A wind-turbine converts part of this power to rotational power at the turbine shaft. The efficiency of the wind-turbine, expressed by the so-called power-coefficient \( c_p \), is a non-linear function depending on wind-speed \( v_w \), mechanical speed \( \omega_m \) and blade-angle \( \beta \).

The Aerodynamic block of Figure 9 models the power conversion from wind-power to mechanical power for a constant, steady-state wind flow according to:

\[
P_v = c_p(\lambda, \beta) P_{v_0}
\]

The area \( A \) is the swept area of the rotor. The variable \( \lambda \) is the tip-speed ratio defined by:

\[
\lambda = \frac{\omega_m R}{v_v}
\]

The Aerodynamic block alone represents an appropriate wind turbine model for all applications, in which the turbine response to mechanical speed variations or to pre-defined wind speed variations (e.g. gusts) is investigated.

For analyzing power fluctuations due to wind-turbulence however, it must be considered that turbulent wind is not constant over the swept area \( A \), but different in every point (see section III-C).

**B. Pitch-Control**

![Fig. 10. Generic pitch-controller model](image)

The pitch-controller limits the generator’s speed to a maximum permitted value \( \omega_{\text{max}} \) by adjusting the pitch-angle \( \beta \). Figure 10 shows a generic pitch-controller model, as it can be used for power system dynamics analysis.

**C. Wind-Speed-Signals**

Wind speed is generally defined by the mean wind-speed \( V_{v_0} \), which represents the average value in e.g. a 10 minutes interval and a turbulent component \( v_t(t) \) (see e.g. [10]).

\[
v_w(t) = V_{v_0} + v_t(t)
\]

For some applications, it is useful to define the turbulent component by a deterministic function, e.g. gusts represented by typical amplitudes and duration or to use measured wind-speed signals.

Besides deterministic wind-speed signals, stochastic models can be applied that are able to predict the occurrence of wind-turbulence and the correlation of wind turbulence at different wind turbines of a windpark.

Turbulence models are based on stochastic signals that can be described by the power spectral density (PSD). A widely used PSD for wind-speed signals is the Kaimal-spectrum, whose two-sided form is given by (e.g. [18]):

\[
S(f) = \frac{\sigma^2}{2 V_{v_0}} \left( \frac{1}{1 + \frac{2}{3} \frac{L}{v_v} f} \right)^\frac{5}{3}
\]

The parameters of the Kaimal-spectrum are defined as follows:

- \( \sigma \): Standard deviation of the wind-speed
- \( V_{v_0} \): Mean wind-speed
- \( L \): Turbulence length-scale

For the length-scale, the following formula can be used [9]:

\[
L = \begin{cases} 
20Z & \text{for } Z < 20 \text{m} \\
600 \text{m} & \text{otherwise}
\end{cases}
\]

\( Z \) is the height in m.

When power fluctuations are simulated in the time-domain, time-series \( v_t(t) \) have to be generated so that its spectra comply with:

\[
V_v(f) V_v^*(f) = S(f)
\]

\( S(f) \) is the Kaimal spectrum according to (5).

However, the turbulent wind-speed signal \( v_t(t) \) just describes a series of wind-speeds at one point of the swept area, e.g. the hub wind-speed.

For calculating the influence of wind turbulence to the mechanical torque generated by the turbine, it has to be considered that the wind speed is not constant over the swept area but different in every point. Therefore, the wind speed in the swept area has to be described by a vector field \( v(r, \theta, t) \) (see figure 11). For a detailed analysis of torques and forces in the rotor, blade iteration techniques need to be applied, as it is described in many standard-books about wind generation (e.g. [6]).

Blade iteration works well if one wind turbine is analyzed in detail. For system analysis however, when hundreds of wind-turbines are simulated, the method is too time consuming.

For system analysis, the equivalent wind-speed method described by Risoe-members in [8], [9] provides a very good compromise between accuracy and calculation time and seems to be the most appropriate at present.

The basic idea of the method is to represent the complete turbulent wind-speed field by one equivalent wind speed that generates the same overall rotor torque as the actual field.

The contribution of each blade to the “turbulent” torque depending on the turbulent wind-speed field can be expressed by ([8], [9]):

\[
M_{\text{th}}(\theta, t) = \int_{r_0}^{R} v(r) v_t(r, \theta, t) dr
\]
The variables used in this expression are:
- \( M_{t,b} \): Torque generated by the turbulent wind-field at blade \( b \).
- \( r_0 \): Inner blade radius
- \( R \): Outer blade radius
- \( \psi(r) \): influence coefficient of the aero load on the blade root moment in radius \( r \) in kN/m (linear approximation).
- \( \theta_b \): Angle of blade \( b \).

According to the above explained idea, the equivalent wind-speed must comply with the following equation:

\[
M_{t,b}(\theta_b, t) = v_{eq}(\theta_b, t) \int_{r_0}^{R} \psi(r)dr
\]  

(9)

Combining (8) and (9) leads to the following definition of the equivalent wind-speed:

\[
v_{eq}(\theta_b, t) = \frac{\int_{r_0}^{R} \psi(r)v_r(r, \theta_b, t)dr}{\int_{r_0}^{R} \psi(r)dr}
\]  

(10)

The equivalent wind-speed becomes independent from the actual turbine design, blade-angle and mechanical speed, if it is assumed that the aero-load depends linearly on the radius \( \psi(r) = K_{ax}(V_{ho}, \omega_t, \beta)r \), which is a further simplification. The dependence on actual turbine design, mechanical speed and blade angle is considered by the factor \( K_{ax} \), which is equivalent to the aerodynamics-model of the previous section (see also Fig. 9).

With this assumption, the equivalent turbulent wind-speed just depends on the inner- and outer blade radius and the blade angle, but not on actual turbine design, pitch angle or mechanical speed:

\[
v_{eq}(\theta_b, t) = \frac{\int_{r_0}^{R} v_r(r, \theta_b, t)dr}{\int_{r_0}^{R} rdr}
\]  

(11)

The equivalent wind-speed considering all three blades is the average of the equivalent wind speeds of every blade. It can be expressed as function of the position of one blade, e.g. blade 1:

\[
v_{eq}(t) = \frac{1}{3} \left( v_{eq1}(\theta_1, t) + v_{eq2}\left(\theta_1 - \frac{2\pi}{3}, t\right) + v_{eq3}\left(\theta_1 - \frac{4\pi}{3}, t\right) \right)
\]  

(12)

For modeling the equivalent wind-speed dependence on \( \theta_1 \), the equivalent wind-speed is further decomposed into a Fourier-series with respect to \( \theta_1 \). Here, only multiples of three need to be considered, because all other components (1p, 2p, etc.) compensate each other in (12):

\[
v_{eq}(t) = \sum_{k=-\infty}^{\infty} \tilde{v}_{eq}(3k, t)e^{j3k\theta_1}
\]  

(13)

Considering the frequency dependent coherence function of the turbulent wind-speed field between different points in the swept area, the PSD of the Fourier-coefficients \( \tilde{v}_{eq}(3k, t) \) can be calculated:

\[
S_{v_{eq}}(3k, f) = F_{v_{eq}}(3k, f)S_c(f)
\]  

(14)

\( S_c(f) \) is the PSD of the turbulent wind-speed in any point in the rotor plane described by the Kaimal spectrum (5).

For time domain simulations, equivalent wind-speed signals have to be generated to comply with:

\[
S_{v_{eq}}(3k, f) = \tilde{v}_{eq}(3k, f)\tilde{v}_{eq}^*(3k, f)
\]

Using the admittance function

\[
F_{v_{eq}}(3k, f) = H(3k, f)H^*(3k, f)
\]  

(15)

equivalent wind-speed signals can be generated with

\[
\tilde{v}_{eq}(3k, f) = H(3k, f)\tilde{v}(f)
\]

(16)

with \( \tilde{v}(f) \) being a wind-speed signal with power spectral density \( S_c(f) \).

In [11] the admittance functions \( H(3k, f) \) have been approximated for various \( k \) by second-order rational functions that can be realized in the time domain by ordinary differential equations.

For power dynamics applications it is sufficient just to consider components of order \( k = 0 \) and \( k = 3 \). Higher order components are very well attenuated and are not significantly noticeable in the power spectrum.

D. Tower Shadow

According to [8], [9], torque variation due to tower-shadow can be included in the turbulent wind-speed considering constant Fourier-coefficients for \( k = 3 \), \( k = 6 \), etc.

The model shown in Figure 12 includes zero- and third order components of the equivalent wind-speed. Tower shadow effect is modeled by the constant third-order Fourier-coefficient \( C_{ts} \). The wind-speed signals \( v_{eq1}(t) \), \( v_{eq2}(t) \) and \( v_{eq3}(t) \) are stochastically independent signals with the same PSD as the hub wind-speed.

\[
\omega_t \rightarrow \frac{1}{s} \rightarrow \theta_t \rightarrow \sin[\omega_t] \rightarrow v_{ts}(t)
\]

\[
v_{eq1}(t) \rightarrow \frac{1}{\sqrt{3}}H[3, f] \rightarrow v_{eq}(t)
\]

\[
v_{eq2}(t) \rightarrow \frac{1}{\sqrt{3}}H[3, f] \rightarrow v_{eq}(t)
\]

\[
v_{eq3}(t) \rightarrow \frac{1}{\sqrt{3}}H[3, f] \rightarrow v_{eq}(t)
\]

\[
C_{ts} \rightarrow \cos[\omega_t] \rightarrow v_{ts}(t)
\]

Fig. 12. Wind-turbulence model including tower-shadow effect

E. Wind-Turbulence in a Wind-Farm

According to IEC 61400-21 [17], it can be assumed that wind-turbulence at different wind turbines in a wind-farm is not correlated. However, measurements have shown that neglecting turbulence correlation leads to underestimating the standard-deviation of power fluctuations of a wind-farm [9].

A suitable model for modeling turbulence correlation in a wind-farm is the complex cross spectral method described in [8], [9].

The cross power spectrum is defined by the Fourier transformation of the correlation function

\[
R_{uv}(t) = E(v(t)v(t-\tau))
\]  

(17)
The function $E$ is the expectancy.

The cross power spectrum matrix of the hub wind-speeds in a wind-farm with $n$ wind turbines is:

$$\mathbf{S}(f) = \begin{bmatrix} S_{11}(f) & \ldots & S_{1n}(f) \\ \vdots & \ddots & \vdots \\ S_{n1}(f) & \ldots & S_{nn}(f) \end{bmatrix}$$  \hspace{1cm} (18)

The main diagonal components $S_{ii}$ are real functions and represent the PSDs of turbulent wind at all wind turbines in the wind farm. The off-diagonal elements consist of the spectral density of the correlation functions according to (17).

The main- and off-diagonal elements of the matrix in (18) can be calculated from the wind-farm layout using correlation functions according to Davenport, as described in [8], [9].

Knowing the cross power matrix of hub wind-speeds according to (18), stochastic wind-speed signals can be generated, whose spectral density function complies with:

$$\mathbf{v}(f) = \mathbf{H}(f)\mathbf{w}(f)$$  \hspace{1cm} (19)

and

$$\mathbf{S}(f) = \mathbf{H}(f)^\top \mathbf{H}(f)$$  \hspace{1cm} (20)

The vector $\mathbf{w}$ is a vector of uncorrelated white noise signals.

For generating time-domain signals $v_i(t)$, there are two possible methods:

- Calculating the cross power spectrum matrix at a number of discrete frequencies and deriving a wind-speed vector $\mathbf{v}(f)$ at discrete frequencies using (19). The white noise is here generated by a vector of constant amplitude and random phase. For obtaining wind speed signals in the time domain, the spectrum of the hub-wind-speed of every turbine $\mathbf{V}(f)$ is transformed into the time domain using inverse FFT.
- Approximating all components of $\mathbf{H}(f)$ by rational functions (of e.g. 2nd order) that can be realized by ordinary differential equations. White noise is here generated in the time domain by a random-signal generator. The white noise passes the transfer function-matrix during the time-domain simulation. At the output of the transfer function-matrix, wind speed signals whose spectrum complies (approximately) with (19) is available. Both methods generate turbulence signals with similar accuracy. The advantage of the first method is that no parameter fitting has to be executed for approximating the functions $\mathbf{H}(f)$ by rational functions. But the resulting signal repeats itself after $T_f = 1/\Delta f$ (with $\Delta f$ being the frequency step) and is therefore not really a stochastic signal. However, when choosing a sufficiently small frequency step for obtaining large periods the quality of the generated turbulence signal is very good.

The complex cross spectral method was implemented in the commercially available software package DigsILENT PowerFactory[12] using DPL (DigsILENT Programming Language).

DPL is an object oriented automation interface for PowerFactory allowing to access every parameter of every PowerFactory model and to execute every PowerFactory command. The DPL-syntax is very similar to C/C++, supports any type of common control structure (e.g. do ...while(), if(...) then(...) else(...) etc.), all common standard functions, arithmetic and logical operations.

The script allows defining a wind-farm by x-y coordinates. Wind speed is characterized by mean wind-speed, turbulence intensity and hub-height. When executing the script, time-sequences of hub wind-speed signals at every wind-turbine are generated, which can directly be used for dynamic simulations.

### IV. Model Aggregation

A large wind farm sometimes consists of hundreds of generators connected by a series of feeders. Only for studies of the wind-farm itself, it is required to analyze the whole wind-farm in detail, representing each individual wind-generator.

For system impact studies however, when the impact of an entire wind-farm to a power transmission system and the interaction of the wind-farm with other power plants is studied, a detailed model of every individual wind turbine would require too much calculation time.

In these studies, the wind-farm should be modeled by one equivalent model representing the entire wind-farm seen from the connection point with highest possible accuracy.

The following sections discuss aggregated models for wind-farms with fixed-speed or variable speed wind generators.

#### A. Fixed-Speed Wind Generator Aggregation

Because the speed deviation between fixed speed wind generators in a wind park are only minor, a wind-farm built by fixed-speed wind generators can be approximated by one equivalent induction generator.

The mechanical torque of all turbines is summed up and drives the equivalent inertia:

$$\sum_i J_i \dot{\omega}_i = \sum_i M_{ti} - \sum_i M_{ei}$$  \hspace{1cm} (21)

If wind-speed differences between different turbines in the park can be assumed to be small, one equivalent aerodynamic model can be used that is driven by an equivalent wind-speed:

$$P_i = n c_p(\beta, \lambda_v) \frac{\rho}{2} A_i^2 v_{eq}^3$$  \hspace{1cm} (22)

with

$$\lambda_v = \frac{\rho \omega_{ti}}{v_{eq}}$$

and

$$v_{eq} = \frac{1}{n} \sum_i v_{\omega i}$$

$n$ is the number of wind generators in the wind-farm. Because the speed is assumed to be the same at any turbine, one equivalent pitch-angle controller is sufficient for representing pitch-control in the aggregated model.

When simulating wind fluctuations, it is recommended not to aggregate aerodynamics. In a model that aggregates the electrical generator but not the mechanical models, equation (21) is still valid but not the equivalent wind-speed approach according to (22). Instead, the mechanical torques $M_i$ are obtained by the power of each wind-turbine:

$$P_i = c_p(\beta, \lambda) \frac{\rho}{2} A_i^2 v_{i}^3$$  \hspace{1cm} (23)

Because generator speeds of all turbines are assumed to be the same in this model too, pitch-angle controllers can be aggregated.

Since shaft oscillations cause considerable power fluctuations in fixed-speed wind-generators, it is recommended use a two-mass-shaft model in the aggregated model as well and not to aggregate turbine inertias but only the generator inertias. Hence, the $n$ turbine inertias oscillate against the aggregated generator inertia.

For an accurate representation of the fault current contribution of a wind-farm the impedance of the grid feeding the generators in a wind farm must be considered as well.

Hence, the short-circuit impedance of the aggregated model must be equal to the actual short-circuit impedance of the wind farm.

When representing the wind-farm by an equivalent induction generator having $n$-times the size of each individual generator but the same p.u.-impedance, the impedance modeling the wind-farm grid needs to be set equal to:

$$Z_{gr \, id} = Z_{gr \, n}^\prime - \frac{1}{n} Z_{gr \, n}^\prime$$  \hspace{1cm} (24)

$Z_{gr \, n}^\prime$ is the short circuit impedance of the wind farm seen from the connection point and $Z_{gr \, n}^\prime$ is the subtransient impedance of each generator.
B. Variable-Speed Wind Generator Aggregation

Full aggregation (generators and mechanical model) of variable-speed wind-generators can only be justified if wind-speeds and mechanical speeds are assumed to be almost equal. This can be a useful assumption when just a rough estimate of the wind-park’s behaviour is required, e.g. in initial planning studies. Also, when carrying out short-term simulations, e.g. in transient stability studies, when the mechanical behaviour has generally no big impact on voltages and power flows at the connection point, a fully aggregated model representing an entire wind farm by one equivalent generator having n-times the size of each individual generator might be a useful approach.

However, when simulating longer term dynamics the fully aggregated, equivalent wind-speed model cannot predict the wind farm’s behaviour with sufficient accuracy, due to the highly nonlinear $c_p(\lambda, \beta)$ and MPT-characteristics.

A very good compromise between model accuracy and calculation speed consists of aggregating just the electrical system, including electrical controls and the electrical part of the generators and to model the mechanical system of each individual turbine and generator. This model maintains the nonlinear aerodynamics- and MPT-characteristics, but it reduces calculation speed considerable compared to a fully detailed, non-aggregated wind farm model. Hence, the proposed aggregated model for variable speed wind generators uses one equivalent model for:

- power electronic converters and controls
- electrical part of the generators

Not part of the aggregation are:

- generator inertia
- aerodynamics
- pitch-controllers

Because only the total electrical wind-farm torque $M_e$ comes out of the equivalent generator, assumptions about how to share the electrical torque amongst the individual generators have to be made:

$$M_{ei} = k_i M_e$$

A reasonable assumption for the participation factor $k_i$ is that the electrical torque is shared according to the power-reference of each individual wind-generator:

$$k_i = \frac{P_{ref,i}}{P_{ref,f}}$$

This results in the following mechanical equation for each generator in the wind-farm:

$$J_i \dot{\omega}_i = M_{ti} - k_i M_e$$

The variables have the following meaning:

- $J_i$: Inertia of generator $i$
- $M_{ti}$: Turbine torque of turbine $i$
- $k_i$: Torque-participation of generator $i$
- $\omega_i$: Mechanical speed of generator $i$.
- $M_e$: Electrical torque of the equivalent generator.

The accuracy of the proposed aggregated and semi-aggregated models are further analyzed in the next section with the help of some examples.

V. CASE STUDIES

All models were implemented and tested using the commercially available power system analysis package DlgSILENT PowerFactory [12].

In order to present results of the wind model and to validate the proposed model aggregation of wind parks for fixed speed and variable speed wind generators for dynamic power system analysis during wind fluctuations and electrical faults, a set of relevant simulation cases was developed.

Fig. 13. Detailed wind farm electrical model

A. Fixed Speed Wind Generation Aggregation

A wind farm consisting of 12 fixed speed stall-controlled wind generators with relatively weak connection to the external system was represented in the simulations using the detailed electrical model presented in figure 13 and an aggregated model according to the proposed method of section IV. The electrical aggregated model is connected to the Main MV bus.

1) Wind Fluctuations: According to the wind parameters and the physical location of the turbines, the time domain behavior of the wind turbulences was estimated at each turbine taking into account the expected turbulence coherence along the farm. The same set of signals $v(t), v_3s(t)$ and $v_3c(t)$ were used with the detailed and the aggregated model to ease the comparison.

Figure 14 presents the electrical magnitudes obtained with both models at the point of connection of the plant with the system. The aggregated model, although containing only one asynchronous machine model, reproduces the power fluctuation due to wind turbulence across the entire park with good accuracy.

The mechanical power of a turbine is presented in figure 15. Due to the aerodynamic characteristics of the rotor blades of stall-control turbines, for large wind speeds (above 14 m/s) the power does not increase. As a consequence, although the last wind turbulences have a symmetrical distribution, the wind power presents an skew distribution truncated at the maximum power the turbine can generate. Besides torsional interaction between the turbine and the generator rotor, the turbine speed is fairly constant.

2) Fault Analysis: The simulation of faults in the electrical system was also explored. For validating the models under most severe
conditions, full voltage-dip-ride-through capability for faults at the connection point was assumed, hence no undervoltage tripping occurs in the simulations.

Figure 16 presents the electrical magnitudes at the point of connection for a three phase close fault in one of the adjacent lines of the Power system cleared in 100 milliseconds. It is assumed that the generators stay connected to the grid. The results match well. It is also important to notice that the representation of individual turbines in the aggregated model as proposed in section IV, does not have a significant impact in this type of simulations. That is, if only the analysis of the system under faults is of interest, the representation of one equivalent turbine for the complete plant is adequate.

B. Variable Speed Wind Generation Aggregation

A wind farm consisting of 12 larger generators was connected to the same power system with a similar arrangement as in Figure 13. The wind generators are variable speed concepts with pitch control. Similarly as with fixed speed generators, simulations were performed with the detailed model of the farm and the aggregated model obtained according to the proposed method in section IV.

1) Wind Turbulences: Using the different physical location of the turbines and similar wind parameters, a new time domain behavior of the wind turbulences at each turbine was estimated. The same wind speed time-series were used with the detailed and the aggregated model to ease the comparison. Figure 17 presents the electrical magnitudes obtained with both models at the point of connection of the plant with the system. Very good accuracy of the aggregated model is observed. The behavior of one turbine is presented in figure 18. Despite the control capabilities of the variable generator, significant power variations can be observed as a consequence of wind variations and the Maximum Power Tracking control of the generator. The action of the pitch control limiting slow speed excursions to 20% above generator rated speed can be observed in the same figure.

2) Fault Analysis: Figure 19 presents the electrical magnitudes at the point of connection for a three phase close fault in one of the adjacent lines of the Power system cleared in 200 milliseconds, assuming that the generators stay connected to the grid (voltage-dip-ride-through capability). The results also match well, indicating that the electrical aggregation of the wind farm is adequate. The representation of individual turbines in the aggregated model, as proposed in section IV, results in different operation points for the turbines if wind turbulences are considered simultaneously or if the initialization intentionally sets the power, and hence speed, of turbines differently. This operating condition is quite realistic.

If the tripping of the units due to overspeed is not a concern, modeling one equivalent turbine for the complete park should suffice for short-term fault simulations. In all cases, in which mechanical speed is of interest, a fully aggregated model leads to highly optimistic results. Therefore the proposed, semi-aggregated model should generally be used for assessing speed variations.

VI. CONCLUSIONS

Fixed speed and variable speed wind generator models for power system dynamics and transient stability analysis were presented. The
models are valid for the simulation of system faults and power fluctuations resulting from wind-turbulence.

For estimating wind-turbulence in wind farms a stochastic turbulence model was presented considering rotational sampling, tower shadow and coherence between different wind-turbines in a wind farm. For dynamic power system analysis of large wind farms, aggregated wind farm models were derived that present the electrical wind-farm consisting of shaft, turbine and pitch controller, aggregation is only valid for short-term stability simulations, and if mechanical speed variations are not a concern.

As a conclusion, models combining an aggregated electrical system with a non-aggregated mechanical system are the most appropriate approach to wind farm models for dynamic simulations of interactions between power systems and wind farms. These models provide high accuracy in many applications like transient and dynamic stability simulation, and they are very efficient with regard to calculation time.

REFERENCES
Fig. 18. Pitch-control turbine behaviour during wind fluctuations


**BIOGRAPHIES**

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